

# We are IntechOpen, the world's leading publisher of Open Access books Built by scientists, for scientists

6,900

Open access books available

185,000

International authors and editors

200M

Downloads

Our authors are among the

154

Countries delivered to

TOP 1%

most cited scientists

12.2%

Contributors from top 500 universities



WEB OF SCIENCE™

Selection of our books indexed in the Book Citation Index  
in Web of Science™ Core Collection (BKCI)

Interested in publishing with us?  
Contact [book.department@intechopen.com](mailto:book.department@intechopen.com)

Numbers displayed above are based on latest data collected.  
For more information visit [www.intechopen.com](http://www.intechopen.com)



# Trajectory tracking control for robot manipulators with no velocity measurement using semi-globally and globally asymptotically stable velocity observers

Farah Bouakrif

*LAMEL Laboratory, University of Jijel  
Algeria*

## 1. Introduction

During the last decade the class of rigid robot systems has been the subject of intensive research in the field of systems and control theory, particularly owing to the inherent nonlinear nature of rigid robots. For the same reason, these systems have widely been used to exemplify general concepts in nonlinear control theory. As a result of this excessive research activity a large variety of control methods for rigid robot systems have been proposed, such as, proportional-integral-derivative (PID) control (Kelly, 1995), computed torque control (Luh et al., 1980), which achieve the trajectory tracking objective by feedback linearization of the nonlinear robot dynamics, adaptive control (Ortega & Spong, 1989), variable structure control (Slotine & Sastry, 1983), fuzzy control (Chang & Chen, 2000), passivity based control (Ryu et al, 2004; Bouakrif et al., 2010) and iterative learning control (Bouakrif et al., 2007; Tayebi, 2007).

Many of these previous controllers require the complete state measurements, that is position and velocity, is available for feedback. Unfortunately, in practice this assumption can only partially be fulfilled for two reasons. First, although robot systems generally are equipped with high precision sensors for position measurements, velocity measurements are often contaminated with a considerable amount of noise. This circumstance may reduce the dynamic performance of the manipulator, since in practice, the values of the controller gain matrices are limited by the noise present in the velocity measurements (Khosla & Kanade, 1988). Second, in robotic applications today velocity sensors are frequently omitted owing to the considerable savings in cost, volume and weight that can be obtained this way. A good solution of this problem is the use of the velocity observers to reconstruct the missing velocity signal starting from the available position measurements. Due to the nonlinear and coupled structure of the robot dynamical model, the problem of designing observers for robots is a very complex one. Recently, exploiting the structural properties of the robot dynamics, a number of conceptually different methods for both regulation and tracking control of robots equipped with only position sensors have been developed (Canudas dewit et al., 1992; Paden & Panja, 1988). (Berghuis & Nijmeijer, 1993) presented a controller-

observer scheme for the global regulation of robots using only position feedback. The PD control with high-gain observer was developed (Yu & Li, 2006), the authors propose to reconstruct the velocity signal via a high-gain observer, but a quite noisy movement of the manipulator, which may be undesirable for greater robots employed for industrial applications.

In this chapter, we want to solve the trajectory tracking problem of rigid robot manipulators which are not equipped with the tachometers (velocity sensors) to avoid the disadvantage mentioned in the previous paragraph. For this purpose, two velocity observers are presented to estimate the missing velocity. Using the first observer, an estimate region of attraction is given. It is important to observe that this region can be made arbitrarily large by increasing the observer gain. This kind of stability is called semi-global. The second is globally asymptotically stable. Thus, there is more freedom to choose the initial states. This presents an advantage of the second observer. Thereafter, these observers are integrated with a nonlinear controller by replacing the velocity in the control law with its estimation yielded by these observers, independently. Furthermore, the semi-global and global asymptotic stability conditions are established of the composite controller consisting of robot manipulator, nonlinear controller and the first and second velocity observer, respectively. This proof is based on Lyapunov theory and using saturation technique for the second observer. Finally, simulation results on two-link manipulator are provided to illustrate the effectiveness of the global velocity observer based trajectory tracking control.

## 2. Dynamic equation for robot manipulators

We consider a robot manipulator that is composed of serially connected rigid links. The motion of the manipulator with  $n$ -links is described by the following dynamic equation:

$$\tau = M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) \quad (1)$$

where  $q(t)$ ,  $\dot{q}(t)$ ,  $\ddot{q}(t) \in R^n$  denote the link position, velocity, and acceleration vectors, respectively,  $M(q(t)) \in R^{n \times n}$  represents the link inertia matrix,  $C(q(t), \dot{q}(t)) \in R^{n \times n}$  represents centripetal-Coriolis matrix,  $G(q(t)) \in R^{n \times 1}$  represents the gravity effects, and  $\tau(t) \in R^{n \times 1}$  represents the torque input vector.

In the sequel,  $q_d(t)$ ,  $\dot{q}_d(t)$ ,  $\ddot{q}_d(t) \in R^n$  denote the desired link position, velocity, and acceleration vectors, respectively.

The dynamic equation (1) has the following properties (Berghuis, 1993; Ortega & Spong, 1989) that will be used in the controller development and analysis.

P 1: The inertia matrix  $M(q(t))$  is symmetric, positive definite and bounded as

$$0_n < M_m < \|M(q)\| < M_M \quad (2)$$

where  $q \in R^n$ , and  $M_M > M_m > 0$ .

P 2:  $\forall i \in \{1, \dots, n\}$ , the  $i^{th}$  element of the vector  $C(q, \dot{q})\dot{q}$  is equal to  $\dot{q}^T N_i(q)\dot{q}$  with  $N_i$  symmetric, continuously differentiable, and such that  $\exists \bar{N}_i > 0$  satisfies

$$\|N_i(q)\| \leq \bar{N}_i \quad \forall q \in \mathbb{R}^n. \quad (3)$$

P 3: Norm of the centripetal-Coriolis is bounded as follows

$$\|C(q, \dot{q})\| \leq C_m \|\dot{q}\|. \quad (4)$$

P 4: The matrix  $\dot{M}(q, \dot{q}) - 2C(q, \dot{q})$  is skew-symmetric, i.e., for all  $X \in \mathbb{R}^n$ ,

$$X^T (\dot{M}(q, \dot{q}) - 2C(q, \dot{q})) X = 0. \quad (5)$$

P 5: For all  $x, y \in \mathbb{R}^n$

$$C(q, x)y = C(q, y)x \quad (6)$$

$$C(q, z + \alpha x)y = C(q, z)y + \alpha C(q, x)y. \quad (7)$$

In this paper, the following lemmas are used.

Lemma 1 (Shim et al., 2001): Consider a  $C^1$  function  $f(x, y): \mathbb{R}^p \times \mathbb{R}^q \rightarrow \mathbb{R}$  which is continuous and well defined on  $X \times \mathbb{R}^q$  where  $X = \{x \in \mathbb{R}^p \mid |x_i| \leq \rho_i, 1 \leq i \leq p\}$  with  $\rho_i > 0$ . Then  $f(\sigma(x), y)$  is globally well defined and equal to  $f(x, y)$  for  $x \in X$ , and where exists  $L(y)$  such that

$$\|f(\sigma(\bar{x}), y) - f(\sigma(\tilde{x}), y)\| \leq L(y) \|\bar{x} - \tilde{x}\|, \forall \bar{x}, \tilde{x} \in \mathbb{R}^p, \forall y \in \mathbb{R}^q \quad (8)$$

where  $\sigma(x)$  is an element-wise saturation function which is saturated outside  $X$ .

Proof

By the Mean Value Theorem, there exists  $z \in \mathbb{R}^p$  such that

$$f(\sigma(\bar{x}), y) - f(\sigma(\tilde{x}), y) = \frac{\partial f}{\partial x}(z, y)(\sigma(\bar{x}) - \sigma(\tilde{x})) \quad (9)$$

which implies

$$\|f(\sigma(\bar{x}), y) - f(\sigma(\tilde{x}), y)\| \leq L(y) \|\sigma(\bar{x}) - \sigma(\tilde{x})\| \quad (10)$$

where  $L(y)$  is the maximum of  $\left\| \frac{\partial f}{\partial x}(z, y) \right\|$  with respect to  $z$  over the compact range of saturation function. Then the claim (8) follows from the fact that  $\|\sigma(\bar{x}) - \sigma(\tilde{x})\| \leq \|\bar{x} - \tilde{x}\|$ .

Lemma 2 “Barbalat’s lemma” (Slotine & Li, 1991): If  $H$  is a continuous function, and it is bounded when  $t \rightarrow \infty$ , and if  $\dot{H}$  is uniformly continuous in time, then  $\dot{H} = 0$ .

In (2), (3), (4) and in the sequel the norm of a vector  $X$  is defined as

$$\|X\| = \sqrt{X^T X} \quad (11)$$

and the norm of a matrix  $A$  as

$$\|A\| = \sqrt{\lambda_{\max}(A^T A)} \quad (12)$$

with  $\lambda_{\max}(\cdot)$  denotes the maximum eigenvalue of  $A$ .

The following assumption is imposed.

Assumption: The robot velocity is bounded by a known constant  $V_m$  such that

$$\|\dot{q}(t)\| \leq V_m \quad \forall t \in \mathbb{R}. \quad (13)$$

Remark 1. This assumption is definitively realistic. In fact, it is reasonable to expect that the joint velocities of a robot will not exceed certain a priori bounds that come from the mechanic limitations of the robot and/or from the way the robot operates. Moreover, this assumption is recurrent in the literature on control for robotic manipulators, for example (Berghuis & Nijmeijer, 1993; Nicosia & Tomei, 1990; Xian et al., 2004).

### 3. Controller-observers design

In this section we present the main results of this chapter, formulated in a lemma and two theorems and their proofs. Indeed, we want to solve the trajectory tracking problem of robot manipulators without using the velocity signal. This signal is reconstructed, firstly by a semi-globally stable velocity observer and secondly by a globally stable velocity observer.

#### 3.1 Semi-globally asymptotically stable observer

Consider the following velocity observer

$$\dot{z} = M^{-1} \left[ \tau - C(q, \dot{q})\dot{q} - G(q) \right] - L\dot{q} \quad (14)$$

$$\dot{\hat{q}} = z + Lq \quad (15)$$

Where  $\dot{\hat{q}}$  represents the estimated velocity,  $z$  is the observer state.  $L = lI_n$ , where  $l > 0$  and  $I_n \in R^{n \times n}$  is an identity matrix.

Lemma 3

If  $L_m > \frac{2C_m V_m}{M_m}$  then  $\lim_{t \rightarrow \infty} \dot{\tilde{q}} = 0$ , and the initial error  $\dot{\tilde{q}}(0)$  belongs to the ball  $B$  defined by:  $B = \left\{ \dot{\tilde{q}} \in R^n \mid \left\| \dot{\tilde{q}}(0) \right\| < \left( \frac{M_m L_m}{C_m} - V_m \right) \sqrt{\frac{M_m}{M_M}} \right\}$ . Where  $L_m$  denotes the minimum eigenvalue of  $L$  and  $\tilde{q} = \dot{q} - \dot{\hat{q}}$ .

Proof

The time-derivative of (15) gives us

$$\ddot{\tilde{q}} = M^{-1} \left[ \tau - C(q, \dot{\hat{q}}) \dot{\hat{q}} - G(q) \right] + L(\dot{q} - \dot{\hat{q}}). \quad (16)$$

From (1), we can write

$$\ddot{q} = M^{-1} \left[ \tau - C(q, \dot{q}) \dot{q} - G(q) \right]. \quad (17)$$

Subtracting (16) from (17), we have

$$\ddot{\tilde{q}} = M^{-1} \left[ -C(q, \dot{q}) \dot{q} + C(q, \dot{\hat{q}}) \dot{\hat{q}} \right] - L \dot{\tilde{q}}. \quad (18)$$

Using the property 5, we obtain

$$\ddot{\tilde{q}} = M^{-1} \left[ -2C(q, \dot{q}) \dot{\tilde{q}} + C(q, \dot{\tilde{q}}) \dot{\tilde{q}} \right] - L \dot{\tilde{q}}. \quad (19)$$

Consider the following Lyapunov function

$$V(\tilde{q}) = \frac{1}{2} \dot{\tilde{q}}^T M(q) \dot{\tilde{q}}. \quad (20)$$

Thus

$$\frac{1}{2} M_m \left\| \dot{\tilde{q}}(t) \right\|^2 \leq V(\tilde{q}(t)) \leq \frac{1}{2} M_M \left\| \dot{\tilde{q}}(t) \right\|^2. \quad (21)$$

The time-derivative of (20) gives us

$$\dot{V}(\tilde{q}) = \dot{\tilde{q}}^T M(q) \ddot{\tilde{q}} + \frac{1}{2} \dot{\tilde{q}}^T \dot{M}(q) \dot{\tilde{q}}. \quad (22)$$

From (19) and (22), we obtain

$$\dot{V}(\tilde{q}) = \tilde{q}^T \left( \frac{1}{2} \dot{M}(q) - C(q, \dot{q}) \right) \dot{\tilde{q}} + \tilde{q}^T \left( C(q, \dot{\tilde{q}}) - C(q, \dot{q}) \right) \dot{\tilde{q}} - \tilde{q}^T M L \tilde{q}. \quad (23)$$

Using the properties 1, 3, 4 and the assumption (13), we have

$$\dot{V} \leq - \left[ M_m L_m - C_m (V_m + \|\tilde{q}\|) \right] \|\tilde{q}\|^2. \quad (24)$$

If

$$L_m > \frac{C_m (V_m + \|\tilde{q}\|)}{M_m} \quad (25)$$

thus

$$L_m > \frac{2C_m V_m}{M_m} \quad (26)$$

then

$$\dot{V} \leq 0. \quad (27)$$

From (25), it comes

$$\|\tilde{q}\| < \frac{M_m L_m}{C_m} - V_m. \quad (28)$$

From (21), (24) and (28), it follows that if

$$\|\tilde{q}(0)\| < \left( \frac{M_m L_m}{C_m} - V_m \right) \sqrt{\frac{M_m}{M_M}}. \quad (29)$$

Then, we have a semi-global asymptotic stability.

**Remark 2.** It is important to observe that the region of attraction can be made arbitrarily large by increasing the observer gain  $L$ . As this region can be increased systematically by the gain  $L$ , we have semi-global asymptotic stability.

Now, we integrate this observer with a nonlinear controller and the semi-global asymptotic stability condition of the closed loop system is given in the following theorem.

*Theorem 1*

Given the robot dynamics (1), and let assumption (13) be satisfied. Under the following control law

$$\tau = M(q) \ddot{q}_d + C(q, \dot{q}) \dot{q}_d + G(q) + K_v (\dot{q}_d - \dot{\hat{q}}) + K_p (q_d - \hat{q}) \quad (30)$$

with  $\dot{\hat{q}}$  is given by (14) and (15). If

$$L_m > \frac{(C_m V_m + K_{vM})^2}{2K_{vm}} M_m^{-1} + 2C_m V_m M_m^{-1}. \quad (31)$$

Then, the closed-loop system is semi-globally asymptotically stable. Hence

$$\lim_{t \rightarrow \infty} E(t) = \lim_{t \rightarrow \infty} \dot{E}(t) = \lim_{t \rightarrow \infty} \ddot{\tilde{q}}(t) = 0. \quad (32)$$

Moreover a region of attraction is given by

$$B = \left\{ y \in R^{3n} \left\| y(0) \right\| < \sqrt{\frac{q_m}{q_M}} \left( \frac{M_m L_m}{C_m} - \frac{(C_m V_m + K_{vM})^2}{2K_{vm} C_m} - V_m \right) \right\}. \quad (33)$$

Where  $K_v$  and  $K_p$  are symmetric, positive definite matrices.  $E(t) = q_d(t) - q(t)$ ,  $\dot{E}(t) = \dot{q}_d(t) - \dot{q}(t)$ .  $y^T = \begin{bmatrix} \dot{E}^T & E^T & \ddot{\tilde{q}}^T \end{bmatrix}$ ,  $Q = \text{diag}\{M(q), K_p, M(q)\}$ ,  $q_m = \lambda_{\min}(Q) = \min\{M_m, K_{pm}\}$ ,  $q_M = \lambda_{\max}(Q) = \max\{M_M, K_{pM}\}$ ,  $K_{pM} = \|K_p\|$  and  $K_{vM} = \|K_v\|$ .  $L_m$ ,  $K_{pm}$  and  $K_{vm}$  denote the minimum eigenvalue of  $L$ ,  $K_p$  and  $K_v$ , respectively.

Proof

The analysis of asymptotic stability is in two parts. In the first part, we demonstrate that the closed-loop system is stable. In the second part, we demonstrate that it is semi-globally asymptotically stable.

Part 1

Let  $\ddot{\tilde{q}} = \dot{q}_d - \dot{\hat{q}} = \ddot{\tilde{q}} + \dot{E}$ .

From (18), we can write

$$M\ddot{\tilde{q}} + C(q, \dot{q})\dot{\tilde{q}} - C(q, \dot{\hat{q}})\dot{\hat{q}} + ML\dot{\tilde{q}} = 0. \quad (34)$$

Subtracting (1) from (30), we find

$$M\ddot{E} + C(q, \dot{\hat{q}})\dot{\hat{q}}_d - C(q, \dot{q})\dot{q} + K_v\dot{\tilde{q}} + K_p E = 0. \quad (35)$$

The sum of (34) and (35) gives us

$$M\ddot{\tilde{q}} + C(q, \dot{\hat{q}})\dot{\tilde{q}} + K_v\dot{\tilde{q}} + ML\dot{\tilde{q}} + K_p E = 0. \quad (36)$$



Thus

$$M\ddot{E} + \left(C(q, \dot{q}) + K_v\right)\dot{E} + K_p E = -M\ddot{\tilde{q}} - \left(C(q, \dot{q}) + K_v\right)\dot{\tilde{q}} - M\dot{L}\dot{\tilde{q}}. \quad (37)$$

From (34), (37) and using the property 5, we obtain

$$M\ddot{E} + \left(C(q, \dot{q}) + K_v\right)\dot{E} + K_p E = C(q, \dot{q})\dot{\tilde{q}} - K_v\dot{\tilde{q}}. \quad (38)$$

Consider the following Lyapunov function

$$H(\dot{E}, E, \dot{\tilde{q}}) = \frac{1}{2}\dot{E}^T M \dot{E} + \frac{1}{2}E^T K_p E + \frac{1}{2}\dot{\tilde{q}}^T M \dot{\tilde{q}}. \quad (39)$$

Hence

$$H(\dot{E}, E, \dot{\tilde{q}}) = \frac{1}{2}y(t)^T Q y(t). \quad (40)$$

It follows that

$$\frac{1}{2}q_m \|y(t)\|^2 \leq H(y(t)) \leq \frac{1}{2}q_M \|y(t)\|^2. \quad (41)$$

The time derivative of (39) evaluated along (34), (38) and using the properties 4 and 5, is

$$\dot{H} = -\dot{E}^T K_v \dot{E} - \dot{E}^T K_v \dot{\tilde{q}} + \dot{E}^T C(q, \dot{q})\dot{\tilde{q}} - \dot{\tilde{q}}^T M \dot{\tilde{q}} - \dot{\tilde{q}}^T C(q, \dot{q})\dot{\tilde{q}} + \dot{\tilde{q}}^T C(q, \dot{q})\dot{\tilde{q}}. \quad (42)$$

Using the properties 1, 3 and the assumption (13), we find

$$\dot{H} \leq -K_{vm} \|\dot{E}\|^2 - M_m L_m \|\dot{\tilde{q}}\|^2 + C_m \left(V_m + \|\dot{\tilde{q}}\|\right) \|\dot{\tilde{q}}\|^2 + \dot{E}^T C(q, \dot{q})\dot{\tilde{q}} - \dot{E}^T K_v \dot{\tilde{q}}. \quad (43)$$

Thus

$$\dot{H} \leq -K_{vm} \|\dot{E}\|^2 - \left(M_m L_m - C_m \left(V_m + \|\dot{\tilde{q}}\|\right)\right) \|\dot{\tilde{q}}\|^2 + \dot{E}^T C(q, \dot{q})\dot{\tilde{q}} - \dot{E}^T K_v \dot{\tilde{q}}. \quad (44)$$

We note that

$$\begin{aligned}
 \dot{E}^T C(q, \dot{q}) \ddot{q} - \dot{E}^T K_v \dot{\ddot{q}} &\leq 2 \|\dot{E}\| \left( \frac{C_m V_m + K_{vM}}{2} \right) \|\ddot{q}\| \\
 &\leq 2 \|\dot{E}\| \left( \frac{C_m V_m + K_{vM}}{2} \right) \left[ \frac{2}{M_m L_m - C_m (V_m + \|\ddot{q}\|)} \right]^{\frac{1}{2}} \left[ \frac{M_m L_m - C_m (V_m + \|\ddot{q}\|)}{2} \right]^{\frac{1}{2}} \|\ddot{q}\| \\
 &\leq \|\dot{E}\|^2 \left( \frac{C_m V_m + K_{vM}}{2} \right)^2 \left[ \frac{2}{M_m L_m - C_m (V_m + \|\ddot{q}\|)} \right] + \|\ddot{q}\|^2 \left[ \frac{M_m L_m - C_m (V_m + \|\ddot{q}\|)}{2} \right].
 \end{aligned} \quad (45)$$

From (44) and (45), we have

$$\dot{H} \leq - \left( K_{vm} - \frac{(C_m V_m + K_{vM})^2}{2 (M_m L_m - C_m (V_m + \|\ddot{q}\|))} \right) \|\dot{E}\|^2 - \frac{1}{2} (M_m L_m - C_m (V_m + \|\ddot{q}\|)). \quad (46)$$

$K_{vm}$  in (46) is chosen as follows

$$K_{vm} > \frac{(C_m V_m + K_{vM})^2}{2 (M_m L_m - C_m (V_m + \|\ddot{q}\|))}. \quad (47)$$

Since the assumption (13) is verified, we have

$$L_m > \frac{(C_m V_m + K_{vM})^2}{2 K_{vm}} M_m^{-1} + 2 C_m V_m M_m^{-1}. \quad (48)$$

Thus

$$\dot{H} \leq -\frac{1}{2} (M_m L_m - 2 C_m V_m) \|\ddot{q}\|^2. \quad (49)$$

If we choose  $L_m > 2 C_m V_m M_m^{-1}$  (it is verified by (48)), then  $\dot{H}$  is a negative semi-definite function, this result is not sufficient to demonstrate the asymptotic stability, and we can conclude only the stability of the system. Nevertheless, it is straightforward to verify that the equilibrium  $(E, \dot{E}, \ddot{q}) = (0, 0, 0)$  is the largest invariant set within the set  $\dot{H} = 0$ . Hence, using La Salle's invariance principle the asymptotic stability of the equilibrium can be proved. Therefore, one must insure that

$$\text{if } \ddot{q}=0 \quad \text{then } E=0 \text{ and } \dot{E}=0. \quad (50)$$

Part 2

When  $\dot{H} = 0$ , it is necessary that  $\ddot{q} = 0$ , in addition  $\ddot{\ddot{q}} = 0$ , therefore (38) will be

$$M\ddot{E} + (C(q, \dot{q}) + K_v)\dot{E} + K_p E = 0. \quad (51)$$

Choosing the following Lyapunov function candidate

$$W(\dot{E}, E) = \frac{1}{2} \dot{E}^T M \dot{E} + \frac{1}{2} E^T K_p E. \quad (52)$$

Using the property 4, the time-derivative of (52) is

$$\dot{W} = -\dot{E}^T K_v \dot{E}. \quad (53)$$

Hence

$$\dot{W} \leq -K_{vm} \|\dot{E}\|^2. \quad (54)$$

$\dot{W}$  is a negative semi-definite function, this result is not sufficient to demonstrate that  $E \rightarrow 0$ . Therefore, the Barbalat's lemma is required to complete the proof of asymptotic stability.

We note that, it is sufficient to show that  $\ddot{W}$  is bounded to conclude that  $\dot{W}$  is uniformly continuous. Indeed, the time-derivative of (53) is

$$\ddot{W} = -2\dot{E}^T K_v \ddot{E}. \quad (55)$$

From (48) and (49), we demonstrated the stability of the system ( $E$  and  $\dot{E}$  are bounded). In addition, from (51), we can conclude that  $\ddot{E}$  is bounded. Then  $W$  and  $\ddot{W}$  are bounded. This result implies that  $\dot{W}$  is uniformly continuous. Therefore, the Barbalat's lemma permits us to conclude that  $\dot{W} = 0$ , then  $\dot{E} = 0$ ,  $\ddot{E} = 0$ , and from (51) we find that  $E = 0$ . Finally, we demonstrated that (50) is verified. Hence, the La Salle's invariance principle is applied, consequently, the equilibrium  $(E, \dot{E}, \ddot{q}) = (0, 0, 0)$  is the largest invariant set within the set  $\dot{H} = 0$ . And the asymptotic stability of the equilibrium is proved.

Since  $\|y\| > \|\dot{\tilde{q}}\|$ , (47) holds if

$$\|y\| < \frac{M_m L_m}{C_m} - \frac{(C_m V_m + K_{vM})^2}{2K_{vm} C_m} - V_m. \quad (56)$$

From (41), (46) and (56), it follows that if

$$\|y(0)\| < \sqrt{\frac{q_m}{q_M}} \left( \frac{M_m L_m}{C_m} - \frac{(C_m V_m + K_{vM})^2}{2K_{vm} C_m} - V_m \right). \quad (57)$$

Then, the closed-loop system is semi-globally asymptotically stable. This completes the proof.

### 3.2 Globally asymptotically stable observer based controller

Now, a second observer is presented to reconstruct the velocity signal in the control law. Hence, the global asymptotic stability of the whole control system (robot plus controller plus observer) is guaranteed. This proof is based on Lyapunov theory and using saturation technique. This result is given in theorem 2.

*Theorem 2*

Given the robot dynamics (1), and let assumption (13) be satisfied. Under the following control law

$$\tau = M(q) \ddot{q}_d + C(q, \text{sat}(\dot{\tilde{q}})) \text{sat}(\dot{\tilde{q}}) + G(q) + K_v (\dot{q}_d - \dot{\tilde{q}}) + K_p E \quad (58)$$

with

$$\dot{\tilde{q}} = z + Lq \quad (59)$$

$$\dot{z} = \ddot{q}_d - L\dot{\tilde{q}} + M^{-1}K_p E. \quad (60)$$

If

$$\begin{aligned} 1. \quad L_m &\geq \frac{4\psi^2}{M_m K_{vm}} + 2(K_{vM} M_m^{-1}) \\ 2. \quad L_m &\geq 2 \frac{\psi}{M_m}. \end{aligned}$$

Then, the closed-loop system is globally asymptotically stable. Hence

$$\lim_{t \rightarrow \infty} E(t) = \lim_{t \rightarrow \infty} \dot{E}(t) = \lim_{t \rightarrow \infty} \dot{\tilde{q}}(t) = 0. \quad (61)$$

Where  $\text{sat}(V)$  represents the saturation for a vector  $V$ , this function is to be defined.  $K_{vM} = \|K_v\|$ .  $L_m$  and  $K_{vm}$  denote the minimum eigenvalue of  $L$  and  $K_v$  respectively.  $\psi$  is positive scalar constant such that  $\left\| \frac{\partial}{\partial \dot{q}} (C(q, \dot{q})\dot{q}) \right\| \leq \psi$ ,  $\forall (q, \dot{q}) \in R^n \times \bar{V}$ , with  $\bar{V} = \left\{ \dot{q} \in R^n \mid |\dot{q}_i| \leq V_i \quad i = 1, \dots, n \right\}$ .

*Proof*

Since assumption (13) holds, in the rest of the proof regard

$$\tau = M(q)\ddot{q} + C(q, \text{sat}(\dot{q}))\text{sat}(\dot{q}) + G(q) \quad (62)$$

as the given dynamic equation instead of (1).

Where the saturation for a vector  $V = [v_1, \dots, v_n]^T \in R^n$  is defined as

$$\text{Sat}(V) = [\text{sat}(v_1), \dots, \text{sat}(v_n)]^T \quad (63)$$

with

$$\text{sat}(v_i) = \begin{cases} v_i & \text{if } |v_i| \leq \bar{v}_i \\ \bar{v}_i & \text{if } v_i > \bar{v}_i \\ -\bar{v}_i & \text{if } v_i < -\bar{v}_i \end{cases} \quad \text{for } i \in \{1 \dots n\} \quad (64)$$

and  $V_m = \left\| [\text{sat}(v_1), \dots, \text{sat}(v_n)]^T \right\|$ .

From (58), (59) and (60) we can eliminate the state  $z$  and write

$$\ddot{\tilde{q}} = M^{-1} \left[ \tau - C(q, \text{sat}(\dot{\tilde{q}}))\text{sat}(\dot{\tilde{q}}) - G(q) - K_v \dot{\tilde{q}} - K_v \dot{E} \right] + L \tilde{q}. \quad (65)$$

From (62), we can write

$$\ddot{q} = M^{-1} [\tau - C(q, \text{sat}(\dot{q}))\text{sat}(\dot{q}) - G(q)]. \quad (66)$$

Subtracting (65) from (66), we obtain

$$M \ddot{\tilde{q}} = -C(q, \text{sat}(\dot{\tilde{q}}))\text{sat}(\dot{\tilde{q}}) + C(q, \text{sat}(\dot{q}))\text{sat}(\dot{q}) + K_v (\dot{E} + \dot{\tilde{q}}) - ML \tilde{q}. \quad (67)$$

Subtracting (62) from (58), we have

$$M\ddot{E} - C(q, \text{sat}(\dot{q}))\text{sat}(\dot{q}) + C(q, \text{sat}(\dot{\hat{q}}))\text{sat}(\dot{\hat{q}}) + K_v(\ddot{\hat{q}} + \dot{E}) + K_p E = 0. \quad (68)$$

Consider the Lyapunov function

$$H(\dot{E}, E, \dot{\hat{q}}) = \frac{1}{2} \dot{E}^T M \dot{E} + \frac{1}{2} E^T K_p E + \frac{1}{2} \dot{\hat{q}}^T M \dot{\hat{q}}. \quad (69)$$

The time-derivative of (69), evaluated along (67) and (68), is

$$\begin{aligned} \dot{H} = & \dot{E}^T \left( C(q, \text{sat}(\dot{q}))\text{sat}(\dot{q}) - C(q, \text{sat}(\dot{\hat{q}}))\text{sat}(\dot{\hat{q}}) \right) \\ & - \dot{\hat{q}}^T M L \dot{\hat{q}} + \dot{\hat{q}}^T \left( C(q, \text{sat}(\dot{\hat{q}}))\text{sat}(\dot{\hat{q}}) - C(q, \text{sat}(\dot{q}))\text{sat}(\dot{q}) \right) \\ & - \dot{E}^T K_v \dot{E} + \dot{\hat{q}}^T K_v \dot{\hat{q}}. \end{aligned} \quad (70)$$

To simplify the notation, let

$$D(q, \dot{q}) = C(q, \dot{q})\dot{q}. \quad (71)$$

Using (P2), it follows that  $\exists \psi > 0$  such that

$$\left\| \frac{\partial}{\partial \dot{q}} (D(q, \dot{q})) \right\| \leq \psi \quad \forall (q, \dot{q}) \in R^n \times \bar{V} \quad (72)$$

with

$$\frac{\partial}{\partial \dot{q}} (D(q, \dot{q})) = 2 \begin{pmatrix} \dot{q}^T N_1(q) \\ \vdots \\ \dot{q}^T N_n(q) \end{pmatrix} \quad (73)$$

and

$$\bar{V} = \left\{ \dot{q} \in R^n \mid |\dot{q}_i| \leq V_i \quad i = 1, \dots, n \right\}. \quad (74)$$

Then, using Lemma 1, we have

$$\dot{H} \leq -K_{vm} \|\dot{E}\|^2 - \left( M_m L_m - K_{vM} \right) \|\dot{\hat{q}}\|^2 + \psi \|\dot{\hat{q}}\|^2 + \psi \|\dot{E}\| \|\dot{\hat{q}}\|. \quad (75)$$

Hence

$$\dot{H} \leq -K_{vm} \|\dot{E}\|^2 - \left( \frac{M_m L_m}{2} - K_{vM} \right) \|\dot{\hat{q}}\|^2 - \left( \frac{M_m L_m}{2} - \psi \right) \|\dot{\hat{q}}\|^2 + \psi \|\dot{E}\| \|\dot{\hat{q}}\|. \quad (76)$$

We note that

$$\begin{aligned}
\psi \|\dot{E}\| \|\ddot{\tilde{q}}\| &\leq 2 \|\dot{E}\| \psi \|\ddot{\tilde{q}}\| \\
&\leq 2 \|\dot{E}\| \psi \left[ \frac{2}{\frac{M_m L_m}{2} - K_{vM}} \right]^{\frac{1}{2}} \left[ \frac{\frac{M_m L_m}{2} - K_{vM}}{2} \right]^{\frac{1}{2}} \|\ddot{\tilde{q}}\| \\
&\leq \|\dot{E}\|^2 \psi^2 \left[ \frac{2}{\frac{M_m L_m}{2} - K_{vM}} \right] + \|\ddot{\tilde{q}}\|^2 \left[ \frac{\frac{M_m L_m}{2} - K_{vM}}{2} \right].
\end{aligned} \tag{77}$$

Therefore, we have

$$\dot{H} \leq - \left( K_{vm} - \frac{4\psi^2}{M_m L_m - 2K_{vM}} \right) \|\dot{E}\|^2 - \frac{1}{2} \left( \frac{M_m L_m}{2} - K_{vM} \right) \|\ddot{\tilde{q}}\|^2 - \left( \frac{M_m L_m}{2} - \psi \right) \|\ddot{\tilde{q}}\|^2. \tag{78}$$

Choosing  $K_{vm}$  and  $L_m$  as follows

$$L_m \geq 2 \frac{\psi}{M_m} \tag{79}$$

and

$$K_{vm} \geq 4 \frac{\psi^2}{M_m L_m - 2K_{vM}}. \tag{80}$$

Hence

$$L_m \geq \frac{4\psi^2}{M_m K_{vm}} + 2(K_{vM} M_m^{-1}). \tag{81}$$

Then, we have

$$\dot{H} \leq - \left( K_{vm} - \frac{4\psi^2}{M_m L_m - 2K_{vM}} \right) \|\dot{E}\|^2 - \frac{1}{2} \left( \frac{M_m L_m}{2} - K_{vM} \right) \|\ddot{\tilde{q}}\|^2. \tag{82}$$

If we choose  $L_m > 2(K_{vM} M_m^{-1})$ , (it is verified by (81)), then  $\dot{H}$  is a negative semi-definite function, this result is not sufficient to demonstrate the asymptotic stability, and we can conclude only the stability of the system. Therefore, the lemma 2 is required to complete the proof of asymptotic stability.

In our case,  $H$  and  $\dot{H}$  are given by (69) and (70) respectively. To conclude that  $\dot{H}$  is uniformly continuous, it is sufficient to show that  $\ddot{H}$  is bounded. The time-derivative of (70) is

$$\begin{aligned}\ddot{H} = & \ddot{E}^T \left( C(q, \text{sat}(\dot{q})) \text{sat}(\dot{q}) - C(q, \text{sat}(\dot{\hat{q}})) \text{sat}(\dot{\hat{q}}) \right) \\ & - 2\ddot{\hat{q}}^T M L \ddot{\hat{q}} + \ddot{\hat{q}}^T \left( C(q, \text{sat}(\dot{\hat{q}})) \text{sat}(\dot{\hat{q}}) - C(q, \text{sat}(\dot{q})) \text{sat}(\dot{q}) \right) \\ & \left( \dot{E}^T + \ddot{\hat{q}}^T \left( \frac{d}{dt} (C(q, \text{sat}(\dot{q})) \text{sat}(\dot{q})) - \frac{d}{dt} (C(q, \text{sat}(\dot{\hat{q}})) \text{sat}(\dot{\hat{q}})) \right) \right) \\ & - 2\dot{E}^T K_v \ddot{E} + 2\ddot{\hat{q}}^T K_v \ddot{\hat{q}}.\end{aligned}\quad (83)$$

From (79), (80), (81) and (82), we demonstrated the stability of the system ( $E$ ,  $\dot{E}$  and  $\ddot{\hat{q}}$  are bounded). Therefore, from (69)  $H$  is bounded. In addition, from (67) and (68) we can conclude that  $\ddot{E}$  and  $\ddot{\hat{q}}$  are bounded, then  $\ddot{H}$  is bounded. This result implies that  $\dot{H}$  is uniformly continuous. Hence, the Barbalat's lemma permits us to conclude that  $\dot{H} = 0$ . Thus, from (82), we have  $\dot{E} = 0, \ddot{\hat{q}} = 0$ , and necessary  $\ddot{E} = 0, \ddot{\hat{q}} = 0$ . Finally from (67) and (68) we find that  $E = 0$ .

Then, the closed-loop system is globally asymptotically stable. This completes the proof.

#### 4. Simulation results

In order to illustrate by simulation the efficiency of our design, we apply in this section the observer-controller laws (55-60) on two-link robot manipulator. The objective of our simulation work is to show that the tracking objective is achieved when an estimated velocity vector is used in the tracking control law.

Consider a two-link manipulator with masses  $m_1, m_2$ , lengths  $l_1, l_2$ , and angles  $q_1, q_2$ ; then the model equations can be written as (1).  $M(q)$ ,  $C(q, \dot{q})$  and  $G(q)$  are given by (Bouakrif et al., 2008):

$$m_{11} = m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2) + (m_1 + m_2) l_1^2, m_{12} = m_{21} = m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2), m_{22} = m_2 l_2^2.$$

$$C_{11} = -m_2 l_1 l_2 \sin(q_2) \dot{q}_2, C_{12} = -m_2 l_1 l_2 \sin(q_2) \dot{q}_2, C_{21} = m_2 l_1 l_2 \sin(q_2) \dot{q}_1, C_{22} = 0.$$

$$G_1 = m_2 l_2 g \cos(q_1 + q_2) + (m_1 + m_2) l_1 g \cos(q_1), G_2 = m_2 l_2 g \cos(q_1 + q_2).$$

The desired trajectories are chosen as:

$$q_{d1}(t) = 2 \cos(4\pi t/3) + \sin(2\pi t/3) \text{ (rad)}, \text{ with } 0 \leq t \leq 5.$$

$$q_{d2}(t) = 1 - 2 \cos(4\pi t/3) - \sin(2\pi t/3) \text{ (rad)}, \text{ with } 0 \leq t \leq 5.$$

Simulation parameters:

$$K_p = \{5000, 5000\}, K_v = \{15, 15\}, L = \{550, 550\},$$

$$m_1 = 0.5[\text{kg}], m_2 = 0.7[\text{kg}], l_1 = 1[\text{m}], l_2 = 1.5[\text{m}].$$

Therefore, we find that  $V_m = 10[\text{rad/s}]$ ,  $M_m = 1[\text{kgm}^2]$ .

The simulation results of the proposed scheme on two-link robot manipulator along a trajectory are shown below. Figure 1 show the observer result, where we can see the convergence of the observed velocity to real velocity, of each joint, in a minimum time.



Figure 2 show the simulation results for real and desired position trajectories, of each joint, when the velocity given by the observer (59) and (60) is used in the control law (58). We can see that the real trajectory follows the desired trajectory without error through time axis. Therefore, it is clear that the control algorithm works well.

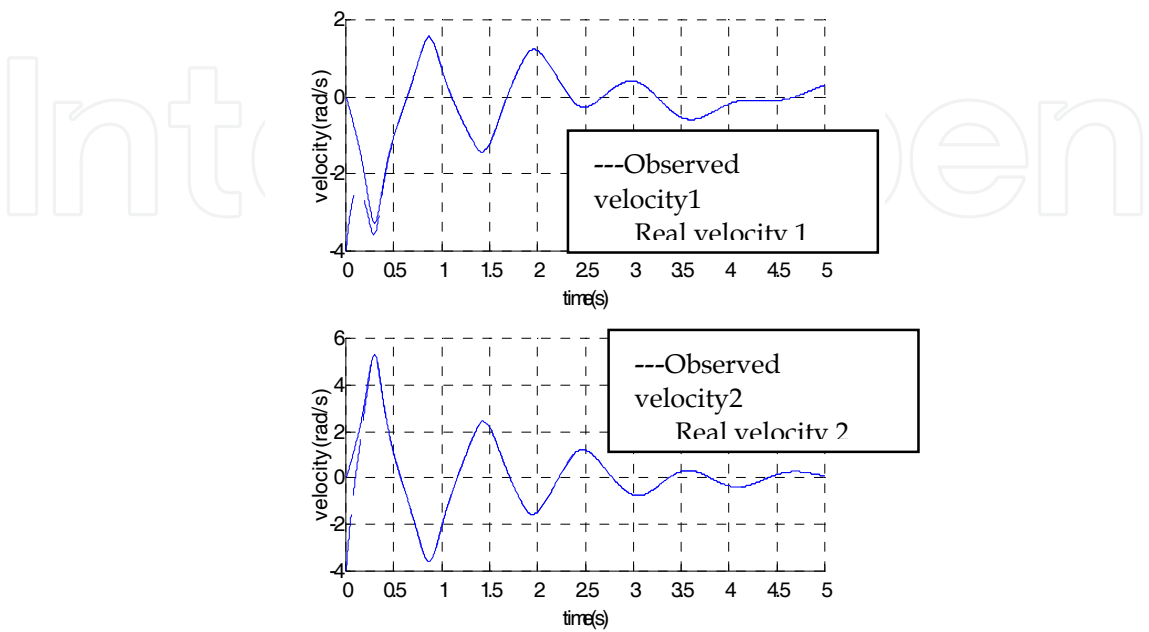


Fig.1. Real and observed velocities of two-link manipulator.

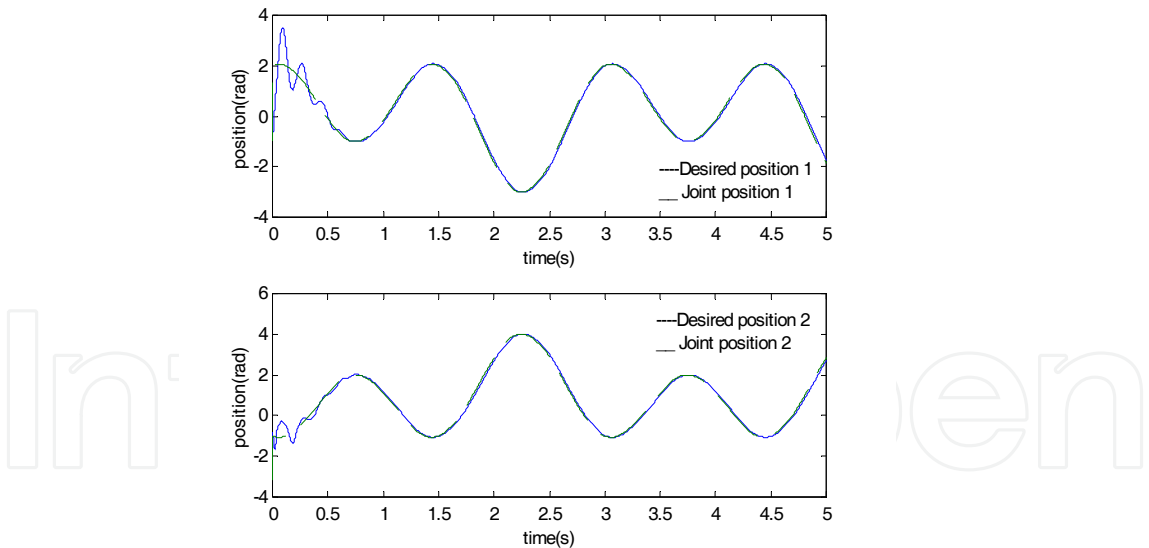


Fig.2. Real and desired position trajectories of two-link manipulator.

5. Conclusion

This chapter has presented two motion control schemes to solve the trajectory tracking problem of rigid-link robot manipulators, when the manipulator’s joint velocities cannot be measured by the control system. The necessity of velocity measurements in the controllers can be removed by replacing the actual velocity signal by an estimate obtained from two

observer systems. The whole control system consisting of robot manipulator, controller and the first observer is semi-globally asymptotically stable and a region of attraction is also given. Using the second observer, the global asymptotic stability of the closed loop system is guaranteed. Hence, there is more freedom to choose the initial states. These proofs are based on Lyapunov theory. Finally, simulation results on two-link manipulator are provided to illustrate the effectiveness of the global velocity observer based trajectory tracking control.

## 6. References

- Berghuis, H. & Nijmeijer, H. (1993). Global regulation of robots using only position measurements, *Syst. and Contr. Lett.*, 21, 1993, pp. 289-293, ISSN 0167-6911.
- Berghuis, H. (1993). Model based control: from theory to practice, Ph.D. thesis, University of Twent, ISBN 90-9006110-X, the Netherlands.
- Bouakrif, F.; Boukhetala, D. & Boudjema, F. (2007). Iterative learning control for robot manipulators, *Archives of Control Sciences*, Vol. 17, No. 1, 2007, pp. 57-69.
- Bouakrif, F.; Boukhetala, D. & Boudjema, F. (2008). Global asymptotic stability of controller-observer for robot manipulators using saturation technique, *The Mediterranean Journal of Measurement and Control*, Vol. 4, No. 1, 2008, ISSN 1743-9310.
- Bouakrif, F.; Boukhetala, D. & Boudjema, F. (2010). Passivity based controller-observer for robot manipulators, *Int. J. Robotics and Automation*, Vol. 25, No. 1, 2010, ISSN 0826-8185.
- Canudas de wit, C.; Fixot, N. & Aström, K.J. (1992). Trajectory tracking in robot manipulators via nonlinear estimated state feedback, *IEEE Trans. Robot. Automat.*, Vol. 8, No. 1, 1992, pp. 138-144, ISSN 1552-3098.
- Chang, Y.C. & Chen, B.S. (2000). Robust tracking designs for both holonomic and nonholonomic constrained mechanical systems: adaptive fuzzy approach, *IEEE Trans. Fuzzy Syst.*, Vol. 8, 2000, pp. 46-66, ISSN 1063-6706.
- Kelly, R. (1995). A tuning procedure of PID control for robot manipulators. *Robotica*, Vol. 13, 1995, pp. 141-148, ISSN 0263-5747.
- Khosla, P.K. & Kanade, T. (1988). Experimental evaluation of nonlinear feedback and feedforward control schemes for manipulators, *Int. J. Robot. Res.*, Vol. 7, 1988, pp. 18-28, ISSN 0278-3649.
- Luh, J.Y.S.; Walker, M.W. & Paul, R.C.P. (1980). Resolved-acceleration control of mechanical manipulators, *IEEE Trans. on Automatic Control*, AC-25(3), pp. 468-474, 1980, ISSN 0018-9286.
- Nicosia, S. & Tomei, P. (1990). Robot control by using only joint position measurements, *IEEE Trans. Automat. Contr.*, Vol. 35, No. 9, 1990, pp. 1058-1061, ISSN 0018-9286.
- Ortega, R. & Spong, M.W. (1989). Adaptive motion control of rigid robots: a tutorial, *Automatica*, Vol. 25, no. 6, 1989, pp. 877-888, ISSN 0005-1098.
- Paden, B & Panja, R. (1988). Globally asymptotically stable 'PD+' controller for robot manipulators, *Int. J. Control*, Vol. 47, No. 6, 1988, pp. 1697-1712, ISSN 0020-7179.
- Ryu, J.H.; Kwon, D.S. & Hannaford, B. (2004). Stable teleoperation with time domain passivity control, *IEEE Trans. Robot. Automat.*, Vol. 20, 2004, pp. 365-373, ISSN 1552-3098.
- Shim, H.; Son, Y. & Seo, J. (2001). Semi-global observer for multi-output nonlinear systems, *Systems and Control Letters*, Vol. 42, No. 3, 2001, pp. 233-44, ISSN 0167-6911.

- Slotine, J.J.E. & Sastry, S.S. (1983). Tracking control of nonlinear systems using sliding surface with application to robot manipulators, *Int. J. Control*, Vol. 38, 1983, pp. 465-492, ISSN 0020-7179.
- Slotine, J.J.E & Li, W. (1991). *Applied Nonlinear Control*", Prentice-Hall. Englewood Cliffs, 1991, ISBN: 0-13-040890-5.
- Tayebi A. (2007). Analysis of two particular iterative learning control schemes in frequency and time domains, *Automatica*, Vol. 43, 2007, pp. 1565-1572, ISSN 0005-1098.
- Xian, B.; Queiroz, M.; Dawson, D. & McIntyre, M. (2004). A discontinuous output feedback controller and velocity observer for nonlinear mechanical systems, *Automatica*, Vol. 40, No. 4, 2004, pp. 695-700, ISSN 0005-1098.
- Yu, W. & Li, X. (2006). PD Control of robot with velocity estimation and uncertainties compensation, *Int. J. of Robot. and Aut.*, Vol. 21, no. 1, 2006, ISSN 0826-8185.

IntechOpen



## **Robot Manipulators New Achievements**

Edited by Aleksandar Lazinica and Hiroyuki Kawai

ISBN 978-953-307-090-2

Hard cover, 718 pages

**Publisher** InTech

**Published online** 01, April, 2010

**Published in print edition** April, 2010

Robot manipulators are developing more in the direction of industrial robots than of human workers. Recently, the applications of robot manipulators are spreading their focus, for example Da Vinci as a medical robot, ASIMO as a humanoid robot and so on. There are many research topics within the field of robot manipulators, e.g. motion planning, cooperation with a human, and fusion with external sensors like vision, haptic and force, etc. Moreover, these include both technical problems in the industry and theoretical problems in the academic fields. This book is a collection of papers presenting the latest research issues from around the world.

### **How to reference**

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Farah Bouakrif (2010). Trajectory Tracking Control for Robot Manipulators with no Velocity Measurement Using Semi-Globally and Globally Asymptotically Stable Velocity Observers, Robot Manipulators New Achievements, Aleksandar Lazinica and Hiroyuki Kawai (Ed.), ISBN: 978-953-307-090-2, InTech, Available from: <http://www.intechopen.com/books/robot-manipulators-new-achievements/trajectory-tracking-control-for-robot-manipulators-with-no-velocity-measurement-using-semi-globally->

**INTECH**  
open science | open minds

### **InTech Europe**

University Campus STeP Ri  
Slavka Krautzeka 83/A  
51000 Rijeka, Croatia  
Phone: +385 (51) 770 447  
Fax: +385 (51) 686 166  
[www.intechopen.com](http://www.intechopen.com)

### **InTech China**

Unit 405, Office Block, Hotel Equatorial Shanghai  
No.65, Yan An Road (West), Shanghai, 200040, China  
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元  
Phone: +86-21-62489820  
Fax: +86-21-62489821

© 2010 The Author(s). Licensee IntechOpen. This chapter is distributed under the terms of the [Creative Commons Attribution-NonCommercial-ShareAlike-3.0 License](https://creativecommons.org/licenses/by-nc-sa/3.0/), which permits use, distribution and reproduction for non-commercial purposes, provided the original is properly cited and derivative works building on this content are distributed under the same license.

IntechOpen

IntechOpen