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Boundary Perturbation Theory for Scattering in Layered Rough Structures

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1. Introduction

The electromagnetic wave interaction with layered structures constitutes a crucial topic of current interest in theoretical and experimental research. Generally speaking, several modelling and design problems, encountered, for instance, in SAR (*Synthetic Aperture Radar*) application, GPR (*Ground Penetrating Radar*) sensing, radar altimeter for planetary exploration, microstrip antennas and MMICs (*Monolithic Microwave Integrated Circuits*), radio-propagation in urban environment for wireless communications, through-the-wall detection technologies, optics, biomedical diagnostic of layered biological tissues, geophysical and seismic exploration, lead to the analysis of the electromagnetic wave interaction with multilayered structure, whose boundaries can exhibit some amount of roughness.

This chapter is aimed primarily at providing a comprehensive analytical treatment of electromagnetic wave propagation and scattering in three-dimensional multilayered structures with rough interfaces. The emphasis is placed on the general formulation of the scattering problem in the analytic framework of the *Boundary Perturbation Theory (BPT)* developed by Imperatore et al. A systematic perturbative expansion of the fields in the layered structure, based on the gently rough interfaces assumption, enables the transferring of the geometry randomness into a non-uniform boundary conditions formulation. Subsequently, the fields' expansion can be analytically evaluated by using a recursive matrix formalism approach encompassing a proper scattered field representation in each layer and a matrix reformulation of non-uniform boundary conditions. A key-point in the development resides in the appropriate exploitation of the *generalized reflection/transmission* notion, which has strong implications in order to make the mathematical treatment manageable and to effectively capture the physics of the problem. Two relevant compact closed-form solutions, derived in the first-order limit of the perturbative development, are presented. They refer to two complementary *bi-static* configurations for the scattering, respectively, from and through layered structures with arbitrary number of rough interfaces. The employed formalism is fully-*polarimetric* and suitable for applications. In addition, it is demonstrated how the symmetrical character of the *BPT* formalism reflects the inherent conformity with the *reciprocity* theorem of the electromagnetic theory.

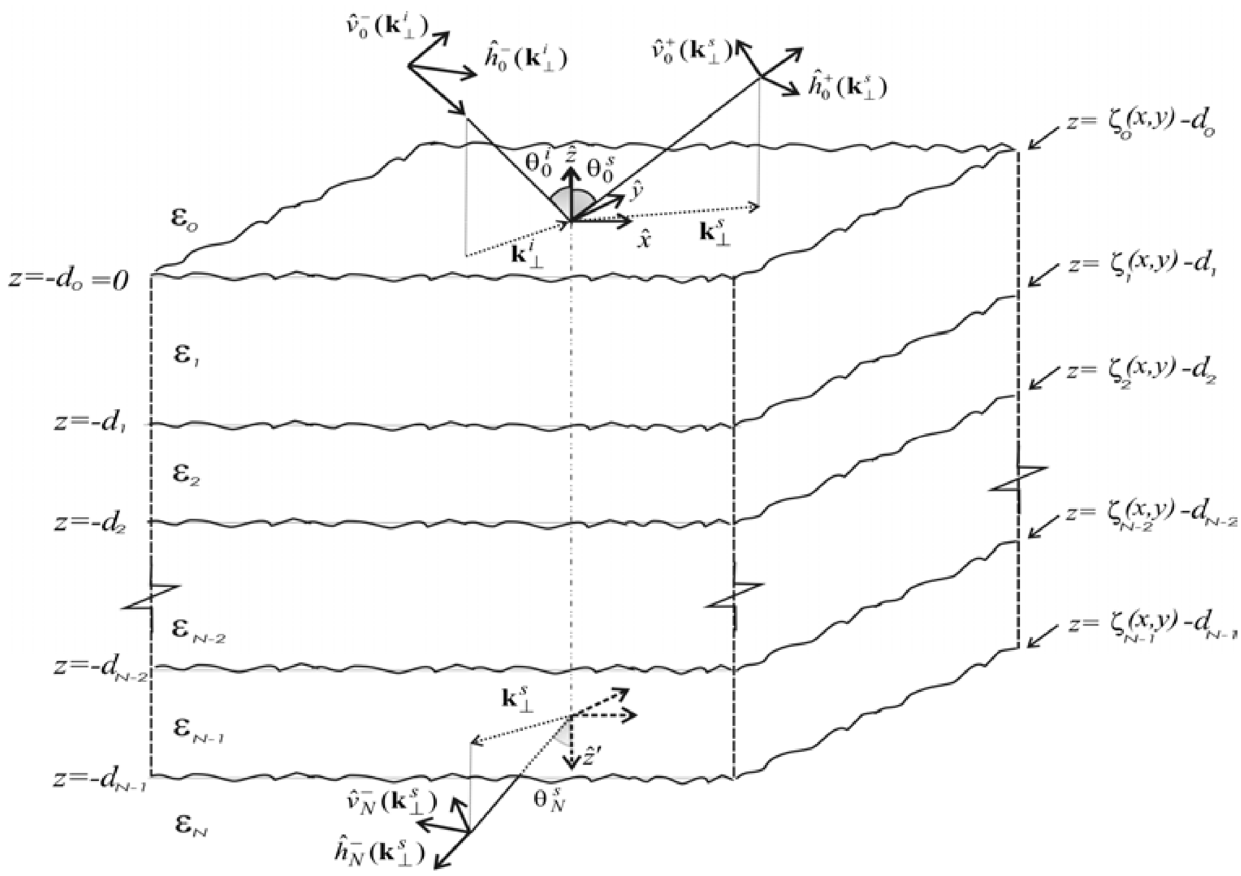


Fig. 1. Geometry for an N-rough boundaries layered medium

2. Statement of the problem

When stratified media with rough interfaces are concerned, the possible approaches to cope with the EM scattering problem fall within three main categories. First, the numerical approaches do not permit to attain a comprehensive understanding of the general functional dependence of the scattering response on the structure parameters, as well as do not allow capturing the physics of the involved scattering mechanisms. In addition, the numerical approach turns out to be feasible for non-fully 3D geometry or configurations in which a very limited number of rough interfaces is accounted for. Layered structures with rough interfaces have been also treated resorting to *radiative transfer theory* (RT). However, coherent effects are not accounted for in RT theory and could not be contemplated without employing full wave analysis, which preserves phase information. Another approach relies on the full-wave methods. Although, to deal with the electromagnetic propagation and scattering in complex random layered media, several analytical formulation involving some idealized cases and suitable approximations have been conducted in last decades, the relevant solutions usually turn out to be too complicated to be generally useful in applications, even if simplified geometries are accounted for. The proliferation of the proposed methods for the simulation of wave propagation and scattering in stratified media and the continuous interest in this topic are indicative of the need of appropriate modelling and interpretation of the complex physical phenomena that take place in layered structures. Indeed, the availability of accurate, sound physical and manageable models turns out still to

be a strong necessity, in perspective to apply them, for instance, in retrieving of add-valued information from the data acquired by microwave sensors.
Generally speaking, an exact analytical solution of *Maxwell* equations can be found only for a few idealized problems. Subsequently, appropriate approximation methods are needed. Regarding the perturbative approaches, noticeable progress has been attained in the analytic investigation on the extension of the classical SPM (small perturbation method) solution for the scattering from rough surface to specific layered configurations. Most of previous existing works analyze different layered configurations in the first-order limit, using procedures, formalisms and final solutions that can appear of difficult comparison (Yarovoy et al., 2000), (Azadegan and Sarabandi, 2003), (Fuks, 2001). All these formulations, which refer to the case of a single rough interface, have been recently unified in (Franceschetti et al, 2008). On the other hand, solution for the case of two rough boundaries has also been proposed in (Tabatabaeenejad and Moghaddam, 2006).
Methodologically, we underline that all the previously mentioned existing perturbative approaches, followed by different authors in analyzing scattering from simplified geometry, imply an inherent analytical complexity, which precludes the treatment to structures with more than one (Fuks, 2001) (Azadegan et al., 2003) (Yarovoy et al., 2000) or two (Tabatabaeenejad er al., 2006) rough interfaces.
The general problem we intend to deal with here refers to the analytical evaluation of the electromagnetic scattering from and through layered structure with an arbitrary number of rough interfaces (see Fig.1). As schematically shown in fig.1, an arbitrary polarized monochromatic plane wave

$$\mathbf{E}_0^i(\mathbf{r}) = [E_0^{ih}\hat{h}_0^-(\mathbf{k}_\perp^i) + E_0^{iv}\hat{v}_0^-(\mathbf{k}_\perp^i)]\, e^{j(\mathbf{k}_\perp^i\cdot\mathbf{r}_\perp - k_{z0}^i z)} \quad (1)$$

is considered to be incident on the layered medium at an angle θ_0^i relative to the $\hat{z}$ direction from the upper half-space, where in the field expression a time factor $exp(-j\omega t)$ is understood, and where, using a spherical frame representation, the incident vector wave direction is individuated by θ_0^i, φ_0^i:

$$k_0\hat{k}_0^i = \mathbf{k}_0^i = \mathbf{k}_\perp^i - \hat{z}k_{z0}^i = k_0(\hat{x}\sin\theta_0^i\cos\varphi_0^i + \hat{y}\sin\theta_0^i\sin\varphi_0^i - \hat{z}\cos\theta_0^i), \quad (2)$$

with

$$\hat{h}_0^-(\mathbf{k}_\perp^i) = \frac{\hat{k}_0^i \times \hat{z}}{\left|\hat{k}_0^i \times \hat{z}\right|} = \sin\varphi_0^i\hat{x} - \cos\varphi_0^i\hat{y}, \quad (3)$$

$$\hat{v}_0^-(\mathbf{k}_\perp^i) = \hat{h}_0^-(\mathbf{k}_\perp^i)\times\hat{k}_0^i = (\hat{x}\cos\varphi_0^i + \hat{y}\sin\varphi_0^i)\cos\theta_0^i + \hat{z}\sin\theta_0^i, \quad (4)$$

where $\mathbf{k}_\perp^i = k_x^i\hat{x} + k_y^i\hat{y}$ is the two-dimensional projection of incident wave-number vector on the plane $z=0$. The parameters pertaining to layer m with boundaries $-d_{m-1}$ and $-d_m$ are distinguished by a subscript m. Each layer is assumed to be homogeneous and characterized by arbitrary and deterministic parameters: the *dielectric relative permittivity* ε_m, the *magnetic relative permeability* μ_m and the *thickness* $\Delta_m = d_m - d_{m-1}$. With reference to Fig.1, it has been assumed that in particular, $d_0=0$. In the following, the symbol $\perp$ denotes the projection of the corresponding vector on the plan $z=0$. Here $\mathbf{r} = (\mathbf{r}_\perp, z)$, so we distinguish the transverse spatial coordinates $\mathbf{r}_\perp = (x, y)$ and the longitudinal coordinate z. In addition, each mth rough interface is assumed to be characterized by a zero-mean two-dimensional *stochastic process* $\zeta_m = \zeta_m(\mathbf{r}_\perp)$ with normal vector $\hat{n}_m$. No constraints are imposed on the degree to which the rough interfaces are correlated.

A general methodology has been developed by *Imperatore et al.* to analytically treat EM bistatic scattering from this class of layered structures that can be described by small changes with respect to an idealized (unperturbed) structure, whose associated problem is exactly solvable. A thorough analysis of the results of this theoretical investigation (BPT), which is based on perturbation of the boundary condition, will be presented in the following, methodologically emphasizing the development of the several inherent aspects.

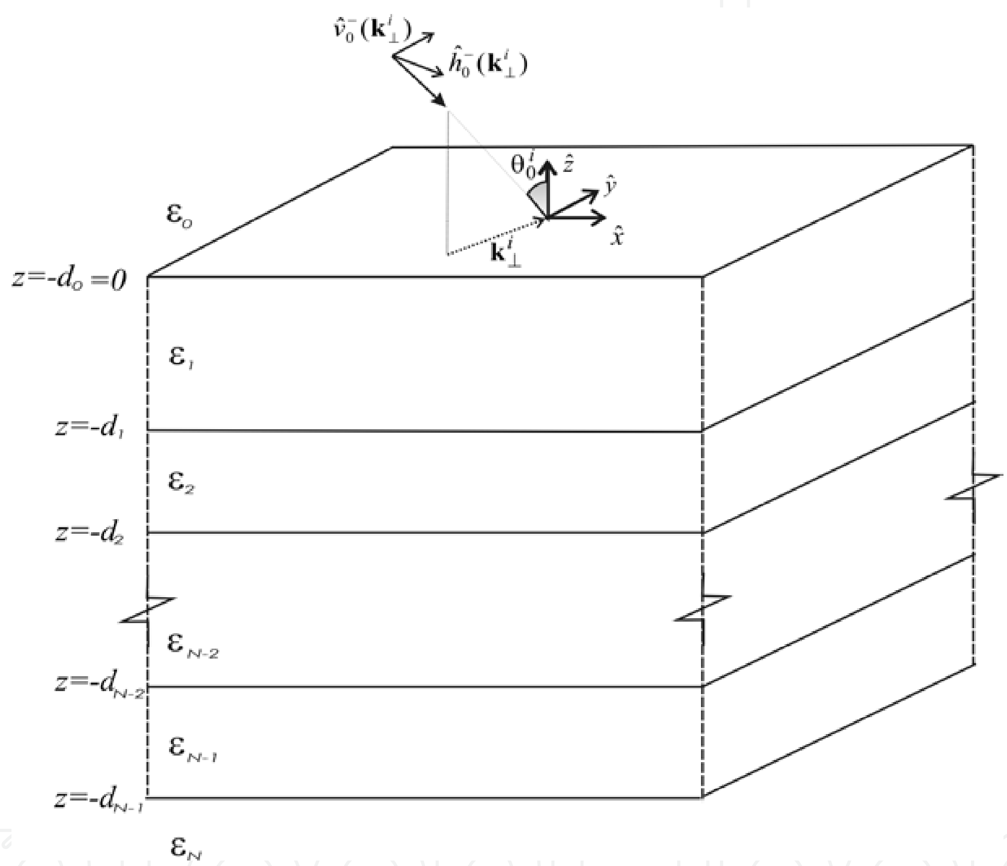


Fig. 2. Geometry for a flat boundaries layered medium

3. Basic definition and notations

This section is devoted preliminary to introduce the formalism used in the following of this chapter. The Flat Boundaries layered medium (*unperturbed structure*) is defined as a stack of parallel slabs (Fig.2), sandwiched in between two half-spaces, whose structure is shift invariant in the direction of x and y (infinite lateral extent in x-y directions). With the notations $T^p_{m|m+1}$ and $R^p_{m|m+1}$, respectively, we indicate the *ordinary transmission* and *reflection coefficients* at the interface between the regions *m-1* and *m+1,*

$$R^h_{m|m+1} = \frac{\mu_{m+1}k_{zm} - \mu_m k_{z(m+1)}}{\mu_{m+1}k_{zm} + \mu_m k_{z(m+1)}}, \tag{5}$$

$$R^v_{m|m+1} = \frac{\varepsilon_{m+1}k_{zm} - \varepsilon_m k_{z(m+1)}}{\varepsilon_{m+1}k_{zm} + \varepsilon_m k_{z(m+1)}}, \tag{6}$$

$$T^h_{m|m+1} = \frac{2\mu_{m+1}k_{zm}}{\mu_{m+1}k_{zm} + \mu_m k_{z(m+1)}}, \tag{7}$$

$$T^v_{m|m+1} = \frac{2\varepsilon_{m+1}k_{zm}}{\varepsilon_{m+1}k_{zm} + \varepsilon_m k_{z(m+1)}}, \tag{8}$$

with the superscript $p \in \{v, h\}$ indicating the polarization state for the incident wave and may stand for *horizontal* (h) or *vertical* (v) polarization (Tsang et al., 1985) (Imperatore et al. 2009a), and where

$$k_{zm} = \sqrt{k_m^2 - |\mathbf{k}_\perp|^2} = k_m \cos\theta_m, \tag{9}$$

where $k_m = k_0\sqrt{\mu_m \varepsilon_m}$ is the *wave number* for the electromagnetic medium in the mth layer, with $k_0 = \omega / c = 2\pi / \lambda$, and where $\mathbf{k}_\perp = k_x\hat{x} + k_y\hat{y}$ is the two-dimensional projection of vector wave-number on the plane $z=0$. In addition, we stress that:

$$T^p_{i|j} = 1 + R^p_{i|j}, \qquad R^p_{i|j} = -R^p_{j|i}, \qquad i=j\pm 1. \tag{10}$$

3.1 Generalized reflection formalism

The *generalized reflection coefficients* $\Re^p_{m-1|m}$ at the interface between the regions (m-1) and m, for the p-polarization, are defined as the ratio of the amplitudes of *upward*- and *downward*-propagating waves immediately above the interface, respectively. They can be expressed by recursive relations as in (Chew W. C., 1997) (Imperatore et al. 2009a):

$$\Re^p_{m-1|m} = \frac{R^p_{m-1|m} + \Re^p_{m|m+1}e^{j2k_{zm}\Delta_m}}{1 + R^p_{m-1|m}\Re^p_{m|m+1}e^{j2k_{zm}\Delta_m}}. \tag{11}$$

Likewise, at the interface between the regions (m+1) and m, $\Re^p_{m+1|m}$ is given by:

$$\Re^p_{m+1|m} = \frac{R^p_{m+1|m} + \Re^p_{m|m-1}e^{j2k_{zm}\Delta m}}{1 + R^p_{m+1|m}\Re^p_{m|m-1}e^{j2k_{zm}\Delta_m}}, \tag{12}$$

Furthermore, it should be noted that the factors

$$\vec{M}_m^p(k_\perp) = 1 - R^p_{m|m-1}\Re^p_{m|m+1}e^{j2k_{zm}\Delta_m}\ , \tag{13}$$

$$\overleftarrow{M}_m^p(k_\perp) = 1 - \Re^p_{m|m-1}R^p_{m|m+1}e^{j2k_{zm}\Delta_m}\ , \tag{14}$$

$$\overleftrightarrow{M}_m^p(k_\perp) = 1 - \Re^p_{m|m-1}\Re^p_{m|m+1}e^{j2k_{zm}\Delta_m}\ , \tag{15}$$

take into account the multiple reflections in the mth layer.

3.2 Generalized transmission formalism

The *generalized transmission coefficients* in *downward* direction $\Im^p_{0|m}$ can be defined as:

$$\Im^p_{0|m}(k_\perp) = \exp\left[j\sum_{n=1}^{m-1}k_{zn}\Delta_n\right]\prod_{n=0}^{m-1}T^p_{n|n+1}\left[\prod_{n=1}^{m}\vec{M}^p_n\right]^{-1}, \tag{16}$$

where $p\in\{v, h\}$. The *generalized transmission coefficients* in *upward* direction $\Im^p_{m|0}$ are then given by

$$\Im^p_{m|0} = \begin{cases} \Im^p_{0|m}\dfrac{\mu_0 k_{zm}}{\mu_m k_{z0}} & for\ p = h \\ \Im^p_{0|m}\dfrac{\varepsilon_0 k_{zm}}{\varepsilon_m k_{z0}} & for\ p = v \end{cases} . \tag{17}$$

As a counterpart of (17), we have

$$\Im^p_{m+1|N} = \begin{cases} \Im^p_{N|m+1}\dfrac{\mu_N k_{z(m+1)}}{\mu_{m+1} k_{zN}} & for\ p = h \\ \Im^p_{N|m+1}\dfrac{\varepsilon_N k_{z(m+1)}}{\varepsilon_{m+1} k_{zN}} & for\ p = v \end{cases} . \tag{18}$$

Equations (17) and (18) formally express the *reciprocity* of the generalized transmission coefficients for an arbitrary flat-boundaries layered structure (Imperatore et al. 2009b).
Here we introduce notion of layered *slab*, which refers to a layered structure sandwiched between two half-spaces. Accordingly, the *generalized transmission coefficients* in *downward* direction for a layered slab between two half-spaces (*0, N*), $\Im^{p(slab)}_{0|N}$, can be defined as:

$$\Im^{p(slab)}_{0|N}(k_\perp) = \exp\left[j\sum_{n=1}^{N-1}k_{zn}\Delta_n\right]\prod_{n=0}^{N-1}T^p_{n|n+1}\left[\prod_{n=1}^{N-1}\vec{M}^p_n\right]^{-1}. \tag{19}$$

It should be noted that the parenthesized superscript *slab* indicates that both the media *0* and *N* are half-space. Similarly, the *generalized transmission coefficients* in *downward* direction for the layered slab between two half-spaces (*m+1, N*), $\Im^{p(slab)}_{m+1|N}$, are defined as:

$$\Im_{m+1|N}^{p(slab)}(k_\perp) = \exp\left[j \sum_{n=m+2}^{N-1} k_{zn}\Delta_n \right] \prod_{n=m+1}^{N-1} T_{n|n+1}^{p} \left[\prod_{n=m+2}^{N-1} \vec{M}_n^p \right]^{-1} . \tag{20}$$

Note also that

$$\Im_{m+1|N}^{p} = [\vec{M}_{m+1}^{p}]^{-1} \Im_{m+1|N}^{p(slab)} . \tag{21}$$

Moreover, we consider the *generalized transmission coefficients* in *upward* direction for the layered *slab* between two half-spaces (*m*, 0), $\Im_{m|0}^{p(slab)}$, which are defined as

$$\Im_{m|0}^{p(slab)}(k_\perp) = \exp\left[j \sum_{n=1}^{m-1} k_{zn}\Delta_n \right] \prod_{n=0}^{m-1} T_{n+1|n}^{p} \left[\prod_{n=1}^{m-1} \vec{M}_n^p \right]^{-1} . \tag{22}$$

Note also that

$$\Im_{m|0}^{p}(k_\perp) = [\vec{M}_m^p(k_\perp)]^{-1} \Im_{m|0}^{p(slab)}(k_\perp) . \tag{23}$$

The *generalized transmission coefficients* in *downward* direction for the layered *slab* between two half-spaces (*0*, *m*), $\Im_{0|m}^{p(slab)}$, can be defined as

$$\Im_{0|m}^{p(slab)}(k_\perp) = \exp\left[j \sum_{n=1}^{m-1} k_{zn}\Delta_n \right] \prod_{n=0}^{m-1} T_{n|n+1}^{p} \left[\prod_{n=1}^{m-1} \vec{M}_n^p \right]^{-1} . \tag{24}$$

It should be noted that the $\Im_{0|m}^{p}$ are distinct from the coefficients $\Im_{0|m}^{p(slab)}$, because in the evaluation of $\Im_{0|m}^{p}$ the effect of all the layers under the layer m is taken into account, whereas $\Im_{0|m}^{p(slab)}$ are evaluated referring to a different configuration in which the intermediate layers *1...m* are bounded by the half-spaces *0* and *m*.
We stress that generalized reflection and transmission coefficients do not depend on the direction of $\mathbf{k}_\perp$. In the following, we shown how the employing the generalized reflection/transmission coefficient notions not only is crucial in obtaining a compact closed-form perturbation solution, but it also permit us to completely elucidate the obtained analytical expressions from a physical point of view, highlighting the role played by the *equivalent reflecting interfaces* and by *the equivalent slabs*, so providing the inherent connection between the global scattering response.

4. Stochastic characterization for the 3-D geometry description

In this section, the focus is on stochastic description for the geometry of the investigated structure, and the notion of wide-sense stationary process is detailed. First of all, when the description of a rough interface by means of deterministic function $\zeta_m(\mathbf{r}_\perp)$ is concerned, the corresponding *ordinary* 2-D *Fourier Transform* pair can be defined as

$$\tilde{\zeta}_m(\mathbf{k}_\perp) = (2\pi)^{-2} \iint d\mathbf{r}_\perp \, e^{-j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \zeta_m(\mathbf{r}_\perp), \tag{25}$$

$$\zeta_m(\mathbf{r}_\perp) = \iint d\mathbf{k}_\perp \, e^{j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \tilde{\zeta}_m(\mathbf{k}_\perp). \tag{26}$$

Let us assume now that $\zeta_m(\mathbf{r}_\perp)$, which describes the generic (mth) rough interface, is a 2-D stochastic process satisfying the conditions

$$< \zeta_m(\mathbf{r}_\perp) > = 0, \tag{27}$$

$$< \zeta_m(\mathbf{r}_\perp + \boldsymbol{\rho}) \zeta_m(\mathbf{r}_\perp) > = B_{\zeta_m}(\boldsymbol{\rho}), \tag{28}$$

where the *angular bracket* denotes statistical ensemble averaging, and where $B_{\zeta_m}(\boldsymbol{\rho})$ is the interface *autocorrelation* function, which quantifies the similarity of the spatial fluctuations with a displacement $\boldsymbol{\rho}$. Equations (27)-(28) constitute the basic assumptions defining a *wide sense stationary* (*WSS*) stochastic process: the statistical properties of the process under consideration are invariant to a spatial shift. Similarly, concerning two mutually correlated random rough interfaces ζ_m and ζ_n, we also assume that they are *jointly WSS*, i.e.

$$< \zeta_m(\mathbf{r}_\perp + \boldsymbol{\rho}) \zeta_n(\mathbf{r}_\perp) > = B_{\zeta_m \zeta_n}(\boldsymbol{\rho}), \tag{29}$$

where $B_{\zeta_m \zeta_n}(\boldsymbol{\rho})$ is the corresponding *cross-correlation function* of the two random processes. It can be readily derived that

$$B_{\zeta_m \zeta_n}(\boldsymbol{\rho}) = B_{\zeta_n \zeta_m}(-\boldsymbol{\rho}). \tag{30}$$

The integral in (25) is a *Riemann* integral representation for $\zeta_m(\mathbf{r}_\perp)$, and it exists if $\zeta_m(\mathbf{r}_\perp)$ is piecewise continuous and *absolutely integrable*. On the other hand, when the spectral analysis of a stationary random process is concerned, the integral (25) does not in general exist in the framework of theory of the ordinary functions. Indeed, a WSS process describing an interface $\zeta_m(\mathbf{r}_\perp)$ of infinite lateral extension, for its proper nature, is not *absolutely integrable*, so the conditions for the existence of the Fourier Transform are not satisfied. In order to obtain a spectral representation for a *WSS* random process, this difficulty can be circumvented by resorting to the more general *Fourier-Stieltjes* integral (Ishimaru, 1978); otherwise one can define space-truncated functions. When a finite patch of the rough interface with area A is concerned, the space-truncated version of (25) can be introduced as

$$\tilde{\zeta}_m(\mathbf{k}_\perp; A) = (2\pi)^{-2} \iint_A d\mathbf{r}_\perp \, e^{-j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \zeta_m(\mathbf{r}_\perp), \tag{31}$$

subsequently, $\tilde{\zeta}_m(\mathbf{k}_\perp) = \lim_{A \to \infty} \tilde{\zeta}_m(\mathbf{k}_\perp; A)$ is not an ordinary function. Nevertheless, we will use again the (25)-(26), regarding them as symbolic formulas, which hold a rigorous mathematical meaning beyond the ordinary function theory (generalized Fourier Transform). We underline that by virtue of the condition (27) directly follows also that $< \tilde{\zeta}_m(\mathbf{k}_\perp) > = 0$. Let us consider

$$<\zeta_m(\mathbf{r}'_\perp)\zeta_n^*(\mathbf{r}''_\perp)> = \iint d\mathbf{k}'_\perp \iint d\mathbf{k}''_\perp\, e^{j(\mathbf{k}'_\perp\cdot\mathbf{r}'_\perp - \mathbf{k}''_\perp\cdot\mathbf{r}''_\perp)} <\tilde{\zeta}_m(\mathbf{k}'_\perp)\tilde{\zeta}_n^*(\mathbf{k}''_\perp)>, \quad (32)$$

where the asterisk denotes the complex conjugated, and where the operations of average and integration have been interchanged. When *jointly WSS* processes ζ_m and ζ_n are concerned, accordingly to (29), the LHS of (32) must be a function of $\mathbf{r}'_\perp - \mathbf{r}''_\perp$ only; therefore, it is required that

$$<\tilde{\zeta}_m(\mathbf{k}'_\perp)\tilde{\zeta}_n^*(\mathbf{k}''_\perp)> = W_{mn}(\mathbf{k}'_\perp)\delta(\mathbf{k}'_\perp - \mathbf{k}''_\perp), \quad (33)$$

where $\delta(\cdot)$ is the *Dirac* delta function, and where $W_{mn}(\boldsymbol{\kappa})$ is called the (spatial) *cross power spectral density* of two interfaces ζ_m and ζ_n, for the spatial frequencies of the roughness. Equation (33) states that the different spectral components of the two considered interfaces must be uncorrelated. This is to say that the (generalized) Fourier transform of jointly WSS processes are *jointly non stationary* white noise with average power $W_{mn}(\mathbf{k}'_\perp)$. Indeed, by using (33) into (32), we obtain

$$<\zeta_m(\mathbf{r}'_\perp)\zeta_n(\mathbf{r}''_\perp)> = \iint d\mathbf{k}''_\perp\, e^{j\cdot\mathbf{k}''_\perp\cdot(\mathbf{r}'_\perp - \mathbf{r}''_\perp)} W_{mn}(\mathbf{k}''_\perp), \quad (34)$$

where the RHS of (34) involves an (ordinary) 2D Fourier Transform. Note also that as a direct consequence of the fact that $\zeta_n(\mathbf{r}_\perp)$ is real we have the relation $\tilde{\zeta}_n(\mathbf{k}_\perp) = \tilde{\zeta}_n^*(-\mathbf{k}_\perp)$. Therefore, setting $\boldsymbol{\rho} = \mathbf{r}'_\perp - \mathbf{r}''_\perp$ in (34), we have

$$B_{\zeta_m\zeta_n}(\boldsymbol{\rho}) = \iint d\boldsymbol{\kappa}\, e^{j\boldsymbol{\kappa}\cdot\boldsymbol{\rho}}\, W_{mn}(\boldsymbol{\kappa}). \quad (35)$$

The *cross-correlation function* $B_{\zeta_m\zeta_n}(\boldsymbol{\rho})$ of two interfaces ζ_m and ζ_n is then given by the (inverse) 2D Fourier Transform of their *(spatial) cross power spectral density*, and Equation (35) together with its Fourier inverse

$$W_{mn}(\boldsymbol{\kappa}) = (2\pi)^{-2} \iint d\boldsymbol{\rho}\, e^{-j\boldsymbol{\kappa}\cdot\boldsymbol{\rho}} B_{\zeta_m\zeta_n}(\boldsymbol{\rho}), \quad (36)$$

may be regarded as the (generalized) *Wiener-Khinchin* theorem. In particular, when *n*=*m*, (33) reduces to

$$<\tilde{\zeta}_m(\mathbf{k}'_\perp)\tilde{\zeta}_m^*(\mathbf{k}''_\perp)> = W_m(\mathbf{k}'_\perp)\delta(\mathbf{k}'_\perp - \mathbf{k}''_\perp), \quad (37)$$

where $W_m(\boldsymbol{\kappa})$ is called the *(spatial) power spectral density* of *n*th corrugated interface ζ_m and can be expressed as the (*ordinary)* 2D Fourier transform of *n*-corrugated interface autocorrelation function, i.e., satisfying the transform pair:

$$W_m(\boldsymbol{\kappa}) = (2\pi)^{-2} \iint d\boldsymbol{\rho}\, e^{-j\boldsymbol{\kappa}\cdot\boldsymbol{\rho}} B_{\zeta_m}(\boldsymbol{\rho}), \quad (38)$$

$$B_{\zeta_m}(\boldsymbol{\rho}) = \iint d\boldsymbol{\kappa}\, e^{j\boldsymbol{\kappa}\cdot\boldsymbol{\rho}}\, W_m(\boldsymbol{\kappa}), \quad (39)$$

which is the statement of the classical *Wiener-Khinchin* theorem. We emphasize the physical meaning of $W_m(\boldsymbol{\kappa})d\boldsymbol{\kappa} = W_m(\kappa_x,\kappa_y)d\kappa_x d\kappa_y$: it represents the power of the spectral components of the mth rough interface having spatial wave number between κ_x and $\kappa_x + d\kappa_x$ and κ_y and $\kappa_y + d\kappa_y$, respectively, in x and y direction. Furthermore, from (30) and (36) it follows that

$$W_{mn}(\boldsymbol{\kappa}) = W_{nm}^{*}(\boldsymbol{\kappa}) . \tag{40}$$

This is to say that, unlike the power spectral density, the cross power spectral density is, in general, neither real nor necessarily positive. Furthermore, it should be noted that the *Dirac*'s delta function can be defined by the integral representation

$$\delta(\boldsymbol{\kappa}) = (2\pi)^{-2} \iint d\boldsymbol{\rho}\, e^{-j\boldsymbol{\kappa}\cdot\boldsymbol{\rho}} = \lim_{A\to\infty} \delta(\boldsymbol{\kappa}; A) . \tag{41}$$

By using in (37) and (33) the relation $\delta(0; A) = A/(2\pi)^2$, we have, respectively, that the (spatial) power *spectral density* of nth corrugated interface can be also expressed as

$$W_m(\boldsymbol{\kappa}) = (2\pi)^2 \lim_{A\to\infty} \frac{1}{A} < \left|\tilde{\zeta}_m(\boldsymbol{\kappa}; A)\right|^2 > , \tag{42}$$

and the (spatial) *cross power spectral density* of two interfaces ζ_m and ζ_n is given by

$$W_{mn}(\boldsymbol{\kappa}) = (2\pi)^2 \lim_{A\to\infty} \frac{1}{A} < \tilde{\zeta}_m(\boldsymbol{\kappa}; A)\tilde{\zeta}_n^{*}(\boldsymbol{\kappa}; A) > . \tag{43}$$

It should be noted that the domain of a rough interface is physically limited by the illumination beamwidth. Note also that the different definitions of the Fourier transform are available and used in the literature: the sign of the complex exponential function are sometimes exchanged and a multiplicative constant $(2\pi)^{-2}$ may appear in front of either integral or its square root in front of each expression (25)-(26). Finally, we recall that the theory of random process predicts only the averages over many realizations.

5. Boundary Perturbation Theory (BPT)

In this section, we first introduce the general perturbative expansion on which the BPT formulation is based. A systematic matrix reformulation, which enables the formal evaluation of pertinent scattered field solutions, is then presented.

5.1 Perturbative formulation

With reference to the geometry of Fig.1, in order to obtain a solution valid in each region of the structure, we have to enforce the continuity of the tangential fields:

$$[\hat{n}_m \times \Delta\mathbf{E}_m]_{z=\zeta_m(\mathbf{r}_\perp)-d_m} = 0 , \tag{44}$$

$$[\hat{n}_m \times \Delta\mathbf{H}_m]_{z=\zeta_m(\mathbf{r}_\perp)-d_m} = 0 , \tag{45}$$

where $\Delta\mathbf{E}_m = \mathbf{E}_{m+1} - \mathbf{E}_m$, $\Delta\mathbf{H}_m = \mathbf{H}_{m+1} - \mathbf{H}_m$, and the surface normal vector is given by:

$$\hat{n}_m = \frac{\hat{z} - \boldsymbol{\gamma}_m}{\sqrt{1-\gamma_m^2}}, \tag{46}$$

with the slope vector $\boldsymbol{\gamma}_m$:

$$\boldsymbol{\gamma}_m = \nabla_\perp \zeta_m = \left[\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y}\right]\zeta_m, \tag{47}$$

and where $\nabla_\perp$ is the *nabla* operator in the *x-y* plane. In order to study the fields $\mathbf{E}_m$ and $\mathbf{H}_m$ within the generic *m*th layer of the structure, we assume then that, for each *m*th rough interface, the deviations and slopes of the interface, with respect to the reference mean plane $z=-d_m$, are small enough in the sense of (Ulaby et al, 1982) (Tsang et al., 1985), so that the fields can be expanded about the reference mean plane. Assume that the fields can be expanded about the reference mean plane $z=-d_m$ as:

$$\Delta\mathbf{E}_m(z) = \Delta\mathbf{E}_m\Big|_{z=-d_m} + \frac{\partial \Delta\mathbf{E}_m}{\partial z}\Big|_{z=-d_m}(z+d_m) + \frac{1}{2}\frac{\partial^2 \Delta\mathbf{E}_m}{\partial z^2}\Big|_{z=-d_m}(z+d_m)^2 + \ldots, \tag{48}$$

$$\Delta\mathbf{H}_m(z) = \Delta\mathbf{H}_m\Big|_{z=-d_m} + \frac{\partial \Delta\mathbf{H}_m}{\partial z}\Big|_{z=-d_m}(z+d_m) + \frac{1}{2}\frac{\partial^2 \Delta\mathbf{H}_m}{\partial z^2}\Big|_{z=-d_m}(z+d_m)^2 + \ldots, \tag{49}$$

where the dependence on $\mathbf{r}_\perp$ is understood. Then (48), (49) are the fields expansions in perturbative orders of the fields and their derivatives at the interfaces of the structure; they can be injected into the boundary conditions (44-45). Retaining only up to the first-order terms with respect to ζ_m and γ_m, we obtain:

$$\hat{z} \times \Delta\mathbf{E}_m\Big|_{z=-d_m} = \nabla_\perp \zeta_m \times \Delta\mathbf{E}_m\Big|_{z=-d_m} - \zeta_m\, \hat{z} \times \frac{\partial \Delta\mathbf{E}_m}{\partial z}\Big|_{z=-d_m}, \tag{50}$$

$$\hat{z} \times \Delta\mathbf{H}_m\Big|_{z=-d_m} = \nabla_\perp \zeta_m \times \Delta\mathbf{H}_m\Big|_{z=-d_m} - \zeta_m\, \hat{z} \times \frac{\partial \Delta\mathbf{H}_m}{\partial z}\Big|_{z=-d_m}. \tag{51}$$

The field solutions can then be represented formally as

$$\mathbf{E}_m(\mathbf{r}_\perp, z) \approx \mathbf{E}_m^{(0)} + \mathbf{E}_m^{(1)} + \mathbf{E}_m^{(2)} + \ldots, \tag{52}$$

$$\mathbf{H}_m(\mathbf{r}_\perp, z) \approx \mathbf{H}_m^{(0)} + \mathbf{H}_m^{(1)} + \mathbf{H}_m^{(2)} + \ldots. \tag{53}$$

where the parenthesized superscript refers to the perturbation field of order *n*: $\mathbf{E}_m^{(0)}, \mathbf{H}_m^{(0)}$ is the unperturbed solution and $\mathbf{E}_m^{(1)}, \mathbf{H}_m^{(1)}$ is correction to the first-order of ζ_m and γ_m. It should be noted that the unperturbed solution represents the field existing in flat boundaries stratification, and satisfying:

$$\hat{z}\times\Delta\mathbf{E}_m^{(0)}\Big|_{z=-d_m}=0\,,\qquad \hat{z}\times\Delta\mathbf{H}_m^{(0)}\Big|_{z=-d_m}=0\,. \tag{54}$$

The fields expansion (52)-(53) can be then injected into the boundary conditions (50)-(51), so that, retaining only up to the first-order terms, the following *nonuniform boundary conditions* can be obtained (Imperatore et al. 2008a) (Imperatore et al. 2008b) (Imperatore et al. 2009a)

$$\hat{z}\times\Delta\mathbf{E}_m^{(1)}\Big|_{z=-d_m}=\nabla_\perp\zeta_m\times\Delta\mathbf{E}_m^{(0)}\Big|_{z=-d_m}-\zeta_m\,\hat{z}\times\frac{\partial\Delta\mathbf{E}_m^{(0)}}{\partial z}\bigg|_{z=-d_m}\,, \tag{55}$$

$$\hat{z}\times\Delta\mathbf{H}_m^{(1)}\Big|_{z=-d_m}=\nabla_\perp\zeta_m\times\Delta\mathbf{H}_m^{(0)}\Big|_{z=-d_m}-\zeta_m\,\hat{z}\times\frac{\partial\Delta\mathbf{H}_m^{(0)}}{\partial z}\bigg|_{z=-d_m}\,. \tag{56}$$

Therefore, the boundary conditions from each *m*th rough interface can be transferred to the associated equivalent flat interface. In addition, the right-hand sides of Eqs. (55) and (56) can be interpreted as effective magnetic ($\mathbf{J}_{Hm}^{p(1)}$) and electric ($\mathbf{J}_{Em}^{p(1)}$) surface current densities, respectively, with *p* denoting the incident polarization; so that we can identify the first-order fluctuation fields as being excited by these effective surface current densities imposed on the unperturbed interfaces. Accordingly, the geometry randomness of each corrugated interfaces is then translated in random current sheets imposed on each reference mean plane (z=-d_m), which radiate in an unperturbed (flat boundaries) layered medium. As a result, within the first-order approximation, the field can be than represented as the sum of an unperturbed part $\mathbf{E}_n^{(0)},\mathbf{H}_n^{(0)}$ and a random part, so that $\mathbf{E}_n(\mathbf{r}_\perp,z)\approx\mathbf{E}_n^{(0)}+\mathbf{E}_n^{(1)}$, $\mathbf{H}_n(\mathbf{r}_\perp,z)\approx\mathbf{H}_n^{(0)}+\mathbf{H}_n^{(1)}$. The first is the primary field, which exists in absence of surface boundaries roughness (flat-boundaries stratification), detailed in (Imperatore et al. 2009a); whereas $\mathbf{E}_n^{(1)},\mathbf{H}_n^{(1)}$ can be interpreted as the superposition of single-scatter fields from each rough interface. In order to perform the evaluation of perturbative development, the scattered field in each region of the layered structure is then represented as the sum of *up*- and *down-going* waves, and the first-order scattered field in each region of the layered structure can be then characterized by adopting the following field *spectral representation* in terms of the unknown coefficients $S_m^{\pm q(1)}(\mathbf{k}_\perp)$:

$$\mathbf{E}_m^{(1)}=\mathbf{E}_m^{-(1)}+\mathbf{E}_m^{+(1)}\,, \tag{57}$$

$$\mathbf{H}_m^{(1)}=\mathbf{H}_m^{-(1)}+\mathbf{H}_m^{+(1)}\,, \tag{58}$$

with

$$\mathbf{E}_m^{\pm(1)}=\sum_{q=h,v}\iint d\mathbf{k}_\perp e^{j\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\hat{q}_m^{\pm}(\mathbf{k}_\perp)S_m^{\pm q(1)}(\mathbf{k}_\perp)e^{\pm jk_{zm}z}\,, \tag{59}$$

$$\mathbf{H}_m^{\pm(1)}=\sum_{q=h,v}\frac{1}{Z_m}\iint d\mathbf{k}_\perp e^{j\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\hat{k}_m^{\pm}\times\hat{q}_m^{\pm}(\mathbf{k}_\perp)S_m^{\pm q(1)}(\mathbf{k}_\perp)e^{\pm jk_{zm}z}\,. \tag{60}$$

where Z_m is the intrinsic impedance of the medium *m*, and where

$$\hat{h}_m^{\pm}(\mathbf{k}_\perp) = \hat{k}_\perp \times \hat{z} = \hat{h} \tag{61}$$

$$\hat{v}_m^{\pm}(\mathbf{k}_\perp) = \mp \frac{k_{zm}}{k_m}\hat{k}_\perp + \frac{k_\perp}{k_m}\hat{z} \tag{62}$$

is a basis for the horizontal/vertical polarization vectors.

5.2 Matrix reformulation

In this section, we reformulate the *non-uniform boundaries condition* (55, 56), reducing the scattering problem to the formal solution of a linear system of equations; the unknowns are the scalar amplitudes, $S_m^{\pm q(1)}(\mathbf{k}_\perp)$, of the scattered fields. Eqs. (55, 56) can be rewritten by using their spectral representation:

$$\hat{z} \times \Delta\mathbf{E}_m^{(1)}\Big|_{z=-d_m} = \iint d\,\mathbf{k}_\perp\, e^{j\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\tilde{\mathbf{J}}_{Hm}^{p(1)}(\mathbf{k}_\perp,\mathbf{k}_\perp^i), \tag{63}$$

$$\hat{z} \times \Delta\mathbf{H}_m^{(1)}\Big|_{z=-d_m} = \iint d\,\mathbf{k}_\perp\, e^{j\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\tilde{\mathbf{J}}_{Em}^{p(1)}(\mathbf{k}_\perp,\mathbf{k}_\perp^i), \tag{64}$$

where the spectral densities $\tilde{\mathbf{J}}_{Em}^{p(1)}, \tilde{\mathbf{J}}_{Hm}^{p(1)}$ are the two-dimensional *Fourier transform* (2D-FT), with respect to $\mathbf{k}_\perp$, of the right-hand sides of (55) and (56), respectively, so that:

$$\tilde{\mathbf{J}}_{Hm}^{p(1)}(\mathbf{k}_\perp,\mathbf{k}_\perp^i) = \tilde{\zeta}_m(\mathbf{k}_\perp - \mathbf{k}_\perp^i)\left\{ j(\mathbf{k}_\perp - \mathbf{k}_\perp^i)\times\Delta\tilde{\mathbf{E}}_m^{(0)}\Big|_{z=-d_m} - \hat{z}\times\frac{\partial\Delta\tilde{\mathbf{E}}_m^{(0)}}{\partial z}\bigg|_{z=-d_m}\right\}, \tag{65}$$

$$\tilde{\mathbf{J}}_{Em}^{p(1)}(\mathbf{k}_\perp,\mathbf{k}_\perp^i) = \tilde{\zeta}_m(\mathbf{k}_\perp - \mathbf{k}_\perp^i)\left\{ j(\mathbf{k}_\perp - \mathbf{k}_\perp^i)\times\Delta\tilde{\mathbf{H}}_m^{(0)}\Big|_{z=-d_m} - \hat{z}\times\frac{\partial\Delta\tilde{\mathbf{H}}_m^{(0)}}{\partial z}\bigg|_{z=-d_m}\right\}, \tag{66}$$

where $\tilde{\zeta}_m(\mathbf{k}_\perp)$ is the *spectral representation* (2D-FT) of the corrugation $\zeta_m(\mathbf{r}_\perp)$, and where $\Delta\tilde{\mathbf{E}}_m^{(0)} = e^{-j\mathbf{k}_\perp^i\cdot\mathbf{r}_\perp}\Delta\mathbf{E}_m^{(0)}$, $\Delta\tilde{\mathbf{H}}_m^{(0)} = e^{-j\mathbf{k}_\perp^i\cdot\mathbf{r}_\perp}\Delta\mathbf{H}_m^{(0)}$, $p \in \{v, h\}$ is associated with the incident field polarization, and where we have taken into account that the 2D-FT of $\nabla_\perp\zeta_m(\mathbf{r}_\perp)$ is $j\mathbf{k}_\perp\tilde{\zeta}_m(\mathbf{k}_\perp)$, and that the 2D-FT of $\zeta_m(\mathbf{r}_\perp)e^{j\mathbf{k}_\perp^i\cdot\mathbf{r}_\perp}$ is $\tilde{\zeta}_m(\mathbf{k}_\perp - \mathbf{k}_\perp^i)$.

Therefore, a solution valid in each region of the layered structure can be obtained from (57)-(62) taking into account the non uniform boundary conditions (63)-(64). In order to solve the scattering problem in terms of the unknown expansion coefficients $S_m^{\pm q(1)}(\mathbf{k}_\perp)$, we arrange their amplitudes in a single vector according to the notation:

$$\mathbf{S}_m^{q(1)}(\mathbf{k}_\perp, d_m) = \begin{bmatrix} S_m^{+q(1)}(\mathbf{k}_\perp)e^{-jk_{zm}d_m} \\ S_m^{-q(1)}(\mathbf{k}_\perp)e^{+jk_{zm}d_m} \end{bmatrix}. \tag{67}$$

Subsequently, the *nonuniform boundary conditions* (63)-(64) can be reformulated by employing a suitable matrix notation, so that for the (q=h) *horizontal* polarized scattered wave we have (Imperatore et al. 2008a) (Imperatore et al. 2009a):

$$\mathbf{S}_m^{h(1)}(\mathbf{k}_\perp, d_m) + \mathbf{\Theta}_m^p(\mathbf{k}_\perp, \mathbf{k}_\perp^i) = \boldsymbol{N}_{m|m+1}^h(k_\perp)\mathbf{S}_{m+1}^{h(1)}(\mathbf{k}_\perp, d_m), \tag{68}$$

where

$$\mathbf{\Theta}_m^p(\mathbf{k}_\perp, \mathbf{k}_\perp^i) = \begin{bmatrix} -\dfrac{k_0 Z_0 \mu_m}{2k_{zm}}(\hat{k}_\perp \times \hat{z}) \cdot \tilde{\mathbf{J}}_{Em}^{p(1)} + \dfrac{1}{2}\hat{k}_\perp \cdot \tilde{\mathbf{J}}_{Hm}^{p(1)} \\ +\dfrac{k_0 Z_0 \mu_m}{2k_{zm}}(\hat{k}_\perp \times \hat{z}) \cdot \tilde{\mathbf{J}}_{Em}^{p(1)} + \dfrac{1}{2}\hat{k}_\perp \cdot \tilde{\mathbf{J}}_{Hm}^{p(1)} \end{bmatrix} \tag{69}$$

is the term associated with the *effective* source distribution, where the expressions of the effective currents $\tilde{\mathbf{J}}_{Em}^{p(1)}$ and $\tilde{\mathbf{J}}_{Hm}^{p(1)}$, imposed on the (flat) unperturbed boundary $z = -d_m$, for an incident polarization $p \in \{v, h\}$ are detailed in (Imperatore et al. 2009a); and where Z_0 is the intrinsic impedance of the vacuum. Furthermore, the fundamental *transfer matrix operator* is given by:

$$\boldsymbol{N}_{m-1|m}^q(k_\perp) = \frac{1}{T_{m-1|m}^q}\begin{bmatrix} 1 & R_{m-1|m}^q \\ R_{m-1|m}^q & 1 \end{bmatrix}, \tag{70}$$

with the superscripts $q \in \{v, h\}$ denoting the polarization. Moreover, it should be noted that on a (kth) flat interface Eq. (68) reduces to the *uniform boundary conditions,* thus getting:

$$\mathbf{S}_k^{h(1)}(\mathbf{k}_\perp, d_k) = \boldsymbol{N}_{k|k+1}^h(k_\perp)\mathbf{S}_{k+1}^{h(1)}(\mathbf{k}_\perp, d_k)\cdot \tag{71}$$

We emphasize that equations (68) state in a simpler form the problem originally set by Eqs. (55)-(56): as matter of fact, solving Eq. (68) $\forall m$ implies dealing with the determination of unknown scalar amplitudes $S_m^{\pm q(1)}(\mathbf{k}_\perp)$ instead of working with the corresponding vector unknowns $\mathbf{E}_m^{(1)}, \mathbf{H}_m^{(1)}$. Therefore, the scattering problem in each mth layer is reduced to the algebraic calculation of the unknown expansion scattering coefficients vector (67). As a result, when a structure with rough interfaces is considered, the enforcement of the original non uniform boundary conditions through the stratification (m=$0, ..., N$-1) can be addressed by writing down a linear system of equations with the aid of the matrix formalism (68)-(69) with m=$0, ..., N$-1. As a result, the formulation of *non-uniform boundary conditions* in matrix notation (68)-(69) enables a systematic method for solving the scattering problem: For the N-layer stratification of Fig.1, we have to find $2N$ unknown expansion coefficients, using N vectorial equations (68), i.e., $2N$ scalar equations. It should be noted that, for the considered configuration, the relevant scattering coefficients $S_N^{+q(1)}(\mathbf{k}_\perp), S_0^{-q(1)}(\mathbf{k}_\perp)$ are obviously supposed to be zero. The scattering problem, therefore, results to be reduced to a formal solution of a linear equation system. It can be demonstrated that, making use of a *recursive approach* involving the key-concept of generalized transmission/reflection (see also Sect. 3), the system of equations (68, 69) is susceptible of a straightforward closed form solution, so that the first-order perturbation fields anywhere in the upper half-space that arise from each mth rough interface can be formally found (Imperatore et al. 2009a) (Imperatore et al. 2009b).

In conclusion, the derivation of scattering field contribution, due to each rough interface, for instance in the upper or the lower half-space, can be then accomplished by avoiding the use of the cumbersome *Green functions* formalism.

We finally emphasize that here we are interested in the scattering from and through the stratification; therefore, the determination of the pertinent unknown expansion coefficients $S_0^{+q(1)}(\mathbf{k}_\perp)$ and $S_N^{-q(1)}(\mathbf{k}_\perp^s)$ of the scattered wave, respectively, into the upper and the lower half-space, is required only. Full expressions for these coefficients are reported in (Imperatore et al. 2009a) (Imperatore et al. 2009b).

6. *BPT* closed-form solutions

The aim of this section is to present the relevant BPT solutions for the scattering from and through the 3-D layered rough structure pictured schematically in Fig.1. We underline that the corresponding first-order solutions refer to two complementary bistatic configuration: in the first case, both the transmitter and the receiver are into the same half-space, whereas, in the second case, each one is located in a different half-space.

6.1 Scattering from layered structure with an arbitrary number of rough interfaces

First we consider the case of one rough interface embedded in the layered structure. The field scattered upward in the upper half-space in the first-order limit can be written in the form (see (57)-(60)):

$$\mathbf{E}_0^{(1)}(\mathbf{r}) = \sum_{q=h,v} \iint d\mathbf{k}_\perp e^{j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \hat{q}_0^+(\mathbf{k}_\perp) S_0^{+q(1)}(\mathbf{k}_\perp) e^{jk_{z0}z} \quad . \tag{72}$$

By employing the *method of stationary phase,* we evaluate the integral (72) in the *far field* zone, obtaining:

$$\mathbf{E}_0^{(1)}(\mathbf{r}) \cdot \hat{q}_0^+(\mathbf{k}_\perp^s) \cong -j2\pi\, k_0 \cos\theta_0^s \, \frac{e^{j\,k_0\,r}}{r} S_0^{+q(1)}(\mathbf{k}_\perp^s) \,, \tag{73}$$

with $q \in \{v, h\}$ is the polarization of the scattered field. Taking into account the expressions for the unknowns expansion coefficients $S_0^{+q(1)}(\mathbf{k}_\perp^s)$ (Imperatore et al. 2009a), we get

$$\mathbf{E}_0^{(1)}(\mathbf{r}) \cdot \hat{q}_0^+(\mathbf{k}_\perp^s) = \pi\, k_0^2 \, \frac{e^{jk_0 r}}{r} \tilde{\alpha}_{qp}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i)\, \tilde{\zeta}_m(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i) \,, \tag{74}$$

wherein

$$\begin{aligned}\tilde{\alpha}_{hh}^{m,m+1} = {} & (\varepsilon_{m+1} - \varepsilon_m) \frac{k_{z0}^s}{k_{zm}^s} (\hat{k}_\perp^s \cdot \hat{k}_\perp^i) \\ & e^{jk_{zm}^s \Delta_m} \Im_{m|0}^h (k_\perp^s)[1 + \Re_{m|m+1}^h (k_\perp^s)]\, e^{jk_{zm}^i \Delta_m} \Im_{0|m}^h (k_\perp^i)[1 + \Re_{m|m+1}^h (k_\perp^i)] \,,\end{aligned} \tag{75}$$

$$\tilde{\alpha}_{vh}^{m,m+1} = (\varepsilon_{m+1} - \varepsilon_m)\frac{k_{z0}^s}{k_0}\hat{z}\cdot(\hat{k}_\perp^i \times \hat{k}_\perp^s) \quad , \tag{76}$$
$$e^{jk_{zm}^s\Delta_m}\Im_{m|0}^v(k_\perp^s)[1-\Re_{m|m+1}^v(k_\perp^s)]\,e^{jk_{zm}^i\Delta_m}\Im_{0|m}^h(k_\perp^i)[1+\Re_{m|m+1}^h(k_\perp^i)]$$

$$\tilde{\alpha}_{hv}^{m,m+1} = (\varepsilon_{m+1} - \varepsilon_m)\frac{k_{z0}^s k_{zm}^i}{k_0 k_{zm}^s \varepsilon_m}\hat{z}\cdot(\hat{k}_\perp^i \times \hat{k}_\perp^s) \tag{77}$$
$$e^{jk_{zm}^s\Delta_m}\Im_{m|0}^h(k_\perp^s)[1+\Re_{m|m+1}^h(k_\perp^s)]\,e^{jk_{zm}^i\Delta_m}\Im_{0|m}^v(k_\perp^i)[1-\Re_{m|m+1}^v(k_\perp^i)] \, ,$$

$$\tilde{\alpha}_{vv}^{m,m+1} = (\varepsilon_{m+1} - \varepsilon_m)\frac{k_{z0}^s}{k_0^2 k_{zm}^s \varepsilon_m}e^{jk_{zm}^s\Delta_m}\Im_{m|0}^v(k_\perp^s)\,e^{jk_{zm}^i\Delta_m}\Im_{0|m}^v(k_\perp^i)$$
$$\{[1+\Re_{m|m+1}^v(k_\perp^s)][1+\Re_{m|m+1}^v(k_\perp^i)]\frac{\varepsilon_m}{\varepsilon_{m+1}}k_\perp^i k_\perp^s \tag{78}$$
$$-[1-\Re_{m|m+1}^v(k_\perp^s)][1-\Re_{m|m+1}^v(k_\perp^i)]k_{zm}^s k_{zm}^i(\hat{k}_\perp^s\cdot\hat{k}_\perp^i)\} \quad ,$$

where $k_{zm}^i = k_{zm}(k_\perp^i)$, $k_{zm}^s = k_{zm}(k_\perp^s)$, $\Im_{0|m}^p$ and $\Im_{m|0}^p$ are, respectively, the generalized transmission coefficients in *downward* direction and the generalized transmission coefficients in *upward* direction (see (16)-(17)), and $\Re_{m|m+1}^p$ are the generalized reflection coefficients (see eq. (11)). The coefficients $\tilde{\alpha}_{qp}^{m,m+1}$ are relative to the *p*-polarized incident wave impinging on the structure from upper half space 0 and to the *q*-polarized scattering contribution from structure into the upper half space, originated from the rough interface between the layers *m, m+1*. Finally, we emphasize that the total scattering from the *N*-rough interfaces layered structure can be straightforwardly obtained, in the first-order approximation, by superposition of the different contributions pertaining each rough interface:

$$\mathbf{E}_0^{(1)}(\mathbf{r})\cdot\hat{q}_0^+(\mathbf{k}_\perp^s) = \pi k_0^2\frac{e^{jk_0 r}}{r}\sum_{m=0}^{N-1}\tilde{\alpha}_{qp}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i)\,\tilde{\zeta}_m(\mathbf{k}_\perp^s-\mathbf{k}_\perp^i)\cdot \tag{79}$$

6.2 Scattering through layered structure with an arbitrary number of rough interfaces

Similarly, when one rough interface embedded in the layered structure is concerned, the field scattered into the last half-space, through the 3-D layered structure, in the first-order limit can be then written in the form:

$$\mathbf{E}_N^{(1)}(\mathbf{r}) = \sum_{q=h,v}\iint d\mathbf{k}_\perp e^{j\mathbf{k}_\perp\cdot\mathbf{r}_\perp}\hat{q}_N^-(\mathbf{k}_\perp)S_N^{-q(1)}(\mathbf{k}_\perp)e^{-jk_{zN}z} \, . \tag{80}$$

In order to evaluate the integral (80) in *far field* zone, we firstly consider a suitable change of variable $\mathbf{r}' = (\mathbf{r}_\perp, z')$, with $z' = -z - d_{N-1}$:

$$\mathbf{E}_N^{(1)}(\mathbf{r}') = \sum_{q=h,v} \iint d\mathbf{k}_\perp e^{j\mathbf{k}_\perp \cdot \mathbf{r}_\perp} \hat{q}_N^-(\mathbf{k}_\perp) S_N^{-q(1)}(\mathbf{k}_\perp) e^{jk_{zN} d_{N-1}} e^{jk_{zN} z'}, \tag{81}$$

then we use the method of *stationary phase* and obtain:

$$\mathbf{E}_N^{(1)}(\mathbf{r}') \cdot \hat{q}_N^-(\mathbf{k}_\perp^s) \cong -j2\pi\, k_N \cos\theta_N^s \frac{e^{jk_N r'}}{r'} S_N^{-q(1)}(\mathbf{k}_\perp^s) e^{jk_{zN}^s d_{N-1}}, \tag{82}$$

with $q \in \{v, h\}$. Taking into account the expressions for the unknowns expansion coefficients $S_N^{-q(1)}(\mathbf{k}_\perp^s)$ (Imperatore et al. 2009b), we get

$$\mathbf{E}_N^{(1)}(\mathbf{r}') \cdot \hat{q}_N^-(\mathbf{k}_\perp^s) = \pi\, k_0^2 \frac{e^{jk_N r'}}{r'} \, {}_N^0\widetilde{\beta}_{qp}^{m,m+1}(\mathbf{k}^s, \mathbf{k}^i)\, \widetilde{\zeta}_m(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i), \tag{83}$$

wherein

$$\begin{aligned}{}_N^0\widetilde{\beta}_{hh}^{m,m+1} &= \left(\varepsilon_{m+1} - \varepsilon_m\right) \frac{k_{zN}^s}{k_{z(m+1)}^s} (\hat{k}_\perp^s \cdot \hat{k}_\perp^i) \\ &[1 + \Re_{m+1|m}^h(k_\perp^s)] e^{jk_{z(m+1)}^s \Delta_{m+1}} \Im_{m+1|N}^h(k_\perp^s) [1 + \Re_{m|m+1}^h(k_\perp^i)] e^{jk_{zm}^i \Delta_m} \Im_{0|m}^h(k_\perp^i),\end{aligned} \tag{84}$$

$$\begin{aligned}{}_N^0\widetilde{\beta}_{vh}^{m,m+1} &= (\varepsilon_{m+1} - \varepsilon_m) \frac{k_{zN}^s}{k_0 \sqrt{\varepsilon_N}} \hat{z} \cdot (\hat{k}_\perp^s \times \hat{k}_\perp^i) \\ &[1 - \Re_{m+1|m}^v(k_\perp^s)] e^{jk_{z(m+1)}^s \Delta_{m+1}} \Im_{m+1|N}^v(k_\perp^s) [1 + \Re_{m|m+1}^h(k_\perp^i)] e^{jk_{zm}^i \Delta_m} \Im_{0|m}^h(k_\perp^i),\end{aligned} \tag{85}$$

$$\begin{aligned}{}_N^0\widetilde{\beta}_{hv}^{m,m+1} &= (\varepsilon_{m+1} - \varepsilon_m) \frac{k_{zN}^s k_{zm}^i \sqrt{\varepsilon_0}}{k_{z(m+1)}^s k_0 \varepsilon_m} \hat{z} \cdot (\hat{k}_\perp^i \times \hat{k}_\perp^s) \\ &[1 + \Re_{m+1|m}^h(k_\perp^s)] e^{jk_{z(m+1)}^s \Delta_{m+1}} \Im_{m+1|N}^h(k_\perp^s) [1 - \Re_{m|m+1}^v(k_\perp^i)] e^{jk_{zm}^i \Delta_m} \Im_{0|m}^v(k_\perp^i),\end{aligned} \tag{86}$$

$$
{}_{N}^{0}\widetilde{\beta}_{vv}^{m,m+1} = (\varepsilon_{m+1}-\varepsilon_m)\frac{k_{zN}^{s}\sqrt{\varepsilon_0}}{k_{z(m+1)}^{s}k_0^2\varepsilon_m\sqrt{\varepsilon_N}}e^{jk_{z(m+1)}^{s}\Delta_{m+1}}\Im_{m+1|N}^{v}(k_\perp^s)\,\Im_{0|m}^{v}(k_\perp^i)e^{jk_{zm}^{i}\Delta_m}
\left\{[1+\Re_{m+1|m}^{v}(k_\perp^s)][1+\Re_{m|m+1}^{v}(k_\perp^i)]k_\perp^i k_\perp^s
+[1-\Re_{m+1|m}^{v}(k_\perp^s)][1-\Re_{m|m+1}^{v}(k_\perp^i)](\hat{k}_\perp^s\cdot\hat{k}_\perp^i)k_{zm}^{i}k_{z(m+1)}^{s}\right\},
\tag{87}
$$

where $k_{zm}^{i}=k_{zm}(k_\perp^i)$, $k_{zm}^{s}=k_{zm}(k_\perp^s)$, $\Im_{0|m}^{p}$ and $\Im_{m+1|N}^{p}$ are, respectively, the generalized transmission coefficients in *downward* direction and the generalized transmission coefficients in *downward* direction given, respectively, by (16) and (21), and $\Re_{m|m+1}^{p}$ are the generalized reflection coefficients (see (11)). The coefficients ${}_{N}^{0}\widetilde{\beta}_{qp}^{m,m+1}$ are relative to the *p*-polarized incident wave impinging on the structure from half-space *0* and to *q*-polarized scattering contribution, originated from the rough interface between the layers *m* and *m+1*, through the structure into last half-space *N*. Finally, we emphasize that the total scattering through the *N*-rough interfaces layered structure can be straightforwardly obtained, in the first order approximation, by superposition of the different contributions pertaining each rough interface:

$$
\mathbf{E}_N^{(1)}(\mathbf{r}')\cdot\hat{q}_N^{-}(\mathbf{k}_\perp^s)=\pi k_0^2\frac{e^{jk_N r'}}{r'}\sum_{m=0}^{N-1}{}_{N}^{0}\widetilde{\beta}_{qp}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i)\,\widetilde{\zeta}_m(\mathbf{k}_\perp^s-\mathbf{k}_\perp^i)\cdot
\tag{88}
$$

As a result, the relevant final solutions (79) and (88) turn out formally identical, provided that the coefficients $\widetilde{\alpha}_{qp}^{m,m+1}$ are replaced with the complementary ones ${}_{N}^{0}\widetilde{\beta}_{qp}^{m,m+1}$.

7. Bi-static scattering cross sections

In this section, we calculate the bi-static scattering cross sections of the layered structure arising from the *BPT* solutions, which have been derived in the first-order approximation in the previous sections. The estimate of the mean power density can be obtained by averaging over an ensemble of statistically identical interfaces.

7.1 Scattering Cross Section of an arbitrary layered structure with an embedded rough interface

In this section, we focus on the scattering property of a single rough interface embedded in the layered structure. The bi-static scattering *cross section* of a generic (*n*th) rough interface embedded in the layered structure can be then defined as

$$
\widetilde{\sigma}_{qp,n}^{0}=\lim_{r\to\infty}\lim_{A\to\infty}\frac{4\pi r^2}{A}<\left|\mathbf{E}_0^{(1)}(\mathbf{r})\cdot\hat{q}_0^{+}(\mathbf{k}_\perp^s)\right|^2>,
\tag{89}
$$

where < > denotes ensemble averaging, where $q \in \{v, h\}$ and $p \in \{v, h\}$ denote, respectively, the polarization of scattered field and the polarization of incident field, and where A is the illuminated surface area. Therefore, by substituting (74) into (89) and considering that the (spatial) *power spectral density* $W_n(\boldsymbol{\kappa})$ of nth corrugated interface is defined as in (42), the *scattering cross section* relative to the contribution of the nth corrugated interface, according to the formalism used in [Franceschetti et al. 2008], can be expressed as

$$\tilde{\sigma}^0_{qp,n} = \pi k_0^4 \left|\tilde{\alpha}^{n,n+1}_{qp}(\mathbf{k}^s, \mathbf{k}^i)\right|^2 W_n(\mathbf{k}^s_\perp - \mathbf{k}^i_\perp) \ , \tag{90}$$

with $p,\ q \in \{v, h\}$ denoting, respectively, the incident and the scattered polarization states, which may stand for *horizontal* polarization (h) or *vertical* polarization (v); Furthermore, we stress when the backscattering case ($\hat{k}^s_\perp \times \hat{k}^i_\perp = 0$) is concerned, our cross-polarized scattering coefficients (75)-(78) evaluated in the plane of incidence vanish, in full accordance with the classical first-order SPM method for a rough surface between two different media (Ulaby et al, 1982) (Tsang et al., 1985).

7.2 Scattering Cross Section into last half-space of an Arbitrary Layered Structure with an Embedded Rough Interface

As counterparts of the configuration considered in the last subsection, we now refer to the complementary one in which the scattering through the structure is concerned. The bi-static scattering cross section into last half-space of the structure with one embedded (nth) rough interface can be defined as

$$\tilde{\sigma}^0_{qp,n} = \lim_{r\to\infty} \lim_{A\to\infty} \frac{4\pi r'^2}{A} < \left|\mathbf{E}^{(1)}_N(\mathbf{r}')\cdot \hat{q}^-_N(\mathbf{k}^s_\perp)\right|^2 > \mathrm{Re}\left\{\sqrt{\frac{\varepsilon_N}{\varepsilon_0}}\right\}, \tag{91}$$

where < > denotes ensemble averaging, where the index $q \in \{v, h\}$ index $p \in \{v, h\}$ and denote, respectively, the polarization of scattered field and the polarization of incident field, A is the surface area, and where we have considered the *Poynthing* power density of the transmitted wave in Nth region normalized to the power density of the incident wave. Therefore, by substituting (82) into (91) and considering that the (spatial) *power spectral density* $W_n(\boldsymbol{\kappa})$ of nth corrugated interface is defined as in (42), as final result, we obtain:

$$\tilde{\sigma}^0_{qp,n} = \pi k_0^4 \left|{}^{0}_{N}\tilde{\beta}^{n,n+1}_{qp}(\mathbf{k}^s, \mathbf{k}^i)\right|^2 W_n(\mathbf{k}^s_\perp - \mathbf{k}^i_\perp)\,\mathrm{Re}\left\{\sqrt{\frac{\varepsilon_N}{\varepsilon_0}}\right\}. \tag{92}$$

7.3 Scattering Cross Section of a Layered Structure with N-rough interfaces

We now show that the solutions, given by the expressions (90) and (92) respectively, are susceptible of a straightforward generalization to the case of arbitrary stratification with N-rough boundaries. Taking into account the contribution of each nth corrugated interface (see (79)), the global *bi-static scattering cross section* of the N-rough interface layered media can be expressed as:

$$\tilde{\sigma}_{qp}^{0} = \pi k_0^4 \sum_{n=0}^{N-1} \left| \tilde{\alpha}_{qp}^{n,n+1}(\mathbf{k}^s,\mathbf{k}^i) \right|^2 W_n(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i) + \pi k_0^4 \sum_{i \neq j} \mathrm{Re}\left\{ \tilde{\alpha}_{qp}^{i,i+1}(\mathbf{k}^s,\mathbf{k}^i) \left[\tilde{\alpha}_{qp}^{j,j+1}(\mathbf{k}^s,\mathbf{k}^i) \right]^* \right\} W_{ij}(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i), \tag{93}$$

with $p, q \in \{v, h\}$, where the asterisk denotes the complex conjugated, where $\tilde{\alpha}_{qp}^{i,i+1}$ are given by (75)-(78), and where the *cross power spectral density* W_{ij}, between the interfaces *i* and *j*, for the spatial frequencies of the roughness is given by (43).
Likewise, the solution given by the expression (92), is susceptible of a straightforward generalization to the case of arbitrary stratification with *N*-rough boundaries. Taking into account the contribution of each *n*th corrugated interface (see (88)), the global *bi-static scattering cross section* into last half-space of the *N*-rough interface layered media can be expressed as:

$$\tilde{\sigma}_{qp}^{0} = \pi k_0^4 \,\mathrm{Re}\left\{ \sqrt{\frac{\varepsilon_N}{\varepsilon_0}} \right\} \sum_{n=0}^{N-1} \left| {}_{N}^{0}\tilde{\beta}_{qp}^{n,n+1}(\mathbf{k}^s,\mathbf{k}^i) \right|^2 W_n(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i) + \pi k_0^4 \,\mathrm{Re}\left\{ \sqrt{\frac{\varepsilon_N}{\varepsilon_0}} \right\} \sum_{i \neq j} \mathrm{Re}\left\{ {}_{N}^{0}\tilde{\beta}_{qp}^{i,i+1}(\mathbf{k}^s,\mathbf{k}^i) \left[{}_{N}^{0}\tilde{\beta}_{qp}^{j,j+1}(\mathbf{k}^s,\mathbf{k}^i) \right]^* \right\} W_{ij}(\mathbf{k}_\perp^s - \mathbf{k}_\perp^i), \tag{94}$$

where $p, q \in \{v, h\}$, where the asterisk denotes the complex conjugated, ${}_{N}^{0}\tilde{\beta}_{qp}^{i,i+1}$ are given by (84)-(87), and where the *cross power spectral density* W_{ij}, between the interfaces *i* and *j*, for the spatial frequencies of the roughness is given by (43).

Some final considerations are now in order. As a matter of fact, the presented closed-form solutions permit the *polarimetric* evaluation of the scattering for a *bi-static* configuration, *from* or *through* the layered rough structure, once the *three-dimensional* layered structure's parameters (shape of the roughness spectra, layers thickness and complex permittivities), the incident field parameters (frequency, polarization and direction) and the observation direction are been specified. As a result, an elegant closed form solution is established, which takes into account parametrically the dependence of scattering properties on structure (geometric and electromagnetic) parameters. Therefore, BPT formulation leads to solutions which exhibit a direct functional dependence (no integral evaluation is required) and, subsequently, permit to show that the scattered field can be parametrically evaluated considering a set of parameters: some of them refer to an unperturbed structure configuration, i.e. intrinsically the physical parameters of the smooth boundary structure, and others which are determined exclusively by (random) deviations of the corrugated boundaries from their reference position. Note also that the coefficients $\tilde{\alpha}_{qp}^{m,m+1}$ and ${}_{N}^{0}\tilde{\beta}_{qp}^{i,i+1}$ depend parametrically on the unperturbed structure parameters only. Procedurally, once the *generalized reflection/transmission coefficients* are recursively evaluated, the coefficients

$\tilde{\alpha}_{qp}^{i,i+1}$ and/or ${}_{N}^{0}\tilde{\beta}_{qp}^{i,i+1}$ can be than directly computed, so that the scattering cross sections (93) and/or (94) for the pertinent structure with rough interfaces can be finally predicted. Furthermore, the scattering from or through the rough layered media is sensitive to the correlation between rough profiles of different interfaces. In fact, a real layered structure will have interfaces cross-correlation somewhere between two limiting situations: perfectly correlated and uncorrelated roughness. Consequently, the degree of correlation affects the phase relation between the fields scattered by each rough interface. Obviously, when the interfaces are supposed to be uncorrelated, the second terms respectively in (93) and (94) vanish and accordingly, in the first-order approximation, the total scattering from or through the structure arises from the incoherent superposition of radiation scattered from each interface. We emphasize that the effects of the interaction between the rough interfaces can limited be treated, in the first-order approximation, only when the rough interfaces exhibit some correlation. In addition, it has been demonstrated that the proposed global solution turns out to be completely interpretable with basic physical concepts, clearly discerning the physics of the involved scattering mechanisms (Imperatore et al 2008c) (Imperatore et al. 2009c). Finally, it should be noted that the method to be applied needs only the classical gently-roughness assumption, without any further approximation.

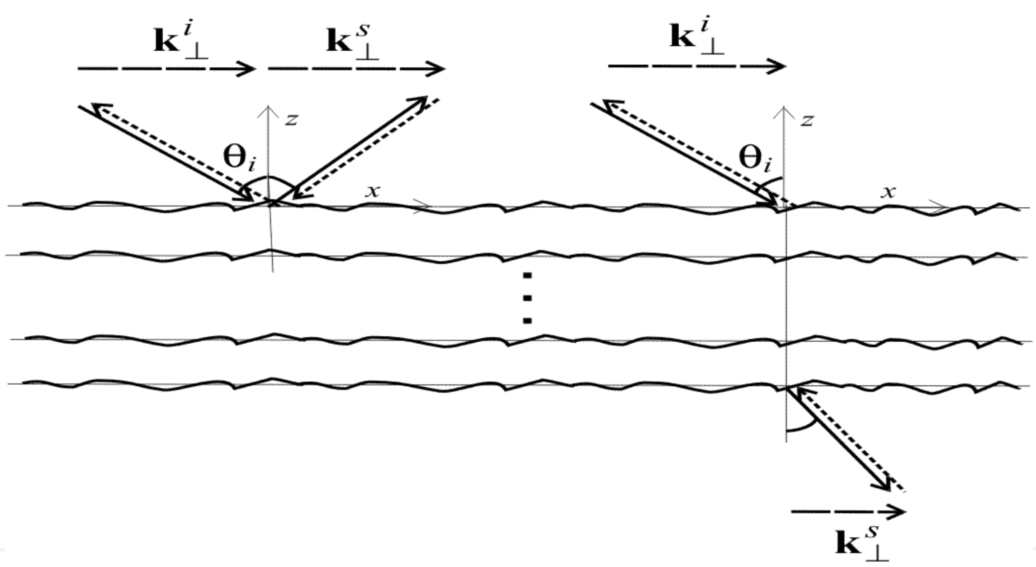


Fig. 3. Reciprocity for scattering from and through a layered structure with rough interfaces.

8. Reciprocal character of the BPT solutions

In this section, the emphasis is placed on the reciprocal character of the final BPT scattering solutions, which evidently constitutes a crucial point in the formal framework of the BPT.

Generally speaking, the reciprocity principle is a statement that expresses some form of symmetry in the laws governing a physical system. Analytically speaking, both the BPT final solutions (79) and (88), respectively, from and through the layered structure with N-rough interfaces can be expressed in a common formal frame exhibiting a symmetric nature:

$$\tilde{\alpha}_{qp}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i) = \tilde{\alpha}_{pq}^{m,m+1}(-\mathbf{k}^i,-\mathbf{k}^s), \tag{95}$$

$${}_{N}^{0}\tilde{\beta}_{qp}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i) = -{}_{0}^{N}\tilde{\beta}_{pq}^{m+1,m}(-\mathbf{k}^i,-\mathbf{k}^s). \tag{96}$$

These formal relations are not only a mere matter of aesthetic; in fact their symmetry inherently reflects the conformity with the reciprocity principle of the electromagnetic theory. We emphasize that the relations (95), (96) imply that the wave amplitude for the scattering process $\mathbf{k}^i \to \mathbf{k}^s$ equals that of reciprocal scattering process $-\mathbf{k}^s \to -\mathbf{k}^i$ .Therefore, (95) and (96) are also *reciprocity* relationships for the scattering, respectively, from and through a layered structure with an (*m*th) embedded rough interface. This is to say that for the presented scattering solutions the role of the source and the receiver can be exchanged (see Fig.3), in conformity with the reciprocity principle of the electromagnetic theory. It should be noted that when the *N*-rough interfaces structure is concerned the properties (95)-(96) are satisfied as well, since the solutions in first-order limit are obtainable by superposition of the contribution of each (*m*th) rough interface. In order to provide general demonstration of these fundamental relationships, we found a more compact expression for (75)-(78) and (84)-(87), respectively. First, we introduce the following suitable notation:

$$\xi_{0\to m}^{\pm p}(k_\perp) = \Im_{0|m}^{p}(k_\perp)\, e^{jk_{zm}\Delta_m}[1 \pm \Re_{m|m+1}^{p}(k_\perp)], \tag{97}$$

$$\xi_{N\to m+1}^{\pm p}(k_\perp) = \Im_{N|m+1}^{p}(k_\perp)\, e^{jk_{z(m+1)}\Delta_{m+1}}[1 \pm \Re_{m+1|m}^{p}(k_\perp)]. \tag{98}$$

Next, when the solution for the scattering from the layered structure with an embedded rough interface is concerned, substituting relations (17) into (75)-(78), we obtain the alternative and more compact expressions for the relevant solution:

$$\begin{aligned}\tilde{\alpha}_{vv}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i) = (\varepsilon_{m+1}-\varepsilon_m)\Bigg[&\frac{\varepsilon_m}{\varepsilon_{m+1}}\frac{k_\perp^s}{k_0\varepsilon_m}\xi_{0\to m}^{+v}(k_\perp^s)\frac{k_\perp^i}{k_0\varepsilon_m}\xi_{0\to m}^{+v}(k_\perp^i)\\ &-\hat{k}_\perp^s\cdot\hat{k}_\perp^i\frac{k_{zm}^s}{k_0\varepsilon_m}\xi_{0\to m}^{-v}(k_\perp^s)\frac{k_{zm}^i}{k_0\varepsilon_m}\xi_{0\to m}^{-v}(k_\perp^i)\Bigg],\end{aligned} \tag{99}$$

$$\tilde{\alpha}_{vh}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i) = (\varepsilon_{m+1}-\varepsilon_m)\hat{z}\cdot(\hat{k}_\perp^i\times\hat{k}_\perp^s)\frac{k_{zm}^s}{k_0\varepsilon_m}\xi_{0\to m}^{-v}(k_\perp^s)\xi_{0\to m}^{+h}(k_\perp^i), \tag{100}$$

$$\tilde{\alpha}_{hv}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i) = (\varepsilon_{m+1}-\varepsilon_m)\,\hat{z}\cdot(\hat{k}_\perp^i\times\hat{k}_\perp^s)\xi_{0\to m}^{+h}(k_\perp^s)\frac{k_{zm}^i}{k_0\varepsilon_m}\xi_{0\to m}^{-v}(k_\perp^i), \tag{101}$$

$$\tilde{\alpha}_{hh}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i) = (\varepsilon_{m+1}-\varepsilon_m)\hat{k}_\perp^s\cdot\hat{k}_\perp^i\,\xi_{0\to m}^{+h}(k_\perp^s)\,\xi_{0\to m}^{+h}(k_\perp^i). \tag{102}$$

Then, by direct inspection of (99)-(102) we ultimately find Eq. (95).

On the other hand, when the solution for the scattering through the layered structure with an embedded rough interface is concerned, we proceed similarly as done previously. Substituting relations (18) into (84)-(87), we obtain the alternative and more compact expressions for the relevant solution:

$$
{}_{N}^{0}\tilde{\beta}_{vv}^{m,m+1}(\mathbf{k}_\perp^s,\mathbf{k}_\perp^i)=(\varepsilon_{m+1}-\varepsilon_m)\frac{\sqrt{\varepsilon_0\varepsilon_N}}{k_0^2\varepsilon_{m+1}\varepsilon_m}
\left\{k_\perp^s k_\perp^i\,\xi_{N\to m+1}^{+v}(k_\perp^s)\xi_{0\to m}^{+v}(k_\perp^i)+(\hat{k}_\perp^s\cdot\hat{k}_\perp^i)\,k_{z(m+1)}^s k_{zm}^i\,\xi_{N\to m+1}^{-v}(k_\perp^s)\xi_{0\to m}^{-v}(k_\perp^i)\right\}. \tag{103}
$$

$$
{}_{N}^{0}\tilde{\beta}_{vh}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i)=(\varepsilon_{m+1}-\varepsilon_m)\,\hat{z}\cdot(\hat{k}_\perp^s\times\hat{k}_\perp^i)\frac{\sqrt{\varepsilon_N}\,k_{z(m+1)}^s}{k_0\varepsilon_{m+1}}\,\xi_{N\to m+1}^{-v}(k_\perp^s)\,\xi_{0\to m}^{+h}(k_\perp^i)\,, \tag{104}
$$

$$
{}_{N}^{0}\tilde{\beta}_{hv}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i)=(\varepsilon_{m+1}-\varepsilon_m)\,\hat{z}\cdot(\hat{k}_\perp^i\times\hat{k}_\perp^s)\frac{\sqrt{\varepsilon_0}\,k_{zm}^i}{k_0\varepsilon_m}\,\xi_{N\to m+1}^{+h}(k_\perp^s)\,\xi_{0\to m}^{-v}(k_\perp^i)\,, \tag{105}
$$

$$
{}_{N}^{0}\tilde{\beta}_{hh}^{m,m+1}(\mathbf{k}^s,\mathbf{k}^i)=(\varepsilon_{m+1}-\varepsilon_m)(\hat{k}_\perp^s\cdot\hat{k}_\perp^i)\,\xi_{N\to m+1}^{+h}(k_\perp^s)\,\xi_{0\to m}^{+h}(k_\perp^i)\,, \tag{106}
$$

Then, by direct inspection of (103)-(106) we ultimately find Eq. (96). This is to say that BPT formalism satisfies reciprocity.

9. Conclusion

The problem of electromagnetic scattering in 3D layered rough structures can be analytical treated by relying on effective results of the *Boundary Perturbation Theory* (*BPT*), whose formulation has been introduced by P. Imperatore and his coauthors in many different papers. A structured presentation of the pertinent theoretical body of results has been provided in this chapter. The first-order scattering models obtained in the framework of the *BPT* allow us to polarimetrically deal with the (bi-static) scattering, from and through three-dimensional layered structures with an arbitrary number of gently rough interfaces.

Analytically speaking, two relevant closed-form solutions, obtained for two different configurations, respectively, for the scattering from and through the structure, are presented in a common formal frame. As a matter of fact, beyond a certain economy and mathematical elegance in the final analytical solutions, their inherent symmetry is intimately related to the reciprocity.

Some remarkable considerations on the meaning of the *BPT* solutions are in order. It can be demonstrated that, beyond the technicalities of the *BPT* formulation, the pertinent analytical results are also susceptible of a powerful physical interpretation; so that the fundamental interactions contemplated by the *BPT* can be revealed, gaining a coherent explanation and a neat picture of the physical meaning of the *BPT* theoretical construct (Imperatore et al 2008c)

(Imperatore et al. 2009c). The consequent phenomenological implications on the practical applications are then considerable.
Therefore, the formally symmetric, physically revealing, and fully polarimetric *BPT solutions* are amenable of direct and parametric numerical evaluation and, therefore, can be effectively applied to several practical situations of interest. We underline that it can be also demonstrated that all the previous existing perturbative scattering models, introduced by other authors to deal with simplified layered geometry with one (Yarovoy et al., 2000), (Azadegan and Sarabandi, 2003), (Fuks, 2001) or two rough interfaces (Tabatabaeenejad and Moghaddam, 2006), can be all rigorously regarded as a special cases of the general BPT solutions (see also Franceschetti et al, 2008). This analytical consistency also provides a unifying perspective on the perturbative approaches. Finally, the body of the *BPT* theoretical results can be also regarded as a generalization to the case of layered media with rough interfaces of the classical *SPM* for rough surface.

10. References

Franceschetti G., Imperatore P., Iodice A., Riccio D., and Ruello G. (2003), Scattering from a Layered Medium with One Rough Interface: Comparison and Physical Interpretation of Different Methods, *Proceedings of IEEE International Geoscience and Remote Sensing Symposium (IGARSS 2003), Toulouse, France,* pp. 2912–2914, Jul. 2003.

Franceschetti G., Imperatore P., Iodice A., Riccio D., and Ruello G. (2008). Scattering form Layered Strucutures with One rough Interface: A Unified Formulation of Perturbative Solutions. *IEEE Trans. on Geoscience and Remote Sensing*, vol.46, no.6, June 2008, pp.1634-1643.

Imperatore P., Iodice A., Riccio D. (2008a), Perturbative Solution for the Scattering from Multilayered Structure with Rough Boundaries, *Proceedings of Microwave Radiometry and Remote Sensing of the Environment* (MICRORAD 2008), pp.1-4, 11-14 March 2008, Florence, Italy.

Imperatore P., Iodice A., Riccio D. (2008b), Small Perturbation Method for Scattering From Rough Multilayers, *Proceedings of IEEE International Geoscience and Remote Sensing Symposium*, 2008 (IGARSS 2008), vol.5, pp.V-271-V-274, Boston 7-11 July 2008.

Imperatore P., Iodice A., Riccio D. (2008c), Interpretation of Perturbative Solution for the Scattering From Layered Structure With Rough Interfaces, *Proceedings of IEEE International Geoscience and Remote Sensing Symposium*, 2008 (IGARSS 2008), vol.4, pp. IV -1141-IV-1144, Boston 7-11 July 2008.

Imperatore P., Iodice A., Riccio D. (2009a). Electromagnetic Wave Scattering From Layered Structures with an Arbitrary Number of Rough Interfaces. *IEEE Trans. on Geoscience and Remote Sensing,* vol.47, no.4, pp.1056-1072, April 2009.

Imperatore P., Iodice A., Riccio D. (2009b). Transmission Through Layered Media With Rough Boundaries: First-Order Perturbative Solution. *IEEE Transaction on Antennas and Propagation*, vol.57, no.5, pp.1481-1494, May 2009.

Imperatore P., Iodice A., Riccio D. (2009c). Physical Meaning of Perturbative Solutions for Scattering from and Through Multilayered Structure with Rough Interfaces. *submitted to IEEE Trans. on Antennas and Propagation*.

Azadegan R. and Sarabandi K. (2003), Analytical formulation of the scattering by a slightly rough dielectric boundary covered with a homogeneous dielectric layer, *Proceedings of IEEE AP-S Int. Symp.*, Columbus, OH, Jun. 2003, pp. 420–423.

Fuks I. M.(1998), Radar contrast polarization dependence on subsurface sensing, *Proceedings of IEEE IGARSS'98*, vol. 3, Seattle, WA, USA, July 6–10, 1998, pp. 1455–1459.

Fuks I.M. (2001). Wave diffraction by a rough boundary of an arbitrary plane-layered medium. *IEEE Transaction on Antennas and Propagation*, vol. 49, no.4, Apr. 2001. pp.630–639.

Yarovoy A.G., de Jongh R.V., Ligthard L.P. (2000). Scattering properties of a statistically rough interface inside a multilayered medium. *Radio Science*, vol.35, n.2, 2000, pp. 445–462.

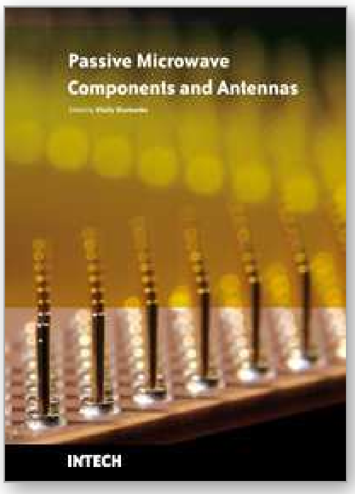
Tabatabaeenejad A., Moghaddam M. (2006). Bistatic scattering from three-dimensional layered rough surfaces. *IEEE Trans. on Geoscience and Remote Sensing*, vol. 44, no. 8, pp. 2102–2114, Aug. 2006.

Chew W. C. (1997). *Waves and Fields in Inhomogeneous Media*. IEEE Press, 1997.

Ishimaru A. (1978). *Wave Propagation and Scattering in Random Media*, vol.2. Academic Press, New York, 1978.

Tsang L., Kong J. A, and Shin R. T. (1985). *Theory of Microwave Remote Sensing*. New York: Wiley, 1985.

Ulaby F. T., Moore R. K., and Fung A. K. (1982). *Microwave Remote Sensing*, vol. I, II, II. Reading, MA: Addison-Wesley, 1982.



Passive Microwave Components and Antennas
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Modelling and computations in electromagnetics is a quite fast-growing research area. The recent interest in this field is caused by the increased demand for designing complex microwave components, modeling electromagnetic materials, and rapid increase in computational power for calculation of complex electromagnetic problems. The first part of this book is devoted to the advances in the analysis techniques such as method of moments, finite-difference time- domain method, boundary perturbation theory, Fourier analysis, mode-matching method, and analysis based on circuit theory. These techniques are considered with regard to several challenging technological applications such as those related to electrically large devices, scattering in layered structures, photonic crystals, and artificial materials. The second part of the book deals with waveguides, transmission lines and transitions. This includes microstrip lines (MSL), slot waveguides, substrate integrated waveguides (SIW), vertical transmission lines in multilayer media as well as MSL to SIW and MSL to slot line transitions.

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