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ROBUST AND REDUCED ORDER H-INFINITY FILTERING VIA LMI APPROACH AND ITS APPLICATION TO FAULT DETECTION

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1. Introduction

Filters that estimate the state variables of a system are important tools for control and signal processing applications. Early work in the area assumed that the system dynamics were known and external disturbances were white noise with known statistical properties. In contrast to traditional Kalman filters, H_∞ filters do not require knowledge of the statistical properties of the noise. H_∞ filters are more robust to disturbances and modeling uncertainties than Kalman filters. Thus, in practical applications where disturbances may not be known exactly and system uncertainties may appear in modeling, the H_∞ technique is often used (Fu et al., 1994).

It is important to consider filter order when fast data processing is necessary. The reduced order filter is often desirable because it reduces the filter complexity and real time computational burdens in many applications. In (Grigoriadis & Watson, 1997) the reduced order H_∞ filtering problem was studied via an LMI (Linear Matrix Inequality) approach, but only for a specific linear time invariant plant model without model uncertainties. In (Bettayeb & Kavranoglu, 1994) the reduced order filter problem was studied in an H_∞ setting, but the H_∞ problem was formulated as distance problem.

Because a wide variety of problems arising in system and control theory can be reduced to an optimization problem involving LMIs and LMIs can be solved numerically very efficiently, LMIs have been used extensively in the controls field. In (Tuan et al., 2000) the robust reduced order filtering problem was studied in an H_2 setting via an LMI approach. In this study, the H_∞ approach is used because it is known that H_∞ approach is more robust to model uncertainties than H_2 (Kim & Watkins, 2006). Less conservative results can be achieved, and a computational example is given to show this.

This filtering technique can be used for fault detection filter design. As science and technology develops, the reliability and security of complex systems becomes more important. Thus, on-line monitoring of faults as they occur during operation of a dynamic

system is necessary. In this study, estimator based fault detection methods will be the focus. The key to estimator based fault detection is to generate a fault indicating signal (residual) using input and output signals from the monitored system (Chen & Zhang, 1991). However, there is always a model-reality mismatch between plant dynamics and the model used for the residual generation (Chen & Patton, 1997). The robustness of residual depends on its fault sensitivity. The residual should be sensitive to faults but insensitive to modeling uncertainties and disturbances (Zhong et al., 2003). To produce the residual signal, an observer is usually used. In the fault detection literature this observer is often called a fault detection filter to emphasize the relationship with the filtering concept. In this study, robust fault detection filter (RFDF) design is formulated as a multi-objective H_∞ optimization for a polytopic uncertain system.

In (Casavola et al., 2005a), RFDF design was formulated as a multi-objective H_∞ optimization only for the full order case. In (Casavola et al., 2005b), RFDF was formulated as a quasi-LMI only for the full order case. In this study, the order of the RFDF is reduced using LMI techniques and the detection performance is compared with the full order filter (Kim & Watkins, 2007).

This paper is organized as follows: In Section 2, notations are introduced. In Section 3, the preliminary and main results for the H_∞ filter design are given. In Section 4, the preliminary and main results for the fault detection filter design are given. Numerical examples of H_∞ filter design and the fault detection filter design are shown in Section 5. Concluding remarks can be found in Section 6.

2. Notation

The notation that is used here is quite standard. R is the field of real numbers, R^n is a real vector with dimension n and $R^{m \times n}$ is a real matrix with dimensions $m \times n$. RH_∞ is the subspace of L_∞ with real and rational functions that are analytic and bounded in the open right-half plane, where L_∞ is the set of functions bounded on $j\omega$ -axis including $\omega = \infty$.

B^T is the transpose of matrix B. Symbol, *, stands for terms that are induced by symmetry, e.g.,

$$\begin{bmatrix} S + (*) & * \\ M & Q \end{bmatrix} K^T \equiv \begin{bmatrix} S + S^T & M^T \\ M & Q \end{bmatrix} K^T$$

BRL stands for Bounded Real Lemma, which is standard in robust control theory (Gahinet et al., 1996).

3. H_∞ Filter design

3.1 Preliminary result

The general filtering configuration can be depicted as in Figure 1 where G is the plant, F is the filter that will be designed, d is an uncertain disturbance that includes process and measurement noise, z is the signal to be estimated, z_F is the estimate of z, e is the estimation error, and y is the measured output.

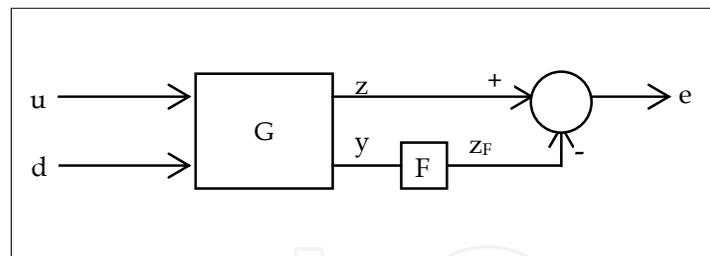


Fig. 1. General filtering configuration

A linear time invariant plant (G) described by

$$G : \begin{cases} \dot{x}(t) = Ax(t) + Bd(t) \\ y(t) = Cx(t) + Dd(t) \\ z(t) = Lx(t) \end{cases} \quad (1)$$

will be considered where $x(t) \in R^n$ is the state, $y(t) \in R^p$ is the measured output, $z(t) \in R^q$ is the estimated output, $d(t) \in R^m$ is the disturbance, and A, B, C, D and L are of appropriate dimensions. To include plant model uncertainties, we formulate them in the form of a polytopic model as follows.

$$\begin{bmatrix} A & B \\ C & D \\ L & 0 \end{bmatrix} \in \left\{ \begin{bmatrix} A(\alpha) & B(\alpha) \\ C(\alpha) & D(\alpha) \\ L(\alpha) & 0 \end{bmatrix} = \sum_{i=1}^s \alpha_i \begin{bmatrix} A_i & B_i \\ C_i & D_i \\ L_i & 0 \end{bmatrix}, \alpha \in \Gamma \right\} \quad (2)$$

where Γ is a unit simplex such that

$$\Gamma \equiv \left\{ (\alpha_1, \dots, \alpha_s) : \sum_{i=1}^s \alpha_i = 1, \alpha_i \geq 0 \right\} \quad (3)$$

The formulation is a convex combination, so it is suitable for the LMI approach. This convex bounded polytopic mathematical description of model uncertainty is sufficiently general to include many uncertain systems with practical appeal. The filter F is attached to the system as follows:

$$F : \begin{cases} \dot{x}_F(t) = A_F x_F(t) + B_F y(t) \\ z_F(t) = L_F x_F(t) \end{cases} \quad (4)$$

where $x_F(t) \in R^k$ is the filter state ($k \leq n$) and $A_F \in R^{k \times k}$, $B_F \in R^{k \times p}$ and $L_F \in R^{q \times k}$ are the filter matrices that are to be synthesized.

Definition 3.1:

The L_2 norm of a vector valued function $f(t)$ is defined:

$$\|f\|_{L_2} = \sqrt{\int_0^{\infty} f^T(t)f(t)dt} \quad \blacksquare \quad (5)$$

The goal of the H_{∞} optimal filtering problem is to find a filter F to minimize the worst case estimation error energy $\|e\|_{L_2}$ over all bounded energy disturbance d , where $e = z - z_F$, that is

$$\min_F \sup_{d \in L_2 - \{0\}} \frac{\|e\|_{L_2}}{\|d\|_{L_2}} \quad (6)$$

Using the induced L_2 -gain property of the H_{∞} norm, this problem is equivalent to the following H_{∞} norm minimization problem

$$\min_F \|T_{ed}\|_{\infty} \quad (7)$$

where T_{ed} is the transfer function from disturbance d to the estimation error e and the H_{∞} norm is defined as the largest gain over all frequencies such that

$$\|T_{ed}(s)\|_{\infty} \equiv \sup_{\omega} \sigma_{\max}(T_{ed}(j\omega)) \quad (8)$$

where σ_{\max} denotes maximum singular value of the given function. The γ -suboptimal H_{∞} filtering problem is to find a filter F such that

$$\|T_{ed}\|_{\infty} < \gamma \quad (9)$$

where γ is a given positive scalar.

To find the transfer function T_{ed} , (1) and (4) can be rewritten in augmented form as (10) and (11).

$$\dot{x}_{cl} = \begin{bmatrix} \dot{x} \\ \dot{x}_F \end{bmatrix} = \begin{bmatrix} Ax + Bd \\ A_F x_F + B_F y \end{bmatrix} = A_{cl} x_{cl} + B_{cl} d \quad (10)$$

$$e = z_{cl} = z - z_F = L_{cl} x_{cl} \quad (11)$$

where

$$x_{cl} \equiv \begin{bmatrix} x \\ x_F \end{bmatrix}, \quad A_{cl} \equiv \begin{bmatrix} A & 0 \\ B_F C & A_F \end{bmatrix}, \quad B_{cl} \equiv \begin{bmatrix} B \\ B_F D \end{bmatrix}, \quad \text{and} \quad L_{cl} \equiv [L \quad -L_F] \quad (12)$$

The transfer function of closed system, T_{ed} , can be found as:

$$\begin{aligned} e &\equiv z_{cl} = L_{cl}x_{cl} \\ &= L_{cl}(sI - A_{cl})^{-1}B_{cl}d \equiv T_{ed}d \end{aligned} \quad (13)$$

where

$$T_{ed} \equiv L_{cl}(sI - A_{cl})^{-1}B_{cl} \quad (14)$$

Using the well-known BRL (Gahinet & Apkarian, 1994), the condition in (9) with (14) can be described as:

$$\begin{bmatrix} PA_{cl} + A_{cl}^T P & PB_{cl} & L_{cl}^T \\ B_{cl}^T P & -\gamma I & 0 \\ L_{cl} & 0 & -\gamma I \end{bmatrix} < 0 \quad (15)$$

where A_{cl}, B_{cl} and L_{cl} are given in (12) and P is a symmetric positive definite matrix variable. Equation (15) satisfies the design requirement as follows:

- a) internal stability: for $d=0$, the state vector x_{cl} of closed loop system tends to zero as time goes to infinity.
- b) performance: The H_∞ norm, $\|T_{ed}\|_\infty$, is less than specified positive scalar γ .

Statement (a) implies quadratic stability and (b) implies quadratic performance. Remember that A_{cl}, B_{cl} and L_{cl} depend on parameter α . Thus, statement (a) should be replaced by quadratic stability in the sense of parameter dependency and statement (b) replaced by quadratic performance in the sense of parameter dependency.

Definition 3.2 (Gahinet et al., 1996):

The system of (10) and (11) is said to have AQS (affine quadratic stability) if there exist a positive symmetric affine-parameter dependent Lyapunov matrix

$$P(\alpha) = \sum_{i=1}^s \alpha_i P_i \quad (16)$$

where $P_i, (i = 1, 2, \dots, s)$ are symmetric, such that

$$A_{cl}(\alpha)^T P(\alpha) + P(\alpha) A_{cl}(\alpha) < 0, \forall \alpha \in \Gamma \quad (17)$$

Definition 3.3 (Gahinet et al., 1996):

The system of (10) and (11) is said to have AQP (affine quadratic H_∞ performance) if there exists a positive symmetric affine-parameter dependent Lyapunov matrix (16) such that

$$\begin{bmatrix} A_{cl}(\alpha)^T P(\alpha) + P(\alpha) A_{cl}(\alpha) & P(\alpha) B_{cl}(\alpha) & L_{cl}(\alpha)^T \\ B_{cl}(\alpha)^T P(\alpha) & -\gamma I & 0 \\ L_{cl}(\alpha) & 0 & -\gamma I \end{bmatrix} < 0 \quad (18)$$

that holds for all admissible parameter $\alpha_i (i = 1, 2, \dots, s)$

3.2 Main result

The reduced-order H_2 filtering problem was studied in (Nagpal et al., 1987), and the reduced-order H_∞ filtering problem for discrete time systems was studied in (O'Brien & Kiriakidis, 2004), but in this section we will develop an H_∞ filtering approach in LMI framework that is applicable and easy to compute. Let's reconsider (18). The matrix inequality in (18) is nonlinear because the terms containing the P , A_{cl} and B_{cl} variables multiply the unknown variable P with the unknowns that are included in A_{cl} and B_{cl} . Finsler's Lemma is a standard tool to separate filter variables from the Lyapunov matrix P (Tuan et al., 2000). The LMI formulation in Theorem 3.4 is very useful for solving the robust filtering problem.

Theorem 3.4 (Tuan et al., 2000):

The LMI

$$P > 0, \begin{bmatrix} A^T P + PA & PB & C^T \\ B^T P & Q_{11} & Q_{12} \\ C & Q_{12}^T & Q_{22} \end{bmatrix} < 0 \quad (19)$$

has a feasible decision variable P if and only if, for any choice of $\mu > 0$, the following LMI is feasible in the decision variables V and P ,

$$\begin{bmatrix} -(V+V)^T & V^T A + P & V^T B & 0 & V^T \\ A^T V + P & -\mu P & 0 & C^T & 0 \\ B^T V & 0 & Q_{11} & Q_{22} & 0 \\ 0 & C & Q_{12}^T & Q_{22} & 0 \\ V & 0 & 0 & 0 & -P/\mu \end{bmatrix} < 0 \quad (20)$$

Proof: refer to (Tuan et al., 2000) ■

From **Theorem 3.4**, (18) can be written as

$$\begin{bmatrix} -(V+V^T) & V^T A_{cl} + P & V^T B_{cl} & 0 & V^T \\ A_{cl}^T V + P & -\mu P & 0 & L_{cl}^T & 0 \\ B_{cl}^T V & 0 & -\gamma I & 0 & 0 \\ 0 & L_{cl} & 0 & -\gamma I & 0 \\ V & 0 & 0 & 0 & -P/\mu \end{bmatrix} < 0 \quad (21)$$

The parameter α is omitted. The slack variable V has been introduced to separate the variable P from the filter design variables.

To proceed further, the V and P variables should be partitioned as

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \quad (22)$$

$$P = \begin{bmatrix} P_1 & P_3^T \\ P_3 & P_2 \end{bmatrix} \quad (23)$$

Because the reduced order case is being considered, the filter order k , will be less than or equal to the plant order n . The partitioned sub-matrices in (22) will be dimensionalized as $V_{11}(n \times n)$, $V_{12}(n \times k)$, $V_{21}(k \times n)$, and $V_{22}(k \times k)$. Now, we need to enforce some special structure on V_{21} as

$$V_{21} = \begin{bmatrix} \tilde{V}_{21} & \mathbf{0}_{k \times (n-k)} \end{bmatrix} \quad (24)$$

where \tilde{V}_{21} is a $k \times k$ matrix. We can replace V, P, A_{cl}, B_{cl} and L_{cl} in (21) with (12), (22), (23) and (24). After that, we perform a congruence transformation with the transformation matrix

$$\text{diag} \left[I \quad V_{22}^{-1} \tilde{V}_{21} \quad I \quad V_{22}^{-1} \tilde{V}_{21} \quad I \quad I \quad I \quad V_{22}^{-1} \tilde{V}_{21} \right] \quad (25)$$

This yields (26)

$$\begin{bmatrix} -(V_{11} + V_{11}^T) & -(S_2^T + \tilde{S}_1^T) & V_{11}^T A + \tilde{B}_F C + \hat{P}_1 & \tilde{A}_F + \hat{P}_3^T & V_{11}^T B + \tilde{B}_F D & 0 & V_{11}^T & \tilde{S}_1^T \\ * & -(S_1 + S_1^T) & S_2 A + \hat{B}_F C + \hat{P}_3 & \hat{A}_F + \hat{P}_2 & S_2 B + \hat{B}_F D & 0 & S_2 & S_1^T \\ * & * & -\mu \hat{P}_1 & -\mu \hat{P}_3^T & 0 & L^T & 0 & 0 \\ * & * & * & -\mu \hat{P}_2 & 0 & -\hat{L}_F^T & 0 & 0 \\ * & * & * & * & -\gamma I & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma I & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_1 & -\frac{1}{\mu} \hat{P}_3^T \\ * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_2 \end{bmatrix} < 0 \quad (26)$$

where

$$S_1 = \tilde{V}_{21}^T V_{22}^{-T} \tilde{V}_{21} \quad (27)$$

$$\tilde{S}_1 = \begin{bmatrix} S_1 & \mathbf{0}_{k \times (n-k)} \end{bmatrix} \quad (28)$$

$$S_2 = \tilde{V}_{21}^T V_{22}^{-T} V_{12}^T \quad (29)$$

$$\hat{B}_F = \tilde{V}_{21}^T B_F \quad (30)$$

$$\tilde{B}_F = \begin{bmatrix} \hat{B}_F \\ 0_{(n-k) \times p} \end{bmatrix} \quad (31)$$

$$\hat{P}_1 = P_1 \quad (32)$$

$$\hat{P}_2 = \tilde{V}_{21}^T V_{22}^{-T} P_2 V_{22}^{-1} \tilde{V}_{21} \quad (33)$$

$$\hat{P}_3^T = P_3^T V_{22}^{-1} \tilde{V}_{21} \quad (34)$$

$$\hat{A}_F = \tilde{V}_{21}^T A_F V_{22}^{-1} \tilde{V}_{21} \quad (35)$$

$$\tilde{A}_F = \begin{bmatrix} \hat{A}_F \\ 0_{(n-k) \times k} \end{bmatrix} \quad (36)$$

$$\hat{L}_F = L_F V_{22}^{-1} \tilde{V}_{21} \quad (37)$$

$$\hat{P} = \begin{bmatrix} \hat{P}_1 & \hat{P}_3^T \\ \hat{P}_3 & \hat{P}_2 \end{bmatrix} \quad (38)$$

Remark 3.5:

The H_∞ filtering solvability condition in (18) is reformulated as the feasibility problem of (26) with respect to \hat{A}_F , \hat{B}_F , \hat{L}_F , S_1 , S_2 , V_{11} , \hat{P} and γ where $\hat{P} > 0$. ■

Remark 3.6:

The filter matrices A_F , B_F and L_F can be derived by means of the following procedure.

- (i) Compute V_{22} and \tilde{V}_{21} by solving the factorization problem

$$S_1 = \tilde{V}_{21}^T V_{22}^{-1} \tilde{V}_{21} \quad (39)$$

- (ii) Compute A_F , B_F and L_F

$$A_F = \tilde{V}_{21}^{-T} \hat{A}_F \tilde{V}_{21}^{-1} V_{22} \quad (40)$$

$$B_F = \tilde{V}_{21}^{-T} \hat{B}_F \quad (41)$$

$$L_F = \hat{L}_F \tilde{V}_{21}^{-1} V_{22} \quad \blacksquare \quad (42)$$

Now, we need to remember that we had a polytopic uncertain system. As explained in (Tuan et al., 2000), a parameter dependent Lyapunov matrix $P(\alpha)$, such as

$$P(\alpha) = \sum_{i=1}^s \alpha_i P_i(\alpha), \alpha \in \Gamma \quad (43)$$

is symmetric positive definite for all admissible values of α , if and only, if this holds for each P_i . Therefore, we need to check the solvability of (18) only at the vertices, $i = 1, 2, \dots, s$. This gives us the final form (44) of the LMI (26).

$$\begin{bmatrix}
 -(V_{11} + V_{11}^T) & -(S_2^T + \tilde{S}_1^T) & V_{11}^T A_i + \tilde{B}_F C_i + \hat{P}_{1,i} & \tilde{A}_F + \hat{P}_{3,i}^T & V_{11}^T B_i + \tilde{B}_F D_i & 0 & V_{11}^T & \tilde{S}_1^T \\
 * & -(S_1 + S_1^T) & S_2 A_i + \hat{B}_F C_i + \hat{P}_{3,i} & \hat{A}_F + \hat{P}_{2,i} & S_2 B_i + \hat{B}_F D_i & 0 & S_2 & S_1^T \\
 * & * & -\mu \hat{P}_{1,i} & -\mu \hat{P}_{3,i}^T & 0 & L_i^T & 0 & 0 \\
 * & * & * & -\mu \hat{P}_{2,i} & 0 & -\hat{L}_F^T & 0 & 0 \\
 * & * & * & * & -\gamma I & 0 & 0 & 0 \\
 * & * & * & * & * & -\gamma I & 0 & 0 \\
 * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_{1,i} & -\frac{1}{\mu} \hat{P}_{3,i}^T \\
 * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_{2,i}
 \end{bmatrix} < 0 \quad (44)$$

($i = 1, 2, \dots, s$)

Consequently, the minimum upper bound γ for the reduced order H_∞ filter can be found by solving the LMI optimization problem

$$\min_{V_{11}, S_1, S_2, A_F, B_F, L_F, \gamma, \hat{P}_i} \{ \gamma : (44) \} \quad (45)$$

In summary, the γ -suboptimal H_∞ reduced order filter for polytopic uncertain system can be solved if and only if (44) holds for all vertices, $i = 1, 2, \dots, s$ and the minimum value can be found from (45). Also, the triple (A_F, B_F, L_F) defining the k_{th} order filter is obtained from (i) and (ii) in *Remark 3.6*.

4. Fault detection filter design

4.1 Preliminary result

Let's consider the following uncertain continuous time linear system described by

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + B_u u(t) + B_f f(t) + B_d d(t) \\ y(t) = Cx(t) + D_u u(t) + D_f f(t) + D_d d(t) \end{cases} \quad (46)$$

where $x(t) \in R^n$ is the state, $y(t) \in R^m$ is the measured output, $f(t) \in R^{n_f}$ is the fault, $d(t) \in R^{n_d}$ is the bounded disturbance, and $u(t) \in R^{n_u}$ is the control signal. Actuator and component faults are modeled by $B_f f(t)$, and sensor faults are modeled by $D_f f(t)$. Plant model uncertainties are modeled in the form of a polytopic model as follows

$$\begin{bmatrix} A & B_u & B_f & B_d \\ C & D_u & D_f & D_d \end{bmatrix} \in \left\{ \begin{bmatrix} A(\alpha) & B_u(\alpha) & B_f(\alpha) & B_d(\alpha) \\ C(\alpha) & D_u(\alpha) & D_f(\alpha) & D_d(\alpha) \end{bmatrix} \right\} = \sum_{i=1}^s \alpha_i \begin{bmatrix} A_i & B_{ui} & B_{fi} & B_{di} \\ C_i & D_{ui} & D_{fi} & D_{di} \end{bmatrix}, \alpha \in \Gamma \quad (47)$$

where Γ is a unit simplex such that

$$\Gamma \equiv \left\{ (\alpha_1, \dots, \alpha_s) : \sum_{i=1}^s \alpha_i = 1, \alpha_i \geq 0 \right\} \quad (48)$$

Because this formulation is a convex combination, it is suitable for an LMI approach.

Here we assume that the above polytopic system possesses the affine quadratic stability that was introduced earlier. Other assumptions that are made for our purpose are that (C, A) is

detectable and $\begin{bmatrix} A - j\omega I & B_d \\ C & D_d \end{bmatrix}$ has full rank for all ω . The assumption that (C, A) is

detectable is standard. The assumption that $\begin{bmatrix} A - j\omega I & B_d \\ C & D_d \end{bmatrix}$ has full row rank for all ω

ensures that $G_{yd} = \begin{bmatrix} A & B_d \\ C & D_d \end{bmatrix}$ has no zeros on the $j\omega$ -axis.

The proposed fault detection filter (FDF) will have the form

$$F : \left\{ \begin{array}{l} \dot{x}_F(t) = A_F x_F(t) + B_F \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \\ z(t) = L_F x_F(t) + H_F \begin{bmatrix} y(t) \\ u(t) \end{bmatrix} \end{array} \right\} \quad (49)$$

The residual $r(t)$ is defined as $r(t) \equiv z(t) - y(t)$. Thus, the RFDF design problem can be described as designing the filter (A_F, B_F, L_F, H_F) such that the residual $r(t)$ is as sensitive as possible to the fault $f(t)$ and as robust as possible to the unknown input $d(t)$, control input $u(t)$ and polytopic model uncertainties.

There are a number of schemes to approach the RFDF problem. In (Ding & Frank, 2003) it is formulated as an optimization problem. Unfortunately, this approach is difficult to extend to systems with modeling uncertainty (Zhong et al., 2003). Another approach is to formulate the RFDF problem as an H_∞ model-matching problem (Casavola et al., 2003). However, this approach may produce a false alarm in a no fault situation (Ding et al., 2000). Thus, we use multi-objective H_∞ optimization (Casavola et al., 2005a, Casavola et al., 2005b) based on the model-matching formulation.

Multi-objective H_∞ optimization based on the model-matching formulation can be described as follows. As previously explained, the generated residual should be sensitive to the faults, but insensitive to the uncertainties and noises. To indicate the residual's sensitivity to the faults, a new H_- index was proposed by (Patton & Hou, 1999). The H_- index is defined as the minimum nonzero singular value of a transfer matrix. However, this is not a norm (Chen & Patton, 1999). Therefore, the model-matching problem needs to be introduced to relate the minimum gain from fault to residual with the residual error (Niemann & Stoustrup, 2000). If we use the residual error instead of the residual, it will be more convenient for optimizing the detection filter F . The residual error is defined as

$$\begin{aligned}
 \tilde{r} &= W(s)f - r \\
 &= W(s)f - \{T_{rf}(s)f + T_{ru}(s)u + T_{rd}(s)d\} \\
 &= \begin{bmatrix} W(s) - T_{rf}(s) & \vdots & -T_{ru}(s) & \vdots & -T_{rd}(s) \end{bmatrix} \begin{bmatrix} f \\ u \\ d \end{bmatrix}
 \end{aligned}
 \tag{50}$$

where $W(s)$ is called reference model (Frisk, 2001) and the transfer matrices, T_{rf} , T_{ru} and T_{rd} , are defined as

$$T_{rf} = F \begin{bmatrix} G_{yf} \\ 0 \end{bmatrix} - G_{yf}, \quad T_{ru} = F \begin{bmatrix} G_{yu} \\ 0 \end{bmatrix} - G_{yu}, \quad T_{rd} = F \begin{bmatrix} G_{yd} \\ 0 \end{bmatrix} - G_{yd}$$

The reference model, $W(s)$, is an RH_∞ transfer matrix (Chen & Patton, 1999). The idea of a reference model has successfully been used to describe signal behavior in other fields like controller design and adaptive control. As discussed in (Frisk, 2001), the main function of the reference model is to describe the desired behavior of the residual vector r with respect to the faults f . For example, if we want to detect faults in the frequency range between 0 and 2 radians/s with a -20dB/decade roll-off at the higher frequencies, the reference model, $W(s)$ is given as $\frac{2}{s+2}$. Using the approach in (Casavola et al., 2005a), $W(s)$ in (50) is placed between the plant and the filter so that the filter can track faults with its specific feature. This can increase the robustness of the filter to the specific faults, if the designer can choose $W(s)$ suitably. Thus, the block diagram for the residual error, $\tilde{r}(t)$, with tracking filter $W(s)$ is shown in Figure 2.

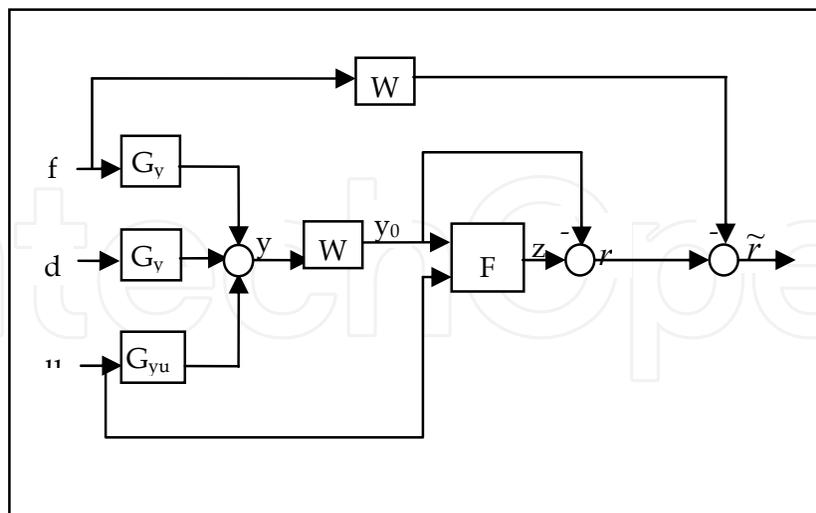


Fig. 2. Block diagram for residual error $\tilde{r}(t)$ with reference model and tracking filter

With this background, we can describe the RFDF design problem using multi-objective H_∞ optimization as follows.

Definition 4.1:

Given positive scalars α_d , α_f and α_u , the RFDF design problem using multi-objective H_∞ optimization is defined as finding $F(s)$ that satisfies $\min_{F(s)}(\alpha_d\gamma_d + \alpha_f\gamma_f + \alpha_u\gamma_u)$ where

$$\begin{aligned} \max_{\alpha \in \Gamma} \left\| F(s) \begin{bmatrix} G_{y_0d}(s) \\ 0 \end{bmatrix} - G_{y_0d}(s) \right\|_\infty &\equiv \max_{\alpha \in \Gamma} \|T_{rd}^0\| \leq \gamma_d \\ \max_{\alpha \in \Gamma} \left\| W(s) - (F(s) \begin{bmatrix} G_{y_0f}(s) \\ 0 \end{bmatrix} - G_{y_0f}(s)) \right\|_\infty &\equiv \max_{\alpha \in \Gamma} \|W(s) - T_{rf}^0\| \leq \gamma_f \\ \max_{\alpha \in \Gamma} \left\| F(s) \begin{bmatrix} G_{y_0u}(s) \\ I \end{bmatrix} - G_{y_0u}(s) \right\|_\infty &\equiv \max_{\alpha \in \Gamma} \|T_{ru}^0\| \leq \gamma_u \end{aligned} \quad (51)$$

and

$$\begin{aligned} G_{yf} &= \begin{bmatrix} A & B_f \\ C & D_f \end{bmatrix}, \quad G_{yd} = \begin{bmatrix} A & B_d \\ C & D_d \end{bmatrix} \quad \text{and} \quad G_{yu} = \begin{bmatrix} A & B_u \\ C & D_u \end{bmatrix}, \quad G_{y_0f} \equiv WG_{yf}, \quad G_{y_0d} \equiv WG_{yd}, \\ G_{y_0u} &\equiv WG_{yu}, \quad T_{rf}^0 \equiv F \begin{bmatrix} G_{y_0f} \\ 0 \end{bmatrix} - G_{y_0f}, \quad T_{rd}^0 \equiv F \begin{bmatrix} G_{y_0d} \\ 0 \end{bmatrix} - G_{y_0d} \quad \text{and} \quad T_{ru}^0 \equiv F \begin{bmatrix} G_{y_0u} \\ 0 \end{bmatrix} - G_{y_0u} \quad \blacksquare \end{aligned}$$

The positive scalars ($\alpha_d, \alpha_f, \alpha_u$) are used to weight the relative importance of tracking and filtering performance.

4.2 Main result

In this section, LMIs will be used to solve the problem formulated in Section 4.1 for an RFDF. To solve the optimization problem in (51), we need the state-space realization of each transfer matrix. The reference model $W(s)$ can be realized as $W(s) = \begin{bmatrix} A_w & B_w \\ C_w & D_w \end{bmatrix}$. After some manipulation, the transfer matrices term in (51) can be realized as

$$F(s) \begin{bmatrix} G_{y_0d}(s) \\ 0 \end{bmatrix} - G_{y_0d}(s) = \begin{bmatrix} A & 0 & 0 & 0 & \vdots & B_d \\ B_w C & A_w & 0 & 0 & \vdots & B_w D_d \\ 0 & 0 & A_w & 0 & \vdots & 0 \\ 0 & B_F \begin{bmatrix} C_w \\ 0 \end{bmatrix} & 0 & A_F & \vdots & 0 \\ \dots & \dots & \dots & \dots & \vdots & \dots \\ 0 & C_w - H_F \begin{bmatrix} C_w \\ 0 \end{bmatrix} & C_w & -L_F & \vdots & -H_F \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix}$$

$$W(s) - (F(s) \begin{bmatrix} G_{y_0f}(s) \\ 0 \end{bmatrix} - G_{y_0f}(s)) =$$

$$\begin{bmatrix} A & 0 & 0 & 0 & \vdots & B_f \\ B_w C & A_w & 0 & 0 & \vdots & B_w D_f \\ 0 & 0 & A_w & 0 & \vdots & B_w \\ 0 & B_F \begin{bmatrix} C_w \\ 0 \end{bmatrix} & 0 & A_F & \vdots & 0 \\ \dots & \dots & \dots & \dots & \vdots & \dots \\ 0 & C_w - H_F \begin{bmatrix} C_w \\ 0 \end{bmatrix} & C_w & -L_F & \vdots & -H_F \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix}$$

$$F(s) \begin{bmatrix} G_{y_0u}(s) \\ I \end{bmatrix} - G_{y_0u}(s) =$$

$$\begin{bmatrix} A & 0 & 0 & 0 & \vdots & B_u \\ B_w C & A_w & 0 & 0 & \vdots & B_w D_u \\ 0 & 0 & A_w & 0 & \vdots & 0 \\ 0 & B_F \begin{bmatrix} C_w \\ 0 \end{bmatrix} & 0 & A_F & \vdots & B_F \begin{bmatrix} 0 \\ I \end{bmatrix} \\ \dots & \dots & \dots & \dots & \vdots & \dots \\ 0 & C_w - H_F \begin{bmatrix} C_w \\ 0 \end{bmatrix} & C_w & -L_F & \vdots & -H_F \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix}$$

(52)

Here, we note that the order of filter is $n_F = 2n_w + n$, where n_F is the filter order, n_w is the tracking filter order, and n is the plant order.

To simplify (52), we let

$$\tilde{A} \equiv \begin{bmatrix} A & 0 & 0 \\ B_w C & A_w & 0 \\ 0 & 0 & A_w \end{bmatrix} \quad \tilde{C} \equiv \begin{bmatrix} 0 & \begin{bmatrix} C_w \\ 0 \end{bmatrix} & 0 \end{bmatrix} \quad \tilde{B}_f \equiv \begin{bmatrix} B_f \\ B_w D_f \\ B_w \end{bmatrix} \quad \tilde{B}_u \equiv \begin{bmatrix} B_u \\ B_w D_u \\ 0 \end{bmatrix} \quad (53)$$

$$\tilde{L}_F \equiv \begin{bmatrix} 0 & C_w - H_F \begin{bmatrix} C_w \\ 0 \end{bmatrix} & C_w \end{bmatrix}$$

Using (53), (52) can be rewritten as

$$\begin{aligned}
 F(s) \begin{bmatrix} G_{y_0d}(s) \\ 0 \end{bmatrix} - G_{y_0d}(s) &= \begin{bmatrix} \tilde{A} & 0 & \vdots & \tilde{B}_d \\ B_F \tilde{C} & A_F & \vdots & 0 \\ \cdots & \cdots & \vdots & \cdots \\ \tilde{L}_F & -L_F & \vdots & -H_F \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix} \\
 W(s) - (F(s) \begin{bmatrix} G_{y_0f}(s) \\ 0 \end{bmatrix} - G_{y_0f}(s)) &= \begin{bmatrix} \tilde{A} & 0 & \vdots & \tilde{B}_f \\ B_F \tilde{C} & A_F & \vdots & 0 \\ \cdots & \cdots & \vdots & \cdots \\ \tilde{L}_F & -L_F & \vdots & -H_F \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix} \\
 F(s) \begin{bmatrix} G_{y_0u}(s) \\ I \end{bmatrix} - G_{y_0u}(s) &= \begin{bmatrix} \tilde{A} & 0 & \vdots & \tilde{B}_u \\ B_F \tilde{C} & A_F & \vdots & B_F \begin{bmatrix} 0 \\ I \end{bmatrix} \\ \cdots & \cdots & \vdots & \cdots \\ \tilde{L}_F & -L_F & \vdots & -H_F \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix}
 \end{aligned}$$

(54)

Applying the BRL to (51) results in

$$\begin{aligned}
 &\left[\begin{array}{ccc} \begin{bmatrix} \tilde{A} & 0 \\ B_F \tilde{C} & A_F \end{bmatrix}^T P + P \begin{bmatrix} \tilde{A} & 0 \\ B_F \tilde{C} & A_F \end{bmatrix} & P \begin{bmatrix} \tilde{B}_d \\ 0 \end{bmatrix} & \begin{bmatrix} \tilde{L}_F^T \\ -L_F^T \end{bmatrix} \\ * & & \begin{bmatrix} -\gamma_d I & -H_F \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix} \\ * & & \begin{bmatrix} * & -\gamma_d I \end{bmatrix} \end{array} \right] < 0 \\
 &\left[\begin{array}{ccc} \begin{bmatrix} \tilde{A} & 0 \\ B_F \tilde{C} & A_F \end{bmatrix}^T P + P \begin{bmatrix} \tilde{A} & 0 \\ B_F \tilde{C} & A_F \end{bmatrix} & P \begin{bmatrix} \tilde{B}_f \\ 0 \end{bmatrix} & \begin{bmatrix} \tilde{L}_F^T \\ -L_F^T \end{bmatrix} \\ * & & \begin{bmatrix} -\gamma_f I & -H_F \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix} \\ * & & \begin{bmatrix} * & -\gamma_f I \end{bmatrix} \end{array} \right] < 0
 \end{aligned}$$

$$\begin{bmatrix}
 \begin{bmatrix} \tilde{A} & 0 \\ B_F \tilde{C} & A_F \end{bmatrix}^T P + P \begin{bmatrix} \tilde{A} & 0 \\ B_F \tilde{C} & A_F \end{bmatrix} & P \begin{bmatrix} \tilde{B}_u \\ B_F \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix} & \begin{bmatrix} \tilde{L}_F^T \\ -L_F^T \end{bmatrix} \\
 * & -\gamma_u I & -H_F \begin{bmatrix} 0 \\ I \end{bmatrix} \\
 * & * & -\gamma_u I
 \end{bmatrix} < 0 \tag{55}$$

where P is $2n_F \times 2n_F$ Lyapunov function matrix and $P > 0$. Using the same procedure as in (Kim & Watkins, 2006), the matrix inequalities in (55) can be rewritten as (56)

$$\begin{bmatrix}
 -(V_{11} + V_{11}^T) & -(S_2^T + S_1^T) & V_{11}^T \tilde{A} + \hat{B}_F \tilde{C} + \hat{P}_1 & \hat{A}_F + \hat{P}_3^T & V_{11}^T \tilde{B}_d & 0 & V_{11}^T & S_1^T \\
 * & -(S_1 + S_1^T) & S_2 \tilde{A} + \hat{B}_F \tilde{C} + \hat{P}_3 & \hat{A}_F + \hat{P}_2 & S_2 \tilde{B}_d & 0 & S_2 & S_1^T \\
 * & * & -\mu \hat{P}_1 & -\mu \hat{P}_3^T & 0 & \tilde{L}_F^T & 0 & 0 \\
 * & * & * & -\mu \hat{P}_2 & 0 & -\hat{L}_F^T & 0 & 0 \\
 * & * & * & * & -\gamma_d I & -[0 \ I] H_F^T & 0 & 0 \\
 * & * & * & * & * & -\gamma_d I & 0 & 0 \\
 * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_1 & -\frac{1}{\mu} \hat{P}_3^T \\
 * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_2
 \end{bmatrix} < 0$$

$$\begin{bmatrix}
 -(V_{11} + V_{11}^T) & -(S_2^T + S_1^T) & V_{11}^T \tilde{A} + \hat{B}_F \tilde{C} + \hat{P}_1 & \hat{A}_F + \hat{P}_3^T & V_{11}^T \tilde{B}_f & 0 & V_{11}^T & S_1^T \\
 * & -(S_1 + S_1^T) & S_2 \tilde{A} + \hat{B}_F \tilde{C} + \hat{P}_3 & \hat{A}_F + \hat{P}_2 & S_2 \tilde{B}_f & 0 & S_2 & S_1^T \\
 * & * & -\mu \hat{P}_1 & -\mu \hat{P}_3^T & 0 & \tilde{L}_F^T & 0 & 0 \\
 * & * & * & -\mu \hat{P}_2 & 0 & -\hat{L}_F^T & 0 & 0 \\
 * & * & * & * & -\gamma_f I & -[0 \ I] H_F^T & 0 & 0 \\
 * & * & * & * & * & -\gamma_f I & 0 & 0 \\
 * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_1 & -\frac{1}{\mu} \hat{P}_3^T \\
 * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_2
 \end{bmatrix} < 0$$

$$\begin{bmatrix}
 -(V_{11} + V_{11}^T) & -(S_2^T + S_1^T) & V_{11}^T \tilde{A} + \hat{B}_F \tilde{C} + \hat{P}_1 & \hat{A}_F + \hat{P}_3^T & V_{11}^T \tilde{B}_u + \hat{B}_F \begin{bmatrix} 0 \\ I \end{bmatrix} & 0 & V_{11}^T & S_1^T \\
 * & -(S_1 + S_1^T) & S_2 \tilde{A} + \hat{B}_F \tilde{C} + \hat{P}_3 & \hat{A}_F + \hat{P}_2 & S_2 \tilde{B}_u + \hat{B}_F \begin{bmatrix} 0 \\ I \end{bmatrix} & 0 & S_2 & S_1^T \\
 * & * & -\mu \hat{P}_1 & -\mu \hat{P}_3^T & 0 & \tilde{L}_F^T & 0 & 0 \\
 * & * & * & -\mu \hat{P}_2 & 0 & -\tilde{L}_F^T & 0 & 0 \\
 * & * & * & * & -\gamma_u I & -[0 \ I] H_F^T & 0 & 0 \\
 * & * & * & * & * & -\gamma_u I & 0 & 0 \\
 * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_1 & -\frac{1}{\mu} \hat{P}_3^T \\
 * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_2
 \end{bmatrix} < 0 \tag{56}$$

where

$$S_1 = V_{21}^T V_{22}^{-T} V_{21}, \quad S_2 = V_{21}^T V_{22}^{-T} V_{12}^T, \quad \hat{B}_F = V_{21}^T B_F, \quad \hat{P}_1 = P_1, \quad \hat{P}_2 = V_{21}^T V_{22}^{-T} P_2 V_{22}^{-1} V_{21}, \quad \hat{P}_3^T = P_3^T V_{22}^{-1} V_{21}, \\
 \hat{A}_F = V_{21}^T A_F V_{22}^{-1} V_{21}, \quad \hat{L}_F = L_F V_{22}^{-1} V_{21} \quad \text{and} \quad \hat{P} = \begin{bmatrix} \hat{P}_1 & \hat{P}_3^T \\ \hat{P}_3 & \hat{P}_2 \end{bmatrix}.$$

To reduce the filter order, we'll need partitions of the V and P variables as in (Kim & Watkins, 2007),

$$V = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix}, \quad P = \begin{bmatrix} P_1 & P_3^T \\ P_3 & P_2 \end{bmatrix} \tag{57}$$

Using the same method as in (Kim & Watkins, 2007), we can solve for the reduced-order filter whose filter order is $k (k \leq n_F)$. The partitioned sub-matrices in (57) will be dimensionalized as $V_{11} (n_F \times n_F)$, $V_{12} (n_F \times k)$, $V_{21} (k \times n_F)$, and $V_{22} (k \times k)$. We need to enforce some special structure on V_{21} as

$$V_{21} = \begin{bmatrix} \tilde{V}_{21} & \mathbf{0}_{k \times (n_F - k)} \end{bmatrix} \tag{58}$$

where \tilde{V}_{21} is a $k \times k$ matrix. Finally, we get three LMIs for the reduced-order RFDF with order k as (59)

$$\begin{bmatrix}
 -(V_{11} + V_{11}^T) & -(S_2^T + \tilde{S}_1^T) & V_{11}^T \tilde{A} + \tilde{B}_F \tilde{C} + \hat{P}_1 & \tilde{A}_F + \hat{P}_3^T & V_{11}^T \tilde{B}_d & 0 & V_{11}^T & \tilde{S}_1^T \\
 * & -(S_1 + S_1^T) & S_2 \tilde{A} + \hat{B}_F \tilde{C} + \hat{P}_3 & \hat{A}_F + \hat{P}_2 & S_2 \tilde{B}_d & 0 & S_2 & S_1^T \\
 * & * & -\mu \hat{P}_1 & -\mu \hat{P}_3^T & 0 & \tilde{L}_F^T & 0 & 0 \\
 * & * & * & -\mu \hat{P}_2 & 0 & -\hat{L}_F^T & 0 & 0 \\
 * & * & * & * & -\gamma_d I & -[0 \ I] H_F^T & 0 & 0 \\
 * & * & * & * & * & -\gamma_d I & 0 & 0 \\
 * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_1 & -\frac{1}{\mu} \hat{P}_3^T \\
 * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_2
 \end{bmatrix} < 0$$

$$\begin{bmatrix}
 -(V_{11} + V_{11}^T) & -(S_2^T + \tilde{S}_1^T) & V_{11}^T \tilde{A} + \tilde{B}_F \tilde{C} + \hat{P}_1 & \tilde{A}_F + \hat{P}_3^T & V_{11}^T \tilde{B}_f & 0 & V_{11}^T & \tilde{S}_1^T \\
 * & -(S_1 + S_1^T) & S_2 \tilde{A} + \hat{B}_F \tilde{C} + \hat{P}_3 & \hat{A}_F + \hat{P}_2 & S_2 \tilde{B}_f & 0 & S_2 & S_1^T \\
 * & * & -\mu \hat{P}_1 & -\mu \hat{P}_3^T & 0 & \tilde{L}_F^T & 0 & 0 \\
 * & * & * & -\mu \hat{P}_2 & 0 & -\hat{L}_F^T & 0 & 0 \\
 * & * & * & * & -\gamma_f I & -[0 \ I] H_F^T & 0 & 0 \\
 * & * & * & * & * & -\gamma_f I & 0 & 0 \\
 * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_1 & -\frac{1}{\mu} \hat{P}_3^T \\
 * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_2
 \end{bmatrix} < 0$$

$$\begin{bmatrix}
 -(V_{11} + V_{11}^T) & -(S_2^T + \tilde{S}_1^T) & V_{11}^T \tilde{A} + \tilde{B}_F \tilde{C} + \hat{P}_1 & \tilde{A}_F + \hat{P}_3^T & V_{11}^T \tilde{B}_u + \tilde{B}_F \begin{bmatrix} 0 \\ I \end{bmatrix} & 0 & V_{11}^T & \tilde{S}_1^T \\
 * & -(S_1 + S_1^T) & S_2 \tilde{A} + \hat{B}_F \tilde{C} + \hat{P}_3 & \hat{A}_F + \hat{P}_2 & S_2 \tilde{B}_u + \hat{B}_F \begin{bmatrix} 0 \\ I \end{bmatrix} & 0 & S_2 & S_1^T \\
 * & * & -\mu \hat{P}_1 & -\mu \hat{P}_3^T & 0 & \tilde{L}_F^T & 0 & 0 \\
 * & * & * & -\mu \hat{P}_2 & 0 & -\hat{L}_F^T & 0 & 0 \\
 * & * & * & * & -\gamma_u I & -[0 \ I] H_F^T & 0 & 0 \\
 * & * & * & * & * & -\gamma_u I & 0 & 0 \\
 * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_1 & -\frac{1}{\mu} \hat{P}_3^T \\
 * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_2
 \end{bmatrix} < 0$$

(59)

where

$$S_1 = \tilde{V}_{21}^T V_{22}^{-T} \tilde{V}_{21}, \quad \tilde{S}_1 = \begin{bmatrix} S_1 & \mathbf{0}_{k \times (n_F - k)} \end{bmatrix}, \quad S_2 = \tilde{V}_{21}^T V_{22}^{-T} V_{12}^T, \quad \hat{B}_F = \tilde{V}_{21}^T B_F, \quad \tilde{B}_F = \begin{bmatrix} \hat{B}_F \\ \mathbf{0}_{(n_F - k) \times m} \end{bmatrix}, \quad \hat{P}_1 = P_1, \\ \hat{P}_2 = \tilde{V}_{21}^T V_{22}^{-T} P_2 V_{22}^{-1} \tilde{V}_{21}, \quad \hat{P}_3^T = P_3^T V_{22}^{-1} \tilde{V}_{21}, \quad \hat{A}_F = \tilde{V}_{21}^T A_F V_{22}^{-1} \tilde{V}_{21}, \quad \tilde{A}_F = \begin{bmatrix} \hat{A}_F \\ \mathbf{0}_{(n_F - k) \times k} \end{bmatrix}, \quad \hat{L}_F = L_F V_{22}^{-1} \tilde{V}_{21}, \quad \text{and} \\ \hat{P} = \begin{bmatrix} \hat{P}_1 & \hat{P}_3^T \\ \hat{P}_3 & \hat{P}_2 \end{bmatrix}.$$

Remark 4.2:

The filter matrices (A_F, B_F, L_F) can be derived by the following procedure:

- (i) Compute V_{22}, \tilde{V}_{21} by solving the factorization problem $S_1 = \tilde{V}_{21}^T V_{22}^{-T} \tilde{V}_{21}$ with the Schur decomposition,
- (ii) Compute $A_F, B_F, L_F : A_F = \tilde{V}_{21}^{-T} \hat{A}_F \tilde{V}_{21}^{-1} V_{22}, B_F = \tilde{V}_{21}^{-T} \hat{B}_F, L_F = \hat{L}_F \tilde{V}_{21}^{-1} V_{22}$ ■

Now, we need to remember that we had a polytopic uncertain system. As explained earlier, the parameter dependent Lyapunov matrix $P(\alpha), P(\alpha) = \sum_{i=1}^s \alpha_i P_i(\alpha), \alpha \in \Gamma$ is symmetric positive definite for all admissible values of α , if and only if, all the P_i 's are positive definite. Therefore, we need to check (59) only at the vertices, $i = 1, 2, \dots, s$. This gives us the final form (60)

$$\begin{bmatrix} -(V_{11} + V_{11}^T) & -(S_2^T + \tilde{S}_1^T) & V_{11}^T \tilde{A}_i + \tilde{B}_F \tilde{C}_i + \hat{P}_{1,i} & \tilde{A}_F + \hat{P}_{3,i}^T & V_{11}^T \tilde{B}_{d,i} & 0 & V_{11}^T & \tilde{S}_1^T \\ * & -(S_1 + S_1^T) & S_2 \tilde{A}_i + \hat{B}_F \tilde{C}_i + \hat{P}_{3,i} & \hat{A}_F + \hat{P}_{2,i} & S_2 \tilde{B}_{d,i} & 0 & S_2 & S_1^T \\ * & * & -\mu \hat{P}_{1,i} & -\mu \hat{P}_{3,i}^T & 0 & \tilde{L}_F^T & 0 & 0 \\ * & * & * & -\mu \hat{P}_{2,i} & 0 & -\hat{L}_F^T & 0 & 0 \\ * & * & * & * & -\gamma_d I & -[0 \ I] H_F^T & 0 & 0 \\ * & * & * & * & * & -\gamma_d I & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_{1,i} & -\frac{1}{\mu} \hat{P}_{3,i}^T \\ * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_{2,i} \end{bmatrix} < 0$$

$$\begin{bmatrix} -(V_{11} + V_{11}^T) & -(S_2^T + \tilde{S}_1^T) & V_{11}^T \tilde{A}_i + \tilde{B}_F \tilde{C}_i + \hat{P}_{1,i} & \tilde{A}_F + \hat{P}_{3,i}^T & V_{11}^T \tilde{B}_{f,i} & 0 & V_{11}^T & \tilde{S}_1^T \\ * & -(S_1 + S_1^T) & S_2 \tilde{A}_i + \hat{B}_F \tilde{C}_i + \hat{P}_{3,i} & \hat{A}_F + \hat{P}_{2,i} & S_2 \tilde{B}_{f,i} & 0 & S_2 & S_1^T \\ * & * & -\mu \hat{P}_{1,i} & -\mu \hat{P}_{3,i}^T & 0 & \tilde{L}_F^T & 0 & 0 \\ * & * & * & -\mu \hat{P}_{2,i} & 0 & -\hat{L}_F^T & 0 & 0 \\ * & * & * & * & -\gamma_f I & -[0 \ I] H_F^T & 0 & 0 \\ * & * & * & * & * & -\gamma_f I & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_{1,i} & -\frac{1}{\mu} \hat{P}_{3,i}^T \\ * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_{2,i} \end{bmatrix} < 0$$

$$\begin{bmatrix} -(V_{11} + V_{11}^T) & -(S_2^T + \tilde{S}_1^T) & V_{11}^T \tilde{A}_i + \tilde{B}_F \tilde{C}_i + \hat{P}_{1,i} & \tilde{A}_F + \hat{P}_{3,i}^T & V_{11}^T \tilde{B}_{u,i} + \tilde{B}_F \begin{bmatrix} 0 \\ I \end{bmatrix} & 0 & V_{11}^T & \tilde{S}_1^T \\ * & -(S_1 + S_1^T) & S_2 \tilde{A}_i + \hat{B}_F \tilde{C}_i + \hat{P}_{3,i} & \hat{A}_F + \hat{P}_{2,i} & S_2 \tilde{B}_{u,i} + \hat{B}_F \begin{bmatrix} 0 \\ I \end{bmatrix} & 0 & S_2 & S_1^T \\ * & * & -\mu \hat{P}_{1,i} & -\mu \hat{P}_{3,i}^T & 0 & \tilde{L}_F^T & 0 & 0 \\ * & * & * & -\mu \hat{P}_{2,i} & 0 & -\hat{L}_F^T & 0 & 0 \\ * & * & * & * & -\gamma_u I & -[0 \ I] H_F^T & 0 & 0 \\ * & * & * & * & * & -\gamma_u I & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_{1,i} & -\frac{1}{\mu} \hat{P}_{3,i}^T \\ * & * & * & * & * & * & * & -\frac{1}{\mu} \hat{P}_{2,i} \end{bmatrix} < 0$$

$(i = 1, 2, \dots, s)$

(60)

Theorem 4.3:

The reduced-order RFDF can be obtained by solving

$$\min_{V_{11}, S_1, S_2, \hat{A}_F, \hat{B}_F, \tilde{L}_F, H_F, \gamma_d, \gamma_f, \gamma_u, \hat{P}} \{ \alpha_d \gamma_d + \alpha_f \gamma_f + \alpha_u \gamma_u : (60) \} \quad (61)$$

where $\hat{P} > 0$ and α_d, α_f , and α_u are given as positive scalars. ■

In summary, the reduced order RFDF filter in the multi-objective H_∞ formulation for polytopic uncertain system can be solved if and only if (60) holds for all vertices, $i = 1, 2, \dots, s$. The minimum value can be found from (61). Also, the filter realization (A_F, B_F, L_F, H_F) defining the k_{th} order filter is obtained from (i) and (ii) in Remark 4.2 and (61).

Another important task for fault detection is the evaluation of the generated residual. An adaptive threshold will be used in this work. The disadvantage of a fixed threshold is that if we fix the threshold too low, it can increase the rate of false alarms. Thus, the optimal choice

of the magnitude of the threshold depends upon the nature of the system uncertainties and varies with the system input. That is called an adaptive threshold. The first step of residual evaluation is to choose an evaluation function and to determine the corresponding threshold. Among a number of residual evaluation functions, the so-called time windowed root mean square (RMS) is often used. The time windowed RMS is represented as

$$\|r(t)\|_{RMS,T} = \sqrt{\frac{1}{T} \int_{t-T}^t r^T(\tau)r(\tau)d\tau} \quad (62)$$

where T is the length of the finite time window. Since an evaluation of the residual signal over the whole time range is impractical, the time windowed RMS evaluation method is used in practice to detect faults as early as possible.

After selecting the evaluation function, we are able to determine the threshold. A major requirement on the fault detection is to reduce or prevent false alarms. Thus, in the absence of any faults, $\|r(t)\|_{RMS,T}$ should be less than the threshold value J_{th} , i.e.,

$$J_{th} = \sup_{\alpha \in \Gamma, f=0} \|r(t)\|_{RMS,T} \quad (63)$$

Under fault-free conditions, (63) can be described as

$$r(s) = T_{ru}^0(s)u(s) + T_{rd}^0(s)d(s) \quad (64)$$

From Parseval's Theorem (Zhou et al., 1996) and the RMS norm relationship (Boyd & Barratt, 1991), the threshold J_{th} can be found as follows

$$\begin{aligned} J_{th} &\equiv \|r\|_{RMS,T,f=0} \\ &= \|T_{ru}^0 u + T_{rd}^0 d\|_{RMS,T} \\ &\leq \|T_{ru}^0\|_{\infty} \|u\|_{RMS,T} + \|T_{rd}^0\|_{\infty} \|d\|_{RMS,T} \\ &= \gamma_u \|u\|_{RMS,T} + \gamma_d \|d\|_{RMS,T} \end{aligned} \quad (65)$$

where γ_u and γ_d come from the optimization problem in (61) and $\|d\|_{RMS,T}$ is calculated or bounded by the worst disturbance acting on the plant. Thus in (65), $\|d\|_{RMS,T}$ is evaluated off-line, while $u(t)$ is assumed to be known and $\|u\|_{RMS,T}$ is calculated on-line.

5. Example

5.1 H_∞ Filter design

In this example, we handle two cases, the full order and the reduced order filter design, and demonstrate the advantages of this study. We use the example from (Tuan et al., 2000) with the plant data

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & -1+0.3\alpha \\ 1 & -0.5 \end{bmatrix} x + \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} w \\ y &= [-100+10\beta \quad 100]x + [0 \quad 1]w \\ z &= [1 \quad 0]x \end{aligned} \tag{66}$$

where the uncertainty parameters, α and β , have four types of uncertainty sets given by

$$|\alpha| \leq 1, |\beta| \leq 1 \tag{67}$$

$$|\alpha| \leq 1, \beta = \alpha \tag{68}$$

$$|\alpha| \leq 3, |\beta| \leq 1 \tag{69}$$

$$|\alpha| \leq 3, \beta = \alpha \tag{70}$$

Table 1 compares results between the filters in (Tuan, 2000, Geromel, 1999, De Souza, 1999) and (45) for the four types of uncertainties in (67)-(70). For the conservative case, a single Lyapunov matrix P is used and for the nonconservative case, four different Lyapunov matrices P are used for each vertex. The calculations were done using (Gahinet). An arbitrary positive scalar for μ should be chosen and the feasibility of the formulation should be checked. If it is feasible, the minimum value of (45) can be found.

Type	Filter order	(Geromel, 1999): $\ T_{ed}\ _2$	(De Souza, 1999): $\ T_{ed}\ _2$	(Tuan, 2000): $\ T_{ed}\ _2$		(45): $\ T_{ed}\ _\infty$	
				Non Conservative	Conservative	Non Conservative	Conservative
$ \alpha \leq 1, \beta \leq 1$	full	5.728	4.867	2.382	5.7495	2.965	3.346
	reduced		4.946	3.001	5.8467	3.179	4.304
$ \alpha \leq 1, \beta = \alpha$	full	4.819	4.373	2.382	4.8841	2.948	3.165
	reduced		4.556	3.079	5.0704	3.163	4.023
$ \alpha \leq 3, \beta \leq 1$	full	$+\infty$	$+\infty$	93.365	$+\infty$	29.106	$+\infty$
	reduced			106.493	$+\infty$	29.679	$+\infty$
$ \alpha \leq 3, \beta = \alpha$	full	$+\infty$	$+\infty$	100.963	$+\infty$	28.808	$+\infty$
	reduced			106.517	$+\infty$	29.732	$+\infty$

Table 1. Performance comparison according to each uncertainty type

The feasibility must be checked because the product term of μ and Lyapunov variable P is nonlinear. We need to tune μ until the LMI solver returns a feasible solution. For any choice of $\mu > 0$, if solvable, it gives a unique solution because of the convexity. For case (67), we found that when $\mu = 7$ (45) results in $\gamma = 2.965$ for the full order case, and when $\mu = 1.451$ (45) results in $\gamma = 3.179$ for the reduced order case. From Table 1, we can see that the optimization in (45) using parameter-dependent Lyapunov functions does not fail in the uncertainty cases of (69) and (70), but the conservative approaches in (De Souza, 1999, Gahinet) failed for the same cases. In the uncertainty cases of (69) and (70) cases, we also found that the conservative application of (Tuan et al., 2000) and our approach in (45), i.e., the usage of a single parameter-independent Lyapunov function, also failed. The filter data in the full and reduced order cases in (67) were found from (40)-(42). The Schur Decomposition method was used to solve the factorization problem in (40). The result is shown in Table 2 for the uncertainty case in (67).

Type	Filter order	A_F	B_F	L_F
$ \alpha \leq 1, \beta \leq 1$	full	$\begin{bmatrix} -23.5163 & 0.6242 \\ 6.3142 & -3.0403 \end{bmatrix}$	$\begin{bmatrix} 0.0006 \\ -0.0031 \end{bmatrix}$	$[588.5549 \quad 6.1988]$
	reduced	-0.8396	0.0022	4.0742

Table 2. Filter synthesis

As we already know, the H_∞ approach in (45) is more robust to uncertainties than the H_2 approach in (Tuan et al., 2000). To illustrate this, we found the filter data from (Tuan et al., 2000) and (45) and used (18) to calculate $\|T_{ed}\|_\infty$ for the uncertainty case in (67). Table 3 shows the result. Therefore, we find that for this example our approach is more robust to model uncertainty than former approaches, (Tuan et al., 2000, Geromel, 1999, Gahinet) and gives a non-conservative result.

Type	Filter order	$\ (Tuan et al., 2000)\ _\infty$	(45)
$ \alpha \leq 3, \beta \leq 1$	Full	11.1939	9.8752
	Reduced	11.3486	10.7081

Table 3. Performance comparison between (Tuan et al., 2000) and (45)

5.2 Fault detection filter design

In this section, a numerical example is given. Consider the uncertain LTI plant that is borrowed from (Nobrega et al., 2000), but is modified to include uncertainties. The plant is

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & -1+0.3\alpha \\ 1 & -0.5 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f + \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix} d \\ y &= [-100+10\beta \quad 100]x + 0.1u + f + 0.05d \end{aligned} \tag{71}$$

where the uncertainty parameters, α and β , have the uncertainty set given by $|\alpha| \leq 1, |\beta| \leq 1$.

The selected reference model is $W(s) = \frac{2}{s+2}$. The fault signal f is simulated as a pulse of unit amplitude that occurs from 20 to 25 seconds and is zero elsewhere. The input u is taken as $1 - e^{-0.01t}$. The unknown input d is assumed to be band-limited white noise with power 0.0005. The upper bound of $\|d\|_{RMS,T}$ is given as 0.15. This value is used to calculate J_{th} of (70).

With this information, we can get the RFDF data from (66). For the full order case, we checked the detection time with weights set at $(\alpha_d = \alpha_f = \alpha_u = 1)$ and $(\alpha_d = \alpha_f = 1, \alpha_u = 100)$. The detection time is defined as the time span where the J -residual (time-windowed residual RMS value) exceeds the threshold, J_{th} . The full order FDF is $4 (= n_F)$. As the order k of the RFDF is reduced from 4 to 1, the effectiveness of the filter is compared by monitoring the detection time. The results are shown in Figures 3 and 4 and Table 4. Table 4 shows a comparison of the results. In this example, we can see that the fault detection time does not change much as the order of the RFDF is reduced.

RFDF Order (k)		Detection time
$k = n_F = 4$ (full)	$\alpha_d = \alpha_f = \alpha_u = 1$	20.705~34.96
	$\alpha_d = \alpha_f = 1, \alpha_u = 100$	22~34.7
$k = 3, (\alpha_d = \alpha_f = \alpha_u = 1)$		22.99~31.36
$k = 2, (\alpha_d = \alpha_f = \alpha_u = 1)$		23.115~31.45
$k = 1, (\alpha_d = \alpha_f = \alpha_u = 1)$		23.58~31.157

Table 4. Comparison of the detection time as the order of RFDF is reduced

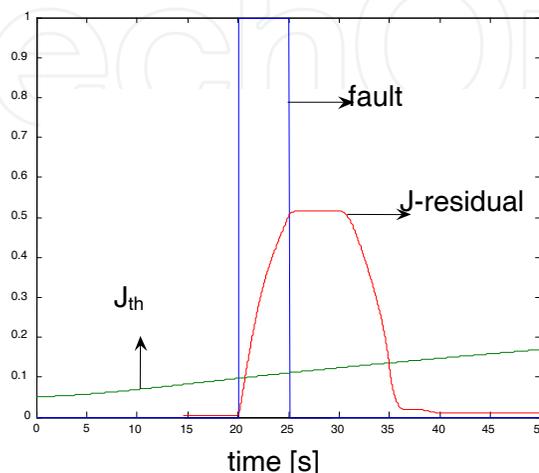


Fig. 3. Full order signals ($\alpha_d = \alpha_f = \alpha_u = 1$)

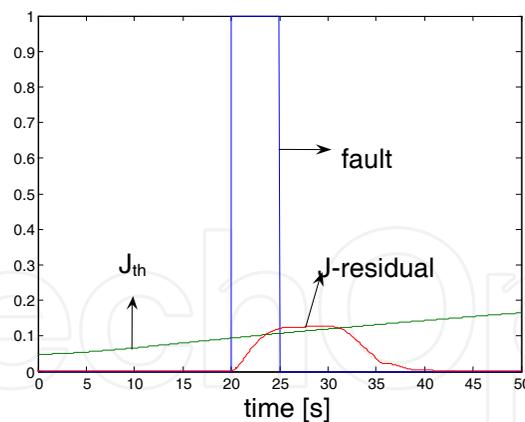


Fig. 4. Reduced order ($k = 1$) signals ($\alpha_d = \alpha_f = \alpha_u = 1$)

6. Conclusion

In this paper, we developed in detail a practical approach for solving the H_∞ reduced filter synthesis problem in an LMI framework and its application for fault detection filter design. The LMI characteristics of the filtering problem were described and formulated with the BRL. To overcome conservatism, parameter-dependent Lyapunov functions were used. A congruence transformation and a change of variable technique were used to linearize the problem. This formulation had formerly been applied to the H_2 problem. As is well-known, the H_∞ approach provides better robustness than the H_2 approach.

The order reduction problem for a robust fault detection filter was considered, and the result was illustrated with a simple example. A reference model was introduced and formulated as a multi-objective optimization for a polytopic uncertain system. The order of the RFDF is reduced using LMI techniques and the detection performance is compared with the full order filter. Using the LMI solver from Mathworks Robust Control Toolbox, the minimum value is found from (61). For the example considered, the detection time did not change much as the filter order was reduced. Thus, our reduced-order RFDF design shows promising results.

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