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Discrete event models for flexible manufacturing cells

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1. Introduction

A manufacturing system includes a set of machines performing different operations, linked by a material handling system. A major consideration in designing a manufacturing system is its availability. When a machine or any other hardware component of the system fails, the system reconfiguration is often less than perfect. It is shown that, if these imperfections constitute even a very small percent of all possible system faults, the availability of the system may be considerably reduced. The system availability is computed as the sum of probabilities of the system operational states. A state is operational when its performance is better than a threshold value. In order to calculate the availability of a manufacturing system, its states (each corresponding to an acceptable system level) are determined. A system level is acceptable when its production capacity is satisfied. To analyze the system with failure/repair process, Markov models are often used. As a manufacturing system includes a large number of components with failure/repair processes, the system-level Markov model becomes computationally intractable. In this paper, a decomposition approach for the analysis of manufacturing systems is decomposed in manufacturing cells. A Markov chain is constructed and solved for each cell *i* to determine the probability of at least N_i operational machines at time *t*. N_i satisfies the production capacity requirement of machine cell *i*.
The probability is determined so that the material handling carriers provide the service required between N_i operational machines in machine cell *i,* and N_{i+1} operational machines in machine cell *i*+1.
The number *i*=1,…,n at time t, where n is the number of machine cells in the decomposed system.
Production lines are sets of machines arranged so as to produce a finished product or a component of a product. Machines are typically unreliable and experience random breakdowns, which lead to unscheduled downtime and production losses. Breakdown of a machine affects all other machines in the system, causing blockage of those upstream and starvation of those downstream. To minimize such perturbations, finite buffers separate the machines. The empty space of buffers protects against blockage and the full space against starvation. Thus, production lines may be modeled as sets of machines and buffers connected according to a certain topology. From a system theoretic perspective, production

lines are discrete event systems. Two basic models of machine reliability are mentioned in the literature: Bernoulli (Jacobs & Meerkov, 1995) and Markov (Gershwin, 1994), (Lim et al., 1990). The first model assumes that the process of Bernoulli trials determines the status of a machine in each cycle (i.e. the time necessary to process a part). In Markov model the state of a machine in a cycle is determined by a conditional probability, with the condition being the state of the machine in the previous cycle. Both model Bernoulli and Markov reflect practical situations: Bernoulli reliability model is more appropriate when downtime is small and comparable with the cycle time. This is often the case in assembly operations where the downtime is due to quality problems. Markov models reflect operations where the downtime is due to mechanical failures, which could be much longer than the cycle time. In this paper we address the Markov model. Intuitively, bottleneck (BN) of a production line is understood as a machine that impedes the system performance in the strongest manner. Some authors define the BN as the machine with the smallest isolation production rate (i.e. the production rate of the machine when no starvation and blockages are present). Others call the BN the machine with the largest inventory accumulated in front of it. Any may identify the machine that affects the bottom line, i.e. the system production rate, because the above definitions are local in nature and do not take into account the total system properties, such as the order of the machines in the production line, capacity of the buffers, etc. Identification of BNs and their optimal capacity for avoiding the machine downtime is considered as one of the most important problems in manufacturing systems. An illustrative example will emphasize our approach.

2. The System Model of Production Lines

The following model of a production line is considered:
1) The system consists of N machines arranged serially and N+1 buffers separating each consecutive pair of machines.
2) Each buffer B_i is characterized by its capacity $C_i < \infty$, $2 \le i \le N$, the first and the last buffer are considered to be of an infinite capacity.
3) Each machine has two states: up and down. When up, the machine produces with the rate of 1 part per unit of time (cycle); when the machine is down, no production takes place.
4) The uptime and the downtime of each machine M_i are random variables distributed exponentially with parameters λ_i and μ_i respectively.
5) Machine M_i is starved at time t if buffer B_{i-1} is empty at time t, machine M_1 is never starved.
6) Machine M_i is blocked at time t if buffer B_{i-1} is full at time t, machine M_N is never blocked.
The isolation production rate of each machine (i.e. the average number of parts produced per unit time if no starvation or blockage takes place) is:

$$\eta_i = \frac{Tup_i}{Tup_i + Tdown_i} = \frac{1}{1 + \frac{\mu_i}{\lambda_i}} \tag{1}$$

Machine M_i is the uptime bottleneck if:

$$\frac{\partial \eta}{\partial Tup_i} > \frac{\partial \eta}{\partial Tup_j}, j \neq i \tag{2}$$

and is the downtime bottleneck if:

$$\left|\frac{\partial \eta}{\partial Tdown_i}\right| > \left|\frac{\partial \eta}{\partial Tdown_j}\right|, j \neq \quad (3)$$

Machine M_i is the bottleneck (BN) if it is both uptime bottleneck and downtime bottleneck.
Let M_i be the bottleneck machine. Then it is referred to as the uptime preventive maintenance bottleneck if:

$$\frac{\partial \eta}{\partial Tup_i} > \left|\frac{\partial \eta}{\partial Tdown_i}\right| \quad (4)$$

If the inequality is reversed, the bottleneck is referred to as the downtime preventive maintenance bottleneck.

Notice: a) The absolute values of $\frac{\partial \eta}{\partial Tdown_i}$ are used because otherwise this number is negative: increase in *Tdown* leads to a decrease of η.
b) In some instances, the downtime of a machine is due to lapses in the performance of manual operators, rather than machine breakdown, thus the identification of downtime bottlenecks provides guidance for the development of production automation.
c) Preventive maintenance, as part of the total production maintenance, leads to both an increased uptime and a decrease of automated machine downtime. Some of the preventive maintenance measures affect the uptime and others the downtime. We refer to them as uptime preventive maintenance and downtime preventive maintenance. Thus, the classification of the bottleneck in either uptime bottleneck or downtime bottleneck has an impact on planning actions that lead to the most efficient system improvement.

2.1 Bottleneck indicators

We are seeking bottlenecks identification tools that are based either on the data available on the factory floor by means of real time measurements (such as average up - and down - time, starvation and blockage time, etc.), or on the data that can be constructively using the machines and buffers parameters (λ_i, μ_i, N_i). We refer to these tools as bottleneck indicators.

2.1.1 A single machine case

A single machine defined by the assumptions made in the second paragraph is uptime bottleneck if *Tup* < *Tdown* and it is downtime bottleneck if *Tdown* < *Tup*.
We can easily show that this assumption is true from (1) since:

$$\left|\frac{\partial \eta}{\partial Tdown}\right| = \frac{Tup}{(Tup + Tdown)^2} \quad (5)$$

and

$$\frac{\partial \eta}{\partial Tup} = \frac{Tdown}{(Tup + Tdown)^2} \tag{6}$$

We may say that the smallest average uptime or down-time of a machine defines its nature as bottleneck. The primary focus of the preventive maintenance and automation should be placed on the downtime further decrease, if *Tdown* < *Tup*. If *Tup* < *Tdown*, the attention should be concentrated on the increase of the uptime. In most practical situations *Tdown* < *Tup,* therefore the above indicator states that the reduction of the downtime is more efficient than a comparable increase of the uptime (Proth. & Xie, 1994).

2.1.2 Two machine cases

It is well known that, given a constant ratio between Tup_i and $Tdown_i$, the machine with the longer up - and down - time is more detrimental to the system's production rate than with a shorter up - and down - time.

In view of this property, one might think that the bottleneck is the machine with the longer up - and down - time. This is not true. The reason is that an improvement of the machine with a shorter up - and down - time leads to a better utilization of the disturbance attenuation capabilities of the buffer than a comparable improvement of the machine with a longer up - and down - time. Therefore, an improvement of the "better" machine is the best for the system as a whole (Chiang et al., 2000).

In a production line with two machines of equal efficiency (i.e., $\frac{Tup_1}{Tdown_1} = \frac{Tup_2}{Tdown_2}$), the machine with the smaller downtime is the bottleneck (Narahari & Viswandham, 1994). If the downtime of this machine is smaller than its uptime, preventive maintenance and automation should be directed toward the downtime decrease. If the downtime is sufficiently longer than the uptime, preventive maintenance and automation should be directed toward the increase of the uptime.

In the most practical situations, the isolation production rate of the machines (i.e., the faction *Tup/(Tup+Tdown*) is greater than 0,5. Therefore, the most usual bottleneck is the downtime bottleneck. To identify the downtime bottleneck in the case of machine with unequal efficiency (i.e. $\frac{Tup_1}{Tdown_1} \neq \frac{Tup_2}{Tdown_2}$) in (Laftit et al., 1992) the following bottleneck indicator is given:

If $mb_1Tup_1Tdown_1 < ms_2Tup_2Tdown_2$, machine M_1 is the downtime bottleneck.

If $mb_1Tup_1Tdown_1 > ms_2Tup_2Tdown_2$, machine M_2 is the downtime bottleneck.

The probability of manufacturing blockage mb_i is defined as:

mb_i = *Prob* ({M_i *is up at time t*} ∩ {B_i *is full at time t*} ∩ {M_{i+1} *fails to take parts from* B_i *at time t*}).

The probability of manufacturing starvation ms_i is defined as:

ms_i = *Prob* ({M_{i-1} *fails to put parts into* B_{i-1} *at time t*} ∩ {B_{i-1} *is empty at time t*} ∩ {M_i *is up at time t*}).

2.2 Extreme status for buffers

In the sequel we will try to determine the bottleneck behavior of the machines as a function of their efficiency correlated with buffer size. We will also try to anticipate the events like buffers full or empty, which determine the bottlenecks. We consider a segment consisting of two machines M_i and M_{i+1} with intermediate storage B_i at any time between successive events. Let TA be the apparent time of an event occurrence at B_i. This event may occur or not if, in the mean time, another cancelling event takes place.

Let P_i be the number of parts which are scheduled in process by M_i until the occurrence of the event. We examine two different situations, which result in a buffer event.

We define the following:

pr_i The nominal production rate of machine M_i, $i = 1,...,N$

$BL(j,t)$ Level of buffer B_j, $j = 2,..., N-1$

$T_{1j}(t)$ Delay time until the next arrival to B_j

$T_{2j}(t)$ Delay time until the next departure from B_j

BC_j The capacity of buffer B_j, $j = 2,..., N$

2.2.1 Buffer-full event

Although the buffer B_i has enough space to accept the parts produced by M_i during the transient time T_{2i}, since M_i produces at a faster rate than M_{i+1} (or the delay time T_{2i} is too long), buffer B_i will be full (see Fig. 1).

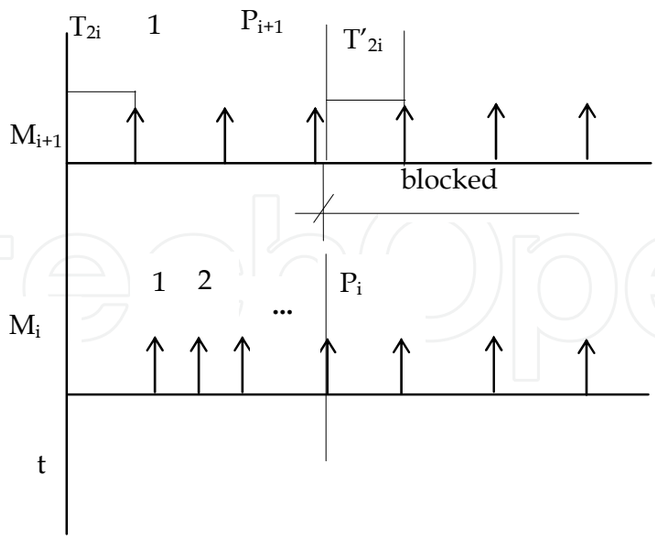


Fig. 1. Buffer - full event

In Fig. 1. the continuous line represents a machine operation on a work-part and the arrows represent arrivals to the succeeding buffer. Blank intervals indicate idle periods due to blockage or starvation of machines. The function depicted in Fig.1. is encountered when:

$$(pr_i > pr_{i+1}) \cap [pr_i (T_{2i} - T_{1i}) > BC_i - BL(i)] \quad (7)$$

The buffer - full event will occur when the P_i-th part leaves from M_i. The number of parts produced by M_i after $t + T_{1i}$ is $P_i - 1$. From Fig. 1. the sequel relations hold:

$$P_i - P_{i+1} = BC_i - BL(i) \quad (8)$$

$$P_i = 1 + (TA - t - T_{1i}).\, pr_i \quad (9)$$

$$P_{i+1} = (TA - t - T_{2i} + T'_{2i}).\, pr_{i+1},\ i = 1,...,N \quad (10)$$

Time interval between departure and the processing end of the first blocked part of M_i, lies in an inter-departure interval of M_{i+1}:

$$\frac{1}{pr_i} \le T'_{2i} \le \frac{1}{pr_{i+1}} \quad (11)$$

From (8) - (11), we obtain:

$$(T_{1i} - T_{2i}) + \frac{BC_i - BL(i)}{pr_{i+1}} < P_i \left(\frac{1}{pr_{i+1}} - \frac{1}{pr_i}\right) \le (T_{1i} - T_{2i}) + \frac{BC_i - BL(i)}{pr_{i+1}} + \frac{1}{pr_{i+1}} - \frac{1}{pr_i} \quad (12)$$

which yields:

$$P_i = 1 + int\{[BC_i - BL(i) + pr_{i+1}(T_{1i} - T_{2i})].\, \frac{pr_i}{pr_i - pr_{i+1}}\} \quad (13)$$

2.2.2 Buffer-empty event

This event is dual to the blockage and analogous results will be derived. The buffer-empty event is encountered when buffer B_i is exhausted and its succeeding machine M_{i+1} has just transmitted a work-part downstream.

Although the buffer B_i has enough parts for the transient period T_{1i}, because machine M_{i+1} produces faster than M_i (see Fig. 2.), or the delay time T_{1i} is too long, finally B_i becomes empty.

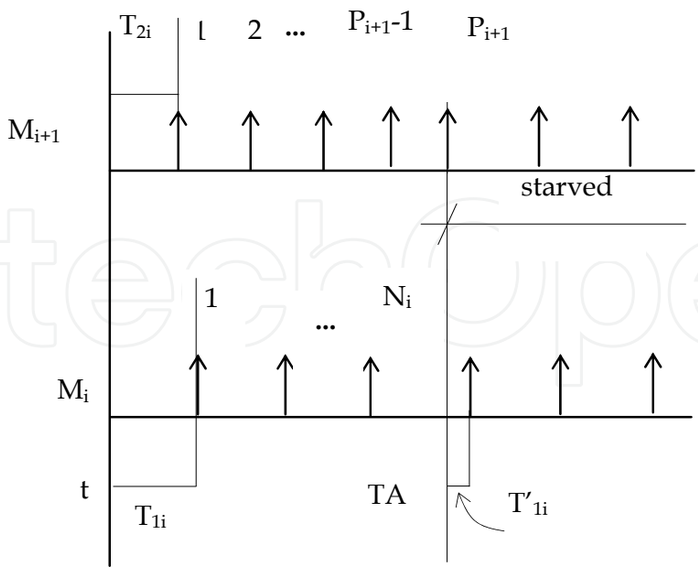


Fig. 2. Buffer - empty event

The situation depicted in Fig. 2. is encountered when:

$$(pr_i < pr_{i+1}) \cap [pr_{i+1}(T_{i1} - T_{2i}) > BL(i)] \tag{14}$$

The inter-departure interval of M_{i+1} just before the occurrence of the empty buffer event satisfies the following:

$$0 \le T'_{i1} \le \frac{1}{pr_i} - \frac{1}{pr_{i+1}} \tag{15}$$

$$P_{i+1} - N_i = BL(i) + 1 \tag{16}$$

$$P_{i+1} = 1 + (TA - t - T_{2i}).\, pr_{i+1} \tag{17}$$

$$P_i = (TA - t - T_{1i} + T'_{1i}).\, pr_i \tag{18}$$

Analogous results with those of the previous section are obtained:

$$P_{i+1}=1+int\,\{[BL(i) + pr_i\,(T_{2i} - T_{1i})].\ \frac{pr_{i+1}}{pr_{i+1} - pr_i}\} \tag{19}$$

3. The System Model of Flexible Manufacturing Cells

In this paper, a flexible manufacturing system (FMS) is treated as a discrete event system and we consider that the system evolution constitutes a discrete state-space stochastic process. In particular, we focus on Markov chain models. Such a model could be generated directly or using higher level models such as stochastic Petri nets or discrete event simulation.

Markov models with absorbing states have a trivial steady-state, namely the chain ends up in some absorbing state, remaining there forever; therefore, transient analysis alone emphasizes the system performance.

We assume that a manufacturing system evolves in time as a homogenous continuous time Markov chain $\{x(t);\ t \leq 0\}$ with state space $S = \{0, 1,...\}$ and infinitesimal generator W. Let $i, j \in S$ and:

$$p_{ij}(t) = P\{x(t) = j;\ x(0) = i\} \tag{20}$$

$$A(t) = [p_{ij}(t)] \tag{21}$$

The forward and backward differential equations that govern the behavior of this Markov chain are respectively given by (Gershwin, 1994):

$$\frac{d}{dt}[A(t)] = A(t).\ W \tag{22}$$

$$\frac{d}{dt}[A(t)]^* = W.\ A(t) \tag{23}$$

with initial conditions $A(0) = I$ in both cases. Note that these are first order, linear, ordinary differential equations. In terms of the individual matrix elements, the above equations become:

$$\frac{d}{dt}[p_{ij}(t)] = w_{ij}.\ p_{ij}(t) + \sum_{k \neq j} w_{kj} \cdot p_{ik}(t) \tag{24}$$

$$\frac{d}{dt}[p_{ij}(t)] = w_{ii}.\ p_{ij}(t) + \sum_{k \neq i} w_{ik} \cdot p_{kj}(t) \tag{25}$$

The forward and backward equations have the same unique solution given by

$$A(t) = e^{Wt} \tag{26}$$

where, e^{Wt} is the exponential matrix defined by the Taylor series.

$$e^{Wt} = \sum_{k=0}^{\infty} \frac{(W \cdot t)^k}{k!} \tag{27}$$

To find out the state probabilities $Y(t) = [p_0(t), p_1(t),...]$ where $p_j(t) = P\{x(t) = j\}$, $j \in S$, we need to solve the differential equation:

$$\frac{d}{dt}[\,Y(t)] = Y(t).\,W \tag{28}$$

The solution is given by:

$$Y(t) = Y(0).\,e^{Wt} \tag{29}$$

3.1 Discrete-event model of a flexible manufacturing cell line

A flexible manufacturing production line is a series arrangement of machines and buffers, as shown in Fig. 3.

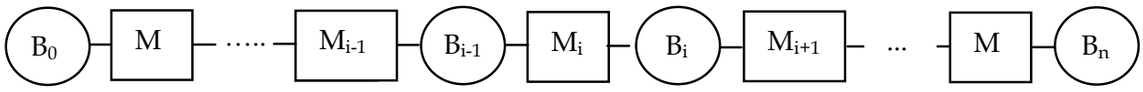


Fig. 3. The flexible manufacturing cell line

The parts enter the first machine and they are processed and transported to the succeeding components, until they finally leave the system. The machines produce at different rates, fail, and are repaired randomly, thus causing changes in the flow of parts. The changes propagate to neighboring machines and may render them starved or blocked. Buffers of finite capacity are inserted in order to reduce these effects. The operation of the production line is ruled by the following:

a) The line consists of $N+1$ buffers. There is one buffer B_0 at the beginning of the line, with finite capacity and another B_n at the end, with unlimited capacity.

b) The uptimes and the downtimes of machines are assumed for convenience to be exponential random variables, although any type of distribution may be considered.

c) In each machine there is space for a single work-part. A machine M_i is starved if it has no part to work on and the inventory of the upstream buffer B_{i-1} has been exhausted. Moreover, M_i is blocked if it is prevented from releasing a finished part downstream because B_i is full.

d) Starved or blocked machines remain forced down until a work-part or a unit space is available. During these idle periods, machines do not deteriorate.

e) Transportation time of work-parts to and from buffers is negligible or is incorporated in the processing time.

As we have seen, absorbing states occur in manufacturing system models that capture phenomena such as deadlocks. Interesting for such systems is the time until an absorbing state is reached. Let $\{X(u);\ u \geq 0\}$ be the Markov chain under consideration. Let the state space be finite and given by $S = \{0, 1,..., m, m+1,..., m+n\}$, where $m \geq 0$, $n > 0$, the first $(m+1)$ states are transient states, and the rest of the states are absorbing states. Let 0 be the initial state and T, the time to reach any absorbing state. Define:

$$p_{ij}(t) = P\{X(t) = j / X(0) = i\} \tag{30}$$

Then, we have, for any $t > 0$

$$P\{T > t\} = P\{X(t) \notin \{m+1,..., m+n\}\} \tag{31}$$

So we have:

$$P\{T > t\} = 1 - \sum_{j=1}^{n} P_{0,m+j}(t) \tag{32}$$

The cumulative distribution function of T is given by

$$F_T(t) = \sum_{j=1}^{n} p_{0,m+j}(t) \tag{33}$$

The individual probabilities $p_{0,m+j}(t)$ have to be computed by solving the differential equations shown in relation (22) or (23).

3.2 The basic cell of the flexible manufacturing system

The basic cell of the proposed model for flexible manufacturing system analysis consists of a machine, for example M_i, its upstream buffer B_{i-1} and its downstream buffer B_i. In Fig. 4 we have the Markov chain representation of the basic cell of our model for flexible manufacturing system analysis and in Fig. 5 we depicted our Makov chain model for flexible manufacturing cell system. The interpretation of basic cell is that the machine M_i is in state 0 when there is no part being processed, but only transfers of the parts from the machine to its buffers. In state 1, when there is a part being processed and a part in state 2, there is a deadlock in the system. The arrival rate of parts is λ_i and the service rate of each part is μ_i.

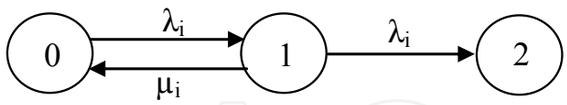


Fig. 4. Markov chain of the model basic cell for flexible manufacturing system analysis

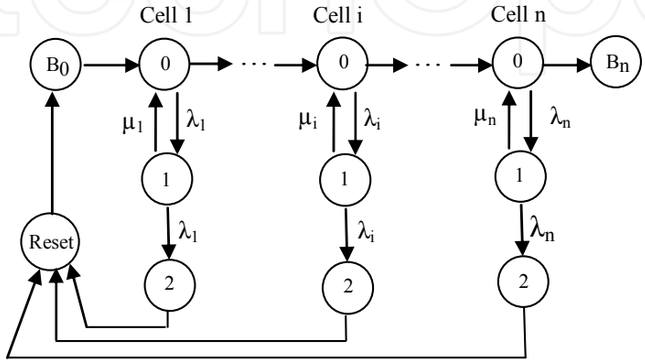


Fig. 5. Markov chain model for flexible manufacturing cells system analysis

Here, the time to absorption is the time elapsed before a deadlock is reached. We know in this case that $F_T(t) = p_{02}(t)$. To compute $p_{02}(t)$, we first write down the infinitesimal generator W of this Markov chain:

$$W = \begin{bmatrix} -\lambda_i & \lambda_i & 0 \\ \mu_i & -(\lambda_i + \mu_i) & \lambda_i \\ 0 & 0 & 0 \end{bmatrix} \tag{34}$$

First, consider the backward equation (6) for $P_{02}(t)$:

$$\frac{d}{dt}(p_{02}(t)) = w_{00}.\ p_{02}(t) + w_{01}.\ p_{12}(t) + w_{02}.\ p_{22}(t) \tag{35}$$

Since $w_{02} = 0$, the above becomes:

$$\frac{d}{dt}(p_{02}(t)) = -\lambda_i p_{02}(t) + \lambda_i p_{12}(t) \tag{36}$$

The backward equation for $p_{12}(t)$ is given by:

$$\frac{d}{dt}(p_{12}(t)) = w_{10}.\ p_{02}(t) + w_{11}.\ p_{12}(t) + w_{12}.\ p_{22}(t) \tag{37}$$

Since $p_{22}(t) = 1$, the above becomes:

$$\frac{d}{dt}(p_{12}(t)) = -\mu_i p_{02}(t) - (\lambda_i + \mu_i) \cdot p_{12}(t) + \lambda_i \tag{38}$$

Let $p_{ij}(s)$ denote the Laplace transform of $p_{ij}(t)$. Taking the transform on either side of the equation above, we get:

$$sP_{02}(s) = -\lambda_i P_{02}(s) + \lambda_i P_{12}(s) \tag{39}$$

$$sP_{12}(s) = \mu_i P_{02}(s) - (\lambda_i + \mu_i) \cdot P_{12}(s) + \frac{\lambda_i}{s} \tag{40}$$

Simplifying using (20) and (21), we get:

$$P_{02}(s) = \frac{\lambda_i^2}{s\left[s^2 + s(2\lambda_i + \mu_i) + \lambda_i^2\right]} \tag{41}$$

Now, $p_{02}(t)$ can be obtained from equation (41) by inverse Laplace transformation:

$$p_{02}(t) = A + B.\ e^{-at} + C.\ e^{-bt} \tag{42}$$

where, the constants are given by:

$$a = \frac{2\lambda_i + \mu_i + \sqrt{\mu_i^2 + 4\lambda_i\mu_i}}{2}; \quad b = \frac{2\lambda_i + \mu_i - \sqrt{\mu_i^2 + 4\lambda_i\mu_i}}{2} \tag{43}$$

$$A = \frac{\lambda_i}{ab}; \; B = \frac{\lambda_i(b-2a)}{ab(b-a)}; \; C = \frac{\lambda_i}{b(b-a)} \tag{44}$$

The works (Viswanadham & Ram, 1994), (Lanzon et al., 1996), (Sethi et al., 1997), contain a similar discussion on computing the mean time to absorption.

3.3 The buffer events

In a production line, the next event of a component depends on its current state and on the state of adjacent components. An interesting quantity to study is the relation between the buffer size and the machine duration of service. This is because a failed machine, being coupled with a large buffer, may be delayed enough so that the blocking event is avoided, if in the mean time the machine is repaired. So, we may say that in a production line the parameter which determines the deadlock of a basic cell, as well as of the entire production line, is the buffer size. The phenomenon of blocked and starved states occurs frequently when a machine produces at a faster rate than its adjacent ones. In this case, machine M_i is located between an empty and a full buffer. It is then forced to wait until a part arrives from the upstream cell and upon completion of part processing it is blocked until an empty space is available in the downstream buffer. We examine two possibilities: a blocked machine empties its upstream buffer (see case A) or a starved machine fills its downstream buffer (see case B). In both situations the event is conventionally encountered when a work-part is released from M_i to the downstream buffer (see Fig. 6 and Fig. 7). A starved or blocked machine alternates between two states for some time until either a not full or a not empty event occurs. We consider the segment of machines M_{i-1}, M_i and M_{i+1}, and buffers B_{i-1} and B_i. Let t be the time when the starved and blocked event is encountered and TA the apparent time of the next event.

$$M(i,t) = \begin{cases} 1, \text{ if machine } M_i \text{ is up} \\ 0, \text{ if } M_i \text{ is under repair} \end{cases}$$

$$B(j,t) = \begin{cases} 0, \text{ if buffer } B_j \text{ is empty} \\ 2, \text{ if buffer } B_j \text{ is full} \\ 1, \text{ otherwise state} \end{cases}$$

$$BE_j(t) = \begin{cases} 1, \text{ if } B_j \text{ empties at time } t \\ 0, \text{ otherwise} \end{cases}$$

We define the following (Narahari & Viswandham, 1994), (Chiang et al., 2000):

μ_i = The nominal production rate (workparts/time-unit) of machine M_i, $i = 1,..., n$
$T_{1j}(t)$ = Delay time until the next arrival to B_j
$T_{2j}(t)$ = Delay time until the next departure from B_j
We discuss the following situations: *Case A:* Machine M_{i+1} is faster than M_{i-1}. This situation is depicted in Fig. 4 and the condition is:

$$(T_{21} > T_{1,i-1} + \frac{1}{\mu_i}) \cap (\mu_{i+1} > \mu_{i-1}) \tag{45}$$

We note that in Fig. 6 and in Fig. 7 the continuous line represents a machine operation on a work-part and the arrows represents arrivals to the succeeding buffer. Blank intervals indicate idle periods due to blockage or starvation of machines. The wavy lines denote a machine under repair. For Fig. 6, buffer B_i is scheduled to switch from full to an intermediate state. The not-full event occurs upon the departure of the last blocked part from M_i. We notice that the end-of-processing time of the $(1 + N_i)^{th}$ work-part in M_i is greater than the time when a single space for this part is available in B_i.

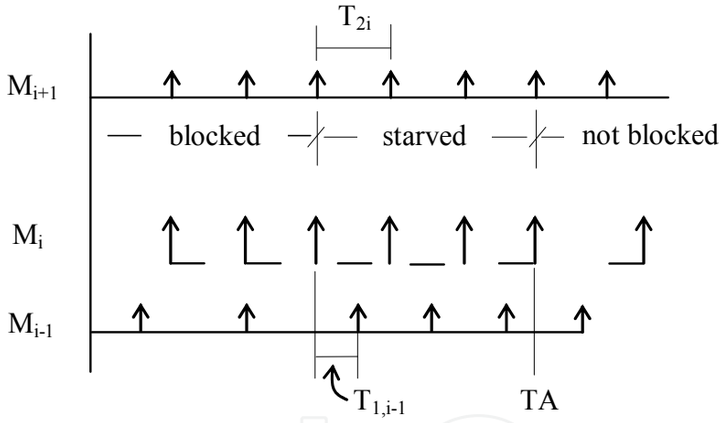


Fig. 6. Starved and blocked machine states when M_{i+1} is faster than M_{i-1}

The opposite holds for the first N_i work-parts. This observation leads to (Ciufudean et al., 2005):

$$t + T_{2i} + \frac{N_i}{\mu_{i+1}} < t + T_{1,i-1} + \frac{N_i}{\mu_{i-1}} + \frac{1}{\mu_i} \tag{46}$$

and

$$TA = t + T_{2i} + \frac{N_i - 1}{\mu_{i+1}} \geq t + T_{1,i-1} + \frac{N_i - 1}{\mu_{i-1}} + \frac{1}{\mu_i} \tag{47}$$

From relation (26) and (27) we compute the parts until next event, for B_i:

$$N_i = 1 + \text{Int}\left\{\frac{T_{2i} - T_{1,i-1} - \frac{1}{\mu_i}}{\frac{1}{\mu_{i-1}} - \frac{1}{\mu_{i+1}}}\right\} \quad (48)$$

Case B: Machine M_{i-1} produces at a faster rate than M_{i+1} (machine M_{i-1} is faster than M_{i+1}) and the starved machine M_i fills its upstream buffer B_i (see Fig. 5). The condition is:

$$(T_{2i} > T_{1,\,i-1} + \frac{1}{\mu_i}) \cap (\mu_{i-1} > \mu_{i+1}) \quad (49)$$

This is dual to case A. After that, machine M_{i-1} processes N_{i-1} parts, a non-empty event will occur.

In Fig. 5 we see that the arrival time of $(1 + N_{i-1})^{th}$ work-part at buffer B_{i-1} is less than the time machine M_i is ready to receive it. The opposite holds for the first N_{i-1} work-parts (Ciufudean, 2008).

This observation leads to:

$$t + T_{2i} + \frac{N_{i-1} - 1}{\mu_{i+1}} > t + T_{1,\,i-1} + \frac{N_{i-1}}{\mu_{i-1}} \quad (50)$$

and

$$TA = t + T_{2i} + \frac{N_{i-1} - 1}{\mu_{i+1}} \leq t + T_{1,\,i-1} + \frac{N_{i-1} - 1}{\mu_{i-1}} \quad (51)$$

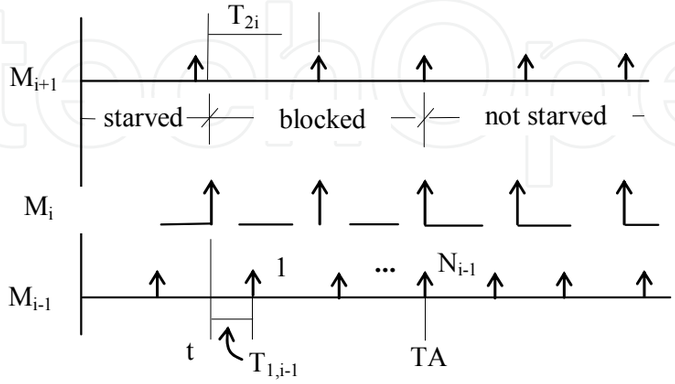


Fig. 7. Starved and blocked machine states when M_{i-1} is faster than M_{i+1}

In a dual situation to case A, we get the parts until the next event, for B_{i-1} (Ciufudean & Filote, 2010):

$$N_{i-1} = 1 + \text{lnt}\left\{\frac{T_{1,i-1} - T_{2i} - \frac{1}{\mu_{i+1}}}{\frac{1}{\mu_{i+1}} - \frac{1}{\mu_{i-1}}}\right\} \tag{52}$$

From relation (48) and (52) we notice that the number of parts until the next event depends on the service rate of each part. We may say that the buffer dimensions can be calculated in such a manner as to avoid the failed state of machines, considering that the time to calculate the number of parts until next event is set to $T_{21} = P_{02}$ in relation (48) and, respectively, to $T_{1,i-1} = P_{02}$ in relation (52); where P_{02} is given by relation (42).
As we discussed before, the failed state of machines can be avoided if the buffer size is bigger than the critical size (the size determined from relation (48) and respectively (52)). The condition to be accomplished is that the average time to repair a machine is less than the average time to fill the upstream buffer of that machine.

4. An Illustrative Example

The manufacturing system considered in this paper consists of two cells linked together by a material system composed of two buffers A and B and a conveyor. Each cell consists of a machine to handle within cell part movement. Pieces enter the system at the load/unload station, where they are released from those two buffers, A and B, and then are sorted in cells (pieces of type "a" in one cell, and pieces of type "b" in the other cell).
We notice that in the buffer A there are pieces of types "a", "b", and others, where the number of pieces "a" is greater than the number of pieces "b". In the buffer B there are pieces of types "a", "b", and others, where the number of pieces "b" is greater than the number of pieces "a". The conveyor moves pieces between the load/unload station of the various cells.
The sorted piece leaves the system, and an unsorted piece enters the system, respectively one of those two buffers A or B. The conveyor along with the central storage incorporates a sufficiently large buffer space, so that it can be thought of as possessing infinite storage capacity. Thus, if a piece routed to a particular cell finds that the cell is full, its entry is refused and it is routed back to the centralized storage area. If a piece routed by the conveyor is of a different type than the required types to be sorted, respectively "a" and "b", then that piece is rejected from the system.
We notice that once a piece is blocked from cell entry, the conveyor does not stop service; instead it proceeds with its operation to the other pieces waiting for transport.
At the system level, we assume that the cells are functionally equivalent, so that each cell can provide the necessary processing for a piece. Hence, one cell is sufficient to maintain production (at a reduced throughput). We say that the manufacturing system is available (or, operational) if the conveyor and at least one of the cells are available. A cell is available if its machine is available (Rodriguez et al., 2010).
Over a specified period of operation, due to the randomly occurring subsystem failures and subsequent repairs, the cellular automated manufacturing system will function in different

configurations and exhibit varying levels of performance over the random residence times in these configurations.
The logical model of our manufacturing system is showed in Fig.8.

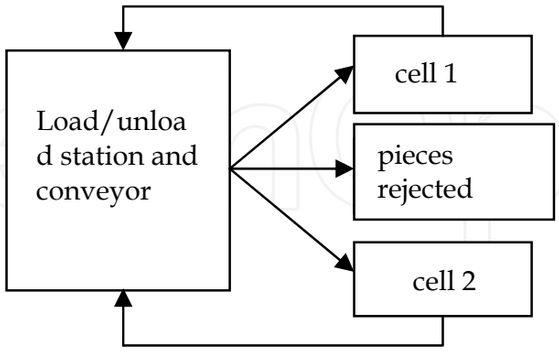


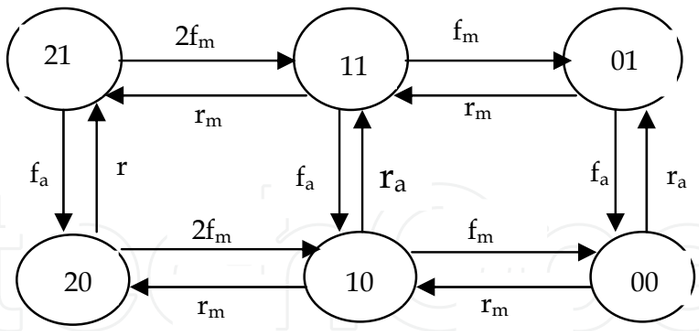
Fig. 8. Logical model for a manufacturing system

4.1 A Markov model for evaluating the system availability

For the flexible manufacturing system depicted in Fig. 8, we assume that the machines are failure-prone, while the load/unload station and the conveyor are extremely reliable. Assuming the failure times and the repair times to be exponentially distributed, we can formulate the state process as a continuous time Markov chain (CTMC). The state process is given by $\{X(u), u \geq 0\}$ with state space $S = \{(ij), i \in \{0,1,2\}, j \in \{0,1\}\}$, where *i* denotes the number of working machines, and *j* denotes the status of the material handling system (load station and conveyor): up "1", and down "0". We consider the state independent (or, time dependent) failure case and the operation dependent failure case separately.

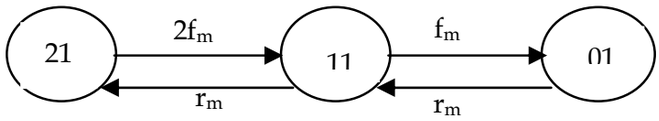
4.1.1 Time dependent failures

In this case, the component fails irrespective of whether the system is operational or not. All failure states are recoverable. Let r_a and r_m denote the repair rates of the material handling system, and a machine respectively. The state process is shown in Fig. 9, a.

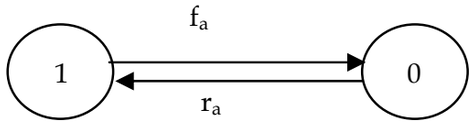


a)

for machines:



for MHS:



b)

Fig. 9. State process of a FMC with time-dependent failures, (a) State process for a state-independent failure model, (b) Decomposed failure/repair process.

Because the failure/repair behavior of the system components are independent, the state process can be decomposed into two CTMCs as shown in Fig. 9, b. Analytically, the state process is expressed by relations: S_0 = {(21), (11)} and S_F = {(20),(10), (00)}. For each state in S_F no production is possible since the M0HS or both machines are down. In Fig. 2, b the failure/repair behavior of each resource type (machines or MHS) is described by a unique Markov chain. Thus, the transient state probabilities, $p_{ij}(t)$ can be obtained from relation:

$$p_{ij}(t) = p_i(t) \cdot p_j(t) \tag{53}$$

where $p_i(t)$ is the probability that i machines are working at time t for i = 0,1,2. The probability pi(t) is obtained by solving (separately) the failure/repair model of the machines. $P_j(t)$ is the probability that j MHS (load/unload station and conveyor) are working at instant t, for j = 0,1. Let fa and fm denote the failure rates of MHS and of a machine respectively.

4.1.2 Operation dependent failures

We assume that when the system is functional, the resources are all fully utilized. Since failures occur only when the system is operational, the state space is: S = {(21), (11), (20), (10), (01)}, with S_0 = {(21), (11)}, S_F = {(20), (10), (01)}. The Markov chain model is shown in Fig. 10. Transitions representing failures will be allowed only when the resource is busy. Transitions rates can however be computed as the product of the failure rates and percentage utilization of the resource. If T_k^{ij} represents the average utilization of the k^{th} resource in the state (i j), the transition rates are given in Fig. 10.

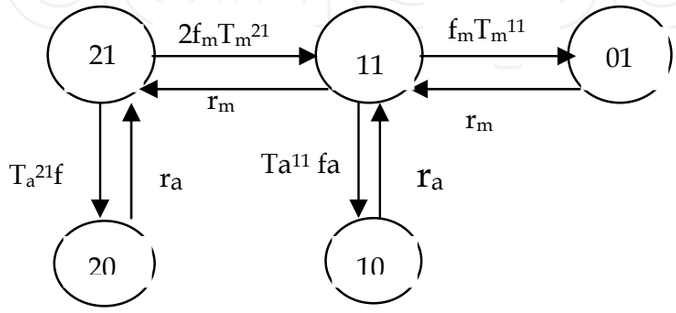


Fig. 10. State process of a FMC with state-dependent failures

4.1.3 A numerical example

For the FMC presented in this paper in Table 1, the failure/repair data of the system components are given. We notice that T_k^{ij} (the system average utilization of the k^{th} resource in state (ij), T_k^{ij} = 1 since the utilization in each operational state is 100% for all i, j, k, i = {0,1,2}, j = {0,1}, k = 4.

The other notations used in Table 1 are: f, the exponential failure rate of resources; r, the exponential repair rate of resources; N_p, the required minimum number of operational machines in cell p; p = {1,2} and n_p, the total number of machines in cell p.

	R	F	N_p	n_p	T_k^{ij}
Machines	1	0,05	1	2	1
MHS	0,2	0,001	1	1	1

Table 1. Data for the numerical study

From Fig. 9 and Fig. 10 we calculate the corresponding infinitesimal generators, and after that, the probability vector of CTMC. With relation (1) we calculate the availability of FMC given in this article.

The computational results are summarized in Table 2 for the state process given in Fig. 9 (FMC with time-dependent failures), and respectively in Table 3 for the state process given in Fig. 10 (FMC with state-dependent failures). We consider the system operation over an interval of 24 hours (three consecutive shifts).

Time [hour]	Machines	MHS	System Availability
0	1.0000	1.0000	1.0000
1	0.9800	0.9548	0.9217
4	0.9470	0.8645	0.7789
8	0.9335	0.8061	0.7025
12	0.9330	0.7810	0.6758
16	0.9331	0.7701	0.6655
20	0.9330	0.7654	0.6623
24	0.9328	0.7648	0.6617

Table 2. Computational results for the FMC in Fig. 9

Time hour	Machines	MHS	System Availability
0	1.0000	1.0000	1.0000
1	0.9580	0.9228	0.9001
4	0.9350	0.8228	0.7362
8	0.9315	0.8039	0.7008
12	0.9310	0.7798	0.6739
16	0.9320	0.7688	0.6632
20	0.9318	0.7639	0.6598
24	0.9320	0.7636	0.6583

Table 3. Computational results for the FMC in Fig. 10

The results of the availability analysis of the flexible manufacturing system are illustrated in Fig.11, which depicts the availability of the system as a time function. The numbers x = 2, 3 indicate the system in Fig. 9, respectively Fig. 10. One can see from Fig.11 that the layout with FMC with time-dependent failures is superior to that with FMC with state-dependent failure.

An analytical technique for the availability evaluation of the flexible manufacturing systems was presented. The novelty of the approach is that the construction of large Markov chains is not required. Using a structural decomposition, the manufacturing system is divided into cells.

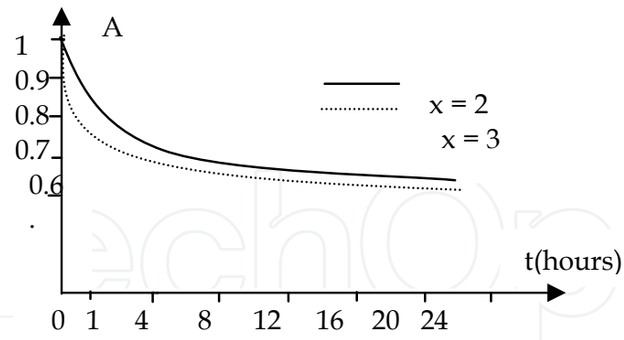


Fig. 11. Availability analysis of the flexible manufacturing system

For each cell, a Markov model was derived and the probability was determined of at least N_i working machines in cell *i,* for *i* = 1,2,..,n and *j* working material handling system at time *t*, where N_i and *j* satisfy the system production capacity requirements. The model presented in this paper can be extended to include other components, e.g., tools, control systems. The results reported here can form the basis of several enhancements, such as conducting throughput studies of cellular flexible manufacturing types with multiple part types.

5. Conclusion

A model for flexible manufacturing cellular systems analysis has been introduced in this paper. Such a model could be generated directly or using higher level models such as stochastic Petri nets or discrete event simulation. A discrete-event system formulation and state partition into basic cells and fast and slow varying section, lead to a reduced computation cost. Further research in this area should focus on systems modeled with Markov chains which exhibit a cut-off phenomenon, as the existence of a cut-off phenomenon is a good indicator to whether a transient or a steady-state analysis is appropriate in a given setting. For example, if the cut-off time is known and the duration of observation is less than the cut-off time, then transient analysis is more meaningful than steady-state analysis.

Identification and measurement of the bottleneck times in production lines has implications for both natures concerning the preventive maintenance and the production automation. In this paper we address the Markov model of production lines with bottlenecks. In lines where machines have identical efficiency, the machine with the smaller downtime is the bottleneck. In two-machine lines, the downtime bottleneck is the machine with the smallest value of $p \cdot Tup \cdot Tdown$, where p is the probability of blockage for the first machine and the probability of starvation for the second. Anticipation of events like full buffer or empty buffer, which determine bottlenecks, has also implications for the preventive maintenance of the manufacturing system. Future work in this area should focus on extensions of the results obtained in manufacturing systems with high failure rates.

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This book is a collection of articles aimed at finding new ways of manufacturing systems developments. The articles included in this volume comprise of current and new directions of manufacturing systems which I believe can lead to the development of more comprehensive and efficient future manufacturing systems. People from diverse background like academia, industry, research and others can take advantage of this volume and can shape future directions of manufacturing systems.

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