Satellite Motion

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1. Introduction

What is satellite?
The word satellite is coming from the Latin language (Latin *satelles* – escort, companion). Satellites are objects that rotate around the planets under the influence of the gravitational force. For example, the Moon is natural satellite of the Earth.
What is the artificial Earth satellite?
The artificial Earth satellites are artificial objects which are launched into orbit around the Earth by a rocket vehicle. This kind of satellites was named, the Human-Made Earth Satellites.
How do rockets function?
Aeroplanes work on the principle of buoyancy difference on their wings. This is the reason why aeroplanes can fly only in the air but not in the vacuum. Thus, an airplane cannot be used for launching satellites in their orbit around the Earth (Fig. 1).

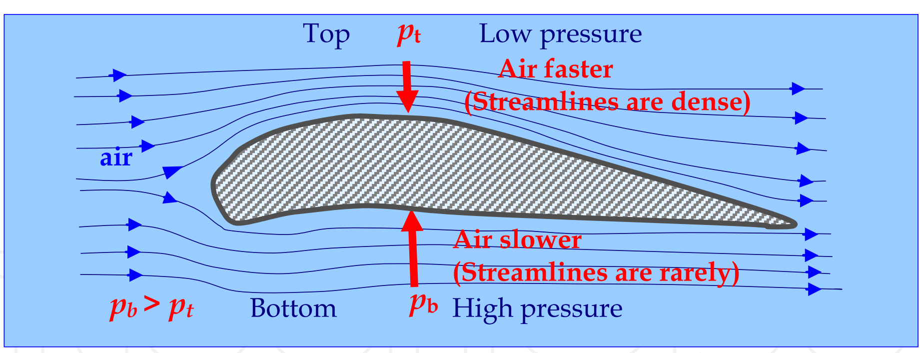


Fig. 1. When the aeroplanes have velocity in the air on the bottom of his wings they have higher pressure then on the top of wings. This difference of pressure is giving the force of buoyancy and the aeroplanes can fly.

It is also not possible to launch a human-made satellite into the orbit around the Earth with a cannon or a gun because a cannon-ball has the velocity of about 0.5 km/s. It means that this velocity of cannon-ball is about 15 times smaller than the first cosmic velocity (7.9 km/s). So for it has been possible to launch a satellite into an orbit around the Earth where there is vacuum is possible only with using rockets.
The word *rocket* comes from the Italian *Rocchetta* (i.e. *little fuse*), a name of a small firecracker. It is commonly accepted that the first recorded use of a rocket in battle was by the Chinese

in 1232 against the Mongol hordes at Kai Feng Fu. The Mongols were the first to have applied rocket technology in Europe as they conquered some parts of China and of Russia, Eastern and Central Europe.
Konstantin Tsiolkovsky (1857-1935) (Fig. 2) from the Imperial Russia and after from the Soviet Union published the first serious scientific work on space travels titled ''*The exploration of Cosmic Space by Means of Reaction Devices*'' in 1903. He is considered by many to be the *father of theoretical astronautics.* He also advocated the use of liquid hydrogen and oxygen for propellant, calculating their maximum exhaust velocity. His work inspired further research, experimentation and the formation of Society for Studies of Interplanetary Travel in 1924. Also in 1924, Tsiolkovsky wrote about multi-stage rockets, in ''*Cosmic Rocket Trains*''.

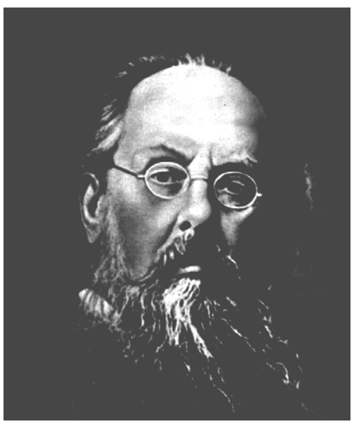


Fig. 2. Konstantin Tsiolkovsky

Fig. 3. Robert Goddard

Fig. 4. Sergei Korolev

Fig. 5. Wernher von Braun

In the USA, Robert Goddard (Fig. 3) began a serious analysis of rockets in 1912. It can thus be concluded that conventional solid-fuel rockets needed to be improved in three ways. One of them is that rockets could be arranged in stages. He also independently developed the mathematics of rocket flight. For his ideas, careful research, and great vision, Goddard was called the *father of modern astronautics.*
After the World War II in the USA Wernher von Braun (Fig. 5) and Sergei Korolev (Fig. 4) in the Soviet Union were the leaders in the advancing rockets technology.
The operational principle a rocket can be explained by means of a balloon (Fig. 6).
In a balloon the pressure of gas is practically equal on all sides (Fig. 6 a). When this balloon has an aperture then a particle of gas on this aperture will under pressure of gas be thrown out with velocity v_g (Fig. 6 b). The pressure in the balloon in opposite direction of the aperture will produce the pressure on the balloon and will give it the velocity v_B. The pressures in the other directions will be mutually cancelled in opposite direction. The rocket operates on this principle.

A rocket travelling in vacuum is accelerated by the high-velocity expulsion of a small part of its mass (gas). The Fig. 7. represents a rocket with the situations before and after the explosion. This is closed material system and for this system linear momentum needs to be conserved. So we can say that the momentum in the beginning position ($M \cdot \mathbf{v}$) is equal to momentum of this system after the explosion when a particle with the mass (dm) is be thrown out with velocity ($\mathbf{u}_{rel}$) in the opposite direction of this rocket velocity.

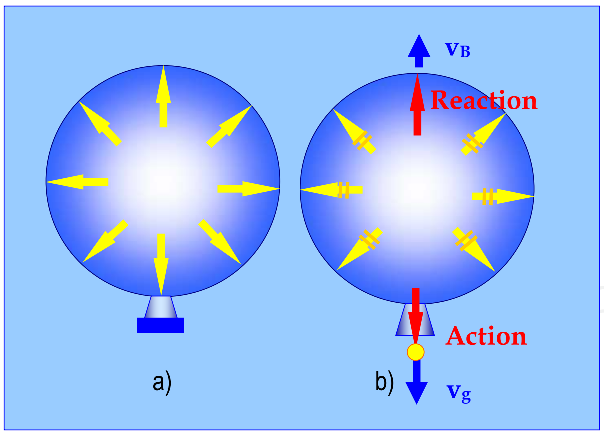


Fig. 6. A balloon closed under gas pressure and after opening an aperture.

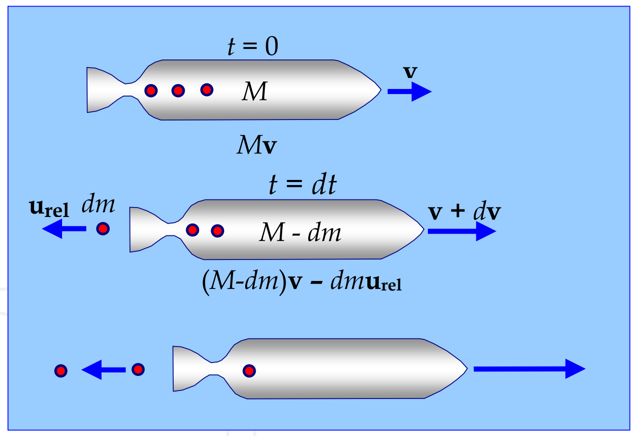


Fig. 7. The principle of a rocket operation.

It means that the momentum for a particle is negative ($-dm\cdot\mathbf{u}_{rel}$) and we can write the equation (Carton, 1965) and (Danby, 1989):

$$M\cdot\mathbf{v}=(M-dm)\cdot(\mathbf{v}+d\mathbf{v})-dm\cdot\mathbf{u}_{\mathrm{rel}}\,. \tag{1}$$

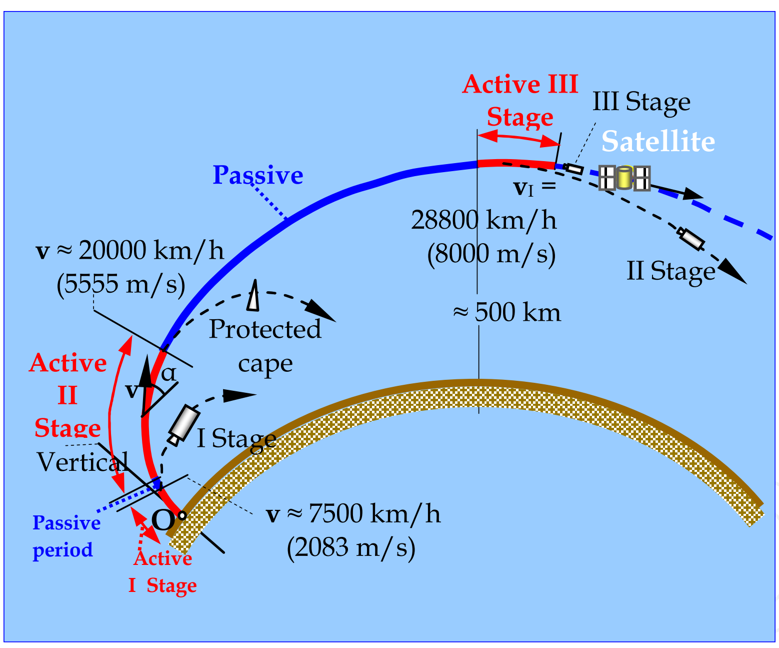


Fig. 8. Imagery of launching a satellite by rocket with the three stages.

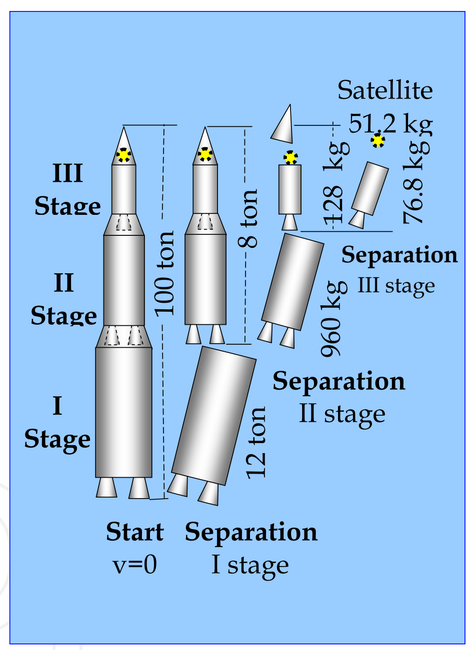


Fig. 9. For example during the start a three stages rocket may have the mass 100 t but mass of a space vehicle will be only 51.2 kg.

It follows from this equation that the velocity of rocket increased for the elementary magnitude

$$d\mathbf{v} = \frac{dm(\mathbf{v} + \mathbf{u}_{\text{rel}})}{M - dm}. \tag{2}$$

From this equation it is possible to see that the increase of the rocket velocity is larger if the velocity $\mathbf{u}_{\text{rel}}$ of the particles of gas is maximally the greater when the mass of particle dm has some magnitude. This is the reason why the constructors of rockets like to make rockets with very high (maximal) velocity of the particles (gas) of the rocket.
Usual rockets have vertical start. Longer delaying of rockets in the Earth gravitation field causes the loss of velocity but also to big thrust during the start is not suitable. So rockets are usually made in same stages (Fig. 8 and 9). During the starts of rockets the consummation of fuel is very large so that a satellite or a space vehicle enters into the orbit with a mass practically next to nothing (see for example Table 1).

Table 1. Data on initial mass of a rocket in a start, spend fuel, thrown parts for a launching space vehicle of the mass 51.2 kg

Stage	1	2	3
Initial mass	100 t	8 t	640 kg
Spend fuel	80 t	6.4 t	512 kg
Thrown part	-	12 t	960 kg
Definitely mass	20 t	1.6 t	128 kg
Velocity	3860 (m/s)	7720 (m/s)	11580 (m/s)

2. Planet Motion

Until the 17th century people were thinking that the Sun and planets are rotated around the Earth by circles. Such opinions were practically usual until Johann Kepler.

2.1 Kepler's Laws of Planetary Motion

Johann Kepler (1546 -1601) discovered the laws of planetary motion empirically from Tycho Brahe's (1546 -1601) astronomical observations of the planet Mars. The first and the second laws he published in *Astronomia Nova (New Astronomy*) in 1609, and the third law in *Harmonices mundi libri V (Harmony of the World*) in 1619.

a) Kepler's First Law of Planetary Motion

This law can be expressed as follows:

The path of each planet describes an ellipse with the Sun located at one of its foci. (The Law of Ellipse)

This first Kepler's Law (Fig. 10 and 11) is sometimes referred to as *the law of ellipse* because planets are orbiting around the Sun in a path described as an ellipse. An ellipse is a special curve in which the sum of the distances from every point on the curve to two other points (foci F_1 and F_2) is a constant.

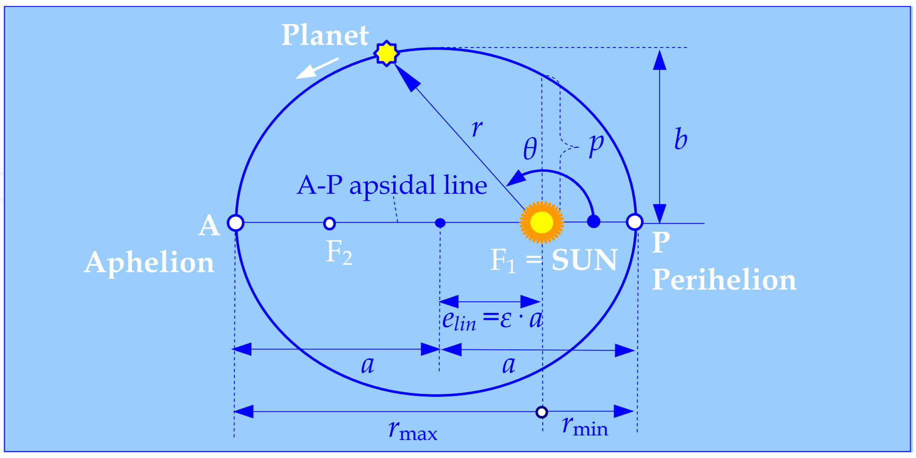


Fig. 10. The Orbit of a planet is an ellipse and its elements.

Before Kepler the Greek astronomer Ptolemy and many others after him were thinking that the Sun and planets travel in circles around the Earth. The ellipse can be mathematically expressed in the polar coordinate system by this equation:

$$r = \frac{p}{1 + \varepsilon \cos\theta}, \tag{3}$$

where (r, θ) are heliocentric polar coordinates for the orbit of planet (r – the distance between the Sun and a planet, θ – angle from the perihelion to the planet as seen from the Sun, respectively known as *the true anomaly*), p is the semi-latus rectum, and ε is the numerical eccentricity.
At $\theta = 0°$ the minimum distance is equal to

$$r_{\min} = \frac{p}{1+\varepsilon}. \tag{4}$$

At $\theta = 90°$ the distance is equal p.
At $\theta = 180°$ the maximum distance is

$$r_{\max} = \frac{p}{1-\varepsilon}. \tag{5}$$

The semi-major axis is the arithmetic mean between $r_{\min}$ and $r_{\max}$:

$$a = \frac{r_{\max} + r_{\min}}{2} = \frac{p}{1-\varepsilon^2}. \tag{6}$$

The semi-minor axis is the geometric mean between $r_{\min}$ and $r_{\max}$:

$$b = \sqrt{r_{\min} \cdot r_{\max}} = \frac{p}{\sqrt{1-\varepsilon^2}} = a\sqrt{1-\varepsilon^2}\ . \tag{7}$$

The semi-latus rectum p is equal to

$$p = \frac{b^2}{a}\ . \tag{8}$$

The area A of an ellipse is

$$A = \pi ab\,. \tag{9}$$

In the special case when $\varepsilon = 0$ then an ellipse turns into a circle where $r = p = r_{\min} = r_{\max} = a = b$ and $A = \pi r^2$.

Using ellipse-related equations Kepler's procedure for calculating heliocentric polar coordinates r, θ, for planetary position as a function of the time t from Perihelion, and the orbital period P, follows four steps:

1. Compute the *mean anomaly* M_a from the equation $M_a = \dfrac{2\pi t}{P}\ .$ (10)
2. Compute the *eccentric anomaly* E by numerically solving Kepler's equation:

$$M_a = E - \varepsilon \sin E\,. \tag{11}$$

3. Compute the *true anomaly* θ by the equation $\tan\dfrac{\theta}{2} = \sqrt{\dfrac{1+\varepsilon}{1-\varepsilon}}\tan\dfrac{E}{2}\ .$ (12)
4. Compute the *heliocentric distance* r from the equation $r = \dfrac{p}{1+\varepsilon\cos\theta}\ .$ (13)

For the circle $\varepsilon = 0$ we have simple dependence $\theta = E = M_a$.

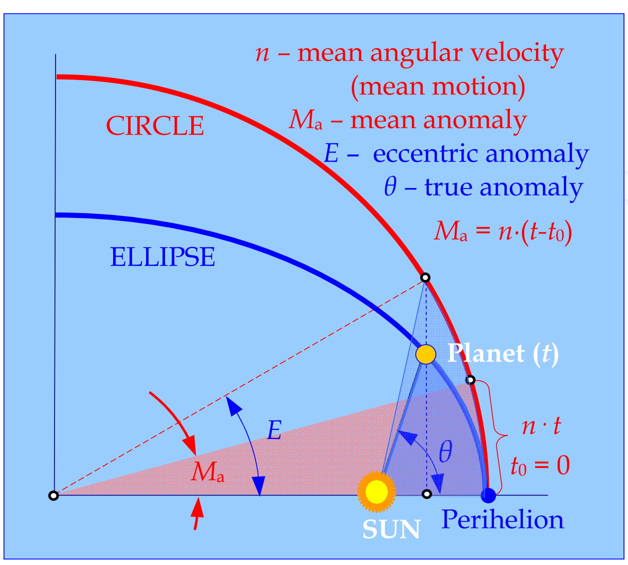


Fig. 11. Elements of parameters of a satellite orbit.

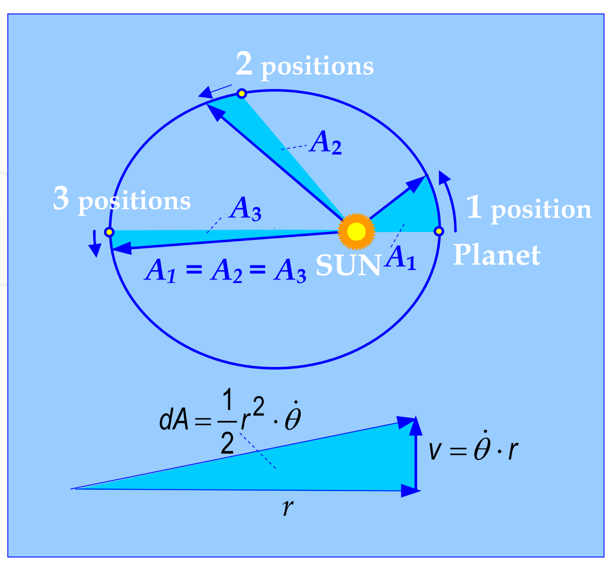


Fig. 12. The radius vector drawn from the Sun to a planet covers equal areas in equal times.

b) The Second Kepler's Law of Planetary Motion
This law can be expressed as follows:

The radius vector drawn from the Sun to a planet covers equal areas in equal times. (The Law of equal areas)

Mathematically this law can be expressed with the equation:

$$\frac{d}{dt}\left(\frac{1}{2}r^2\dot{\theta}\right) = 0\,, \tag{14}$$

where $\dot{\theta}$ is angular velocity of true anomaly, $\frac{1}{2}r^2\dot{\theta}$ is the "*areal velocity*" that the radius vector *r* drawn from the Sun to the planet sweeps in one second (Fig. 12).

From this law it follows that the speed at which any planet moves through space is continuously by changing. A planet moves most quickly when it's closer to the Sun and more slowly when it is further from the Sun.

c) The Third Kepler's Law of Planetary Motion
This law can be expressed as follows:

The squares of the periodic times of the planets are proportional to the cubes of the semi-major axes of their orbits. (The harmonic law)

This law is giving the relationship between the distance of planets from the Sun and their orbital periods. Mathematically and symbolically it's possible to express as follows:

$$P^2 \propto a^3\,,$$

where *P* is the orbital period of circulate planet around the Sun and *a* is the semi-major axis of this orbit. Because this proportionality is the same for any planet which rotates around the Sun it's possible to write the next equation:

$$\frac{P_{planet1}^2}{a_{planet1}^3} = \frac{P_{planet2}^2}{a_{planet2}^3}\,, \quad \text{namely} \quad \frac{P_{planet1}^2}{P_{planet2}^2} = \frac{a_{planet1}^3}{a_{planet2}^3}\,. \tag{15}$$

3. The Physical Laws of Motions

Sir Isaac Newton's formulated three fundamental laws of the classical mechanics and the law of gravitation in his great work *Philosophieæ Naturalis (Principia Mathematica)* published on July 5, 1687. Before Isaac Newton the great contribution to the advance of mechanic was given by Galileo, Kepler and Huygens.

3.1 The First Law of Motion – Law of Inertia

This law can be expressed as follows:

Everybody persists in its state of being at rest or moving uniformly straight forward, except insofar as it is compelled to change its state by force impressed.

Newton's first law of motion is also called *the law of inertia*. It states that if the vector sum of all forces acting on an object is zero, then the acceleration of the object is zero and its velocity is constant. Consequently:

- An object that is at rest will stay at rest until a balanced force acts upon it.
- An object that is in motion will not change its velocity until a balanced force acts upon it.

If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion).

3.2 The Second Law of Motion – Law of Force

This law can be expressed as follows:

Force equals mass times acceleration.
If the resultant force acting on a particle is not zero, the particle will have acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

This law may be expressed by the equation:

$$\mathbf{F} = m\mathbf{a}\,, \tag{16}$$

where $\mathbf{F}$ is the vector of force, m the mass of particle and $\mathbf{a}$ is the vector of acceleration (Fig. 13. a).

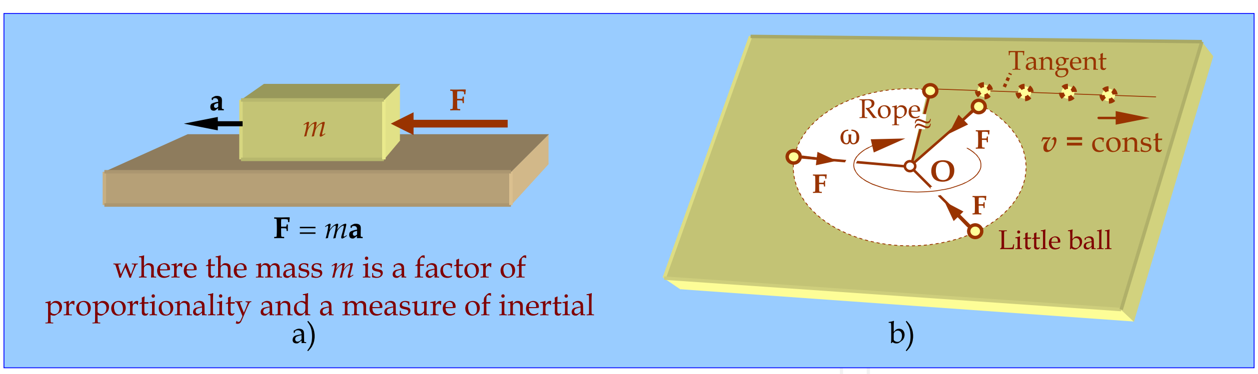


Fig. 13. a) An acceleration **a** of a free body on a horizontal plane under influence of a force **F**, b) In the rotation when the rope is broken, a little ball shall start moving with constant velocity along the line of tangent in the horizontal plane.

Really, this is differential equation which represents a basic equation of motion or basic equation of dynamic.
Alternatively this law can be expressed by the equation:

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v})\,, \tag{17}$$

where the product $m\mathbf{v}$ is the momentum: m - the mass of particle and $\mathbf{v}$ - the velocity. So, we can say:

The force is equal to the time derivative of the body's momentum.

3.3 The Third Law – Law of action and reaction

This law can be expressed in the following way:

To every action there is an equal by magnitude and opposite reaction (Fig. 14. a).

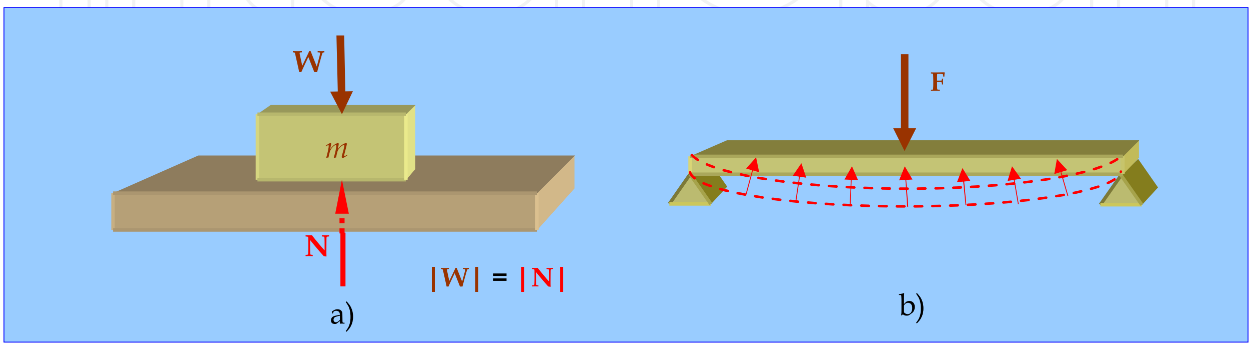


Fig. 14. a) Under the influence of the weight **W** of a body, a normal reaction of its support occurs, b) A beam under loading by a force **F** will be deformed as reaction to an active force **F**.

This law can be also expressed:

The forces of action and reaction between bodies in contact have the same magnitude, same line of action and opposite sense.

3.4 The Law of Gravitation – The Law of Universal Gravitation

Isaac Newton stated that two particles at the distance r from each other and, respectively, of mass M and m, attract each other with equal and opposite forces $\mathbf{F_1}$ and $\mathbf{F_2}$ directed along the line joining the particles (Fig. 15). The common magnitude F of these two forces is:

$$F = G\frac{Mm}{r^2}, \qquad (18)$$

where G is the *universal constant of gravitation* $G \approx 6.67428 \times 10^{-11}\ m^3kg^{-1}s^{-2}$),or approximately $3.44\times10^{-8} ft^4/lb\text{-}sec^4$ in British gravitational system of units (Beer & Johnston 1962).

The force of attraction exerted by the Earth on body of the mass m located on or near its surface is defined as the weight (W) of the body (Fig. 16)

$$W = mg, \qquad (19)$$

where g is the acceleration of gravity, being also the acceleration of force of weight.

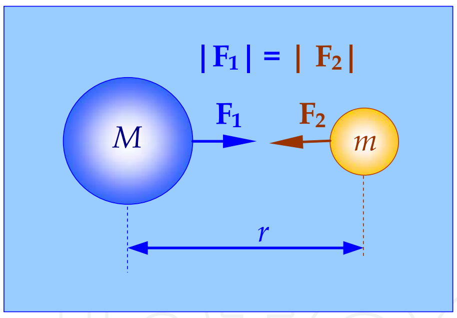


Fig. 15. Newton's Law of the universal gravitation.

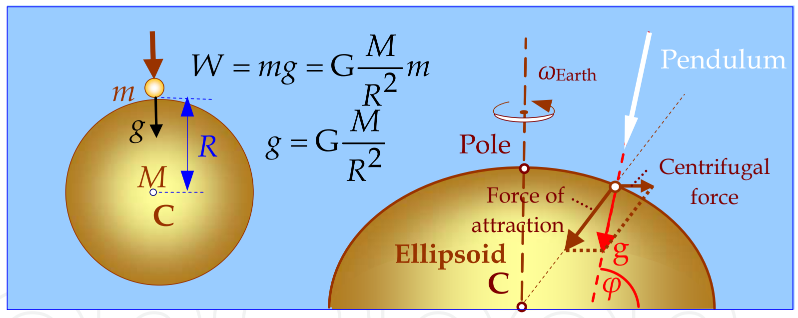


Fig. 16. The weight of a body on the surface of the Earth and the influence of centrifugal forces.

Because this force is really the force of universal gravitation it's possible to say

$$W = mg = G\frac{M}{R^2}m\,. \tag{20}$$

From this equation it follows that the acceleration of gravity is

$$g = \frac{GM}{R^2}\,. \tag{21}$$

The Earth is not truly spherical so the distance R from the centre of the Earth depends on the point selected on its surface. This will be the reason why the weight of the same body will also not be the same weight on different geographical latitude and altitude of the considered point. For more accurate definition of the weight of a body it's necessary to include a component representing the centrifugal force due to the rotation of the Earth. So, the values of g for a body in rest at the sea level vary from 9.780 m/s² (32.09 ft/s²) at the equator to 9.832 m/s² (32.26 ft/s²) at the poles.

3.5 D'Alembert's Principle

Jean le Rond d' Alembert (1717-1783) postulated the principle called by his name from the basic equation of dynamic:

$$\mathbf{F} = m\cdot\mathbf{a}\,. \tag{22}$$

This equation can be written in this form:

$$\mathbf{F} + (-m\cdot\mathbf{a}) = 0\,. \tag{23}$$

From this equation the magnitude $(-m\cdot\mathbf{a})$ is called *inertial force*. So, this equation represents an equation of fictive equilibrium where **F** represents resultant of all active and reactive forces, and inertial force which has the magnitude $m{\cdot}a$, but in opposite direction of the acceleration **a**. This equation is named *equation of dynamic equilibrium.*

For example, when a body is rotating in a circle with constant velocity v restrained by a rope length R then centrifugal force appears (Fig. 17).

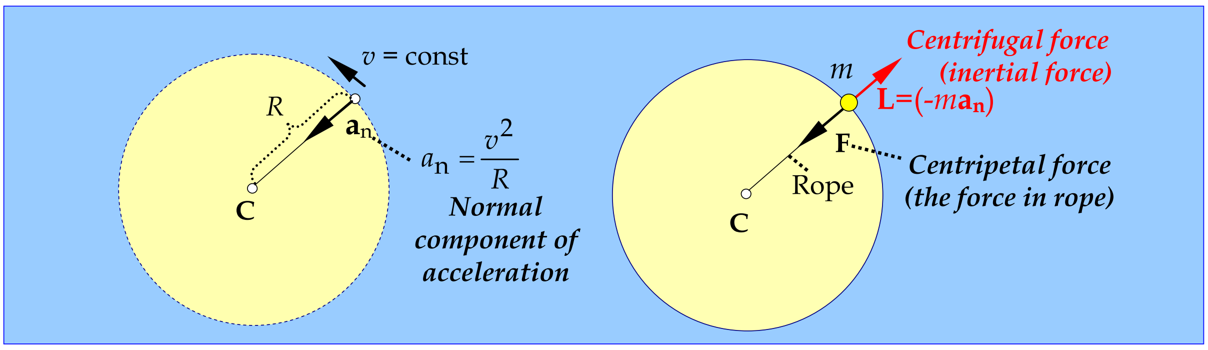


Fig. 17. Centrifugal force at rotation with constant magnitude of velocity v along a circle, and then centrifugal force **L** appears.

3.6 Potential due to a Spherical Shell

A basic result proved by Isaac Newton is that spherical shell which is homogeneous (with constant density) attracts an exterior point with mass m=1 as if all of the mass M of the spherical shell concentrated at its centre C (Fig. 18). This is the same, as if we have homogenous concentric layers but with different densities and whole masses M then an exterior mass point m attracts as if all of the mass M of the spheres was concentrated at its centre (Fig. 19).

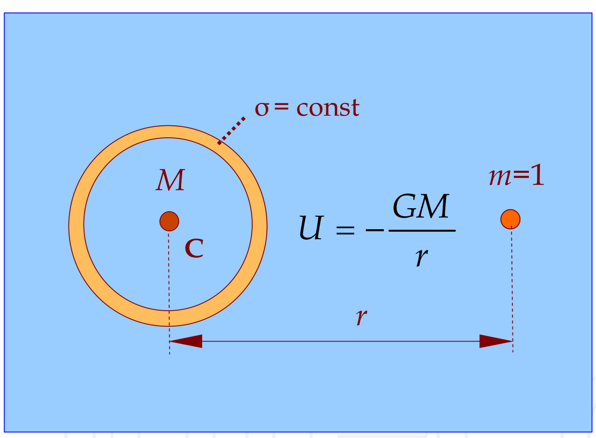


Fig. 18. The potential due to the solid spherical shell.

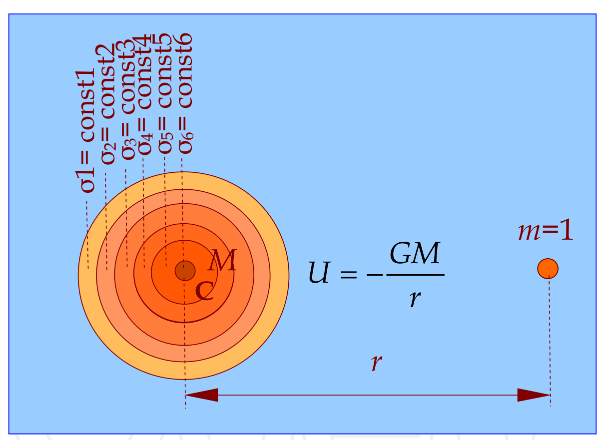


Fig. 19. The potential due to the concentric solid homogeneous spherical shells.

This fundamental result allows us to consider that the attraction between the Earth and the Sun, for example, to be equivalent to that between two mass points.
So we can say:

The solid sphere of constant density attracts an exterior unit mass though all of its masses were concentrated at the centre.

The potential, therefore, due to a spherical body homogeneous in concentric layers, for a point outside the sphere is

$$U = -\frac{GM}{r}\,, \tag{24}$$

where r is the distance from the point with mass m to the centre C of the mass of the homogeneous sphere or to the centre C of the concentric homogeneous spheres.

4. Determination of Orbits

Jacques Philippe Marie Binet (1786-1856) derived the differential equation in the polar coordinate system of the motion free material particle under action of the central force when areal velocity by the second Kepler's law is constant. This Binet's differential equation can be put down in writing

$$-mC^2u^2\left[\frac{d^2u}{d\theta^2}+u\right] = F_{\mathrm{rad}}\,, \tag{25}$$

where is $u = \frac{1}{r}$, C - double areal velocity ($C = \dot{\theta}r^2$), F_{rad} gravitation force of the central body with mass M on the free particle with mass m ($F_{\mathrm{rad}} = -GMmu^2$), where minus sign indicates that this is an attracting force, and plus sign stands for the repulsive force.
This differential equation is equation of free particle motion in a plane displayed in the polar coordinate system. Thus, inhomogeneous differential equation is obtained

$$\underbrace{\left[\frac{d^2u}{d\theta^2}+u\right]}_{\substack{\text{Homogeneous}\\ \text{Part}}} = \underbrace{\frac{GM}{C^2}}_{\substack{\text{Inhomogeneous}\\ \text{Part}}}\,. \tag{26}$$

The solution for the homogeneous part of this equation is

$$u_1 = B\cos(\theta - \theta_0)\,, \tag{27}$$

where B and θ_0 are the constants of integration. Choosing the polar axis so that $\theta_0 = 0$ we can write

$$u_1 = B\cos(\theta) \tag{28}$$

and for the inhomogeneous part of the equation

$$u_2 = \frac{GM}{C^2} = \frac{1}{p}\,. \tag{29}$$

The solution of this inhomogeneous differential equation (26) is

$$u = u_2 + u_1 = \frac{1}{p}\left[1 + \frac{C^2}{GM} B\cos\left(\theta\right)\right]. \tag{30}$$

The equation for the ellipse and for the other conic section in the polar coordinate system can be written in the form

$$\frac{1}{r} = \frac{1}{p}\left[1 + \varepsilon\cos\theta\right]. \tag{31}$$

After comparing the equations (30) and (31) it is possible to see that the equation (30) truly represents the equation of a conic section. The product of the constants B and $C^2/(GM)$ defines the *eccentricity* ε of the conic section. So it can be expressed by the equation (Beer & Johnston, 1962)

$$\varepsilon = \frac{B}{\frac{GM}{C^2}} = \frac{BC^2}{GM}. \tag{32}$$

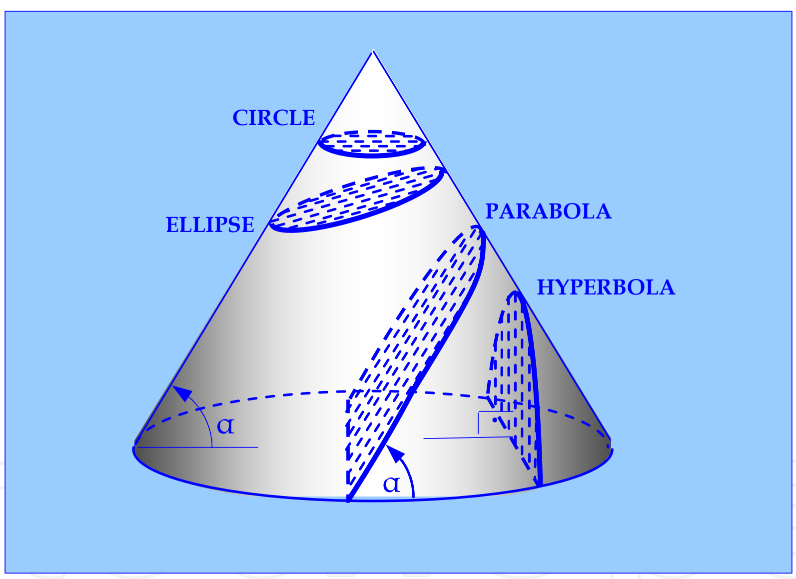


Fig. 20. The conic sections: circle, ellipse, parabola and hyperbola.

Four cases may be distinguished for different eccentricities (Fig. 20).
1) The conic section is a *circle* when is $\varepsilon = 0$.
2) The conic section is an *ellipse* when $0 < \varepsilon < 1$.
3) The conic section is a *parabola* when $\varepsilon = 1$.
4) The conic section is a *hyperbola* when $\varepsilon > 1$.
Of cause for the planets and for the satellites orbits can be only circulars or ellipses.

5. The Two-Body Problem

It is possible to investigate the motion of two bodies that are only under their mutual attraction. It can also be assumed that the bodies are symmetrical and homogeneous and that they can be considered to be point masses. So we can do analysis of the motion of planets and the Sun.

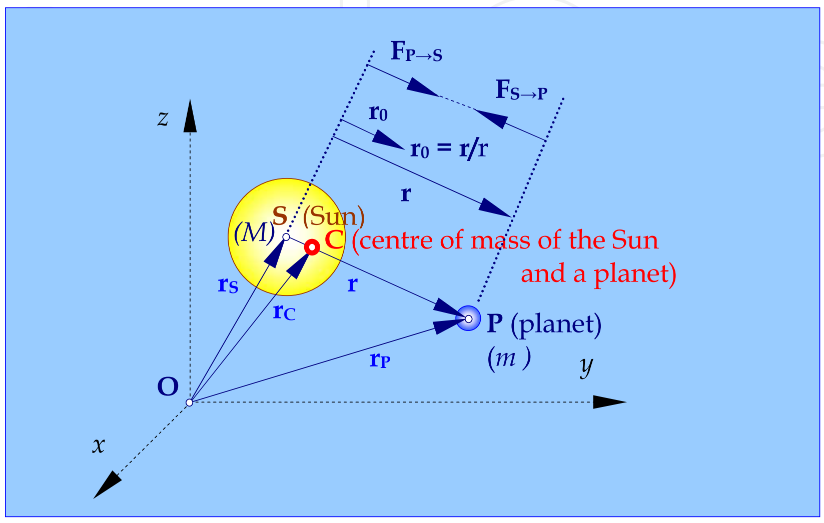


Fig. 21. Motion of the Sun and a planet in two-body problem.

The differential equation of the Sun motion (Fig. 21) is

$$M\frac{d^2\mathbf{r}_S}{dt^2} = +G\frac{Mm}{r^2}\frac{\mathbf{r}}{r}. \qquad (33)$$

The sign + is because the force $\mathbf{F}_{P\to S}$ has the same orientation as the vector $\mathbf{r}_0$. The differential equation of the planet motion is

$$m\frac{d^2\mathbf{r}_P}{dt^2} = -G\frac{Mm}{r^2}\frac{\mathbf{r}}{r}. \qquad (34)$$

The sign – is because the force $\mathbf{F}_{S\to P}$ has the same orientation as the vector $\mathbf{r}_0$. After summing up the equations (33) and (34) we can write

$$M\frac{d^2\mathbf{r}_S}{dt^2} + m\frac{d^2\mathbf{r}_P}{dt^2} = 0 \quad \text{or} \quad \frac{d^2}{dt^2}\left(M\mathbf{r}_S + m\mathbf{r}_P\right) = 0. \qquad (35)$$

From the static it is known that the sum of the moment forces is equal to the moment of resultant. So we can say that the sum of the moment masses is equal to the moment of resultant mass. Now it is possible to write

$$M\mathbf{r}_S + m\mathbf{r}_P = (M+m)\mathbf{r}_C. \qquad (36)$$

After the first and the second derivation we have

$$\frac{d}{dt}\left(M\mathbf{r}_S + m\mathbf{r}_P\right) = \frac{d}{dt}\left[(M+m)\mathbf{r}_C\right] \qquad \text{and}$$

$$\frac{d^2}{dt^2}\left(M\mathbf{r}_S + m\mathbf{r}_P\right) = \frac{d^2}{dt^2}\left[(M+m)\mathbf{r}_C\right] = \frac{d^2\mathbf{r}_C}{dt^2}(M+m). \tag{37}$$

From the equations (35) and (37) it follows

$$\frac{d^2\mathbf{r}_C}{dt^2}(M+m) = 0 \quad \text{or} \quad \frac{d^2\mathbf{r}_C}{dt^2} = 0\,. \tag{38}$$

Therefore the centre of this material system has no acceleration, namely this material system is in the inertial motion with the possibility to move with constant velocity $\mathbf{v}_C$, or remain at rest.
The equation (33) may be multiplied by m, and equation (34) by M and after subtracting the equation (33) from (34) we can write

$$Mm\left[\frac{d^2\mathbf{r}_P}{dt^2} - \frac{d^2\mathbf{r}_S}{dt^2}\right] = -G\frac{mM}{r^2}\frac{\mathbf{r}}{r}\left(M+m\right), \tag{39}$$

$$Mm\frac{d^2}{dt^2}\left[\mathbf{r}_P - \mathbf{r}_S\right] = -G\frac{mM}{r^2}\frac{\mathbf{r}}{r}\left(M+m\right) \tag{40}$$

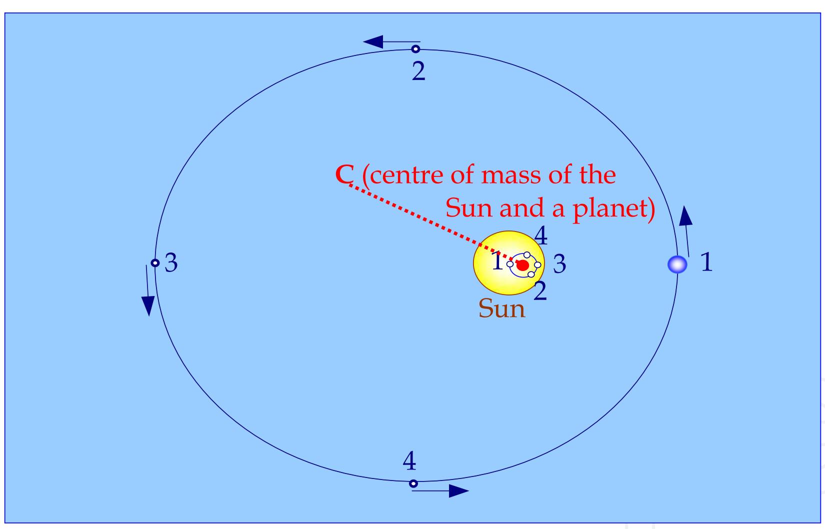


Fig. 22. The Sun is moving also around the centre C of mass of the Sun and a planet by a small ellipse and a planet is moving about the same centre C by the bigger ellipse, not around the geometrical centre of Sun.

After dividing the equation (40) by M and taking from Fig. 21 that $\mathbf{r}=\mathbf{r}_P-\mathbf{r}_S$ we can write

$$m\frac{d^2\mathbf{r}}{dt^2} = -G\frac{m(M+m)}{r^2}\frac{\mathbf{r}}{r}\,. \tag{41}$$

This is differential equation (41) of the planet motion when taken into account and the planet acting on the Sun. It is easy to prove that this planet is really rotating around the centre of mass (C) of the Sun and the planet. Also the Sun's geometrical center is rotating by the small ellipse around the centre of mass (C) (Fig. 22).
Hence:

The planet is rotating around the centre of masses (C) of the Sun and the planet by the bigger ellipse. The geometrical centre of the Sun also rotates around the centre of masses (C) by a small ellipse.

6. Satellite Motion

The problem, of two bodies is solved exactly in the celestial mechanics, but only in the special case if both bodies are having small dimensions, i.e. if the Sun and a planet can be thought of as particles. In this special case the motions of particles around the body with finite dimensions is also included, if this body with finite dimensions has the central spherical field of forces. (For example, as a homogeneous ball (Fig. 18) or concentric solid homogeneous spherical shells with different densities (Fig. 19)). Just because our Earth is not a ball and with homogeneous masses some discrepancies appear at satellite motions around the Earth from the exact solutions of two bodies when we imagine whole mass of the Earth as concentrated in it the centre of mass. For the solution of the problem of the motion of bodies (two particles) exactly valuable are three Kepler's laws from which fallow that the satellite would be moving constantly in the same plane by the ellipse with constant areal velocity.

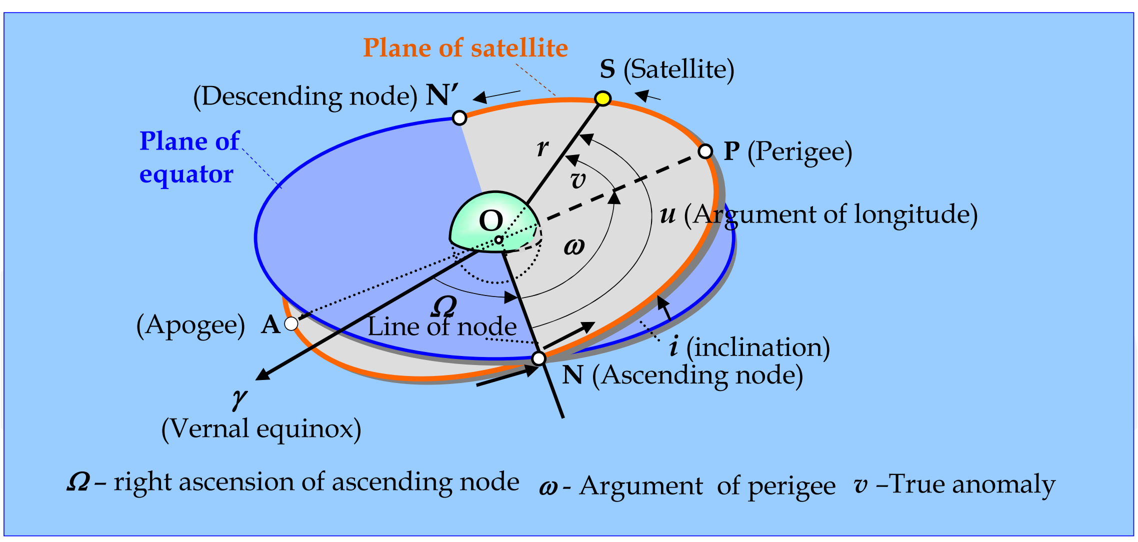


Fig. 23. Keplerian orbital parameters.

The positions of satellites are determined with six Keplerian orbital parameters: Ω, i, ω, a, e and v or t (Fig. 23):

- *The orientation of orbits* in space is determined by:

Ω - *the right ascension of ascending node* (the angle measured in the equator plane between the directions to the vernal equinox and ascending node N where the satellite crosses equatorial plane from the south to the north celestial sphere), i – *the inclination of orbit*, (the angle between the equatorial plane and orbital plane) and ω – *the argument of perigee* (the angle between the ascending node and the direction to perigee (as the nearest point of satellite)).

- *The dimensions of orbit* are determined by: a - *the semi-major axis* and ε – *the numerical eccentricity* of an ellipse.
- *The position of satellite on its orbits* is determined by: v - *the true anomaly* (as the angle between the directions to perigee and instantaneous position of satellite) or by t - the difference of time in instantaneous position and the time in perigee.

All Kepler's laws and Newton's laws for a planet motion are valued also for the Earth's satellites motion but at satellites there are some more perturbations.

6.1 Required Velocity for a Satellite

A body will be a satellite in a circular orbit around the Earth if it has velocity in the horizontal line so that centrifugal force is equal to centripetal force which is produced by the Earth's gravitation attraction (Fig. 24). So it can be expressed with the equation:

$$m\frac{v_I^2}{R+H_s} = G\frac{mM}{(R+H_s)^2}, \tag{42}$$

where: m - mass of a satellite, M – the Earth's mass, R - radius of the Earth, H_s - altitude of a satellite above the surface of the Earth, G constant of universal gravitation and v_I velocity of a body which will become the satellite.

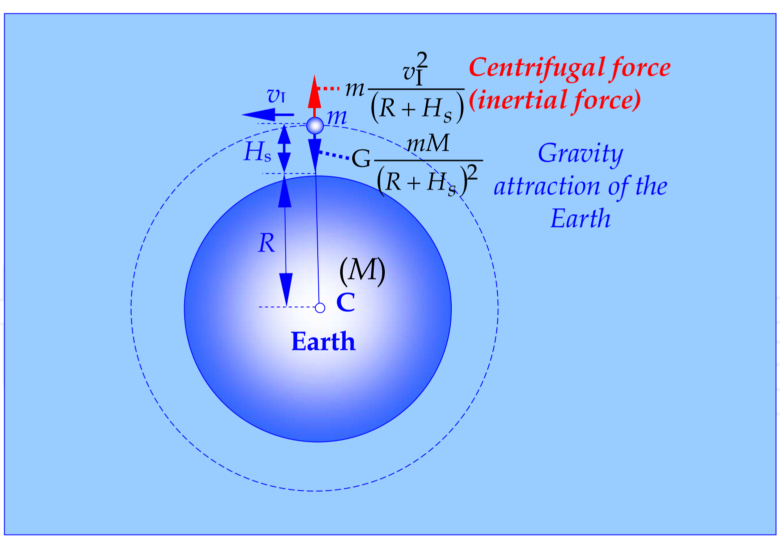


Fig. 24. On the satellite in orbit act the Earth's gravity attraction and the centrifugal force.

From this equation (42) next the equation follows

$$v_I = \sqrt{\frac{GM}{R+H_s}}. \tag{43}$$

The equation (21) can be expressed as

$$GM = gR^2 \tag{44}$$

and after including this equation (44) into (43) we have

$$v_\mathrm{I} = \sqrt{\frac{gR^2}{R + H_\mathrm{S}}} . \tag{45}$$

This velocity v_I is sometimes called the *first cosmic velocity,* see Table 2.

Table 2. The first cosmic velocity depending on the heights over the Earth.

The heights over the Earth (km)	200	300	400	500	1000	3000	5000
Velocity v_I (m/s)	7791	7732	7675	7619	7356	6525	5924

6.2 Period of Circulation a Satellite around the Earth by a Circle

Since a satellite makes a circle $2\pi(R + H_\mathrm{S})$ long around the Earth in a time P (period) and because its velocity is constant then the equation (45) can be expressed as

$$2\pi(R + H_\mathrm{s}) = \sqrt{\frac{gR^2}{R + H_\mathrm{s}}}\, P. \tag{46}$$

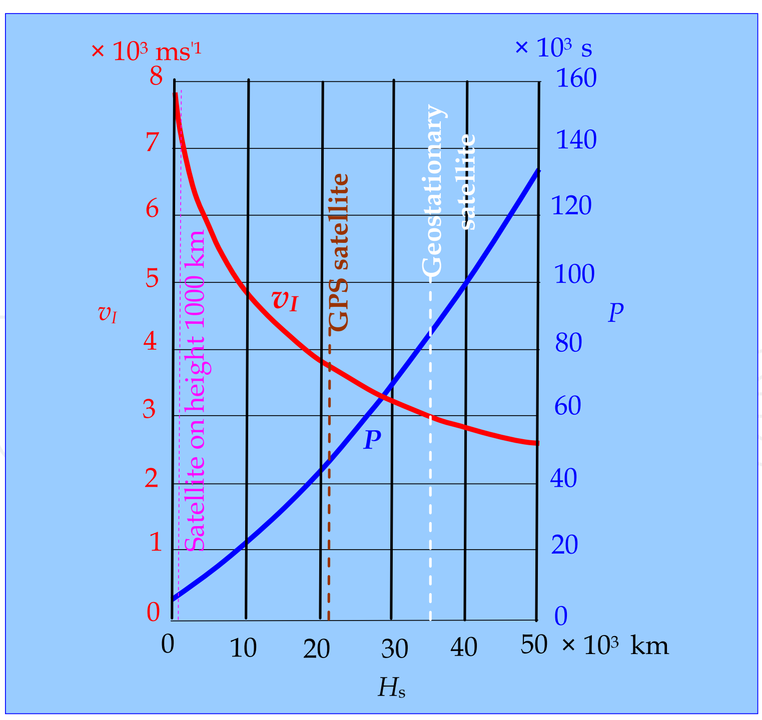


Fig. 25. Dependence the velocity v_I on the circular orbit and the period P of satellite height over the Earth's surface.

From this equation it follows

$$P = 2\pi\sqrt{\frac{(R+H_S)^3}{gR^2}} \,. \tag{47}$$

With the help of the equations (45) and (47) graphs on the Fig. 25 are constructed.

6.3 Escape Velocity

The escape velocity is minimum velocity of a body that a spacecraft must have in order to escape from the Earth's gravitation field or of other celestial bodies.
In order to launch a spacecraft into the space out of the Earth's gravitation field, it's necessary to do mechanical work for lifting a spacecraft from r=R to r=∞ (Fig. 26).

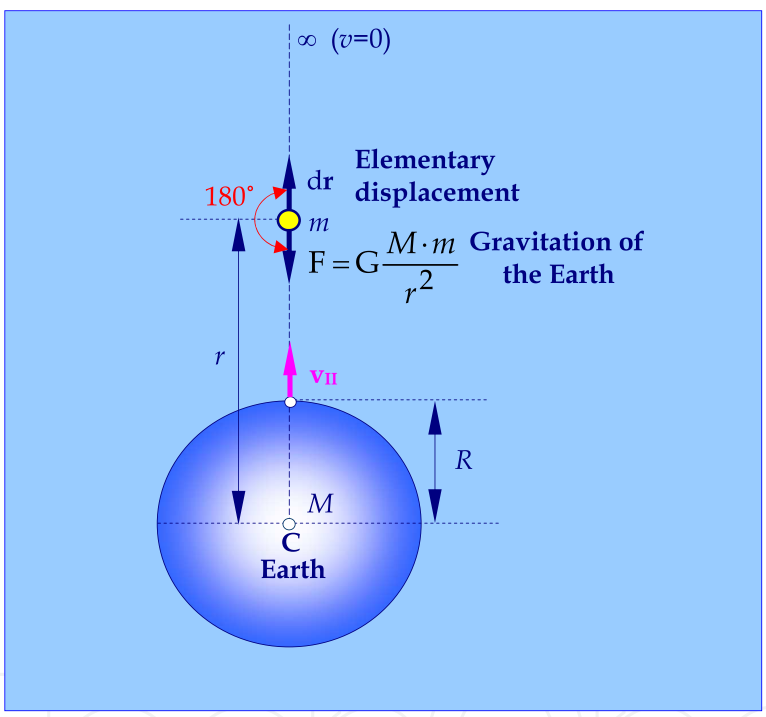


Fig. 26. Launching a spacecraft in the space out of Earth gravitation field (the second cosmic velocity).

The elementary work of the gravitation force **F** during the displacement $d\mathbf{r}$ is defined as the scalar quantity

$$dW = F \cdot dr\cos 180^{\circ} = -F \cdot dr \,. \tag{47}$$

For the work needed by the a spacecraft from R to the infinite ∞ we need to do the integration

$$W = -GMm\int_R^{\infty}\frac{dr}{r^2} = -GMm\frac{1}{R} \,. \tag{48}$$

The kinetic energy of a particle at the end minus the kinetic energy at the start is equal to the work, so we have

$$0 - \frac{mv_{II}^2}{2} = -GMm\frac{1}{R}, \quad \text{namely} \quad v_{II} = \sqrt{2GM\frac{1}{R}}. \tag{49}$$

Since is $GM = gR^2$, according to the equation (44), the equation (49) can be written as

$$v_{II} = \sqrt{2gR}. \tag{50}$$

When a body has the velocity v_{II} then it will abandon gravitation field of the bigger body. This velocity v_{II} is called by the some people the *second cosmic velocity*.
From the equation (50) for the Earth it follows that the second cosmic velocity is 11.2 km/s (6.96 mi/second) and in this case spacecrafts will abandon gravitation field of the Earth.

6.4 The Potential of a Body with Irregular Disposition of Mass

The elementary potential of an attraction force of a parcel P_1 with mass dM in the point P (Fig. 27) is equal

$$dU = G\frac{dM}{\Delta}. \tag{51}$$

where: G – the universal gravitational constant and Δ the distance point P of P_1.

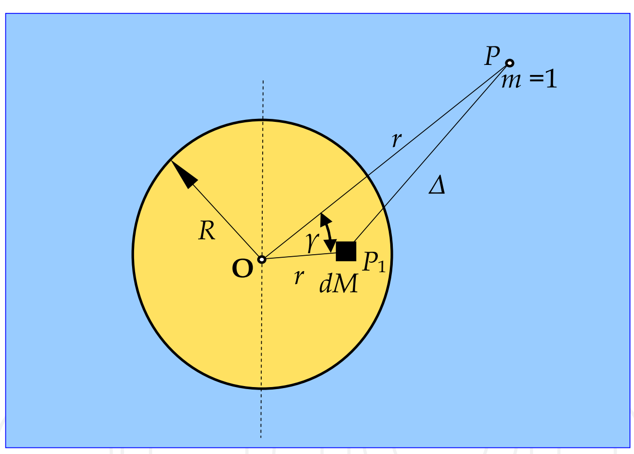


Fig. 27. The elementary potential of an attraction force of a body with arbitrary disposition of mass.

For a body with finite dimensions and with arbitrary disposition of mass it can be written in this form

$$U = G\int_M \frac{dM}{\Delta}. \tag{52}$$

This equation can be expressed by spherical functions in the form

$$U = G\frac{M}{r}\left[1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^n P_{nm}(\sin\phi)\left(c_{nm}\cos m\lambda + s_{nm}\sin m\lambda\right)\right], \tag{53}$$

where the potential is evaluated at a point whose geocentric spherical coordinates are:
r – the geocentric distance, φ – the geocentric latitude, λ – the longitude measured eastwards and R - the Earth's equatorial radius, c_{nm} and s_{nm} – the coefficients of the Earth's harmonics. The origin of the coordinate system is in the centre of Earth's mass, the geocentric longitude λ is calculated starting from Greenwich meridian and can have the values $0 \le \lambda \le 2\pi$, the geocentric latitude φ can have the values $(+\pi/2) \ge \varphi \ge (-\pi/2)$. In the equation (53) the expression P_{nm} (sin φ), is spherical function of sin φ which is determined by this expression

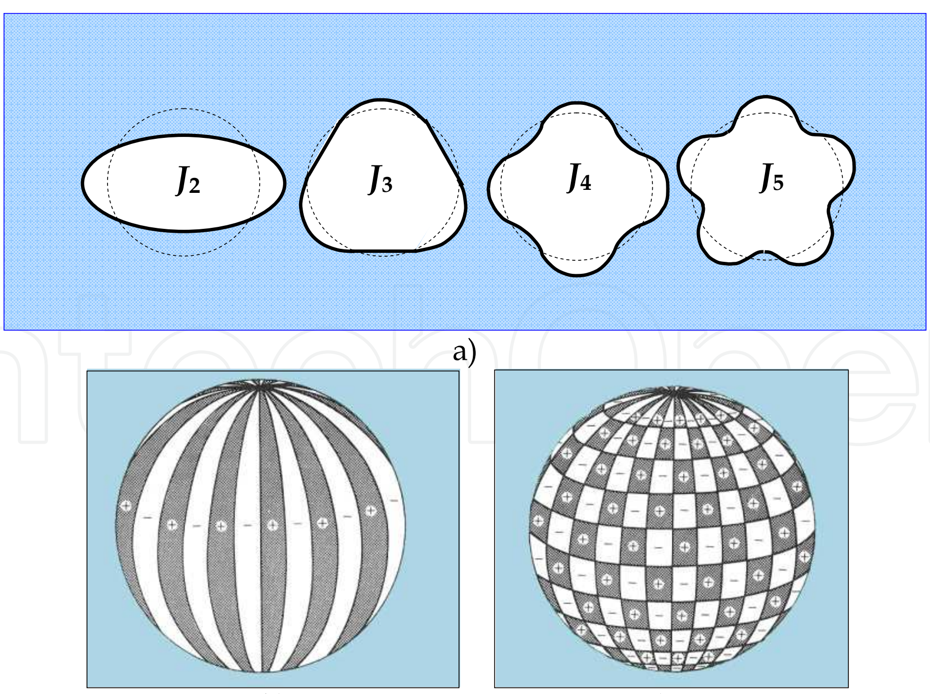
$$P_{nm}(\sin\phi) = \left(1-\sin^2\phi\right)^{m/2} \frac{d^m P_{n0}(\sin\phi)}{d(\sin\phi)^m}, \tag{54}$$

where is

$$P_{n0}(\sin\phi) = \frac{1}{2^n n!} \frac{d^n\left(\sin^2\phi - 1\right)^n}{d(\sin\phi)^n}. \tag{55}$$

The functions:

- $P_{n0}(\sin\varphi)$ are the *zonal spherical function* or shorter *zonal harmonics* (Fig. 28. a),
- $P_{nn}(\sin\varphi)\cos n\lambda$ and $P_{nn}(\sin\varphi)\sin n\lambda$ are *sectorial spherical harmonics* or sectorial harmonics (Fig. 28. b) and
- $P_{nm}(\sin\varphi)\cos m\lambda$ and $P_{nm}(\sin\varphi)\sin m\lambda$ are *tesseral spherical harmonics* or tesseral harmonics (Fig. 28. c).



a)

b) c)

Fig. 28. Images of influences a) zonal hamonics, b) sectorial harmonics and c) tesseral harmonics.

If we suppose that the disposition of the mass in interior of the Earth is symmetrical with respect to the rotation axis than we obtain a somewhat simpler equation:

$$U = G\frac{M}{r}\left[1 + \sum_{n=2}^{\infty} J_n \left(\frac{R}{r}\right)^n P_{n0}\left(\sin\phi\right)\right], \tag{56}$$

where $J_n = -c_{n0}$. Hence, than only zonal harmonics remain. The asymmetry of the north and of the south hemisphere is characterised by the coefficients of the odd zonal harmonics. If we suppose that the Earth is symmetrical in respect of the equator than will coefficients of the odd zonal harmonics disappear. So, it can be expressed with the equation

$$U = G\frac{M}{r}\left[1 - J_2\left(\frac{R}{r}\right)^2 P_{20}\left(\sin\phi\right) - J_4\left(\frac{R}{r}\right)^4 P_{40}\left(\sin\phi\right) - \ldots.\right]. \tag{57}$$

6.5 Influence of the Earth's Gravitation on Movement of Artificial Satellites

Because the Earth is not a ball, i. e. not homogeneous ball, and also because it does not consist of concentric homogeneous spherical shells, the deviations of satellites orbits from an ellipse which corresponds to Kepler's laws will appear. Consequently to the second Newton's law can be written following the differential equations of satellites motion in rectangular coordinate system:

$$\ddot{x} = \frac{\partial U}{\partial x}, \qquad \ddot{y} = \frac{\partial U}{\partial y}, \qquad \ddot{z} = \frac{\partial U}{\partial z}. \tag{58}$$

Only the gravitation force of the Earth is taken into account but the other influences are neglected (the resistances of the air, the attraction of the Sun and the Moon, the radiation pressure of the Sun and so on). In this case the force of disturbance is

$$R^* = U - \frac{GM}{r} \tag{59}$$

and the equations (58) can be written in the form

$$\frac{d^2x}{dt^2} + \frac{\mu x}{r^3} = \frac{\partial R^*}{\partial x}, \qquad \frac{d^2y}{dt^2} + \frac{\mu y}{r^3} = \frac{\partial R^*}{\partial y}, \qquad \frac{d^2z}{dt^2} + \frac{\mu z}{r^3} = \frac{\partial R^*}{\partial z}, \tag{60}$$

where $\mu = G(1+M)$. The solution of these systems of differential equations and replacing the coordinates by the elliptical elements yields the parameters of orbit depending on time and the position of satellite.

Very complicated equations are obtained that can be found in the professional experts literature (Kozai 1959 and 1966), (McCuskey 1963). They are made of the starting or the mean magnitude of orbital parameters, their secular perturbations (which are linearly progressive in the time), the long-periodic and the short-periodic perturbations (Fig. 29) (Escobal, (1965).

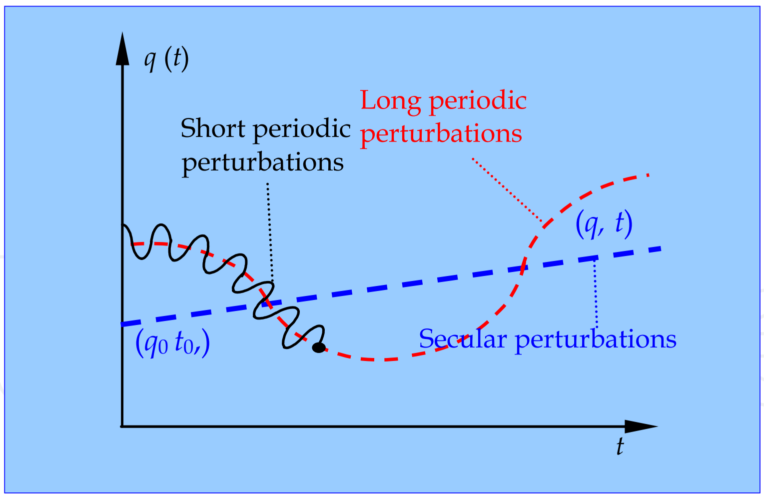


Fig. 29. The typical changes of orbital elements ω, Ω and M_0 by time.

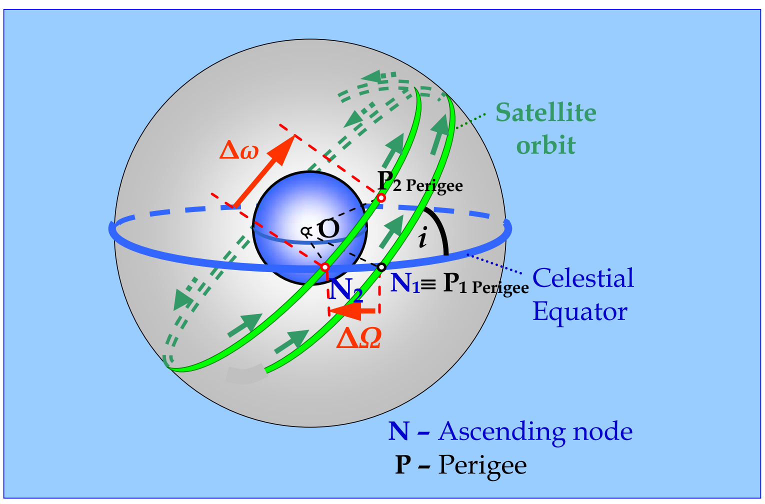


Fig. 30. Perturbations of the ascending node and perigee.

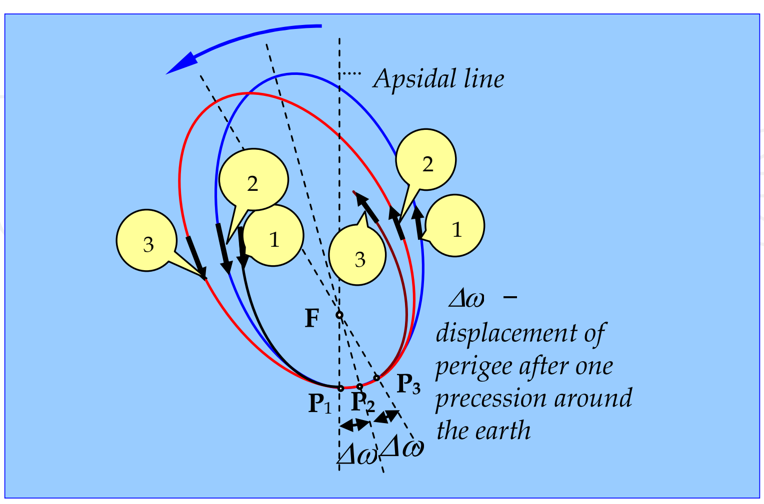


Fig. 31. Perturbations of the perigee P.

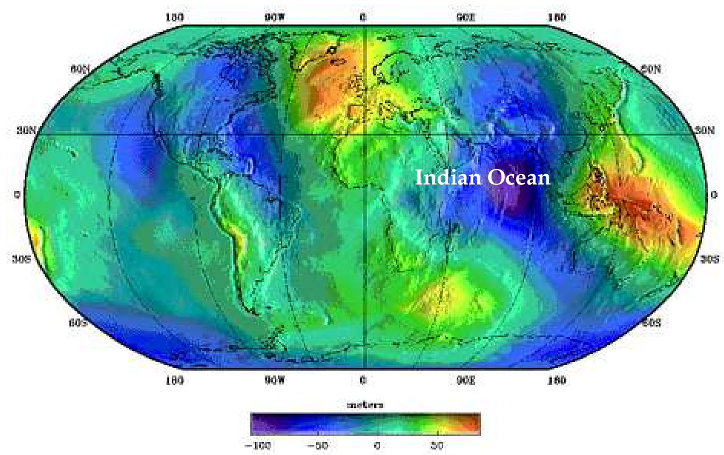
The parameter a has only short periodic perturbations, the parameters ε and i have long and short periodic perturbations and ω, Ω and M_a have secular, long and short periodic perturbations (Hofmann-Wellenhof et al. 1994) and (Kaula 1966).
From this solutions it follow that the ascending node will change position from the point N_1 in the point N_2 after one satellite precession around the Earth and so the longitude of node will dislocate for the angle $\Delta\Omega$ (Fig. 30). This phenomenon is named the precession. Also the point of perigee $N_1 \equiv P_1$ will change position in the point P_2 (Fig. 30 and Fig. 31) and thus, it will arise the argument of perigee for the angle $\Delta\omega$. Also the apsidal line will rotate.

The secular perturbation of node longitude and the argument of perigee can be determined from approximate equations

$$\dot{\Omega} = -\frac{3}{2} n a_{\mathrm{E}}^2 \frac{\cos i}{a^2\left(1-\varepsilon^2\right)^2} J_2 + \ldots\ldots , \tag{61}$$

$$\dot{\omega} = \frac{3}{4} n a_{\mathrm{E}}^2 \frac{5\cos^2 i - 1}{a^2\left(1-\varepsilon^2\right)^2} J_2 + \ldots\ldots , \tag{62}$$

where are: $\dot{\Omega}$ and $\dot{\omega}$ are the temporal changes of angles in the degrees per day, n – the mean angular satellite velocity ($n = 2\pi / P = \sqrt{GM / a^3}$, G – the universal constant of Gravitation, M – the Earth's mass), a_{E} – the semi-major axis of the Earth, a – the semi-major axis of satellite orbit, ε – the numerical eccentricity of elliptic orbit, i – the inclination of orbital plane and J_2 – the zonal coefficient of Earth's harmonics. This coefficient is about 500 and more time bigger than other coefficients and characterizes the flatness on poles and the bulginess on equator, which is caused by the Earth's rotation.
From the equation (61) it is possible to make the decision that $\dot{\Omega}$ =0°/day for i = 90° when satellites are in polar orbits. Then the ascending node will not move. From the eqution (61) it also thus follows that $\dot{\Omega}$ has maximal value for i = 0° when the satellite rotates in the equatorial plane.
From the equation (62) it is possible to see that $\dot{\omega}$ =0°/day for $\cos^2 i$ = 0.2 and thus, it is obtained that the inclination of orbit is i = 63.4° or i = 166.6°. The maximal value of $\dot{\omega}$ is when is i = 90° that is when the satellite rotates in the polar orbit.
In the equations (61) and (62) there is ratio a_E^2/a^2 which is in every case less than 1. The satellites with bigger distance from the Earth's centre have the lower value of ratio a_E^2/a^2 and then the dislocations of nodes and apsidal lines will be much smaller when the satellites are farther from the Earth.
The zonal coefficient J_2 has the largest influence on satellites motion. The other coefficients have considerably smaller influence but in spite of this fact the artificial satellites made possible to determine the true shape of the Earth that is the geoid (Fig. 32). For this purpose usually the low satellites were used (altitudes about 1000 km), but before calculation we are obliged to remove the other influences which is a very complicated mode of proceedings. In the recent time the satellite GOCE was launched for the determination Earth's gravitation field with the very high accuracy.



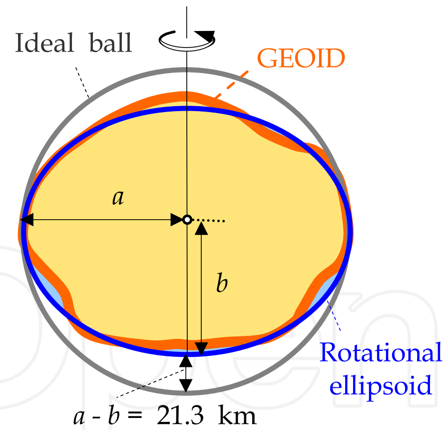


Fig. 32 The geoid is the true shape of the Earth – the maximal elevation over the reference ellipsoid is 85.4 m and the largest hollow 107.0 m in Indian Ocean (Geoid 050 NASA).

6.6 Influence of the Atmosphere on the Satellite Motion

The influence of atmosphere on the satellite motions is bigger if the distance of satellite from the Earth's surface is smaller. This effect is the biggest immediately after the effects of Earth's flatness for the low Earth satellites. The force of satellite retardation (F_0) can be determined from the equation

$$F_0 = -0.5C_D \rho v_{rel}^2 \frac{A}{m}, \tag{63}$$

where: C_D – the aerodynamic coefficient, ρ – the density of atmosphere, v_{rel} – the relative velocity of a satellite in respect to the atmosphere, A – the area of satellite cross section and m – the mass of a satellite.

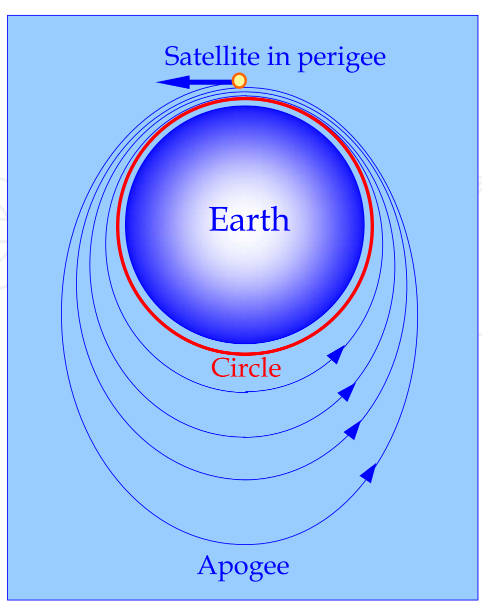


Fig. 33. Image of influence the resistance

The density of atmosphere is changeable and depends of altitudes but bigger changes caused by the activity of the Sun (phenomenon of the Sun spot). Because the resistance of the air depends on its density which is changeable and depends on more factors, this influence can be taken in account with more limited accuracy. The eccentricity of elliptic orbits in low orbits is progressively smaller (Fig. 33).
From the parameters of satellites motions in the low Earth orbit the effect of air resistance can be removed only with limited accuracy.

6.7 The Influence of Gravitation of Moon and Sun

The gravitation influence of the Moon and the Sun can be eliminated by equations which are obtained solving the differential equations of satellite motion in which the functions of the Moon and the Sun attractions forces are inserted.

Secular motions due to the lunar and solar attractions are denoted by $\Delta\omega_{\mathrm{LS}}$ and $\Delta\Omega_{\mathrm{LS}}$ are expressed as,

$$\Delta\omega_{\mathrm{LS}} = \frac{1}{2}\psi\left(-1+5\theta_i^2+\varepsilon^2\right) \quad \text{and} \quad \Delta\Omega_{\mathrm{LS}} = \Psi\left((1+\frac{1}{2}\varepsilon^2\right), \tag{64}$$

where: ε is the numerical eccentricity of the satellite orbit at the beginning for $t = 0$,
$\theta_i = \cos(i)$ for inclination of the satellite orbit i at beginning for $t = 0$,

$$\Psi = \frac{3}{4}\frac{1}{n^2\sqrt{1-\varepsilon^2}}\left[n_\Theta^2\left(1-\frac{3}{2}\sin^2\zeta\right)+n_\Diamond^2 m_\Diamond\left\{1-\frac{3}{4}\sin^2 J\left(1+\cos^2\zeta\right)-\frac{3}{2}\sin^2\zeta\cos^2 J\right\}\right]. \tag{65}$$

In this expression n_Θ an*d* $n_\Diamond$ are mean motions of the Sun and the Moon, respectively, $m_\Diamond$ is the mass of the Moon expressed in units of the mass of the Earth, ζ is then obliquity, that is the inclination between the ecliptic and the equator, and J is the inclination of the lunar orbit with reference to the ecliptic and is about 5°. When n is given in degrees per day, Ψ, the dimensionless quantity, takes the following value (Kozai 1959 and Kozai 1966):

$$\Psi = \frac{1.762}{n^2\sqrt{1-\varepsilon^2}}\,. \tag{66}$$

6.8 Tidal Effects

The Sun and the Moon deform the Earth by their tidal forces not only in the ocean but also in the body (the land). In this manner the Earth's shape is temporally changeable and the tidal effects are apparent. Merson, R. H.; Kaula, W. M.; Newton, R. R. and Kozai, Y. are the first who examined the tidal effects on the motion of polar Earth's satellites and they found out that there are small changes in inclinations of the polar satellites orbits but this results were not consistent. Today the instrument for determining the positions of satellites are more precise and this effect needs to be taken into account for better determination parameters of satellite's orbit (Hofmann-Wellenhof et al. 1994).

6.9 Solar Radiation Pressure (The Pressure of Light)

In the first time when the first satellites were launched in the Earth's orbit solar radiation (the pressure of light) was neglected. The motions examinations of the satellite "Vangard 1" indicated that there are some discrepancies. These perturbations were possible to explain by the pressure of light. The pressure of light on the Earth's surface is very tiny and really practically zero, but in the orbit this effect can change position of satellite especially if the mass of satellite is small and its effective cross section is very big. So this effect could be studied better at the satellite EHO 1 which really was a weightless balloon with small mass and big cross section. The module of acceleration ''a_a'' of a satellite under influence of the solar radiation pressure can be determined by the equation

$$a_{\mathrm{a}} = P_{\mathrm{r}} \frac{A}{m} . \tag{67}$$

where: P_r - the force of light pressure on the unit of area, A – the area of satellite cross section and m – the mass of satellite.

It should be noted here that the satellites enter into shadow of the Earth and so the problems with the pressure of light are more complex, although small.
The solar radiation pressure which is reflected back from the Earth's surface causes an effect called albedo. The pressure of reflected light also affects on the satellites but this effect can be for the most number of cases practically neglected. From the perturbations of Earth's satellite motion it was determined in the paper (Sehnal 1978) which parts of the Earth's surface give bigger or smaller albedo.
The solar radiation pressure which is reflected to the Earth's satellite from the Moon also affects on the Earth's satellite but this effect is smaller than this of Earth's albedo.

6.10 The other influences

▪ The low satellites have very big velocities something more of 7 km/s but this velocities are relatively small comparing to the velocity of light (300,000 km/s). Meanwhile, this *relativistic effect* needs also to be taken in the accurate calculation of parameters of the satellite orbits. This effect can cause the perturbing acceleration results in an order of $3 \cdot 10^{-10}$ ms^{-2}.
▪ The influences of planets gravitations can be neglected because the perturbation of planets is really too small.
▪ On the satellites there are different electronic equipments which have proper magnetic and electric fields. Since the satellites are in the motion in the Earth's the magnetic field the perturbation forces appear which are provoked by the acting of this field. Such effects are very small and poorly examined.

6.11 Sorts of Satellite Orbits

Satellites are launched into orbits, which is to say that they are shot up into the sky on rockets to get them up above the atmosphere where there is no friction. The idea is to get them flying so fast, that when they fall back to the Earth, they fall towards the Earth at the same rate the Earth's surface falls away from them. When an object's path around the Earth ''trajectory'' matches the Earth curvature, the object is said to be ''in orbit''.

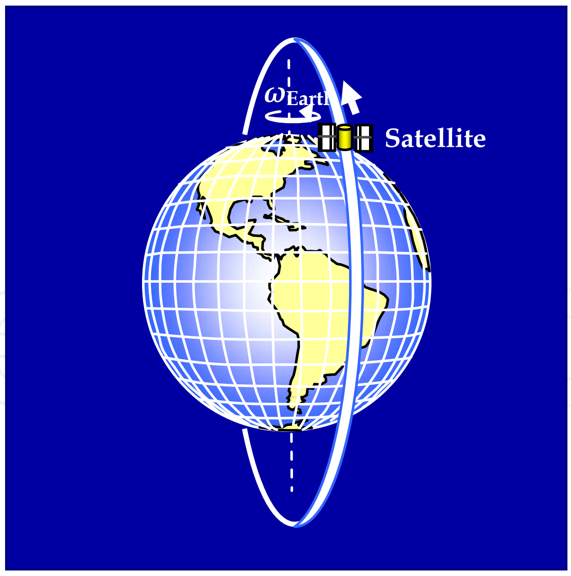


Fig. 34 Polar orbit of a satellite.

The orbits are defined by 3 factors:
▪ The first is the ***shape of the orbit,*** which can be
- *circular* (with equal distance from the Earth's centre) or
- *elliptic* (an oval shaped orbit with the *perigee* as the closet point to the Earth and the *apogee* as the farthest point from the Earth).

▪ The second is the ***altitude of the orbit.*** The altitude is constant for a circular orbit but changes constantly for an elliptical orbit.
▪ The third factor is the ***angle of the inclination,*** the angle which plane of satellite orbit makes with the plane of equator. An orbit when a satellite is passing over the Earth's poles or close to it has a large angle of inclination (i = 90˚ or near 90˚). These satellites are called the *polar satellites* (Fig. 34). An orbit that makes the satellite stay close to the equator has a small angle of inclination (i = 0˚ or near 0˚). These satellites are called the *equatorial satellites* (Fig. 35).

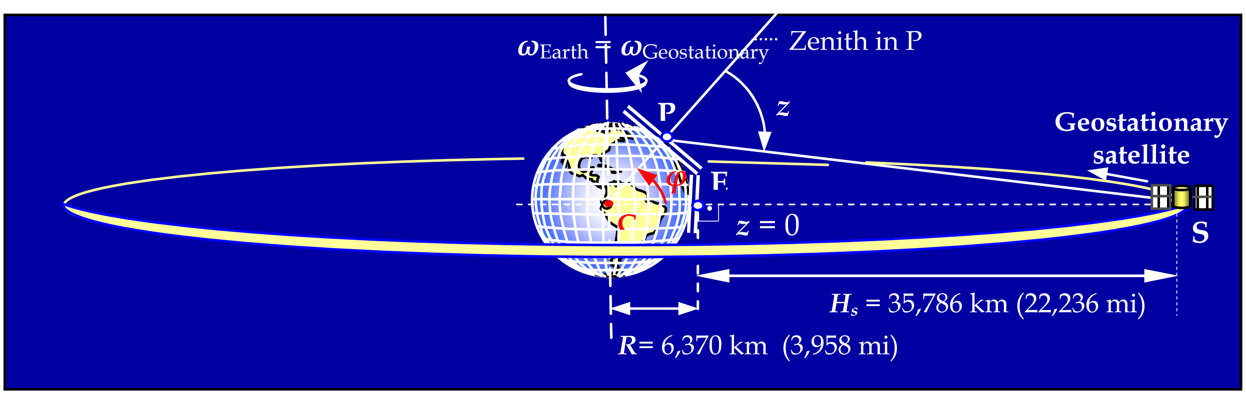


Fig. 35. Imagery of a geostationary satellite in the equatorial orbit.

□ ***Low Earth Orbits* (LEO)**
Satellites in low Earth orbit (LEO) orbit around the Earth at altitudes of less than 2000 km (1240 miles). Communication satellites in LEO can give clearer surveillance images and require much less power to transmit their data to the Earth.

For example by the *Iridium satellite network* voice and data messages can be routed anywhere in the world. This Iridium satellite constellation was originally planned to have 77 active satellites and was redesigned to on the 66 active satellites of day. These satellites are in six polar low Earth's orbital plane at a height of approximately 781 km (485 miles) and have inclinations i = 86,4˚. Each satellite completely orbits the Earth in 100 minutes travelling. From the horizon to the horizon it takes 10 minutes. As satellites move out of view from the satellite phone user the call is handed over the next satellite coming into view (URL 4) and (URL 5).
The ***apparent trajectory*** for LEO satellites in their circular orbits or elliptic orbits for a motionless spectator on the Earth looks like as a straight line or a mild curved line.

□ ***Medium Earth Orbits (*MEO)**
Satellites in medium Earth orbit (LEO) circling around the Earth are at altitudes of about 2000 km (1240 miles) to just below geosynchronous orbit at 35,786 km (22,240 miles). These kinds of orbits are known as *intermediate circular orbits*.

□ ***Geosynchronous satellites***
The geosynchronous satellites can be: the geostationary satellites and the 24-hours Earth synchronous satellites.

- ***Geostationary satellites* (GEO)**
A satellite in geostationary orbit circles the Earth in exactly 1 day and is placed above the equator. The geostationary satellites have angles of inclination of orbits equal 0˚ and they are at the elevation of approximately 35,786 km (22,236 miles) above the ground of Earth (Fig. 35). These satellites have the same directions of rotation as the Earth and so follow the Earth's rotation. For one full orbit they need 23 hours, 56 minutes, 4.091 seconds (one sidereal day). As a result the satellite seems to the man who is standing on the Earth that the satellite (GEO) is all the time in the same position without motion.

The geostationary satellites are usually launched as the first in an elliptical orbit (Fig. 36) and after when the satellite is in the apogee of this orbit it obtains velocity necessary for maintaining the height over the Earth.

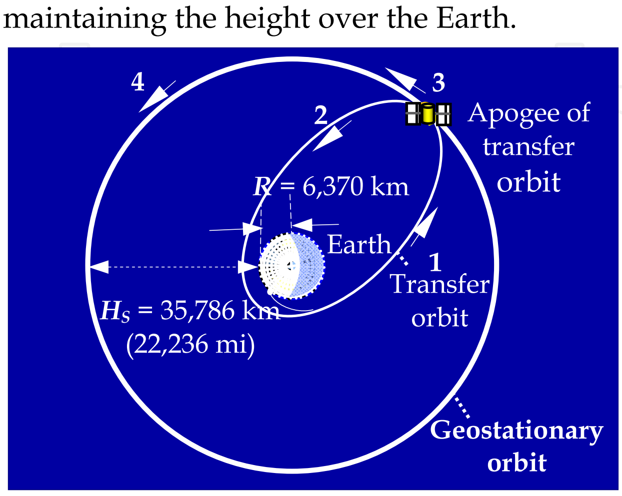


Fig. 36. Launching a satellite in the geostationary orbit with the help of an elliptic transfer orbit (Bazjanac 1977).

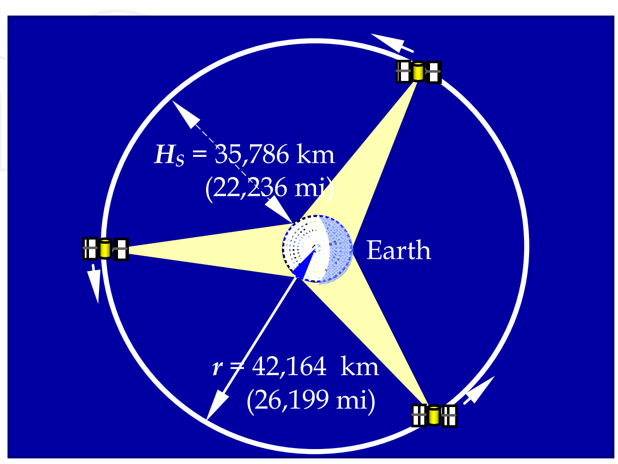


Fig. 37. All the Earth's surface is covered with only three geostationary satellites.

These satellites are usually used for communications, satellite TV, meteorology and remote sensing. With only three geostationary satellites it is practically possible to cover all the Earth's surface (Fig. 37) from the equator to the geographical latitudes +81.3° and – 81.3°.
A geostationary satellite can be immovable in the zenith only in a point on the equator, but in the point on the Earth with higher latitudes it is not possible to see immovable satellites in the zenith. This means that a satellite cannot be geostationary in the zenith at any point on the Earth with higher geographic latitudes. The planes of satellites need to pass through the centre of Earth's mass and this is the reason why the satellites cannot orbit around the Earth in the plane of parallel and to be all the time immovable in the zenith of this point on the parallel.

- ***Geosynchronous 24 Hour Earth Satellites***

These kinds of satellites have the same orbital period as the geostationary satellites but their orbital plane is not placed in the equator plane, namely they have an inclination greater than 0°. This orbit of 24-hour Earth satellite in reality may be a circular or an elliptic.

a) Apparent Motion of 24-Hour Earth Satellites in Circular Orbits

The 24-hour satellites in the circular orbits and under certain inclination angles i in relation to equatorial plane can be described as a *number eight* on the sky for the "standing spectator" on the Earth. This apparent motion of 24-hour satellites is caused by combining rotation of the Earth around its axis and rotation of the satellite around the Earth around the axis inclined under the angle i in relation to the equatorial plane.
This occurrence can be best explained by Fig. 38. In the initial moment $t = 0$ let the 24-hour satellite be in the equatorial plane. After the time t, the Earth will turn around its rotation axis for the angle β_{Earth}. At the same time t 24-hour satellites will rotate around their orbit for the equal angle β_{Sa}. Namely, angle speed of the Earth ω_{Earth} and angle speed of 24-hour satellite ω_{Sa} are equal, so the banking angles of longitude plane and of 24-hour satellite in its plane will be equal, that is $\beta_{\text{Earth}} = \beta_{\text{Sa}}$.
However, these inclination angles β_{Earth} and β_{Sa} do not lie in the same plane, but their planes close between themselves an inclination angle i of the 24-hour satellite orbit. This is why the projection of banking angle of the satellite in the plane of the satellite's orbit β_{Sa} will be projected in the equatorial plane in the S′ spot, e.g. it will be shorter from β_{Earth}. This means that a spectator on the Earth will at first see 24-hour satellite lapses on the west side. It will remain this way until the plane of the Earth's longitude does not turn for 90°, when the 24-hour satellite appears in that displaced longitude's plane.
In order to mathematically determine this apparent motion of 24-hour satellites, one can observe the spherical triangle E_1SS' and calculate the size of the projection of the spherical side angle β_{Sa} in the equatorial plane, which has been marked with letter S′ on the Fig. 38. In this way we get:

$$s = \arctan\left[\tan(\beta_{\text{Sa}}) \cdot \cos(i)\right]. \tag{68}$$

So the lagging angle of the 24-hour satellite in the equatorial plane is:

$$\Delta\beta = \beta_{\text{Earth}} - s. \tag{69}$$

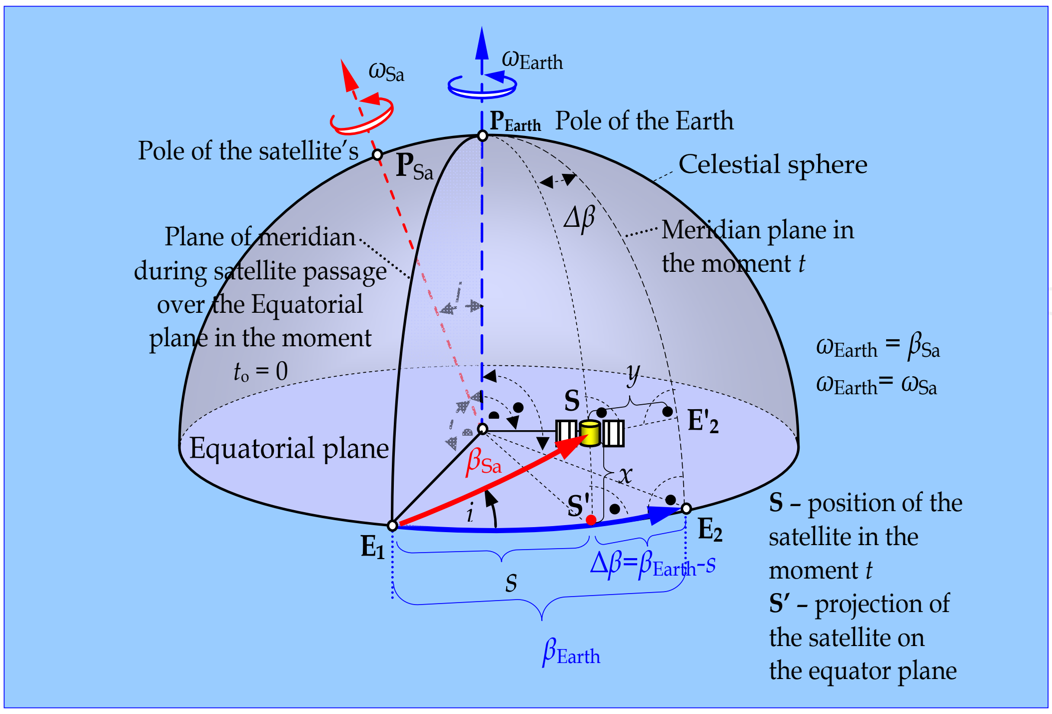


Fig. 38. Graphic illustration of why the spectator on the Earth sees relatively (apparent) motion of 24-hour satellites in circular orbit in relation to ''still'' Earth.

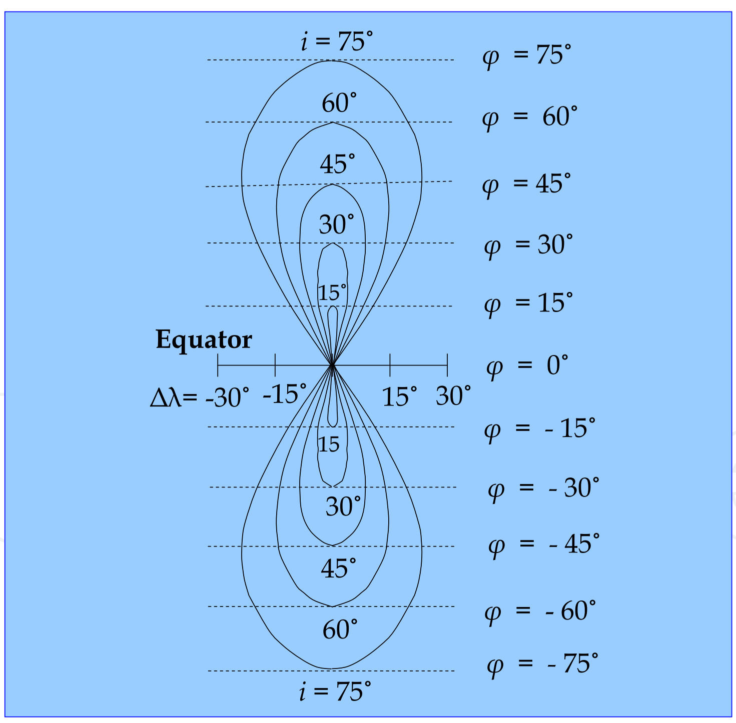


Fig. 39. Graphical illustration of apparent orbits of relative motion of 24-hour satellites on their circular orbit for different inclinations of their orbits, as a spectator on the Earth sees them.

Now it can be calculated, from the very same spherical triangle E_1SS' the value of angle distance x from the equatorial plane and the plane the satellite passes through. In this way, we get:

$$x = \arctan[\sin(s) \cdot \tan(i)]. \tag{70}$$

The coordinate y expressed in degrees can be calculated from the spherical triangle $P_{Earth}SE'_2$. We get:

$$y = \arctan[\cos(x) \cdot \tan(\Delta\beta)]. \tag{71}$$

For different inclinations i of the satellite's orbit, with the help of equations (68), (69), (70) and (71), angle values of x and y were calculated for drawing of graphical display of orbits of apparent (relative) motion of 24-hour satellites, as it is seen on the Earth, which is shown in Fig. 39 (Solarić, 2007).

From the chart in the Fig. 39, it is obvious that 24-hour satellite in its apparent relative motion reaches latitudes that are equal to their orbital inclinations. Besides, these *symmetrical eights* become greater when the inclination i of the satellite's orbit are greater.

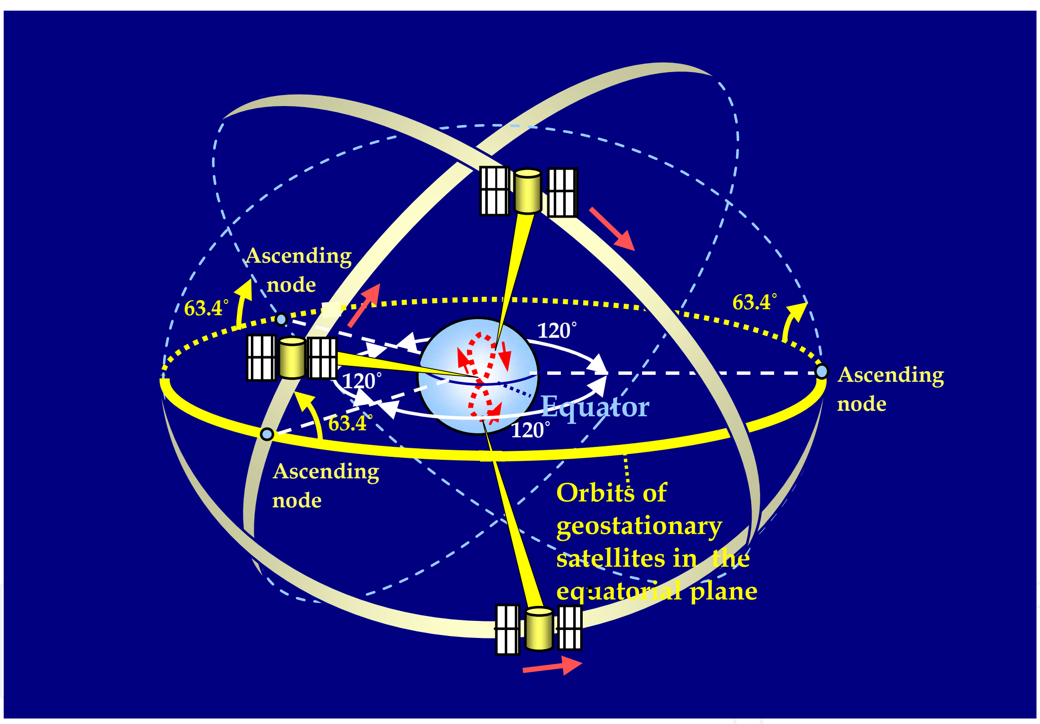


Fig. 40. Imaging in three dimensions of a satellite system with three satellite in three orbital planes.

b) Apparent Motion of 24-Hours Earth Satellites in Elliptic Orbits

The geosynchronous satellites on elliptical orbits with a high inclination (usually near 63.4°) and an orbital period of one sidereal day (23 hours, 56 minutes, 4.091 seconds) called *Tundra* (URL 7). A satellite placed in this orbit spends most of its time over a chosen area of the Earth.

From the second Kepler's law that satellite's velocity is lowest in the apogee, the greatest in the perigee, while the velocity of the satellite in any other point of elliptical orbit is between these values, that is: $v_{perigee} > v > v_{apogee}$. So the satellites in the apogee point of an elliptic orbits spend the most time, and because they have usually the critical inclinations i = 63.4° they have not the secular perturbation of the argument of perigee caused by the Earth's equatorial bulge.

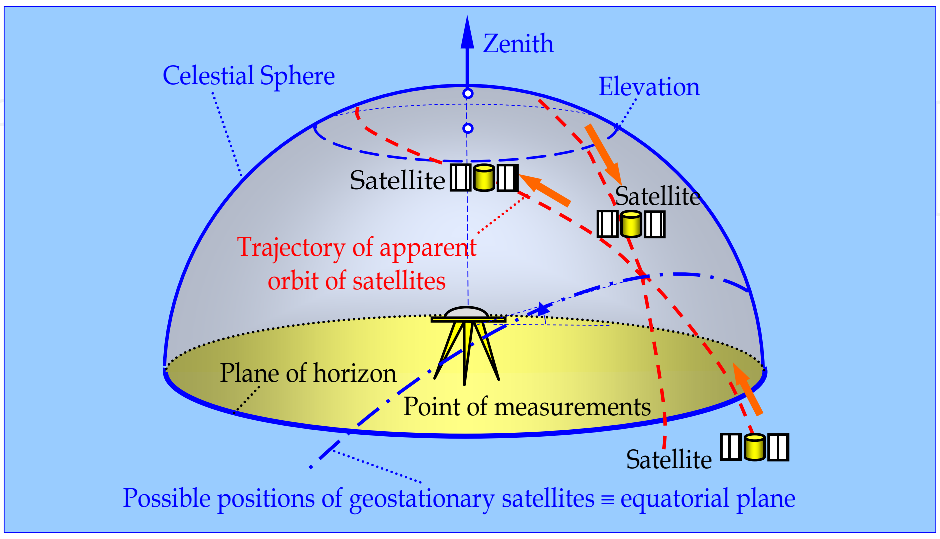


Fig. 41. View of celestial sphere on the point on the north hemisphere with apparent (relative) orbits a satellite system with the 24-hour satellites in three different orbits, as the standing spectator sees them. The possible positions of geostationary satellites have been represented as well.

The apparent ground track of a satellite in *a tundra orbit* is a closed ''*asymmetrical– eight*'' lemniscate's.
The Sirius Satellite Radio (the USA) uses the tundra orbits in three orbital planes with the inclination i = 63.4° and the eccentricity ε = 0.2684 so that their ascending nodes in the equatorial plane are moved on the longitude for 120°. Also the difference of mean anomaly of each satellite is 120°. When one satellites moves out of position, another has passed perigee and is ready to take functions over the territory of the USA and the Canada. Every satellite of these satellites is every day 16 hours over the north hemisphere and 8 hours over the south hemisphere. (Simile as at Fig. 40.)
In order to numerically determine apparent trajectory of tundra orbits with elliptical orbit, as a "standing" spectator sees it on the Earth, one should:

- project position of a satellite from the elliptical orbit to sphere,
- out of these coordinates from the sphere, according to equations (68) – (71), determine coordinates x and y the point of tundra orbits, expressed in degrees,
- graphically demonstrate sequential positions of satellite's apparent orbit.

Japan decided to set up their satellite communication and positioning system called ''*Quasi Zenith Satellite system*'' (abbreviated QZSS) or in Japanese ''Jun-Ten-Cho''. As expected this satellite system will up to schedule operate in 2013 (URL 8).
It is being anticipated that 3 of these 24-hour QZSS satellites would be substituting every 8 hours on the near zenith above Japan. In this way, there will always be one satellite approximately in zenith, and another QZSS satellite would be on somewhat greater zenith distance. The third QZSS satellite would then be somewhere on the south, so it can be for

shorter time under the horizon in Tokyo. This could be done in the way that every QZSS satellite has its own orbital plane and that their ascending nodes in the equatorial plane are moved on the longitude for 120° (Fig. 42). The Japanese called this satellite system quasi-zenith because there will always be one QZSS satellite near the zenith.

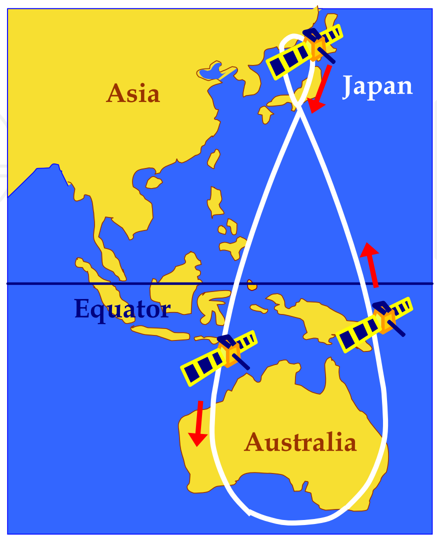


Fig. 42. Apparent orbits of three planned 24-hour QZSS-satellites with inclination i = 45˚ and eccentricity e = 0.099.

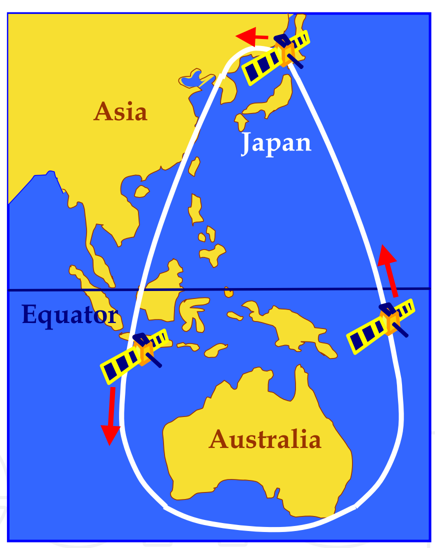


Fig. 43. Apparent orbits of three planned 24-hour QZSS-satellites with inclination i = 42.5˚ and eccentricity e = 0.21.

In the article (Petrovski et al. 2003), experiments for possible usage of 24-hour satellites' orbits with different eccentricities ε and for different inclinations of orbits' planes i, have been conducted.

Orbit of 24-hour satellite will have different form for different values of eccentricities of apparent motion:

a) For the inclination of orbital plane i = 45° and eccentricity ε = 0.099 apparent orbit of 24-hour satellite will have the shape of *asymmetrical eight* (Fig. 42), while they are symmetrical with the circular orbits of 24-hour satellites.
b) For the grater eccentricity of elliptical orbit, apparent orbit of 24-hour satellite motion will no longer have the shape of an asymmetrical eight, but it will assume the shape of *rain drops*. It can be seen in Fig. 43. for the eccentricity of elliptical orbits of 24-hour satellites ε = 0.21 and orbital inclination i = 45°. Apparent motion orbit of QZSS satellite will expand even more for somewhat greater eccentricity ε. This kind of orbit of a QZSS satellite would be appropriate for an area wider than Japan, so, for this case, possibilities with 3 and 4 QZSS satellites have been tested.
Clear spatial image could also be constructed for this case, when QZSS has three QZSS satellites in elliptical orbits, as it is demonstrated in Fig. 40, for circular orbits of QZSS satellite.

□ ***Semi - Synchronous Orbit***
The satellite in the semi-synchronous orbit has the period of the 0.5 sidereal day (about 11 hours, 58 minutes) of travelling about the Earth. A Molniya orbit is a type of highly elliptical orbit with an inclination of 63.4˚ and an orbital period of precisely one half of sidereal day. Soviet/Russian communications satellites use these kinds of orbits which have been using this type of orbit since the mid 1960s. (Molniya (Russian mean: ''lightning'')). A satellite in a Molniya orbit spends most of its time over the Earth as a result of ''apogee dwell'' (Fig. 44). Only three such satellites are needed to give 24 hour coverage.
Also the satellites from the global position system (GPS) have the semi-synchronous period of orbits around the Earth.

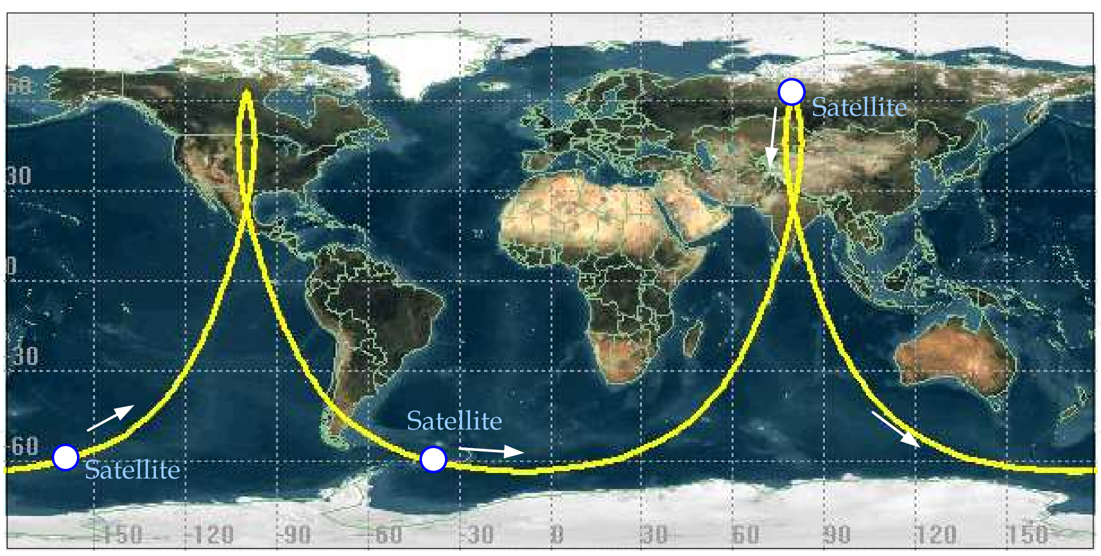


Fig. 44. Apparent trajectory of the satellites Molniya (URL 6).

□ ***Sun - Synchronous Orbit (SO)***
A Sun-synchronous orbit of the Earth's satellite is a geometric orbit which combines altitude and inclination in such a way that an object on that orbit ascends or descends over any given point of the Earth's surface at the same local mean solar time. In this way the Earth's surface illumination angle will be nearly in the same time.

Because the Earth is bulgy on the equator it follows from the equation (61) that the right ascension of ascending node has a constant negative velocity. In this way the plane of satellite orbit does not remain in the same position. So the ascending node is moving for approximately one degree per day which is equal to the angle of the Earth's shift during one day on the Earth orbit around the Sun.
Typical Sun-synchronous orbits are about 600-800 km in altitude, with periods in the 96-100 minute range, and inclinations of around 98˚. Variations on this type of orbit are possible; a satellite could have a highly eccentric sun-synchronous orbit.

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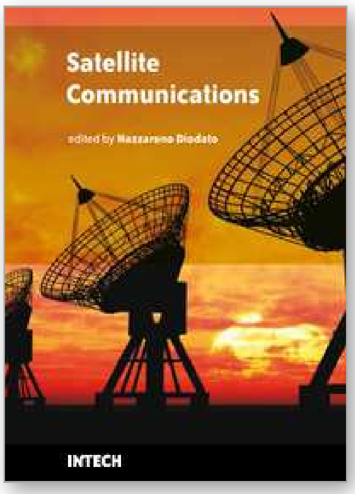

Satellite Communications

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This study is motivated by the need to give the reader a broad view of the developments, key concepts, and technologies related to information society evolution, with a focus on the wireless communications and geoinformation technologies and their role in the environment. Giving perspective, it aims at assisting people active in the industry, the public sector, and Earth science fields as well, by providing a base for their continued work and thinking.

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