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Structure and Property of the Singularity Loci of Gough-Stewart Manipulator

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1. Introduction

During the past two decades, parallel manipulator system has become one of the research attentions in robotics. This popularity has been motivated by the fact that parallel manipulators possess some specific advantages over serial manipulators, i.e., higher rigidity and load-carrying capacity, better dynamic performance and a simpler inverse position kinematics, etc. Among various manipulators, the best-known is the Gough-Stewart Platform (GSP) that was introduced as a tire performance (Gough 1956-57) and an aircraft simulator (Stewart 1965).

One of the important problems in robot kinematics is special configuration or singularity. As to parallel manipulators, in such configurations, the end-effector keeps at least one remnant freedom while all the actuators are locked. This transitorily puts the end-effector out of control. Meanwhile, the articular forces may go to infinity and cause mechanical damages.

Determination of the special configurations of the six-DOF Gough-Stewart parallel manipulators is a very important problem. It is one of the main concerns in the analysis and design of manipulators. The singularity analysis of parallel manipulators has attracted a great deal of attention in the past two decades. Hunt (1983) first discovered a special configuration for this manipulator that occurs when the moving triangle-platform is coplanar with two legs meeting at a vertex of the triangle, and all the six segments associated with six prismatic actuators intersect a common line. Fichter (1986) discovered a singularity of the parallel manipulator. That occurs when the moving platform rotates $\psi = \pm\pi/2$ around Z-axis, whatever the position of the moving platform is. That mechanism has a triangular mobile platform and a hexagonal base platform. It may be named a 3/6-GSP. Huang and Qu (1987) and Huang, Kong and Fang (1997) also studied the singularity of the parallel manipulator, whose moving and basic platforms are both semi-regular hexagons (6/6-GSP). It also occurs when $\psi = \pm\pi/2$. Merlet (1988, 1989) studied the singularity of the six-DOF 3/6-GSP more systematically based on Grassman line geometry. He discovered many new singularities including 3c, 4b, 4d, 5a and 5b. 3c occurs when four lines of the six legs intersect at a common point; 4b occurs when five lines are concurrent with two skew lines; 4d occurs when all the five lines are in one plane or pass through one common point

in that plane; 5a is in general complex; 5b occurs when the six segments cross the same line. Based on line geometry, wrench singularity analyses for platform devices have been presented by Collins & Long (1995), and Hao & McCarthy (1998). Gosselin and Angeles (1990) pointed that singularities of closed-loop mechanisms can be classified into three different groups based on the Jacobian matrices. This classification was further discussed by Zlatanov, Fenton and Benhabib (1994, 1995). Zlatanov, Bonev and Gosselin (2002) discussed constraint singularities. Ma and Angeles (1991) studied architecture singularities of parallel manipulators. Kong (1998) also discussed architecture singularities of the general GSP. McAree and Daniel (1999) discussed the singularity and motion property of a 3/3-parallel manipulator. Karger and Husty (1998), Karger (2001) described the singular positions and self-motions of a special class of planar parallel manipulators where the platform is similar to the base one. It is shown that it has no self-motions unless it is architecturally singular. Kong (1998), Kong and Gosselin (2002) also studied self-motion. Chan and Ebert-Uphoff (2000) studied the nature of the kinematic deficiency in a singular configuration by calculating the nullspace of the Jacobian matrix. Di Gregorio (2004) studied the SX-YS-ZS Structures and Singularity.

Many researches dealt only with isolated singular points in space. However, in the practical configuration space of parallel manipulators the singularity configuration should be a continuous singularity curve or even be high-dimension surface. One of the main concerns is further to find out its singularity loci and their graphical representations, as well as the structure and property of the singularity loci. That is of great significance in a context of analysis and design since it allows one to obtain a complete picture of the location of the singular configurations in the workspace. For a given practical application, it is therefore easy to decide whether the singularities can be avoided. Sefrioui and Gosselin (1994, 1995) studied singularity loci of planar and spherical parallel mechanisms. Wang and Gosselin (1996, 1997) used the numerical method to study the singularity loci of spatial four- and five-DOF parallel manipulators. Collins and McCarthy (1997, 1998) studied singularity loci of the planar 3-RPR parallel manipulator, and 2-2-2 and 3-2-1 platforms and obtained cubic singularity surfaces. For the six-DOF Gough-Stewart Platform, however, the singularity expression generally is quite complicated, and difficult to analyze. Recently, Wang (1998) presented a method to analyze the singularity of a special form of the GSP and derived corresponding analytical singularity conditions. Di Gregorio (2001, 2002) also discussed the singularity loci of 3/6 and 6/6 fully-parallel manipulators. In particular, Mayer St-Onge and Gosselin (2000) analyzed the Jacobian matrix of general Stewart manipulators by two different new approaches. They derived a simpler explicit expression from the Jacobian matrix, and pointed out that the singularity locus of the general Gough-Stewart manipulator should be a polynomial expression of degree three. They also gave the graphical representations of the singularity loci.

For practical application, we want to obtain a simpler algebra expression of the singularity loci, their accurate graphical representations and know whether it consists of some typical geometrical figures. But this is very difficult for the Gough-Stewart manipulator. Huang et al. (1999, 2003) studied the singularity kinematics principle of parallel manipulators, and proved a new kinematics sufficient and necessary condition to determine the singularity. Using this method he discovered the characters of singularity locus of the 3/6-Gough-Stewart platform firstly. It shows that the singularity locus of the 3/6-Gough-Stewart platform is resolvable and consists of two typical geometrical graphs, a plane and a

hyperbolic paraboloid, for the special orientations: $\phi=\pm30°$, $\pm90°$, or $\pm150°$. However, the singularity locus expression of degree three is irresolvable, and the locus graph in infinite parallel principal sections includes a parabola, four pairs of intersecting straight lines and infinite hyperbolas for the general orientations: $\phi\neq\pm30°$, $\pm90°$, and $\pm150°$.

For the singularity loci of the 6/6-GSP which is a more general structure form and widely used in practice, its graphical representations of the singularity loci for different orientations are quite various and complex. Huang and Cao (2005) analyzed the singularity loci both in 3-D space and in the principal-section on which the moving platform lies. The singularity locus equation of this class of Gough–Stewart manipulators in three-dimensional space is also irresolvable, and the curves in infinite parallel principal sections of the singularity loci also contains one parabola, four pairs of intersecting straight lines, and infinite hyperbolas. We also found out an incredible phenomenon, in that special configuration six lines associated with the six extensible links of the 6/6-Gough-Stewart manipulator can intersect the same common line and the remnant instantaneous motion of the manipulator is a pure rotation.

All the above-mentioned analyses are only about positional singularity when the orientation of the moving platform is specified and invariable. On the other hand, there is a need to further find out the orientation-singularity space when the position of the moving platform is specified and invariable. Some researchers began to study the issue, such as Pernkopf and Husty (2002); Cao, Huang & Ge (2006). Of course, for this topic there is still much work to be done in depth.

2. The kinematics principle and linear-complex classification

2.1 The classification of singularity by linear-complex

A general algebraic equation for a linear complex (Hunt 1978; Ball 1900) is:

$$a_1P + a_2Q + a_3R + a_4L + a_5M + a_6N = 0 \tag{1}$$

where the six coefficients denote a twist screw

$$\$^m = (a_1\ a_2\ a_3 ; a_4\ a_5\ a_6) \tag{2}$$

Its pitch is

$$h^m = \frac{a_1a_4 + a_2a_5 + a_3a_6}{a_1^2 + a_2^2 + a_3^2} \tag{3}$$

Its reciprocal screw satisfying Eq. (1) is

$$\$ = (L\ M\ N ; P\ Q\ R) \tag{4}$$

and we have

$$LP + MQ + NR = 0 \tag{5}$$

where $\$$ denotes a line vector. The infinite line vectors satisfying Eq. (1) composed a line complex.
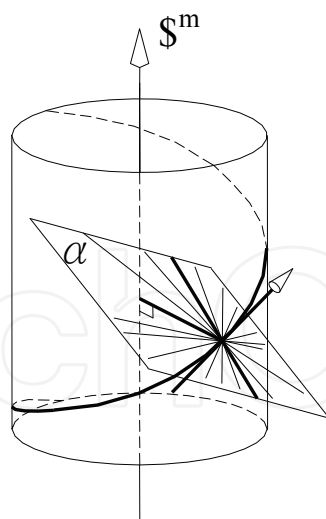
Fig. 1. Linear complex

In a linear complex (Hunt 1978), those lines that pass through any pole must all lie in the same polar plane; those lines that lie in any polar plane must all intersect the same point. Fig. 1 shows the pole and polar plane of a linear complex with the pencil of lines in α. All the lines that pass through the pole are normal to the helix. The linear complex can be divided into three parts according to its pitch h^m: when h^m is finite and nonzero, it is a general linear complex; when $h^m=0$, this is the first special linear complex, in which all the coaxial helices collapse into homocentric circles with a common axis $\$^m$ and all the lines of the complex intersect $\$^m$ or parallel to it; and when $h^m=\infty$, this is the second special linear complex, in which all the lines of the complex comprise planar fields of lines in all planes normal to the direction $\$^m$, and $\$^m$ is no longer occupying a specific line. The last two forms are associated respectively with pure rotation and pure translation.

 All singularities of the Gough-Stewart parallel mechanism belong to the linear-complex singularity. From this point of view, the singularity can be divided into three kinds with different instant output motion:

(1) The general Linear-Complex Singularity. The possible motion of end-effector is a twist with h^m is finite and nonzero;

(2) The First Special Linear-Complex Singularity. The possible motion of end-effector is a pure rotation with $h^m=0$;

(3) The Second Special Linear-Complex Singularity. The possible motion of end-effector is a pure translation with $h^m=\infty$.

2.2 The kinematic principle of singularity

First of all, let us discuss the velocity relationship of three points in a moving body. The following issue is to introduce the principle of a novel method analyzing the singularities of parallel manipulators (Huang et al. 1999; 2003; Ebert-Uphoff et al., 2000; Kong and Gosselin 2001). Let us consider any non-collinear three points in a rigid body, and then we may deduce the following theorem:

Theorem 1: Three velocities of three points in a moving body have three normal planes at the corresponding three points. In general, the three planes intersect at a common point, and the intersecting point necessarily lies in the plane determined by the three points.
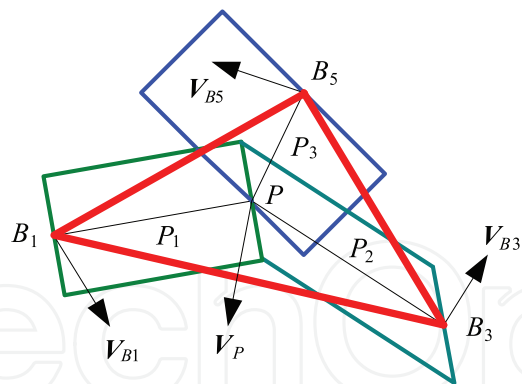
Fig. 2 The velocity relationship of three non collinear points in a moving body

Theorem 2: When three velocity directions of three points in a rigid body are given, then three normal planes of the three velocities are determined. If the intersecting point of the three planes lies in the plane determined by the three points, the three velocities can determine a twist; otherwise, the given velocities are improper and cannot determine a twist of that body.

The thinking of the velocity analysis in the proof of Theorem 2 itself is also useful for singularity study of the 3/6-GSP.

The 3/6-GSP is a typical manipulator which many authors paid attention to. The 3/6-GSP is represented schematically in Fig.3. It consists of a mobile platform $B_1B_3B_5$, equilateral triangle; a base platform $C_1 \dots C_6$, semi-regular hexagon; and they are connected via six extensible prismatic actuators.

When all the legs of 3/6-GSP are locked, the three normal planes of three velocities V_{B1}, V_{B3} and V_{B5} are respectively $B_1C_1C_2$, $B_3C_3C_4$ and $B_5C_5C_6$ (Fig.3). According to Theorem 2, we may educe the following deduction to determine the singularity of 3/6-GSP. Let us firstly define a "Star-frame C-$B_1B_3B_5$" in the moving platform. It is constructed by using three ray lines passing three points, B_1, B_2 and B_3, of the triangle $B_1B_3B_5$ and intersecting at a common point C called the center of Star-Frame.

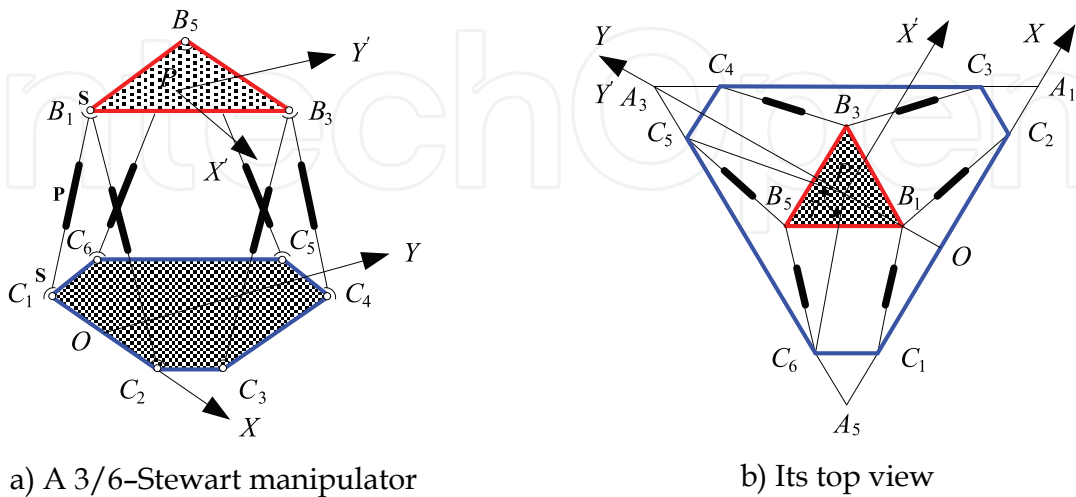a) A 3/6–Stewart manipulator

b) Its top view

Fig. 3. A 3/6-Stewart parallel manipulator

Theorem 3: A necessary and sufficient condition that the three velocities of three points in a rigid body can express that the body has a possible twist motion is that the intersecting point of three normal planes of the three velocities lies in the plane determined by the three points.

3. Structure and property of singularity loci of 3/6-Gough-Stewart for special orientations

The kinematics method can determine the singularity of the manipulator. If the six extensible legs of the GSP are locked and the mechanism has an instantaneous freedom, the manipulator is singular. Now, let us firstly discuss the kinematics properties of the typical singularity structures including singularities: 3c, 4b, 4d, 5b (Merlet 1988; 1989) and others.

3.1 Singularity hyperbola equation derived in an oblique plane

Our task is to find the whole singularity loci of the GSP and identify their structure and property. It is of an important and difficult issue. Here three Euler angles ϕ, θ and ψ are used to represent orientation of the mobile in terms of a rotation ϕ about Z-axis, then a rotation θ about the new Y'-axis, and finally a rotation ψ about the new Z''-axis.

In order to find the whole singularity loci and solve the issue, we first study the singularity equation in a special plane (Huang et al. 2003). The issue is divided into two parts:

(1) When the first Euler angle ϕ is equal to one of the following values, $\pm 30°$, $\pm 90°$, and $\pm 150°$, it is a special orientation cases and easier to analyze.

(2) When ϕ is any value with the exception of $\pm 30°$, $\pm 90°$, and $\pm 150°$, this is the general case.

Now, we solve the equation for singularity curve of the 3/6-GSP in a certain plane while the orientation of the mobile is provisionally set to ϕ=90°, ψ=0 and θ is any finite nonzero value.

The parameters of the parallel manipulator are as follows. The circumcircle radius of the basic hexagon platform is R_a, and the one of the triangle mobile is R_b; β_0 denotes the central angle of the circumcircle of the basic hexagon corresponding to side C_1C_2. Point P is the geometric center of the mobile (Fig. 3). The stationary frame O-XYZ is fixed to the base and the moving frame P-$X'Y'Z'$ is attached to the mobile.

Fig. 4 shows the position after the mobile rotates (90°, θ, 0**)**. The oblique plane in which the moving platform lies intersects the basic plane at line UV, which is parallel to axis X. For the orientation, $B_1P(Y')$ is parallel to A_5A_1 (X). At first, providing that point P is located at a special point C_0 in the perpendicular bisector of UV, and the distance from O_2 to point C_0 is equal to that between point O_2 and A_3, then we deduce that C_0B_3 and A_3A_1 intersect at point V, and C_0B_5 and A_3A_5 intersect at U. In that case, the mechanism is singular according to Deduction 2. The included angle between the oblique plane and the basic one is θ. In order to conveniently express the oblique plane below, we call it θ-plane. Let us suppose that the mobile translates to the position $B_{11}B_{31}B_{51}$ in θ-plane and line $B_{11}P$ intersects line O_2C_0 at C. If line $B_{31}C$ intersects A_1A_3 at point V, and line C_1B_{51} intersects A_3A_5 at point U, the mechanism is also singular (Deduction 2). We can prove that the center of Star-frame always lies in line O_2C_0 for the orientation. In general, the singularity is a general-linear-complex singularity. Based on the analysis above, we study the singularities of 3/6–GSP when the mobile translates arbitrarily in θ-plane. The coordinates of point C_0 and O_2 with respect to the fixed frame are (0, Y_0, Z_0) and (0, u, 0), respectively. The frame O_2-xyz is attached to θ-plane. It should be noticed that angle θ as shown in Fig. 5 about the Y'-axis is negative.
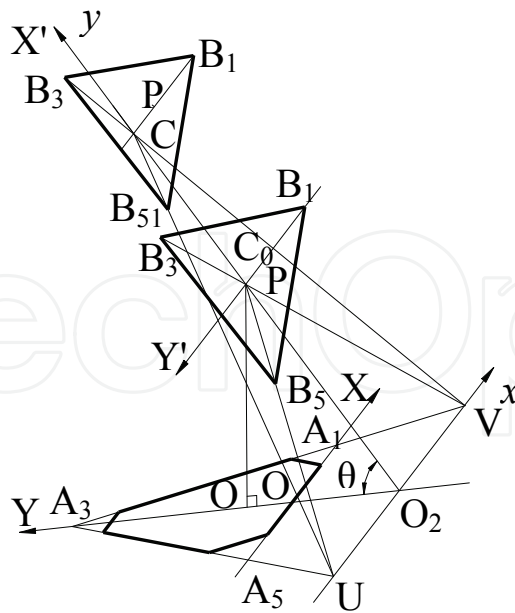
Fig. 4. θ- Oblique plane for the orientation $(90°, \theta, 0)$

The coordinates of points P, C, B_{31} and V with respect to O_2-xyz are

$$
\begin{aligned}
&P : (x, \ y, \ 0) \\
&C : (0, \ y, \ 0) \\
&B_{31} : (x - \frac{R_b}{2}, \ y + \frac{\sqrt{3}}{2} R_b, \ 0) \\
&V : (-\frac{\sqrt{3}}{3} \frac{Z_0}{\sin\theta}, \ 0, \ 0)
\end{aligned}
\tag{6}
$$

Considering $O_2C_0 = O_2A_3$, we can obtain

$$
O_2O_1 - OO_1 + OA_3 = O_2C_0
\tag{7}
$$

namely

$$
3R_a \cos(\beta_0 / 2) - Z_0 \frac{\cos\theta}{\sin\theta} - Y_0 = -\frac{Z_0}{\sin\theta}
\tag{8}
$$

In the right-angled triangle $\Delta O_1O_2C_0$, we obtain

$$
Y_0 - u = -Z_0 \frac{\cos\theta}{\sin\theta}
\tag{9}
$$

Solving Eqs. (8) and (9) for Y_0 and Z_0, we obtain

$$
\begin{aligned}
Y_0 &= u(1 - \cos\theta) + 3R_a \cos(\beta_0 / 2)\cos\theta \\
Z_0 &= u\sin\theta - 3R_a \cos(\beta_0 / 2)\sin\theta
\end{aligned}
\tag{10}
$$

Provided that the coordinates of an arbitrary point in line $B_{31}V$ are $(x_x, \ y_y, \ 0)$, its equation is written as

$$\frac{y_y - y - \frac{\sqrt{3}}{2}R_b}{-y - \frac{\sqrt{3}}{2}R_b} = \frac{x_x - x + \frac{R_b}{2}}{-\frac{\sqrt{3}}{3}\frac{Z_0}{\sin\theta} - x + \frac{R_b}{2}} \tag{11}$$

Since point C lies in line $B_{31}V$, substitute the coordinates of point C ($x_x = 0$ and $y_y = y$) into Eq. (9) and simplify as

$$xy - \frac{R_b}{2}y - \frac{Z_0 R_b}{2\sin\theta} = 0 \tag{12}$$

Substituting Eq. (10) into Eq. (11) and eliminating Z_0, yield

$$xy - \frac{R_b}{2}y + \frac{(3R_a\cos(\beta_0/2) - u)R_b}{2} = 0 \tag{13}$$

Eq. (13) denotes a hyperbola and is independent of the Euler angle θ. The coordinates of its center are $(R_b/2, 0)$, and its vertical and horizontal asymptotes are $x= R_b/2$, $y=0$

This is an important conclusion, as we have known that the singularity equation of GSP in 3-dimension space is a polynomial expression of degree three. However, equation (13) is only a quadratic equation in the special θ-plane. Eq. (13) only contains variables x and y, so it denotes the positions of point p when the mechanism is singular. The equation is termed the equation of the singularity curve in θ-plane.
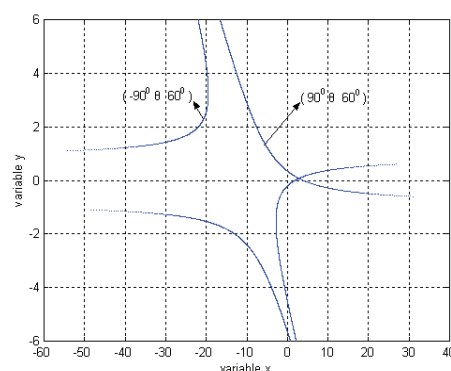
When orientation of the mobile is given by three Euler angles (-90°, θ, 0), the singularity equation can also be obtained in θ-plane with respect to the frame O_2-xyz, the same as in Fig. 4.

$$xy + \frac{R_b}{2}y - \frac{(3R_a\cos(\beta_0/2) - u)R_b}{2} = 0 \tag{14}$$

When parameters of the mechanism are set to $R_a = \sqrt{2}$, $R_b = 1$, $\beta_0 = 90^0$ and $u = -2$, the hyperbolas denoted by Eqs. (13) and (14) are illustrated in Fig. 6(a). Since the result comes from the above-mentioned Theorem and satisfies the necessary and sufficient condition of singularity, so that there is no any singularity except the points on hyperbolas in that θ-plane.

(a) For the orientation $(\pm 90^0 \ \theta \ 0)$ (b) For the orientation $(\pm 90^0 \ \theta \ 60^0)$

Fig. 5. The singularity curve in θ- plane

3.2 The singularity equation derived in three-dimensional space

Eqs. (13) and (14) are deduced by geometric method in the oblique plane. By Theorem 3, we can analyze the distribution properties of the singularities of 3/6-GSP in three-dimensional space.

The coordinates of point B_i (i=1, 2, 3) of the mobile are denoted as B_i':(B_{ix}', B_{iy}', B_{iz}') in the moving frame, and B_i:(B_{ix}, B_{iy}, B_{iz}) in the fixed frame; the coordinates of point C_j are denoted as (C_{jx}, C_{jy}, C_{jz}) in the fixed frame.

The transformation matrix $\mathbf{T}$ of the moving frame with respect to the fixed one can be written using Euler angles ϕ, θ and ψ as

$$[\mathbf{T}] = \begin{bmatrix} \cos\phi\cos\theta\cos\psi - \sin\phi\sin\psi & -\cos\phi\cos\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta & X \\ \sin\phi\cos\theta\cos\psi + \cos\phi\sin\psi & -\sin\phi\cos\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta & Y \\ -\sin\theta\cos\psi & \sin\theta\sin\psi & \cos\theta & Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (15)$$

where(X, Y, Z) are the coordinates of point p with respect to the fixed frame. The coordinates of point B_i in the mobile with respect to the fixed frame are

$$\begin{Bmatrix} B_{ix} \\ B_{iy} \\ B_{iz} \\ 1 \end{Bmatrix} = [\mathbf{T}] \begin{Bmatrix} B_{ix}' \\ B_{iy}' \\ B_{iz}' \\ 1 \end{Bmatrix}, \quad i = 1, 2, 3 \quad (16)$$

3.2.1 Singularity equation for orientation (90°, θ, 0)

When three Euler angles are 90^0, θ and 0, respectively, from Eq. (16), we can obtain coordinates of three points $B_i (i = 1, 2, 3)$ in the mobile with respect to the fixed frame. Thus three equations of three normal planes $B_1C_1C_2$, $B_3C_3C_4$ and $B_5C_5C_6$ and the one that the mobile belongs to can be written by the coordinates of the three corresponding points. The equation of plane $B_1C_1C_2$ is

$$\begin{vmatrix} x - B_{1x} & y - B_{1y} & z - B_{1z} \\ C_{1x} - B_{1x} & C_{1y} - B_{1y} & C_{1z} - B_{1z} \\ C_{2x} - B_{1x} & C_{2y} - B_{1y} & C_{2z} - B_{1z} \end{vmatrix} = 0 \quad (17)$$

where x, y and z are the coordinates of moving point in plane $B_1C_1C_2$ with respect to the fixed frame. Substituting coordinates of points B_1, C_1 and C_2 into the above equation, we obtain

$$Z y - Y z = 0 \quad (18)$$

Similarly, the equation of plane $B_3C_3C_4$ can be obtained

$$(-3R_b \sin\theta + 2\sqrt{3}Z)x + (2Z - \sqrt{3}R_b \sin\theta)y + (-2Y + 6R_a \cos(\beta_0 / 2) - \sqrt{3}R_b \cos\theta$$
$$+ \sqrt{3}R_b - 2\sqrt{3}X)z - 6ZR_a \cos(\beta_0 / 2) + 3\sqrt{3}R_aR_b \sin\theta\cos(\beta_0 / 2) = 0 \quad (19)$$

The one of plane $B_5C_5C_6$ is:

$$(-3R_b\sin\theta - 2\sqrt{3}Z)x + (2Z + \sqrt{3}R_b\sin\theta)y + (-2Y + 6R_a\cos(\beta_0/2) + \sqrt{3}R_b\cos\theta - \sqrt{3}R_b$$
$$+2\sqrt{3}X)z - 6ZR_a\cos(\beta_0/2) - 3\sqrt{3}R_aR_b\sin\theta\cos(\beta_0/2) = 0 \tag{20}$$

The one of plane $B_1B_3B_5$ is

$$(\sin\theta)y + (\cos\theta)z - (\sin\theta)Y - (\cos\theta)Z = 0 \tag{21}$$

Note that, the equations of these planes are on the same condition that point P (X, Y, Z) is located at some point and the orientation is denoted by three Euler angles ($90°$, θ, 0).

Solving Eqs. (18), (19) and (20) for x, y and z, then substituting them into Eq. (21) and eliminating x, y and z, we obtain

$$\left[(\sin\theta)\,Y + (\cos\theta)\,Z\right]\left[2XZ + R_b(\sin\theta)Y + R_b(\cos\theta)Z - R_b\,Z - 3R_bR_a\sin\theta\cos(\beta_0/2)\right] = 0 \tag{22}$$

According to Theorem 3, Eq. (22) denotes the singularity locus of point P for the orientation ($90°$, θ, 0). Obviously, it includes a plane and a conicoid. The plane equation is

$$(\sin\theta)\,Y + (\cos\theta)Z = 0 \tag{23}$$

Eq. (23) denotes that singularity locus of point P is a plane containing line C_1C_2 or A_5A_1, namely, X-axis. As the plane and plane $B_1B_3B_5$ denoted by Eq. (23) have the same normal vector, and when plane $B_1B_3B_5$ translates and coincides with plane expressed by Eq.(23), the configuration is singular. The case belongs to the Hunt's singularity and is the first special-linear-complex singularity explained in Case 5. Eq. (23) shows that the mechanism is singular, wherever point P locates in the plane.

The conicoid equation is

$$2XZ + R_b(\sin\theta)\,Y + R_b((\cos\theta) - 1)Z - 3R_bR_a\sin\theta\cos(\beta_0/2) = 0 \tag{24}$$

When θ is constant, Eq. (24) denotes a hyperbolic paraboloid and we will explain later. Eq. (28) also represents a hyperbolic paraboloid.

3.2.2 Singularity equation for the orientation ($\pm90°$, θ, ψ)

3.2.2.1. The Derivation of the Equation

For the orientation ($\pm90°$, θ, ψ), the transformation matrix $\mathbf{T}$ is

$$\left[\mathbf{T}\right] = \begin{bmatrix} -c & -d & 0 & X \\ bd & -bc & a & Y \\ -ad & ac & b & Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{25}$$

where

$$a = \sin\theta\,;\ b = \cos\theta\,;\ c = \sin\psi\,;\ d = \cos\psi \tag{26}$$

Using the same method above, the equations of the three normal planes can be obtained.

$$(acR_b - Z)y + (bcR_b + Y)z = 0 \tag{27}$$

$$\sqrt{3}(\sqrt{3}adR_b - acR_b - 2Z)x + (\sqrt{3}adR_b - 2Z - acR_b)y + (2Y + 2\sqrt{3}X + \sqrt{3}bdR_b - 3cR_b - \sqrt{3}dR_b - bcR_b - 6R_a\cos(\beta_0/2))z + 3R_a\cos(\beta_0/2)(2Z + acR_b - \sqrt{3}adR_b) = 0 \tag{28}$$

$$\sqrt{3}(acR_b + 2Z + \sqrt{3}adR_b)x - (2Z + \sqrt{3}adR_b + acR_b)y + (2Y - bcR_b - 6R_a\cos(\beta_0/2) - \sqrt{3}bdR_b - 3cR_b + \sqrt{3}dR_b - 2\sqrt{3}X)z + 3R_a\cos(\beta_0/2)(2Z + acR_b + \sqrt{3}adR_b) = 0 \tag{29}$$

The equation of plane $B_1B_3B_5$ is

$$ay + bz - aY - bZ = 0 \tag{30}$$

Solving Eqs. (27), (28) and (29) for x, y and z, and then substituting them into Eq. (30), the singularity equation is

$$\left[(\sin\theta)Y + (\cos\theta)Z\right](eZ^2 - fXZ + gYZ + hX - iY + jZ + k) = 0 \tag{31}$$

Eq. (31) shows that the singular loci include a plane and a conicoid. The plane equation is the same as Eq. (25). It also represents that in this case all the six lines cross a common line. This case belongs to the first special-linear-complex singularity. The quadratic equation is

$$eZ^2 - fXZ + gYZ + hX - iY + jZ + k = 0 \tag{32}$$

Eq. (32) is a singularity equation with respect to the fixed frame $O\text{-}XYZ$. When the mobile shown in Fig. 5 rotates an angle ψ about Z''-axis again, its orientation is $(90°, \theta, \psi)$.

The plane in which the mobile lies is still θ-plane. After the coordinate transformation, the equation of the singularity curve in θ-plane with respect to the frame $O_2\text{-}xyz$ is

$$\begin{cases} 2(\sin\psi)y^2 + 2(\cos\psi)xy + R_b\sin(2\psi)x + (-2u\sin\psi + 6R_a\sin\psi\cos(\beta_0/2) \\ -R_b\cos(2\psi))y - R_b^2\sin\psi + R_b\cos(2\psi)(3R_a\cos(\beta_0/2) - u) = 0 \\ z = 0 \end{cases} \tag{33}$$

It is also a hyperbola. In addition, Eq. (33) is independent of the Euler angle θ.

3.2.2.2. Analysis of the Singularity Property

The four invariants Δ, D, I and J of Eq. (32) are

$$\Delta = \begin{vmatrix} 0 & 0 & -\dfrac{f}{2} & \dfrac{h}{2} \\ 0 & 0 & \dfrac{g}{2} & -\dfrac{i}{2} \\ -\dfrac{f}{2} & \dfrac{g}{2} & e & \dfrac{j}{2} \\ \dfrac{h}{2} & -\dfrac{i}{2} & \dfrac{j}{2} & k \end{vmatrix} = \frac{R_b^2\sin^6\theta\cos^2 3\psi}{4} \geq 0 \tag{34}$$

$$D = \begin{vmatrix} 0 & 0 & -\dfrac{f}{2} \\ 0 & 0 & \dfrac{g}{2} \\ -\dfrac{f}{2} & \dfrac{g}{2} & e \end{vmatrix} = 0 \tag{35}$$

$$I = 2\sin\psi(1+\cos\theta) \,, \quad J = -\sin^2\theta \tag{36}$$

The following cases are discussed according to its invariants, in which D is always zero whatever θ and ψ are.

1. If $\theta \neq 0$, $\psi \neq \pm 30°$, $\pm 90°$, and $\pm 150°$, then $D=0$, $\Delta>0$, the singular locus denoted by Eq. (32) is a hyperbolic paraboloid. Generally, the six lines 1, 2, …, 6 belong to a general linear complex when point P locates at the surface.

2. If $\theta=0$, Eq. (32) can be written as

$$4(\sin\psi)Z^3 = 0 \tag{37}$$

    a. When $\psi=0$ and $Z \neq 0$, namely, the orientation is (90°, 0, 0), Eq. (37) is an identical equation and the mechanism is singular whatever the position of point P in three-dimensional space is. This is the Fichter's singular configuration and all the six lines belong to a general linear complex.

    b. When $Z=0$, the moving platform and the base are coplanar. The mechanism is also singular whatever Euler angle ψ is. The mechanism holds three remnant freedoms when all the legs are locked. In this case, there exist the first and the second special-linear-complex singularities.

3. If $\theta \neq 0$, $\psi = \pm 30°$, $\pm 90°$, or $\pm 150°$, then $D=0$, $\Delta=0$ and $J \neq 0$, and the conicoid degenerates into a pair of intersecting planes. For instance, when $\psi=\pm 30°$, two equations are

$$2Z - R_b \sin\theta = 0 \tag{38}$$

$$\sqrt{3}(\sin\theta)X - (\sin\theta)Y - (1+\cos\theta)Z - R_b \sin\theta + 3R_a \sin\theta\cos(\beta_0/2) = 0 \tag{39}$$

When $\psi=-30°$, $\pm 90°$, or ± 150, the conicoid also degenerates into two planes. The singularity cases are similar to the above.

3.2.2.3. Analysis of Other Singularities

The singularities discussed above are all for the orientations, ($\pm 90°$, θ, ψ), of the mobile. In these cases, the intersecting lines between the oblique moving plane and the basic one are parallel to line C_1C_2 or A_1A_5, one of the three sides of the triangle $A_1A_3A_5$.

The similar singularities with a plane equation and a quadratic one can also occur when the orientations are as follows

1. The Euler angles are

(–150°, θ, ψ) or (30°, θ, ψ)

All the intersecting lines between the oblique mobile and the base are parallel to line C_3C_4 or A_1A_3.

2. The Euler angles are

(150°, θ, ψ) or (–30°, θ, ψ)

All the intersecting lines between the oblique mobile and the base are parallel to line C_5C_6 or A_3A_5.

For the two cases, the singularity equation can also resolve into two parts: one is a plane equation containing the corresponding side C_iC_j, another is a hyperbolic paraboloid equation, too. When $\psi = \pm30°$, $\pm90°$, or $\pm150°$, the hyperbolic paraboloid also degenerates into two planes.
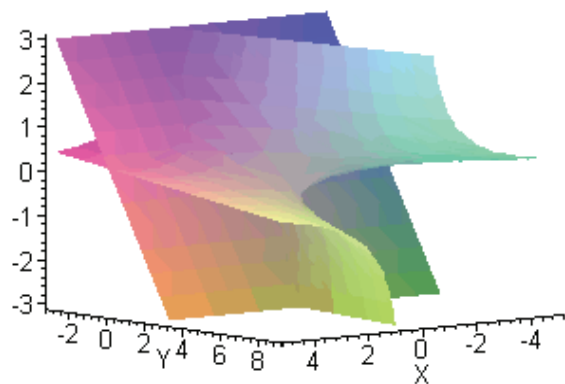
However, when the orientation is

$$(\phi, \theta, \pm30°), (\phi, \theta, \pm90°) \text{ or } (\phi, \theta, \pm150°)$$

in which ϕ and θ, can be arbitrary values, the singularity locus also consists of two parts: One is a plane; another is also a hyperbolic paraboloid. When point P translates in the plane, two of three points B_1, B_3 and B_5 lie in the basic plane.

3.3 Singularity distribution in three-dimensional space

According to the analysis method above, we may easily know the distribution characteristics of the singularity loci of the 3/6-Gough-Stewart manipulator, and draw their singularity surface in three-dimensional space for some different orientations of the mobile in frame $O\text{-}XYZ$. Here, the parameters of the mechanism are set to $R_a = \sqrt{2}$, $R_m = 1m$ and $\beta_0 = 90°$, and the surfaces are shown in Fig. 6.
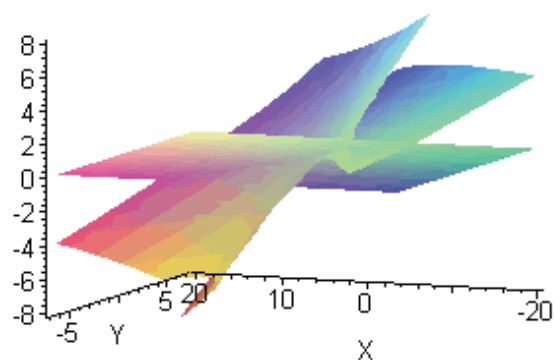

(a) The orientation (90° 45° 0)

(b) The orientation (90° 30° 60°)

(c) The orientation (90° 45° 30°)

(d) The orientation (45° 25° 30°)

Fig. 6. The singularity loci for 3/6—Stewart parallel manipulator

The readers may wonder that the singularity loci are so huge and completed and ask how can the GSP work? In practice, if you notice the position of the origin point of the O-XYZ system in Figures, and the magnitudes of the parameters, $R_a = \sqrt{2}$ and R_m=1m, you can find that the workspace of the manipulator is smaller relative the singularity loci shown in figures. You can easily design the manipulator making its workspace locate over the singularity loci and avoiding singularity.
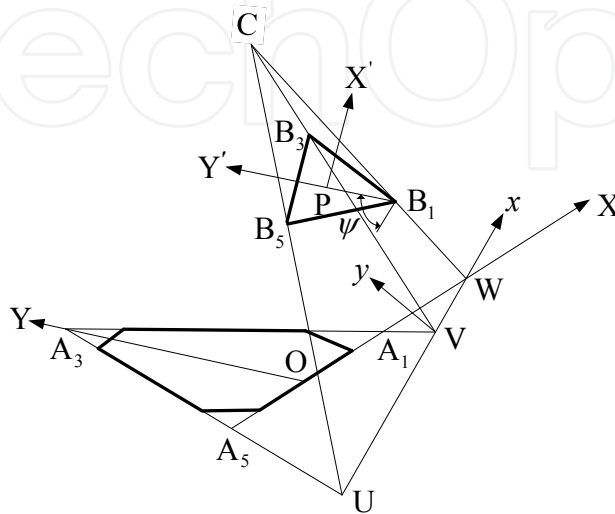

Fig. 7. The general case

4. Structure and property of the singularity loci of the 3/6-Gough-Stewart for general orientations

When ϕ takes any value with the exception of ±30°, ±90°, or ±150°, this is the general orientation case of the mobile of the GSP, and the analysis of the singularity loci is more difficult. In this case, UV is not parallel to any side of triangle $A_1A_3A_5$, as shown in Fig. 7.

4.1 Singularity equation based on Theorem 3 for general orientations
For the most general orientations of the mobile, $\phi \neq$ ±30°, ±90°, and ±150°, the singularity equation can be directly obtained by using Theorem 3. The equation of normal plane $B_1C_1C_2$ is

$$\begin{vmatrix} x' - B_{1x} & y' - B_{1y} & z' - B_{1z} \\ C_{1x} - B_{1x} & C_{1y} - B_{1y} & C_{1z} - B_{1z} \\ C_{2x} - B_{1x} & C_{2y} - B_{1y} & C_{2z} - B_{1z} \end{vmatrix} = 0 \tag{40}$$

where (x', y', z') denotes coordinates of the moving point on plane $B_1C_1C_2$ in the fixed frame. This gives

$$Fy' + Gz' = 0 \tag{41}$$

Similarly, equations of three planes $B_3C_3C_4$, $B_5C_5C_6$ and $B_1B_3B_5$ can be obtained as well. According to Theorem 3, solving the linear equation system of the four planes for intersecting point C, the singularity locus equation for general orientations is as follows

$$f_1Z^3 + f_2XZ^2 + f_3YZ^2 + f_4X^2Z + f_5Y^2Z + f_6XYZ + f_7Z^2 + f_8X^2$$
$$+f_9Y^2 + f_{10}XY + f_{11}XZ + f_{12}YZ + f_{13}Z + f_{14}X + f_{15}Y + f_{16} = 0 \tag{42}$$

where (X, Y, Z) are the coordinates of center point P. It is a polynomial expression of degree three. The equation is still very complicated and difficult to further analyze, but it is very simple in the following special cases.

When $\phi \neq \pm30°$, $\pm90°$, and $\pm150°$ and ψ is one of the values $\pm30°$, $\pm90°$, or $\pm150°$, Eq. (42) degenerates into a plane and a hyperbolic paraboloid as well. For example, when $\psi=90°$, the singularity equation is

$$(2Z + R_b \sin\theta)(a_{11}X^2 + a_{22}Y^2 + a_{33}Z^2 + 2a_{23}YZ + 2a_{31}ZX + 2a_{12}XY +$$
$$2a_{14}X + 2a_{24}Y + 2a_{34}Z + a_{44}) = 0 \tag{43}$$

where these coefficients are listed in the Appendix 2. Eq. (43) indicates a plane and a hyperbolic paraboloid. The first factor forms a plane equation

$$2Z + R_b \sin\theta = 0 \tag{44}$$

which is parallel to the basic plane. When point P lies in the plane, the mechanism is singular for orientation $(\phi, \theta, 90°)$, because points B_3 and B_5 lie in the basic plane. This is similar to Case 6. All the six lines cross the same line C_1C_2.

4.2 Singularity analysis using singularity-equivalent-mechanism

The singularity locus expression (Eq. (43)) for general orientations has been derived by Theorem 3. But it is still quite complicated, and we are not sure whether it consists of some typical geometrical figures. Meanwhile the property of singularity loci is unknown yet. In order to reply this question, a "Singularity-Equivalent-Mechanism" which is a planar mechanism is proposed. Thus the troublesome singularity analysis of the GSP can be transformed into a position analysis of the simpler planar mechanism.

4.2.1 The parallel case

4.2.1.1 The Singularity-Equivalent-Mechanism

In the parallel case, the three Euler angles of the mobile platform are $(90°, \theta, \psi)$, while θ and ψ can be any nonzero value. The mobile plane of the mechanism lies on θ-plane (Fig. 5).

The corresponding imaginary planar singularity-equivalent-mechanism is illustrated in Fig. 8. Where R denotes a revolute pair and P a prismatic pair, triangle $B_1B_3B_5$ is connected to ground by three kinematic chains, RPP, PPR and RPR. The latter two pass through two points U and V, respectively, while the first one slides along the vertical direction and keeps $L_1C//UV$. Three slotted links, L_1, L_2 and L_3, intersect at a common point C. In order to keep the three links always intersecting at a common point and satisfying Deduction 2, a concurrent kinematic chain PRPRP is used. It consists of five kinematic pairs, where two R pairs connect three sliders. The three sliders and three slotted links form three P pairs. The PRPRP chain coincides with a single point C from top view. Based on the Grübler-Kutzbach criterion, the mobility of the mechanism is two.

It is evident that the planar mechanism can guarantee that the three lines passing through three vertices intersect at a common point, and these three lines can always intersect the

corresponding sides of the basic triangle. From Deduction 2, every position of the planar mechanism corresponds to a special configuration of the original GSP. So we call it a "singularity-equivalent-mechanism". Thus the position solution of the planar mechanism expresses the singularity of the original mechanism.

4.2.1.2 Forward Position Analysis of the Singularity -Equivalent-Mechanism

The frames are set as the same as in Fig. 5 and Fig. 10. The coordinates of point P in frame O_2-xy are (x, y). ψ indicates the orientation of the triangle $B_1B_3B_5$ in θ-plane. In order to obtain the locus equation of point P, firstly we can set three equations of three lines passing through the three vertices, and substitute the coordinates of points B_1, B_3 and B_5 into the equations, then (x, y) and ψ can be obtained.
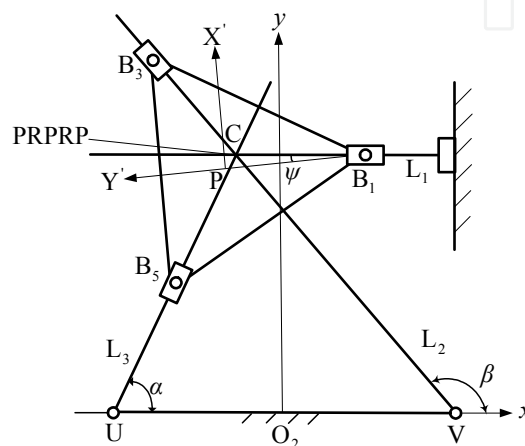

Fig. 8. The singularity-equivalent-mechanism for (90°, θ, ψ)

Considering that the mobility of this mechanism is two, there need two inputs α and β. Three equations of three lines CU, CV and CB_1 in reference frame O_2-xy are respectively

$$Y = (\tan\alpha)(X + a/2) \tag{45}$$

$$Y = (\tan\beta)(X - a/2) \tag{46}$$

and

$$Y = -\frac{a\tan a\tan\beta}{\tan\alpha - \tan\beta} \tag{47}$$

Solving Eqs. (52), (53) and (54) yields

$$x = \frac{R_b\cos\psi J_1 - (\sqrt{3}R_b\sin\psi + a)J_3}{2(\tan a - \tan\beta)} \tag{48}$$

$$y = \frac{R_b\sin\psi J_2 - \sqrt{3}R_b J_3\cos\psi - 2a\tan a\tan\beta}{2(\tan a - \tan\beta)} \tag{49}$$

and

$$\tan\psi = \frac{(\tan\beta + \tan\alpha)}{\sqrt{3}\tan\alpha - \sqrt{3}\tan\beta - 2\tan\alpha\tan\beta} \tag{50}$$

where $J_1 = \tan a - \tan \beta - 2\sqrt{3}$, $J_2 = \tan a - \tan \beta - 2\sqrt{3} \tan a \tan \beta$, $J_3 = \tan \alpha + \tan \beta$, Eqs. (48), (49) and (50) denote direct kinematics of the mechanism.

4. 2. 1. 3. Singularity Equation in the θ- plane

Once the orientation (90°, θ, ψ)of the mobile platform is specified, in Fig.10, Euler angle ψ is an invariant. So it only needs to choose one input in this case. From Eq. (65) one obtains

$$\tan\beta = \frac{\tan\alpha(\sqrt{3}\tan\psi\text{-}1)}{\sqrt{3}\tan\psi + 2\tan\alpha\,\tan\psi + 1} \tag{51}$$

So the singularity equation in θ- plane for the orientation (90°, θ, ψ) can be obtained from Eqs. (48), (49) and (50) by eliminating parameters α andβ

$$2(\sin\psi)y^2 + 2(\cos\psi)xy + R_b \sin(2\psi)x + $$
$$(\sqrt{3}a\sin\psi - R_b\cos(2\psi))y - R_b^2\sin\psi + \sqrt{3}aR_b\cos(2\psi)\,/\,2 = 0 \tag{52}$$

where $a = 2(3R_a\cos(\beta_0\,/\,2) - u)\,/\,\sqrt{3}$. Eq. (52) denotes a hyperbola. Especially, when $\psi = \pm 90°$, Eq. (52) degenerates into a pair of intersecting straight lines respectively. Two of the four equations are

$$y - R_b/2 = 0 \text{ , } y + R_b/2 = 0 \tag{53}$$

In both cases, two points B_3 and B_5 lie in line UV. So that four lines are coplanar with the base plane. This is the singularity of Case 6. The similar situation is for $\psi = 30°, \psi = \text{-}150°$, $\psi = \text{-}30°$ and $\psi = 150°$.

4.2.2 The general case
When $\psi \neq \pm 30°$, $\pm 90°$, and $\pm 150°$, the intersecting line UVW between θ- plane and the base one is not parallel to any side of triangle $A_1A_3A_5$. This is the most general and also the most difficult case.

4.2.2.1 The Singularity-Equivalent-Mechanism

Fig. 11 shows the singularity-equivalent-mechanism. The triangle $B_1B_3B_5$ is connected to the ground passing through three points W, V and U by three RPR kinematic chains. The three points U, V and W, as shown in Fig. 9, are three intersecting points between θ-plane and sides A_3A_5, A_1A_3 and A_1A_5, respectively. Three slotted links L_1, L_2 and L_3 intersect at a common point C. In order to keep the three links always intersecting at a common point, a concurrent kinematic chain, PRPRP, is used as well. Therefore, all the configurations of the equivalent mechanism satisfying Deduction 2 are special configurations of the Gough-Stewart manipulator. So we can analyze direct kinematics of the equivalent mechanism to find singularity loci of the manipulator.
Similarly the mobility of the equivalent mechanism is two, and it needs two inputs when analyzing its position.

4.2.2.2 Forward Position Analysis of the Singularity–Equivalent -Mechanism

The frames are set as shown in Fig. 11. Similar to section 4.2.1.2, we may set three equations of three straight lines passing through three vertices, and substitute the coordinates of points B_1, B_3 and B_5 into the equations, then solutions, (x, y)and ψ, can be obtained
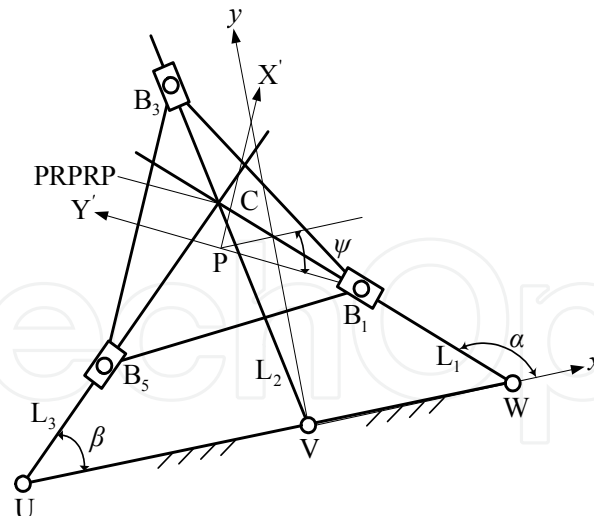
Fig. 9. The singularity-equivalent-mechanism for general case

$$x = -(3R_b \sin\psi - 2R_b \tan\alpha \cos\psi + 2w \tan\alpha + \sqrt{3}R_b \cos\psi$$
$$-R_b \tan\beta \cos\psi + \sqrt{3}R_b \tan\beta \sin\psi - 2u \tan\beta)/(2\tan\beta - 2\tan\alpha) \tag{54}$$

$$y = (-R_b \tan\alpha \sin\psi - \sqrt{3}R_b \tan\alpha \cos\psi + 3R_b \tan\alpha \tan\beta \cos\psi + 2u \tan\alpha \tan\beta$$
$$-\sqrt{3}R_b \tan\alpha \tan\beta \sin\psi - 2R_b \tan\beta \sin\psi - 2w \tan\alpha \tan\beta)/(2\tan\beta - 2\tan\alpha) \tag{55}$$

$$\tan\psi = \frac{2\sqrt{3}\, w \tan\alpha - 3u \tan\alpha \tan\beta - \sqrt{3}u \tan\beta}{\tan\beta\,(-2\sqrt{3}\, w \tan\alpha + \sqrt{3}u \tan\alpha - 3u)} \tag{56}$$

where u indicates the distance from point U to V, and w the distance from V to W. Substituting Eq. (56) into Eqs. (55) and (54), and eliminating ψ, the relations between (x, y) and the inputs α, β can be obtained. This is direct kinematics of the equivalent mechanism.

4.2.2.3 Singularity Equation in the θ- plane

Under a general case, Euler angle ϕ can be any value with the exception of ±30°, ±90°, or ±150°. From Eq (56) one obtains

$$\tan\beta = \frac{2\sqrt{3}w \tan\alpha}{-2\sqrt{3}w \tan\alpha \tan\psi + \sqrt{3}u \tan\alpha \tan\psi - 3u \tan\psi + 3u \tan\alpha + \sqrt{3}u} \tag{57}$$

In the case of some specified ψ, there are the same three particular situations that is B_1 and B_5, B_1 and B_3, or B_3 and B_5 lie in the line UV, respectively. The singularity loci are three pairs of intersecting straight lines.
In order to use the above-mentioned formulas, u and w in Eq. (57) should be calculated in advance. They depend on their relative positions in UV, as shown in Fig. 10.
The distance w between V and W is

$$|w| = |WV| = \left| \frac{3R_a \cos(\beta_0/2) - \sqrt{3}x_V}{\cos\phi} \right| \tag{68}$$
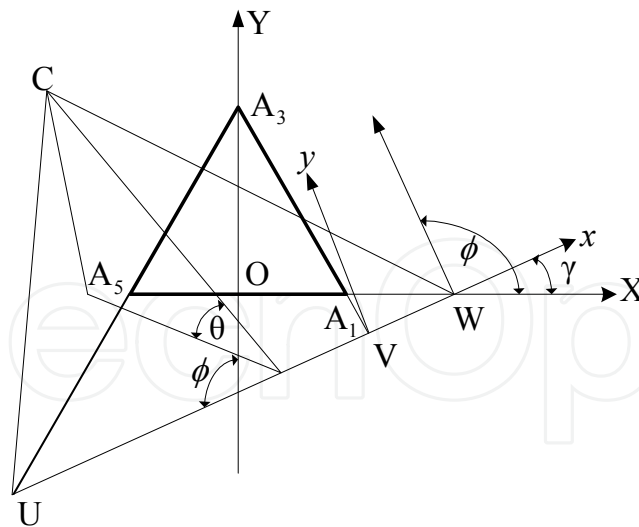
Fig. 10. The Intersecting Line UW of two planes

The distance u between U and V is

$$|u| = |UV| = \left| \frac{2\sqrt{3}x_V}{(\sqrt{3} + \cot\phi)\sin\phi} \right| \tag{59}$$

The sign of w is positive when point W is on the right side of V, and it is negative when W is on the left side of V. It is similar for the sign of u.

For a given x_v, the singularity equation in θ-plane can be obtained by eliminating the parameter α

$$bxy + cy^2 + dx + ey + f = 0 \tag{60}$$

The two invariants D, δ of Eq. (60) are

$$D = \begin{vmatrix} 0 & b/2 & d/2 \\ b/2 & c & e/2 \\ d/2 & e/2 & f \end{vmatrix} = -\frac{1}{4}(b^2f + d^2c - bde) \tag{61}$$

and

$$\delta = \begin{vmatrix} 0 & b/2 \\ b/2 & c \end{vmatrix} = -\frac{1}{4}b^2 < 0 \tag{62}$$

Generally, $D \neq 0$ and $\delta < 0$ for a general value of x_v, so Eq. (60) indicates a set of hyperbolas.

4.3 Five special cases of the singularity equation

There are five special cases. For the given parameters (R_a, R_b, β_0) and (ϕ, θ, ψ), D is a quartic equation while δ a quadratic equation with respect to the single variable x_v, respectively. Generally, there are four real roots of x_v when $D=0$ and $\delta \neq 0$, and Eq. (60) degenerates into four pairs of intersecting straight lines. For the same reason, there is one real root of multiplicity 2 when $\delta=0$ and $D \neq 0$, and Eq. (60) degenerates into a parabola.
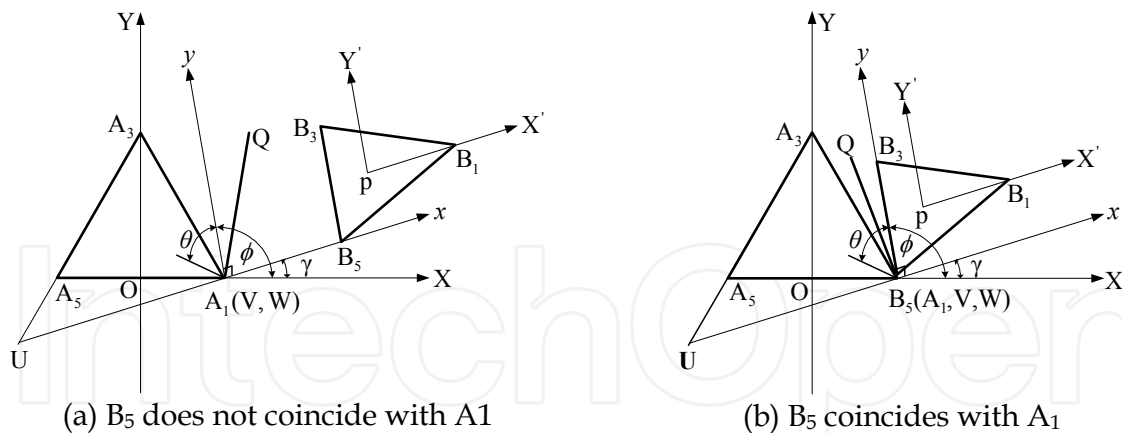
(a) B$_5$ does not coincide with A1   (b) B$_5$ coincides with A$_1$

Fig. 11. UV passes through the points A$_1$

Case 1. The line UV passes through point A$_1$, as shown in Fig. 11. In this case $x_v = \sqrt{3}R_a\cos(\beta_0/2)$, two points W and V coincide with point A$_1$. The singularity equation denoted by Eq. (75) degenerates into a pair of intersecting straight lines

$$[y - R_b\sin(\psi + 60°)][(-\sqrt{3}\sin(\psi) + \cos(\psi))x + (\sqrt{3}\cos(\psi) + \sin(\psi))y + R_b] = 0 \qquad (63)$$

One of them is

$$y - R_b\sin(\psi + 60°) = 0 \qquad (64)$$

Case 2. UV passes through point A$_3$. In this case x_v=0, two points U and V coincide with point A$_3$. Eq. (60) degenerates into a pair of intersecting straight lines either

$$[(y + R_b\sin(\psi)][x\cos(\psi) + y\sin(\psi) - R_b/2] = 0 \qquad (65)$$

The first part of Eq. (65) indicates a straight line parallel to x-axis. Similarly when B$_1$ coincides with point A$_3$, the singularity of this point is the first special-linear-complex singularity and the instantaneous motion is a pure rotation. When B$_1$ does not coincide with A$_3$, the singularities of points lying in this straight line are the general-linear-complex singularity and its instantaneous motion is a twist with $h^m \neq 0$.

The second part of Eq. (62) denotes another straight line. The singularities of points lying in this straight line are all the general-linear-complex singularity.

Case 3. UV passes point A$_5$. In this case

$$x_v = \sqrt{3}R_a\cos(\beta_0/2)(\sqrt{3} + \cot\phi)/(\sqrt{3} - \cot\phi)$$

two points U and W coincide with point A$_5$. Eq. (60) degenerates into a pair of intersecting straight lines.

$$\begin{aligned}&[y - R_b\sin(\psi - 60°)][(\sqrt{3}\sin(\psi) + \cos(\psi))x + (-\sqrt{3}\cos(\psi) + \sin(\psi))y \\ &+ R_b - 2\sqrt{3}\cos(\beta_0/2)\sin(\psi + 60°)/\sin(\varphi - 60°)] = 0\end{aligned} \qquad (66)$$

The first factor indicates a straight line parallel to the x-axis. Similarly when B$_3$ coincides with A$_5$, the singularity of this point is the first special-linear-complex singularity. When B$_3$ does not coincide with A$_5$, the singularities of points lying in this straight line are the general-linear-complex singularity.

Similarly the second factor of Eq. (66) denotes another straight line. The singularities of points lying in this straight line are all the general-linear-complex singularity.

Case 4. When

$$x_v = \frac{(-1 + 2\cos(2\phi))(R_b \cos\phi - 2R_a \cos(\beta_0/2)\cos\psi)}{(2(\sqrt{3}\sin\phi - \cos\phi)\sin(\phi + \psi))} \tag{67}$$

Eq. (67) degenerates into a pair of intersecting straight lines as well

$$(R_a \cos((\beta_0 - 6\psi)/2) - R_b \cos(\phi - 2\psi) + R_a \cos((\beta_0 + 6\psi)/2) \\ -2y\sin(\phi + \psi))(ax + by + c) = 0 \tag{68}$$

For the first straight line when $\beta_0 = 90°$, $(\phi, \theta, \psi) = (60°, 30°, 0)$, and the coordinates of point P_6 are $x = R_b/2$, $y = (2\sqrt{2}R_a - R_b)/2\sqrt{3}$, point B_5 lies in the intersecting line of two normal planes $B_1A_1A_5$ and $B_3A_1A_3$. Therefore, the six lines associated with the six extensible links of the 3/6-GSP intersect a common line B_5A_1. It is the first special-linear-complex singularity. The instantaneous motion is a pure rotation about line B_5A_1. The singularities of points lying in the first line with the exception of the above-mentioned point and the points lying in the second line are all belong to the general-linear-complex singularity.

Case 5. When

$$x_v = R_a \cos(\beta_0/2)\cos\psi(\cos\varphi + \sqrt{3}\sin\phi)/\sin(\phi + \psi), \ \delta = 0 \text{ and } D \neq 0,$$

Eq. (60) degenerates into a parabola.

$$cy^2 + dx + ey + f = 0 \tag{69}$$

According to the analysis mentioned above, it is shown that the singularity expression in θ-plane is not cubic but always quadratic. This indicates the θ-plane is a very special cross section of the singularity surface, so the special θ-plane can be called the principal section.

Generally speaking, the singularity loci of the 3/6-GSP for the most general orientations are different from those for some special orientations. The singularity loci in infinite parallel principal sections are all quadratic equations. The structure of the singularity loci in the principal sections of the cubic singularity surface includes a parabola, four pairs of intersecting straight lines and infinity of hyperbolas. The singularity loci in three-dimensional space are illustrated in Fig. 12.

In addition, it should be pointed out that once the mechanism is singular at the orientation (ϕ, θ, ψ), any orientation with different variable θ is singular as well (Huang at el. 2003).

5. Structure and property of the singularity loci of the 6/6-Gough-Stewart

Base on the above-mentioned analysis of the 3/6-GSP, here we focus on the most difficult issue, the singularity locus analysis of the 6/6-GSP including the singularity equation and the structure of singularity surface. The 6/6-GSP is typical manipulator. The 6/6-GSP is represented schematically in Fig. 13. It consists of two semi-regular hexagons: a mobile platform $B_1B_3...B_6$ and a base platform $C_1...C_6$. They are connected via six extensible prismatic actuators.
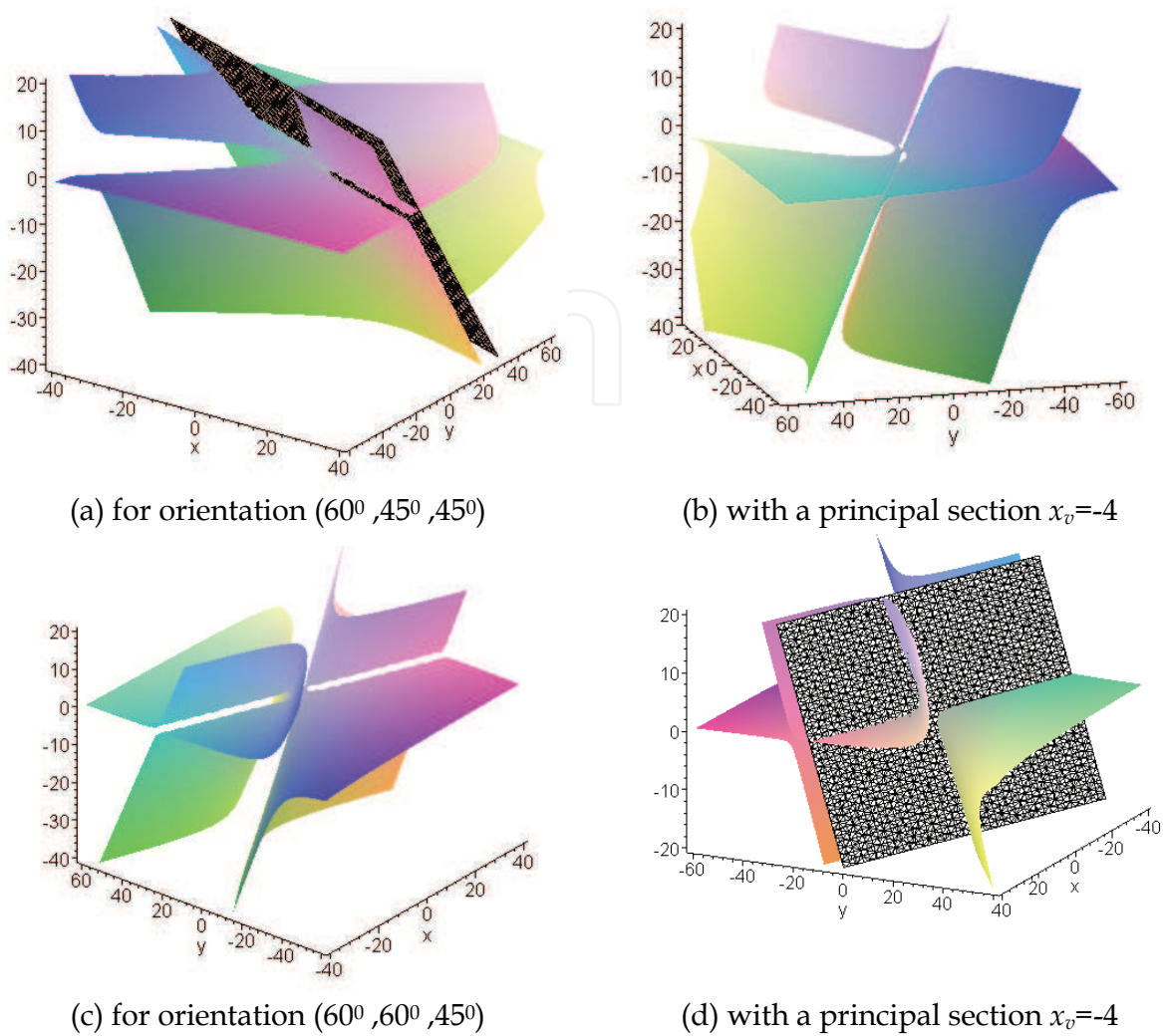
(a) for orientation $(60^0, 45^0, 45^0)$

(b) with a principal section $x_v = -4$

(c) for orientation $(60^0, 60^0, 45^0)$

(d) with a principal section $x_v = -4$

Fig. 12. The singularity loci in three-dimensional space for the general orientations

5.1 The Jacobian matrix

The Jacobian matrix of this class of the Gough-Stewart manipulators can be constructed as follows according to the theory of static equilibrium (Huang and Qu 1987)

$$
[\mathbf{J}]^T = \begin{bmatrix} \$_1 & \$_2 & \$_3 & \$_4 & \$_5 & \$_6 \end{bmatrix} = \begin{pmatrix} S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \\ S_{O1} & S_{O2} & S_{O3} & S_{O4} & S_{O5} & S_{O6} \end{pmatrix}
$$

$$
= \begin{pmatrix} \dfrac{(B_1 - C_1)}{\|B_1 - C_1\|} & \dfrac{(B_2 - C_2)}{\|B_2 - C_2\|} & \dfrac{(B_3 - C_3)}{\|B_3 - C_3\|} & \dfrac{(B_4 - C_4)}{\|B_4 - C_4\|} & \dfrac{(B_5 - C_5)}{\|B_5 - C_5\|} & \dfrac{(B_6 - C_6)}{\|B_6 - C_6\|} \\ \dfrac{(C_1 \times B_1)}{\|B_1 - C_1\|} & \dfrac{(C_2 \times B_2)}{\|B_2 - C_2\|} & \dfrac{(C_3 \times B_3)}{\|B_3 - C_3\|} & \dfrac{(C_4 \times B_4)}{\|B_4 - C_4\|} & \dfrac{(C_5 \times B_5)}{\|B_5 - C_5\|} & \dfrac{(C_6 \times B_6)}{\|B_6 - C_6\|} \end{pmatrix} \quad (70)
$$

where vectors, B_i, C_i ($i=1, 2, \ldots, 6$), respectively denote the vertex vectors of the moving and base platforms with respect to the fixed frame , Fig. 15; $\$_i$ ($i=1, 2, \ldots, 6$) is a line vector of the corresponding extensible link, and its Plücker coordinates are as follows $\$_i=(S_i; S_{Oi})=(L_i, M_i, N_i; P_i, Q_i, R_i)$ where the subscript i ($i=1, 2, \ldots, 6$) indicates the ith limb connected by two vertices B_i, C_i of the moving and base platforms of the manipulator. S_i is a unit vector

specifying the direction of line vector $\$_i$, and S_{Oi} is a vector indicating the position of the line vector together with S_i.

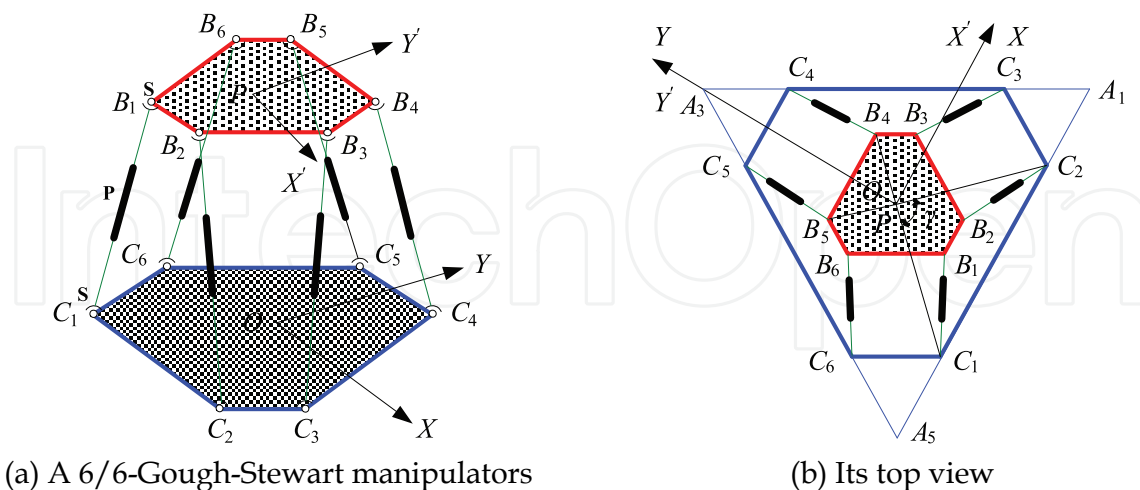(a) A 6/6-Gough-Stewart manipulators   (b) Its top view

Fig. 13. Schematic of a class of the Gough-Stewart manipulators

5.2 Singularity analysis in three-dimensional space

A moving reference frame $P\text{-}X'Y'Z'$ and a fixed one $O\text{-}XYZ$ are respectively attached to the moving platform and the base platform of the manipulator, as shown in Fig. 15, where origins P and O are corresponding geometric center of the moving and base platforms. The position of the moving platform is given by the position of point P with respect to the fixed frame, designated by (X, Y, Z), and the orientation of the moving platform is represented by the standard $Z\text{-}Y\text{-}Z$ Euler angles (ϕ, θ, ψ). Furthermore, geometric parameters of the manipulator can be described as follows. The circumcircle radius of the base hexagon is R_a, and that of the mobile hexagon is R_b. β_0 denotes the central angle of circumcircles of the hexagons corresponding to sides C_1C_2 and B_1B_6, as shown in Fig. 15. The coordinates of six vertices, B_i (i=1, 2, …, 6), of the moving platform are denoted by B_i' with respect to the moving frame, and B_i with respect to the fixed frame. Similarly, C_i and A_j represent coordinates of vertices, C_i (i=1, 2, …, 6) and A_j (j=1, 3, 5), of the base platform with respect to the fixed frame.

Gosselin and Angeles (1990) pointed out that singularities of parallel manipulators could be classified into three different types, i.e., inverse kinematic singularity, direct kinematic singularity and architecture singularity. Here we only discuss the direct kinematic singularity of this class of 6/6-Gough-Stewart manipulators, which occurs when the determinant of the Jacobian matrix of the manipulator is equal to zero, i.e., $\det(J)=\det(J^T)=0$. Expanding and factorizing the determinant of the Jacobian matrix, the singularity locus equation of the manipulator can be written as

$$
\begin{aligned}
& f_1Z^3 + f_2XZ^2 + f_3YZ^2 + f_4X^2Z + f_5Y^2Z + f_6XYZ + f_7Z^2 + f_8X^2 + \\
& f_9Y^2 + f_{10}XY + f_{11}XZ + f_{12}YZ + f_{13}Z + f_{14}X + f_{15}Y + f_{16} = 0
\end{aligned}
\tag{71}
$$

Eq. (71) represents the constant-orientation singularity locus of this class of the Gough-Stewart manipulators in the Cartesian space for a constant orientation (ϕ, θ, ψ). It is a polynomial expression of degree three in the moving platform position parameters XYZ.

Coefficients of Eq. (71), f_i (i=1, 2, ..., 15, 16), are all functions of geometric parameters, R_a, R_b and β_0, and orientation parameters, (ϕ, θ, ψ), of the manipulator.

Graphical representations of the constant-orientation singularity locus of the manipulator for different orientations are given to illustrate the result, as shown in Fig. 14. Geometric parameters used here are given as R_b=2, R_a=1.5, β_0=π/2.
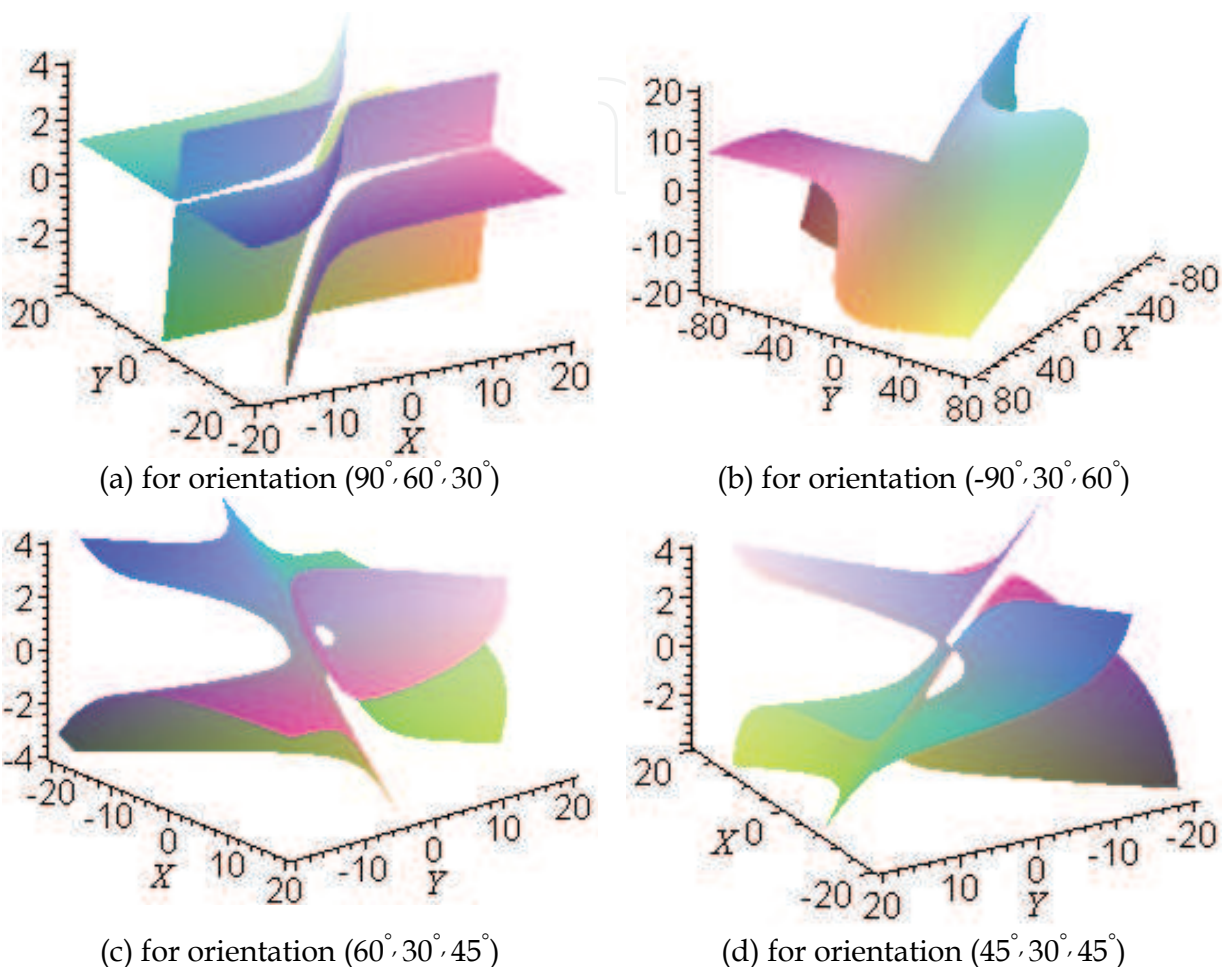

(a) for orientation (90°, 60°, 30°)

(b) for orientation (-90°, 30°, 60°)

(c) for orientation (60°, 30°, 45°)

(d) for orientation (45°, 30°, 45°)

Fig. 14. Singularity loci for different orientations

From Figure 14, it can be clearly seen that the singularity loci for different orientations are quite different, and they are complex and various. Among them, the most complicated graph of the singularity loci looks like a trifoliate surface, whose two branches are of the shape of a horn with one hole (Figure 14 (c) and (d)).

5.3 Singularity analysis in parallel principal-sections
5.3.1 Singularity locus equation in θ-plane

Huang, Chen and Li (2003) pointed out that the cross-sections of the cubic singularity locus equation of the 3/6-GSP in parallel θ-planes are all quadratic expressions that include a parabola, four pairs of intersecting lines and infinite hyperbolas. This conclusion is of great importance for the property identification of the singularity loci of the 3/6-GSP. Similarly, in order to identify the characteristics of singularity loci of this class of the 6/6-GSP, singularity loci of the manipulator in parallel θ-planes will also be discussed in this section. Fig. 16

shows the position of the manipulator for orientation (ϕ, θ, ψ). The oblique plane is θ-plane on which the moving platform lies.
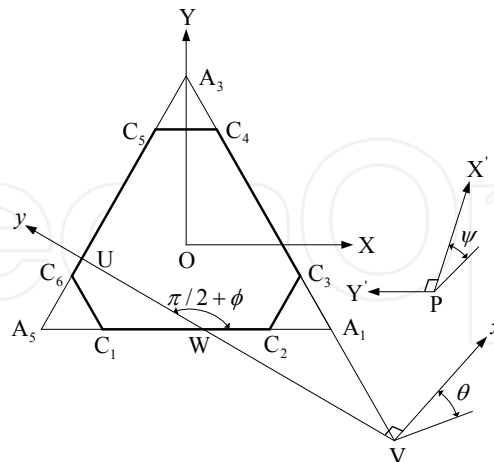

Fig. 15. The position of the manipulator for orientation (ϕ, θ, ψ)

When $\theta \neq 0$, the moving platform is not parallel to the base one. θ-plane intersects the base plane at a line UWV, where points U, W, V are intersecting points between θ-plane and three sides, A_3A_5, A_1A_5, and A_3A_1, of the base hexagon, as shown in Fig. 15. We set another moving reference frame **V**-xy in θ-plane, and the coordinates of point P in this moving frame **V**-xy are denoted by (x, y).

Equations of three lines, A_3A_5, A_1A_5, and A_3A_1, in the fixed frame O-XYZ can be easily written. Owing to space limitations, we do not present these equations here. As point **V**, i.e., origin of the moving frame **V**-xy, lies on line A_3A_1, and it can be assumed that the coordinates of point V with respect to the fixed frame O-XYZ are

$$V : (X_V, \ Y_V, 0)$$

where X_V is a variable indicating the position of θ-plane, i.e., the position of the moving platform for any given geometric and orientation parameters, and Y_V can be established by the following expression

$$Y_V = 2R_a \cos(\beta_0 / 2) - \sqrt{3}X_V \tag{72}$$

So, the equation of line UV can be written as follows

$$Y_{UV} - 2R_a \cos(\beta_0 / 2) + \sqrt{3}X_V = -\cot(\phi)(X_{UV} - X_V) \tag{73}$$

Therefore, coordinates of points U and W can be easily obtained. The coordinates of point P designated by (X, Y, Z) with respect to the fixed frame and (x, y) in the moving frame **V**-xy satisfy the following expression

$$\begin{aligned}
X &= \cos\phi\cos\theta\, x - \sin\phi\, y + X_V \\
Y &= \sin\phi\cos\theta\, x + \cos\phi\, y + Y_V \\
Z &= -\sin\theta\, x
\end{aligned} \tag{74}$$

Substituting Eq. (81) into Eq. (78) and after some rearrangements and factorizations, the singularity locus equation of the manipulator in θ-plane can be written as follows

$$\sin^3\theta\left(ax^2 + 2bxy + cy^2 + 2dx + 2ey + f\right) = 0 \tag{75}$$

Since $\theta\neq0$, the singularity locus equation of the manipulator with respect to θ-plane becomes

$$ax^2 + 2bxy + cy^2 + 2dx + 2ey + f = 0 \tag{76}$$

It can be proved that the coefficient c is always equal to zero, so Eq. (76) is a quadratic polynomial expression with respect to x and y, and the maximum degree of variable x is 2 and y is 1. Coefficients a, b, d, e, f of Eq. (76) are all functions of geometric parameters R_a, R_b and β_0, Euler angles (ϕ, ψ) and X_V. They are all independent of Euler angle θ. Generally, the intersecting curve between a cubic surface and a plane is also a cubic expression that may also contain a closed-loop curve. For example, when R_a=2, R_b=1.5, β_0=п/2, (ϕ, θ, ψ)=(п/3, п/6, п/4), intersecting curves between the corresponding singularity locus surface and the following two planes, Z=-Y/3 and Z=-4(X-14)/45, are respectively presented as follows

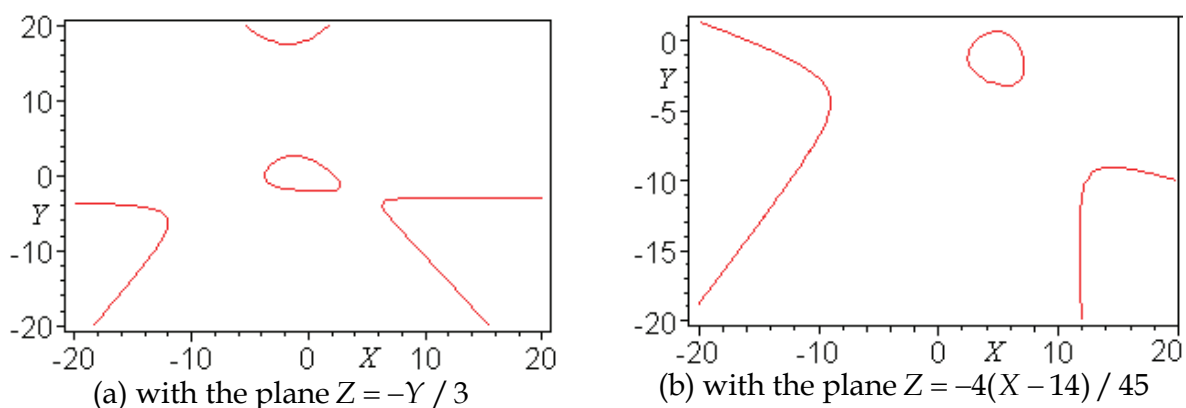(a) with the plane $Z = -Y/3$ (b) with the plane $Z = -4(X - 14)/45$

Fig. 16. Intersecting curves of closed-loop with different sections

Obviously, the intersecting curves between the singularity locus surface of the manipulator and the two aforementioned planes are actually cubic expressions, which contain a closed-loop curve, Fig.18. However, Eq. (76) is always a quadratic polynomial expression; it is worth noting the same conclusion presented above holds for any manipulator of this class of the Gough-Stewart manipulators considered in the current study. Therefore, θ-plane reflects characteristics of the singularity loci of this class of the Gough-Stewart manipulators, and that is why we call it the principal-section.

5.3.2 Property identification of the singularity loci in parallel principal-sections

The property of the singularity loci of the manipulator in parallel principal-sections can be analyzed by two invariants, D and δ, of Eq. (76)

$$\delta = \begin{vmatrix} a & b \\ b & c \end{vmatrix} = ac - b^2 = -b^2 \tag{77}$$

$$D = \begin{vmatrix} a & b & d \\ b & c & e \\ d & e & f \end{vmatrix} = -(ae^2 + fb^2 - 2bde) \tag{78}$$

For any given geometric parameters and orientation parameters, generally, $D \neq 0$ and $\delta < 0$ for general values of X_V, so Eq. (76) indicates a set of hyperbolas shown in Fig. 19. Geometric parameters and orientation parameters used in these examples are given as R_a=2, R_b=1.5, β_0=π/2, (ϕ, θ, ψ)=(π/3, π/6, 0).

Further research shows that for any given geometric parameters, R_a, R_b and β_0, and orientation parameters, (ϕ, θ, ψ), of the manipulator, D is a quartic expression while δ is a quadratic expression with respect to the single variable X_V. Generally, there are four real roots when D=0 and $\delta \neq 0$, and, in each of the four cases, Eq. (76) degenerates into two intersecting lines. For the same reason, there is one real root of multiplicity two when δ=0 and D=0; in this case, Eq. (83) degenerates into a parabola. In order to demonstrate the aforementioned theoretical results, a 6/6-GSP will be studied, whose geometric parameters and the orientation parameters are given as follows: R_a=2, R_b=1.5, β_0=π/2, (ϕ, θ, ψ)=(π/3, π/6, 0). Please note that the following calculations are all based on these parameters.

(a) for orientation $(60°, 30°, 0°)$, $X_V = 0$

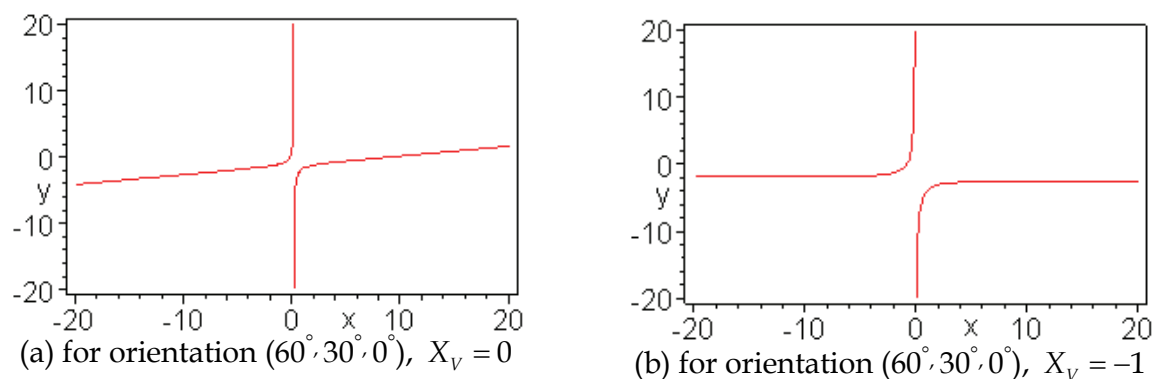(b) for orientation $(60°, 30°, 0°)$, $X_V = -1$

Fig. 17. Singularity loci in parallel principal-planes for general values of X_V
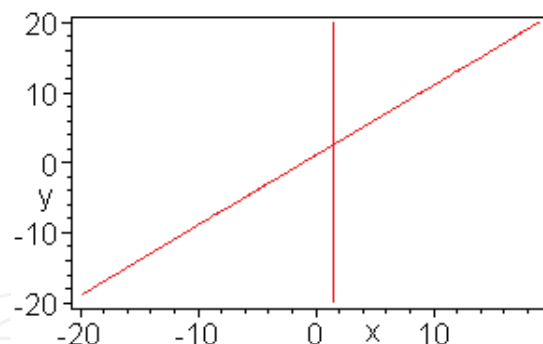

Fig. 18. The first case of two intersecting lines

5.3.2.1 First Case of Two Intersecting Lines

The intersecting line UV passes through point C_3 and then point V coincides with point C_3. When $X_{V1} = (\sqrt{6} + \sqrt{2})/2$, Eq. (76) degenerates into two intersecting lines, as shown in Fig. 18.

$$(8x - 3(\sqrt{6} + \sqrt{2}))(x + k_1 y + c_1) = 0 \qquad (79)$$

5.3.2.2 Second Case of Two Intersecting Lines

The intersecting line UV passes through point C_4 and then point V coincides with point C_4. When

$$X_{V2} = (\sqrt{6} - \sqrt{2}) / 2 \qquad (80)$$

Eq. (76) also degenerates into two intersecting lines

$$(8x - 3(\sqrt{6} - \sqrt{2}))(x + k_2 y + c_2) = 0 \qquad (81)$$

The first part of Eq. (81) is

$$8x - 3(\sqrt{6} - \sqrt{2}) = 0 \qquad (82)$$

which is a line parallel to y-axis. Meanwhile, it can be proved that point B_1 is located on the base plane. Similarly, when B_1 coincides with C_4, the singularity of this point is of the first special-linear-complex singularity. Similarly, when B_1 does not coincide with C_4, singularities corresponding to points lying in line of Eq. (82) are of the general-linear-complex singularity.

The second part of Eq. (81) denotes another line. Singularities corresponding to points lying in this line are all of the general-linear-complex singularity.

5.3.2.3 Third Case of Two Intersecting Lines

When $X_{V3} = 3(\sqrt{6} + \sqrt{2}) / 2$, D=0 and $\delta \neq 0$, Eq. (76) degenerates into two intersecting lines

$$(8x + (9\sqrt{2} + 3\sqrt{6}))(x + k_3 y + c_3) = 0 \qquad (83)$$

5.3.2.4 The Fourth Case of Two Intersecting Lines

When $X_{V4} = (11\sqrt{6} - 3\sqrt{2}) / 12$, D=0 and $\delta \neq 0$, Eq. (76) degenerates into two intersecting lines

$$(24x - (15\sqrt{2} + 5\sqrt{6}))(x + k_4 y + c_4) = 0 \qquad (84)$$

The first part of Eq. (84) is

$$24x - (15\sqrt{2} + 5\sqrt{6}) = 0 \qquad (85)$$

which is a line parallel to y-axis of frame **V**-xy. In particular, there are three special points at which all the segments associated with the six extensible links of the manipulator intersect one common line, respectively

5.3.2.5 One Case of a Parabola

When $X_{V5} = (7\sqrt{6} + 3\sqrt{2}) / 6$, δ=0 and $D \neq 0$, Eq. (76) degenerates into a parabola, as shown in Fig. 23

$$(528\sqrt{6} - 912\sqrt{2})x^2 + (1423\sqrt{3} - 2472)x + (504 - 288\sqrt{3})y + 513\sqrt{6} - 909\sqrt{2} = 0 \qquad (86)$$

The manipulator is always singular corresponding to points lying in the parabola. Similarly, there are three special points at which all the segments associated with the six extensible links of the manipulator intersect one common line.

Based on the analyses described above, it can be concluded that the singularity loci of this class of the 6/6-Gough-Stewart manipulators in parallel principal-sections are always

quadratic expressions that generally include infinite hyperbolas. However, for four parallel locations of the principal-section, the quadratic expression degenerates into two intersecting lines respectively, and in one location the quadratic expression is a parabola when $\theta \neq 0$.
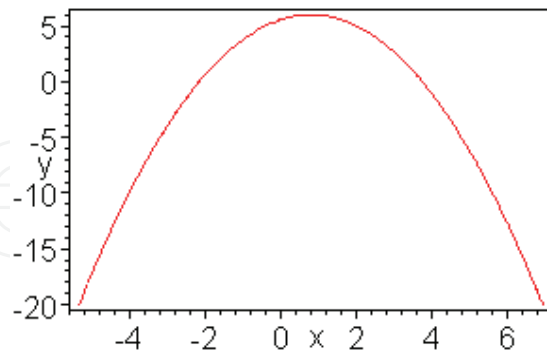

Fig. 19. One case of a parabola

5.3.3 Singularity analysis when $\theta=0$

When $\theta=0$, the moving platform is parallel to the base one. Meanwhile, Eq. (94) can be reduces as follows

$$Z^3 \cos(\phi + \psi) = 0 \tag{87}$$

1. When $Z=0$, the moving and the base platforms are coincident. In this special configuration, the manipulator has three DOFs: two rotational freedoms and one translational freedom.
2. When $(\phi + \psi) = \pm 90°$, for the 6/6-Gough-Stewart manipulator, it is the singularity proposed by Fichter (1986).

As we have discussed, the singularity loci of this class of the 6/6-Gough-Stewart manipulators in parallel principal-sections include infinite hyperbolas, four cases of two intersecting lines, and one case of a parabola when $\theta=0$.

From analytic geometry, there are five different types of quadric surface with hyperbola sections: hyperbolic cylinder, hyperbolic paraboloid, hyperboloid of one sheet, hyperboloid of two sheets, and conic surface. However, none of these can contain infinite hyperbolas, one case of a parabola, and four cases of two intersecting lines simultaneously. Therefore, the singularity locus equation of this class of 6/6-Gough-Stewart manipulators considered in three-dimensional space is a special irresolvable polynomial expression of degree three, whose cross-sections in parallel principal-sections contain one case of a parabola, four cases of two intersecting lines and infinite hyperbolas.

6. Conslusion

6.1 A necessary and sufficient condition that the three velocities of three non-collinear points in a rigid body can reflect that the body has a possible twist motion is that the intersecting point of three normal planes of three velocities lies in the plane determined by the three points. This is also the necessary and sufficient condition of occuring singularity for a parallel mechanism, when all actuators are locked.
6.2 Based on the singularity kinematics principle and Singularity-Equivalent-Mechanism method, the structure and property of the singularity surface of 3/6-Gough-Stewart platform for all different orientations (ϕ, θ, ψ)can be finally concluded as follows

6.2.1 When θ=0, the mobile platform and the base one are parallel with each other. The special configuration of the mechanism is (90°, 0, 0)which is proposed by Fichter (1986). The mechanism is singular whatever the position of the mobile platform is. It belongs to the general-linear-complex singularity.

6.2.2 When θ=0, Z=0, the mobile platform and the base one are coincident. The mechanism is singular whatever the other two Euler angles are. The mechanism has three DOF: two rotational freedoms and one translational freedom. They belong to the first or the second special-linear-complex singularity, respectively (Huang; Chen; Li 2003).

6.2.3 When θ≠0, ϕ≠±30°, ±90°, and ±150° and ψ≠±30°, ±90°, and ±150°, the singularity loci for this orientation include a plane (Hunt plane) and a hyperbolic paraboloid (Huang; Chen; Li 2003). The intersecting line UV between the mobile platform and the base one is parallel to some side of the basic triangle $A_1A_3A_5$.

6.2.4 When θ≠0, ϕ≠±30°, ±90°, and ±150° and ψ=±30°, ±90°, or ±150°, the singularity loci are three intersecting planes (Huang; Chen; Li 2003).

6.2.5 When θ≠0, ϕ≠±30°, ±90°, and ±150° and ψ=±30°, ±90°, or ±150°, the singularity loci for this orientation also include a plane and a hyperbolic paraboloid (Huang; Chen; Li 2003).

6.2.6 When θ=0, neither ϕ nor ψ is equal to any one of the angles, ±30°, ±90°, ±150°. This is the most general case for 3/6-GSP. The singularity equation for this orientation is a special irresolvable polynomial expression of degree three, and the structure of the singularity loci in infinite parallel principal sections includes a parabola, four pairs of intersecting straight lines and infinity of hyperbolas (Huang and Cao 2006).

6.2.7 There is only one instantaneous freedom forming a twist with h^m≠0, appearing when 3/6-GSP is singular for infinite general orientations. But there are seven special situations in which the instantaneous motion is a pure rotation with h^m=0. However, under the case θ=0, Z=0 the instantaneous motion is a pure translation with h^m=∞.

6.2.8 According to the singularity-equivalent-mechanism, the singularity can occur at any point all over the θ-plane. That is to say, the singularity can occur everywhere in the workspace of the mechanism. But the singularity orientation of the platform in different points may be different.

6.2.9 The planar section parallel to the mobile platform is named principal section of the singularity surface. The singularity expressions in infinite parallel principal sections are always quadratic.

6.3 The structure and property of the singularity loci of this class of 6/6-Gough-Stewart manipulators for all different orientations can be finally concluded as follows (Huang and Cao 2005)

6.3.1 The singularity locus equation of degree three, is of a special irresolvable polynomial expression whose cross-sections in parallel principal-sections contain one parabola, four pairs of intersecting lines, and infinite hyperbolas.

6.3.2 The graphical representations of the singularity locus of this class of the 6/6-Gough-Stewart manipulators are quite complex and various for different orientations. The most complex graphic of the singularity loci looks like a trifoliate surface with two holes.

6.3.3 We find an incredible phenomenon, for this class of the 6/6-Gough-Stewart manipulators, there are also some special singularity cases where six lines associated with the six extensible links of the manipulator can intersect one common line and the remnant motion of the manipulator is a pure rotational motion. Even for the same orientation of the manipulator, there are two or more positions of the manipulator at which the six lines of the manipulator all intersect one common line simultaneously.

6.3.4 When θ=0, Z=0, the manipulator also has three remnant DOFs: two rotational freedoms and one translational freedom.

7. Acknowledgment

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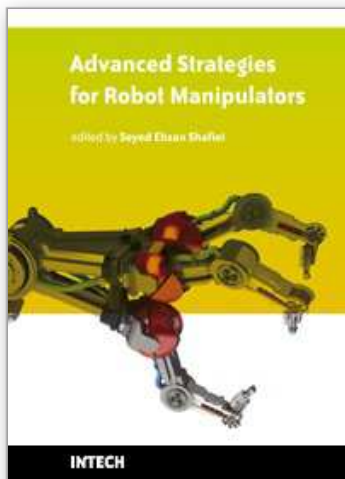
Advanced Strategies for Robot Manipulators

Edited by S. Ehsan Shafiei

Amongst the robotic systems, robot manipulators have proven themselves to be of increasing importance and are widely adopted to substitute for human in repetitive and/or hazardous tasks. Modern manipulators are designed complicatedly and need to do more precise, crucial and critical tasks. So, the simple traditional control methods cannot be efficient, and advanced control strategies with considering special constraints are needed to establish. In spite of the fact that groundbreaking researches have been carried out in this realm until now, there are still many novel aspects which have to be explored.

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