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Optimal Filtering for Linear States over Polynomial Observations

Joel Perez[1], Jose P. Perez[1] and Rogelio Soto[2]
[1] *Department of Physical and Mathematical Sciences, Autonomous University of Nuevo Leon*
[2] *Mechatronics and Automation Department, Monterrey Institute of Technology and Higher Education*
Mexico

1. Introduction

Although the general optimal solution of the filtering problem for nonlinear state and observation equations confused with white Gaussian noises is given by the equation for the conditional density of an unobserved state with respect to observations (see (1–6)), there are a very few known examples of nonlinear systems where that equation can be reduced to a finite-dimensional closed system of filtering equations for a certain number of lower conditional moments (see (7–10) for more details). Some relevant results on filtering for nonlinear stochastic systems can be found in (11–14). There also exists a considerable bibliography on robust filtering for the "general situation" systems (see, for example, (15–23)). Apart form the "general situation," the optimal finite-dimensional filters have recently been designed for certain classes of polynomial system states over linear observations with invertible ((24; 25; 27; 28)) or non-invertible ((26; 29)) observation matrix. However, the cited papers never consider filtering problems with nonlinear, in particular, polynomial observations.

This work presents the optimal finite-dimensional filter for linear system states over polynomial observations, continuing the research in the area of the optimal filtering for polynomial systems, which has been initiated in ((24–27; 29)). Designing the optimal filter over polynomial observations presents a significant advantage in the filtering theory and practice, since it enables one to address some filtering problems with observation nonlinearities, such as the optimal cubic sensor problem (30). The optimal filtering problem is treated proceeding from the general expression for the stochastic Ito differential of the optimal estimate and the error variance (31). As the first result, the Ito differentials for the optimal estimate and error variance corresponding to the stated filtering problem are derived. It is then proved that a closed finite-dimensional system of the optimal filtering equations with respect to a finite number of filtering variables can be obtained for a polynomial observation equation, additionally assuming a conditionally Gaussian initial condition for the higher degree states. This assumption is quite admissible in the filtering framework, since the real distribution of the entire state vector is actually unknown. In this case, the corresponding procedure for designing the optimal filtering equations is established.

As an illustrative example, the closed system of the optimal filtering equations with respect to two variables, the optimal estimate and the error variance, is derived in the explicit form for the particular case of the third degree polynomial observations. This filtering problem generalizes the optimal cubic sensor problem stated in (30), where nonexistence of a closed-form solution is indicated for the "general situation" case, without any assumptions for the third order state distribution. In our paper, taking into account that the real distributions of the first and third degree states are unknown, a conditionally Gaussian initial condition is additionally assumed for the third degree state. The resulting filter yields a reliable and rapidly converging estimate, in spite of a significant difference in the initial conditions between the state and estimate and very noisy observations, in the situation where the unmeasured state itself is a time-shifted Wiener process and the extended Kalman filter (EKF) approach fails.

2. Filtering Problem for Linear States over Polynomial Observations

Let (Ω, F, P) be a complete probability space with an increasing right-continuous family of σ-algebras $F_t, t \geq t_0$, and let $(W_1(t), F_t, t \geq t_0)$ and $(W_2(t), F_t, t \geq t_0)$ be independent Wiener processes. The F_t-measurable random process $(x(t), y(t)$ is described by a linear differential equation for the system state

$$dx(t) = (a_0(t) + a(t)x(t))dt + b(t)dW_1(t), \quad x(t_0) = x_0, \tag{1}$$

and a nonlinear polynomial differential equation for the observation process

$$dy(t) = h(x,t)dt + B(t)dW_2(t). \tag{2}$$

Here, $x(t) \in R^n$ is the state vector and $y(t) \in R^m$ is the observation vector. The initial condition $x_0 \in R^n$ is a Gaussian vector such that x_0, $W_1(t)$, and $W_2(t)$ are independent. It is assumed that $B(t)B^T(t)$ is a positive definite matrix. All coefficients in (1)–(2) are deterministic functions of time of appropriate dimensions. The nonlinear function $h(x,t)$ forms the drift in the observation equation (2).

The nonlinear function $h(x,t)$ is considered a polynomial of n variables, components of the state vector $x(t) \in R^n$, with time-dependent coefficients. Since $x(t) \in R^n$ is a vector, this requires a special definition of the polynomial for $n > 1$. In accordance with (27), a p-degree polynomial of a vector $x(t) \in R^n$ is regarded as a p-linear form of n components of $x(t)$

$$h(x,t) = \alpha_0(t) + \alpha_1(t)x + \alpha_2(t)xx^T + \ldots + \alpha_p(t)x \ldots_{p\ times} \ldots x, \tag{3}$$

where $\alpha_0(t)$ is a vector of dimension n, α_1 is a matrix of dimension $n \times n$, α_2 is a 3D tensor of dimension $n \times n \times n$, α_p is an $(p+1)$D tensor of dimension $n \times \ldots_{(p+1)\ times} \ldots \times n$, and $x \times \ldots_{p\ times} \ldots \times x$ is a pD tensor of dimension $n \times \ldots_{p\ times} \ldots \times n$ obtained by p times spatial multiplication of the vector $x(t)$ by itself (see (27) for more definition). Such a polynomial can also be expressed in the summation form

$$h_k(x,t) = \alpha_{0\ k}(t) + \sum_i \alpha_{1\ ki}(t)x_i(t) + \sum_{ij} \alpha_{2\ kij}(t)x_i(t)x_j(t) + \ldots$$
$$+ \sum_{i_1 \ldots i_p} \alpha_{p\ ki_1 \ldots i_p}(t)x_{i_1}(t) \ldots x_{i_p}(t), \quad k, i, j, i_1, \ldots, i_p = 1, \ldots, n.$$

The estimation problem is to find the optimal estimate $\hat{x}(t)$ of the system state $x(t)$, based on the observation process $Y(t) = \{y(s), 0 \leq s \leq t\}$, that minimizes the Euclidean 2-norm

$$J = E[(x(t) - \hat{x}(t))^T(x(t) - \hat{x}(t)) \mid F_t^Y]$$

at every time moment t. Here, $E[\xi(t) \mid F_t^Y]$ means the conditional expectation of a stochastic process $\xi(t) = (x(t) - \hat{x}(t))^T(x(t) - \hat{x}(t))$ with respect to the σ - algebra F_t^Y generated by the observation process $Y(t)$ in the interval $[t_0, t]$. As known (31), this optimal estimate is given by the conditional expectation

$$\hat{x}(t) = m_x(t) = E(x(t) \mid F_t^Y)$$

of the system state $x(t)$ with respect to the σ - algebra F_t^Y generated by the observation process $Y(t)$ in the interval $[t_0, t]$. As usual, the matrix function

$$P(t) = E[(x(t) - m_x(t))(x(t) - m_x(t))^T \mid F_t^Y]$$

is the estimation error variance.
The proposed solution to this optimal filtering problem is based on the formulas for the Ito differential of the optimal estimate and the estimation error variance (cited after (31)) and given in the following section.

3. Optimal Filter for Linear States over Polynomial Observations

Let us reformulate the problem, introducing the stochastic process $z(t) = h(x, t)$. Using the Ito formula (see (31)) for the stochastic differential of the nonlinear function $h(x, t)$, where $x(t)$ satisfies the equation (1), the following equation is obtained for $z(t)$

$$dz(t) = \frac{\partial h(x,t)}{\partial x}(a_0(t) + a(t)x(t))dt + \frac{\partial h(x,t)}{\partial t}dt + \tag{4}$$

$$\frac{1}{2}\frac{\partial^2 h(x,t)}{\partial x^2}b(t)b^T(t)dt + \frac{\partial h(x,t)}{\partial x}b(t)dW_1(t), \quad z(0) = z_0.$$

Note that the addition $\frac{1}{2}\frac{\partial^2 h(x,t)}{\partial x^2}b(t)b^T(t)dt$ appears in view of the second derivative in x in the Ito formula.
The initial condition $z_0 \in R^n$ is considered a conditionally Gaussian random vector with respect to observations. This assumption is quite admissible in the filtering framework, since the real distributions of $x(t)$ and $z(t)$ are actually unknown. Indeed, as follows from (32), if only two lower conditional moments, expectation m_0 and variance P_0, of a random vector $[z_0, x_0]$ are available, the Gaussian distribution with the same parameters, $N(m_0, P_0)$, is the best approximation for the unknown conditional distribution of $[z_0, x_0]$ with respect to observations. This fact is also a corollary of the central limit theorem (33) in the probability theory.
A key point for further derivations is that the right-hand side of the equation (4) is a polynomial in x. Indeed, since $h(x, t)$ is a polynomial in x, the functions $\frac{\partial h(x,t)}{\partial x}$, $\frac{\partial h(x,t)}{\partial x}x(t)$, $\frac{\partial h(x,t)}{\partial t}$, and $\frac{\partial^2 h(x,t)}{\partial x^2}$ are also polynomial in x. Thus, the equation (4) is a polynomial state equation with a polynomial multiplicative noise. It can be written in the compact form

$$dz(t) = f(x,t)dt + g(x,t)dW_1(t), \quad z(t_0) = z_0, \tag{5}$$

where

$$f(x,t) = \frac{\partial h(x,t)}{\partial x}(a_0(t) + a(t)x(t)) + \frac{\partial h(x,t)}{\partial t} +$$
$$\frac{1}{2}\frac{\partial^2 h(x,t)}{\partial x^2}b(t)b^T(t), \quad g(x,t) = \frac{\partial h(x,t)}{\partial x}b(t).$$

In terms of the process $z(t)$, the observation equation (2) takes the form

$$dy(t) = z(t)dt + B(t)dW_2(t). \tag{6}$$

The reformulated estimation problem is now to find the optimal estimate $[m_z(t), m_x(t)]$ of the system state $[z(t), x(t)]$, based on the observation process $Y(t) = \{y(s), 0 \leq s \leq t\}$. This optimal estimate is given by the conditional expectation

$$m(t) = [m_z(t), m_x(t)] = [E(z(t) \mid F_t^Y), E(x(t) \mid F_t^Y)]$$

of the system state $[z(t), x(t)]$ with respect to the σ - algebra F_t^Y generated by the observation process $Y(t)$ in the interval $[t_0, t]$. The matrix function

$$P(t) = E[([z(t), x(t)] - [m_z(t), m_x(t)]) \times$$
$$([z(t), x(t)] - [m_z(t), m_x(t)])^T \mid F_t^Y]$$

is the estimation error variance for this reformulated problem.
The obtained filtering system includes two equations, (4) (or (5)) and (1), for the partially measured state $[z(t), x(t)]$ and an equation (6) for the observations $y(t)$, where $z(t)$ is a measured polynomial state with polynomial multiplicative noise, $x(t)$ is an unmeasured linear state, and $y(t)$ is a linear observation process directly measuring the state $z(t)$. Hence, the optimal filter for the polynomial system states with unmeasured linear part and polynomial multiplicative noise over linear observations, obtained in (29), can be applied to solving this problem. Indeed, as follows from the general optimal filtering theory (see (31)), the optimal filtering equations take the following particular form for the system (5), (1), (6)

$$dm(t) = E(\bar{f}(x,t) \mid F_t^Y)dt + \tag{7}$$
$$P(t)[I,0]^T(B(t)B^T(t))^{-1}(dy(t) - m_z(t)dt),$$
$$dP(t) = (E(([z(t), x(t)] - m(t))(\bar{f}(x,t))^T \mid F_t^Y) + \tag{8}$$
$$E(\bar{f}(x,t)([z(t), x(t)] - m(t))^T) \mid F_t^Y) +$$
$$E(\bar{g}(x,t)\bar{g}^T(x,t) \mid F_t^Y) -$$
$$P(t)[I,0]^T(B(t)B^T(t))^{-1}[I,0]P(t))dt +$$
$$E((([z(t), x(t)] - m(t))([z(t), x(t)] - m(t)) \times$$
$$([z(t), x(t)] - m(t))^T \mid F_t^Y) \times$$
$$[I,0]^T(B(t)B^T(t))^{-1}(dy(t) - m_z(t)dt),$$

where $\bar{f}(x,t) = [f(x,t), a_0(t) + a(t)x(t)]$ is the polynomial drift term and $\bar{g}(x,t) = [g(x,t), b(t)]$ is the polynomial diffusion (multiplicative noise) term in the entire system of the state equations (4), (1), and the last term should be understood as a 3D tensor (under the expectation sign) convoluted with a vector, which yields a matrix. The matrix $[I,0]$ is the $m \times (n+m)$ matrix composed of the $m \times m$-dimensional identity matrix and $m \times n$-dimensional zero matrix. The equations (7), (8) should be complemented with the initial conditions $m(t_0) = [m_z(t_0), m_x(t_0)] = E([z_0, x_0] \mid F_{t_0}^Y)$ and $P(t_0) = E[([z_0, x_0] - m(t_0)([z_0, x_0] - m(t_0)^T \mid F_{t_0}^Y]$.
The result given in (27; 29) claims that a closed system of the filtering equations can be obtained for the state $[z(t), x(t)]$ over the observations $y(t)$, in view of the polynomial properties

of the functions in the right-hand side of the equation (4). Indeed, since the observation matrix in (6) is the identity one, i.e., invertible, and the initial condition z_0 is assumed conditionally Gaussian with respect to observations, the random variable $z(t) - m_z(t)$ is conditionally Gaussian with respect to the observation process $y(t)$ for any $t \geq t_0$ ((27; 29)). Moreover, the random variable $x(t) - m_x(t)$ is also conditionally Gaussian with respect to the observation process $y(t)$ for any $t \geq t_0$, because $x(t)$ is Gaussian, in view of (1), and $y(t)$ depends only on $z(t)$, in view of (6), and the assumed conditional Gaussianity of the initial random vector z_0 ((26; 29)). Hence, the entire random vector $[z(t), x(t)] - m(t)$ is conditionally Gaussian with respect to the observation process $y(t)$ for any $t \geq t_0$, and the following considerations outlined in (26; 27; 29) are applicable.

First, since the random variable $x(t) - m(t)$ is conditionally Gaussian, the conditional third moment $E((([z(t), x(t)] - m(t))([z(t), x(t)] - m(t))([z(t), x(t)] - m(t))^T \mid F_t^Y)$ with respect to observations, which stands in the last term of the equation (8), is equal to zero, because the process $[z(t), x(t)] - m(t)$ is conditionally Gaussian. Thus, the entire last term in (8) is vanished and the following variance equation is obtained

$$\begin{aligned} dP(t) = (E(([z(t), x(t)] - m(t))(\bar{f}(x,t))^T \mid F_t^Y) + \\ E(\bar{f}(x,t)([z(t), x(t)] - m(t))^T) \mid F_t^Y) + \\ E(\bar{g}(x,t)\bar{g}^T(x,t) \mid F_t^Y) - \\ P(t)[I,0]^T(\bar{B}(t)\bar{B}^T(t))^{-1}[I,0]P(t))dt, \end{aligned} \tag{9}$$

with the initial condition $P(t_0) = E[([z_0, x_0] - m(t_0)([z_0, x_0] - m(t_0)^T \mid F_{t_0}^Y]$.

Second, if the functions $\bar{f}(x,t)$ and $\bar{g}(x,t)$ are polynomial functions of the state x with time-dependent coefficients, the expressions of the terms $E(\bar{f}(x,t) \mid F_t^Y)$ in (4) and $E(([z(t), x(t)] - m(t))\bar{f}^T(x,t)) \mid F_t^Y)$ and $E(\bar{g}(x,t)\bar{g}^T(x,t) \mid F_t^Y)$, which should be calculated to obtain a closed system of filtering equations (see (31)), would also include only polynomial terms of x. Then, those polynomial terms can be represented as functions of $m(t)$ and $P(t)$ using the following property of Gaussian random variable $[z(t), x(t)] - m(t)$: all its odd conditional moments, $m_1 = E[([z(t), x(t)] - m(t)) \mid Y(t)], m_3 = E[([z(t), x(t)] - m(t))^3 \mid Y(t)], m_5 = E[([z(t), x(t)] - m(t))^5 \mid Y(t)], ...$ are equal to 0, and all its even conditional moments $m_2 = E[([z(t), x(t)] - m(t))^2 \mid Y(t)], m_4 = E[([z(t), x(t)] - m(t))^4 \mid Y(t)], ...$ can be represented as functions of the variance $P(t)$. For example, $m_2 = P, m_4 = 3P^2, m_6 = 15P^3, ...$ etc. After representing all polynomial terms in (7) and (9), that are generated upon expressing $E(\bar{f}(x,t) \mid F_t^Y)$, $E(([z(t), x(t)] - m(t))\bar{f}^T(x,t)) \mid F_t^Y)$, and $E(\bar{g}(x,t)\bar{g}^T(x,t) \mid F_t^Y)$, as functions of $m(t)$ and $P(t)$, a closed form of the filtering equations would be obtained. The corresponding representations of $E(f(x,t) \mid F_t^Y)$, $E(([z(t), x(t)] - m(t))(f(x,t))^T \mid F_t^Y)$ and $E(\bar{g}(x,t)\bar{g}^T(x,t) \mid F_t^Y)$ have been derived in (24–27; 29) for certain polynomial functions $f(x,t)$ and $g(x,t)$.

In the next example section, a closed form of the filtering equations will be obtained for a particular case of a scalar third degree polynomial function $h(x,t)$ in the equation (2). It should be noted, however, that application of the same procedure would result in designing a closed system of the filtering equations for any polynomial function $h(x,t) \in R^n$ in (2).

4. Example: Third Degree Sensor Filtering Problem

This section presents an example of designing the optimal filter for a linear state over third degree polynomial observations, reducing it to the optimal filtering problem for a second degree

polynomial state with partially measured linear part and second degree polynomial multiplicative noise over linear observations, where a conditionally Gaussian state initial condition is additionally assumed.
Let the unmeasured scalar state $x(t)$ satisfy the trivial linear equation

$$dx(t) = dt + dw_1(t), \quad x(0) = x_0, \tag{10}$$

and the observation process be given by the scalar third degree sensor equation

$$dy(t) = (x^3(t) + x(t))dt + dw_2(t), \tag{11}$$

where $w_1(t)$ and $w_2(t)$ are standard Wiener processes independent of each other and of a Gaussian random variable x_0 serving as the initial condition in (10). The filtering problem is to find the optimal estimate for the linear state (10), using the third degree sensor observations (11).
Let us reformulate the problem, introducing the stochastic process $z(t) = h(x,t) = x^3(t) + x(t)$. Using the Ito formula (see (31)) for the stochastic differential of the cubic function $h(x,t) = x^3(t) + x(t)$, where $x(t)$ satisfies the equation (10), the following equation is obtained for $z(t)$

$$dz(t) = (1 + 3x(t) + 3x^2(t))dt + (3x^2(t) + 1)dw_1(t),\ z(0) = z_0. \tag{12}$$

Here, $\frac{\partial h(x,t)}{\partial x} = 3x^2(t) + 1$, $\frac{1}{2}\frac{\partial^2 h(x,t)}{\partial x^2} = 3x(t)$, and $\frac{\partial h(x,t)}{\partial t} = 0$; therefore, $f(x,t) = 1 + 3x(t) + 3x^2(t)$ and $g(x,t) = 3x^2(t) + 1$. The initial condition $z_0 \in R$ is considered a conditionally Gaussian random vector with respect to observations (see the paragraph following (4) for details). This assumption is quite admissible in the filtering framework, since the real distributions of $x(t)$ and $z(t)$ are unknown. In terms of the process $z(t)$, the observation equation (11) takes the form

$$dy(t) = z(t)dt + dw_2(t). \tag{13}$$

The obtained filtering system includes two equations, (12) and (10), for the partially measured state $[z(t), x(t)]$ and an equation (13) for the observations $y(t)$, where $z(t)$ is a completely measured quadratic state with multiplicative quadratic noise, $x(t)$ is an unmeasured linear state, and $y(t)$ is a linear observation process directly measuring the state $z(t)$. Hence, the designed optimal filter can be applied for solving this problem. The filtering equations (7),(9) take the following particular form for the system (12),(10),(13)

$$dm_1(t) = (1 + 3m_2(t) + 3m_2^2(t) + 3P_{22}(t))dt + \tag{14}$$
$$P_{11}(t)[dy(t) - m_1(t)dt],$$
$$dm_2(t) = 1 + P_{12}(t)[dy(t) - m_1(t)dt], \tag{15}$$

with the initial conditions $m_1(0) = E(x_0 \mid y(0)) = m_{10}$ and $m_2(0) = E(x_0^3 \mid y(0)) = m_{20}$,

$$\dot{P}_{11}(t) = 12(P_{12}(t)m_2(t)) + 6P_{12}(t) + 27P_{22}^2(t) + \tag{16}$$
$$54P_{22}(t)m_2^2(t) + 9m_2^4(t) + 6P_{22}(t) + 6m_2^2 + 1 - P_{11}^2(t),$$
$$\dot{P}_{12}(t) = 6(P_{22}(t)m_2(t)) + 3P_{22}(t) + \tag{17}$$
$$3(m_2^2(t) + P_{22}(t)) + 1 - P_{11}(t)P_{12}(t),$$

$$\dot{P}_{22}(t) = 1 - P_{12}^2(t), \tag{18}$$

with the initial condition $P(0) = E(([x_0, z_0]^T - m(0))([x_0, z_0]^T - m(0))^T \mid y(0)) = P_0$. Here, $m_1(t)$ is the optimal estimate for the state $z(t) = x^3(t) + x(t)$ and $m_2(t)$ is the optimal estimate for the state $x(t)$.

Numerical simulation results are obtained solving the systems of filtering equations (14)–(18). The obtained values of the state estimate $m_2(t)$ satisfying the equation (15) are compared to the real values of the state variable $x(t)$ in (10).

For the filter (14)–(18) and the reference system (12),(10),(13) involved in simulation, the following initial values are assigned: $x_0 = z_0 = 0$, $m_2(0) = 10$, $m_1(0) = 1000$, $P_{11}(0) = 15$, $P_{12}(0) = 3$, $P_{22}(0) = 1$. Gaussian disturbances $dw_1(t)$ and $dw_2(t)$ are realized using the built-in MatLab white noise functions. The simulation interval is $[0, 0.05]$.

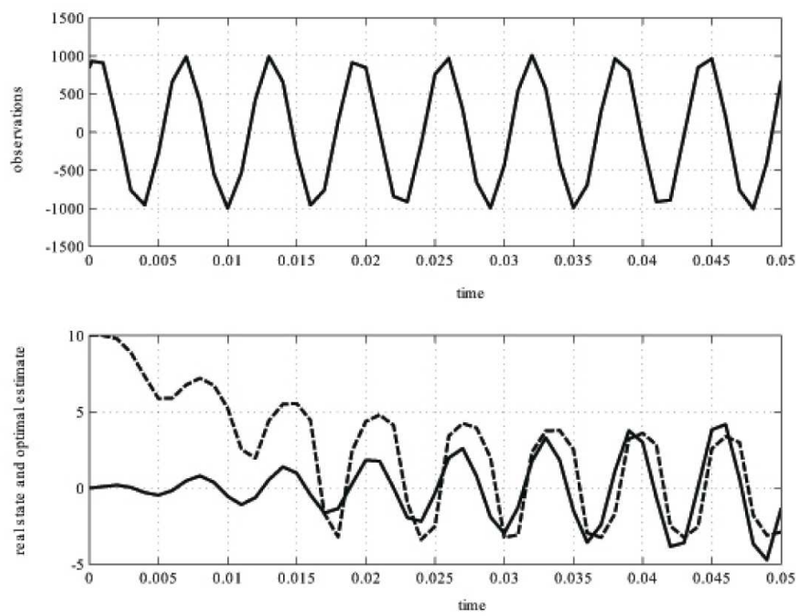

Fig. 1. **Above.** Graph of the observation process $y(t)$ in the interval $[0, 0.05]$. **Below.** Graphs of the real state $x(t)$ (solid line) and its optimal estimate $m_2(t)$ (dashed line) in the interval $[0, 0.05]$.

Figure 1 shows the graphs of the reference state variable $x(t)$ (10) and its optimal estimate $m_2(t)$ (15), as well as the observation process $y(t)$ (11), in the entire simulation interval from $t_0 = 0$ to $T = 0.05$. It can be observed that the optimal estimate given by (14)–(18) converges to the real state (10) very rapidly, in spite of a considerable error in the initial conditions, $m_2(0) - x_0 = 10$, $m_1(0) - z_0 = 1000$, and very noisy observations which do not even reproduce the shape of $z(t) = x^3(t) + x(t)$. Moreover, the estimated signal $x(t)$ itself is a time-shifted Wiener process, i.e., the integral of a white Gaussian noise, which makes the filtering problem even more difficult. It should also be noted that the extended Kalman filter (EKF) approach fails for the system (10),(11), since the linearized value $\partial z/\partial x = 3x^2(t) + 1$ at zero is the unit-valued constant, therefore, the observation process would consist of pure noise.

Thus, it can be concluded that the obtained optimal filter (14)–(18) solves the optimal third degree sensor filtering problem for the system (10),(11) and yields a really good estimate of the unmeasured state in presence of quite complicated observation conditions. Subsequent discussion of the obtained results can be found in Conclusions.

5. Conclusions

This paper presents the optimal filter for linear system states over nonlinear polynomial observations. It is shown that the optimal filter can be obtained in a closed form for any polynomial function in the observation equation. Based on the optimal filter for a bilinear state, the optimal solution is obtained for the optimal third degree sensor filtering problem, assuming a conditionally Gaussian initial condition for the third degree state. This assumption is quite admissible in the filtering framework, since the real distributions of the first and third degree states are unknown. The resulting filter yields a reliable and rapidly converging estimate, in spite of a significant difference in the initial conditions between the state and estimate and very noisy observations, in the situation where the unmeasured state itself is a time-shifted Wiener process and the extended Kalman filter (EKF) approach fails. Although this conclusion follows from the developed theory, the numerical simulation serves as a convincing illustration.

6. References

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