

We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

186,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?
Contact book.department@intechopen.com

Numbers displayed above are based on latest data collected.
For more information visit www.intechopen.com



Shear Elastic Wave Refraction on a Gap between Piezoelectric Crystals with Uniform Relative Motion

Nick Shevyakhov¹ and Sergey Maryshev²

¹*Kotel'nikov Institute of Radio Engineering and Electronics of Russian Academy of Sciences, Ul'yanovsk Branch*

²*Moscow Institute of Physics and Technology
Russia*

1. Introduction

The ability of acoustic waves to pass through a slot or vacuum (air) gap between piezoelectric crystals which are not contacting with each other, for the first time has noted Kaliski (Kaliski, 1966). In those years the interest of a researchers to the phenomenon first of all was connected to the being widely discussed problem of generation and amplification of ultrasonic waves that propagate along boundary of a piezoelectric medium adjoining semiconductor with a drift current (Gulyaev, 2005). More important it turned out to use Kaliski's idea of acoustic wave passage through a gap of piezoelectric crystals in metrological purposes. Firstly for development of contactless measurements of electro-acoustic fields in crystals, at which are excluded or minimized the distortions caused by own loading action of transducer. Secondly for search of effective ways of contactless excitation of acoustic oscillations in solids.

Because of anisotropy and of weakly expressed transversal piezoelectricity the case of cubic piezoelectric crystals that has considered Kaliski for shear waves of horizontal polarization (Kaliski, 1966) have not shown the proper efficiency of wave passage through a gap even with very small thickness and under condition of almost sliding incidence. Therefore in subsequent this phenomenon due to similarity to tunnel transition in the quantum mechanics (Landau & Lifshitz, 1991) named by acoustic tunneling (Balakirev & Gilinskii, 1982), began to be considered for more suitable crystals of tetragonal and hexagonal systems. The being reviewed cycle of investigations for case of strictly plane boundaries (a Balakirev & Gorchakov, 1977; Balakirev & Gorchakov, 1986), was finished (Balakirev et al., 1978) by experimental detection of effect.

By common result of the quoted works was the conclusion that the efficiency of acoustic tunneling is caused essentially by electromechanical coupling factor of crystals \mathcal{K} , and with growth of thickness of a gap is very decreasing. The passage of an acoustic wave through a gap will be especially appreciable at angle of incidence $\alpha \sim \pi/2 - \mathcal{K}^2(1 + \mathcal{K}^2)^{-1}$. So, even for such strong piezoelectric, as BaTiO₃, we have $\mathcal{K}^2 < 0.4$ (Royer & Dieulesaint, 2000) with a following from here estimation $\alpha > 75^\circ$. Therefore the opportunity of acoustic tunneling to using is very being complicated.

Source: Acoustic Waves, Book edited by: Don W. Dissanayake,
ISBN 978-953-307-111-4, pp. 466, September 2010, Sciyo, Croatia, downloaded from SCIYO.COM

The first attempt of overcoming this difficulty of practical realization of the phenomenon of acoustic tunneling was connected with known opportunity to control by coupling of acoustic and electric oscillation in crystal with high electrostriction by an external electrical field (Gulyaev, 1967; a Gulyaev & Plessky, 1977). In particular, for shear waves with horizontal polarization of displacement (SH-waves) that propagate across applied field, its action is similar to piezoelectricity of 6mm (4mm)-class crystals with piezoelectric modulus $e_{15}=aE_0/2$, where a is the coefficient of electrostriction, and E_0 is the strength of electric field. However the simple reproduction of the above results for this case takes place (Filippov, 1985) as the increase of piezoelectric activity of crystal with escalating of an electrical field almost up to a voltage of dielectric breakdown only a few decrease the suitable angles of incidence. The case, when the incident shear wave has vertical polarization and the external field is lying in a plane of incidence, also was considered by Filippov (Filippov, 1985). It is more interesting as in such conditions the acceptable for practical purposes angles of incidence can be lowered up to forty degrees.

Results received in (Filippov, 1985), have encouraged the researchers of the phenomenon of acoustic tunneling, but have not brought the complete satisfaction because of necessity in using a source of a high voltage. The attention has been addressed to other opportunity to increase the efficiency of tunneling of waves through a gap not only on intermediate angles of incidence, but also small ones. In its basis is laid the account of resonant properties of a gap as a waveguide of the slotted electroacoustic wave (Balakirev & Gorchakov, 1977 b; Gulyaev & Plessky, 1977 b). For achievement of declared object it was necessary to change resonant properties of a gap appreciably. As an effective way it was offered the using of piezocrystals with a periodic shape of surfaces (Gulyaev & Plessky, 1978) or with periodic inertial loading in form of guideway layer from other dielectric material (Gulyaev et al., 1978).

At a geometrical resonance of incident wave with the period of profile or loading impedance of boundaries the effective excitation of the appropriate mode of a slotted electroacoustic wave took place. In a result the complete passage through a gap, possible on conditions of excitation even at normal incidence, will be achieved. As in a gap there are two modes of slotted electroacoustic wave (Gulyaev & Plessky, 1977 b), for the given configuration of slotted structure it was possible to determine two frequencies ensuring for a wave the complete passage through a gap. In case of guideway boundary layers of an other dielectric with periodic inertial loading (Gulyaev et al., 1978) slotted electroacoustic waves of a gap are being replaced, as a matter of fact, by surface Love waves (Royer & Dieulesaint, 2000), which connect through a gap by an electrical field. The advantage of use of surface Love waves before slotted electroacoustic waves consists in much stronger boundary localization and, as a consequence, in their ability to form on appreciably smaller distances along guideway boundary. Due to this the resonant tunneling of waves through a gap "adjusted" on Love waves, can be carried out with the appreciably smaller apertures of an incident acoustic beam. Idea to take advantage of resonant properties of a gap for achievement of complete passage of a wave through a gap experimentally was realized in work (Grigor'evskii, 1987) for waves of vertical polarization, when resonant modes of a gap with a periodic profile of boundaries are surface Releigh-type waves. It is necessary to note, that in this experiment the passage of a wave through a gap of piezoelectrics with rectangular grooves was not quite complete. The authors have explained it by partial transformation in transversal waves of a longitudinal wave, which is falling normally on a gap with periodic grooves.

The concept of acoustic tunneling is successfully applied now to interpretation of transfer effects of wave disturbances between phononic crystals (Qui et al, 2005; Van Der Biest et al, 2005; Pennec et al, 2009). Strictly speaking, there is not here the obvious analogy to acoustic tunneling of waves through a slot between piezoelectric crystals because of absence of a vacuum gap. Instead of it in respect to tunneling phonons consider the forbidden zone of phononic crystals, and role of the electric-field coupling between piezoelectric crystals begin to play the allowed states, which arise in the forbidden zone because of infringements of Bragg interference requirements by discrepancy of the periods of lattices or by introduction of artificial defects of periodic structure of phononic crystals.

Other direction of modern researches of acoustic tunneling, which directly continues early works (Balakirev & Gorchakov, 1977 a; Balakirev & Gorchakov, 1986; Balakirev et al, 1978; Filippov, 1985), is connected with taking into account the relative longitudinal displacement of piezoelectric crystals divided by a gap. As we know, earlier in all works on acoustic tunneling in piezoelectric layered structures with a gap the crystals always relied fixed. There are, however, some reasons, on which the acoustic tunneling in conditions of relative longitudinal displacement of piezoelectric crystals represents doubtless interest. So, in practice the relative moving of bodies is one of the main occasions for using of contactless ways of introduction of acoustic oscillations. On the other hand, in sphere of high technologies (robotics, mechatronics) the important place occupies monitoring relative moving of elements of designs that, in particular, means development of sensor controls using piezoelectric effect. At last, the relative longitudinal displacement of piezoelectrics in slot-type structures is possible to consider and as the additional factor of the processing of signal information by standard means of acoustoelectronics.

Present article is written on materials of the publications (Gulyaev et al, 2007 a; Gulyaev et al, 2007 b; Maryshev & Shevyakhov, 2007), concerning only of case of shear waves of horizontal polarization in piezoelectric crystals of some classes of crystallographic symmetry with ideal plane boundaries of a gap, which are not subjected to any periodic impedance loading. The appropriate generalizations, for example, account not only electrical, but also magnetic connection of crystals (piezomagnetism) by fields through a gap are represented by matter of the nearest future. Confirming it we shall refer to work (Vilkov et al, 2009), in which tunneling of shear magnetoelastic waves through a gap of ferromagnetic crystals testing relative longitudinal displacement recently was considered.

2. Tunneling of a shear wave through a gap of piezoelectric crystals with relative longitudinal motion

2.1 Shear elastic wave in a moving crystal

The typical geometry of boundary problem of acoustic tunneling through a gap of pair piezoelectric crystals with relative longitudinal motion is submitted on Fig. 1. On it one of crystals (bottom) moves with the given constant velocity V , whereas other (top) is in rest. Generally crystals can differ in the parameters, have various orientations of crystallographic axes and belong to various classes of crystal symmetry. However, it is important, that falling on a gap on the part of immobile crystal the acoustic wave was a piezoactive wave, i.e. was accompanied in its deformations by electric fields, and the surfaces of crystals – boundaries of a gap, were not covered with metal electrodes.

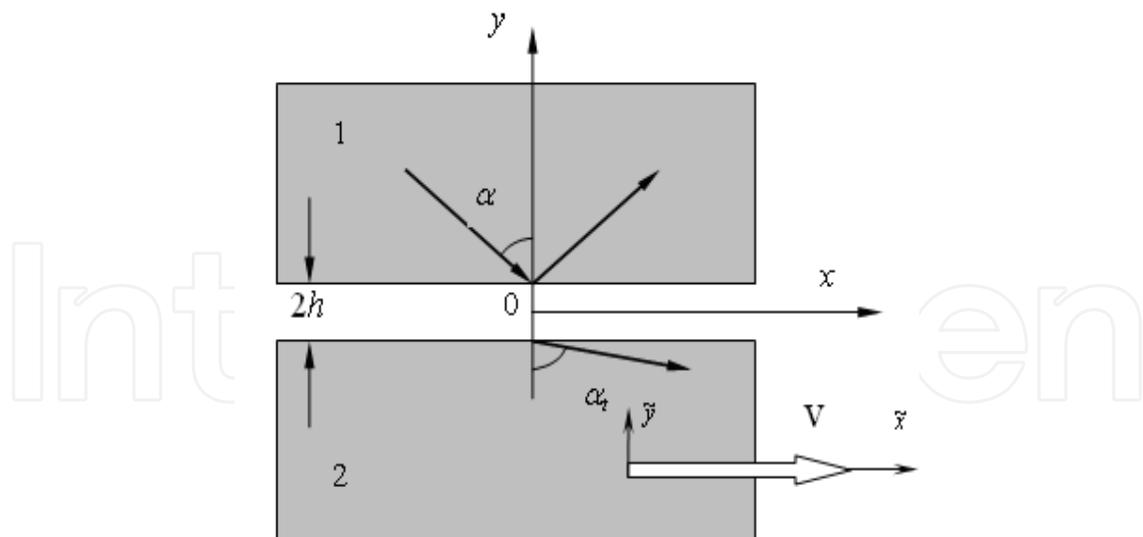


Fig. 1. Geometry of acoustic wave tunneling through a gap of piezoelectric crystals with relative longitudinal motion

From elementary reasons the following way of consideration of acoustic tunneling in conditions of relative longitudinal motion of piezoelectric crystals arises. It is necessary to connect with each of crystal own system of coordinates. For immobile crystal "1" it is the laboratory system of coordinates $x_0y_0z_0$. For a moving crystal "2" it is the passing coordinate system $\tilde{x}_0\tilde{y}_0\tilde{z}_0$. The propagation of acoustic waves in own coordinate systems of crystals, where both of them are immobile, is being described, obviously, by the standard manner (Balakirev & Gilinskii, 1982; Royer & Dieulesaint, 2000). However, because of coupling by electric fields through a gap it is impossible already to consider these wave processes as isolated ones and refraction of waves by a gap must be described in one common coordinate system. For this purpose any of own coordinate systems of crystals is suitable, but more preferably to use the laboratory system of coordinates, as according with logic of subject matter just with the one are connected the acoustic radiator and detector of a reflected wave. Thus, we need to describe waves, passing into a moving crystal, with a point of the observer of laboratory system of reference. In language of mathematics it means that in equations for moving crystal we transfer from coordinates $\tilde{x}, \tilde{y}, \tilde{z}$ to coordinates x, y, z .

If to accept reasonable restriction $V \ll c$, where c is velocity of light, the transfer of disturbances by electric fields through a gap can rely instantaneous. It will be first and foremost is in accordance with usually used quasistatic approximation for determination of the electric fields, which accompany the acoustic waves in piezoelectric crystals (Balakirev & Gilinskii, 1982; Royer & Dieulesaint, 2000). Secondly, we then may be limited by mechanical relativity and to use for connection of coordinate systems $x_0y_0z_0$ and $\tilde{x}_0\tilde{y}_0\tilde{z}_0$ Galilean transformation

$$x = \tilde{x} + V\tilde{t}, \quad y = \tilde{y}, \quad z = \tilde{z}, \quad t = \tilde{t}. \quad (1)$$

Here \tilde{t} and t is the time, which has equal duration in both systems of reference.

Let's consider identical piezoelectric crystals of a class 6 (4, 6mm, 4mm, ∞ m) with common orientation of axes of symmetry of high order 6 (4), along coordinate directions z and \tilde{z} . In shear waves of horizontal polarization the elastic displacement u_j (here and everywhere are

lower $j=1, 2$ is the number of a crystal) also are parallel these directions: $\mathbf{u}_1=\mathbf{u}_1(x,y,t) \parallel z$ and $\mathbf{u}_2=\mathbf{u}_2(\tilde{x},\tilde{y},\tilde{t}) \parallel \tilde{z}$. Therefore according to the equations of state for a piezoelectric material the working components of stress tensor T_{ik} and vector of an electrical induction \mathbf{D} for a moving crystal have in the passing system of coordinates the form (Balakirev & Gilinskii, 1982; Royer & Dieulesaint, 2000)

$$T_{xz}^{(2)} = T_{zx}^{(2)} = \lambda \frac{\partial u_2}{\partial \tilde{x}} + e_{15} \frac{\partial \varphi_2}{\partial \tilde{x}} - e_{14} \frac{\partial \varphi_2}{\partial \tilde{y}}, \quad T_{yz}^{(2)} = T_{zy}^{(2)} = \lambda \frac{\partial u_2}{\partial \tilde{y}} + e_{15} \frac{\partial \varphi_2}{\partial \tilde{y}} + e_{14} \frac{\partial \varphi_2}{\partial \tilde{x}}, \quad (2)$$

$$D_x^{(2)} = e_{15} \frac{\partial u_2}{\partial \tilde{x}} + e_{14} \frac{\partial u_2}{\partial \tilde{y}} - \varepsilon \frac{\partial \varphi_2}{\partial \tilde{x}}, \quad D_y^{(2)} = e_{15} \frac{\partial u_2}{\partial \tilde{y}} - e_{14} \frac{\partial u_2}{\partial \tilde{x}} - \varepsilon \frac{\partial \varphi_2}{\partial \tilde{y}}. \quad (3)$$

The similar expressions, but already in laboratory system of coordinates, turn out for the immobile crystal. They directly follow from the formulas (2), (3), if in them to change number of a crystal $j=2$ for number $j=1$ and instead of coordinates \tilde{x}, \tilde{y} to use accordingly laboratory coordinates x, y . In the formulas (2), (3) φ is the potential of an electrical field, λ is shear modulus, e_{15} and e_{14} is the piezoelectric modules of longitudinal and transverse piezoelectric effect, ε is the permittivity of a crystal.

By the initial equations for electroelastic fields of SH-waves propagating in a plane $\tilde{x}0\tilde{y}$ of a moving piezoelectric crystal, are the equations

$$\rho \frac{\partial^2 u_2}{\partial \tilde{t}^2} = \frac{\partial T_{ik}^{(2)}}{\partial \tilde{x}_k}, \quad \frac{\partial D_i^{(2)}}{\partial \tilde{x}_i} = 0, \quad (4)$$

where ρ is density. First of them represents the equation of elastic medium motion, and second expresses the fact of absence of free carriers of a charge in piezoelectric crystal and in quasi-static approximation with high accuracy replaces with itself complete system of the Maxwell equations for determination of an electrical field. For immobile piezoelectric in view of the above-stated replacement of number of a crystal and use of laboratory coordinates we have the similar equations. Let's remind that partial derivatives on spatial variables in the equations (4) are summarized on a repeating index, forming tensor convolutions.

Result of substitution of expressions (2), (3) in the equations (4) will be well known (Balakirev & Gilinskii, 1982; Royer & Dieulesaint, 2000) the equations of piezocrystal acoustics for waves of a SH-type

$$\rho \frac{\partial^2 u_2}{\partial \tilde{t}^2} = \lambda \tilde{\nabla}^2 u_2 + e_{15} \tilde{\nabla}^2 \varphi_2, \quad \frac{e_{15}}{\varepsilon} \tilde{\nabla}^2 u_2 = \tilde{\nabla}^2 \varphi_2. \quad (5)$$

Similarly for immobile crystal is received

$$\rho \frac{\partial^2 u_1}{\partial t^2} = \lambda \nabla^2 u_1 + e_{15} \nabla^2 \varphi_1, \quad \frac{e_{15}}{\varepsilon} \nabla^2 u_1 = \nabla^2 \varphi_1. \quad (6)$$

The difference between the equations (5), (6) is defined by differences in pairs of differential operators: $\partial/\partial \tilde{t}$, $\partial/\partial t$ and $\tilde{\nabla}^2 = \partial^2/\partial \tilde{x}^2 + \partial^2/\partial \tilde{y}^2$, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$. By rule of indirect differentiation of functions with many variables the connection between them it is possible to open, using relations

$$\begin{aligned}\frac{\partial}{\partial \tilde{x}} &= \left(\frac{\partial x}{\partial \tilde{x}}\right) \frac{\partial}{\partial x} + \left(\frac{\partial y}{\partial \tilde{x}}\right) \frac{\partial}{\partial y} + \left(\frac{\partial t}{\partial \tilde{x}}\right) \frac{\partial}{\partial t}, \\ \frac{\partial}{\partial \tilde{y}} &= \left(\frac{\partial x}{\partial \tilde{y}}\right) \frac{\partial}{\partial x} + \left(\frac{\partial y}{\partial \tilde{y}}\right) \frac{\partial}{\partial y} + \left(\frac{\partial t}{\partial \tilde{y}}\right) \frac{\partial}{\partial t}, \\ \frac{\partial}{\partial \tilde{t}} &= \left(\frac{\partial x}{\partial \tilde{t}}\right) \frac{\partial}{\partial x} + \left(\frac{\partial y}{\partial \tilde{t}}\right) \frac{\partial}{\partial y} + \left(\frac{\partial t}{\partial \tilde{t}}\right) \frac{\partial}{\partial t}.\end{aligned}\quad (7)$$

As from (1) follows, that $\frac{\partial x}{\partial \tilde{x}} = 1$, $\frac{\partial y}{\partial \tilde{y}} = 1$, $\frac{\partial x}{\partial \tilde{t}} = V$, $\frac{\partial t}{\partial \tilde{t}} = 1$ and all others derivative, included in (7) are equal to zero, on the basis (7) we come to equalities

$$\frac{\partial}{\partial \tilde{x}} = \frac{\partial}{\partial x}, \quad \frac{\partial}{\partial \tilde{y}} = \frac{\partial}{\partial y}, \quad \frac{\partial}{\partial \tilde{t}} = V \frac{\partial}{\partial x} + \frac{\partial}{\partial t}.$$

From here it is visible, that in the equations (5) transitions from coordinates of passing system of reference to laboratory coordinates are reduced to the following replacement of the differential operators: $\tilde{\nabla}^2 \rightarrow \nabla^2$, $\partial/\partial \tilde{t} \rightarrow \partial/\partial t + V\partial/\partial x$. On this basis the equations (5) can give a form

$$\rho \left(\frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right)^2 u_2 = \lambda \nabla^2 u_2 + e_{15} \nabla^2 \varphi_2, \quad \frac{e_{15}}{\varepsilon} \nabla^2 u_2 = \nabla^2 \varphi_2. \quad (8)$$

We shall be interested in propagation of plane monochromatic SH-waves in moving piezoelectric from a position of the observer to laboratory system of reference, not accepting, while, in attention limitation of the sizes of a crystal. Then according to second of the equations (8) we have $\varphi_2 = u_2 (e_{15}/\varepsilon_2)$, where $u_2 \sim \exp[i(\mathbf{k}_2 \mathbf{r} - \omega t)]$ is the solution of the wave equation

$$\left(\frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right)^2 u_2 = c_t^2 \nabla^2 u_2, \quad c_t = \sqrt{\frac{\lambda^*}{\rho}}, \quad \lambda^* = \lambda + \frac{e_{15}^2}{\varepsilon}. \quad (9)$$

Further, noticing, that $\mathbf{V} \parallel x$ and using replacements $\frac{\partial}{\partial t} = -i\omega$, $V \frac{\partial}{\partial x} = \mathbf{V} \frac{\partial}{\partial \mathbf{r}} = i\mathbf{V} \mathbf{k}_2$, $\nabla^2 = -c_t^2 \mathbf{k}_2^2$, we come to next dispersion relation for SH-waves in a moving crystal

$$c_t^2 \mathbf{k}_2^2 = (\omega - \mathbf{k}_2 \mathbf{V})^2. \quad (10)$$

In expressions (9), (10) c_t is the velocity of shear waves in a piezoelectric material, λ^* is the shear modulus modified by piezoelectric effect.

The formula (10) establishes connection of a SH-wave frequency in laboratory system of reference ω with wave number k_2 , and also shows dependence of phase velocity of a wave $v_2 = \omega/k_2$ from a direction of propagation in relation to a direction of a crystal motion. Thus, a consequence of a crystal motion concerning the observer is the anisotropy of propagation of SH-waves. If the left side of equality (10) to transfer to the right, the dispersion relation will accept a form of a difference of two squares with a zero right part:

$$(\omega - \mathbf{k}_2 \mathbf{V})^2 - (\mathbf{k}_2 c_t)^2 = [(\omega - \mathbf{k}_2 \mathbf{V}) + c_t \sqrt{\mathbf{k}_2^2}] [(\omega - \mathbf{k}_2 \mathbf{V}) - c_t \sqrt{\mathbf{k}_2^2}] = 0 .$$

Accordingly, it will break up to two independent equations

$$\omega = \mathbf{k}_2 \mathbf{V} \pm c_t \sqrt{\mathbf{k}_2^2} . \quad (11)$$

The presence of two various dispersion branches has basic meaning for understanding of specificity of acoustic tunneling of waves through a gap of piezoelectrics, undergoing the relative longitudinal displacement. In the beginning we shall notice, that the ray or group velocity of waves can be defined on known (Balakirev & Gilinskii, 1982; Royer & Dieulesaint, 2000) to the formula $\mathbf{V}_g = \partial\omega / \partial\mathbf{k}_2$. From (11) differentiation ω on \mathbf{k}_2 and taking into account, that $k_2 = \sqrt{\mathbf{k}_2^2}$, we receive

$$\mathbf{V}_g = \mathbf{V} \pm c_t \mathbf{n}_2 , \quad (12)$$

where the value $\mathbf{n}_2 = \mathbf{k}_2 / k_2$ is the vector of wave normal of a SH-wave. On the other hand, expression (11) it is possible to write as $\omega = \mathbf{k}_2 \mathbf{V} \pm c_t k_2$ and after division on k_2 to receive expression for phase velocity

$$v_2 = \mathbf{V} \mathbf{n}_2 \pm c_t . \quad (13)$$

Multiplying both sides (13) on \mathbf{n}_2 , we come to a conclusion, that the phase velocity of a wave $\mathbf{v}_2 = v_2 \mathbf{n}_2$ coincides with its group velocity \mathbf{V}_g and for the observer of laboratory system of reference represents expected result of Galilean addition of velocity of wave propagation concerning a crystal with velocity of moving of the crystal.

Pair of signs in expressions (11) - (13) should not cause bewilderment, as the propagation of plane monochromatic waves along any elected direction in a crystal can occur by a counter manner. For an immobile crystal direct and return propagation of waves ($+\mathbf{n}_2$ is the wave normal for a wave direct, and $-\mathbf{n}_2$ - for a wave of return propagation) are made equally with velocity c_t . The crystal motion brings in a difference to their propagation, indicating about acquisition by a crystal of such quality, as nonreciprocity of propagation. An evident picture of nonreciprocity of wave propagation because of a crystal motion demonstrate on Fig. 2, 3 polar curves of the reduced phase velocity

$$\frac{v_2}{c_t} = \beta \cos\theta \pm 1 , \quad \beta = \frac{V}{c_t} , \quad (14)$$

where θ is the angle between vectors \mathbf{n}_2 and \mathbf{V} . At construction the polar curves we were guided by a rule to correlate to waves of direct propagation orientation wave normal in side from pole and, opposite, for waves of return propagation to consider as it oriented along a direction of wave propagation in the side of a pole. We agree also to represent the polar curves of phase velocity of waves of direct propagation by continuous lines, and polar curves of phase velocity of waves of return propagation - dashed lines. Let's notice, that the equality (13), resulting to the formula (14), represents balance of projections of velocities participating in Galilean addition, on a direction of wave propagation. In this connection the value v_2 for waves of return propagation turned out negative, and at construction of the polar curves its modulus was used.

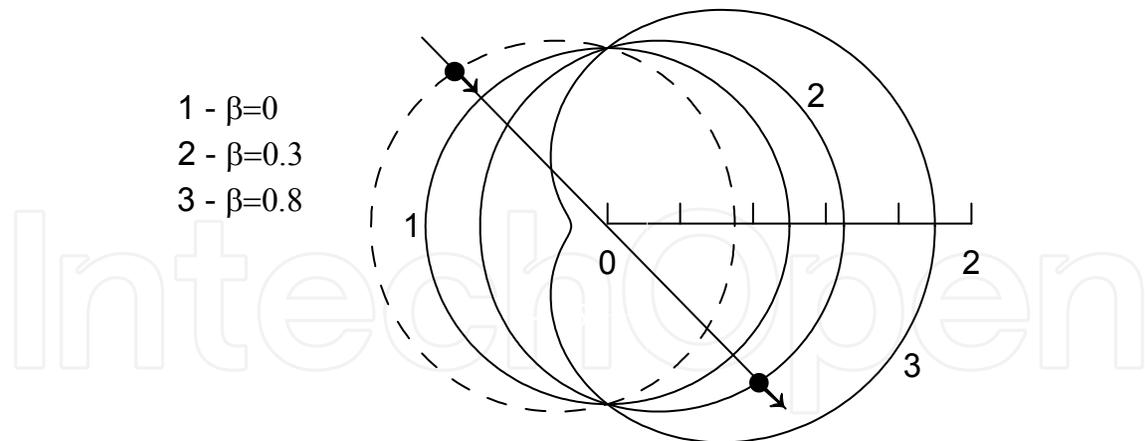


Fig. 2. Polar curves of phase velocity of SH-wave propagation at subsonic velocities of a crystal motion

The accepted way of graphic representation of polar curves excludes mess in definition of types of waves (direct or return propagation) and choice of the appropriate orientation wave normal. For an example, on Fig. 2 thin straight line allocates a direction of propagation of a SH-wave, which in the top point is crossed with dashed polar curve of return propagation ($v_2 < 0$) for $\beta = 0.3$. Thus, this point we correlate a return wave with wave normal, as shown by arrow directed to a pole. The same wave, but only direct propagation, we have the right to connect with the bottom point laying in crossing of a line of propagation with curve 2, which is mirror reflection of dashed polar curve concerning a vertical line passing through a pole. Last circumstance is a geometrical consequence of rearrangement by places of waves of direct and return propagation at inversion of velocity of a crystal motion of what it is uneasy to be convinced by substitution $\beta \rightarrow -\beta$ in (13), (14).

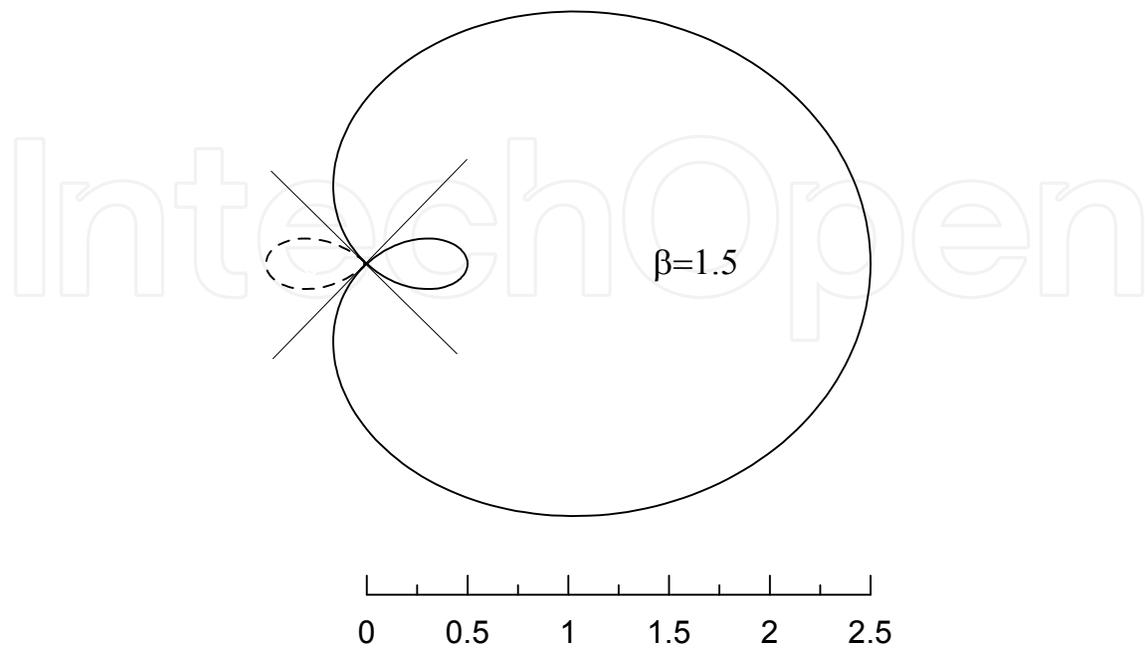


Fig. 3. Polar curve of phase velocity of a SH-wave propagation at supersonic crystal motion

At subsonic velocities of a crystal motion ($\beta < 1$, Fig. 2) circular at $\beta = 0$ polar curve 1, identical at direct and return propagation, is horizontal stretched (is compressed) for waves of direct (return) propagation in sector of polar angles $|\theta| < \pi/2$. In sector of angles $|\theta| > \pi/2$ takes place opposite. Such deformation of polar curves, reflecting property of nonreciprocity of SH-wave propagation in a moving crystal, is expressed, naturally, the more strongly, than above velocity of a crystal. At supersonic velocities ($\beta > 1$, Fig. 3) the change of polar curves by motion of a crystal is complicated by an opportunity of mutual transformation of waves of direct and return propagation. On mathematical language it will be expressed by change of sign of phase velocity v_2 in the formula (14). So, at $\beta > 1$ and angles $\theta > \pi/2$ the first term $\beta \cos \theta$ of the right side of expression for the phase velocity of a wave of direct propagation will be negative and, since an angle of transformation $\theta^* = \arccos(-1/\beta)$, begins to surpass in magnitude unit. Thus instead of former values $v_2 > 0$ we shall receive, as the certificate of the come transformation of a wave in a wave of return propagation, $v_2 < 0$. On the polar curves of phase velocity the range of transformation will settle down of a symmetrically horizontal axis and it will found in sector of obtuse (acute) angles between two thin straight lines, crossed in a pole, on Fig. 3 for waves of direct (return) propagation.

The parts of polar curves of phase velocity appropriate to the transformed waves, look like petals. On Fig. 3 such petals appropriate to transformation of a wave of direct propagation to a wave of return propagation, is shown by a dashed line. Instead of it the directly propagating waves receive mirror imaged concerning a vertical and shown continuous line a petal of waves, which are transformed by a crystal motion from waves of return propagation in waves of direct propagation. Certainly, that in the crystal, i.e. in a passing system of coordinates any transformation of waves does not occur. It appears possible only with transition to a position of the observer of laboratory system of reference, and in this sense is effect typically of a relativistic nature. As at transformation of a wave there is an inversion its wave normal, this phenomenon can be classified as specific, relativistic version of the known phenomenon of conjugation of wave front (Fisher, 1983; Brysev et al, 1998; Fink et al, 2000). But if in a basis of the processes, described in the literature, the parametrical effects put, nonlinear first of all, here conjugation of wave front is provided with the linear laws of Galilean kinematics.

2.2 Refractive properties of a gap

After we have established characteristics of shear wave propagation caused by relative motion of a crystal, it is possible to begin definition of those waves, which arise in crystals on the different sides of a gap under action of a wave, falling on it. As shown in Fig. 1, we shall believe, that the incidence of a shear wave on a gap occurs on the side of immobile crystal. Then, it is necessary to understand frequency ω as a frequency of incident wave. Standard for the wave refraction problems (Balakirev & Gilinskii, 1982; Royer & Dieulesaint, 2000) the need of phase conjugation of harmonic fields on boundaries of a gap $y = \pm h$ follows from boundary conditions (will be discussed more in details in section 2.3) and means identical concurrence of phases of oscillations in all arising waves and near-boundary electrical fields with a phase of oscillations of incident wave. Or else, if the incident wave has a phase multiplier $\exp[i(k_x x - \omega t)]$, the same phase multiplier will characterize oscillations with change of longitudinal coordinate x and time t in all other arising waves. Accordingly, the law of wave refraction is formulated as equality of frequencies of waves to frequency of incident wave

$$\omega_i = \omega_R = \omega_T \equiv \omega, \quad (15)$$

and as the predefiniteness of projections of wave vectors to a direction of boundaries of a gap by a projection of a wave vector of incident wave

$$k_{\parallel}^{(i)} = k_{\parallel}^{(R)} = k_{\parallel}^{(T)} \equiv k_x. \quad (16)$$

In expressions (15), (16) indexes i, R, T show an belonging of examined parameter according to incident, reflected (arising in immobile crystal) and refracted (arising in a moving crystal) to waves.

In view of equality (6) for waves in immobile crystal we have a dispersion relation

$$c_t^2 \mathbf{k}_1^2 = \omega^2. \quad (17)$$

From two its possible branches $c_t k_1 = \pm \omega$ for incident wave we, actually, elect a branch of directly propagating wave $c_t k_1 = \omega$. It specifies a positive sign in (15). Thus, in the subsequent transformations with use of expressions (15), (16) we accept, that $\omega > 0$, $k_1 > 0$ and accordingly $k_x = k_1 \sin \alpha > 0$, where α is the angle of incidence (see Fig. 1).

As $\mathbf{k}_1 = \mathbf{n}_1 \omega / c_t$, where \mathbf{n}_1 is the vector of a wave normal, the refractive curve, described by a vector $\mathbf{k}_1 = \mathbf{k}_1(\mathbf{n}_1)$ in the incident plane xOy , has for waves in immobile crystal the form of a circle of radius ω / c_t . In particular, the incident wave has the wave normal $\mathbf{n}_1^{(i)} = (\sin \alpha, -\cos \alpha)$. In view of (16) a wave normal $\mathbf{n}_1^{(R)} = (n_x^{(R)}, n_y^{(R)})$ any other wave arising in immobile crystal, also is characterized by value $n_x^{(R)} = \sin \alpha$ and noticing further, that $n_x^{(R)2} + n_y^{(R)2} = 1$, we have $n_y^{(R)} = \pm \cos \alpha$. The negative sign here actually is already used for an incident wave, so on reasons connected with causality, we are compelled to stop the choice on a positive sign. Thus, in immobile crystal in addition to the incident wave there is only one reflected wave with the wave normal $\mathbf{n}_1^{(R)} = (\sin \alpha, \cos \alpha)$, which is propagated in side from the boundary $y = h$. It is obvious, that in complete conformity with Mandelstam' principle of radiation the following from dispersion relation (17) the expression for the group velocity of waves in immobile crystal $\mathbf{V}_g^{(i)} = \mathbf{n}_1 c_t$ is confirmed with ability of this wave to take aside energy from boundary.

For waves arising in a moving crystal under action of incident wave, it is easier all to proceed from expression (13) and formula for a wave vector $\mathbf{k}_2 = \omega \mathbf{n}_2^{(T)} / v_2$, where $\mathbf{n}_2 = (n_x^{(T)}, n_y^{(T)})$ - vector of wave normal. Meaning, that in examined case $\mathbf{V} \mathbf{n}_2 = V n_x^{(T)} = V \sin \alpha_t$ (α_t - angle of refraction), we receive

$$\mathbf{k}_2 = \begin{cases} \mathbf{k}_2^+ = \frac{\omega}{c_t} \frac{\mathbf{n}_2}{1 + \beta \sin \alpha_t}, & \beta \sin \alpha_t > -1, \\ \mathbf{k}_2^- = \frac{\omega}{c_t} \frac{\mathbf{n}_2}{\beta \sin \alpha_t - 1}, & \beta \sin \alpha_t > 1. \end{cases} \quad (18)$$

According to (18) there are two refraction branches, appropriate to signs "plus" (\mathbf{k}_2^+) and "minus" (\mathbf{k}_2^-) in the formulas (12) - (14). The conditions of existence of the branches express the mentioned above requirement of positive values of wave numbers $k_2^{\pm} > 0$ at the elected way of representation of problem solution in laboratory system of reference by means of waves of direct propagation $\omega > 0$. In this sense the formula (18) does not add the new information that was received in the previous section, and only translates its in the terms of wave vectors more convenient for consideration of refractive effects.

The first refraction branch with wave number k_2^+ we arrange to name as a usual branch, as for it the waves in a moving crystal represent waves of a direct propagation irrespective of a choice of system of reference. Really, if, using (1) to compare phases of oscillations of a wave in passing $\exp[ik_x\tilde{x}-\Omega t]$ and laboratory system of reference $\exp[ik_x x-\omega t]$, for frequency of a wave in passing system of reference it is not difficult to receive expression

$$\Omega = \omega - \mathbf{k}_2 \mathbf{V} . \tag{19}$$

It shows Doppler shift of frequency of a wave and at substitution \mathbf{k}_2^+ from (18) determines always positive values of frequencies $\Omega_+ = \Omega(\mathbf{k}_2^+) = \omega(1 + \beta \sin \alpha_t)^{-1}$. On the contrary, at the substitution in (19) \mathbf{k}_2^- , we receive $\Omega_- = \Omega(\mathbf{k}_2^-) = -\omega(\beta \sin \alpha_t - 1)^{-1}$ and for the second refraction branch we have $\Omega_- < 0$, whereas $\omega > 0$. Thus, in case of this refraction branch, the waves, refracted in a moving crystal, are in relation to the crystal waves with the reversed wave front, but are perceived in laboratory system of reference as waves of direct distribution. Therefore it is possible to name a refraction branch \mathbf{k}_2^- as a reverse refraction branch.

As against known results (Fisher, 1983; Brysev et al, 1998; Fink et al, 2000) the phenomenon of conjugation of wave front, examined by us, has of a purely kinematic origin. It is caused by drift action of a medium moving at a transonic velocity along the wave incident from the immobile crystal, which exhaustively compensates the reverse propagation of a refracted wave relative to the crystal and eventually provides its spatial synchronism (by means of electrical fields induced via the gap) with waves that are true of direct propagation in the immobile piezoelectric crystal.

On Fig. 4, 5 solid lines show typical refraction curves of direct propagating waves which are described by the ends of wave vectors \mathbf{k}_2 from (18) at change of a direction of a vector wave normal \mathbf{n}_2 in a plane of incidence. They correspond to two qualitatively different cases of SH-wave refraction by a gap at subsonic ($\beta < 1$, Fig. 4) and very supersonic ($\beta > 2$, Fig. 5) velocities of relative crystal motion. Simultaneously with it the dashed circles represent on Fig. 4, 5 dependences $\mathbf{k}_1(\mathbf{n}_1)$ for SH-waves in immobile crystal. At $\beta < 1$ takes place only usual refraction (refraction curve is marked "plus"). The incident wave with a wave vector

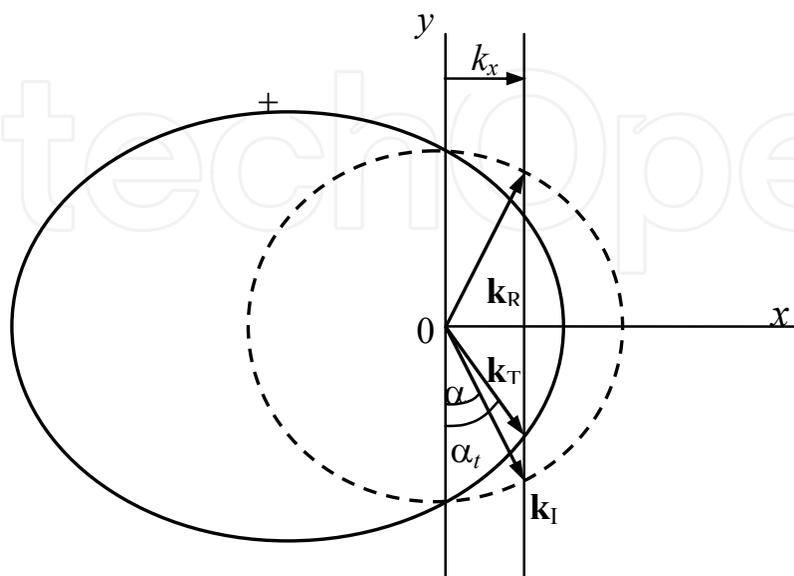


Fig. 4. Polar curves of refraction for the case $\beta < 1$.

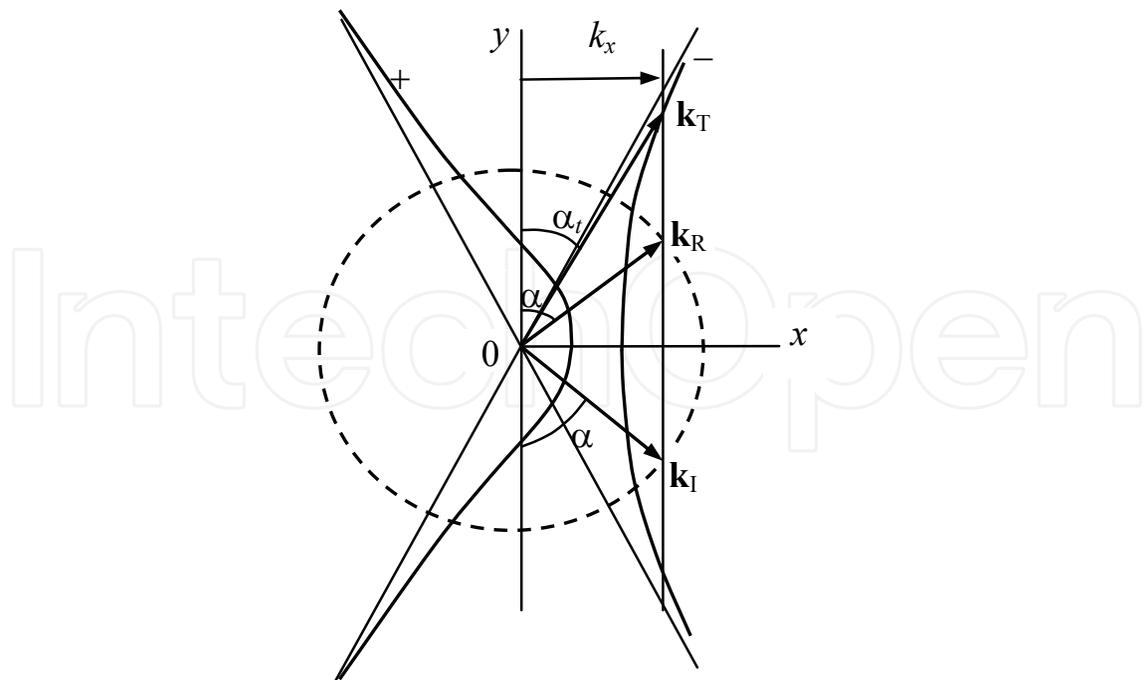


Fig. 5. Polar curves of refraction for the case $\beta > 2$.

$\mathbf{k}_I = (k_1 \sin \alpha, -k_1 \cos \alpha)$ defines valid (16) identical in all other waves a horizontal projection k_x . The wave vectors reflected \mathbf{k}_R and refracted in a moving crystal \mathbf{k}_T of waves will be, therefore, are directed from the origin 0 to points of crossing of appropriate refraction curves by a thin vertical line cutting on a horizontal a segment, equal k_x , so that the energy was removed by waves on a direction of their propagation from boundaries of crystals. Thus, we have $\mathbf{k}_R = (k_1 \sin \alpha, k_1 \cos \alpha)$, $\mathbf{k}_T = (k_2 \sin \alpha_t, -k_2 \cos \alpha_t)$.

In case of $\beta > 1$ branch usual refraction exists in intervals $0 < \theta < \theta_1^*$ and $\theta_2^* < \theta < 2\pi$ of polar angle $\theta = \pi/2 - \alpha_t$, where $\theta_2^* = 2\pi - \theta_1^*$, $\theta_1^* = \arccos(-1/\beta)$. In addition to it, as shown in Fig. 5, in the sector of angles $|\theta| < \arccos(1/\beta)$ there is a branch inversed refraction, marked by sign "minus". However, if $\beta < 2$, its curve lays more to the right of a dashed circle for a refraction curve of immobile crystal. For this reason appropriate inversed refraction of a wave are not capable to be raised in a moving crystal by incident wave and refraction picture does not differ that is submitted on Fig. 4. At velocities of relative motion of crystals is twice higher sound usual refraction will be replaced, as shown in Fig. 5, inversed refraction. It will take place, since the angle of incidence α_0 , at which

$$\sin \alpha_0 = \frac{1}{\beta - 1}. \quad (20)$$

In order to conclude this condition in expression (18) for wave number of the inversed wave k_2^- it is necessary to accept $\alpha_t = \pi/2$ and to take into account following from (16) equality $k_2^- = k_1 \sin \alpha$. In passing we shall notice, that in a regime of sliding propagation $\alpha_t = \pi/2$ difference of longitudinal projections k_x of wave vectors for inversed and usual refracted waves is given by the formula

$$\Delta k_x = k_x^- - k_x^+ = \frac{2k_1}{(\beta - 1)^2}. \quad (21)$$

From (21) we have $\Delta k_x > 0$ at any finite values of quantity β . On geometry this fact means absence of crossing of usual and inversed refraction curves. Physically it shows existence of the refracted wave always in a form of single wave, first (at $\alpha < \alpha_0$) as usual, and then (at $\alpha > \alpha_0$, if $\alpha_0 \in [0, \pi/2]$), - as the inversed wave. As the transition from usual to inversed refraction is reached by change of a sign $\cos \alpha_t$ (at an invariance of all other parameters of a wave), at construction of the solution there is a temptation to describe it in the terms of usual refraction, not resorting to consideration of two separate solutions. By an implicit manner such opportunity contains in refractive relations. Really, at usual refraction from (16), (18) the expression turns out

$$\cos \alpha_t = \frac{\sqrt{(1 - \beta \sin \alpha)^2 - \sin^2 \alpha}}{1 - \beta \sin \alpha}. \quad (22)$$

According to the requirement $k_2^- > 0$, that is equivalent also to following from (16), (18) condition $\beta \sin \alpha > 1$, the actual inclusion by the formula (22) case not only usual, but also inversed refraction ($\cos \alpha_t \rightarrow -\cos \alpha_t$) is obvious. Thus, not ordering beforehand to $\cos \alpha_t$ of a negative sign, i.e. describing refraction of a SH-wave in a moving crystal as usual, with use of the formula (22) it is possible automatically to take into account transition to inversed refraction.

2.3 Solution of a boundary problem

The connection between crystals is carried out by electrical fields penetrating through a gap. Therefore it is necessary to consider the equations (6), (8) together with the Laplace equation for potential φ of an electrical field in a gap

$$\nabla^2 \varphi = 0. \quad (23)$$

It is got, if, considering a gap as very rarefied material medium with permeability ε_g , instead of the equations (4) to use in laboratory system of coordinates the equation $\nabla \mathbf{D} = 0$, where $\mathbf{D} = \varepsilon_g \mathbf{E}$ is the induction, and $\mathbf{E} = -\nabla \varphi$ is the strength of a field. According to the equation (6) and accepted on a Fig. 1 picture of incidence, for immobile crystal we have

$$\begin{aligned} u_1 &= U \exp[i(k_x x - \omega t)] [\exp(-ik_y^{(1)} y) + R \exp(ik_y^{(1)} y)], \quad \varphi_1 = \frac{e_{15}}{\varepsilon} u_1 + \Phi_1, \\ \Phi_1 &= F_1 \exp[i(k_x x - \omega t) \exp(-k_x y)], \quad k_x = \frac{\omega}{c_t} \sin \alpha, \quad k_y^{(1)} = \frac{\omega}{c_t} \cos \alpha. \end{aligned} \quad (24)$$

In the moving crystal on base of equations (8) and stated above idea to consider the tunneling wave as a single wave of usual refraction, we have

$$\begin{aligned} \varphi_2 &= \frac{e_{15}}{\varepsilon} u_2 + \Phi_2, \quad \Phi_2 = F_2 \exp[i(k_x x - \omega t)] \exp(-k_x y), \\ u_2 &= UT \exp[i(k_x x - \omega t)] \exp(-ik_y^{(2)} y), \\ k_y^{(2)} &= \sqrt{k_2^2 - k_x^2} = k_1 \sqrt{(1 - \beta \sin \alpha)^2 - \sin^2 \alpha}. \end{aligned} \quad (25)$$

To the expressions (24), (25) we shall add expression for an electric field potential in a gap

$$\varphi = \exp[i(k_x x - \omega t)][C \exp(k_x y) + D \exp(-k_x y)]. \quad (26)$$

This expression follows from the equation (23).

In the formulas (24) - (26) values Φ_j represent potentials of fields of near-boundary electrical oscillations, U is the known amplitude of incident wave. The coefficients of reflection (R) and passage of incident wave through the gap (T), and also amplitude of potentials of near-boundary electrical oscillations F_1, F_2, C, D are subject still to determination. With this purpose we use boundary conditions of a problem, which mean a continuity of electrical potentials, y -components of an electrical induction and absence of shear stresses T_{zy} at $y=\pm h$. As the values $D_y^{(2)}, T_{yz}^{(2)}$ included in boundary conditions, do not contain derivative on time, they will not change at transitions from passing system of reference to laboratory system of reference. In result the boundary conditions will accept in laboratory system of reference the form

$$\begin{aligned} \left(\frac{e_{15}}{\varepsilon} u_j + \Phi_j \right) \Big|_{y=(-1)^{j+1}h} &= \varphi \Big|_{y=(-1)^{j+1}h}, \\ \left(e_{14} \frac{\partial u_j}{\partial x} + \varepsilon \frac{\partial \Phi_j}{\partial y} \right) \Big|_{y=(-1)^{j+1}h} &= \varepsilon_g \frac{\partial \varphi}{\partial y} \Big|_{y=(-1)^{j+1}h}, \\ \left[\lambda^* \frac{\partial u_j}{\partial y} + \frac{e_{14} e_{15}}{\varepsilon} \frac{\partial u_j}{\partial x} + \left(e_{15} \frac{\partial \Phi_j}{\partial y} + e_{14} \frac{\partial \Phi_j}{\partial x} \right) \right] \Big|_{y=(-1)^{j+1}h} &= 0. \end{aligned} \quad (27)$$

After substitution (24) - (26) in (27) and solution of forming system of the nonhomogeneous algebraic equations we shall receive representing for us interest coefficients

$$R = \frac{\frac{k_y^{(1)} k_y^{(2)}}{k_x^2} + \left[\Delta_a \Delta_s + i \frac{(k_y^{(1)} - k_y^{(2)})}{2k_x} (\Delta_a + \Delta_s) \right]}{\frac{k_y^{(1)} k_y^{(2)}}{k_x^2} - \left[\Delta_a \Delta_s + i \frac{(k_y^{(1)} - k_y^{(2)})}{2k_x} (\Delta_a + \Delta_s) \right]}, \quad (28)$$

$$T = - \frac{i \bar{\varepsilon} \mathcal{K}^2 \left(\frac{k_y^{(2)} + k_y^{(1)}}{k_x} \right)}{(1 + \bar{\varepsilon}^2) \sinh(\xi) + 2 \bar{\varepsilon} \cosh(\xi)} \Big/ \frac{\frac{k_y^{(1)} k_y^{(2)}}{k_x^2} - \left[\Delta_a \Delta_s + i \frac{(k_y^{(1)} - k_y^{(2)})}{2k_x} (\Delta_a + \Delta_s) \right]}{k_x}, \quad (29)$$

where we have

$$\begin{aligned} \Delta_a &= \frac{\mathcal{K}^2 - \bar{\varepsilon} \tanh(\xi/2) \mathcal{K}_\perp^2}{1 + \bar{\varepsilon} \tanh(\xi/2)}, & \Delta_s &= \frac{\mathcal{K}^2 \tanh(\xi/2) - \bar{\varepsilon} \mathcal{K}_\perp^2}{\bar{\varepsilon} + \tanh(\xi/2)}, \\ \mathcal{K}^2 &= \frac{e_{15}^2}{\varepsilon \lambda + e_{15}^2}, & \mathcal{K}_\perp^2 &= \frac{e_{14}^2}{\varepsilon \lambda + e_{15}^2}. \end{aligned} \quad (30)$$

In these formulas, \mathcal{K}^2 and \mathcal{K}_\perp^2 are the square coefficients of electromechanical coupling for the longitudinal and transverse piezoeffect respectively, $\xi=k_x h$ is wave half-width of the gap, and $\bar{\varepsilon} = \varepsilon/\varepsilon_g$. In a particular case $\beta=0$ when the relative longitudinal motion of piezoelectric crystals is absent, we have $k_y^{(1)}=k_y^{(2)}\equiv k_y$, $k_y/k_x=\tan\theta$ ($\theta=\pi/2-\alpha$ is the glancing angle) the expression (28)-(30) leads in earlier known results (Balakirev & Gilinskii, 1982).

2.4 Discussion of results

The main attention we shall concentrate here on angular spectra of coefficients of reflection and passage of waves through a gap. For the beginning we shall notice, that in limiting cases $h\rightarrow\infty$ and $\varepsilon_g\rightarrow\infty$ ($\bar{\varepsilon}\rightarrow 0$) the expressions (28) - (30) show absence of passage $T\rightarrow 0$. In the first case it is caused by the disappearance of coupling of crystals by electrical fields through a gap in process of increase of its thickness. In the second case takes place a shielding of fields of a gap due to metallization of crystal surfaces.

Typical behaviour of angular dependences of modules of reflection coefficient $|R|$ and the passage coefficient $|T|$, calculated on the formulas (28) - (30) for pair of crystals LiIO_3 with parameters $\mathcal{K}^2=0.38$, $\mathcal{K}_\perp^2=0.002$, $\bar{\varepsilon}=8.2$, demonstrate Fig. 6 and 7. As can be seen, a general tendency in the case of usual refraction is a decrease in the extent of wave tunneling into the moving crystal with increasing angle of incidence. This trend is more pronounced in the angular dependences of the reflection coefficient R . Indeed, even at relatively small velocities, the opposite (antiparallel) relative longitudinal displacement (RLD) ($\beta=-0.05$, see curve 1 in Fig. 6) lead to extension of the wedge of transparency (depicted by the dashed curve in the region of large α) by more than a half toward greater angles ($|R|_{\min}>0.6$). However, a nearly complete extension of this wedge (Fig. 7, curve 3) takes place only for

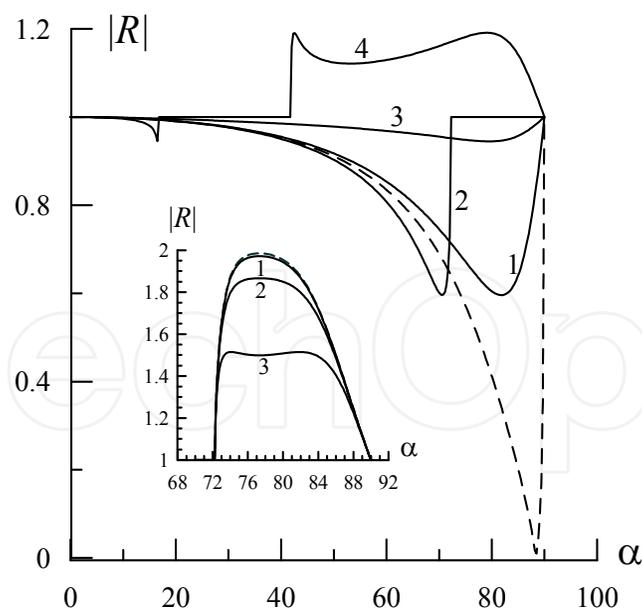


Fig. 6. Plots of reflection coefficient $|R|$ versus angle of incidence α for a pair of piezoelectric LiIO_3 with an extremely thin ($\xi=10^{-6}$) gap for an RLD velocity of $\beta=-0.05$ (1), 0.05 (2), -2.5 (3), and 2.5 (4). The inset shows the angular dependence of the reflection coefficient in the case of reverse refraction for $\beta=2.05$ and various gap thicknesses $\xi=10^{-3}$ (1), 10^{-2} (2), 0.06 (3), and 10^{-6} (dashed curve).

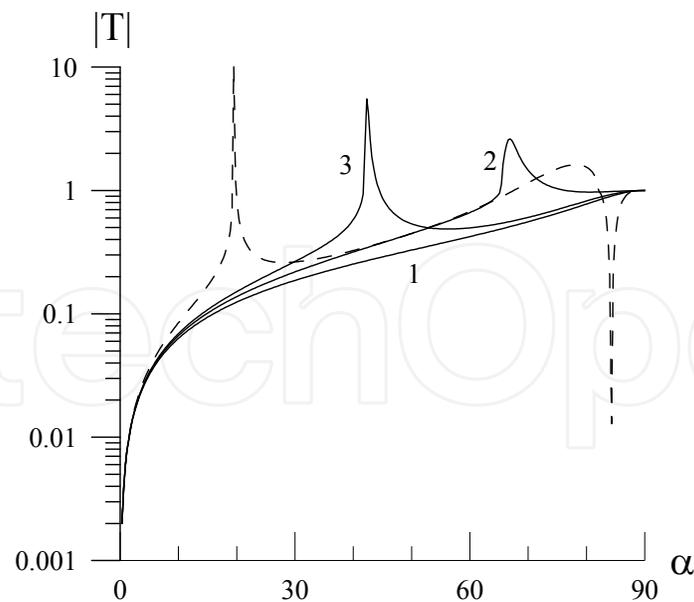


Fig. 7. Plots of the transmission coefficient $|T|$ versus angle of incidence α for a pair of piezoelectric LiIO_3 crystals with a thin ($\xi=10^{-3}$) gap for an RLD velocity of $\beta=-0.5$ (1), 0.1 (2), 0.5 (3) and 2.01 (dashed curve).

ultrahigh velocities of the opposite RLD ($\beta < 0$, $|\beta| > 2$). However, a comparison of curves 1 – 3 in Fig. 7 shows that no significant decrease in the transmission of waves through the gap takes place and the possibility of practical application of the effect of wave tunneling is retained.

In the case of parallel RLDs ($\beta > 0$) the transparency wedge under the usual refraction conditions is not only extended with increasing β , but is additionally shifted toward smaller incidence angles by the appearing region of total reflection (Fig. 6, curves 2). The angular dependences of transmission (Fig. 7, curves 2 and 3) show well-pronounced peaks at the limiting angles α^* of total reflection ($\sin \alpha^* = (1 + \beta)^{-1}$). The left sides of these peaks apparently correspond to the conditions of effective tunneling of incident wave into the moving crystal. However, it should be taken into account that, in view of the proximity to α^* , the tunneling waves will have very small transverse components ($k_y^{(2)} \geq 0$) of the wave vector. Thus, the effective tunneling of waves into the moving crystal is possible, but only for small (or very small) angles of refraction for moderate (Fig. 7, curve 3) and even small (Fig. 7, dashed curve) angles of incidence.

In the latter case, ultra-high RLD velocities ($\beta > 2$) are necessary, which make possible the reverse refraction. As for the phenomenon of tunneling as such, the region of reverse refraction $\alpha > \alpha^{**}$ ($\sin \alpha^{**} = (\beta - 1)^{-1}$, $\alpha^{**} \sim 82^\circ$ for the dashed curve in Fig. 7) does not present much interest because formula (13) implies "closing" of the gap for $k_y^{(1)} + k_y^{(2)} = 0$ with significant decrease in the transmission coefficient $|T|$ in the vicinity of the corresponding incidence angle. On the other hand, there is an attractive possibility of enhancement of the reflected wave for $|R| > 1$ (see Fig. 6, curve 4 and the inset to Fig. 6, curves 1 – 3), which is related to the fact that the wave in a moving crystal in the case of reverse refraction propagated (as indicated by dashed arrow in Fig. 1) toward the gap and carries the energy in the same direction. Naturally, an increase in the gap width leads to decrease in electric coupling between crystals and in the enhancement of reflection (see the inset to Fig. 6, curves 1 – 3).

3. Tunneling of shear waves by a vacuum gap of piezoelectric 6- and 222-class crystal pair at the uniform relative motion

In this section we consider the effect of tunneling of shear waves in the layered structure of piezoelectric crystals with a gap for the crystal pair of 6 (6mm, 4, 4mm, ∞m) and 222 (422, 622, 4 2m, 4 3m, 23) class symmetry, undergoing relative longitudinal motion. This case allows, to estimate influence of elastic and electric anisotropy on tunneling of SH-waves in a moving crystal in conditions of difference of its symmetry from symmetry of an immobile crystal. We assume that the shear wave falls on the part of the immobile crystal of a class 6. Now, instead (8) we shall have in laboratory system of reference the equations

$$\begin{aligned} \rho_2 \left(\frac{\partial}{\partial t} + V \frac{\partial}{\partial x} \right)^2 u_2 &= \lambda_{55}^{(2)} \frac{\partial^2 u_2}{\partial x^2} + \lambda_{44}^{(2)} \frac{\partial^2 u_2}{\partial y^2} + (e_{14}^{(2)} + e_{25}^{(2)}) \frac{\partial^2 \varphi_2}{\partial x \partial y}, \\ (e_{14}^{(2)} + e_{25}^{(2)}) \frac{\partial^2 u_2}{\partial x \partial y} &= \varepsilon_1^{(2)} \frac{\partial^2 \varphi_2}{\partial x^2} + \varepsilon_2^{(2)} \frac{\partial^2 \varphi_2}{\partial y^2}. \end{aligned} \quad (31)$$

The equations (6) remain in force, but with a clause, that in them all parameters of a crystal are marked by an index "1", i.e. $\rho \rightarrow \rho_1$, $\lambda \rightarrow \lambda_{55}^{(1)}$, $e_{15} \rightarrow e_{15}^{(1)}$ and $\varepsilon \rightarrow \varepsilon_1^{(1)}$.

Following from (6), (31) the dispersion relation of SH-waves and Snell's condition (16) allow to establish the refraction law in form of the inverse dependence

$$\sin \alpha = \frac{v_1 \sin \alpha_t}{V \sin \alpha_t \pm v_{2||} \sqrt{(\sin^2 \alpha_t + a \cos^2 \alpha_t)(1 + Q^2(\alpha_t) \sin^2 \alpha_t)}}. \quad (32)$$

Here $v_1 = (\lambda_{55}^{(1)*} / \rho_1)^{1/2}$ is the velocity of SH-waves in immobile crystal, $\lambda_{55}^{(1)*} = \lambda_{55}^{(1)} + e_{15}^{(1)2} / \varepsilon_1^{(1)}$, $v_{2||} = (\lambda_{55}^{(2)} / \rho_2)^{1/2}$ is the velocity of SH-wave propagation in a moving crystal along [100]-direction (axis \tilde{x}), $a = \lambda_{44}^{(2)} / \lambda_{55}^{(2)}$ is the elastic anisotropy factor of moving crystal. Function $Q^2(\alpha_t)$, determined by equality

$$Q^2(\alpha_t) = \frac{(e_{14}^{(2)} + e_{25}^{(2)})^2 \cos^2 \alpha_t}{(\lambda_{55}^{(2)} \sin^2 \alpha_t + \lambda_{44}^{(2)} \cos^2 \alpha_t)(\varepsilon_1^{(2)} \sin^2 \alpha_t + \varepsilon_2^{(2)} \cos^2 \alpha_t)}, \quad (33)$$

is the square of electromechanical coupling factor for SH-waves propagating in (001)-plane of a crystal.

The expression (32) shows that at subsonic velocities of crystal motion there exists only usual refraction, corresponding to the top sign. It is not accompanied by the inversion of wave fronts and has the top threshold of incident angle α^* , such that $\sin \alpha^* = v_1 / (V + v_{2||})$. At the supersonic velocities of crystal motion $V > v_{2||}$ total reflection for the usual refraction ($\alpha^* < \alpha < \alpha^{**}$) becomes possible even at smaller rigidity of a moving crystal. Second refraction branch appropriate to the bottom sign in formula (32) and accompanied by the inversion of wave fronts, is possible only at supersonic velocities of crystal motion and additional condition $V > v_1 + v_{2||}$. The bottom threshold of this branch α^{**} exceeds the value α^* is determined by equality $\sin \alpha^{**} = v_1 / (V - v_{2||})$. On Fig. 8, 9 the curves usual and inversed refraction, received by calculation under the formulas (32), (33) for pair of crystals $\text{Pb}_5\text{Ge}_3\text{O}_{11}$ – Rochell salt with parameters taken from (Royer & Dieulesaint, 2000; Shaskolskaya, 1982) are submitted accordingly.

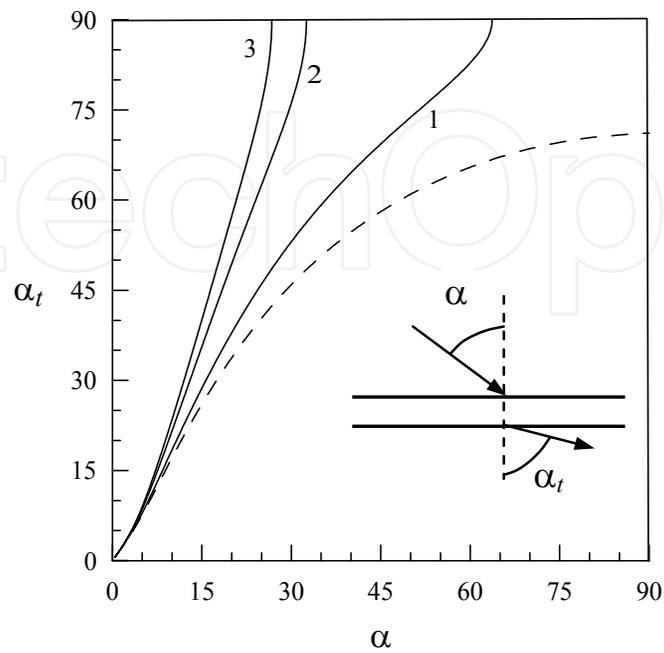


Fig. 8. Curves usual refraction of a wave by a gap $\text{Pb}_5\text{Ge}_3\text{O}_{11}$ - Rochell salt: 1 - $\beta=V/v_{2||}=0.5$, 2 - $\beta=1.5$, 3 - $\beta=1.8$ ($\beta=0$ - dashed curve).

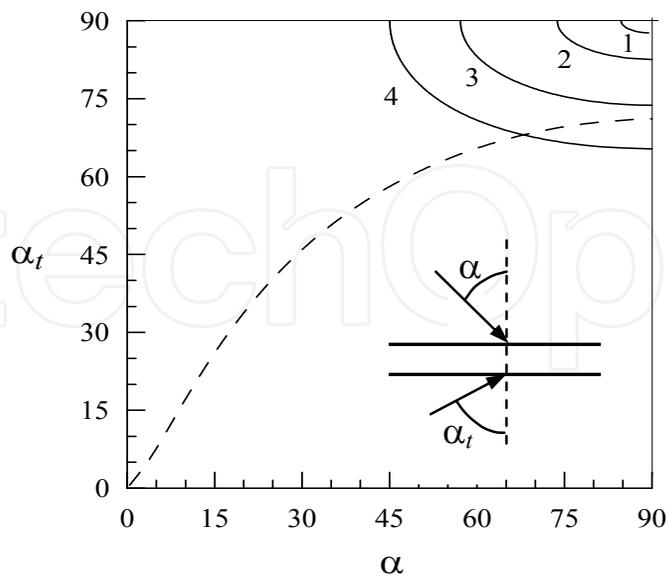


Fig. 9. Curves reversed refraction of a wave by a gap $\text{Pb}_5\text{Ge}_3\text{O}_{11}$ - Rochell salt: 1 - $\beta=V/v_{2||}=2.35$, 2 - $\beta=2.4$, 3 - $\beta=2.5$, 4 - $\beta=2.6$ ($\beta=0$ - dashed curve).

The solutions of the equations (6), (20) will keep the form (24), (26), and instead of (25) from the equations (31) we shall receive

$$u_2 = U \exp(i\phi)[T \exp(-ik_y^{(2)}y) + A \exp(sy)],$$

$$\phi_2 = \frac{ik_x s(e_{14}^{(2)} + e_{25}^{(2)})}{\varepsilon_2^{(2)}s^2 - \varepsilon_1^{(2)}k_x^2} UA \exp(i\phi) \exp(sy) - \frac{ik_x k_y^{(2)}(e_{14}^{(2)} + e_{25}^{(2)})}{\varepsilon_2^{(2)}k_y^{(2)2} + \varepsilon_1^{(2)}k_x^2} UT \exp(i\phi) \exp(-ik_y^{(2)}y). \quad (34)$$

The values $k_y^{(2)}$ and s in expressions (34) are accordingly imaginary $q = -ik_y^{(2)}$ (for solution (34) in writing we chose the case of usual refraction) and real $q = s$ a root of the characteristic equation $[(\omega - k_x V)^2 v_{21}^{-1} a^{-1} + q^2 - ak_x^2](bk_x^2 - q^2) + Q_0^2 k_x^2 q^2 = 0$, where $b = \varepsilon_1^{(2)} / \varepsilon_2^{(2)}$ is the factor of electric anisotropy of a crystal, and $Q_0 = Q(0)$. As against the solution (25) for pair of identical hexagonal crystals the near-boundary oscillations any more are not only electrical. They are the connected electro-elastic oscillations, which are made with amplitude A and phase $\phi = k_x x - \omega t$.

The physical sense of boundary conditions will not change. For the top boundary $y = h$ on former it is possible to use conditions (27). On the bottom boundary $y = -h$ their change will be caused by the appropriate differences of the state equations for 222-class crystals from the equations (2), (3) (Royer & Dieulesaint, 2000). After substitution of expressions (24), (26), (34) in boundary conditions and solutions of system of the algebraic equations we shall receive expressions for amplitude coefficients. For example, in the case of a very thin gap ($k_x h \rightarrow 0$) we have

$$R = \frac{1 - \tan \alpha Q_1^2 \Psi}{1 + \tan \alpha Q_1^2 \Psi}, \quad T = -\frac{2(e_{15}^{(1)} + ie_{14}^{(1)}) \tan \alpha}{\lambda_{44}^{(2)}(1 + i\Delta)(1 + \tan \alpha Q_1^2 \Psi)} \frac{e_{25}^{(2)} b k_x^2 - e_{14}^{(2)} [k_y^{(2)2} + 2k_x^2(1 - b)]}{k_y^{(2)2} + k_x^2 [b + Q_0^2 f_2 (1 + f_2)^{-1}]}.$$

The value Ψ characterizes mutual piezoelectric connection of crystals through a gap and is defined by equalities

$$\Psi = i \frac{f_1^2 \varepsilon_1^{(1)} \Gamma - \varepsilon_2^{(2)}}{\varepsilon_2^{(2)} + \varepsilon_1^{(1)} \Gamma}, \quad \Gamma = \frac{k_x s (1 + i\Delta)(1 + f_2)}{b k_x^2 (1 + i s k_y^{-1} \Delta) + f_2 s^2 (1 - i k_y^{(2)} s^{-1} \Delta)}, \quad \Delta = \frac{(s^2 - b k_x^2)(1 + f_2) - k_x^2 f_2 Q_0^2}{(k_y^{(2)2} + b k_x^2)(1 + f_2) + k_x^2 f_2 Q_0^2}.$$

There are $f_1 = e_{14}^{(1)} / e_{15}^{(1)}$, $f_2 = e_{14}^{(2)} / e_{25}^{(2)}$, and $Q_1^2 = e_{15}^{(1)2} / [\varepsilon_1^{(1)} \lambda_{55}^{(1)*}]$.

The numerical accounts show, that elastic and electrical anisotropy of a moving crystal does not cause essential changes in angular spectra of reflection and passage of SH-waves through a gap. The distinctions of symmetry of the crystals in addition to their relative motion are reduced by efficiency of acoustic tunneling. Thus, the assumption, that in a slot structure of crystals, from which one with strong longitudinal, and another with strong transverse piezoelectric effect, is possible appreciable shift of effective acoustic tunneling in area of moderate incident angles, has not found confirmation. The amplitude A of near-boundary electro-elastic oscillations is usually small and does not vary almost under influence of crystal motion. In a considered case of crystals of various classes of symmetry amplification the reflected wave in conditions inversed refraction (superreflection) also takes place. However, similarly to acoustic tunneling the superreflection appears well appreciable only at sliding angles of incidence.

4. Conclusion

In this article we have touched upon the poorly investigated problem of refraction of acoustic waves by a gap of piezoelectric crystals with relative longitudinal motion. By the basic result was the conclusion about existence not only usual, but also so-called inversed refraction, capable to replace the usual refraction at superfast motion of a crystal with velocity twice above velocity of a sound. We have shown, that if usual refraction underlies representations about the tunneling of acoustic waves through a gap, with the inversed refraction the opportunity of amplification of reflection is connected.

Both these phenomena, however, provide essential changes of a level of the reflected signals because of a crystal motion (it is interesting to applications), only at the sliding angles of incidence. It is represented, therefore, most urgent search of conditions and means, which would allow to advance in area of moderate or small angles of incidence. With this purpose, as we have found out, is unpromising to use anisotropy of elastic and electrical properties of a moving crystal or distinction in classes of symmetry of crystals.

We believe that there are two approaches to the decision of a problem. It is, first, search and use of hexagonal piezoelectric crystals with equally strong both longitudinal, and trasverse piezoactivity. Secondly, it is the application already of known piezoelectric materials, but having not a plane, and periodically profiled boundaries of a gap. It is doubtless, that the appropriate theoretical researches of effects acoustic refraction by a gap of piezoelectric crystals with relative motion are required. In particular, it is desirable to consider a case of refraction of piezoactive acoustic waves of vertical polarization. We hope, that present article will serve as stimulus for the further study of acoustic refraction in layered structures of piezoelectric crystals with relative motion.

5. References

- Balakirev, M. K. & Gorchakov, A. V. (1977 a). The leakage of elastic wave through a gap between piezoelectrics. *Fiz. Tverd. Tela*, Vol. 19, No 2, pp. 571-572, ISSN 0367-3294.
- Balakirev, M. K. & Gorchakov, A. V. (1977 b). Connected surface wave in piezoelectrics. *Fiz. Tverd. Tela*, Vol. 19, No 2, pp. 613-614, ISSN 0367-3294.
- Balakirev, M. K.; Bogdanov, S. V & Gorchakov, A. V. (1978). Tunneling of ultrasonic wave through a gap between jodat lithium crystals. *Fiz. Tverd. Tela*, Vol. 20, No 2, pp. 588-590, ISSN 0367-3294.
- Balakirev, M. K. & Gilinskii, I. A. (1982). *Waves in Piezoelectric Crystals*, Nauka, Novosibirsk.
- Balakirev, M. K. & Gorchakov, A. V. (1986). Volume wave refraction and surface waves in the piezoelectric - gap - piezoelectric system. *Surface*, No 5, pp. 80-85, ISSN 0207-3528 .
- Brysev, A.P.; Krutyansky, L.M. & Preobrazhensky, V.L. (1998) Wave phase conjugation of ultrasonic beams. *Phys. Uspekhi*, Vol. 41, No 8, pp. 793-806, ISSN 1063-7869.
- Filippov, V. V. (1985). Leakage of an elastic wave through a slot between two media, caused by electrostriction. *Techn. Phys.*, Vol. 55, No 5, pp. 975-979, ISNN 0044-4642.
- Fink, M; Casserau, D.; Derode, A; Prada, C.; Roux, P; Tanter, M; Thomas, J.-L. & Wu, F (2000) Time-reversed acoustics. *Rep. Progr. Phys.*, Vol. 63, No 12, pp. 1893-1995, ISSN 0034-4885.

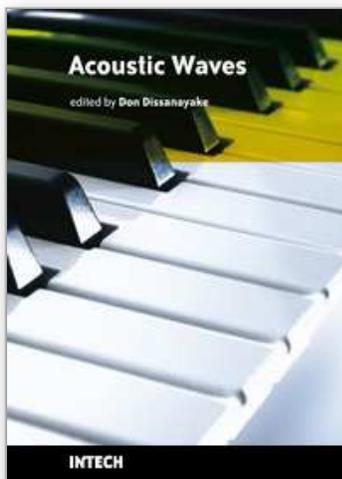
- Fisher, R.A. (1983) *Optical Phase Conjugation*, Academic Press, New York.
- Grigor'evskii, V.I.; Gulyaev, Yu.V.; Kotelyanskii, I.M.; Pashkevich, G.A. & Plessky, V.P. (1987). Acoustic wave transmission through a slot in a piezoelectric. *Akust. Zhurn.*, Vol. 33, No 2, pp. 360-362, ISSN 0320-7919.
- Gulyaev, Yu.V. (1967). To the question on electron-phonon interaction proportional to applied external field. *Fiz. Tverd. Tela*, Vol. 9, No 6, pp. 1816-1818, ISSN 0367-3294.
- Gulyaev, Yu. V. & Plessky, V.P. (1977 a). Shear surface acoustic wave in dielectrics in the presence of an electric field. *Phys. Lett.*, Vol. 56A, No 6, pp. 491-492, ISSN 0375-9601.
- Gulyaev, Yu. V. & Plessky, V.P. (1977 b). Surface waves propagating along a slot in piezoelectric materials. *Akust. Zhurn.*, Vol. 23, No 5, pp. 716-723, ISSN 0320-7919.
- Gulyaev, Yu. V. & Plessky, V.P. (1978). Resonant penetration of bulk acoustic wave through the vacuum gap between the piezoelectrics. *Fiz. Tverd. Tela*, Vol. 20, No 1, pp. 133-136, ISSN 0367-3294.
- Gulyaev, Yu. V.; Nikitov, S.A. & Plessky, V.P. (1978). Penetration of the acoustic wave through the gap between the piezoelectrics, covered by dielectric layers. *Fiz. Tverd. Tela*, Vol. 20, No 5, pp. 1580-1581, ISSN 0367-3294.
- Gulyaev, Yu. V. (2005). Acoustoelectronics (historical review). *Phys. Uspekhi*, Vol. 48, No 8, pp. 847-855, ISSN 1063-7869.
- Gulyaev, Yu. V.; Maryshev, S. N. & Shevyakhov, N. S. (2007 a). Shear wave passage through a vacuum gap between hexagonal piezoelectric crystals with relative longitudinal displacement. *Techn. Phys. Lett.*, Vol. 33, No 9, pp. 799-803, ISSN 1063-7850.
- Gulyaev, Yu. V.; Maryshev, S. N. & Shevyakhov, N. S. (2007 b). Tunneling of shear waves by a vacuum gap of piezoelectric 6- and 222-class crystal pair under the uniform relative motion. Proceedings of the XIX Session of the Russian Acoustic Society, pp. 27-30, ISBN 8-85118-383-8, Nizhny Novgorod, September 2007, GEOS, Moscow.
- Kaliski, S. (1966). The passage of an ultrasonic wave across a contactless junction between two piezoelectric bodies. *Proc. Vib. Probl.*, Vol. 7, No 2, pp. 95-104, ISSN 0001-6705.
- Landau, L. D. & Lifshitz, E.M. (1991). *Quantum Mechanics: Non-Relativistic Theory*, Pergamon Press, ISBN 0-08-020-940-8, New York.
- Maryshev, S. N. & Shevyakhov, N. S. (2007). Acoustic wave tunneling in a gap between hexagonal piezoelectric crystals with relative longitudinal displacement. *Techn. Phys. Lett.*, Vol. 33, No 5, pp. 368-372, ISSN 1063-7850.
- Pennec, Y.; Djafari-Rouhani, B.; Larabi, H.; Vasseur, J. & Hladky-Hennion, A.-Chr. (2009). Phononic crystals and manipulation of sound. *Phys. Status Solidi C*, Vol. 6, No. 9, pp. 2080-2085, ISSN 1610-1642.
- Qiu, Ch. L.; Zhengyou, M. & Jun, S.J. (2005). Mode-selecting acoustic filter by using resonant tunneling of two-dimensional double phononic crystals. *Appl. Phys. Letters*, Vol. 87, No 10, pp. 104101-104101-3, ISSN 0003-6951.
- Royer, D. & Dieulesaint Eu. (2000). *Elastic Waves in Solids I*. Springer, ISBN 3-540-65932-3, Berlin-Heidelberg-New York.
- Shaskolskaya, M.P. (1982). *Acoustic Crystals: A Handbook*. Nauka, Moscow.
- Van Der Biest, F.; Sukhovich, A.; Tourin, A.; Page, J. H.; Van Tiggelen, B. A.; Liu, Z. & Fink, M. (2005). Resonant tunneling of acoustic waves through a double barrier

consisting of two phononic crystals. *Europhys.Letters.*, Vol. 71, No 1, pp. 63-69, ISSN 0295-50075.

Vilkov, E.A.; Moiseev, A.V. & Shavrov, V.G. (2009). Magnetoelastic wave tunneling via a gap between ferromagnetic crystals with relative longitudinal displacement. *Techn. Phys. Lett.*, Vol. 35. No 9. pp. 876-879, ISSN 1063-7850.

IntechOpen

IntechOpen



Acoustic Waves

Edited by Don Dissanayake

ISBN 978-953-307-111-4

Hard cover, 434 pages

Publisher Sciyo

Published online 28, September, 2010

Published in print edition September, 2010

SAW devices are widely used in multitude of device concepts mainly in MEMS and communication electronics. As such, SAW based micro sensors, actuators and communication electronic devices are well known applications of SAW technology. For example, SAW based passive micro sensors are capable of measuring physical properties such as temperature, pressure, variation in chemical properties, and SAW based communication devices perform a range of signal processing functions, such as delay lines, filters, resonators, pulse compressors, and convolvers. In recent decades, SAW based low-powered actuators and microfluidic devices have significantly added a new dimension to SAW technology. This book consists of 20 exciting chapters composed by researchers and engineers active in the field of SAW technology, biomedical and other related engineering disciplines. The topics range from basic SAW theory, materials and phenomena to advanced applications such as sensors actuators, and communication systems. As such, in addition to theoretical analysis and numerical modelling such as Finite Element Modelling (FEM) and Finite Difference Methods (FDM) of SAW devices, SAW based actuators and micro motors, and SAW based micro sensors are some of the exciting applications presented in this book. This collection of up-to-date information and research outcomes on SAW technology will be of great interest, not only to all those working in SAW based technology, but also to many more who stand to benefit from an insight into the rich opportunities that this technology has to offer, especially to develop advanced, low-powered biomedical implants and passive communication devices.

How to reference

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Nick Shevyakhov and Sergey Maryshev (2010). Shear Elastic Wave Refraction on a Gap between Piezoelectric Crystals with Uniform Relative Motion, *Acoustic Waves*, Don Dissanayake (Ed.), ISBN: 978-953-307-111-4, InTech, Available from: <http://www.intechopen.com/books/acoustic-waves/shear-elastic-wave-refraction-on-a-gap-between-piezoelectric-crystals-with-uniform-relative-motion>

INTECH
open science | open minds

InTech Europe

University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia

InTech China

Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元

www.intechopen.com

Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

Phone: +86-21-62489820
Fax: +86-21-62489821

IntechOpen

IntechOpen

© 2010 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](#), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen