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Dynamic Modelling, Tracking Control and Simulation Results of a Novel Underactuated Wheeled Manipulator (WAcrobot)

Mohsen Moradi Dalvand and Bijan Shirinzadeh
Robotics and Mechatronics Research Laboratory (RMRL), Department of Mechanical and Aerospace Engineering, Monash University, Clayton, Victoria Australia

1. Introduction

Being an inherently open loop unstable mechanical system with highly nonlinear dynamics and with the number of actuators less than the number of degrees of freedom, the inverted pendulum system is a perfect benchmark for the design of a wide range of classical and contemporary control techniques. There are a number of different versions of the inverted pendulum systems offering a variety of control challenges. The most common types are the single inverted pendulum on a cart (Ohsumi & Izumikawa, 1995; Åström & Furuta, 2000; Yoshida, 1999), the double inverted pendulum on a cart (Zhong & Rock, 2001), the double inverted pendulum with an actuator at the first joint only (Pendubot) (Spong, 1996; Graichen & Zeitz, 2005; Fantoni et al., 2000), the double inverted pendulum with an actuator at the second joint only (Acrobot) (Spong, 1994; 1995; Hauser & Murray, 1990), the rotational single-arm pendulum (Furuta et al., 1991; 1992) and the rotational two-arm pendulum (Yamakita & Furuta, 1999). Beyond non-mobile inverted pendulum robots, wheeled inverted pendulum robots or commonly known as balancing robots (e.g., Segway (Browning et al., 2005), Quasimoro (Salerno & Angeles, 2003), and Joe (Grasser et al., 2002)) have induced much interests by researchers.

The control techniques involved in various types of inverted pendulum systems are also numerous, ranging from simple conventional controllers to advanced control techniques based on modern nonlinear control theory. A vast range of contributions exists for the stabilization of different types of inverted pendulums (Mori et al., 1976; Chaturvedi et al., 2008; Angeli, 2001). Besides the stabilization aspect, the swing-up of various types of single and double inverted pendulum(s) is also addressed in the literature. Examples include classic single pendulum on a cart (Åström et al., 2008; Åström & Furuta, 2000), Acrobot and Pendubot (Fantoni et al., 2000; Spong, 1994; 1995; Graichen et al., 2007; Brown & Passino, 1997) and the rotary double inverted pendulum (Yamakita et al., 1993; 1995). In addition to the stabilization and swing-up of different kinds of inverted pendulum robots, trajectory tracking of these underactuated systems has gained attention by researches (Cho & Jung, 2003; Chanchareon et al., 2006; Hung et al., 1997; Magana & Holzapfel, 1998). There are two major approaches to construct the trajectory tracking controller for such nonlinear systems. The first one is based on system inversion (Devasia et al., 1996; Wang & Chen, 2006) and the

second approach is based on output regulation theory (Isidori & Byrnes, 1990; Qian & Lin, 2002; Hirschorn & Aranda-Bricaire, 1998). Extensive controller developments have also been achieved by researchers for mobile inverted pendulum robots over the last decade (Salerno & Angeles, 2007; Pathak et al., 2005; Tsuchiya et al., 1999).

This chapter studies a novel underactuated wheeled manipulator (WAcrobot) comprising an underactuated 2-DOF planar manipulator or an unstable double inverted pendulum (Acrobot) combined with a balancing robot. The WAcrobot has two independent driving wheels in same axis, and two gyro type sensors to determine the inclination angular velocity of two arms and rotary encoders to know wheels and arms rotation individually. Due to its configuration with two coaxial wheels, each wheel is coupled to a geared dc motor. The manipulator is able to do stationary U-turns while keeping balance and manipulating. Such manipulator is of interest because it has a small foot-print and can turn on dime. The design, dynamic modeling and tracking control of this novel mobile manipulator is discussed in this chapter for the first time. This chapter aims at achieving three different types of trajectory tracking control tasks for *a*) wheels, *b*) first or second arm and *c*) wheels and one of the arms simultaneously, while the WAcrobot stabilization is guaranteed by the system internal equilibria calculation. The tracking controller is designed using the Gain Scheduling method that is based on the idea of the linearisation of the system equations around certain operating points and design of a linear controller for each region of operation (Lawrence & Rugh, 1993; Shamma & Athans, 1990a)]. For the design of the linear controller, we consider the Linear Quadratic Regulator (LQR) model to stabilize the WAcrobot around any point over the equilibrium manifold. We verified the effectiveness of the designed control system via numerical simulation visualized by graphical simulation to illustrate the physical response of the WAcrobot.

In the following sections of this chapter the dynamic model of the wheeled manipulator (WAcrobot) is firstly presented. Then the equilibrium manifold of the WAcrobot is investigated. After that the stabilization controller based on LQR technique is proposed. Then by employing Gain Scheduling method, for any given trajectory of wheels and/or arm(s), the trajectories of the rest of DOF of the WAcrobot is determined such that during the trajectory tracking the WAcrobot system is stabilized. Numerical and graphical simulations for three types of tracking control tasks are given to show the effectiveness of the proposed scheme.

2. Dynamics of WAcrobot

The mechanism of the WAcrobot is shown in Figure 1 schematically. The WAcrobot (Wheeled Acrobot) is an underactuated mechanical system consisting of an underactuated planar manipulator (Acrobot), a double inverted pendulum robot with an actuator at the second joint only (Figure 1-a), which is combined with a balancing robot (Figure 1-b) or equipped with two actuated wheels and has the capability to be as an underactuated wheeled manipulator. The mathematical model of the WAcrobot can be derived using the Euler-Lagrange equation. The form of the Euler-Lagrangian equation used here is:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}}\right] - \frac{\partial L}{\partial q} = \tau \tag{1}$$

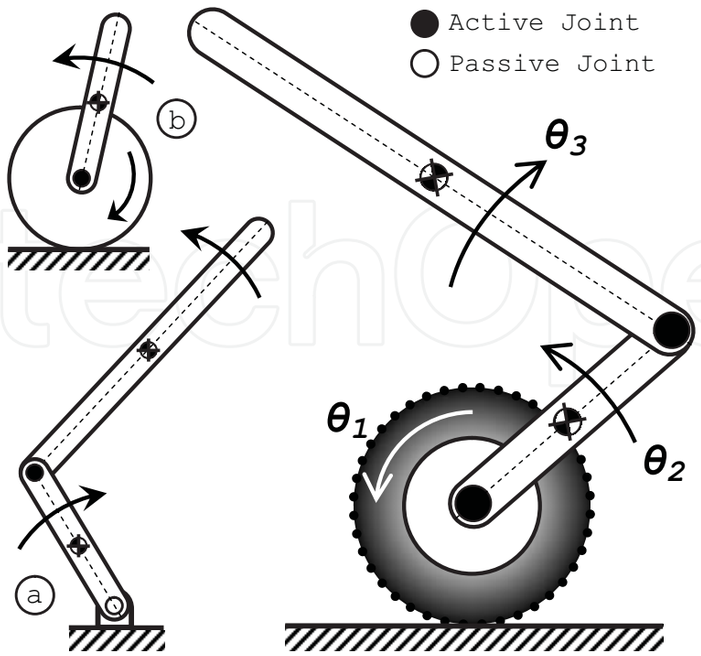

Fig. 1. WAcrobot, Acrobot (a) and Wheeled Inverted Pendulum (b)

where $L = T - V$ is a Lagrangian, T is kinetic energy, V is potential energy, $\tau = [\tau_1\ 0\ \tau_2]^T$ is the input generalized force vector produced by two actuators at wheels and second arm, $q = [q_1\ q_2\ q_3]^T \epsilon\ \mathbb{R}^3$ is generalized coordinate vector which is selected as $q = [\theta_1\ \theta_2\ \theta_3]^T$ where θ_1, θ_2 and θ_3 are angular positions of wheels, first arm, and second arm of the WAcrobot, respectively. The kinetic and potential energies of the WAcrobot's components in terms of generalized coordinates can be determined as:

$$
\begin{aligned}
&{}^1/_2(I_1 + (m_1 + m_2 + m_3)l_1^2)\dot{\theta}_1^2 + {}^1/_2(m_3 l_{c3}^2 + I_3)\dot{\theta}_3^2 + m_3 l_{c3} l_2 \dot{\theta}_2 \dot{\theta}_3 cos(\theta_3) \quad = \quad T \qquad (2)\\
&+{}^1/_2\,(2m_3 cos(\theta_3) l_2 l_{c3} + m_2 l_{c2}^2 + m_3 l_2^2 + m_3 l_{c3}^2 + I_2 + I_3)\dot{\theta}_2^2 + m_3 l_{c3} l_{c3} \dot{\theta}_2 \dot{\theta}_3\\
&+l_1((m_2 l_{c2} + m_3 l_2)cos(\theta_2) + m_3 l_{c3} cos(\theta_2 + \theta_3))\dot{\theta}_1 \dot{\theta}_2 + m_3 l_1 l_{c3} cos(\theta_2 + \theta_3)\dot{\theta}_1 \dot{\theta}_3\\
&(m_1 + m_2 + m_3) l_1 g + m_3 l_{c3} cos(\theta_2 + \theta_3) g + (m_2 l_{c2} + m_3 l_2) cos(\theta_2) g \quad = \quad V \qquad (3)
\end{aligned}
$$

Differentiating the Lagrangian ($L = T - V$) by generalized coordinate vector θ and $\dot{\theta}$ yields Euler-Lagrange Equation (1) as:

$$
\begin{aligned}
&((m_1 + m_2 + m_3)l_1^2 + I_1)\ddot{\theta}_1 + l_1(l_{c3} m_3 cos(\theta_2 + \theta_3) + cos(\theta_2)(m_2 l_{c2} + m_3 l_2))\ddot{\theta}_2 \quad = \quad \tau_1 \qquad (4)\\
&+m_3 l_1 l_{c3} cos(\theta_2 + \theta_3)\ddot{\theta}_3 - l_1(l_{c3} m_3 sin(\theta_2 + \theta_3) + (l_2 m_3 + l_{c2} m_2) sin(\theta_2))\dot{\theta}_2^2\\
&-l_1 l_{c3} m_3 sin(\theta_2 + \theta_3)\dot{\theta}_3^2 - 2 l_1 l_{c3} m_3 sin(\theta_2 + \theta_3)\dot{\theta}_2 \dot{\theta}_3
\end{aligned}
$$

$$l_1(l_{c3}m_3cos(\theta_2+\theta_3)+(m_2l_{c2}+m_3l_2)cos(\theta_2))\ddot{\theta}_1+m_3l_{c3}(l_{c3}+l_2cos(\theta_3))\ddot{\theta}_3 \quad = \quad 0 \tag{5}$$

$$+(m_2l_{c2}^2+m_3(l_{c3}^2+l_2^2)+I_2+I_3+2m_3l_2l_{c3}cos(\theta_3))\ddot{\theta}_2-m_3l_2l_{c3}sin(\theta_3)\dot{\theta}_3^2$$

$$-(m_3l_{c3}sin(\theta_2+\theta_3)+(m_3l_2+m_2l_{c2})sin(\theta_2))g-2m_3l_2l_{c3}sin(\theta_3)\dot{\theta}_2\dot{\theta}_3$$

$$l_1l_{c3}m_3cos(\theta_2+\theta_3)\ddot{\theta}_1+m_3l_{c3}(l_{c3}+l_2cos(\theta_3))\ddot{\theta}_2+(m_3l_{c3}^2+I_3)\ddot{\theta}_3 \quad = \quad \tau_2 \tag{6}$$

$$+l_2l_{c3}m_3sin(\theta_3)\dot{\theta}_2^2-l_{c3}m_3sin(\theta_2+\theta_3)g$$

Equations (4), (5) and (6) can be put into the frequently used compact form (Spong & Block, 1995):

$$M(\theta)\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+G(\theta)=\tau \tag{7}$$

where $\theta=[\theta_1\ \theta_2\ \theta_3]^T\in\mathbb{R}^3$ is the generalized coordinate vector, $M(\theta)\in\mathbb{R}^{3\times3}$ is the symmetric positive definite inertia matrix, $C(\theta,\dot{\theta})\,\dot{\theta}\in\mathbb{R}^3$ contains Coriolis and centrifugal terms, $G(\theta)\in\mathbb{R}^3$ contains gravitational terms and $\tau=[\tau_1\,0\ \tau_2]^T$ is the input generalized force vector. Furthermore,

$$M(q)=\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}, \tag{8}$$

where

$$\begin{aligned}
M_{11} &= (m_1+m_2+m_3)l_1^2+I_1 \\
M_{12} &= M_{21}=l_1(l_{c3}m_3cos(\theta_2+\theta_3)+cos(\theta_2)(m_2l_{c2}+m_3l_2)) \\
M_{13} &= M_{31}=m_3l_1l_{c3}cos(\theta_2+\theta_3) \\
M_{22} &= m_2l_{c2}^2+m_3(l_{c3}^2+l_2^2)+I_2+I_3+2m_3l_2l_{c3}cos(\theta_3) \\
M_{23} &= M_{32}=m_3l_{c3}(l_{c3}+l_2cos(\theta_3)) \\
M_{33} &= m_3l_{c3}^2+I_3
\end{aligned}$$

and

$$C(\theta,\dot{\theta})\dot{\theta}=[H_1\,H_2\,H_3]^T \tag{9}$$

where

$$\begin{aligned}
H_1 &= -l_1(l_{c3}m_3sin(\theta_2+\theta_3)+(l_2m_3+l_{c2}m_2)sin(\theta_2))\dot{\theta}_2^2 \\
&\quad -l_1l_{c3}m_3sin(\theta_2+\theta_3)\dot{\theta}_3^2-2l_1l_{c3}m_3sin(\theta_2+\theta_3)\dot{\theta}_2\dot{\theta}_3 \\
H_2 &= -2m_3l_2l_{c3}sin(\theta_3)\dot{\theta}_2\dot{\theta}_3-m_3l_2l_{c3}sin(\theta_3)\dot{\theta}_3^2 \\
H_3 &= +l_2l_{c3}m_3sin(\theta_3)\dot{\theta}_2^2
\end{aligned}$$

and

$$G(\theta) = [G_1\, G_2\, G_3]^T \tag{10}$$

where

$$G_1 = 0$$

$$G_2 = -(m_3 l_{c3} sin(\theta_2 + \theta_3) + (m_3 l_2 + m_2 l_{c2}) sin(\theta_2))g$$

$$G_3 = -l_{c3} m_3 sin(\theta_2 + \theta_3)g$$

and g is the gravitational acceleration. The parameters of the WAcrobot are defined in Table 1. Equation (7) represents the underactuated and nonlinear system of the WAcrobot including two input torques applied to wheels and second arm (τ_1 and τ_2), two active DOFs (θ_1 and θ_3) and one passive DOF (θ_2).

θ_i (i = 1,2, 3)	Angular rotation of the wheels and arms
m_i (i = 1,2, 3)	Mass of wheels and arms
l_{ci}(i = 2, 3)	Length from the joint to the center of the gravity of the arms
l_i(i = 1, 2,3)	Radius of the wheels and length of arms
I_i(i = 1,2,3)	Inertia moment around the center of gravity

Table 1. Definition of Parameters

3. Tracking control

The tracking controller of the WAcrobot is designed using the Gain Scheduling method based on the linearisation of the system equations around certain equilibrium points in a first stage followed by the design of a linear controller for each region of tracking operation in a second stage. For the design of the linear controller, we consider the Linear Quadratic Regulator (LQR) model to stabilize the WAcrobot around any operating point over the equilibrium manifold.

3.1 Equilibrium manifold

Underactuated mechanical systems generally have equilibria which depend on both their kinematic and dynamic parameters (Bortoff & Spong, 1992). In these systems, to track a trajectory while balancing is guaranteed, it is vital to consider the equilibrium manifold. Beyond the unforced equilibria of the WAcrobot, $(\theta, \dot{\theta}) = (\theta_1, \pi, 0, 0, 0, 0)$ (lower or pendent equilibrium) and $(\theta, \dot{\theta}) = (\theta_1, 0, 0, 0, 0, 0)$ (upper or inverted equilibrium), it has a manifold of forced equilibrium points. Generally the WAcrobot is at rest or particularly at equilibrium point whenever $\theta_{1_{eq}}, \dot{\theta}_{2_{eq}}$ and $\dot{\theta}_{3_{eq}}$ are zero and the joint torque $\tau_{eq} = [\tau_{1eq}\ 0\ \tau_{2eq}]^T$ is such that to equalize $G(\theta)$ in Equation (7). So this set of equilibrium points consists of all states where

$$\ddot{\theta}_{eq} = \dot{\theta}_{eq} = 0 \tag{11}$$

$$G(\theta_{eq}) = \tau_{eq} \tag{12}$$

If the outputs that are required to track a trajectory include the first arm, it follows from Equations (7), (10), (11) and (12) that:

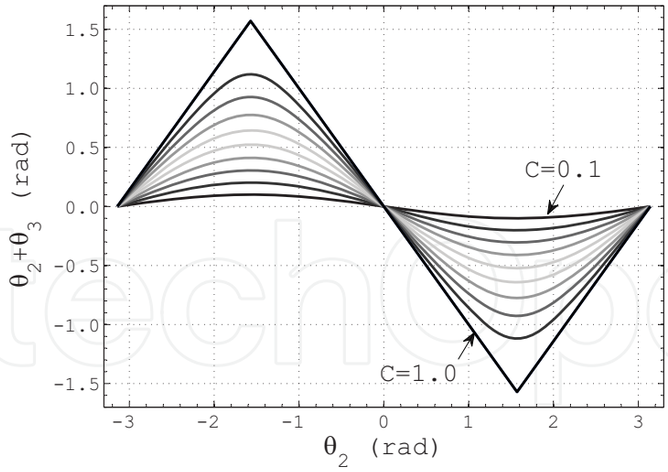

Fig. 2. Equilibrium set points according to the different values of parameter C

$$\tau_{1_{eq}} = 0$$

$$\tau_{2_{eq}} = (m_3 l_2 + m_2 l_{c2}) g sin(\theta_{2_{eq}}) \tag{13}$$

$$\theta_{3_{eq}} + \theta_{2_{eq}} = arcsin[-(\frac{m_3 l_2 + m_2 l_{c2}}{m_3 l_{c3}}) sin(\theta_{2_{eq}})] \tag{14}$$

Since the value of the absolute angular position of the second arm with respect to the vertical direction $\theta^a_{3_{eq}} = \theta_{3_{eq}} + \theta_{2_{eq}}$ cannot be imaginary, the condition for the existence of the equilibrium from Equation (14) may be written:

$$C = \frac{m_3 l_2 + m_2 l_{c2}}{m_3 l_{c3}} \leq 1 \tag{15}$$

Figure 2 illustrates equilibrium set points derived from Equation (14) according to different values of parameter C from Equation (15). It demonstrates how value of parameter C affects $\theta^a_{3_{eq}}$ corresponding to any given $\theta_{2_{eq}}$ in which the WAcrobot is stabilized. By decreasing parameter C, the required $\theta^a_{3_{eq}}$, corresponding to the desired $\theta_{2_{eq}}$ to stabilize the robot, decreases and to decrease parameter C, the second arm should be long and heavy which is not suitable. On the other hand, for any given angular position of the first arm, if the second arm is long and heavy, it needs smaller angular changes to stabilize the WAcrobot and vice versa. Therefore there needs to be a trade-off between the ranges of the rotational motions of arms and the volume and weight of the WAcrobot. In particular, for any given m_3, if the specification of the first arm (m_2, l_2 and l_{c2}) are given, Equation (15) is only true if $l_{c3} \geq (m_3 l_2 + m_2 l_{c2})/m_3$ and for any given l_{c3}, it is only true if $m_3 \geq m_2 l_{c2}/(l_{c3} - l_2)$. Considering $l_2 = 2l_{c2}$ and $l_3 = 2l_{c3}$, we can simplify Equation (15) as:

$$\frac{l_3}{l_2} \geq (2 + \frac{m_2}{m_3}) \tag{16}$$

From the other point of view, if the trajectory tracking of the second arm is desired, it follows from Equations (7), (10), (11) and (12) that:

$$\tau_{1_{eq}} = 0$$

$$\tau_{2_{eq}} = -m_3 l_{c3} g sin(\theta^a_{3_{eq}}) \tag{17}$$

$$\theta_{2_{eq}} = arcsin[-(\frac{m_3 l_{c3}}{m_3 l_2 + m_2 l_{c2}})sin(\theta^a_{3_{eq}})] \tag{18}$$

Since the value of the angular position of the first arm ($\theta_{2_{eq}}$) cannot be imaginary, the condition for the existence of the equilibrium from Equation (18) is:

$$C^{-1} = \frac{m_3 l_{c3}}{m_3 l_2 + m_2 l_{c2}} \leq 1 \tag{19}$$

Considering $l_2 = 2l_{c2}$ and $l_3 = 2l_{c3}$, we can simplify Equation (19) as:

$$\frac{l_3}{l_2} \leq (2 + \frac{m_2}{m_3}) \tag{20}$$

3.2 Stabilization

The balancing controller is designed using the well known Linear Quadratic Regulator (LQR) method based on the linearised plant model around any equilibrium point. The LQR is a controller for state variable feedback in such a way that $u = -Kx$ is the input so that the value of K is obtained from minimization of the cost function $J = \int_0^\infty (x'Qx + u'Ru)dt$ where matrix Q and R are positive semidefinite matrix and symmetric positive definite matrix that penalize the state error and the control effort, respectively.

3.3 Gain scheduling

Jacobian linearisation or linearisation about an equilibrium point is the technique for transforming original system models into equivalent models with simpler form. Since the linearization is about a single point, trajectory tracking can only be guaranteed in a sufficiently small region of states about that point. There are several methods for circumventing this problem; one of the most common is Gain Scheduling (Shamma & Athans, 1990b). Control of nonlinear systems by Gain Scheduling is based on the idea of the linearising the system equations around certain operating points, and the design of a linear controller for each region of operation over the entire motion envelope (Cloutier et al., 1996; Dorato et al., 1994; Langson, 1997). The controller coefficients are varied continuously according to the value of the scheduling variable. In fact, this can be performed in a more or less continuous fashion using a technique called extended linearisation (Baumann & Rugh, 1986).

In broad terms, according to (WJ & Shamma, 2000), the design of a gain scheduled controller for nonlinear plant of the WAcrobot can be described with a six-step procedure, though

various technical methods are available in each step. The first step involves finding $\theta_{3_{eq}}^{a}$ or $\theta_{2_{eq}}$ in all operating points, for each desired $\theta_{2_{eq}}$ or $\theta_{3_{eq}}^{a}$, using Equations 14 or 18 respectively. The second step is the calculation of the joint torque required to keep the WAcrobot at desired $\theta_{2_{eq}}$ (or $\theta_{3_{eq}}^{a}$) and calculated $\theta_{3_{eq}}^{a}$ (or $\theta_{2_{eq}}$). The third step is the computation of a linear parameter varying model for the plant. The most common approach is to linearise the nonlinear plant around a selection of equilibrium points. This results in a family of operating points. The fourth step is to design a family of controllers for the linearised models in each operating point. Because of the linearised model, linear controller design methods such as LQR can be used to stabilize the system around the operating point. The fifth step is the actual Gain Scheduling. Gain Scheduling involves the implementation of the family of linear controllers such that the controller coefficients are scheduled according to the current value of the scheduling variables which are $\theta_{2_{eq}}$ or $\theta_{3_{eq}}^{a}$. The last step is the performance assessment that can be performed analytically or by using extensive computational analysis and simulation.

3.4 Computational analysis and simulation

In order to verify the validity of the Gain Scheduling method for trajectory tracking of different types of reference trajectories in the WAcrobot, we carried out computational analyses and visual simulations using *MATLAB/Simulink®* package integrated with *ADAMS®* simulation software. Three types of tracking control tasks for wheels and/or arm(s) have been evaluated which are presented in this section. The simulations are performed with the following parameters given in Table 2.

Wheels/Arms	Wheels	First arm	Second arm
$m_i\,[kg]$	1.22	0.28	0.72
$l_i\,[m]$	0.05	0.15	0.45
$l_{ci}\,[kg]$	—	0.075	0.225
$I_i\,[kg.m^2]$	1.53E-003	5.98E-004	1.3138E-002

Table 2. Parameters of the WAcrobot

In table 2, parameter *l*, for wheels, means radius while for arms means length. From Equation (15) and Table 2, we obtain C=0.763. It is supposed that the WAcrobot starts the trajectory tracking from its unforced inverted equilibrium position. Therefore the initial conditions are as follow:

$$\theta_1 = 0 \quad \theta_2 = 0 \quad \theta_3 = 0 \quad \dot{\theta}_1 = 0 \quad \dot{\theta}_2 = 0 \quad \dot{\theta}_3 = 0 \quad \tau_1 = 0 \quad \tau_2 = 0$$

Q and R in the optimal regulators for simulations are designed as:

$$Q = diag([10, 100, 100, 0, 0,0])$$

$$R = diag([0.1, 0.1])$$

It must be noted that in order to have better sense of motion, the angular position and velocity of the second arm are considered as absolute states and are plotted with respect to the vertical direction not relative to the first arm.

3.4.1 Wheels tracking

The control objective is that the wheels to follow a trajectory with linear segments and parabolic blends while both arms are balancing inverted close to their initial positions. Practically this task is that the WAcrobot smoothly starts moving at x = 0 (m) and gently stops at x = 1.5 (m) while both arms are stabilized during the movement. Figure 3 shows the computational analysis results. In this figure the responses of angular position and velocity of the wheels and arms as well as applied torques to actuated DOFs are shown respectively. Tracking errors are calculated for the linear position and velocity of the WAcrobot, as shown in Figure 4. It should be noted that this control task can be defined as tracking problem for wheels while the arms, instead of being at inverted position, are at any point over the equilibrium manifold. Assume that the WAcrobot is balanced while the first arm is at 1 (*rad*). In this case the absolute angular position of the second arm and the input torque for the second joint required to keep the arms balanced at the specified angular positions, are calculated using Equations (13) and (14). Therefore the initial conditions for this simulation are as follow:

$$\theta_1 = 0 \quad \theta_2 = 1 \quad \theta_3 = -0.7343 \quad \dot{\theta}_1 = 0 \quad \dot{\theta}_2 = 0 \quad \dot{\theta}_3 = 0 \quad \tau_1 = 0 \quad \tau_2 = -1.065$$

Figure 5 demonstrates a superimposed snapshot of the graphical simulation for two tracking problems of the WAcrobot's wheels while the arms are at the inverted position and are at another point over the equilibrium manifold. It is clear from both numerical and graphical simulations that the WAcrobot's wheels track a specified trajectory while both arms are close to the inverted position or a defined position over the equilibrium manifold at all times during the movement.

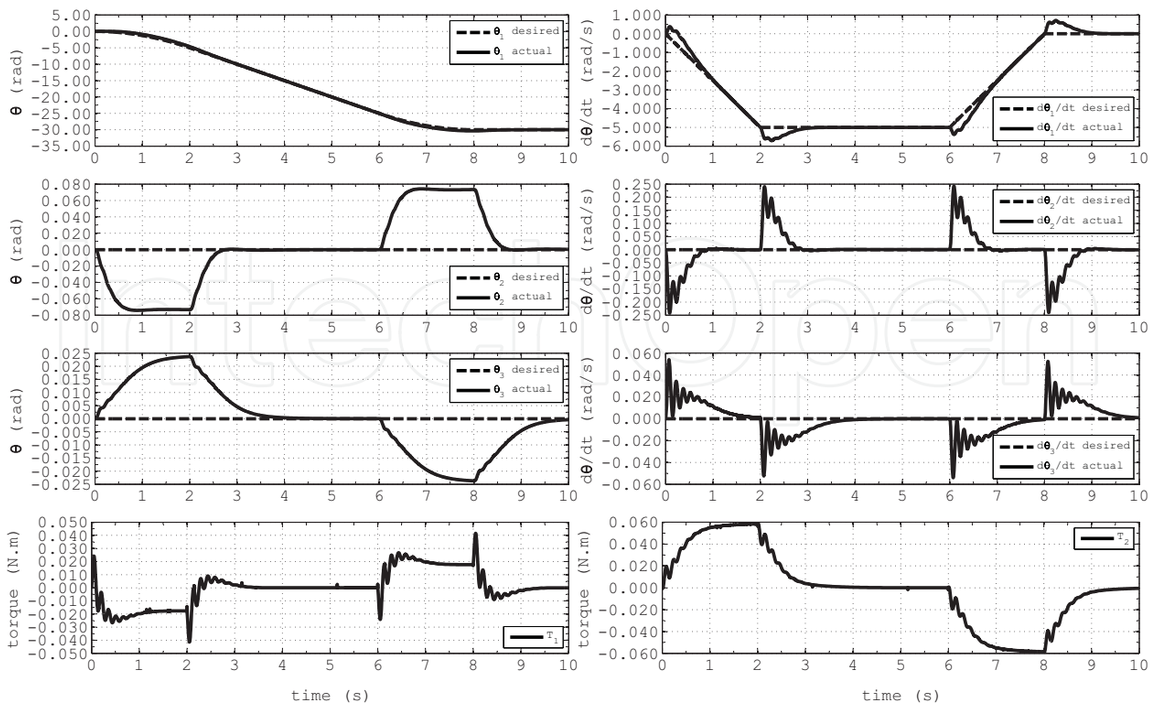

Fig. 3. The simulation responses of positions, velocities and torques for the wheels tracking

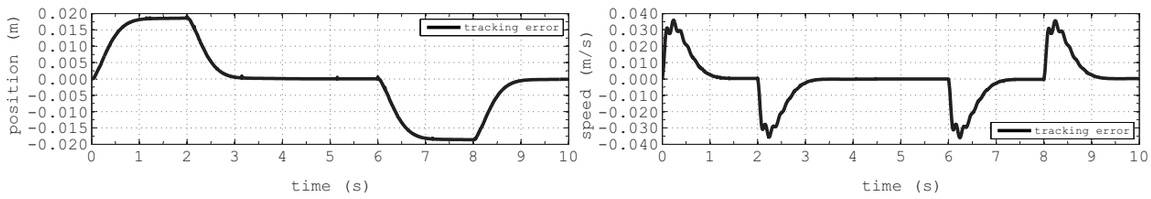

Fig. 4. Tracking errors for the linear position and velocity of the wheels for wheels tracking

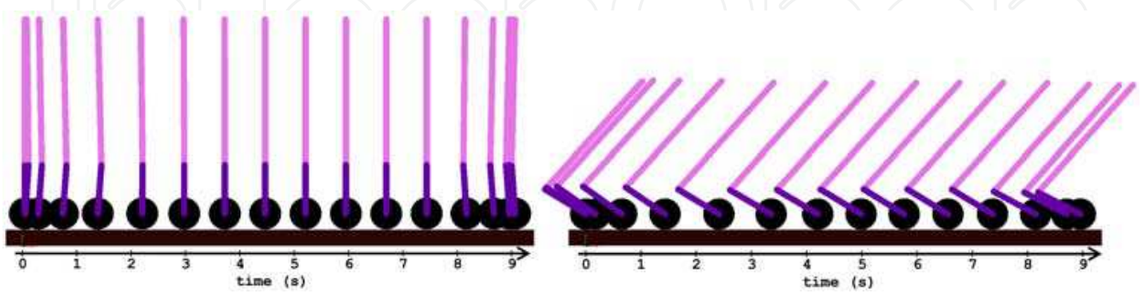

Fig. 5. Superimposed snapshot of the visualized simulation of the wheels tracking task while the arms are in inverted position (left) and are not in inverted position (right)

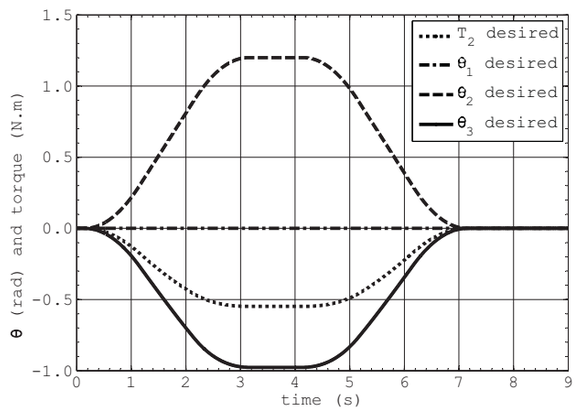

Fig. 6. The desired and calculated trajectories for the wheels, arms and time-varying torque

3.4.2 Arms tracking

Another tracking control objective defined for the WAcrobot is that the first arm to follow a trajectory with linear segments and parabolic blends while the WAcrobot's wheels are fixed in the initial position. Practically this task is that the first arm to smoothly start rotating from the inverted position and stop at a special angular position while wheels have no rotation during the tracking. To make the first arm to track the desired trajectory, the second arm should also track a calculated trajectory to make the WAcrobot stabilized during the tracking motion. Therefore the tracking problem for the first arm is also a tracking problem for the second arm. Figure 6 shows the desired and calculated trajectories for the first and second arm as well as the calculated time-varying torque required to be applied at the second joint to keep the WAcrobot balanced. Tracking errors of the linear and angular positions and velocities of the wheels, first arm and second arm are displayed in Figure 7 from top to bottom, respectively.

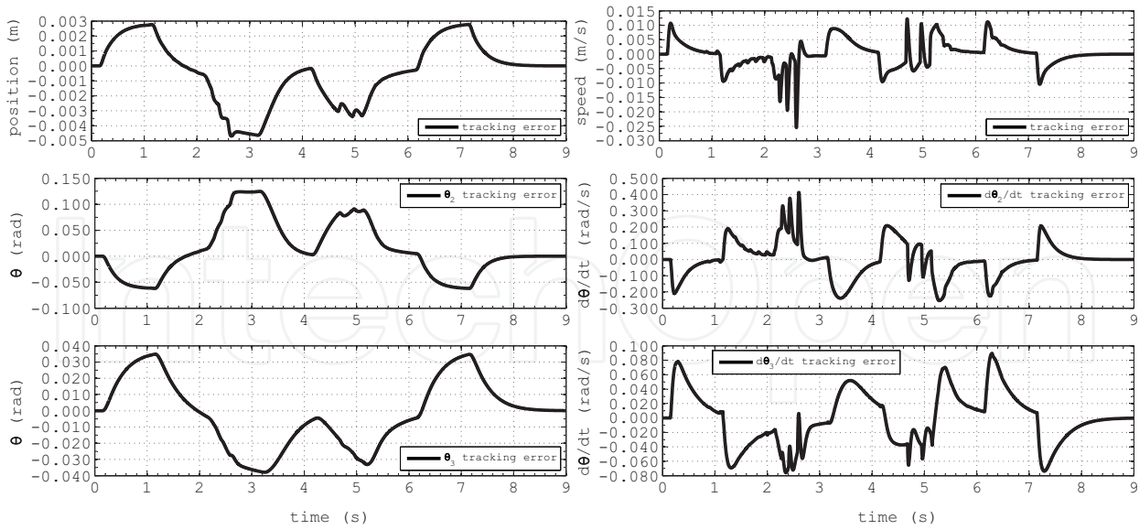

Fig. 7. Tracking errors of the positions and velocities of the wheels and arms for arms tracking

Simulation results are shown in Figure 8. In this figure the simulation responses of angular positions and velocities of wheels and arms as well as applied torques to actuated degrees of freedom are demonstrated, respectively. To show the correlation between the computational analysis results and the WAcrobot physical response, the graphical simulation is prepared and is shown in Figure 9.

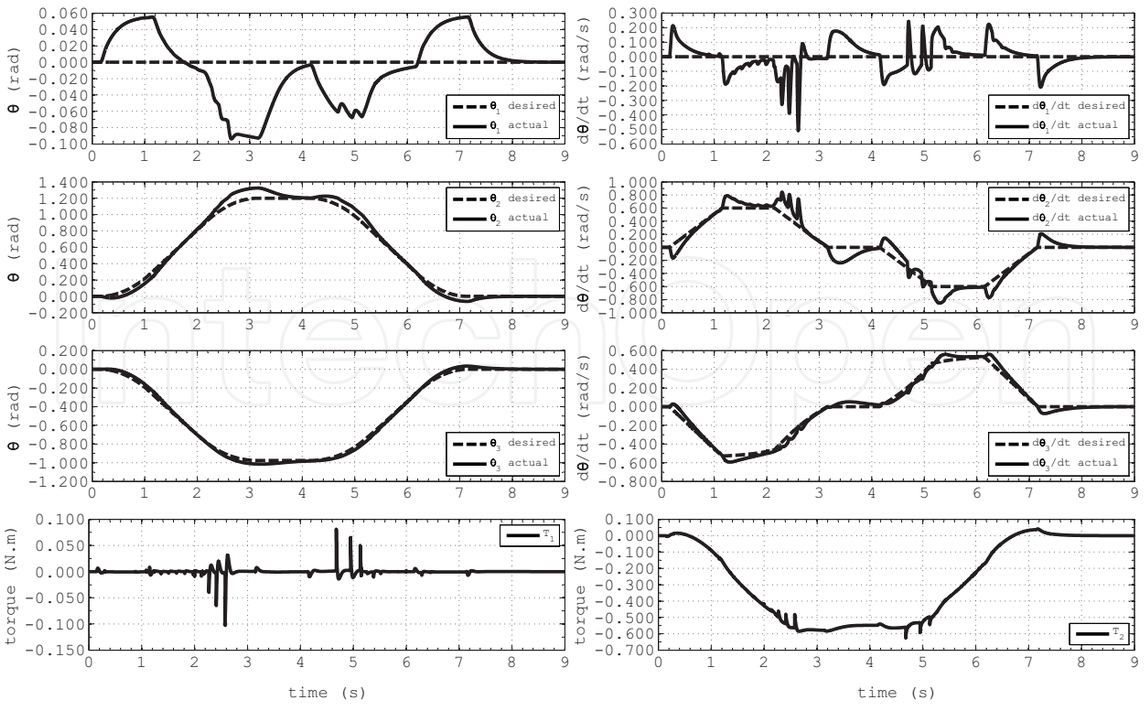

Fig. 8. The simulation responses of positions, velocities and torques for the first arm tracking

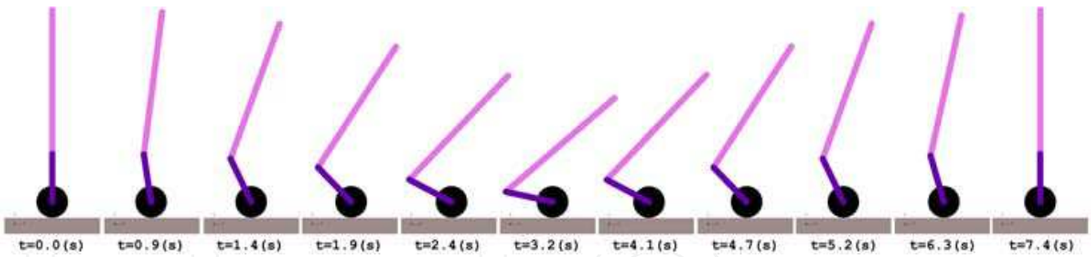

Fig. 9. Snapshots of the visualized simulation for the arms tracking task

3.4.3 Wheels and arm tracking

The most complicated task is that the first arm to follow a trajectory while the wheels are tracking another specified trajectory. In other words, this task is that the first arm to track the trajectory while the WAcrobot starts moving from the initial position (x = 0 (m)) and stop at x = 1.6 (m) smoothly. Both specified trajectories are linear segments with parabolic blends and are shown in Figure 10. Also the calculated trajectories for the angular position of the second arm as well as the input torque required at the second joint are also plotted in Figure 10.

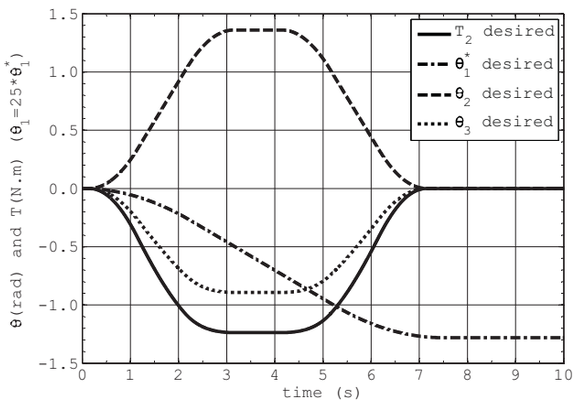

Fig. 10. The desired and calculated trajectories for the wheels, arms and time-varying torque

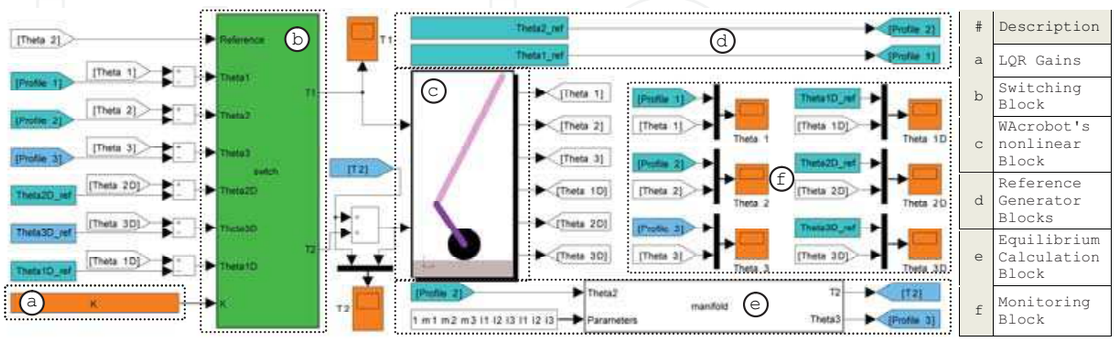

Fig. 11. The *Simulink*® block diagram of the WAcrobot system with tracking controller

The WAcrobot system with gain scheduling tracking controller simulated in *Simulink*® is illustrated in Figure 11. Figure 12 shows the simulation results of the wheels and arm tracking task. In this figure the simulation responses of the angular positions and velocities

of wheels and arms as well as applied torques to actuated DOFs are shown. Figure 13 respectively displays the tracking errors of the linear and angular positions and velocities of the wheels and arms from top to bottom. Figure 14 shows a superimposed snapshot of the visualized simulation of the WAcrobot while wheels are tracking the specified trajectory and the first arm is tracking another specified trajectory simultaneously. The simulation results illustrate the effectiveness of the proposed control methodology and the developed theory.

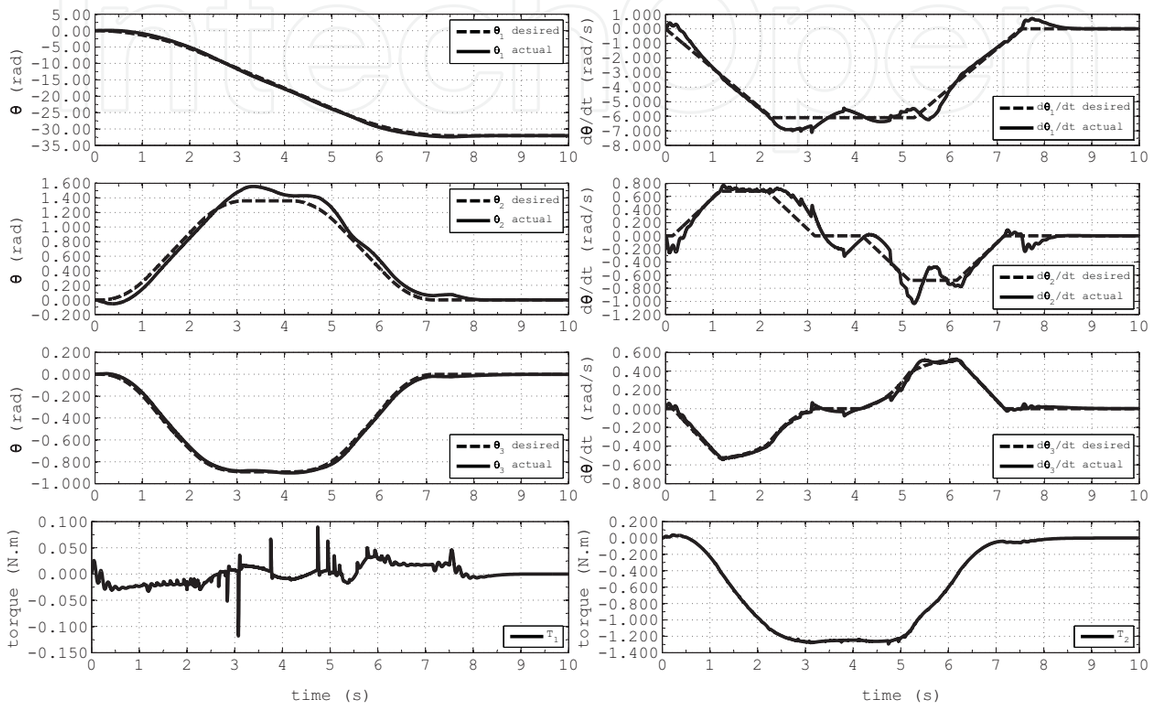

Fig. 12. The simulation responses of positions, velocities and torques for the manipulator

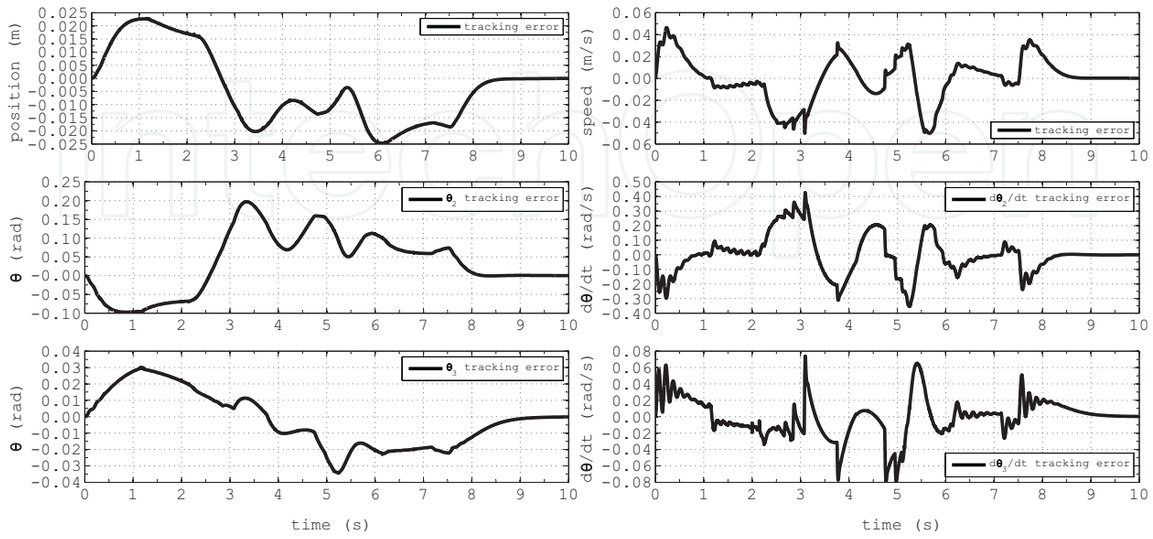

Fig. 13. Tracking errors of the positions and velocities of the wheels and arms

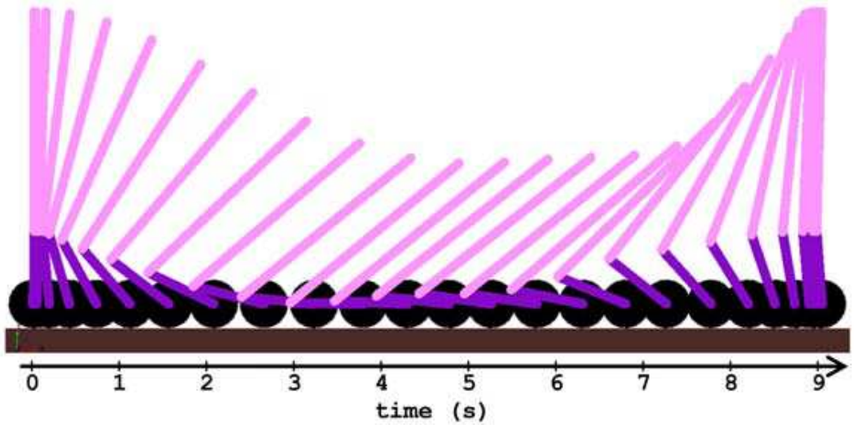

Fig. 14. Superimposed snapshot of the graphical simulation of the wheels and arms tracking

4. Conclusion

In this chapter the WAcrobot, a novel underactuated manipulator which is the combination of a well-known double inverted pendulum (Acrobot) and a wheeled inverted pendulum, was proposed and the tracking control algorithm of this mobile manipulator was investigated. The balancing controller is designed using the well known Linear Quadratic Regulator (LQR) method and the tracking controller was designed on the basis of the Gain Scheduling control strategy. Three different types of trajectory tracking tasks were investigated including tracking of *a*) wheels, *b*) first or second arm and *c*) wheels and first or second arm simultaneously.

This chapter also provided numerical and graphical simulation results to validate the obtained theoretical results and to demonstrate the correlation between the numerical results and the WAcrobot physical response. Simulation results illustrated good performance results for different tracking controls designed based on the Gain Scheduling method.

Research into the control of this novel robotic system is just in the beginning and there are a number of research problems that remain to be addressed. It would be desirable to develop the theory of robust and adaptive controller for swing-up control problem of the WAcrobot.

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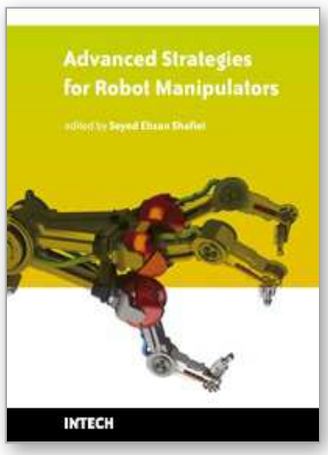

Advanced Strategies for Robot Manipulators

Edited by S. Ehsan Shafiei

ISBN 978-953-307-099-5
Hard cover, 428 pages
Publisher Sciyo
Published online 12, August, 2010
Published in print edition August, 2010

Amongst the robotic systems, robot manipulators have proven themselves to be of increasing importance and are widely adopted to substitute for human in repetitive and/or hazardous tasks. Modern manipulators are designed complicatedly and need to do more precise, crucial and critical tasks. So, the simple traditional control methods cannot be efficient, and advanced control strategies with considering special constraints are needed to establish. In spite of the fact that groundbreaking researches have been carried out in this realm until now, there are still many novel aspects which have to be explored.

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Mohsen Moradi Dalvand and Bijan Shirinzadeh (2010). Dynamic Modelling, Tracking Control and Simulation Results of a Novel Underactuated Wheeled Manipulator (WAcrobot), Advanced Strategies for Robot Manipulators, S. Ehsan Shafiei (Ed.), ISBN: 978-953-307-099-5, InTech, Available from: http://www.intechopen.com/books/advanced-strategies-for-robot-manipulators/dynamic-modelling-tracking-control-and-simulation-results-of-a-novel-underactuated-wheeled-manipulat

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