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Crystal-like Symmetric Sensor Arrangements for Blind Decorrelation of Isotropic Wavefield

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1. Introduction

Sensor array technique has been widely used for measuring various types of wavefields such as acoustic waves, mechanical vibrations, and electromagnetic waves (1). A common goal of array signal processing is estimating locations of sources or separating source signals based on multiple observations. For obtaining efficient spatial information, the geometrical arrangement of sensors is one of the significant issues in this field. An uniform linear array is the most popular and fundamental one (2; 3), and suiting with purposes, various types of arrays have been considered such as circular, planar, cross-shaped, cylindrical, and spherical arrays.

In this chapter, we discuss the sensor arrangements from a new viewpoint: correlation between channels. Generally, multiply-observed signals have correlation each other, and it becomes larger especially in a small-sized array. In the case, observed signals themselves are not efficient representation due to redundancy between channels. Although they are uncorrelated by appropriate basis transformation, which is corresponding to the diagonalization of the covariance matrix, it depends on the observed wavefield.

However, in isotropic wavefield, there exist special geometrical sensor arrangements, and observed signals by them are commonly uncorrelated by a fixed basis transform. The significances of isotropic wavefield decorrelation are as follows.

- If there is no a priori knowledge to wavefield, the isotropic assumption is simple and natural. It means spatial stationarity.

- It is well known that Fourier coefficients of a temporally stationary periodic signal are uncorrelated each other. The isotropic wavefield decorrelation can be considered as a spatial version of it and decorrelated components represent something like *spatial spectra*.

- The decorrelated representation are also useful for encoding because redundancy between channels is removed.

- It can be applied for several kinds of estimation methods in isotropic noise field such as power spectrum estimation (4), noise reduction (5), and inverse filtering (6).

- The isotropy assumption can be valid even if wavefield is disturbed by sensor array itself. Suppose that microphone array is mounted on a rigid sphere. Although the rigid sphere disturbs acoustic field, due to the symmetry of sphere, the isotropy is still hold.
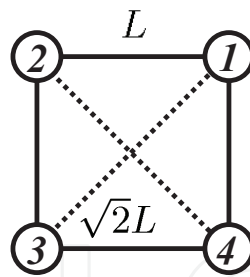
Fig. 1. Square

Although our main concern lies on microphone array, this technique can be applied for different kinds of wavefield sensing. In the following, we mathematically discuss possible sensor arrangements for blind decorrelation.

2. Problem Formulation

Let's consider isotropic wavefield is observed by M sensors. Let $x_m(t)$ be a signal observed by the mth sensor, $X_m(\omega)$ be its Fourier transform, and $\boldsymbol{X}(\omega) = (X_1(\omega)\ X_2(\omega)\ \cdots\ X_M(\omega))^t$ be the vector representation, respectively, where ^{t} denotes transpose operation. The isotropic assumption leads: 1) the power spectrum is the same on each sensor, and 2) the cross spectrum is determined by only a distance between sensors. Under them, by normalizing diagonal elements to unit, the covariance matrix $V(\omega)$ of the observation vector $\boldsymbol{X}(\omega)$ is represented as

$$V(\omega) = E[\boldsymbol{X}(\omega)\boldsymbol{X}(\omega)^h] = \begin{pmatrix} 1 & \Gamma(r_{12},\omega) & \cdots & \Gamma(r_{1n},\omega) \\ \Gamma(r_{21},\omega) & 1 & \cdots & \Gamma(r_{2n},\omega) \\ \vdots & \vdots & \ddots & \vdots \\ \Gamma(r_{n1},\omega) & \Gamma(r_{n2},\omega) & \cdots & 1 \end{pmatrix}, \quad (1)$$

where $E[\cdot]$ denotes expectation operation, ^{h} denotes Hermite transpose, r_{ij} is the distance between sensor i and j, and $\Gamma(r,\omega)$ represents the spatial coherence function of the wavefield (3). Under the isotropic assumption, $V(\omega)$ is a symmetry matrix since $r_{ij} = r_{ji}$. Then, there exist an orthogonal matrix U for diagonalizing $V(\omega)$. Our goal here is to find special sensor arrangements and corresponding unitary matrices U such that $U^t V(\omega) U$ is constantly diagonal for any coherence function $\Gamma(r,\omega)$. We call this kind of decorrelation *blind decorrelation* because we don't have to know each element of $V(\omega)$ and the diagonalization matrix U is determined by only sensor arrangements. For simplicity, we hereafter omit ω and represents the covariance matrix of the observation vector by just V.

Intuitively, it seems to be impossible since a diagonalization matrix U generally depends on the elements of V. But suppose that four sensors are arrayed at vertices of a square. There are only two distances among the vertices in a square: one is the length of a line L, another is the length of a diagonal $\sqrt{2}L$. Then, numbering sensors circularly shown in Fig. 1 and letting $a = \Gamma(L,\omega)$ and $b = \Gamma(\sqrt{2}L,\omega)$, the covariance matrix is represented as the following form

$$V = \begin{pmatrix} 1 & a & b & a \\ a & 1 & a & b \\ b & a & 1 & a \\ a & b & a & 1 \end{pmatrix} \quad (2)$$

for any ω and any coherence function $\Gamma(r, \omega)$. Since it is a circulant matrix, it is diagonalized by the fourth order DFT matrix Z_4 or its real-valued version $\tilde{Z}_4$ defined by

$$
\tilde{Z}_4 = \begin{pmatrix}
\frac{1}{2}\cos\frac{2\pi \cdot 0 \cdot 0}{4} & \frac{1}{\sqrt{2}}\cos\frac{2\pi \cdot 1 \cdot 0}{4} & \frac{1}{\sqrt{2}}\sin\frac{2\pi \cdot 1 \cdot 0}{4} & \frac{1}{2}\cos\frac{2\pi \cdot 2 \cdot 0}{4} \\
\frac{1}{2}\cos\frac{2\pi \cdot 0 \cdot 1}{4} & \frac{1}{\sqrt{2}}\cos\frac{2\pi \cdot 1 \cdot 1}{4} & \frac{1}{\sqrt{2}}\sin\frac{2\pi \cdot 1 \cdot 1}{4} & \frac{1}{2}\cos\frac{2\pi \cdot 2 \cdot 1}{4} \\
\frac{1}{2}\cos\frac{2\pi \cdot 0 \cdot 2}{4} & \frac{1}{\sqrt{2}}\cos\frac{2\pi \cdot 1 \cdot 2}{4} & \frac{1}{\sqrt{2}}\sin\frac{2\pi \cdot 1 \cdot 2}{4} & \frac{1}{2}\cos\frac{2\pi \cdot 2 \cdot 2}{4} \\
\frac{1}{2}\cos\frac{2\pi \cdot 0 \cdot 3}{4} & \frac{1}{\sqrt{2}}\cos\frac{2\pi \cdot 1 \cdot 3}{4} & \frac{1}{\sqrt{2}}\sin\frac{2\pi \cdot 1 \cdot 3}{4} & \frac{1}{2}\cos\frac{2\pi \cdot 2 \cdot 3}{4}
\end{pmatrix} \tag{3}
$$

$$
= \begin{pmatrix}
1/2 & 1/\sqrt{2} & 0 & 1/2 \\
1/2 & 0 & 1/\sqrt{2} & -1/2 \\
1/2 & -1/\sqrt{2} & 0 & 1/2 \\
1/2 & 0 & -1/\sqrt{2} & -1/2
\end{pmatrix} \tag{4}
$$

such as

$$
\tilde{Z}_4^t V \tilde{Z}_4 = \begin{pmatrix}
2a+b+1 & 0 & 0 & 0 \\
0 & -b+1 & 0 & 0 \\
0 & 0 & -b+1 & 0 \\
0 & 0 & 0 & -2a+b+1
\end{pmatrix}. \tag{5}
$$

This diagonalization can be performed at any frequency ω because $\tilde{Z}_4$ is independent of a and b. It means the following basis-transformed observations:

$$
\begin{align}
y_1(t) &= x_1(t) + x_2(t) + x_3(t) + x_4(t) \tag{6} \\
y_2(t) &= x_1(t) - x_3(t) \tag{7} \\
y_3(t) &= x_2(t) - x_4(t) \tag{8} \\
y_4(t) &= x_1(t) - x_2(t) + x_3(t) - x_4(t) \tag{9}
\end{align}
$$

are uncorrelated each other in any isotropic field. The problem we concern here is a generalization of it.

If $U^t V U$ is diagonalized as

$$
U^t V U = \begin{pmatrix}
\gamma_1 & 0 & \cdots & 0 \\
0 & \gamma_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \gamma_M
\end{pmatrix}, \tag{10}
$$

V is represented as

$$
V = U \begin{pmatrix}
\gamma_1 & 0 & \cdots & 0 \\
0 & \gamma_2 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & \gamma_M
\end{pmatrix} U^t. \tag{11}
$$

Then, for blind decorrelation, one of the necessary conditions is that V is represented by only M parameters $(\gamma_1 \cdots \gamma_M)$ at most. It means there should exist at most M kinds of distances between sensors. Generally, when sensor arrangement has some symmetry, the number of kinds
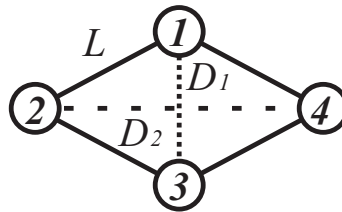
Fig. 2. Argyle

of distances between sensors is smaller. But what kind of symmetry the sensor arrangement should have for blind decorrelation is not trivial. For instance, suppose an argyle arrangement shown in Fig. 2. An argyle is one of symmetrical shapes and there are three kinds of distances among sensors. In arranging sensors shown in Fig. 2, the covariance matrix has the following form:

$$V = \begin{pmatrix} 1 & a & b & a \\ a & 1 & a & c \\ b & a & 1 & a \\ a & c & a & 1 \end{pmatrix}. \tag{12}$$

Despite of the symmetry of argyle, there are no matrices U for diagonalizing V in eq. (12) independent of a, b and c. It can be easily checked as the following (7). V in eq. (12) is decomposed as

$$V = I + aP_1 + bP_2 + cP_3 \tag{13}$$

where I is an identity matrix and

$$P_1 = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}, \tag{14}$$

$$P_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{15}$$

$$P_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}. \tag{16}$$

For diagonalizing V by an unitary matrix U independently of a, b and c, it is necessary that P_1, P_2 and P_3 have to be jointly diagonalized, which is equivalent to the condition that P_1, P_2 and P_3 are commutative each other. However,

$$P_1 P_2 - P_2 P_1 = \begin{pmatrix} 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ -1 & 0 & -1 & 0 \end{pmatrix}, \tag{17}$$

which means P_1 and P_2 are not commutative. Therefore, there are no matrices U to jointly diagonalize P_1 and P_2. More rigorous mathematical discussion is described in (7).

Note that the finding possible sensor arrangements for blind decorrelation includes two kinds of problems. One is what a matrix represented by several parameters is diagonalized independently of the values of the parameters, and the other is whether a corresponding sensor arrangement to the matrix exists or not. For example,

$$V = \begin{pmatrix} 1 & a & a & a & a \\ a & 1 & a & a & a \\ a & a & 1 & a & a \\ a & a & a & 1 & a \\ a & a & a & a & 1 \end{pmatrix} \tag{18}$$

is diagonalized by the DFT matrix Z_5 independently of a since V in eq. (18) is a kind of circulant matrix. However, eq. (18) means that each different pair of five sensors has the same distance, which cannot be realized in 3-D space.

3. Crystal Arrays

3.1 Necessary Condition

First, we begin with the following lemma.

Lemma 1. *A necessary condition for V defined by eq. (1) to be diagonalized by an unitary matrix U for any function $\Gamma(r, \omega)$, is that a set of distances from the sensor i to others: $\{r_{i1}, r_{i2}, \cdots, r_{in}\}$ is identical for any i.*

Proof: If V is diagonalized by an unitary matrix U without dependence on $\Gamma(r, \omega)$, the matrix I_n, of which all elements are identical to 1, is also diagonalized by U since I_n is obtained by letting $\Gamma(r, \omega) = 1$. Then, V and I_n are commutative. From (i, j) element of $VI_n = I_nV$, we see that

$$\sum_{k=1}^{n} \Gamma(\omega, r_{ik}) = \sum_{k=1}^{n} \Gamma(\omega, r_{jk}) \tag{19}$$

has to be an identical equation of r_{ij}s. It means that a distance set: $\{r_{ij} | j = 1, 2, \cdots n\}$ must be identical for any i. ∎

A square arrangement surely satisfies Lemma 1 since a set of distances from the sensor i to others is represented as $\{0, L, L, \sqrt{2}L\}$, which is identical to any i ($i = 1, 2, 3, 4$). While, in an argyle arrangement, a set of distances is $\{0, L, L, D_1\}$ from the sensor 1, and it is $\{0, L, L, D_2\}$ from the sensor 2. Thus, an argyle arrangement does't satisfy Lemma 1.

Lemma 1 directly gives a necessary condition of sensor arrangements for the blind decorrelation, but it is not a sufficient condition. Actually, there exist arrangements which satisfies Lemma 1 but cannot be used for the blind decorrelation. An example is shown in Fig. 3. The shape is obtained by merging vertices of two triangles with the same center and a different angle in the same plane, denoted as a bi-triangle.

In a bi-triangle arrangement, there are four kinds of distances between sensors: a short and a long line, and two kind of diagonals. The corresponding covariance matrix V is represented by

$$V = \begin{pmatrix} 1 & a & a & b & c & d \\ a & 1 & a & d & b & c \\ a & a & 1 & c & d & b \\ b & d & c & 1 & a & a \\ c & b & d & a & 1 & a \\ d & c & b & a & a & 1 \end{pmatrix}. \tag{20}$$
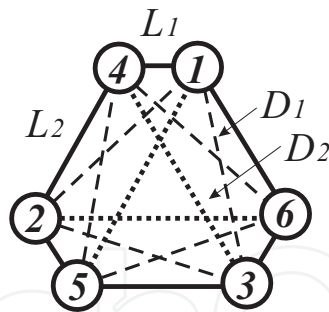
Fig. 3. Bi-triangle

This arrangement obviously satisfies Lemma 1 since a set of distances from a sensor to others is identically represented as $\{0, L_1, L_2, L_2, D_1, D_2, D_2\}$, but there is no matrices for diagonalizing U in eq. (20).

Although it is not straightforward from lemma 1 to a specific sensor arrangement, we have found five classes of sensor arrangements for blind decorrelation up to now (4; 8). According to the geometrical resemblance with crystals, we call them *crystal arrays*.

3.2 Five classes of crystal arrays
1) Regular polygons
Let *circ* denote a circulant matrix as

$$\text{circ}(1, a, b) = \begin{pmatrix} 1 & a & b \\ b & 1 & a \\ a & b & 1 \end{pmatrix}. \tag{21}$$

In arraying sensors on vertices of a n-sided regular polygon, circularly numbering them as shown in Fig. 4 yields a circulant $V = \text{circ}(1\, a_1\, a_2 \cdots a_2\, a_1)$. As well known, it is diagonalized by n-th order DFT matrix Z_n (9). Note that as a matrix to diagonalize V, we can choose a real-valued version of Z_n as shown in eq. (4), instead of Z_n itself, which leads simple basis transform in time domain discussed in section 2.
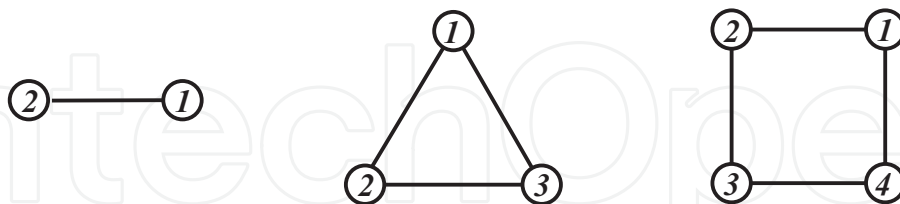

Fig. 4. Regular polygons

2) Rectangular
The second class consists of only a rectangular. Under numbering sensors as shown in Fig. 5, V has a block-circulant structure as

$$V = \begin{pmatrix} F_1 & F_2 \\ F_2 & F_1 \end{pmatrix}, \tag{22}$$

where F_1 and F_2 are 2×2 circulant matrices. It is diagonalized by $U = Z_2 \otimes Z_2$.
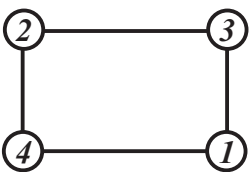
Fig. 5. Rectangular

3) Regular polygonal prisms

The regular polygonal prism arrangement is given by merging vertices of two parallel n-sided polygons with the same center axis. As the rectangular case, V has a block-circulant structure as

$$V = \begin{pmatrix} F_1 & F_2 \\ F_2 & F_1 \end{pmatrix}, \tag{23}$$

where F_1 and F_2 are $n \times n$ circulant matrices. It is diagonalized by

$$U = Z_n \otimes Z_2 = \begin{pmatrix} Z_n & Z_n \\ Z_n & -Z_n \end{pmatrix}. \tag{24}$$

The two parallel n-sided polygon may have a certain different angle, which yields a twisted prism as shown in Fig. 6. In $n = 2$, any angles are allowable, which the matrix structure is invariant for. In $n \geq 3$, only the rotation with π/n is allowable, where V becomes simply circular by alternative numbering in the top and the bottom n-sided polygon as shown in Fig. 6.
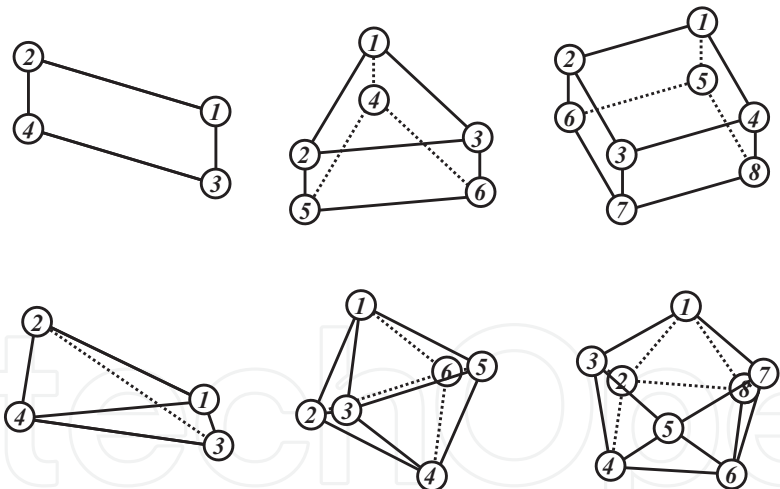
Fig. 6. Regular polygonal prisms (upper) and their twisted versions (lower)

4) Rectangular solid

In related to a rectangular, a rectangular solid forms another class. By numbering sensors shown in Fig. 7, V has the following structure:

$$V = \begin{pmatrix} F_1 & F_2 & F_3 & F_4 \\ F_2 & F_1 & F_4 & F_3 \\ F_3 & F_4 & F_1 & F_2 \\ F_4 & F_3 & F_2 & F_1 \end{pmatrix}, \tag{25}$$

where F_i $(i = 1, 2, 3, 4)$ are 2×2 circulant matrices. V itself is not circulant but it has recursively circulant structure. Hence, it is diagonalized by $U = Z_2 \otimes Z_2 \otimes Z_2$.
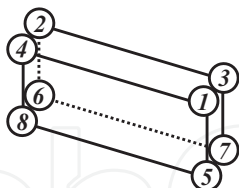

Fig. 7. A rectangular solid

5) Regular polyhedrons

As well known, there are only five polyhedrons in a 3D space: tetrahedron, octahedron, hexahedron, icosahedron, and dodecahedron, and they form the last class. From the viewpoint of the covariance matrix form, the tetrahedron is a special case of a twisted 2-sided polygonal prism, while the octahedron and the hexahedron are a special case of twisted 3-sided and 4-sided polygonal prisms, respectively. The most difficult cases are given by the icosahedron and the dodecahedron arrangements.
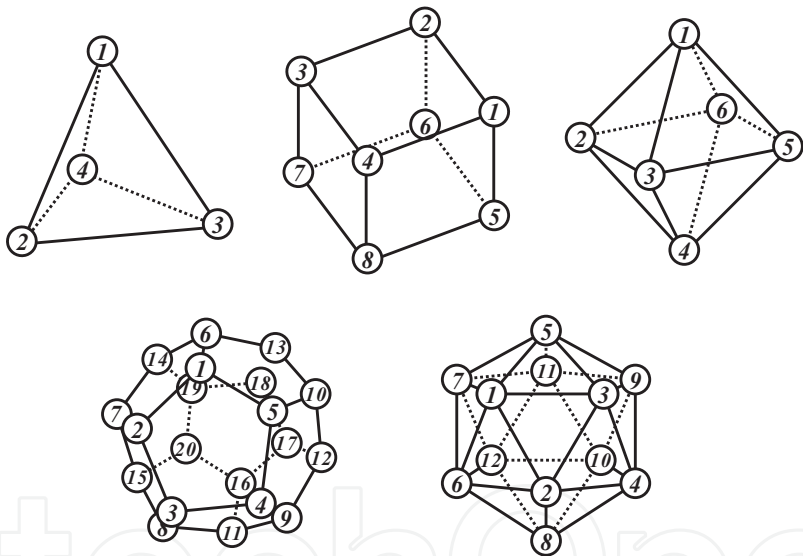
Fig. 8. Polyhedrons

An icosahedron has twenty equilateral triangular faces. Let two opposed triangles be the top and the bottom faces. Then, all vertices lie in four parallel planes. Numbering vertices circularly in the top plane, and then, from the top to the bottom in order as shown in Fig. 8, we have

$$V = \begin{pmatrix} F_1 & F_2 & F_3 & F_4 \\ F_2 & F_5 & F_6 & F_3 \\ F_3 & F_6 & F_5 & F_2 \\ F_4 & F_3 & F_2 & F_1 \end{pmatrix}, \tag{26}$$

where

$$F_1 = \text{circ}(1\ a\ a), \quad F_2 = \text{circ}(b\ a\ a), \tag{27}$$
$$F_3 = \text{circ}(a\ b\ b), \quad F_4 = \text{circ}(c\ b\ b), \tag{28}$$
$$F_5 = \text{circ}(1\ b\ b), \quad F_6 = \text{circ}(c\ a\ a). \tag{29}$$

Unlike the other cases, V doesn't have the circulant structure. Taking into consideration that 1) F_i ($1 \leq i \leq 6$) is diagonalized by Z_3 (the 3rd order DFT matrix) and 2) the block structure is different between the first, fourth columns and the second, third columns, we assume that U has the following form:

$$U = \begin{pmatrix} Z_3 & Z_3 & Z_3 & Z_3 \\ Z_3 P_3 & Z_3 Q_3 & -Z_3 R_3 & -Z_3 S_3 \\ Z_3 P_3 & Z_3 Q_3 & Z_3 R_3 & Z_3 S_3 \\ Z_3 & Z_3 & -Z_3 & -Z_3 \end{pmatrix}, \tag{30}$$

where P_3, Q_3, R_3, and S_3 are diagonal matrices. Eq. (30) yields

$$Z^H V Z = \begin{pmatrix} K_1 & A & O & O \\ A & K_2 & O & O \\ O & O & K_3 & B \\ O & O & B & K_4 \end{pmatrix}, \tag{31}$$

where K_i ($1 \leq i \leq 4$) are diagonal matrices with the size of 3×3 and

$$\begin{aligned} A &= (G_1 + G_2 Q_3 + G_3 Q_3 + G_4) + P_3(G_2 + G_5 Q_3 + G_6 Q_3 + G_3) \\ &\quad + P_3(G_3 + G_6 Q_3 + G_5 Q_3 + G_2) + (G_4 + G_3 Q_3 + G_2 Q_3 + G_1), \end{aligned} \tag{32}$$

$$\begin{aligned} B &= (G_1 - G_2 S_3 + G_3 S_3 - G_4) - R_3(G_2 - G_5 S_3 + G_6 S_3 - G_3) \\ &\quad + R_3(G_3 - G_6 S_3 + G_5 S_3 - G_2) - (G_4 - G_3 S_3 + G_2 S_3 - G_1), \end{aligned} \tag{33}$$

$$G_1 = \text{diag}(1 + 2a\ 1 - a\ 1 - a), \tag{34}$$
$$G_2 = \text{diag}(2a + b\ b - a\ b - a), \tag{35}$$
$$G_3 = \text{diag}(a + 2b\ a - b\ a - b), \tag{36}$$
$$G_4 = \text{diag}(2b + c\ c - b\ c - b), \tag{37}$$
$$G_5 = \text{diag}(1 + 2b\ 1 - b\ 1 - b), \tag{38}$$
$$G_6 = \text{diag}(2a + c\ c - a\ c - a), \tag{39}$$

where *diag* denote a diagonal matrix as

$$\text{diag}(a, b, c) = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}. \tag{40}$$

From $A=0$, we have

$$2(1 + c)(1 + p_1 q_1) + 2(a + b)(2 + 3(p_1 + q_1) + 2 p_1 q_1) = 0, \tag{41}$$
$$2(1 + c - a - b)(1 + p_2 q_2) = 0, \tag{42}$$
$$2(1 + c - a - b)(1 + p_3 q_3) = 0, \tag{43}$$

where p_i and q_i $(i = 1, 2, 3)$ are diagonal components of P_3 and Q_3, respectively. For satisfying them for any a‚ b‚ and c, there are ambiguities on determining p_2, q_2, p_3, q_3 since the conditions for them are only $p_2q_2 = p_3q_3 = 1$. Determining them the most simply, we choose

$$p_1 = p_2 = p_3 = 1, \tag{44}$$
$$q_1 = q_2 = q_3 = -1. \tag{45}$$

While, $B = 0$ yields

$$2(1-c)(1+r_1s_1) + 2(a-b)(2-(r_1+s_1) - 2r_1s_1) = 0, \tag{46}$$
$$2(1-c)(1+r_2s_2) - 2(a-b)(1-2(r_2+s_2) - r_2s_2) = 0, \tag{47}$$
$$2(1-c)(1+r_3s_3) - 2(a-b)(1-2(r_3+s_3) - r_3s_3) = 0, \tag{48}$$

where r_i and s_i $(i = 1, 2, 3)$ are diagonal components of R_3 and S_3, respectively. In the same way as p_i and q_i, we have

$$r_1s_1 = -1, \quad r_1 + s_1 = 4, \tag{49}$$
$$r_2s_2 = -1, \quad r_2 + s_2 = 1, \tag{50}$$
$$r_3s_3 = -1, \quad r_3 + s_3 = 1. \tag{51}$$

Solving them,

$$r_1 = \gamma_+^2 + \gamma_+, \quad s_1 = \gamma_-^2 + \gamma_-, \tag{52}$$
$$r_2 = r_3 = \gamma_+, \quad s_2 = s_3 = \gamma_-, \tag{53}$$

where r_i and s_i $(i = 1, 2, 3)$ are diagonal components of R_3 and S_3, respectively, and

$$\gamma_+ = (1 + \sqrt{5})/2, \quad \gamma_- = (1 - \sqrt{5})/2. \tag{54}$$

Consequently,

$$P_3 = \mathrm{diag}(1\ 1\ 1), \tag{55}$$
$$Q_3 = -\mathrm{diag}(1\ 1\ 1), \tag{56}$$
$$R_3 = \mathrm{diag}(\gamma_+^2 + \gamma_+\ \ \gamma_+\ \ \gamma_+), \tag{57}$$
$$S_3 = \mathrm{diag}(\gamma_-^2 + \gamma_-\ \ \gamma_-\ \ \gamma_-), \tag{58}$$

in eq. (30) gives us U to diagonalize eq. (26).

By the similar numbering to the icosahedron shown in Fig. 8, V in the dodecahedron has the same block structure as eq. (26) where

$$F_1 = \mathrm{circ}(1\ a\ b\ b\ a), \quad F_2 = \mathrm{circ}(a\ b\ c\ c\ b), \tag{59}$$
$$F_3 = \mathrm{circ}(d\ c\ b\ b\ c), \quad F_4 = \mathrm{circ}(e\ d\ c\ c\ d), \tag{60}$$
$$F_5 = \mathrm{circ}(1\ b\ d\ d\ b), \quad F_6 = \mathrm{circ}(e\ c\ a\ a\ c). \tag{61}$$

The form of U is also the same structure as eq. (30), just replacing the subscript 3 by 5, where

$$P_5 = \mathrm{diag}(1\ \ \gamma_-^2\ \ \gamma_+^2\ \ \gamma_+^2\ \ \gamma_-^2), \tag{62}$$
$$Q_5 = -\mathrm{diag}(1\ \ \gamma_+^2\ \ \gamma_-^2\ \ \gamma_-^2\ \ \gamma_+^2), \tag{63}$$
$$R_5 = \mathrm{diag}(\gamma_+^2 + \gamma_+\ \ \gamma_+\ \ \gamma_+\ \ \gamma_+\ \ \gamma_+), \tag{64}$$
$$S_5 = \mathrm{diag}(\gamma_-^2 + \gamma_-\ \ \gamma_-\ \ \gamma_-\ \ \gamma_-\ \ \gamma_-). \tag{65}$$

4. Conclusions

In this paper, we discussed geometrical sensor arrangements for the blind decorrelation of isotropic wavefield. Based on a necessary condition, we showed specific five classes of sensor arrangements: 1) regular polygons, 2) rectangular, 3) regular polygonal prisms, 4) rectangular solid, and 5) polyhedrons, the first two of which have two dimensional, and other three have three dimensional geometries, respectively. Specific orthogonal matrices corresponding to the sensor arrangements are also derived.

Finding all possible sensor arrangements for blind decorrelation is still an open problem and we are investigating the relationship with the group theory in mathematics, especially, *a point group* (10).

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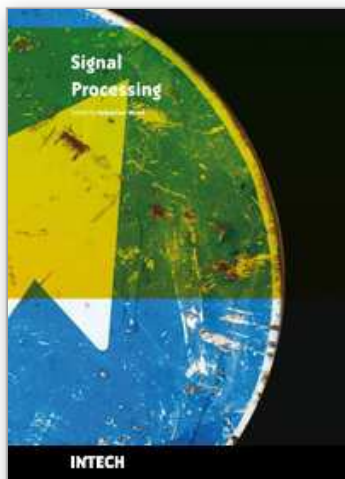
Signal Processing

Edited by Sebastian Miron

This book intends to provide highlights of the current research in signal processing area and to offer a snapshot of the recent advances in this field. This work is mainly destined to researchers in the signal processing related areas but it is also accessible to anyone with a scientific background desiring to have an up-to-date overview of this domain. The twenty-five chapters present methodological advances and recent applications of signal processing algorithms in various domains as telecommunications, array processing, biology, cryptography, image and speech processing. The methodologies illustrated in this book, such as sparse signal recovery, are hot topics in the signal processing community at this moment. The editor would like to thank all the authors for their excellent contributions in different areas of signal processing and hopes that this book will be of valuable help to the readers.

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