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# New Adaptive Algorithms for the Rapid Identification of Sparse Impulse Responses

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## 1. Introduction

It is well known that the convergence of the adaptive filtering algorithms becomes slow when the number of coefficients is very large. However, in many applications, such as digital network and acoustical echo cancelers, the system being modeled presents sparse impulse response, that is, most of its coefficients have small magnitudes. The classical adaptation approaches, such as the least-mean square (LMS) and recursive least squares (RLS) algorithms, do not take into account the sparseness characteristics of such systems.

In order to improve the convergence for these applications, several algorithms have been proposed recently, which employ individual step-sizes for the updating of the different coefficients. The adaptation step-sizes are made larger for the coefficients with larger magnitudes, resulting in a faster convergence for the most significant coefficients. Such idea was first introduced in (Duttweiler, 2000) resulting in the so-called proportionate normalized least mean square (PNLMS) algorithm. However, the performance of the PNLMS algorithm for the identification of non-sparse impulse response can be very poor, even slower than that of the conventional LMS algorithm. An improved version of such algorithm, which employs an extra parameter to control the amount of proportionality in the step-size normalization, was proposed in (Benesty & Gay, 2002).

An observed characteristic of the PNLMS algorithm is a rapid initial convergence, due to the fast adaptation speed of the large value coefficients, followed by an expressive performance degradation, owing to the small adaptation speed of the small value coefficients. Such behavior is more significant in the modeling of not very sparse impulse responses. In order to reduce this problem, the application of a non-linear function to the coefficients in the step-size normalization was proposed in (Deng & Doroslovacki, 2006).

The well-known slow convergence of the gradient algorithms for colored input signals is also observed in the proportionate-type NLMS algorithms. Implementations that combine the ideas of the PNLMS and transform-domain adaptive algorithms were proposed in (Deng & Doroslovacki, 2007) and (Petraglia & Barboza, 2008) for accelerating the convergence for colored input signals.

In this chapter, we give an overview of the most important adaptive algorithms developed for the fast identification of systems with sparse impulse responses. The convergence of the proposed algorithms are compared through computer simulations for the identification of the channel impulse responses in a digital network echo cancellation application.

2. Sparse Impulse Response Systems

Sparse impulse responses are encountered in several applications, such as in acoustic and digital network echo cancelers. The adaptive filters employed in the modeling of the unknown system in such applications present a small number of coefficients with significant magnitude. Figure 1 illustrates the modeling of an unknown system  $\mathbf{w}^o$ , which is assumed to be linear, time-invariant and of finite impulse response length ( $N$ ), by an adaptive filter. The vector containing the adaptive filter coefficients is denoted as  $\mathbf{w}(n) = [w_0(n) \ w_1(n) \cdots w_{N-1}(n)]^T$  and its input vector as  $\mathbf{x}(n) = [x(n) \ x(n-1) \cdots x(n-N+1)]^T$ . The adaptive filter output is denoted as  $y(n)$ , the desired response as  $d(n)$  and the estimation error as  $e(n)$ . One of the most used adaptation techniques is the normalized least mean-square (NLMS) algorithm, shown in Table 1, where  $\beta$  is a fixed step-size factor and  $\delta$  is a small constant needed in order to avoid division by zero.

As shown in Table 1 for the NLMS algorithm, typical initialization parameters are given for all algorithms studied in this chapter.

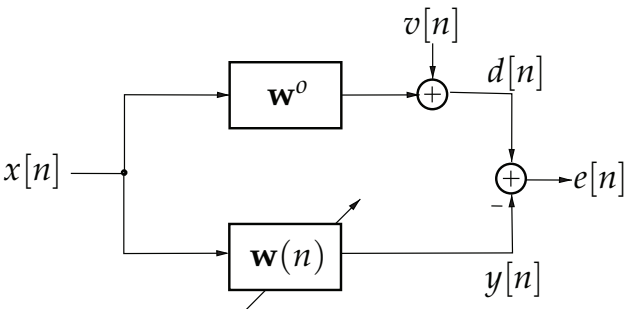


Fig. 1. System identification through adaptive filtering.

Initialization (typical values)
$\delta = 0.01, \beta = 0.25$
$\mathbf{w}(0) = [ \ w_0(0) \ \ w_1(0) \ \ \cdots \ \ w_{N-1}(0) \ ]^T = \mathbf{0}$
Processing and Adaptation
For $n = 0, 1, 2, \cdots$
$\mathbf{x}(n) = [ \ x(n) \ \ x(n-1) \ \ \cdots \ \ x(n-N+1) \ ]^T$
$y(n) = \mathbf{x}^T(n)\mathbf{w}(n)$
$e(n) = d(n) - y(n)$
$\mathbf{w}(n+1) = \mathbf{w}(n) + \beta \frac{\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{x}(n) + \delta}$
End

Table 1. NLMS Algorithm

Described in the next sections, adaptive algorithms that take into account the sparseness of the unknown system impulse response have been recently developed. The convergence behavior

of such algorithms depends on how sparse the modeled impulse response is. A sparseness measure of an  $N$ -length impulse response  $\mathbf{w}$  was proposed in (Hoyer, 2004) as

$$\zeta_{\mathbf{w}} = \frac{N}{N - \sqrt{N}} \left( 1 - \frac{\|\mathbf{w}\|_1}{\sqrt{N}\|\mathbf{w}\|_2} \right) \quad (1)$$

where  $\|\mathbf{w}\|_l$  is the  $l$ -norm of the vector  $\mathbf{w}$ . It should be observed that  $0 \leq \zeta_{\mathbf{w}} \leq 1$ , and that  $\zeta_{\mathbf{w}} = 0$  when all elements of  $\mathbf{w}$  are equal in magnitude (non-sparse impulse response) and  $\zeta_{\mathbf{w}} = 1$  when only one element of  $\mathbf{w}$  is non-zero (the sparsest impulse response).

In the simulations presented throughout this chapter, the identification of the digital network channels of ITU-T Recommendation G.168 (G.168, 2004), by an adaptive filter with  $N = 512$  coefficients, will be considered. Figures 2(a) and 2(b) show the impulse responses of the most and least sparse digital network channel models (gm1 and gm4, respectively) described in (G.168, 2004). Figure 2(c) presents the gm4 channel impulse response with a white noise (uniformly distributed in  $[-0.05, 0.05]$ ) added to it, such as to simulate a non-sparse system. The corresponding sparseness measures are  $\zeta_{\mathbf{w}} = 0.8970$  for the gm1 channel,  $\zeta_{\mathbf{w}} = 0.7253$  for the gm4 channel and  $\zeta_{\mathbf{w}} = 0.2153$  for the gm4 plus noise channel.

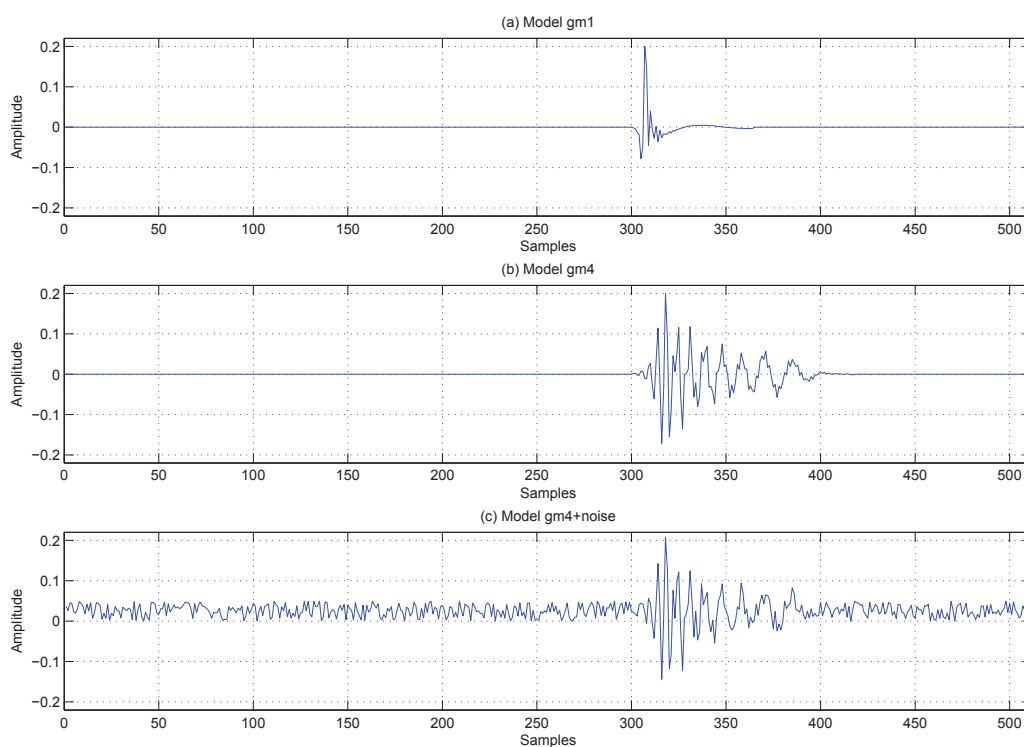


Fig. 2. Channel impulse responses: (a) gm1, (b) gm4 and (c) gm4+noise.

### 3. Proportionate-type NLMS Algorithms

The proportionate-type NLMS algorithms employ a different step-size for each coefficient, such that larger adjustments are applied to the larger coefficients (or active coefficients), re-

sulting in faster convergence rate when modeling systems with sparse impulse responses. The main algorithms of such family are described next.

3.1 PNLMS Algorithm

For an adaptive filter with coefficients  $w_i(n)$ , for  $1 \leq i \leq N - 1$ , the proportionate normalized least mean-square (PNLMS) algorithm is presented in Table 2. In this algorithm, a time-varying step-size control matrix  $\Gamma(n)$ , whose elements are roughly proportional to the absolute values of the corresponding coefficients, is included in the update equation (Duttweiler, 2000). As a result, the large coefficients at a given iteration get significantly more update energy than the small ones. The parameter  $\beta$  is a fixed step-size factor,  $\delta$  is a small constant needed in order to avoid division by zero, and  $\delta_p$  and  $\rho$  are small positive constants which are important when all the coefficients are zero (such as in the beginning of the adaptation process) or when a coefficient is much smaller than the largest one.

Initialization (typical values)
$\delta_p = \delta = 0.01, \beta = 0.25, \rho = 0.01$
$\mathbf{w}(0) = [w_0(0) \quad w_1(0) \quad \cdots \quad w_{N-1}(0)]^T = \mathbf{0}$
Processing and Adaptation
For $n = 0, 1, 2, \cdots$
$\mathbf{x}(n) = [x(n) \quad x(n - 1) \quad \cdots \quad x(n - N + 1)]^T$
$y(n) = \mathbf{x}^T(n)\mathbf{w}(n)$
$e(n) = d(n) - y(n)$
$\gamma_{min}(n) = \rho \max\{\delta_p,  w_0(n) , \cdots,  w_{N-1}(n) \}$
For $i = 0, 1, \cdots, N - 1$
$\gamma_i(n) = \max\{\gamma_{min}(n),  w_i(n) \}$
End
For $i = 0, 1, \cdots, N - 1$
$g_i(n) = \frac{\gamma_i(n)}{\frac{1}{N} \sum_{j=0}^{N-1} \gamma_j(n)}$
End
$\Gamma(n) = \text{diag}\{g_0(n), \cdots, g_{N-1}(n)\}$
$\mathbf{w}(n + 1) = \mathbf{w}(n) + \beta \frac{\Gamma(n)\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\Gamma(n)\mathbf{x}(n) + \delta}$
End

Table 2. PNLMS Algorithm

Figure 3 displays the experimental MSE evolutions of the PNLMS and NLMS algorithms for the three channels of Fig. 2 with white Gaussian noise input. In all experiments a white Gaussian measurement noise of variance  $\sigma_v^2 = 10^{-6}$  was added to the desired signal. It can

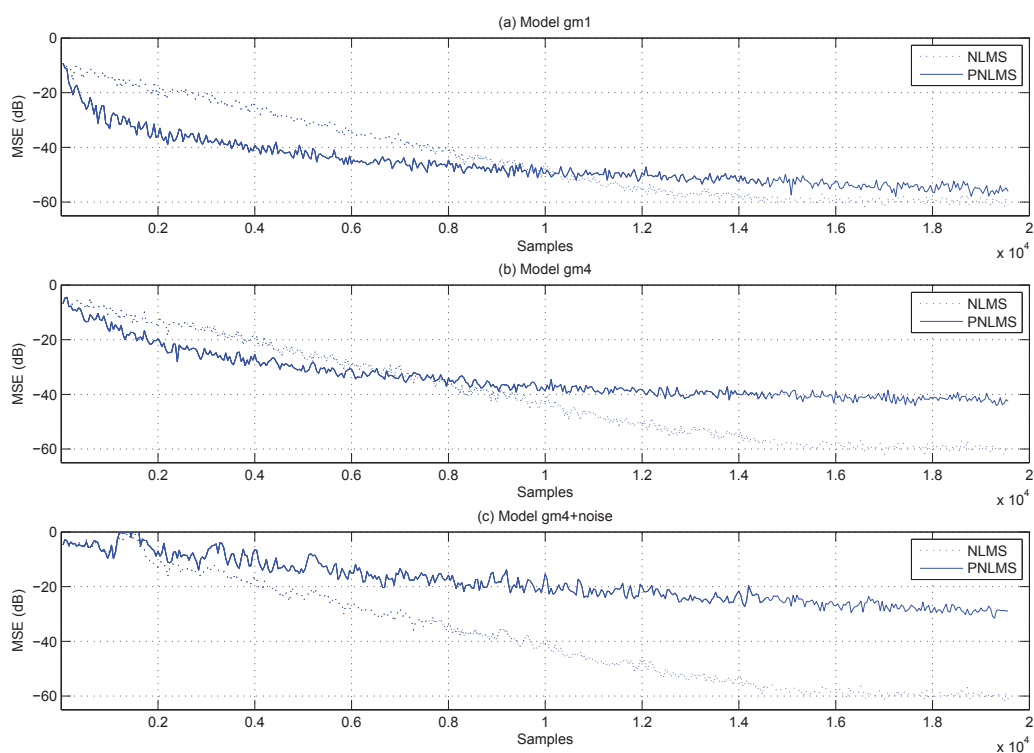


Fig. 3. MSE evolution for the PNLMS and NLMS algorithms for white noise input and channels (a) gm1, (b) gm4 and (c) gm4+noise.

be observed in Fig. 3 that the PNLMS algorithm converges much faster than the NLMS algorithm for the sparse channel gm1. However, for the dispersive channel gm4+noise the PNLMS behaves much worse than the NLMS. For channel gm4 the PNLMS algorithm presents a fast initial convergence, which is significantly reduced after 2000 iterations, becoming slower than that of the NLMS algorithm.

3.2 IPNLMS Algorithm

In the improved proportionate normalized least mean-square (IPNLMS) algorithm, the individual step-sizes are a compromise between the NLMS and the PNLMS step-sizes, resulting in a better convergence for different degrees of sparseness of the impulse response (Benesty & Gay, 2002). The IPNLMS algorithm is listed in Table 3. It can be observed that for  $\alpha = -1$  the step-size control matrix  $\Gamma(n)$  reduces to  $\frac{1}{N}\mathbf{I}$  and hence the IPNLMS and NLMS algorithms turn identical. For  $\alpha = 1$ , the elements of  $\Gamma(n)$  become proportional to the absolute values of the coefficients, in which case the IPNLMS and PNLMS algorithms show practically the same behavior. A typical value for this parameter is  $\alpha = -0.5$ . Figure 4 presents the experimental MSE evolutions of the IPNLMS and NLMS algorithms for the three channels of Fig. 2 with white Gaussian noise input. From this figure, it can be observed that for the sparse channel gm1, the IPNLMS algorithm produces similar performance as the PNLMS algorithm, that is, significantly better than the NLMS algorithm. For the dispersive channel gm4+noise, the IPNLMS performance is similar to that of the NLMS algorithm, not presenting the severe convergence degradation of the PNLMS algorithm. For channel

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Initialization (typical values)
$\delta = 0.01, \epsilon = 0.001, \beta = 0.25, \alpha = -0.5$
$\mathbf{w}(0) = [w_0(0) \ w_1(0) \ \cdots \ w_{N-1}(0)]^T = \mathbf{0}$
Processing and Adaptation
For $n = 0, 1, 2, \dots$
$\mathbf{x}(n) = [x(n) \ x(n-1) \ \cdots \ x(n-N+1)]^T$
$y(n) = \mathbf{x}^T(n)\mathbf{w}(n)$
$e(n) = d(n) - y(n)$
For $i = 0, 1, \dots, N-1$
$g_i(n) = \frac{1-\alpha}{2N} + \frac{(1+\alpha) w_i(n) }{2\sum_{j=0}^{N-1} w_j(n)  + \epsilon}$
End
$\mathbf{\Gamma}(n) = \text{diag}\{g_0(n), \dots, g_{N-1}(n)\}$
$\mathbf{w}(n+1) = \mathbf{w}(n) + \beta \frac{\mathbf{\Gamma}(n)\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{\Gamma}(n)\mathbf{x}(n) + \delta}$
End

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Table 3. IPNLMS Algorithm

gm4, the IPNLMS algorithm does not present the performance degradation (after the initial convergence period) observed in the PNLMS algorithm; however, there is almost no gain in the initial convergence speed when compared to the NLMS algorithm.

### 3.3 MPNLMS and SPNLMS Algorithms

In the  $\mu$ -law improved proportionate normalized least mean-square (MPNLMS) algorithm, the step-sizes are optimal in the sense of minimizing the convergence rate (considering white noise input signal) (Deng & Doroslovacki, 2006). The resulting algorithm employs a non-linear (logarithm) function of the coefficients in the step-size control. A simplified version of the MPNLMS, referred to as the segmented PNLMS (SPNLMS) algorithm, also proposed in (Deng & Doroslovacki, 2006), employs a segmented linear function in order to reduce its computational complexity. These two algorithms are presented in Table 4, where the function  $F(\cdot)$  is defined as

$$F(x) = \ln(1 + \mu x) \quad (2)$$

for the MPNLMS algorithm and

$$F(x) = \begin{cases} 600x, & x < 0.005 \\ 3, & x \geq 0.005 \end{cases} \quad (3)$$

for the SPNLMS algorithm (Deng & Doroslovacki, 2006).

Figure 5 shows the experimental MSE evolutions of the MPNLMS and NLMS algorithms for the three channels of Fig. 2 with white Gaussian noise input. From this figure, it can be noticed



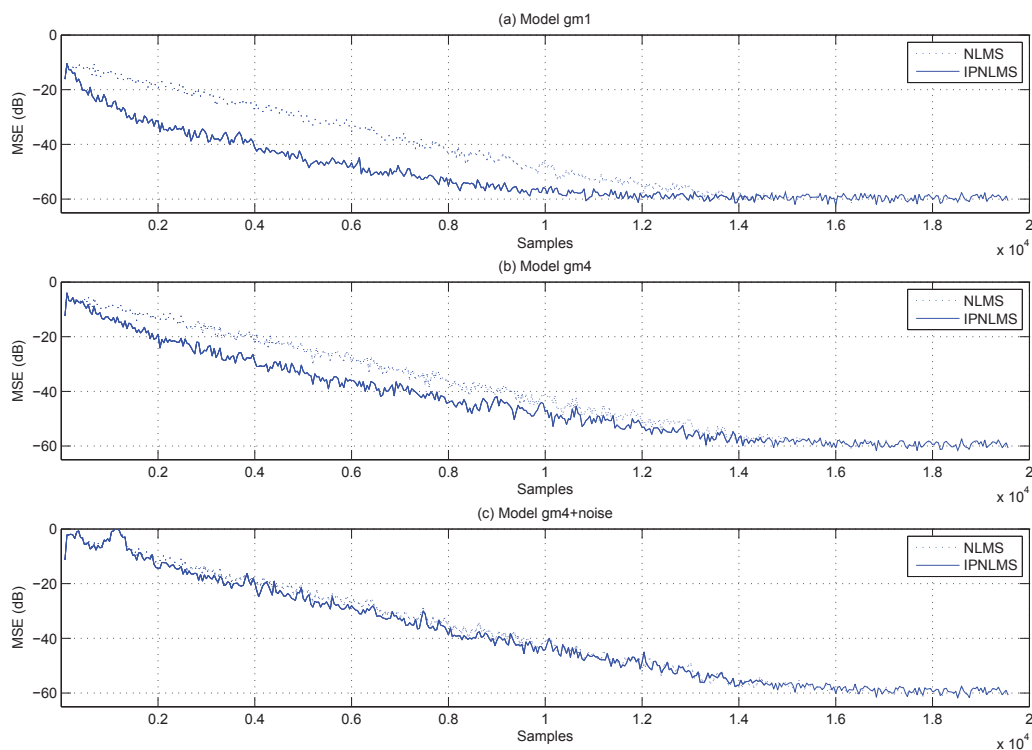


Fig. 4. MSE evolution for the IPNLMS and NLMS algorithms for white noise input and channels (a) gm1, (b) gm4 and (c) gm4+noise.

that the MPNLMS algorithm presents significantly faster convergence, when compared to the NLMS, PNLMS and IPNLMS algorithms, mainly for the sparse channels gm1 and gm4. However, for the dispersive channel gm4+noise, its convergence is severely degraded, being much slower than that of the NLMS algorithm.

Figure 6 presents the experimental MSE evolutions of the SPNLMS and NLMS algorithms for the three channels of Fig. 2 with white Gaussian noise input. Comparing Figs. 5 and 6, it can be verified that the use of the simplified non-linear function does not deteriorate meaningfully the performance of the MPNLMS algorithm.

### 3.4 Variable-Parameter IMPNLMS Algorithm

The variable-parameter improved  $\mu$ -law PNLMS (IMPNLMS) algorithm was proposed in (L. Liu & Saiki, 2008). In this algorithm, the channel sparseness measure of Eq. (1) was incorporated into the  $\mu$ -law PNLMS algorithm in order to improve the adaptation convergence for dispersive channels. Since the real channel coefficients are not available, the corresponding sparseness measure is estimated recursively using the current adaptive filter coefficients. The resulting algorithm is summarized in Table 5, where the parameter  $\alpha(n)$ , which in the improved PNLMS algorithm of Table 4 was a predetermined fixed factor, is made variable and related to the estimated impulse response sparseness measure  $\zeta_{\mathbf{w}}(n)$ . In addition, also proposed in (L. Liu & Saiki, 2008), was the use of the line segment function

$$F(x) = \begin{cases} 400x, & x < 0.005 \\ 8.51|x| + 1.96, & x \geq 0.005 \end{cases} \quad (4)$$



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Initialization (typical values)
$\delta_p = \delta = 0.01, \beta = 0.25, \rho = 1/N$
$\mathbf{w}(0) = [w_0(0) \ w_1(0) \ \cdots \ w_{N-1}(0)]^T = \mathbf{0}$
Processing and Adaptation
For $n = 0, 1, 2, \dots$
$\mathbf{x}(n) = [x(n) \ x(n-1) \ \cdots \ x(n-N+1)]^T$
$y(n) = \mathbf{x}^T(n)\mathbf{w}(n)$
$e(n) = d(n) - y(n)$
$\gamma_{min}(n) = \rho \max\{\delta_p, F( w_0(n) ), \dots, F( w_{N-1}(n) )\}$
For $i = 0, 1, \dots, N-1$
$\gamma_i(n) = \max\{\gamma_{min}(n), F( w_i(n) )\}$
End
For $i = 0, 1, \dots, N-1$
$g_i(n) = \frac{\gamma_i(n)}{\frac{1}{N} \sum_{j=0}^{N-1}  \gamma_j(n) }$
End
$\mathbf{\Gamma}(n) = \text{diag}\{g_0(n), \dots, g_{N-1}(n)\}$
$\mathbf{w}(n+1) = \mathbf{w}(n) + \beta \frac{\mathbf{\Gamma}(n)\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{\Gamma}(n)\mathbf{x}(n) + \delta}$
End

---

Table 4. MPNLMS and SPNLMS Algorithms

with which the steady-state misalignment is decreased in comparison to those of the MPNLMS and SPNLMS algorithms (Eqs. (2) and (3)), which place too much emphasis on small coefficients.

Figure 7 presents the experimental MSE evolutions of the IMPNLMS and NLMS algorithms for the three channels of Fig. 2 with white Gaussian noise input. The good convergence behavior of the IMPNLMS algorithm for the sparse and dispersive channels can be observed in this figure.

#### 4. Wavelet-based proportionate-type NLMS Algorithms

Although the proportionate-type NLMS algorithms produce better convergence than the NLMS algorithm when modeling sparse impulse responses with white noise inputs, they suffer from the same performance degradation as the NLMS when the excitation signal is colored. Figure 8 illustrates the performance of the NLMS, MPNLMS and IMPNLMS algorithms for a colored input signal, generated by passing a white Gaussian noise with zero-mean and unit variance through the filter with transfer function

$$H(z) = \frac{0.25\sqrt{3}}{1 - 1.5z^{-1} - 0.25z^{-2}}. \quad (5)$$

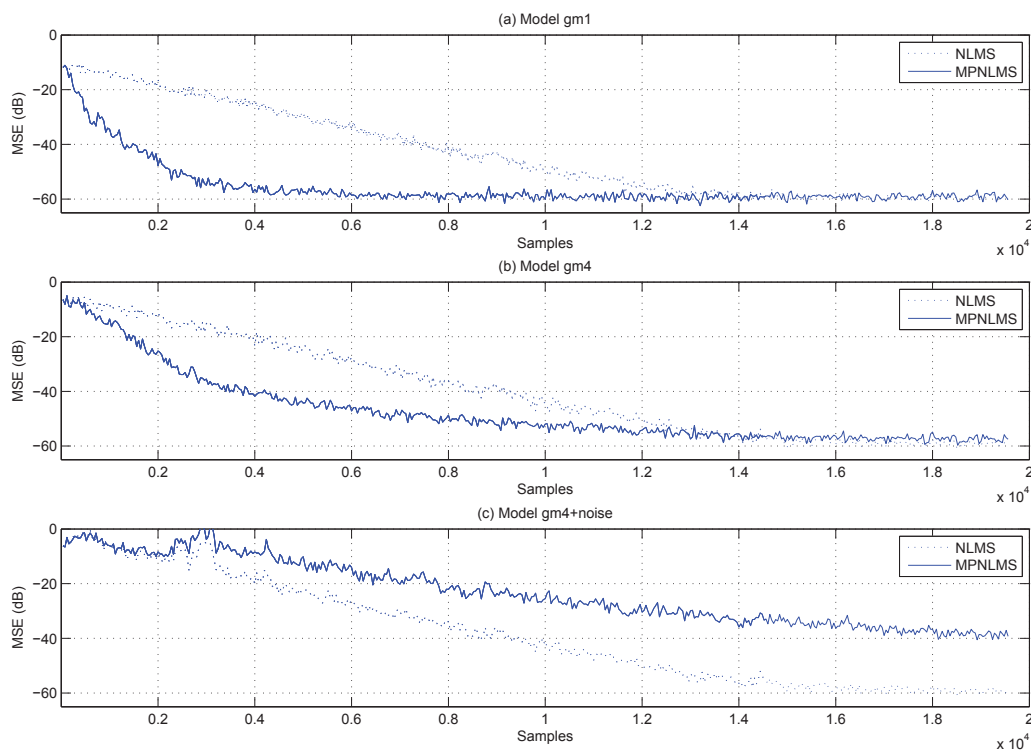


Fig. 5. MSE evolution for the MPNLMS and NLMS algorithms for white noise input and channels (a) gm1, (b) gm4 and (c) gm4+noise.

Such input signal has power spectrum similar to speech signal (Ikeda & Sugiyama, 1994). In order to improve the adaptation speed of these algorithms in dispersive channels with colored input signals, the use of wavelet transform was proposed independently in (Deng & Doroslovacki, 2007) and (Petraglia & Barboza, 2008). The resulting algorithms are described next.

#### 4.1 Wavelet-based MPNLMS algorithm (Transform-Domain)

The transform-domain proportionate technique presented in (Deng & Doroslovacki, 2007) employs the  $\mu$ -law PNLMS algorithm in the wavelet-domain. Besides improving the convergence of the conventional algorithms owing to power normalization of the step-sizes, the wavelet-domain approach may be advantageous in the modeling of non-sparse impulse responses, since they usually become more sparse in the transformed domain. The resulting algorithm, termed as wavelet-based MPNLMS in the transform-domain (WMPNLMS-TD), is described in Table 6. The transformation matrix  $\mathbf{T}$  is formed by the coefficients of the wavelet filters, as defined in (Attallah, 2000), the vector  $\mathbf{z}(n) = [z_0(n) \cdots z_{N-1}(n)]^T = \mathbf{T}\mathbf{x}(n)$  is the transformed input vector and  $p_i(n)$  is the power estimate of  $z_i(n)$ . The matrix  $\mathbf{D}(n)$  introduced in the update equation assigns a different step-size normalization to each coefficient.

Figure 9 presents the experimental MSE evolutions of the WMPNLMS-TD algorithm for the three channels of Fig. 2 and colored noise input with the following wavelet functions: Haar, Daubechies 2 (Db2) and Daubechies 4 (Db4). Comparing the simulation results of the

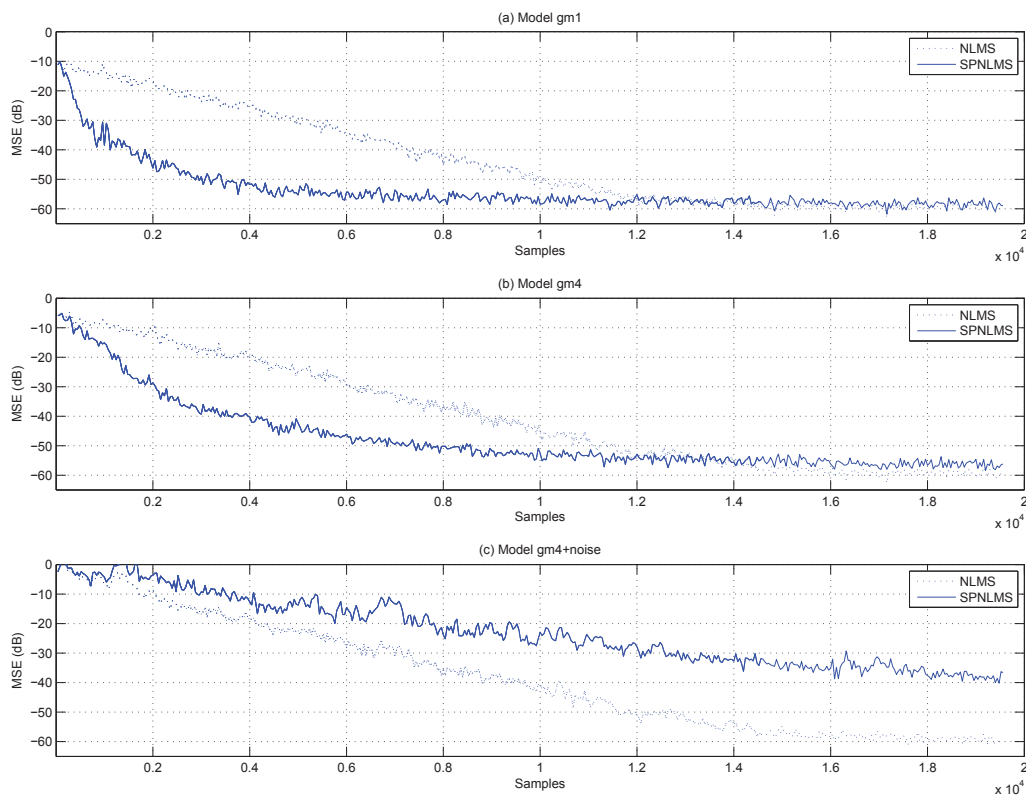


Fig. 6. MSE evolution for the SPNLMS and NLMS algorithms for white noise input and channels (a) gm1, (b) gm4 and (c) gm4+noise.

WMPNLMS-TD algorithm with those of Fig. 8, it can be observed that, for colored input, there is a significant improvement in the performance of the MPNLMS algorithm when it is applied in the wavelet-domain. The more selective wavelet (Daubechies 4) produced the fastest convergence, as expected.

#### 4.2 Wavelet-based MPNLMS Algorithm (Sparse Filters)

The wavelet-based proportionate NLMS algorithm proposed in (Petraglia & Barboza, 2008) employs a wavelet transform and sparse adaptive filters. Illustrated in Fig. 10, the wavelet transform is represented by a non-uniform filter bank with analysis filters  $H_k(z)$ , and sparse adaptive subfilters  $G_k(z^{L_k})$  (Petraglia & Torres, 2002). For an octave-band wavelet, the equivalent analysis filters of the  $M$ -channel filter bank are (Vaidyanathan, 1993)

$$\begin{aligned}
 H_0(z) &= \prod_{j=0}^{M-2} H^0(z^{2^j}), \\
 H_k(z) &= H^1(z^{2^{M-1-k}}) \prod_{j=0}^{M-k-2} H^0(z^{2^j}), \quad k = 1, \dots, M-1,
 \end{aligned} \tag{6}$$

---

Initialization (typical values)
$\delta = 0.01, \epsilon = 0.001, \beta = 0.25, \lambda = 0.1, \xi(-1) = 0.96$
$\mathbf{w}(0) = [w_0(0) \ w_1(0) \ \cdots \ w_{N-1}(0)]^T = \mathbf{0}$
Processing and Adaptation
For $n = 0, 1, 2, \dots$
$\mathbf{x}(n) = [x(n) \ x(n-1) \ \cdots \ x(n-N+1)]^T$
$y(n) = \mathbf{x}^T(n)\mathbf{w}(n)$
$e(n) = d(n) - y(n)$
$\xi_{\mathbf{w}}(n) = \frac{N}{N - \sqrt{N}} \left( 1 - \frac{\sum_{j=0}^{N-1}  w_j(n) }{\sqrt{N \sum_{j=0}^{N-1}  w_j(n) ^2}} \right)$
$\xi(n) = (1 - \lambda)\xi(n-1) + \lambda\xi_{\mathbf{w}}(n)$
$\alpha(n) = 2\xi(n) - 1$
For $i = 0, 1, \dots, N-1$
$g_i(n) = \frac{1 - \alpha(n)}{2N} + \frac{(1 + \alpha(n))F( w_i(n) )}{2 \sum_{j=0}^{N-1} F( w_j(n) ) + \epsilon}$
End
$\mathbf{\Gamma}(n) = \text{diag}\{g_0(n), \dots, g_{N-1}(n)\}$
$\mathbf{w}(n+1) = \mathbf{w}(n) + \beta \frac{\mathbf{\Gamma}(n)\mathbf{x}(n)e(n)}{\mathbf{x}^T(n)\mathbf{\Gamma}(n)\mathbf{x}(n) + \delta}$
End

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Table 5. Variable-Parameter IMPNLMS Algorithms

where  $H^0(z)$  and  $H^1(z)$  are, respectively, the lowpass and high-pass filters associated with the wavelet functions (Vaidyanathan, 1993). The sparsity factors are

$$L_0 = 2^{M-1}, \quad L_k = 2^{M-k}, \quad k = 1, \dots, M-1, \quad (7)$$

and the delays  $\Delta_k$  in Fig. 10, introduced for the purpose of matching the delays of the different length analysis filters, are given by  $\Delta_k = N_{H_0} - N_{H_k}$ , where  $N_{H_k}$  is the length of the  $k$ th analysis filter. This structure yields an additional system delay (compared to a direct-form FIR structure) equal to  $\Delta_D = N_{H_0}$ . For the modeling of a length  $N$  FIR system, the number of adaptive coefficients of the subfilters  $G_k(z)$  (non-zero coefficients of  $G_k(z^{L_k})$ ) should be at least

$$N_k = \left\lceil \frac{N + N_{F_k}}{L_k} \right\rceil \quad (8)$$

where  $N_{F_k}$  are the lengths of the corresponding synthesis filters which, when associated to the analysis filters  $H_k(z)$ , lead to perfect reconstruction.

The resulting proportionate-type NLMS algorithm, referred here as wavelet-based MPNLMS with sparse filters (WMPNLMS-SF), is presented in Table 7, where  $x_k(n)$  is the input signal of

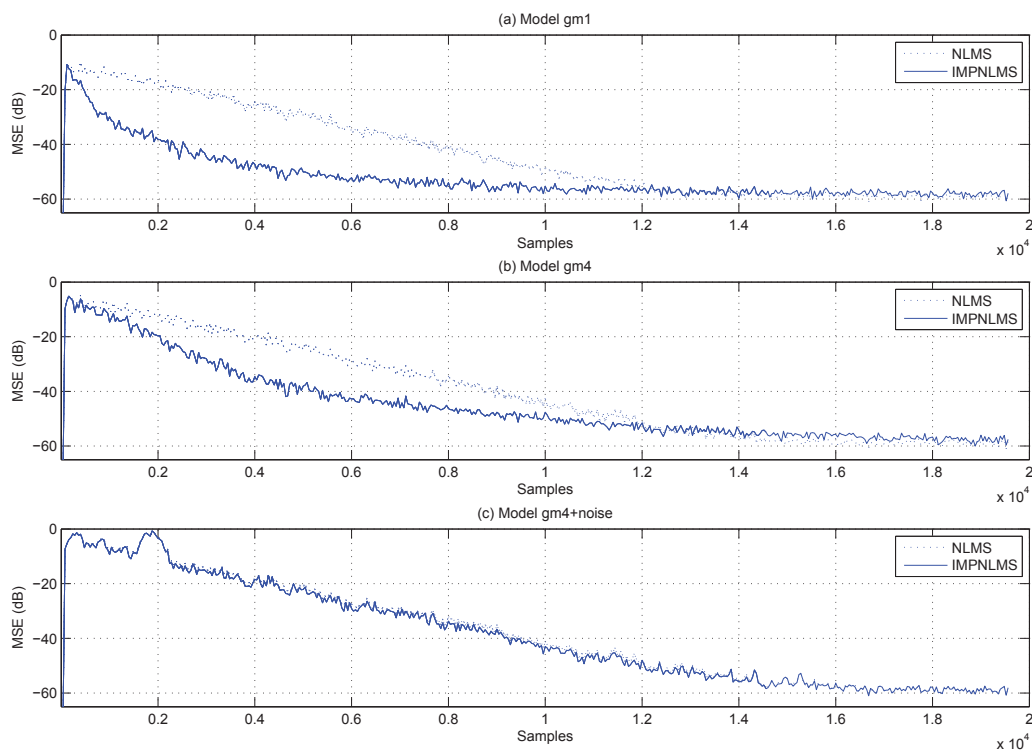


Fig. 7. MSE evolution for the IMPNLMS and NLMS algorithms for white noise input and channels (a) gm1, (b) gm4 and (c) gm4+noise.

the  $k$ -th subband ( $x(n)$  filtered by  $H_k(z)$ ) and  $w_{k,i}$  is the  $i$ -th coefficient of  $G_k(z)$ . For colored input signals, the WMPNLMS-SF algorithm presents faster convergence than the time-domain MPNLMS algorithm, since its step-size normalization strategy uses the input power at the different frequency bands. It should be observed that the step-size normalization takes into account the absolute value of each coefficient in comparison to the values of the corresponding subfilter coefficients (and not to all coefficients, as is done in the WMPNLMS-TD algorithm (Deng & Doroslovacki, 2007)). As a result, the large coefficients of a given subfilter get significantly more of the update energy assigned to the corresponding subband than the small ones.

Figure 11 shows the experimental MSE evolution of the WMPNLMS-SF algorithm for a two-level decomposition ( $M = 3$  subbands) using the following wavelet functions: Haar, Daubechies 2, Daubechies 4 and Biorthogonal 4.4 (Bior4.4). With such wavelets, the increase in the complexity (compared to the MPNLMS algorithm) and the delay introduced by the decomposition are not very large, owing to the reduced orders of the corresponding prototype filters.

From Figs. 8 and 11 we conclude that, for the colored input signal employed in the simulations, the use of the very simple Haar wavelet transform improves significantly the convergence rate of the MPNLMS algorithm. The fastest convergence of the WMPNLMS-SF algorithm was obtained with the more selective Daubechies 4 and Biorthogonal 4.4 wavelets.

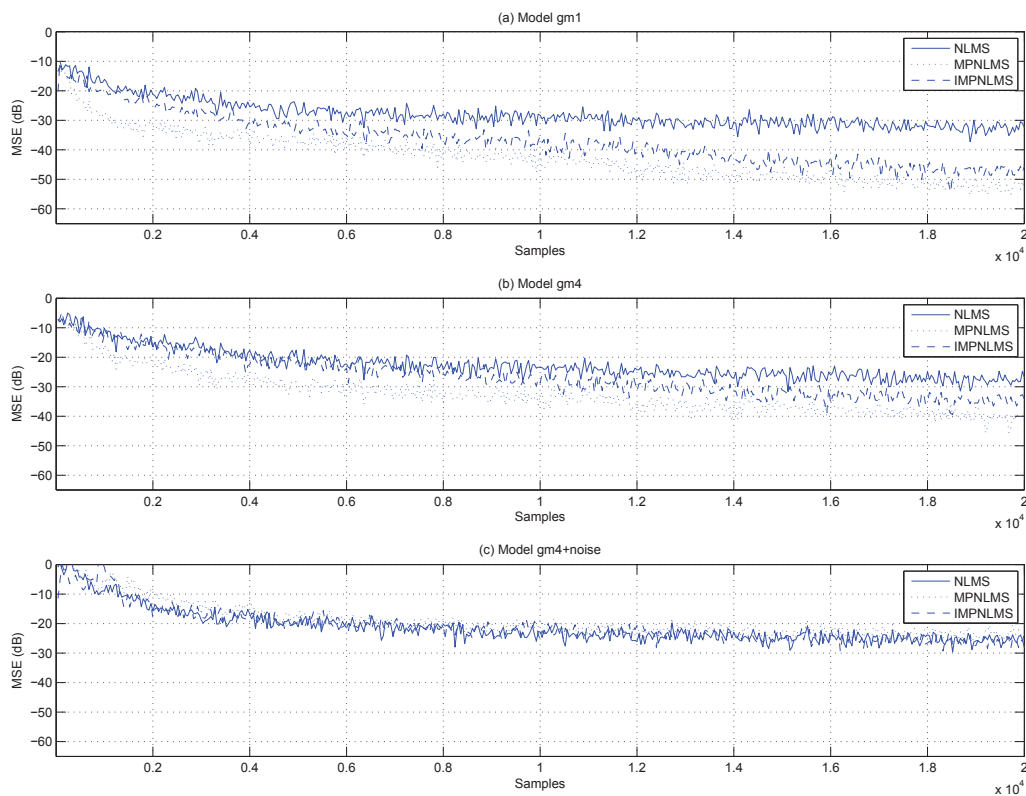


Fig. 8. MSE evolution for the NLMS, MPNLMS and IMPNLMS algorithms for colored noise input and channels (a) gm1, (b) gm4 and (c) gm4+noise.

Comparisons with Figs. 9 and 11 indicate that the step-size normalization strategy adopted by the WMPNLMS-SF method is advantageous when compared to that of the WMPNLMS-TD method.

The convergence performance of the WMPNLMS-SF algorithm for non-sparse channels can be improved by using the IMPNLMS algorithm independently for each adaptive subfilter. Figure 12 shows the MSE evolution of the resulting algorithm, referred therein as WIMPNLMS-SF, for different wavelets with colored input signal. The improvement in the convergence rate for the dispersive channel gm4+noise can be observed by comparing Figs. 11(c) and 12(c).

## 5. Conclusions

In this chapter we presented a family of algorithms developed in the last years for improving the convergence of adaptive filters when modeling sparse impulse responses. The performances of the described techniques, known as proportionate-type LMS algorithms, were illustrated through computer simulations in the identification of the digital network channels of ITU-T recommendation G.168. The first proposed approach, the PNLMS algorithm, was shown to produce fast initial convergence for sparse impulse responses, followed by a significant reduction after the fast initial period. Also, its performance was poor for non-sparse impulse responses. Improved versions of the PNLMS algorithm were then described and

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Initialization (typical values)

**T**: wavelet transform matrix

$\delta_p = \delta = 0.01, \beta = 0.25/N, \rho = 0.01, \alpha = 0.9$

$\mathbf{w}(0) = [w_0(0) \ w_1(0) \ \cdots \ w_{N-1}(0)]^T = \mathbf{0}$

Processing and Adaptation

For  $n = 0, 1, 2, \dots$

$\mathbf{x}(n) = [x(n) \ x(n-1) \ \cdots \ x(n-N+1)]^T$

$\mathbf{z}(n) = \mathbf{T}\mathbf{x}(n)$

$y(n) = \mathbf{z}^T(n)\mathbf{w}(n)$

$e(n) = d(n) - y(n)$

$\gamma_{\min}(n) = \rho \max\{\delta_p, F(|w_0(n)|), \dots, F(|w_{N-1}(n)|)\}$

For  $i = 0, 1, \dots, N-1$

$\gamma_i(n) = \max\{\gamma_{\min}(n), F(|w_i(n)|)\}$

End

For  $i = 0, 1, \dots, N-1$

$g_i(n) = \frac{\gamma_i(n)}{\frac{1}{N} \sum_{j=0}^{N-1} |\gamma_j(n)|}$

$p_i(n) = \alpha p_i(n-1) + (1-\alpha) * z_i^2(n)$

End

$\mathbf{D}(n) = \text{diag}\{1/p_0(n), \dots, 1/p_{N-1}(n)\}$

$\mathbf{\Gamma}(n) = \text{diag}\{g_0(n), \dots, g_{N-1}(n)\}$

$\mathbf{w}(n+1) = \mathbf{w}(n) + \beta \mathbf{D}(n) \mathbf{\Gamma}(n) \mathbf{z}(n) e(n)$

End

---

Table 6. WMPNLMS-TD Algorithm

their advantages were verified in the simulation results. Whereas the IPNLMS algorithm produced enhanced convergence performance when modeling dispersive impulse responses, the MPNLMS employed a non-linear function of the coefficients in the step-size normalization in order to obtain optimal convergence rate. The combination of these two techniques and the use of a sparseness measure of the impulse response resulted in the variable-parameter IMPNLMS algorithm. The fast convergence rate of the proportionate-type algorithms was limited to white input signals. In order to extend their performance advantages to colored input signals, wavelet-domain algorithms, whose step-size normalization takes into account the value of each coefficient as well as the input signal power in the corresponding frequency band, were described. Simulations showed that the wavelet-domain PNLMS methods present significantly faster convergence rate than do the time-domain PNLMS ones for applications in which the system has sparse impulse responses and is excited with colored input signal.



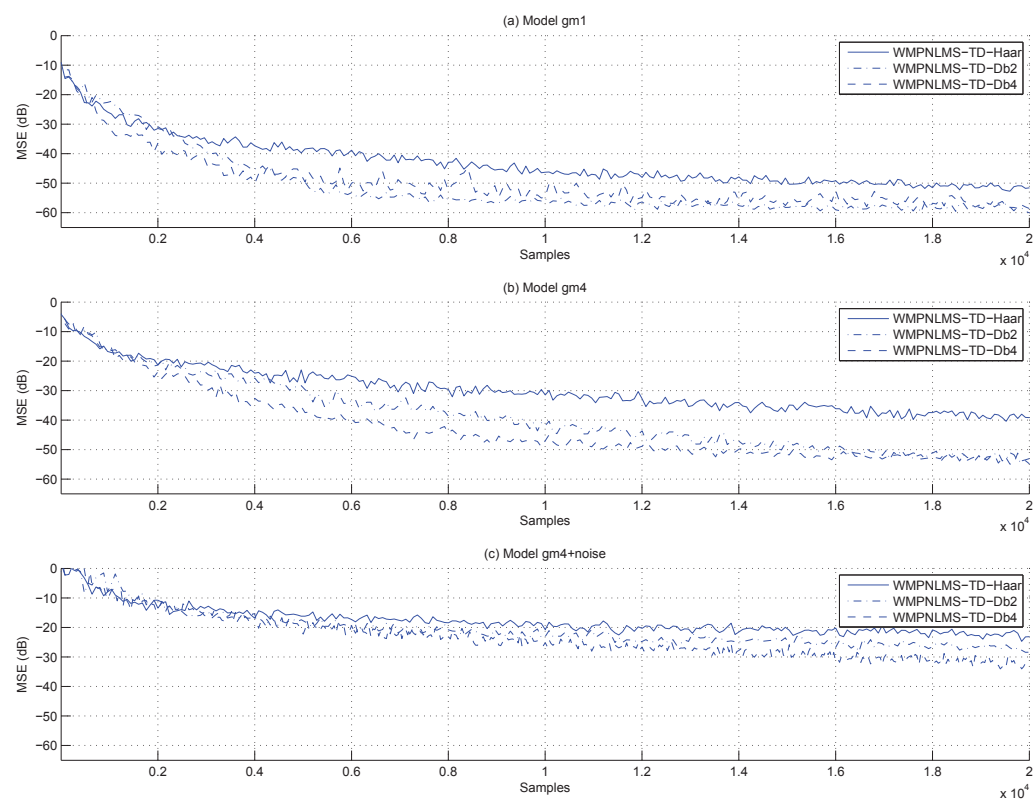


Fig. 9. MSE evolution for the WMPNLMS-TD with Haar, Db2 and Db4 wavelets for colored noise input and channels (a) gm1, (b) gm4 and (c) gm4+noise.

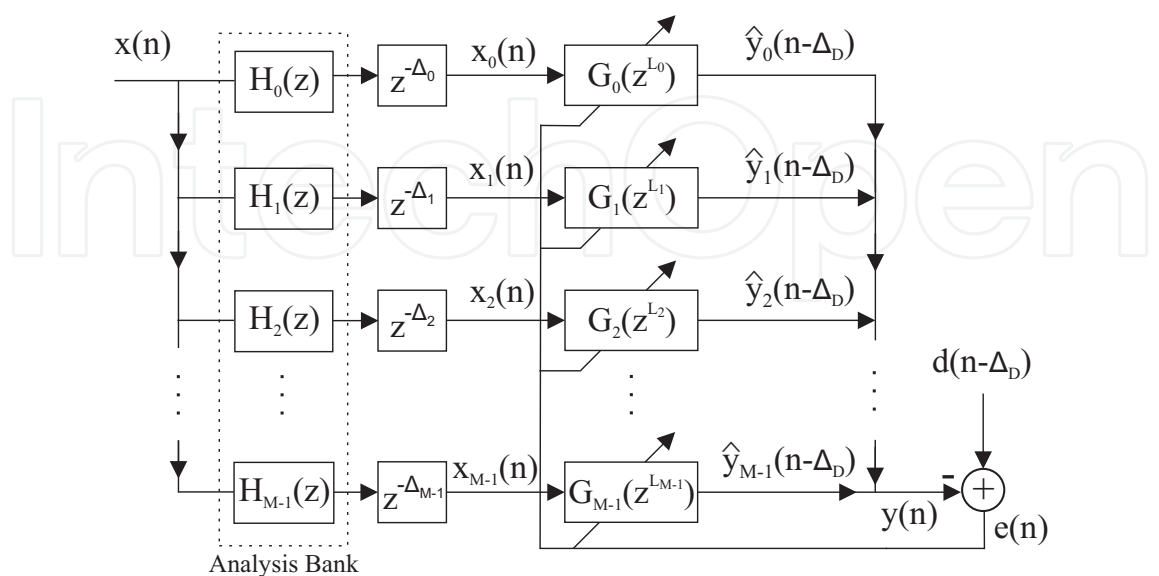


Fig. 10. Adaptive subband structure composed of a wavelet transform and sparse subfilters.

---

**Initialization**

$$\delta_p = \delta = 0.01, \beta = 0.25, \rho = 0.01$$

For  $k = 0, 1, \dots, M - 1$

$$\mathbf{w}_k(0) = [w_{k,0}(0) \ w_{k,1}(0) \ \cdots \ w_{k,N_k-1}(0)]^T = \mathbf{0}$$

End

**Processing and Adaptation**

For  $n = 0, 1, 2, \dots$

For  $k = 0, 1, \dots, M - 1$

$$x_k(n) = \sum_{i=0}^{N_{H_k}-1} h_k(i)x(n-i)$$

$$\mathbf{x}_k(n) = [x_k(n) \ x_k(n-L_k) \ \cdots \ x_k(n-(N_k-1)L_k)]^T$$

$$\hat{y}_k(n - \Delta_D) = \mathbf{x}_k^T(n) \mathbf{w}_k(n)$$

End

$$y(n) = \sum_{k=0}^{M-1} \hat{y}_k(n - \Delta_D)$$

$$e(n) = d(n - \Delta_D) - y(n)$$

For  $k = 0, 1, \dots, M - 1$

$$\gamma_{min,k}(n) = \rho \max\{\delta_p, F(|w_{k,0}(n)|), \dots, F(|w_{k,N_k}(n)|)\}$$

For  $i = 0, 1, \dots, N_{k-1}$

$$\gamma_{k,i}(n) = \max\{\gamma_{min,k}(n), F(|w_{k,i}(n)|)\}$$

End

For  $i = 0, 1, \dots, N_{k-1}$

$$g_{k,i}(n) = \frac{\gamma_{k,i}(n)}{\frac{1}{N_k} \sum_{j=0}^{N_k-1} \gamma_{k,j}(n)}$$

End

$$\mathbf{\Gamma}_k(n) = \text{diag}\{g_{k,0}(n), \dots, g_{k,N}(n)\}$$

$$\mathbf{w}_k(n+1) = \mathbf{w}_k(n) + \beta \frac{\mathbf{\Gamma}_k(n) \mathbf{x}_k(n) e(n)}{\mathbf{x}_k^T(n) \mathbf{\Gamma}_k(n) \mathbf{x}_k(n) + \delta}$$

End

End

---

Table 7. WMPNLMS-SF Algorithm

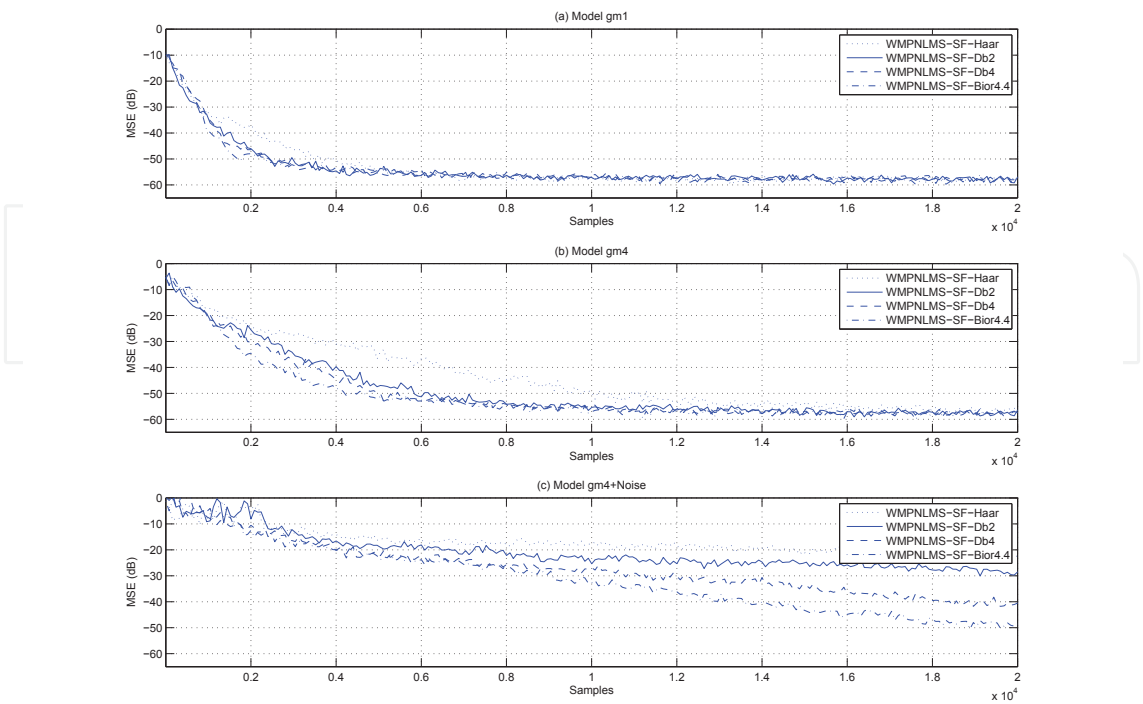


Fig. 11. MSE evolution for the WMPNLMS-SF algorithm with Haar, Db2, Db4 and Bior4.4 wavelets and  $M = 3$ , for colored noise input and channels (a) gm1, (b) gm4 and (c) gm4+noise.

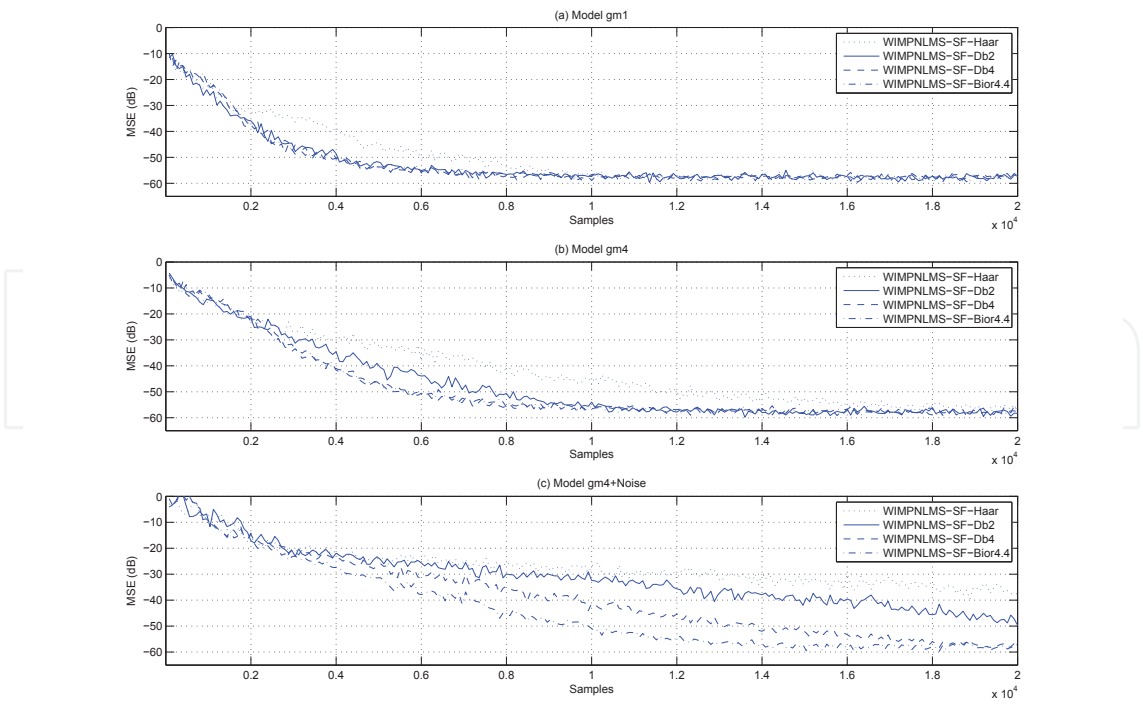
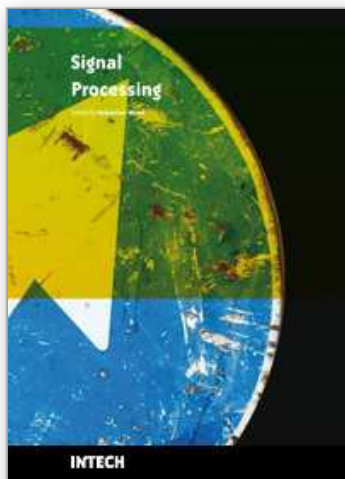


Fig. 12. MSE evolution for the WIMP NLMS-SF algorithm with Haar, Db2, Db4 and Bior4.4 wavelets and  $M = 3$ , for colored noise input and channels (a) gm1, (b) gm4 and (c) gm4+noise.

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## **Signal Processing**

Edited by Sebastian Miron

ISBN 978-953-7619-91-6

Hard cover, 528 pages

**Publisher** InTech

**Published online** 01, March, 2010

**Published in print edition** March, 2010

This book intends to provide highlights of the current research in signal processing area and to offer a snapshot of the recent advances in this field. This work is mainly destined to researchers in the signal processing related areas but it is also accessible to anyone with a scientific background desiring to have an up-to-date overview of this domain. The twenty-five chapters present methodological advances and recent applications of signal processing algorithms in various domains as telecommunications, array processing, biology, cryptography, image and speech processing. The methodologies illustrated in this book, such as sparse signal recovery, are hot topics in the signal processing community at this moment. The editor would like to thank all the authors for their excellent contributions in different areas of signal processing and hopes that this book will be of valuable help to the readers.

### **How to reference**

In order to correctly reference this scholarly work, feel free to copy and paste the following:

Mariane R. Petraglia (2010). New Adaptive Algorithms for the Rapid Identification of Sparse Impulse Responses, Signal Processing, Sebastian Miron (Ed.), ISBN: 978-953-7619-91-6, InTech, Available from: <http://www.intechopen.com/books/signal-processing/new-adaptive-algorithms-for-the-rapid-identification-of-sparse-impulse-responses>

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