28

Mean-variance hedging under partial information

M. Mania[1),2)], R. Tevzadze[1),3)] and T. Toronjadze[1),2)]
[1)] *Georgian American University, Business School*
[2)] *A. Razmadze Mathematical Institute*
[3)] *Institute of Cybernetics*
Georgia

Abstract

We consider the mean-variance hedging problem under partial information. The underlying asset price process follows a continuous semimartingale, and strategies have to be constructed when only part of the information in the market is available. We show that the initial mean-variance hedging problem is equivalent to a new mean-variance hedging problem with an additional correction term, which is formulated in terms of observable processes. We prove that the value process of the reduced problem is a square trinomial with coefficients satisfying a triangle system of backward stochastic differential equations and the filtered wealth process of the optimal hedging strategy is characterized as a solution of a linear forward equation.
2000 Mathematics Subject Classification: 90A09, 60H30, 90C39.
Key words and phrases: Backward stochastic differential equation, semimartingale market model, incomplete markets, mean-variance hedging, partial information.

1. Introduction

In the problem of derivative pricing and hedging it is usually assumed that the hedging strategies have to be constructed by using all market information. However, in reality, investors acting in a market have limited access to the information flow. For example, an investor may observe just stock prices, but stock appreciation rates depend on some unobservable factors; one may think that stock prices can be observed only at some time intervals or up to some random moment before an expiration date, or an investor would like to price and hedge a contingent claim whose payoff depends on an unobservable asset, and he observes the prices of an asset correlated with the underlying asset. Besides, investors may not be able to use all available information even if they have access to the full market flow. In all such cases, investors are forced to make decisions based on only a part of the market information.

We study a mean-variance hedging problem under partial information when the asset price process is a continuous semimartingale and the flow of observable events do not necessarily contain all information on prices of the underlying asset.

We assume that the dynamics of the price process of the asset traded on the market is described by a continuous semimartingale $S = (S_t, t \in [0, T])$ defined on a filtered probability space $(\Omega, \mathcal{A}, (\mathcal{A}_t, t \in [0, T]), P)$, satisfying the usual conditions, where $\mathcal{A} = \mathcal{A}_T$ and $T < \infty$ is the fixed time horizon. Suppose that the interest rate is equal to zero and the asset price process satisfies the structure condition; i.e., the process S admits the decomposition

$$S_t = S_0 + N_t + \int_0^t \lambda_u d\langle N\rangle_u, \quad \langle \lambda \cdot N\rangle_T < \infty \quad \text{a.s.}, \tag{1.1}$$

where N is a continuous $\mathcal{A}$-local martingale and λ is an $\mathcal{A}$-predictable process.
Let G be a filtration smaller than $\mathcal{A}$: $G_t \subseteq \mathcal{A}_t$ for every $t \in [0, T]$.
The filtration G represents the information that the hedger has at his disposal; i.e., hedging strategies have to be constructed using only information available in G.
Let H be a P-square integrable $\mathcal{A}_T$-measurable random variable, representing the payoff of a contingent claim at time T.
We consider the mean-variance hedging problem

$$\text{to minimize} \quad E[(X_T^{x,\pi} - H)^2] \quad \text{over all} \quad \pi \in \Pi(G), \tag{1.2}$$

where $\Pi(G)$ is a class of G-predictable S-integrable processes. Here $X_t^{x,\pi} = x + \int_0^t \pi_u dS_u$ is the wealth process starting from initial capital x, determined by the self-financing trading strategy $\pi \in \Pi(G)$.
In the case $G = \mathcal{A}$ of complete information, the mean-variance hedging problem was introduced by Föllmer and Sondermann (Föllmer & Sondermann, 1986) in the case when S is a martingale and then developed by several authors for a price process admitting a trend (see, e.g., (Duffie & Richardson, 1991), (Hipp, 1993), (Schweizer, 1992), (Schweizer, 1994), (Schäl, 1994), (Gourieroux et al., 1998), (Heath et al., 2001)).
Asset pricing with partial information under various setups has been considered. The mean-variance hedging problem under partial information was first studied by Di Masi, Platen, and Runggaldier (Di Masi et al., 1995) when the stock price process is a martingale and the prices are observed only at discrete time moments. For general filtrations and when the asset price process is a martingale, this problem was solved by Schweizer (Schweizer, 1994) in terms of G-predictable projections. Pham (Pham, 2001) considered the mean-variance hedging problem for a general semimartingale model, assuming that the observable filtration contains the augmented filtration F^S generated by the asset price process S

$$F_t^S \subseteq G_t \quad \text{for every} \quad t \in [0, T]. \tag{1.3}$$

In this paper, using the variance-optimal martingale measure with respect to the filtration G and suitable Kunita–Watanabe decomposition, the theory developed by Gourieroux, Laurent, and Pham (Gourieroux et al., 1998) and Rheinländer and Schweizer (Rheinlander & Schweizer, 1997) to the case of partial information was extended.
If G is not containing F^S, then S is not a G-semimartingale and the problem is more involved.
Let us introduce an additional filtration $F = (F_t, t \in [0, T])$, which is an augmented filtration generated by F^S and G.
Then the price process S is a continuous F-semimartingale, and the canonical decomposition of S with respect to the filtration F is of the form

$$S_t = S_0 + \int_0^t \widehat{\lambda}_u^F d\langle M\rangle_u + M_t, \tag{1.4}$$

where $\widehat{\lambda}^F$ is the F-predictable projection of λ and

$$M_t = N_t + \int_0^t [\lambda_u - \widehat{\lambda}_u^F] d\langle N\rangle_u$$

is a continuous F-local martingale. Besides $\langle M \rangle = \langle N \rangle$, and these brackets are F^S-predictable. Throughout the paper we shall make the following assumptions:
(A) $\langle M \rangle$ is G-predictable and $d\langle M \rangle_t dP$ a.e. $\widehat{\lambda}^F = \widehat{\lambda}^G$; hence P-a.s. for each t

$$E(\lambda_t | F^S_{t-} \vee G_t) = E(\lambda_t | G_t);$$

(B) any G-martingale is an F-local martingale;
(C) the filtration G is continuous; i.e., all G-local martingales are continuous;
(D) there exists a martingale measure for S (on F_T) that satisfies the reverse Hölder condition.
Remark. It is evident that if $F^S \subseteq G$, then $\langle M \rangle$ is G-predictable. Besides, in this case $G = F$, and conditions (A) and (B) are satisfied.
We shall use the notation $\widehat{Y}_t$ for the process of the G-projection of Y (note that under the present conditions, for all processes we consider, the optional projection coincides with the predictable projection, and therefore we use for them the same notation). Condition (A) implies that

$$\widehat{S}_t = E(S_t | G_t) = S_0 + \int_0^t \widehat{\lambda}_u d\langle M \rangle_u + \widehat{M}_t.$$

Let

$$H_t = E(H|F_t) = EH + \int_0^t h_u dM_u + L_t \quad \text{and} \quad H_t = EH + \int_0^t h^G_u d\widehat{M}_u + L^G_t$$

be the Galtchouk–Kunita–Watanabe (GKW) decompositions of $H_t = E(H|F_t)$ with respect to local martingales M and $\widehat{M}$, where h and h^G are F-predictable processes and L and L^G are local martingales strongly orthogonal to M and $\widehat{M}$, respectively.
We show (Theorem 3.1) that the initial mean-variance hedging problem (1.2) is equivalent to the problem to minimize the expression

$$E\left[\left(x + \int_0^T \pi_u d\widehat{S}_u - \widehat{H}_T\right)^2 + \int_0^T \left(\pi_u^2\left(1 - \rho_u^2\right) + 2\pi_u \widetilde{h}_u\right) d\langle M \rangle_u\right] \tag{1.5}$$

over all $\pi \in \Pi(G)$, where

$$\widetilde{h}_t = \widehat{h^G_t} \rho_t^2 - \widehat{h}_t \quad \text{and} \quad \rho_t^2 = \frac{d\langle \widehat{M} \rangle_t}{d\langle M \rangle_t}.$$

Thus, the problem (1.5), equivalent to (1.2), is formulated in terms of G-adapted processes. One can say that (1.5) is the mean-variance hedging problem under complete information with an additional correction term.
Let us introduce the value process of the problem (1.5):

$$V^H(t,x) = \operatorname*{ess\,inf}_{\pi \in \Pi(G)} E\left[\left(x + \int_t^T \pi_u d\widehat{S}_u - \widehat{H}_T\right)^2 + \int_t^T \left[\pi_u^2\left(1 - \rho_u^2\right) + 2\pi_u \widetilde{h}_u\right] d\langle M \rangle_u | G_t\right]. \tag{1.6}$$

We show in Theorem 4.1 that the value function of the problem (1.5) admits a representation

$$V^H(t,x) = V_t(0) - 2V_t(1)x + V_t(2)x^2,$$

where the coefficients $V_t(0), V_t(1)$, and $V_t(2)$ satisfy a triangle system of backward stochastic differential equations (BSDEs). Besides, the filtered wealth process of the optimal hedging strategy is characterized as a solution of the linear forward equation

$$\widehat{X}_t^* = x - \int_0^t \frac{\rho_u^2 \varphi_u(2) + \widehat{\lambda}_u V_u(2)}{1-\rho_u^2+\rho_u^2 V_u(2)} \widehat{X}_u^* d\widehat{S}_u + \int_0^t \frac{\rho_u^2 \varphi_u(1) + \widehat{\lambda}_u V_u(1) + \widetilde{h}_u}{1-\rho_u^2+\rho_u^2 V_u(2)} d\widehat{S}_u. \tag{1.7}$$

Note that if $F^S \subseteq G$, then

$$\rho = 1, \quad \tilde{h} = 0, \quad \widehat{M} = M, \quad \text{and} \quad \widehat{S} = S. \tag{1.8}$$

In the case of complete information ($G = \mathcal{A}$), in addition to (1.8) we have $\widehat{\lambda} = \lambda$ and $\widehat{M} = N$, and (1.7) gives equations for the optimal wealth process from (Mania & Tevzadze, 2003).
In section 5 we consider a diffusion market model, which consists of two assets S and η, where S_t is a state of a process being controlled and η_t is the observation process. Suppose that S_t and η_t are governed by

$$dS_t = \mu_t dt + \sigma_t dw_t^0, \qquad d\eta_t = a_t dt + b_t dw_t,$$

where w^0 and w are Brownian motions with correlation ρ and the coefficients μ, σ, a, and b are $\mathcal{F}^\eta$-adapted. In this case $\mathcal{A}_t = \mathcal{F}_t = \mathcal{F}_t^{S,\eta}$, and the flow of observable events is $\mathcal{G}_t = \mathcal{F}_t^\eta$.
As an application of Theorem 4.1 we also consider a diffusion market model with constant coefficients and assume that an investor observes the price process S only up to a random moment τ before the expiration date T. In this case we give an explicit solution of (1.2).

2. Main Definitions and Auxiliary Facts

Denote by $\mathcal{M}^e(F)$ the set of equivalent martingale measures for S, i.e., the set of probability measures Q equivalent to P such that S is a F-local martingale under Q.
Let

$$\mathcal{M}_2^e(F) = \{Q \in \mathcal{M}^e(F) : EZ_T^2(Q) < \infty\},$$

where $Z_t(Q)$ is the density process (with respect to the filtration F) of Q relative to P. We assume that $\mathcal{M}_2^e(F) \neq \emptyset$.

Remark 2.1. Note that $\mathcal{M}_2^e(\mathcal{A}) \neq \emptyset$ implies that $\mathcal{M}_2^e(F) \neq \emptyset$ (see Remark 2.1 from Pham (Pham, 2001).
It follows from (1.4) and condition (A), that the density process $Z_t(Q)$ of any element Q of $\mathcal{M}^e(F)$ is expressed as an exponential martingale of the form

$$\mathcal{E}_t(-\widehat{\lambda} \cdot M + L),$$

where L is a F-local martingale strongly orthogonal to M and $\mathcal{E}_t(X)$ is the Doleans–Dade exponential of X.

If the local martingale $Z_t^{\min} = \mathcal{E}_t(-\widehat{\lambda} \cdot M)$ is a true martingale, $dQ^{min}/dP = Z_T^{min}$ defines the minimal martingale measure for S.
Recall that a measure Q satisfies the reverse Hölder inequality $R_2(P)$ if there exists a constant C such that

$$E\left(\frac{Z_T^2(Q)}{Z_\tau^2(Q)}|F_\tau\right) \leq C, \quad P\text{-a.s.}$$

for every F-stopping time τ.

Remark 2.2. If there exists a measure $Q \in \mathcal{M}^e(F)$ that satisfies the reverse Hölder inequality $R_2(P)$, then according to Theorem 3.4 of Kazamaki (Kazamaki, 1994) the martingale $M^Q = -\widehat{\lambda} \cdot M + L$ belongs to the class BMO and hence $-\widehat{\lambda} \cdot M$ also belongs to BMO, i.e.,

$$E\left(\int_\tau^T \widehat{\lambda}_u^2 d\langle M\rangle_u | F_\tau\right) \le \text{const} \tag{2.1}$$

for every stopping time τ. Therefore, it follows from Theorem 2.3 of (Kazamaki, 1994) that $\mathcal{E}_t(-\widehat{\lambda} \cdot M)$ is a true martingale. So, condition (D) implies that the minimal martingale measure exists (but Z^{min} is not necessarily square integrable).

Let us make some remarks on conditions (B) and (C).

Remark 2.3. Condition (B) is satisfied if and only if the σ-algebras $F_t^S \vee G_t$ and G_T are conditionally independent given G_t for all $t \in [0, T]$ (see Theorem 9.29 from Jacod (Jacod, 1979)).

Remark 2.4. Condition (C) is weaker than the assumption that the filtration F is continuous. The continuity of the filtration F and condition (B) imply the continuity of the filtration G, but the converse is not true in general. Note that filtrations F and F^S can be discontinuous. Recall that the continuity of a filtration means that all local martingales with respect to this filtration are continuous.

By μ^K we denote the Dolean measure of an increasing process K. For all unexplained notations concerning the martingale theory used below, we refer the reader to (Dellacherie & Meyer, 1980), (Liptser & Shiryaev, 1986), (Jacod, 1979).
Let $\Pi(F)$ be the space of all F-predictable S-integrable processes π such that the stochastic integral

$$(\pi \cdot S)_t = \int_0^t \pi_u dS_u, \quad t \in [0, T],$$

is in the $\mathcal{S}^2$ space of semimartingales, i.e.,

$$E\left(\int_0^T \pi_s^2 d\langle M\rangle_s\right) + E\left(\int_0^T |\pi_s \widehat{\lambda}_s| d\langle M\rangle_s\right)^2 < \infty.$$

Denote by $\Pi(G)$ the subspace of $\Pi(F)$ of G-predictable strategies.

Remark 2.5. Since $\widehat{\lambda} \cdot M \in BMO$ (see Remark 2.2), it follows from the proof of Theorem 2.5 of Kazamaki (Kazamaki, 1994) that

$$E\left(\int_0^T |\pi_u \widehat{\lambda}_u| d\langle M\rangle_u\right)^2 = E\langle |\pi| \cdot M, |\widehat{\lambda}| \cdot M\rangle_T^2 \le 2\|\widehat{\lambda} \cdot M\|_{\text{BMO}} E\int_0^T \pi^2 d\langle M\rangle_u < \infty.$$

Therefore, under condition (D) the G-predictable (resp., F-predictable) strategy π belongs to the class $\Pi(G)$ (resp., $\Pi(F)$) if and only if $E\int_0^T \pi_s^2 d\langle M\rangle_s < \infty$.

Define $J_T^2(F)$ and $J_T^2(G)$ as spaces of terminal values of stochastic integrals, i.e.,

$$J_T^2(F) = \{(\pi \cdot S)_T : \pi \in \Pi(F)\}, \qquad J_T^2(G) = \{(\pi \cdot S)_T : \pi \in \Pi(G)\}.$$

For convenience we give some assertions from (Delbaen et al., 1997), which establishes necessary and sufficient conditions for the closedness of the space $J_T^2(F)$ in L^2.

Proposition 2.1. *Let S be a continuous semimartingale. Then the following assertions are equivalent:*

(1) *There is a martingale measure $Q \in \mathcal{M}^e(F)$, and $J_T^2(F)$ is closed in L^2.*

(2) *There is a martingale measure $Q \in \mathcal{M}^e(F)$ that satisfies the reverse Hölder condition $R_2(P)$.*

(3) *There is a constant C such that for all $\pi \in \Pi(F)$ we have*

$$\|\sup_{t\le T}(\pi\cdot S)_t\|_{L^2(P)} \le C\|(\pi\cdot S)_T\|_{L^2(P)}.$$

(4) *There is a constant c such that for every stopping time τ, every $A \in \mathcal{F}_\tau$, and every $\pi \in \Pi(F)$, with $\pi = \pi I_{]\tau,T]}$, we have*

$$\|I_A - (\pi\cdot S)_T\|_{L^2(P)} \ge cP(A)^{1/2}.$$

Note that assertion (4) implies that for every stopping time τ and for every $\pi \in \Pi(G)$ we have

$$E\left(\left(1+\int_\tau^T \pi_u dS_u\right)^2 \Big/ F_\tau\right) \ge c. \tag{2.2}$$

Now we recall some known assertions from the filtering theory. The following proposition can be proved similarly to (Liptser & Shiryaev, 1986)(the detailed proof one can see in (Mania et al., 2009)).

Proposition 2.2. *If conditions* (A), (B)*, and* (C) *are satisfied, then for any continuous F-local martingale M, with $M_0 = 0$, and any G-local martingale m^G*

$$\widehat{M}_t = E(M_t|G_t) = \int_0^t \frac{d\widehat{\langle M, m^G\rangle}_u}{d\langle m^G\rangle_u} dm_u^G + L_t^G, \tag{2.3}$$

where L^G is a local martingale orthogonal to m^G.

It follows from this proposition that for any G-predictable, M-integrable process π and any G-martingale m^G

$$\langle\widehat{(\pi\cdot M)}, m^G\rangle_t = \int_0^t \pi_u \frac{d\widehat{\langle M, m^G\rangle}_u}{d\langle m^G\rangle_u} d\langle m^G\rangle_u = \int_0^t \pi_u d\langle \widehat{M}, m^G\rangle_u = \langle \pi\cdot\widehat{M}, m^G\rangle_t.$$

Hence, for any G-predictable, M-integrable process π

$$\widehat{(\pi\cdot M)}_t = E\left(\int_0^t \pi_s dM_s | G_t\right) = \int_0^t \pi_s d\widehat{M}_s. \tag{2.4}$$

Since π, λ, and $\langle M\rangle$ are G-predictable, from (2.4) we have

$$\widehat{(\pi\cdot S)}_t = E\left(\int_0^t \pi_u dS_u | G_t\right) = \int_0^t \pi_u d\widehat{S}_u, \tag{2.5}$$

where

$$\widehat{S}_t = S_0 + \int_0^t \widehat{\lambda}_u d\langle M\rangle_u + \widehat{M}_t.$$

3. Separation Principle: The Optimality Principle

Let us introduce the value function of the problem (1.2) defined as

$$U^H(t,x) = \operatorname*{ess\,inf}_{\pi \in \Pi(G)} E\left(\left(x + \int_t^T \pi_u dS_u - H\right)^2 |G_t\right). \tag{3.1}$$

By the GKW decomposition

$$H_t = E(H|F_t) = EH + \int_0^t h_u dM_u + L_t \tag{3.2}$$

for a F-predictable, M-integrable process h and a local martingale L strongly orthogonal to M. We shall use also the GKW decompositions of $H_t = E(H|F_t)$ with respect to the local martingale $\widehat{M}$

$$H_t = EH + \int_0^t h_u^G d\widehat{M}_u + L_t^G, \tag{3.3}$$

where h^G is a F-predictable process and L^G is a F-local martingale strongly orthogonal to $\widehat{M}$. It follows from Proposition 2.2 (applied for $m^G = \widehat{M}$) and Lemma A.1 that

$$\langle E(H|G_\cdot), \widehat{M}\rangle_t = \int_0^t \widehat{h_u^G} \rho_u^2 d\langle M\rangle_u. \tag{3.4}$$

We shall use the notation

$$\widetilde{h}_t = \widehat{h_t^G} \rho_t^2 - \widehat{h}_t. \tag{3.5}$$

Note that $\widetilde{h}$ belongs to the class $\Pi(G)$ by Lemma A.2.

Let us introduce now a new optimization problem, equivalent to the initial mean-variance hedging problem (1.2), to minimize the expression

$$E\left[\left(x + \int_0^T \pi_u d\widehat{S}_u - \widehat{H}_T\right)^2 + \int_0^T \left(\pi_u^2\left(1-\rho_u^2\right) + 2\pi_u \tilde{h}_u\right) d\langle M\rangle_u\right] \tag{3.6}$$

over all $\pi \in \Pi(G)$. Recall that $\widehat{S}_t = E(S_t|G_t) = S_0 + \int_0^t \widehat{\lambda}_u d\langle M\rangle_u + \widehat{M}_t$.

Theorem 3.1. *Let conditions* (A), (B)*, and* (C) *be satisfied. Then the initial mean-variance hedging problem* (1.2) *is equivalent to the problem* (3.6)*. In particular, for any $\pi \in \Pi(G)$ and $t \in [0,T]$*

$$E\left[\left(x + \int_t^T \pi_u dS_u - H\right)^2 |G_t\right] = E\left[\left(H - \widehat{H}_T\right)^2 |G_t\right]$$
$$+ E\left[\left(x + \int_t^T \pi_u d\widehat{S}_u - \widehat{H}_T\right)^2 + \int_t^T \left(\pi_u^2\left(1-\rho_u^2\right) + 2\pi_u \tilde{h}_u\right) d\langle M\rangle_u |G_t\right]. \tag{3.7}$$

Proof. We have

$$E\left[\left(x + \int_t^T \pi_u dS_u - H\right)^2 |G_t\right] = E\left[\left(x + \int_t^T \pi_u d\widehat{S}_u - H + \int_t^T \pi_u d\left(M_u - \widehat{M}_u\right)\right)^2 |G_t\right]$$

$$= E\left[\left(x+\int_t^T \pi_u d\widehat{S}_u - H\right)^2 |G_t\right] + 2E\left[\left(x+\int_t^T \pi_u d\widehat{S}_u - H\right)\left(\int_t^T \pi_u d\left(M_u - \widehat{M}_u\right)\right)|G_t\right]$$

$$+ \; E\left[\left(\int_t^T \pi_u d\left(M_u - \widehat{M}_u\right)\right)^2 |G_t\right] = I_1 + 2I_2 + I_3. \tag{3.8}$$

It is evident that

$$I_1 = E\left[\left(x+\int_t^T \pi_u d\widehat{S}_u - \widehat{H}_T\right)^2 |G_t\right] + E\left[\left(H - \widehat{H}_T\right)^2 |G_t\right]. \tag{3.9}$$

Since $\pi, \widehat{\lambda}$, and $\langle \widehat{M} \rangle$ are G_T-measurable and the σ-algebras $F_t^S \vee G_t$ and G_T are conditionally independent given G_t (see Remark 2.3), it follows from (2.4) that

$$\begin{aligned}
&E\left[\int_t^T \pi_u \widehat{\lambda}_u d\langle M\rangle_u \int_t^T \pi_u d\left(M_u - \widehat{M}_u\right)|G_t\right] \\
&\quad = E\left[\int_t^T \pi_u \widehat{\lambda}_u d\langle M\rangle_u \int_0^T \pi_u d\left(M_u - \widehat{M}_u\right)|G_t\right] \\
&\quad\quad - E\left[\int_t^T \pi_u \widehat{\lambda}_u d\langle M\rangle_u \int_0^t \pi_u d\left(M_u - \widehat{M}_u\right)|G_t\right] \\
&\quad = E\left[\int_t^T \pi_u \widehat{\lambda}_u d\langle M\rangle_u E\left(\int_0^T \pi_u d\left(M_u - \widehat{M}_u\right)|G_T\right)|G_t\right] \\
&\quad\quad - E\left[\int_t^T \pi_u \widehat{\lambda}_u d\langle M\rangle_u |G_t\right] E\left[\int_0^t \pi_u d\left(M_u - \widehat{M}_u\right)|G_t\right] = 0.
\end{aligned} \tag{3.10}$$

On the other hand, by using decomposition (3.2), equality (3.4), properties of square characteristics of martingales, and the projection theorem, we obtain

$$\begin{aligned}
E\left[H\int_t^T \pi_u d\left(M_u - \widehat{M}_u\right)|G_t\right] &= E\left[H\int_t^T \pi_u dM_u|G_t\right] - E\left[\widehat{H}_T\int_t^T \pi_u d\widehat{M}_u|G_t\right] \\
&= E\left[\int_t^T \pi_u d\langle M, E(H|F_\cdot)\rangle_u |G_t\right] - E\left[\int_t^T \pi_u d\langle \widehat{H}, \widehat{M}\rangle_u |G_t\right] \\
&= E\left[\int_t^T \pi_u h_u d\langle M\rangle_u |G_t\right] - E\left[\int_t^T \pi_u \widehat{h_u^G} \rho_u^2 d\langle M\rangle_u |G_t\right] \\
&= E\left[\int_t^T \pi_u \left(\widehat{h}_u - \widehat{h_u^G}\rho_u^2\right) d\langle M\rangle_u |G_t\right] = -E\left[\int_t^T \pi_u \widetilde{h}_u d\langle M\rangle_u |G_t\right].
\end{aligned} \tag{3.11}$$

Finally, it is easy to verify that

$$\begin{aligned}
&2E\left[\int_t^T \pi_u \widehat{M}_u \int_t^T \pi_u d\left(M_u - \widehat{M}_u\right)|G_t\right] + E\left[\left(\int_t^T \pi_u d\left(M_u - \widehat{M}_u\right)\right)^2 |G_t\right] \\
&\quad = E\left[\left(\int_t^T \pi_u^2 d\langle M\rangle_u - \int_t^T \pi_u^2 d\langle \widehat{M}\rangle_u\right)|G_t\right] = E\left[\int_t^T \pi_u^2\left(1-\rho_u^2\right) d\langle M\rangle_u |G_t\right].
\end{aligned} \tag{3.12}$$

Therefore (3.8), (3.9), (3.10), (3.11), and (3.12) imply the validity of equality (3.7). □

Thus, it follows from Theorem 3.1 that the optimization problems (1.2) and (3.6) are equivalent. Therefore it is sufficient to solve the problem (3.6), which is formulated in terms of G-adapted processes. One can say that (3.6) is a mean-variance hedging problem under complete information with a correction term and can be solved by using methods for complete information.
Let us introduce the value process of the problem (3.6)

$$V^H(t,x) = \operatorname*{ess\,inf}_{\pi\in\Pi(G)} E\left[\left(x+\int_t^T \pi_u d\widehat{S}_u - \widehat{H}_T\right)^2 + \int_t^T \left[\pi_u^2\left(1-\rho_u^2\right)+2\pi_u\tilde{h}_u\right] d\langle M\rangle_u | G_t\right]. \tag{3.13}$$

It follows from Theorem 3.1 that

$$U^H(t,x) = V^H(t,x) + E\left[(H-\widehat{H}_T)^2 | G_t\right]. \tag{3.14}$$

The optimality principle takes in this case the following form.

Proposition 3.1 (optimality principle)**.** *Let conditions* (A), (B) *and* (C) *be satisfied. Then*

(a) *for all $x \in R$, $\pi \in \Pi(G)$, and $s \in [0,T]$ the process*

$$V^H\left(t, x+\int_s^t \pi_u d\widehat{S}_u\right) + \int_s^t \left[\pi_u^2\left(1-\rho_u^2\right) + 2\pi_u\tilde{h}_u)\right] d\langle M\rangle_u$$

is a submartingale on $[s,T]$, admitting an right continuous with left limits (RCLL) modification.

(b) *π^* is optimal if and only if the process*

$$V^H\left(t, x+\int_s^t \pi_u^* d\widehat{S}_u\right) + \int_s^t \left[(\pi_u^*)^2\left(1-\rho_u^2\right) + 2\pi_u^*\tilde{h}_u\right] d\langle M\rangle_u$$

is a martingale.

This assertion can be proved in a standard manner (see, e.g., (El Karoui & Quenez, 1995), (Kramkov, 1996)). The proof more adapted to this case one can see in (Mania & Tevzadze, 2003).
Let

$$V(t,x) = \operatorname*{ess\,inf}_{\pi\in\Pi(G)} E\left[\left(x+\int_t^T \pi_u d\widehat{S}_u\right)^2 + \int_t^T \pi_u^2\left(1-\rho_u^2\right) d\langle M\rangle_u | G_t\right]$$

and

$$V_t(2) = \operatorname*{ess\,inf}_{\pi\in\Pi(G)} E\left[\left(1+\int_t^T \pi_u d\widehat{S}_u\right)^2 + \int_t^T \pi_u^2\left(1-\rho_u^2\right) d\langle M\rangle_u | G_t\right].$$

It is evident that $V(t,x)$ (resp., $V_t(2)$) is the value process of the optimization problem (3.6) in the case $H=0$ (resp., $H=0$ and $x=1$), i.e.,

$$V(t,x) = V^0(t,x) \quad \text{and} \quad V_t(2) = V^0(t,1).$$

Since $\Pi(G)$ is a cone, we have

$$V(t,x) = x^2 \operatorname*{ess\,inf}_{\pi\in\Pi(G)} E\left[\left(1+\int_t^T \frac{\pi_u}{x} d\widehat{S}_u\right)^2 + \int_t^T \left(\frac{\pi_u}{x}\right)^2\left(1-\rho_u^2\right) d\langle M\rangle_u | G_t\right] = x^2 V_t(2). \tag{3.15}$$

Therefore from Proposition 3.1 and equality (3.15) we have the following.

Corollary 3.1. (a) *The process*

$$V_t(2)\left(1+\int_s^t \pi_u d\widehat{S}_u\right)^2+\int_s^t (\pi_u)^2(1-\rho_u^2)d\langle M\rangle_u,$$

$t \geq s$, is a submartingale for all $\pi \in \Pi(G)$ and $s \in [0,T]$.
(b) *π^* is optimal if and only if*

$$V_t(2)\left(1+\int_s^t \pi_u^* d\widehat{S}_u\right)^2+\int_s^t (\pi_u^*)^2(1-\rho_u^2)d\langle M\rangle_u,$$

$t \geq s$, is a martingale.

Note that in the case $H=0$ from Theorem 3.1 we have

$$E\left[\left(1+\int_t^T \pi_u dS_u\right)^2|G_t\right]=E\left[\left(1+\int_t^T \pi_u d\widehat{S}_u\right)^2+\int_t^T \pi_u^2\left(1-\rho_u^2\right)d\langle M\rangle_u|G_t\right] \tag{3.16}$$

and, hence,

$$V_t(2)=U^0(t,1). \tag{3.17}$$

Lemma 3.1. *Let conditions* (A)–(D) *be satisfied. Then there is a constant $1 \geq c > 0$ such that $V_t(2) \geq c$ for all $t \in [0,T]$ a.s. and*

$$1-\rho_t^2+\rho_t^2 V_t(2) \geq c \quad \mu^{\langle M\rangle} a.e. \tag{3.18}$$

Proof. Let

$$V_t^F(2)=\operatorname*{ess\,inf}_{\pi\in\Pi(F)} E\left[\left(1+\int_t^T \pi_u dS_u\right)^2|F_t\right].$$

It follows from assertion (4) of Proposition 2.1 that there is a constant $c>0$ such that $V_t^F(2) \geq c$ for all $t \in [0,T]$ a.s. Note that $c \leq 1$ since $V^F \leq 1$. Then by (3.17)

$$\begin{aligned} V_t(2)=U^0(t,1)&=\operatorname*{ess\,inf}_{\pi\in\Pi(G)} E\left[\left(1+\int_t^T \pi_u dS_u\right)^2|G_t\right] \\ &=\operatorname*{ess\,inf}_{\pi\in\Pi(G)} E\left[E\left(\left(1+\int_t^T \pi_u dS_u\right)^2|F_t\right)|G_t\right] \geq E(V_t^F(2)|G_t) \geq c. \end{aligned}$$

Therefore, since $\rho_t^2 \leq 1$ by Lemma A.1,

$$1-\rho_t^2+\rho_t^2 V_t(2) \geq 1-\rho_t^2+\rho_t^2 c \geq \inf_{r\in[0,1]}(1-r+rc)=c.$$

4. BSDEs for the Value Process

Let us consider the semimartingale backward equation

$$Y_t = Y_0 + \int_0^t f(u, Y_u, \psi_u) d\langle m \rangle_u + \int_0^t \psi_u dm_u + L_t \tag{4.1}$$

with the boundary condition

$$Y_T = \eta, \tag{4.2}$$

where η is an integrable G_T-measurable random variable, $f : \Omega \times [0,T] \times R^2 \to R$ is $\mathcal{P} \times \mathcal{B}(R^2)$ measurable, and m is a local martingale. A solution of (4.1)–(4.2) is a triple (Y, ψ, L), where Y is a special semimartingale, ψ is a predictable m-integrable process, and L a local martingale strongly orthogonal to m. Sometimes we call Y alone the solution of (4.1)–(4.2), keeping in mind that $\psi \cdot m + L$ is the martingale part of Y.

Backward stochastic differential equations have been introduced in (Bismut, 1973) for the linear case as the equations for the adjoint process in the stochastic maximum principle. The semimartingale backward equation, as a stochastic version of the Bellman equation in an optimal control problem, was first derived in (Chitashvili, 1983). The BSDE with more general nonlinear generators was introduced in (Pardoux & Peng, 1990) for the case of Brownian filtration, where the existence and uniqueness of a solution of BSDEs with generators satisfying the global Lifschitz condition was established. These results were generalized for generators with quadratic growth in (Kobylanski, 2000), (Lepeltier & San Martin, 1998) for BSDEs driven by a Brownian motion and in (Morlais, 2009), (Tevzadze, 2008) for BSDEs driven by martingales. But conditions imposed in these papers are too restrictive for our needs. We prove here the existence and uniqueness of a solution by directly showing that the unique solution of the BSDE that we consider is the value of the problem.

In this section we characterize optimal strategies in terms of solutions of suitable semimartingale backward equations.

Theorem 4.1. *Let H be a square integrable F_T-measurable random variable, and let conditions* (A), (B), (C), *and* (D) *be satisfied. Then the value function of the problem* (3.6) *admits a representation*

$$V^H(t,x) = V_t(0) - 2V_t(1)x + V_t(2)x^2, \tag{4.3}$$

where the processes $V_t(0), V_t(1)$, and $V_t(2)$ satisfy the following system of backward equations:

$$Y_t(2) = Y_0(2) + \int_0^t \frac{\left(\psi_s(2)\rho_s^2 + \widehat{\lambda}_s Y_s(2)\right)^2}{1-\rho_s^2+\rho_s^2 Y_s(2)} d\langle M \rangle_s + \int_0^t \psi_s(2) d\widehat{M}_s + L_t(2), \quad Y_T(2) = 1, \tag{4.4}$$

$$Y_t(1) = Y_0(1) + \int_0^t \frac{\left(\psi_s(2)\rho_s^2 + \widehat{\lambda}_s Y_s(2)\right)\left(\psi_s(1)\rho_s^2 + \widehat{\lambda}_s Y_s(1) - \tilde{h}_s\right)}{1-\rho_s^2+\rho_s^2 Y_s(2)} d\langle M \rangle_s$$
$$+ \int_0^t \psi_s(1) d\widehat{M}_s + L_t(1), \quad Y_T(1) = E(H|G_T), \tag{4.5}$$

$$Y_t(0) = Y_0(0) + \int_0^t \frac{\left(\psi_s(1)\rho_s^2 + \widehat{\lambda}_s Y_s(1) - \tilde{h}_s\right)^2}{1-\rho_s^2+\rho_s^2 Y_s(2)} d\langle M \rangle_s$$
$$+ \int_0^t \psi_s(0) d\widehat{M}_s + L_t(0), \quad Y_T(0) = E^2(H|G_T), \tag{4.6}$$

where $L(2), L(1)$, and $L(0)$ are G-local martingales orthogonal to $\widehat{M}$.

Besides, the optimal filtered wealth process $\widehat{X}_t^{x,\pi^*} = x + \int_0^t \pi_u^* d\widehat{S}_u$ *is a solution of the linear equation*

$$\widehat{X}_t^* = x - \int_0^t \frac{\rho_u^2 \psi_u(2) + \widehat{\lambda}_u Y_u(2)}{1-\rho_u^2+\rho_u^2 Y_u(2)} \widehat{X}_u^* d\widehat{S}_u + \int_0^t \frac{\psi_u(1)\rho_u^2 + \widehat{\lambda}_u Y_u(1) - \tilde{h}_u}{1-\rho_u^2+\rho_u^2 Y_u(2)} d\widehat{S}_u. \tag{4.7}$$

Proof. Similarly to the case of complete information one can show that the optimal strategy exists and that $V^H(t,x)$ is a square trinomial of the form (4.3) (see, e.g., (Mania & Tevzadze, 2003)). More precisely the space of stochastic integrals

$$J_{t,T}^2(G) = \left\{ \int_t^T \pi_u dS_u : \pi \in \Pi(G) \right\}$$

is closed by Proposition 2.1, since $\langle M \rangle$ is G-predictable. Hence there exists optimal strategy $\pi^*(t,x) \in \Pi(G)$ and $U^H(t,x) = E[|H - x - \int_t^T \pi_u^*(t,x) dS_u|^2 | G_t]$. Since $\int_t^T \pi_u^*(t,x) dS_u$ coincides with the orthogonal projection of $H - x \in L^2$ on the closed subspace of stochastic integrals, then the optimal strategy is linear with respect to x, i.e., $\pi_u^*(t,x) = \pi_u^0(t) + x\pi_u^1(t)$. This implies that the value function $U^H(t,x)$ is a square trinomial. It follows from the equality (3.14) that $V^H(t,x)$ is also a square trinomial, and it admits the representation (4.3).
Let us show that $V_t(0), V_t(1)$, and $V_t(2)$ satisfy the system (4.4)–(4.6). It is evident that

$$V_t(0) = V^H(t,0) = \operatorname*{ess\,inf}_{\pi \in \Pi(G)} E\left[\left(\int_t^T \pi_u d\widehat{S}_u - \widehat{H}_T \right)^2 + \int_t^T [\pi_u^2 \left(1-\rho_u^2\right) + 2\pi_u \tilde{h}_u] d\langle M \rangle_u | G_t \right] \tag{4.8}$$

and

$$V_t(2) = V^0(t,1) = \operatorname*{ess\,inf}_{\pi \in \Pi(G)} E\left[\left(1 + \int_t^T \pi_u d\widehat{S}_u \right)^2 + \int_t^T \pi_u^2 \left(1-\rho_u^2\right) d\langle M \rangle_u | G_t \right]. \tag{4.9}$$

Therefore, it follows from the optimality principle (taking $\pi = 0$) that $V_t(0)$ and $V_t(2)$ are RCLL G-submartingales and

$$V_t(2) \le E(V_T(2)|G_t) \le 1, \qquad V_t(0) \le E(E^2(H|G_T)|G_t) \le E(H^2|G_t).$$

Since

$$V_t(1) = \frac{1}{2}(V_t(0) + V_t(2) - V^H(t,1)), \tag{4.10}$$

the process $V_t(1)$ is also a special semimartingale, and since $V_t(0) - 2V_t(1)x + V_t(2)x^2 = V^H(t,x) \ge 0$ for all $x \in R$, we have $V_t^2(1) \le V_t(0)V_t(2)$; hence

$$V_t^2(1) \le E\left(H^2|G_t\right).$$

Expressions (4.8), (4.9), and (3.13) imply that $V_T(0) = E^2(H|G_T)$, $V_T(2) = 1$, and $V^H(T,x) = (x - E(H|G_T))^2$. Therefore from (4.10) we have $V_T(1) = E(H|G_T)$, and $V(0), V(1)$, and $V(2)$ satisfy the boundary conditions.
Thus, the coefficients $V_t(i), i = 0,1,2$, are special semimartingales, and they admit the decomposition

$$V_t(i) = V_0(i) + A_t(i) + \int_0^t \varphi_s(i) d\widehat{M}_s + m_t(i), \quad i = 0,1,2, \tag{4.11}$$

where $m(0), m(1)$, and $m(2)$ are G-local martingales strongly orthogonal to $\widehat{M}$ and $A(0)$, $A(1)$, and $A(2)$ are G-predictable processes of finite variation.
There exists an increasing continuous G-predictable process K such that

$$\langle M\rangle_t = \int_0^t \nu_u dK_u, \quad A_t(i) = \int_0^t a_u(i) dK_u, \quad i = 0,1,2,$$

where ν and $a(i), i = 0,1,2$, are G-predictable processes.
Let $\widehat{X}^{x,\pi}_{s,t} \equiv x + \int_s^t \pi_u d\widehat{S}_u$ and

$$Y^{x,\pi}_{s,t} \equiv V^H\left(t, \widehat{X}^{x,\pi}_{s,t}\right) + \int_s^t \left[\pi_u^2\left(1-\rho_u^2\right) + 2\pi_u \tilde{h}_u\right] d\langle M\rangle_u.$$

Then by using (4.3), (4.11), and the Itô formula for any $t \geq s$ we have

$$\left(\widehat{X}^{x,\pi}_{s,t}\right)^2 = x + \int_s^t \left[2\pi_u \widehat{\lambda}_u \widehat{X}^{x,\pi}_{s,u} + \pi_u^2 \rho_u^2\right] d\langle M\rangle_u + 2\int_s^t \pi_u \widehat{X}^{x,\pi}_{s,u} d\widehat{M}_u \tag{4.12}$$

and

$$\begin{aligned} Y^{x,\pi}_{s,t} - V^H(s,x) = &\int_s^t \left[\left(\widehat{X}^{x,\pi}_{s,u}\right)^2 a_u(2) - 2\widehat{X}^{x,\pi}_{s,u} a_u(1) + a_u(0)\right] dK_u \\ &+ \int_s^t \left[\pi_u^2\left(1-\rho_u^2+\rho_u^2 V_{u-}(2)\right) + 2\pi_u \widehat{X}^{x,\pi}_{s,u}\left(\widehat{\lambda}_u V_{u-}(2) + \varphi_u(2)\rho_u^2\right)\right. \\ &- \left. 2\pi_u\left(V_{u-}(1)\widehat{\lambda}_u + \varphi_u(1)\rho_u^2 - \tilde{h}_u\right)\right] \nu_u dK_u + m_t - m_s, \end{aligned} \tag{4.13}$$

where m is a local martingale.
Let

$$\begin{aligned} G(\pi,x) = G(\omega,u,\pi,x) = &\pi^2\left(1-\rho_u^2+\rho_u^2 V_{u-}(2)\right) + 2\pi x\left(\widehat{\lambda}_u V_{u-}(2) + \varphi_u(2)\rho_u^2\right) \\ &- 2\pi(V_{u-}(1)\widehat{\lambda}_u + \varphi_u(1)\rho_u^2 - \tilde{h}_u). \end{aligned}$$

It follows from the optimality principle that for each $\pi \in \Pi(G)$ the process

$$\int_s^t \left[\left(\widehat{X}^{x,\pi}_{s,u}\right)^2 a_u(2) - 2\widehat{X}^{x,\pi}_{s,u} a_u(1) + a_u(0)\right] dK_u + \int_s^t G\left(\pi_u, \widehat{X}^{x,\pi}_{s,u}\right) \nu_u dK_u \tag{4.14}$$

is increasing for any s on $s \leq t \leq T$, and for the optimal strategy π^* we have the equality

$$\int_s^t \left[\left(\widehat{X}^{x,\pi^*}_{s,u}\right)^2 a_u(2) - 2\widehat{X}^{x,\pi^*}_{s,u} a_u(1) + a_u(0)\right] dK_u = -\int_s^t G\left(\pi^*_u, \widehat{X}^{x,\pi^*}_{s,u}\right) \nu_u dK_u. \tag{4.15}$$

Since $\nu_u dK_u = d\langle M\rangle_u$ is continuous, without loss of generality one can assume that the process K is continuous (see (Mania & Tevzadze, 2003) for details). Therefore, by taking in (4.14) $\tau_s(\varepsilon) = \inf\{t \geq s : K_t - K_s \geq \varepsilon\}$ instead of t, we have that for any $\varepsilon > 0$ and $s \geq 0$

$$\frac{1}{\varepsilon}\int_s^{\tau_s(\varepsilon)} \left[\left(\widehat{X}^{x,\pi}_{s,u}\right)^2 a_u(2) - 2\widehat{X}^{x,\pi}_{s,u} a_u(1) + a_u(0)\right] dK_u \geq -\frac{1}{\varepsilon}\int_s^{\tau_s(\varepsilon)} G\left(\pi_u, \widehat{X}^{x,\pi}_{s,u}\right) \nu_u dK_u. \tag{4.16}$$

By passing to the limit in (4.16) as $\varepsilon \to 0$, from Proposition B of (Mania & Tevzadze, 2003) we obtain

$$x^2 a_u(2) - 2x a_u(1) + a_u(0) \geq -G(\pi_u, x)\nu_u, \qquad \mu^K\text{-a.e.},$$

for all $\pi \in \Pi(G)$. Similarly from (4.15) we have that μ^K-a.e.

$$x^2 a_u(2) - 2x a_u(1) + a_u(0) = -G(\pi_u^*, x)\nu_u$$

and hence

$$x^2 a_u(2) - 2x a_u(1) + a_u(0) = -\nu_u \operatorname*{ess\,inf}_{\pi \in \Pi(G)} G(\pi_u, x). \tag{4.17}$$

The infimum in (4.17) is attained for the strategy

$$\hat{\pi}_t = \frac{V_t(1)\widehat{\lambda}_t + \varphi_t(1)\rho_t^2 - \tilde{h}_t - x(V_t(2)\widehat{\lambda}_t + \varphi_t(2)\rho_t^2)}{1 - \rho_t^2 + \rho_t^2 V_t(2)}. \tag{4.18}$$

From here we can conclude that

$$\operatorname*{ess\,inf}_{\pi \in \Pi(G)} G(\pi_t, x) \geq G(\hat{\pi}_t, x) = -\frac{\left(V_t(1)\widehat{\lambda}_t + \varphi_t(1)\rho_t^2 - \tilde{h}_t - x\left(V_t(2)\widehat{\lambda}_t + \varphi_t(2)\rho_t^2\right)\right)^2}{1 - \rho_t^2 + \rho_t^2 V_t(2)}. \tag{4.19}$$

Let $\pi_t^n = I_{[0,\tau_n[}(t)\hat{\pi}_t$, where $\tau_n = \inf\{t : |V_t(1)| \geq n\}$.
It follows from Lemmas A.2, 3.1, and A.3 that $\pi^n \in \Pi(G)$ for every $n \geq 1$ and hence

$$\operatorname*{ess\,inf}_{\pi \in \Pi(G)} G(\pi_t, x) \leq G(\pi_t^n, x)$$

for all $n \geq 1$. Therefore

$$\operatorname*{ess\,inf}_{\pi \in \Pi(G)} G(\pi_t, x) \leq \lim_{n \to \infty} G(\pi_t^n, x) = G(\hat{\pi}_t, x). \tag{4.20}$$

Thus (4.17), (4.19), and (4.20) imply that

$$\begin{aligned} &x^2 a_t(2) - 2x a_t(1) + a_t(0) \\ &\qquad = \nu_t \frac{(V_t(1)\widehat{\lambda}_t + \varphi_t(1)\rho_t^2 - \tilde{h}_t - x(V_t(2)\widehat{\lambda}_t + \varphi_t(2)\rho_t^2))^2}{1 - \rho_t^2 + \rho_t^2 V_t(2)}, \quad \mu^K\text{-a.e.}, \end{aligned} \tag{4.21}$$

and by equalizing the coefficients of square trinomials in (4.21) (and integrating with respect to dK) we obtain

$$A_t(2) = \int_0^t \frac{\left(\varphi_s(2)\rho_s^2 + \widehat{\lambda}_s V_s(2)\right)^2}{1 - \rho_s^2 + \rho_s^2 V_s(2)} d\langle M \rangle_s, \tag{4.22}$$

$$A_t(1) = \int_0^t \frac{\left(\varphi_s(2)\rho_s^2 + \widehat{\lambda}_s V_s(2)\right)\left(\varphi_s(1)\rho_s^2 + \widehat{\lambda}_s V_s(1) - \tilde{h}_s\right)}{1 - \rho_s^2 + \rho_s^2 V_s(2)} d\langle M \rangle_s, \tag{4.23}$$

$$A_t(0) = \int_0^t \frac{\left(\varphi_s(1)\rho_s^2 + \widehat{\lambda}_s V_s(1) - \tilde{h}_s\right)^2}{1 - \rho_s^2 + \rho_s^2 V_s(2)} d\langle M \rangle_s, \tag{4.24}$$

which, together with (4.11), implies that the triples $(V(i), \varphi(i), m(i))$, $i = 0, 1, 2$, satisfy the system (4.4)–(4.6).

Note that $A(0)$ and $A(2)$ are integrable increasing processes and relations (4.22) and (4.24) imply that the strategy $\hat{\pi}$ defined by (4.18) belongs to the class $\Pi(G)$.
Let us show now that if the strategy $\pi^* \in \Pi(G)$ is optimal, then the corresponding filtered wealth process $\widehat{X}_t^{\pi^*} = x + \int_0^t \pi_u^* d\widehat{S}_u$ is a solution of (4.7).
By the optimality principle the process

$$Y_t^{\pi^*} = V^H\left(t, \widehat{X}_t^{\pi^*}\right) + \int_0^t \left[(\pi_u^*)^2\left(1-\rho_u^2\right) + 2\pi_u^*\tilde{h}_u\right] d\langle M\rangle_u$$

is a martingale. By using the Itô formula we have

$$Y_t^{\pi^*} = \int_0^t \left(\widehat{X}_u^{\pi^*}\right)^2 dA_u(2) - 2\int_0^t \widehat{X}_u^{\pi^*} dA_u(1) + A_t(0) + \int_0^t G\left(\pi_u^*, \widehat{X}_u^{\pi^*}\right) d\langle M\rangle_u + N_t,$$

where N is a martingale. Therefore by applying equalities (4.22), (4.23), and (4.24) we obtain

$$Y_t^{\pi^*} = \int_0^t \left(\pi_u^* - \frac{V_u(1)\widehat{\lambda}_u + \varphi_u(1)\rho_u^2 - \tilde{h}_u}{1-\rho_u^2+\rho_u^2 V_u(2)} \right.$$
$$\left. + \widehat{X}_u^{\pi^*}\frac{V_u(2)\widehat{\lambda}_u + \varphi_u(2)\rho_u^2}{1-\rho_u^2+\rho_u^2 V_u(2)}\right)^2 \left(1-\rho_u^2+\rho_u^2 V_u(2)\right) d\langle M\rangle_u + N_t,$$

which implies that $\mu^{\langle M\rangle}$-a.e.

$$\pi_u^* = \frac{V_u(1)\widehat{\lambda}_u + \varphi_u(1)\rho_u^2 - \tilde{h}_u}{1-\rho_u^2+\rho_u^2 V_u(2)} - \widehat{X}_u^{\pi^*}\frac{\left(V_u(2)\widehat{\lambda}_u + \varphi_u(2)\rho_u^2\right)}{1-\rho_u^2+\rho_u^2 V_u(2)}.$$

By integrating both parts of this equality with respect to $d\widehat{S}$ (and adding then x to the both parts), we obtain that $\widehat{X}^{\pi^*}$ satisfies (4.7). □

The uniqueness of the system (4.4)–(4.6) we shall prove under following condition (D*), stronger than condition (D).
Assume that

$$(D^*) \qquad \int_0^T \frac{\widehat{\lambda}_u^2}{\rho_u^2} d\langle M\rangle_u \le C.$$

Since $\rho^2 \le 1$ (Lemma A.1), it follows from (D*) that the mean-variance tradeoff of S is bounded, i.e.,

$$\int_0^T \widehat{\lambda}_u^2 d\langle M\rangle_u \le C,$$

which implies (see, e.g., Kazamaki (Kazamaki, 1994)) that the minimal martingale measure for S exists and satisfies the reverse Hölder condition $R_2(P)$. So, condition (D*) implies condition (D). Besides, it follows from condition (D*) that the minimal martingale measure $\widehat{Q}^{min}$ for $\widehat{S}$

$$d\widehat{Q}^{min} = \mathcal{E}_T\left(-\frac{\widehat{\lambda}}{\rho^2}\cdot\widehat{M}\right)$$

also exists and satisfies the reverse Hölder condition. Indeed, condition (D*) implies that $\mathcal{E}_t(-2\frac{\widehat{\lambda}}{\rho^2}\cdot\widehat{M})$ is a G-martingale and hence

$$E\left(\mathcal{E}_{tT}^2\left(-\frac{\widehat{\lambda}}{\rho^2}\cdot\widehat{M}\right)|G_t\right)=E\left(\mathcal{E}_{tT}\left(-2\frac{\widehat{\lambda}}{\rho^2}\cdot\widehat{M}\right)e^{\int_t^T\frac{\widehat{\lambda}_u^2}{\rho_u^2}d\langle M\rangle_u}G_t\right)\le e^C.$$

Recall that the process Z belongs to the class D if the family of random variables $Z_\tau I_{(\tau\le T)}$ for all stopping times τ is uniformly integrable.

Theorem 4.2. *Let conditions* (A), (B), (C), *and* (D*) *be satisfied. If a triple* $(Y(0),Y(1),Y(2))$, *where* $Y(0)\in D$, $Y^2(1)\in D$, *and* $c\le Y(2)\le C$ *for some constants* $0<c<C$, *is a solution of the system* (4.4)–(4.6), *then such a solution is unique and coincides with the triple* $(V(0),V(1),V(2))$.

Proof. Let $Y(2)$ be a bounded strictly positive solution of (4.4), and let

$$\int_0^t\psi_u(2)d\widehat{M}_u+L_t(2)$$

be the martingale part of $Y(2)$.
Since $Y(2)$ solves (4.4), it follows from the Itô formula that for any $\pi\in\Pi(G)$ the process

$$Y_t^\pi=Y_t(2)\left(1+\int_s^t\pi_u d\widehat{S}_u\right)^2+\int_s^t\pi_u^2\left(1-\rho_u^2\right)d\langle M\rangle_u, \tag{4.25}$$

$t\ge s$, is a local submartingale.
Since $\pi\in\Pi(G)$, from Lemma A.1 and the Doob inequality we have

$$\begin{aligned}&E\sup_{t\le T}\left(1+\int_0^t\pi_u d\widehat{S}\right)^2\\&\le\text{const}\left(1+E\int_0^T\pi_u^2\rho_u^2d\langle M\rangle_u\right)+E\left(\int_0^T|\pi_u\widehat{\lambda}_u|d\langle M\rangle_u\right)^2<\infty.\end{aligned} \tag{4.26}$$

Therefore, by taking in mind that $Y(2)$ is bounded and $\pi\in\Pi(G)$ we obtain

$$E(\sup_{s\le u\le T}Y_u^\pi)^2<\infty,$$

which implies that $Y^\pi\in D$. Thus Y^π is a submartingale (as a local submartingale from the class D), and by the boundary condition $Y_T(2)=1$ we obtain

$$Y_s(2)\le E\left(\left(1+\int_s^T\pi_u d\widehat{S}_u\right)^2+\int_s^T\pi_u^2\left(1-\rho_u^2\right)d\langle M\rangle_u|G_s\right)$$

for all $\pi\in\Pi(G)$ and hence

$$Y_t(2)\le V_t(2). \tag{4.27}$$

Let

$$\tilde{\pi}_t=-\frac{\widehat{\lambda}_tY_t(2)+\psi_t(2)\rho_t^2}{1-\rho_t^2+\rho_t^2Y_t(2)}\mathcal{E}_t\left(-\frac{\widehat{\lambda}Y(2)+\psi(2)\rho^2}{1-\rho^2+\rho^2Y(2)}\cdot\widehat{S}\right).$$

Since $1+\int_0^t \tilde{\pi}_u d\widehat{S}_u = \mathcal{E}_t(-\frac{\widehat{\lambda}Y(2)+\psi(2)\rho^2}{1-\rho^2+\rho^2 Y(2)}\cdot\widehat{S})$, it follows from (4.4) and the Itô formula that the process $Y^{\tilde{\pi}}$ defined by (4.25) is a positive local martingale and hence a supermartingale. Therefore

$$Y_s(2) \geq E\left(\left(1+\int_s^T \tilde{\pi}_u d\widehat{S}_u\right)^2 + \int_s^T \tilde{\pi}_u^2\left(1-\rho_u^2\right) d\langle M\rangle_u | G_s\right). \tag{4.28}$$

Let us show that $\tilde{\pi}$ belongs to the class $\Pi(G)$.
From (4.28) and (4.27) we have for every $s\in[0,T]$

$$E\left(\left(1+\int_s^T \tilde{\pi}_u d\widehat{S}_u\right)^2 + \int_s^T \tilde{\pi}_u^2\left(1-\rho_u^2\right) d\langle M\rangle_u | G_s\right) \leq Y_s(2) \leq V_s(2) \leq 1 \tag{4.29}$$

and hence

$$E\left(1+\int_0^T \tilde{\pi}_u d\widehat{S}_u\right)^2 \leq 1, \tag{4.30}$$

$$E\int_0^T \tilde{\pi}_u^2\left(1-\rho_u^2\right) d\langle M\rangle_u \leq 1. \tag{4.31}$$

By (D*) the minimal martingale measure $\widehat{Q}^{min}$ for $\widehat{S}$ satisfies the reverse Hölder condition, and hence all conditions of Proposition 2.1 are satisfied. Therefore the norm

$$E\left(\int_0^T \tilde{\pi}_s^2\rho_s^2 d\langle M\rangle_s\right) + E\left(\int_0^T |\tilde{\pi}_s\widehat{\lambda}_s| d\langle M\rangle_s\right)^2$$

is estimated by $E(1+\int_0^T \tilde{\pi}_u d\widehat{S}_u)^2$ and hence

$$E\int_0^T \tilde{\pi}_u^2\rho_u^2 d\langle M\rangle_u < \infty, \quad E\left(\int_0^T |\tilde{\pi}_s\widehat{\lambda}_s| d\langle M\rangle_s\right)^2 < \infty.$$

It follows from (4.31) and the latter inequality that $\tilde{\pi}\in\Pi(G)$, and from (4.28) we obtain

$$Y_t(2) \geq V_t(2),$$

which together with (4.27) gives the equality $Y_t(2) = V_t(2)$.
Thus $V(2)$ is a unique bounded strictly positive solution of (4.4). Besides,

$$\int_0^t \psi_u(2) d\widehat{M}_u = \int_0^t \varphi_u(2) d\widehat{M}_u, \quad L_t(2) = m_t(2) \tag{4.32}$$

for all t, P-a.s.
Let $Y(1)$ be a solution of (4.5) such that $Y^2(1)\in D$. By the Itô formula the process

$$\begin{aligned} R_t = {} & Y_t(1)\mathcal{E}_t\left(-\frac{\varphi(2)\rho^2+\widehat{\lambda}V(2)}{1-\rho^2+\rho^2 V(2)}\cdot\widehat{S}\right) \\ & + \int_0^t \mathcal{E}_u\left(-\frac{\varphi(2)\rho^2+\widehat{\lambda}V(2)}{1-\rho^2+\rho^2 V(2)}\cdot\widehat{S}\right)\frac{(\varphi_u(2)\rho_u^2+\widehat{\lambda}_u V_u(2))\tilde{h}_u}{1-\rho_u^2+\rho_u^2 V_u(2)} d\langle M\rangle_u \end{aligned} \tag{4.33}$$

is a local martingale. Let us show that R_t is a martingale.

As was already shown, the strategy

$$\tilde{\pi}_u = \frac{\psi_u(2)\rho_u^2 + \widehat{\lambda}_u Y_u(2)}{1-\rho^2+\rho^2 Y_u(2)} \mathcal{E}_u \left(-\frac{\psi(2)\rho^2 + \widehat{\lambda} Y(2)}{1-\rho^2+\rho^2 Y(2)} \cdot \widehat{S} \right)$$

belongs to the class $\Pi(G)$.
Therefore (see (4.26)),

$$E \sup_{t \le T} \mathcal{E}_t^2 \left(-\frac{\psi(2)\rho^2 + \widehat{\lambda} Y(2)}{1-\rho^2+\rho^2 Y(2)} \cdot \widehat{S} \right) = E \sup_{t \le T} \left(1 + \int_0^t \tilde{\pi}_u d\widehat{S} \right)^2 < \infty, \tag{4.34}$$

and hence

$$Y_t(1) \mathcal{E}_t \left(-\frac{\varphi(2)\rho^2 + \widehat{\lambda} V(2)}{1-\rho^2+\rho^2 V(2)} \cdot \widehat{S} \right) \in D.$$

On the other hand, the second term of (4.33) is the process of integrable variation, since $\tilde{\pi} \in \Pi(G)$ and $\tilde{h} \in \Pi(G)$ (see Lemma A.2) imply that

$$E \int_0^T \left| \mathcal{E}_u \left(-\frac{\varphi(2)\rho^2 + \widehat{\lambda} V(2)}{1-\rho^2+\rho^2 V(2)} \cdot \widehat{S} \right) \frac{(\varphi_u(2)\rho_u^2 + \widehat{\lambda}_u V_u(2))\tilde{h}_u}{1-\rho_u^2+\rho_u^2 V_u(2)} \right| d\langle M \rangle_u$$
$$= E \int_0^T |\tilde{\pi}_u \tilde{h}_u| d\langle M \rangle_u \le E^{1/2} \int_0^T \tilde{\pi}_u^2 d\langle M \rangle_u E^{1/2} \int_0^T \tilde{h}_u^2 d\langle M \rangle_u < \infty.$$

Therefore, the process R_t belongs to the class D, and hence it is a true martingale. By using the martingale property and the boundary condition we obtain

$$Y_t(1) = E \Bigg(\widehat{H}_T \mathcal{E}_{tT} \left(-\frac{\varphi(2)\rho^2 + \widehat{\lambda} V(2)}{1-\rho^2+\rho^2 V(2)} \cdot \widehat{S} \right)$$
$$+ \int_t^T \mathcal{E}_{tu} \left(-\frac{\varphi(2)\rho^2 + \widehat{\lambda} V(2)}{1-\rho^2+\rho^2 V(2)} \cdot \widehat{S} \right) \frac{(\varphi_u(2)\rho_u^2 + \widehat{\lambda}_u V_u(2))\tilde{h}_u}{1-\rho_u^2+\rho_u^2 V_u(2)} d\langle M \rangle_u | G_t \Bigg). \tag{4.35}$$

Thus, any solution of (4.5) is expressed explicitly in terms of $(V(2), \varphi(2))$ in the form (4.35).
Hence the solution of (4.5) is unique, and it coincides with $V_t(1)$.
It is evident that the solution of (4.6) is also unique. □

Remark 4.1. In the case $F^S \subseteq G$ we have $\rho_t = 1, \tilde{h}_t = 0$, and $\widehat{S}_t = S_t$, and (4.7) takes the form

$$\widehat{X}_t^* = x - \int_0^t \frac{\psi_u(2) + \widehat{\lambda}_u Y_u(2)}{Y_u(2)} \widehat{X}_u^* dS_u + \int_0^t \frac{\psi_u(1) + \widehat{\lambda}_u Y_u(1)}{Y_u(2)} dS_u.$$

Corollary 4.1. *In addition to conditions* (A)–(C) *assume that ρ is a constant and the mean-variance tradeoff $\langle \widehat{\lambda} \cdot M \rangle_T$ is deterministic. Then the solution of* (4.4) *is the triple $(Y(2), \psi(2), L(2))$, with $\psi(2) = 0, L(2) = 0$, and*

$$Y_t(2) = V_t(2) = \nu \left(\rho, 1-\rho^2 + \langle \widehat{\lambda} \cdot M \rangle_T - \langle \widehat{\lambda} \cdot M \rangle_t \right), \tag{4.36}$$

where $\nu(\rho, \alpha)$ is the root of the equation

$$\frac{1-\rho^2}{x} - \rho^2 \ln x = \alpha. \tag{4.37}$$

Besides,

$$\begin{aligned} Y_t(1) = E\Bigg(& H\mathcal{E}_{tT}\left(-\frac{\widehat{\lambda}V(2)}{1-\rho^2+\rho^2 V(2)}\cdot\widehat{S}\right) \\ & + \int_t^T \mathcal{E}_{tu}\left(-\frac{\widehat{\lambda}V(2)}{1-\rho^2+\rho^2 V(2)}\cdot\widehat{S}\right)\frac{\lambda_u V_u(2)\tilde{h}_u}{1-\rho^2+\rho^2 V_u(2)}d\langle M\rangle_u | G_t\Bigg) \end{aligned} \tag{4.38}$$

uniquely solves (4.5)*, and the optimal filtered wealth process satisfies the linear equation*

$$\widehat{X}_t^* = x - \int_0^t \frac{\widehat{\lambda}_u V_u(2)}{1-\rho^2+\rho^2 V_u(2)}\widehat{X}_u^* d\widehat{S}_u + \int_0^t \frac{\varphi_u(1)\rho^2 + \widehat{\lambda}_u V_u(1) - \tilde{h}_u}{1-\rho^2+\rho^2 V_u(2)} d\widehat{S}_u. \tag{4.39}$$

Proof. The function $f(x) = \frac{1-\rho^2}{x} - \rho^2 \ln x$ is differentiable and strictly decreasing on $]0, \infty[$ and takes all values from $]-\infty, +\infty[$. So (4.37) admits a unique solution for all α. Besides, the inverse function $\alpha(x)$ is differentiable. Therefore $Y_t(2)$ is a process of finite variation, and it is adapted since $\langle\widehat{\lambda}\cdot M\rangle_T$ is deterministic.
By definition of $Y_t(2)$ we have that for all $t \in [0, T]$

$$\frac{1-\rho^2}{Y_t(2)} - \rho^2 \ln Y_t(2) = 1-\rho^2 + \langle\widehat{\lambda}\cdot M\rangle_T - \langle\widehat{\lambda}\cdot M\rangle_t.$$

It is evident that for $\alpha = 1-\rho^2$ the solution of (4.37) is equal to 1, and it follows from (4.36) that $Y(2)$ satisfies the boundary condition $Y_T(2) = 1$. Therefore

$$\begin{aligned} \frac{1-\rho^2}{Y_t(2)} - \rho^2 \ln Y_t(2) - \left(1-\rho^2\right) &= -\left(1-\rho^2\right)\int_t^T d\frac{1}{Y_u(2)} + \rho^2 \int_t^T d\ln Y_u(2) \\ &= \int_t^T \left(\frac{1-\rho^2}{Y_u^2(2)} + \frac{\rho^2}{Y_u(2)}\right) dY_u(2) \end{aligned}$$

and

$$\int_t^T \frac{1-\rho^2+\rho^2 Y_u(2)}{Y_u^2(2)} dY_u(2) = \langle\widehat{\lambda}\cdot M\rangle_T - \langle\widehat{\lambda}\cdot M\rangle_t$$

for all $t \in [0, T]$. Hence

$$\int_0^t \frac{1-\rho^2+\rho^2 Y_u(2)}{Y_u^2(2)} dY_u(2) = \langle\widehat{\lambda}\cdot M\rangle_t,$$

and, by integrating both parts of this equality with respect to $Y(2)/(1-\rho^2+\rho^2 Y(2))$, we obtain that $Y(2)$ satisfies

$$Y_t(2) = Y_0(2) + \int_0^t \frac{Y_u^2(2)\widehat{\lambda}_u^2}{1-\rho^2+\rho^2 Y_u(2)} d\langle M\rangle_u, \tag{4.40}$$

which implies that the triple $(Y(2), \psi(2) = 0, L(2) = 0)$ satisfies (4.4) and $Y(2) = V(2)$ by Theorem 4.2. Equations (4.38) and (4.39) follow from (4.35) and (4.7), respectively, by taking $\varphi(2) = 0$. □

Remark 4.2. In case $F^S \subseteq G$ we have $\widehat{M} = M$ and $\rho = 1$. Therefore (4.40) is linear and $Y_t(2) = e^{\langle \widehat{\lambda} \cdot M \rangle_t - \langle \widehat{\lambda} \cdot M \rangle_T}$. In the case $\mathcal{A} = G$ of complete information, $Y_t(2) = e^{\langle \lambda \cdot N \rangle_t - \langle \lambda \cdot N \rangle_T}$.

5. Diffusion Market Model

Example 1. Let us consider the financial market model

$$d\tilde{S}_t = \tilde{S}_t \mu_t(\eta)dt + \tilde{S}_t \sigma_t(\eta)dw_t^0,$$
$$d\eta_t = a_t(\eta)dt + b_t(\eta)dw_t,$$

subjected to initial conditions. Here w^0 and w are correlated Brownian motions with $Edw_t^0 dw_t = \rho dt, \rho \in (-1, 1)$.

Let us write

$$w_t = \rho w_t^0 + \sqrt{1 - \rho^2} w_t^1,$$

where w^0 and w^1 are independent Brownian motions. It is evident that $w^\perp = -\sqrt{1-\rho^2} w^0 + \rho w^1$ is a Brownian motion independent of w, and one can express Brownian motions w^0 and w^1 in terms of w and $w^\perp$ as

$$w_t^0 = \rho w_t - \sqrt{1-\rho^2} w_t^\perp, \quad w_t^1 = \sqrt{1-\rho^2} w_t + \rho w_t^\perp. \tag{5.1}$$

Suppose that $b^2 > 0$, $\sigma^2 > 0$, and coefficients μ, σ, a, and b are such that $F_t^{S,\eta} = F_t^{w^0,w}$ and $F_t^\eta = F_t^w$.

We assume that an agent would like to hedge a contingent claim H (which can be a function of S_T and η_T) using only observations based on the process η. So the stochastic basis will be $(\Omega, \mathcal{F}, F_t, P)$, where F_t is the natural filtration of (w^0, w) and the flow of observable events is $G_t = F_t^w$.

Also denote $dS_t = \mu_t dt + \sigma_t dw_t^0$, so that $d\tilde{S}_t = \tilde{S}_t dS_t$ and S is the return of the stock.

Let $\tilde{\pi}_t$ be the number of shares of the stock at time t. Then $\pi_t = \tilde{\pi}_t \tilde{S}_t$ represents an amount of money invested in the stock at the time $t \in [0, T]$. We consider the mean-variance hedging problem

$$\text{to minimize} \quad E\left[\left(x + \int_0^T \tilde{\pi}_t d\tilde{S}_t - H\right)^2\right] \quad \text{over all} \quad \tilde{\pi} \quad \text{for which} \quad \tilde{\pi}\tilde{S} \in \Pi(G), \tag{5.2}$$

which is equivalent to studying the mean-variance hedging problem

$$\text{to minimize} \qquad E\left[\left(x + \int_0^T \pi_t dS_t - H\right)^2\right] \quad \text{over all} \quad \pi \in \Pi(G).$$

Remark 5.1. Since S is not G-adapted, $\widetilde{\pi}_t$ and $\widetilde{\pi}_t \widetilde{S}_t$ cannot be simultaneously G-predictable and the problem

$$\text{to minimize} \qquad E\left[\left(x + \int_0^T \tilde{\pi}_t d\tilde{S}_t - H\right)^2\right] \quad \text{over all} \quad \tilde{\pi} \in \Pi(G)$$

is not equivalent to the problem (5.2). In this setting, condition (A) is not satisfied, and it needs separate consideration.

By comparing with (1.1) we get that in this case

$$M_t = \int_0^t \sigma_s dw_s^0, \quad \langle M \rangle_t = \int_0^t \sigma_s^2 ds, \quad \lambda_t = \frac{\mu_t}{\sigma_t^2}.$$

It is evident that w is a Brownian motion also with respect to the filtration F^{w^0,w^1} and condition (B) is satisfied. Therefore by Proposition 2.2

$$\widehat{M}_t = \rho \int_0^t \sigma_s dw_s.$$

By the integral representation theorem the GKW decompositions (3.2) and (3.3) take the following forms:

$$c_H = EH, \quad H_t = c_H + \int_0^t h_s \sigma_s dw_s^0 + \int_0^t h_s^1 dw_s^1, \tag{5.3}$$

$$H_t = c_H + \rho \int_0^t h_s^G \sigma_s dw_s + \int_0^t h_s^\perp dw_s^\perp. \tag{5.4}$$

By putting expressions (5.1) for w^0 and w^1 in (5.3) and equalizing integrands of (5.3) and (5.4), we obtain

$$h_t = \rho^2 h_t^G - \sqrt{1-\rho^2}\frac{h_t^\perp}{\sigma_t}$$

and hence

$$\widehat{h}_t = \rho^2 \widehat{h_t^G} - \sqrt{1-\rho^2}\,\frac{\widehat{h}_t^\perp}{\sigma_t}.$$

Therefore by the definition of $\widetilde{h}$

$$\widetilde{h}_t = \rho^2 \widehat{h_t^G} - \widehat{h}_t = \sqrt{1-\rho^2}\,\frac{\widehat{h}_t^\perp}{\sigma_t}. \tag{5.5}$$

By using notations

$$Z_s(0) = \rho\sigma_s\varphi_s(0),\ Z_s(1) = \rho\sigma_s\varphi_s(1),\ Z_s(2) = \rho\sigma_s\varphi_s(2),\ \ \theta_s = \frac{\mu_s}{\sigma_s},$$

we obtain the following corollary of Theorem 4.1.

Corollary 5.1. *Let H be a square integrable F_T-measurable random variable. Then the processes $V_t(0), V_t(1)$, and $V_t(2)$ from (4.3) satisfy the following system of backward equations:*

$$V_t(2) = V_0(2) + \int_0^t \frac{(\rho Z_s(2) + \theta_s V_s(2))^2}{1-\rho^2+\rho^2 V_s(2)} ds + \int_0^t Z_s(2) dw_s, \quad V_T(2) = 1, \tag{5.6}$$

$$\begin{aligned} V_t(1) = V_0(1) &+ \int_0^t \frac{(\rho Z_s(2) + \theta_s V_s(2))\left(\rho Z_s(1) + \theta_s V_s(1) - \sqrt{1-\rho^2}\,\widehat{h}_s^\perp\right)}{1-\rho^2+\rho^2 V_s(2)} ds \\ &+ \int_0^t Z_s(1) dw_s, \quad V_T(1) = E(H|G_T), \end{aligned} \tag{5.7}$$

$$\begin{aligned} V_t(0) = V_0(0) &+ \int_0^t \frac{\left(\rho Z_s(1) + \theta_s V_s(1) - \sqrt{1-\rho^2}\,\widehat{h}_s^\perp\right)^2}{1-\rho^2+\rho^2 V_s(2)} ds \\ &+ \int_0^t Z_s(0) dw_s, \quad V_T(0) = E^2(H|G_T). \end{aligned} \tag{5.8}$$

Besides, the optimal wealth process $\widehat{X}^*$ satisfies the linear equation

$$\widehat{X}^*_t = x - \int_0^t \frac{\rho Z_s(2) + \theta_s V_s(2)}{1-\rho^2+\rho^2 V_s(2)} \widehat{X}^*_s (\theta_s ds + \rho dw_s)$$
$$+ \int_0^t \frac{\rho Z_s(1) + \theta_s V_s(1) - \sqrt{1-\rho^2}\, \widehat{h}^{\perp}_s}{1-\rho^2+\rho^2 V_s(2)} (\theta_s ds + \rho dw_s). \tag{5.9}$$

Suppose now that θ_t and σ_t are deterministic. Then the solution of (5.6) is the pair $(V_t(2), Z_t(2))$, where $Z(2) = 0$ and $V(2)$ satisfies the ordinary differential equation

$$\frac{dV_t(2)}{dt} = \frac{\theta_t^2 V_t^2(2)}{1-\rho^2+\rho^2 V_t(2)}, \quad V_T(2) = 1. \tag{5.10}$$

By solving this equation we obtain

$$V_t(2) = \nu\left(\rho, 1-\rho^2 + \int_t^T \theta_s^2 ds\right) \equiv \nu_t^{\theta,\rho}, \tag{5.11}$$

where $\nu(\rho,\alpha)$ is the solution of (4.37). From (5.10) it follows that

$$\left(\ln \nu_t^{\theta,\rho}\right)' = \frac{\theta_t^2 \nu_t^{\theta,\rho}}{1-\rho^2+\rho^2\nu_t^{\theta,\rho}} \quad \text{and} \quad \ln\frac{\nu_s^{\theta,\rho}}{\nu_t^{\theta,\rho}} = \int_t^s \frac{\theta_r^2 \nu_r^{\theta,\rho} dr}{1-\rho^2+\rho^2\nu_r^{\theta,\rho}}. \tag{5.12}$$

If we solve the linear BSDE (5.7) and use (5.12), we obtain

$$V_t(1) = E\left[\widehat{H}_T(w)\mathcal{E}_{tT}\left(-\int_0^\cdot \frac{\theta_r \nu_r^{\theta,\rho}}{1-\rho^2+\rho^2\nu_r^{\theta,\rho}}(\theta_r dr + \rho dw_r)\right)|G_t\right],$$
$$\int_t^T \frac{\theta_s \nu_s^{\theta,\rho}\sigma_s}{1-\rho^2+\rho^2\nu_s^{\theta,\rho}} E\left[\tilde{h}_s(w)\mathcal{E}_{ts}\left(-\int_0^\cdot \frac{\theta_r \nu_r^{\theta,\rho}}{1-\rho^2+\rho^2\nu_r^{\theta,\rho}}(\theta_r dr + \rho dw_r)\right)|G_t\right] ds$$
$$= \nu_t^{\theta,\rho} E\left[\widehat{H}_T(w)\mathcal{E}_{tT}\left(-\int_0^\cdot \frac{\theta_r \nu_r^{\theta,\rho}}{1-\rho^2+\rho^2\nu_r^{\theta,\rho}}\rho dw_r\right)|G_t\right]$$
$$+\ \nu_t^{\theta,\rho}\int_t^T \frac{\mu_s}{1-\rho^2+\rho^2\nu_s^{\theta,\rho}} E\left[\tilde{h}_s(w)\mathcal{E}_{ts}\left(-\int_0^\cdot \frac{\theta_r \nu_r^{\theta,\rho}}{1-\rho^2+\rho^2\nu_r^{\theta,\rho}}\rho dw_r\right)|G_t\right] ds.$$

By using the Girsanov theorem we finally get

$$V_t(1) = \nu_t^{\theta,\rho} E\left[\widehat{H}_T\left(\rho\int_0^\cdot \frac{\theta_r \nu_r^{\theta,\rho}}{1-\rho^2+\rho^2\nu_r^{\theta,\rho}}dr + w\right)|G_t\right]$$
$$+\nu_t^{\theta,\rho}\int_t^T \frac{\mu_s}{1-\rho^2+\rho^2\nu_s^{\theta,\rho}} E\left[\tilde{h}_s\left(\rho\int_0^\cdot \frac{\theta_r \nu_r^{\theta,\rho}}{1-\rho^2+\rho^2\nu_r^{\theta,\rho}}dr + w\right)|G_t\right] ds. \tag{5.13}$$

Besides, the optimal strategy is of the form

$$\pi_t^* = -\frac{\theta_t V_t(2)}{(1-\rho^2+\rho^2 V_t(2))\sigma_t}\widehat{X}^*_t + \frac{\rho Z_t(1) + \theta_t V_t(1) - \sqrt{1-\rho^2}\,\widehat{h}^{\perp}_t}{(1-\rho^2+\rho^2 V_t(2))\sigma_t}.$$

If in addition μ and σ are constants and the contingent claim is of the form $H = \mathcal{H}(S_T, \eta_T)$, then one can give an explicit expressions also for $\tilde{h}$, $\widehat{h}^{\perp}$, $\widehat{H}$, and $Z(1)$.

Example 2. In Frey and Runggaldier (Frey & Runggaldier, 1999) the incomplete-information situation arises, assuming that the hedger is unable to monitor the asset continuously but is confined to observations at discrete random points in time $\tau_1, \tau_2, \ldots, \tau_n$. Perhaps it is more natural to assume that the hedger has access to price information on full intervals $[\sigma_1, \tau_1], [\sigma_2, \tau_2], \ldots, [\sigma_n, \tau_n]$. For the models with nonzero drifts, even the case $n = 1$ is non-trivial. Here we consider this case in detail.

Let us consider the financial market model

$$d\widetilde{S}_t = \mu \widetilde{S}_t dt + \sigma \widetilde{S}_t dW_t, \quad S_0 = S,$$

where W is a standard Brownian motion and the coefficients μ and σ are constants. Assume that an investor observes only the returns $S_t - S_0 = \int_0^t \frac{1}{\widetilde{S}_u} d\widetilde{S}_u$ of the stock prices up to a random moment τ before the expiration date T. Let $\mathcal{A}_t = F_t^S$, and let τ be a stopping time with respect to F^S. Then the filtration G_t of observable events is equal to the filtration $F^S_{t\wedge\tau}$.
Consider the mean-variance hedging problem

$$\text{to minimize} \quad E\left[\left(x + \int_0^T \pi_t dS_t - H\right)^2\right] \quad \text{over all} \quad \pi \in \Pi(G),$$

where π_t is a dollar amount invested in the stock at time t.
By comparing with (1.1) we get that in this case

$$N_t = M_t = \sigma W_t, \quad \langle M \rangle_t = \sigma^2 t, \quad \lambda_t = \frac{\mu}{\sigma^2}.$$

Let $\theta = \frac{\mu}{\sigma}$. The measure Q defined by $dQ = \mathcal{E}_T(\theta W)dP$ is a unique martingale measure for S, and it is evident that Q satisfies the reverse Hölder condition. It is also evident that any G-martingale is F^S-martingale and that conditions (A)–(C) are satisfied. Besides,

$$E(W_t|G_t) = W_{t\wedge\tau}, \quad \widehat{S}_t = \mu t + \sigma W_{t\wedge\tau} \quad \text{and} \quad \rho_t = I_{\{t\le\tau\}}. \tag{5.14}$$

By the integral representation theorem

$$E\left(H|F_t^S\right) = EH + \int_0^t h_u \sigma dW_u \tag{5.15}$$

for F-predictable W-integrable process h. On the other hand, by the GKW decomposition with respect to the martingale $W^\tau = (W_{t\wedge\tau}, t \in [0, T])$,

$$E\left(H|F_t^S\right) = EH + \int_0^t h_u^G \sigma dW_u^\tau + L_t^G \tag{5.16}$$

for F^S-predictable process h^G and F^S martingale L^G strongly orthogonal to W^τ. Therefore, by equalizing the right-hand sides of (5.15) and (5.16) and taking the mutual characteristics of both parts with W^τ, we obtain $\int_0^{t\wedge\tau} (h_u^G \rho_u^2 - h_u) du = 0$ and hence

$$\int_0^t \widetilde{h}_u du = \int_0^t \left(\widehat{h}_u^G I_{(u\le\tau)} - \widehat{h}_u\right) du = -\int_0^t I_{(u>\tau)} E\left(h_u|F_\tau^S\right) du. \tag{5.17}$$

Therefore, by using notations

$$Z_s(0) = \rho\sigma\varphi_s(0),\ Z_s(1) = \rho\sigma\varphi_s(1),\ Z_s(2) = \rho\sigma\varphi_s(2),$$

it follows from Theorem 4.1 that the processes $(V_t(2), Z_t(2))$ and $(V_t(1), Z_t(1))$ satisfy the following system of backward equations:

$$\begin{aligned} V_t(2) &= V_0(2) + \int_0^{t\wedge\tau} \frac{\big(Z_s(2)+\theta V_s(2)\big)^2}{V_s(2)} ds \\ &\quad + \int_{t\wedge\tau}^t \theta^2 V_s^2(2) ds + \int_0^{t\wedge\tau} Z_s(2) dW_s, \quad V_T(2) = 1, \end{aligned} \tag{5.18}$$

$$\begin{aligned} V_t(1) &= V_0(1) + \int_0^{t\wedge\tau} \frac{\big(Z_s(2)+\theta V_s(2)\big)\big(Z_s(1)+\theta V_s(1)\big)}{V_s(2)} ds \\ &\quad + \int_{t\wedge\tau}^t \theta V_s(2) \left(\theta V_s(1) + E\left(h_s|F_\tau^S\right)\right) ds + \int_0^{t\wedge\tau} Z_s(1) dW_s, \ \ V_T(1) = E(H|G_T). \end{aligned} \tag{5.19}$$

Equation (5.18) admits in this case an explicit solution. To obtain the solution one should solve first the equation

$$U_t = U_0 + \int_0^t \theta^2 U_s^2 ds, \quad U_T = 1, \tag{5.20}$$

in the time interval $[\tau, T]$ and then the BSDE

$$V_t(2) = V_0(2) + \int_0^t \frac{\big(Z_s(2)+\theta V_s(2)\big)^2}{V_s(2)} ds + \int_0^t Z_s(2) dW_s \tag{5.21}$$

in the interval $[0, \tau]$, with the boundary condition $V_\tau(2) = U_\tau$. The solution of (5.20) is

$$U_t = \frac{1}{1+\theta^2(T-t)},$$

and the solution of (5.21) is expressed as

$$V_t(2) = \frac{1}{E\big((1+\theta^2(T-\tau))\mathcal{E}_{t,\tau}^2(-\theta W)|F_t^S\big)}$$

(this can be verified by applying the Itô formula for the process $V_t^{-1}(2)\mathcal{E}_t^2(-\theta W)$ and by using the fact that this process is a martingale). Therefore

$$V_t(2) = \begin{cases} \dfrac{1}{1+\theta^2(T-t)} & \text{if} \quad t \geq \tau, \\ \dfrac{1}{E\big((1+\theta^2(T-\tau))\mathcal{E}_{t,\tau}^2(-\theta W)|F_t^S\big)} & \text{if} \quad t \leq \tau. \end{cases} \tag{5.22}$$

According to (4.37), taking in mind (5.14), (5.17), and the fact that $e^{-\int_t^T \theta^2 V_u(2) du} = \frac{1}{1+\theta^2(T-t)}$ on the set $t \geq \tau$, the solution of (5.19) is equal to

$$\begin{aligned} V_t(1) &= E\bigg(\frac{H}{1+\theta^2(T-t)} + \int_t^T \frac{\theta V_u(2) h_u du}{1+\theta^2(T-u)} |F_\tau^S\bigg) I_{(t>\tau)} \\ &+ E\left(\mathcal{E}_{t,\tau}\left(-\frac{\varphi(2)+\lambda V(2)}{V(2)}\cdot S\right)\left(\frac{H}{1+\theta^2(T-\tau)} + \int_\tau^T \frac{\theta V_u(2) h_u du}{1+\theta^2(T-u)}\right)|F_t^S\right) I_{(t\leq\tau)}. \end{aligned} \tag{5.23}$$

By Theorem 4.1 the optimal filtered wealth process is a solution of a linear SDE, which takes in this case the following form:

$$\widehat{X}_t^* = x - \int_0^{t\wedge\tau} \frac{\varphi_u(2)+\theta V_u(2)}{V_u(2)} \widehat{X}_u^*(\theta du + dW_u) - \int_{t\wedge\tau}^t \theta^2 V_u(2)\widehat{X}_u^* du$$
$$+ \int_0^{t\wedge\tau} \frac{\varphi_u(1)+\theta V_u(1)}{V_u(2)}(\theta du + dW_u) + \int_{t\wedge\tau}^t \left(\theta^2 V_u(1) + \mu E\left(h_u|F_\tau^S\right)\right) du. \quad (5.24)$$

The optimal strategy is equal to

$$\pi_t^* = \left[-\frac{\varphi_t(2)+\theta V_t(2)}{V_t(2)} I_{(t\le\tau)} - \theta^2 V_t(2) I_{(t>\tau)}\right]\widehat{X}_t^*$$
$$+ \frac{\varphi_t(1)+\theta V_t(1)}{V_t(2)} I_{(t\le\tau)} + \left(\theta^2 V_t(1) + \mu E\left(h_t|F_\tau^S\right)\right) I_{(t>\tau)}, \quad (5.25)$$

where $\widehat{X}_t^*$ is a solution of the linear equation (5.24), $V(2)$ and $V(1)$ are given by (5.22) and (5.23), and $\varphi(2)$ and $\varphi(1)$ are integrands of their martingale parts, respectively. In particular the optimal strategy in time interval $[\tau, T]$ (i.e., after interrupting observations) is of the form

$$\pi_t^* = -\theta^2 V_t(2)\widehat{X}_t^* + \theta^2 V_t(1) + \mu E\left(h_t|F_\tau^S\right), \quad (5.26)$$

where

$$\widehat{X}_t^* = \frac{\widehat{X}_\tau^*}{1+\theta^2(t-\tau)} - \int_\tau^t \left(\theta^2 V_u(1) - \mu E\left(h_u|F_\tau^S\right)\right)\frac{1}{1+\theta^2(t-u)} du.$$

For instance, if τ is deterministic, then $V_t(2)$ is also deterministic:

$$V_t(2) = \begin{cases} \dfrac{1}{1+\theta^2(T-t)} & \text{if} \quad t \ge \tau, \\ \dfrac{1}{1+\theta^2(T-t)} e^{-\theta^2(\tau-t)} & \text{if} \quad t \le \tau, \end{cases}$$

and $\varphi(2) = 0$.
Note that it is not optimal to do nothing after interrupting observations, and in order to act optimally one should change the strategy deterministically as it is given by (5.26).

Appendix

For convenience we give the proofs of the following assertions used in the paper.

Lemma A.1. *Let conditions* (A)–(C) *be satisfied and* $\widehat{M}_t = E(M_t|G_t)$. *Then* $\langle\widehat{M}\rangle$ *is absolutely continuous w.r.t.* $\langle M\rangle$ *and* $\mu^{\langle M\rangle}$ *a.e.*

$$\rho_t^2 = \frac{d\langle\widehat{M}\rangle_t}{d\langle M\rangle_t} \le 1.$$

Proof. By (2.4) for any bounded G-predictable process h

$$E\int_0^t h_s^2 d\langle\widehat{M}\rangle_s = E\left(\int_0^t h_s d\widehat{M}_s\right)^2 = E\left(E\left(\int_0^t h_s dM_s|G_t\right)\right)^2$$
$$\le E\left(\int_0^t h_s dM_s\right)^2 = E\int_0^t h_s^2 d\langle M\rangle_s, \quad (A.1)$$

which implies that $\langle \widehat{M} \rangle$ is absolutely continuous w.r.t. $\langle M \rangle$, i.e.,

$$\langle \widehat{M} \rangle_t = \int_0^t \rho_s^2 d\langle M \rangle_s$$

for a G-predictable process ρ. □

Moreover (A.1) implies that the process $\langle M \rangle - \langle \widehat{M} \rangle$ is increasing and hence $\rho^2 \le 1$ $\mu^{\langle M \rangle}$ a.e.

Lemma A.2. *Let $H \in L^2(P, F_T)$, and let conditions* (A)–(C) *be satisfied. Then*

$$E \int_0^T \tilde{h}_u^2 d\langle M \rangle_u < \infty.$$

Proof. It is evident that

$$E \int_0^T (h_u^G)^2 d\langle \widehat{M} \rangle_u < \infty, \qquad E \int_0^T h_u^2 d\langle M \rangle_u < \infty.$$

Therefore, by the definition of $\tilde{h}$ and Lemma A.1,

$$\begin{aligned} E \int_0^T \tilde{h}_u^2 d\langle M \rangle_u &\le 2E \int_0^T \widehat{h}_u^2 d\langle M \rangle_u + 2E \int_0^T \left(\widehat{h_u^G}\right)^2 \rho_u^4 d\langle M \rangle_u \\ &\le 2E \int_0^T h_u^2 d\langle M \rangle_u + 2E \int_0^T \left(h_u^G\right)^2 \rho_u^2 d\langle \widehat{M} \rangle_u < \infty. \end{aligned}$$

Thus $\tilde{h} \in \Pi(G)$ by Remark 2.5. □

Lemma A.3. (a) *Let $Y = (Y_t, t \in [0, T])$ be a bounded positive submartingale with the canonical decomposition*

$$Y_t = Y_0 + B_t + m_t,$$

where B is a predictable increasing process and m is a martingale. Then $m \in BMO$.
(b) *In particular the martingale part of $V(2)$ belongs to BMO. If H is bounded, then martingale parts of $V(0)$ and $V(1)$ also belong to the class BMO, i.e., for $i = 0, 1, 2$,*

$$E\left(\int_\tau^T \varphi_u^2(i) \rho_u^2 d\langle M \rangle_u | G_\tau\right) + E\left(\langle m(i) \rangle_T - \langle m(i) \rangle_\tau | G_\tau\right) \le C \tag{A.2}$$

for every stopping time τ.

Proof. By applying the Itô formula for $Y_T^2 - Y_\tau^2$ we have

$$\langle m \rangle_T - \langle m \rangle_\tau + 2\int_\tau^T Y_u dB_u + 2\int_\tau^T Y_u dm_u = Y_T^2 - Y_\tau^2 \le \text{const} \tag{A.3}$$

Since Y is positive and B is an increasing process, by taking conditional expectations in (A.3) we obtain

$$E(\langle m \rangle_T - \langle m \rangle_\tau | F_\tau) \le \text{const}$$

for any stopping time τ, and hence $m \in BMO$.
(A.2) follows from assertion (a) applied for positive submartingales $V(0), V(2)$, and $V(0) + V(2) - 2V(1)$. For the case $i = 1$ one should take into account also the inequality

$$\langle m(1) \rangle_t \le \text{const}(\langle m(0) + m(2) - 2m(1) \rangle_t + \langle m(0) \rangle_t + \langle m(2) \rangle_t).$$

6. Acknowledgments

This work was supported by Georgian National Science Foundation grant STO09-471-3-104.

7. References

Bismut, J. M. (1973) Conjugate convex functions in optimal stochastic control, *J. Math. Anal. Appl.*, Vol. 44, 384–404.

Chitashvili, R. (1983) Martingale ideology in the theory of controlled stochastic processes, In: *Probability theory and mathematical statistics* (Tbilisi, 1982), 73–92, Lecture Notes in Math., 1021, Springer, Berlin.

Di Masi, G. B.; Platen, E.; Runggaldier, W. J. (1995) Hedging of options under discrete observation on assets with stochastic volatility, *Seminar on Stochastic Analysis, Random Fields and Applications*, pp. 359–364, Ascona, 1993, Progr. Probab., 36, Birkhäuser, Basel.

Delbaen, F.: Monat, P.: Schachermayer, W.: Schweizer, W. & Stricker, C. (1997) Weighted norm inequalities and hedging in incomplete markets, *Finance Stoch.*, Vol. 1, 181–227.

Dellacherie, C. & Meyer, P.-A. (1980) *Probabilités et Potentiel*, II, Hermann, Paris.

Duffie, D. & Richardson, H. R. (1991) Mean-variance hedging in continuous time, *Ann. Appl. Probab.*, Vol. 1, No. 1, 1–15.

El Karoui, N. & Quenez, M.-C. (1995) Dynamic programming and pricing of contingent claims in an incomplete market, *SIAM J. Control Optim.*, Vol. 33, No. 1, 29–66.

Föllmer, H. & Sondermann, D. (1986) Hedging of nonredundant contingent claims, In: *Contributions to mathematical economics,* 205–223, North-Holland, Amsterdam.

Frey, R. & Runggaldier, W. J. (1999) Risk-minimizing hedging strategies under restricted information: the case of stochastic volatility models observable only at discrete random times. Financial optimization, *Math. Methods Oper. Res.*, Vol. 50, No. 2, 339–350.

Gourieroux, C.: Laurent, J. P. & Pham, H. (1998) Mean-variance hedging and numeraire, *Math. Finance*, Vol. 8, No. 3, 179–200.

Heath, D.: Platen, E. & Schweizer, M. (2001) A comparison of two quadratic approaches to hedging in incomplete markets, *Math. Finance*, Vol. 11, No. 4, 385–413.

Hipp, C. (1993) Hedging general claims, In: *Proceedings of the 3rd AFIR Colloquium*, 2, pp. 603–613, Rome.

Jacod, J. (1979) *Calcul Stochastique et Problèmes de Martingales*, Lecture Notes in Mathematics, 714. Springer, ISBN 3-540-09253-6, Berlin.

Kazamaki, N. (1994) *Continuous Exponential Martingales and BMO*, Lecture Notes in Mathematics, 1579. Springer-Verlag, ISBN 3-540-58042-5, Berlin.

Kramkov, D. O. (1996) Optional decomposition of supermartingales and hedging contingent claims in incomplete security markets, *Probab. Theory Related Fields*, Vol. 105, No. 4, 459–479.

Kobylanski, M. (2000) Backward stochastic differential equations and partial differential equations with quadratic growth, *Ann. Probab.*, Vol. 28, No. 2, 558–602.

Lepeltier, J.-P. & San Martin, J. (1998) Existence for BSDE with superlinear-quadratic coefficient, *Stochastics Stochastics Rep.*, Vol. 63, No. 3-4, 227–240.

Liptser, R. Sh. & Shiryaev, A. N. (1986) *Martingale Theory*, Probability Theory and Mathematical Statistics, "Nauka", Moscow (in Russian).

Mania, M., Tevzadze, R. & Toronjadze, T. (2009) L^2-approximating Pricing under Restricted Information *Applied Mathematics and Optimization*, Vol.60, 39–70.

Mania, M. & Tevzadze, R. (2003) Backward stochastic PDE and imperfect hedging, *Int. J. Theor. Appl. Finance*, Vol. 6, No. 7, 663–692.

Morlais, M.-A. (2009) Quadratic BSDEs driven by a continuous martingale and applications to the utility maximization problem, *Finance Stoch.*, Vol. 13, No. 1, 121–150.

Pardoux, É. & Peng, S. G. (1990) Adapted solution of a backward stochastic differential equation, *Systems Control Lett.*, Vol. 14, No. 1, 55–61.

Pham, H. (2001) Mean-variance hedging for partially observed drift processes, *Int. J. Theor. Appl. Finance*, Vol. 4, No. 2, 263–284.

Rheinlander, T. & Schweizer, M. (1997) On L^2-projections on a space of stochastic integrals. *Ann. Probab.*, Vol. 25, No. 4, 1810–1831.

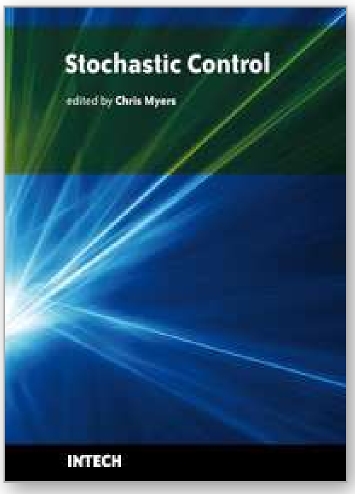
Schäl, M. (1994) On quadratic cost criteria for option hedging, *Math. Oper. Res.*, Vol. 19, No. 1, 121–131.

Schweizer, M. (1992) Mean-variance hedging for general claims, *Ann. Appl. Probab.*, Vol. 2, No. 1, 171–179.

Schweizer, M. (1994) Approximating random variables by stochastic integrals, *Ann. Probab.*, Vol. 22, No. 3, 1536–1575.

Schweizer, M. (1994) Risk-minimizing hedging strategies under restricted information, *Math. Finance*, Vol. 4, No. 4, 327–342.

Tevzadze, R. (2008) Solvability of backward stochastic differential equations with quadratic growth, *Stochastic Process. Appl.*, Vol. 118, No. 3, 503–515.



Stochastic Control
Edited by Chris Myers

ISBN 978-953-307-121-3
Hard cover, 650 pages
Publisher Sciyo
Published online 17, August, 2010
Published in print edition August, 2010

Uncertainty presents significant challenges in the reasoning about and controlling of complex dynamical systems. To address this challenge, numerous researchers are developing improved methods for stochastic analysis. This book presents a diverse collection of some of the latest research in this important area. In particular, this book gives an overview of some of the theoretical methods and tools for stochastic analysis, and it presents the applications of these methods to problems in systems theory, science, and economics.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Michael Mania, Revaz Tevzadze and Teimuraz Toronjadze (2010). Mean-Variance Hedging Under Partial Information, Stochastic Control, Chris Myers (Ed.), ISBN: 978-953-307-121-3, InTech, Available from: http://www.intechopen.com/books/stochastic-control/mean-variance-hedging-under-partial-information

InTech Europe
University Campus STeP Ri
Slavka Krautzeka 83/A
51000 Rijeka, Croatia
Phone: +385 (51) 770 447
Fax: +385 (51) 686 166
www.intechopen.com

InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821