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The Extended Integral Equation Model IEM2M for topographically modulated rough surfaces

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1. Introduction

Remote sensing of terrain and ocean surfaces is circumscribed in the physical domain of electromagnetic scattering by rough surfaces. The development of accurate models has gathered a great deal of efforts since the 80's. Until that moment there were two classical approaches to be applied to two different asymptotic cases: the surfaces with small roughness and those having long correlation length. The first situation was dealt successfully via the small perturbation method (SPM)) whereas the second one was the target of the Kirchhoff approximation (KA). In effect, the abundance of models in the last two decades has made it very difficult for the Earth Observation practitioner to properly classify them and choose between them. The most important effort to that purpose was made by Tanos Elfouhaily in Elfouhaily & Guerin (2004), and we refer to his work for those interested in having a comprehensive account of the available methods for the problem. We focus here on the model that has arguably awakened the largest share of interest within the remote sensing community, that is, the Integral Equation Model (IEM) presented by Fung and Pan in Fung & Pan (1986) and later corrected in a long series of amendments by the same authors Fung (1994); Hsieh et al. (1997); Chen et al. (2000); Fung et al. (2002); Chen et al. (2003); Fung & Chen (2004); Wu & Chen (2004); Wu et al. (2008). In effect, there has been a number of issues that made the model theoretically inconsistent, even if each amendment was accompanied by properly suiting numerically simulated results. In 2001 the author of this chapter carried out a complete revision of Fung's work and proposed a corrected IEM that successfully achieved one of the objectives of the rough surface scattering models developed so far: to unify in a single equation both the SPM and the KA in the most general case of bistatic scattering. This corrected IEM was named IEM with proper inclusion of multiple scattering at second order or IEM2M.
This chapter aim is twofold: on the one had a quick summary of the IEM2M is given and on the other an extension of it is proposed to include those surfaces comprising both a zero-mean height, random component and a deterministic component that we call here "topographical".

2. Summary of the IEM2M for surfaces with zero height mean

The rationale of the IEM and therefore of the IEM2M is to perform a second iteration in the integral equations describing the rough surface electromagnetic scattering problem, as given in Poggio and Miller Poggio & Miller (1973). The first iteration corresponds to the KA, where each point on the surface is locally surrounded by neighbouring points lying on a flat surface, which is equivalent to the assumption of a low curvature. As a matter of fact, the proper in-

clusion of this second or complementary term coming from a second iteration bridges the gap between SPM and KA since it includes the local effects due to these neighbouring points to the extent which is necessary to meet the SPM limit. Second order effects describe the interaction of points on the surface, considered in pairs, just like third order effects would include interactions among sets of points taken in triads. This second-order contribution happens to contribute to the first-order, KA term with a non-zero addend when the limit of two points approaching to each other is taken. Even if full detail of IEM2M is given in Alvarez-Perez (2001), we summarize here the results regarding the complete first-order model that includes the KA term plus aforementioned correction coming from the limit of the second-order where pairs of point approach to one another. Unlike in Alvarez-Perez (2001), this first-order IEM2M is spelled out in a completely explicit form that easies its direct implementation in a computer code. Thus, we have for the first-order scattering coefficient the following formula, which contains new terms over the KA owing to the limit phenomena explained above

$$\begin{aligned}\sigma_{qp}^{o} =& \frac{1}{2} k_1^2 e^{-\sigma^2 (k_{sz}-k_z)^2} \\ & \times \sum_{n=1}^{\infty} \frac{\sigma^{2n}}{n!} \left| I_{qp}^{(n)} \right|^2 W_1^{(n)} (k_{sx}-k_x, k_{sy}-k_y)\end{aligned} \tag{1}$$

where

$$I_{qp}^{(n)} = (k_{sz}-k_z)^n f_{qp} + \frac{1}{4}[\mathrm{i}_1 + \mathrm{i}_2 + \mathrm{i}_{1'} + \mathrm{i}_{2'} + \mathrm{i}_{3'} + \mathrm{i}_{4'}] \tag{2}$$

with

$$\begin{aligned}\mathrm{i}_1 &= (k_{sz}+k_z)^{n-1} F_{qp}^1 (k_x, k_y, -k_z)\, e^{-\sigma^2 (k_{sz}+k_z)^2} \\ \mathrm{i}_2 &= [-(k_{sz}+k_z)]^{n-1} F_{qp}^1 (k_{sx}, k_{sy}, -k_{sz})\, e^{-\sigma^2 (k_{sz}+k_z)^2} \\ \mathrm{i}_{1'} &= (k_{sz}-k_z^{(2)})^{n-1} F_{qp}^2 (k_x, k_y, k_z^{(2)}) \\ &\quad \times e^{-\sigma^2 [k_z^{(2)2} - (k_{sz}+k_z) k_z^{(2)}]}\, e^{-\sigma^2 k_{sz} k_z} \\ \mathrm{i}_{2'} &= (k_{sz}+k_z^{(2)})^{n-1} F_{qp}^2 (k_x, k_y, -k_z^{(2)}) \\ &\quad \times e^{-\sigma^2 [k_z^{(2)2} + (k_{sz}+k_z) k_z^{(2)}]}\, e^{-\sigma^2 k_{sz} k_z} \\ \mathrm{i}_{3'} &= (k_{sz}^{(2)}-k_z)^{n-1} F_{qp}^2 (k_{sx}, k_{sy}, k_{sz}^{(2)}) \\ &\quad \times e^{-\sigma^2 [k_{sz}^{(2)2} - (k_{sz}+k_z) k_{sz}^{(2)}]}\, e^{-\sigma^2 k_{sz} k_z} \\ \mathrm{i}_{4'} &= [-(k_{sz}^{(2)}+k_z)]^{n-1} F_{qp}^2 (k_{sx}, k_{sy}, -k_{sz}^{(2)}) \\ &\quad \times e^{-\sigma^2 [k_{sz}^{(2)2} + (k_{sz}+k_z) k_{sz}^{(2)}]}\, e^{-\sigma^2 k_{sz} k_z}\end{aligned} \tag{3}$$

and

$$\begin{aligned}W_1^{(n)} (k_{sx}-k_x, k_{sy}-k_y) =& \\ & \frac{1}{2\pi} \int d\xi\, d\eta\, \rho^n (\xi, \eta)\, e^{-j[(k_{sx}-k_x)\xi + (k_{sy}-k_y)\eta]}\end{aligned} \tag{4}$$

$$k_z^{(2)} = (k_2^2 - k_x^2 - k_y^2)^{1/2}$$

$$k_{sz}^{(2)} = (k_2^2 - k_{sx}^2 - k_{sy}^2)^{1/2} \tag{5}$$

The symbols in equation (1) are: $\vec{k}^i = (k_x, k_y, k_z)$ represents the incident wave vector upon the scattering surface, $\vec{k}^s = (k_{sx}, k_{sy}, k_{sz})$ is the scattering wave vector, k_1 is the wave number of the incident medium (above the surface), k_2 is the wave number of the scattering medium (below the surface), σ is the standard deviation of the surface height and ρ is the correlation function of the surface height. The F_{qp} coefficients are given in Alvarez-Perez (2001). They, in turn, depend on some coefficients named as $C_i(\vec{k}^i, \vec{k}^s, \vec{l}_m^{(r)}); i = 1, \ldots, 4$, where $\vec{l}_m^{(r)}$ represents the effective interaction vector of a second-order scattering event, with r representing its upwards (+1) or downwards (-1) character and m the medium through which the second-order interaction takes place. For the first-order reduction IEM2M this vector $\vec{l}_m^{(r)}$ reduces to a few possible values, as explained in Alvarez-Perez (2001). These C coefficients are provided in Alvarez-Perez (2001) in a very formal way that may pose a difficulty for those not familiar with surface geometry. Therefore, a more user-friendly version is given in Appendix A at the end of this chapter. Also some remarks on its implementation by other authors are given.

3. IE2M Scattering Coefficient for Topographical Surfaces

3.1 Average Coherent Power

The average coherent power density over an ensemble of statistically equivalent surfaces is the modulus of the Poynting vector for the coherently scattered field

$$S_{qp}^c = \frac{1}{2} \mathrm{Re}\{1/\eta_1\} \left\langle \vec{E}_{qp}^s \right\rangle \left\langle \vec{E}_{qp}^{s\,*} \right\rangle \tag{6}$$

where η_1 is the impedance of the incident medium. It is common to assume far-zone fields to have a plane wave front. Although this is a valid approximation for incoherent scattering, it is now more convenient to replace the usual approximation

$$\frac{e^{jk_1|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \simeq \frac{e^{jk_1 r}}{r} e^{-jk_1 \hat{r}\cdot\vec{r}'} \tag{7}$$

by

$$\frac{e^{jk_1|\vec{r}-\vec{r}'|}}{|\vec{r}-\vec{r}'|} \simeq \frac{e^{jk_1 r}}{r} e^{-jk_1 \hat{r}\cdot\vec{r}'} e^{j\frac{r'^2}{2r}} \tag{8}$$

in the derivation of the Stratton-Chu-Silver integral. The reason to include the second order term in r'^2 in the phase of the spherical wave function is the higher sensitivity of a coherent interference to the wave front shape. Likewise, it is appropriate to assume a spherical incident front from the source of the incident field

$$\frac{e^{jk_1|\vec{r}_s-\vec{r}'|}}{|\vec{r}_s-\vec{r}'|} \simeq \frac{e^{jk_1 r_s}}{r_s} e^{-jk_1 \hat{r}_s\cdot\vec{r}'} e^{j\frac{r'^2}{2r_s}} \tag{9}$$

where $\vec{r}_s$ is the position vector of the source. We will assume that the incident field is Gaussian modulated along the direction given by $\vec{r}_s$, according to the window

$$\begin{aligned} w_G(x,y) &= e^{-g_0^2(x^2\cos^2\theta + y_2)} \\ g_0 &= \frac{1}{r_s \beta_0} \end{aligned} \tag{10}$$

where β_0 is the one-sided beamwidth of the transmitter. By placing the origin of coordinates on the plane to which the average rough surface belongs but far from the illuminated area, the following approximation can be made both in (8) and (9)

$$r'^2 = x'^2 + y'^2 + h'^2(x', y') \simeq x'^2 + y'^2 \tag{11}$$

With the inclusion of these changes plus the introduction of a shadowing function (see next section) and assuming $r_s = r$, the Kirchhoff far-zone scattered field can be written as

$$(E^s_{qp})_k = \frac{jk_1 E_o}{4\pi} \frac{e^{jk_1 r}}{r^2} \int_S \hat{f}_{qp}\, e^{j\frac{k_1(x'^2+y'^2)}{2r}}\, e^{-g_0^2(x'^2\cos^2\theta + y'^2)}\, e^{-j[(\vec{k}^s - \vec{k}^i)\cdot\vec{r}']}\, dx'dy' \tag{12}$$

where we have "dressed" the factor f_{qp} to include the shadowing function

$$\hat{f}_{qp} = \mathcal{S}(\hat{k}^i, \hat{k}^s)\, f_{qp} \tag{13}$$

Then, the coherently scattered power takes the form

$$S^c_{qp} = \frac{1}{2}\,\mathrm{Re}\{1/\eta_1\} \left(\frac{k_1 E_o \hat{f}_{qp}}{4\pi r^2}\right)^2 \left| \int_S e^{jk_1(x'^2+y'^2)/2r}\, e^{-g_0^2(x'^2\cos^2\theta + y'^2)}\, e^{-j[(k_{sx}-k_x)x' + (k_{sy}-k_y)y']} \left\langle e^{-j[(k_{sz}-k_z)z']} \right\rangle dx'dy' \right|^2 \tag{14}$$

To calculate the averages comprised in the integrand of (14), we compute

$$\left\langle e^{-j(k_{sz}-k_z)z'} \right\rangle = e^{-j(k_{sz}-k_z)\bar{z}(x',y')}\, e^{-(k_{sz}-k_z)^2(\sigma^2/2)} \tag{15}$$

Hence,

$$S^c_{qp} = \frac{1}{2}\,\mathrm{Re}\{1/\eta_1\} \left(\frac{k_1 E_o \hat{f}_{qp}}{\pi r^2}\right)^2 e^{-(k_{sz}-k_z)^2\sigma^2} \left|W_0(k_{sx}-k_x, k_{sy}-k_y)\right|^2 \tag{16}$$

where

$$W_0(k_{sx}-k_x, k_{sy}-k_y) = \int e^{-j2[(k_{sx}-k_x)x' + (k_{sy}-k_y)y']}\, e^{x'^2(jk_1/2r - g_0^2\cos^2\theta) + y'^2(jk_1/2r - g_0^2)} e^{-j(k_{sz}-k_z)\bar{z}(x',y')}\, dx'\, dy' \tag{17}$$

Integral W_0 has the shape of a Gabor transform, that is, of a Fourier transform with a Gaussian window included in the integrand.

3.2 Average Incoherent Power

The average incoherent power density over an ensemble of statistically equivalent surfaces is the modulus of the Poynting vector for the diffuse field

$$S_{qp}^{d} = \frac{1}{2}\,\mathrm{Re}\{1/\eta\}\left(\langle \vec{E}_{qp}^{\,s}\,\vec{E}_{qp}^{\,s*}\rangle - \langle \vec{E}_{qp}^{\,s}\rangle\langle \vec{E}_{qp}^{\,s*}\rangle\right) \tag{18}$$

where $\mathrm{Re}\{1/\eta_1\}$ is the real part of the inverse of the magnetic permeability in the incidence medium and * is the symbol for complex conjugate. Separating the scattered field into the Kirchhoff and complementary terms, we obtain

$$\begin{aligned} S_{qp}^{d} =& \frac{1}{2}\,\mathrm{Re}\{1/\eta\}\left\{\langle E_{qp}^{sk}E_{qp}^{sk*}\rangle - \langle E_{qp}^{sk}\rangle\langle E_{qp}^{sk*}\rangle\right. \\ &+ 2\,\mathrm{Re}\left\{\langle E_{qp}^{sc}E_{qp}^{sk*}\rangle - \langle E_{qp}^{sc}\rangle\langle E_{qp}^{sk*}\rangle\right\} \\ &\left.+ \langle E_{qp}^{sc}E_{qp}^{sc*}\rangle - \langle E_{qp}^{sc}\rangle\langle E_{qp}^{sc*}\rangle\right\} \end{aligned} \tag{19}$$

The analysis of (19) will be carried out by considering separately three terms, namely, the Kirchhoff term, the complementary term and the "interference" term between both, which will be named the cross term.

To perform the averages in (19), we need to know the statistics of the ensemble of surfaces. We select the ensemble of surfaces such that it follows a joint Gaussian distribution with a constant variance across the surface. This assumption greatly simplifies the computation of the averaging. However, the random surfaces included in the aforementioned ensemble will be allowed to have nonzero means at each point.

3.2.1 Kirchhoff Incoherent Power

Once the shadowing effects are included, the Kirchhoff diffuse power density can be written as

$$\begin{aligned} S_{qp}^{dk} &= \frac{1}{2}\,\mathrm{Re}\{1/\eta_1\}\left\{\langle E_{qp}^{sk}E_{qp}^{sk*}\rangle - \langle E_{qp}^{sk}\rangle\langle E_{qp}^{sk*}\rangle\right\} \\ &= \frac{\left|K\,E_o\,\hat{f}_{qp}\right|^2}{2}\,\mathrm{Re}\{1/\eta_1\}\left(\left\langle \int_S e^{-j(\hat{k}_s-\hat{k}_i)\cdot(\vec{r}\,'-\vec{r}\,'')}\,dx'dy'dx''dy''\right\rangle\right. \\ &\left.- \left|\left\langle \int_S e^{-j(\hat{k}_s-\hat{k}_i)\cdot\vec{r}\,'}\,dx'dy'\right\rangle\right|^2\right) \end{aligned} \tag{20}$$

The averages in (20) are readily evaluated

$$\left\langle e^{-jp_z z'}\right\rangle = e^{-jp_z\bar{z}(x',y')}\,e^{-p_z^2(\sigma^2/2)} \tag{21a}$$

$$\left\langle e^{-jp_z(z'-z'')}\right\rangle = e^{-jp_z(\bar{z}(x',y')-\bar{z}(x'',y''))}\,e^{-p_z^2\sigma^2[1-\rho(x'-x'',y'-y'')]} \tag{21b}$$

$$p_z = k_{sz} - k_z$$

Substituting now (21a) and (21b) into (20) and using the integration variables $\xi = x' - x''$ and $\eta = y' - y''$ instead of x' and y'', we have

$$S_{qp}^{dk} = \frac{\left|K\,E_o\,\hat{f}_{qp}\right|^2}{2}\,\mathrm{Re}\{1/\eta_1\}\,e^{-p_z^2\sigma^2}\iint d\xi d\eta\,(e^{p_z^2\sigma^2\rho(\xi,\eta)}-1)\,D_1(\xi,\eta;p_z)\,e^{-j(p_x\xi+p_y\eta)} \tag{22}$$

where $p_x = k_{sx} - k_x$, $p_y = k_{sy} - k_y$ and $D_1(\xi,\eta;p_z)$ is

$$D_1(\xi,\eta;p_z) = \iint dx''dy'' e^{-jp_z[\bar{z}(x''+\xi,y''+\eta)-\bar{z}(x'',y'')]} \tag{23}$$

and represents the autocorrelation of the phase $e^{-jp_z\bar{z}(x'',y'')}$ over the surface.

3.2.2 Cross Incoherent Power

The incoherently scattered power for the cross term is given by

$$\begin{aligned} S_{qp}^{dkc} &= \mathrm{Re}\{1/\eta_1\}\,\mathrm{Re}\Big\{\big\langle E_{qp}^{sc}E_{qp}^{sk*}\big\rangle - \big\langle E_{qp}^{sc}\big\rangle\big\langle E_{qp}^{sk*}\big\rangle\Big\} \\ &= \frac{\left|KE_o\right|^2}{8\pi^2}\mathrm{Re}\{1/\eta_1\}\sum_{m=1,2}\mathrm{Re}\Big\{\hat{f}_{qp}^{*}\int_{\mathbb{R}^2}du\,dv\int_{S^3}dx'dy'dx''dy''dx'''dy''' \\ &\quad e^{j[u(x'-x'')+v(y'-y'')]}\,e^{-j[k_{sx}(x'-x''')+k_{sy}(y'-y''')]}\,e^{j[k_x(x''-x''')+k_y(y''-y''')]} \\ &\quad \cdot\Big[\Big\langle e^{-jk_{sz}(z'-z''')}e^{jk_z(z''-z''')}e^{jq_m|z'-z''|}\hat{F}_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{g}_m)\Big\rangle \\ &\quad -\Big\langle e^{-jk_{sz}z'}e^{jk_zz''}e^{jq_m|z'-z''|}\hat{F}_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{g}_m)\Big\rangle \\ &\quad \cdot\Big\langle e^{j(k_{sz}-k_z)z'''}\Big\rangle\Big]\Big\} \end{aligned} \tag{24}$$

where factors F_{qp}^m have been "dressed" to include the shadowing function

$$\hat{F}_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{g}_m) = \mathcal{S}_m(\vec{k}^i,\vec{g}_m,\vec{k}^s)\,F_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{g}_m) \tag{25}$$

On the other hand, factors $\hat{F}_{qp}^m$ have been included within the averages since they depend on $(z'-z'')/|z'-z''|$. To compute these averages we will make use of the invariance of the formalism under the change

$$G_m(\vec{r}',\vec{r}'') = G_m^{retarded}(\vec{r}',\vec{r}'') \longrightarrow G_m^{*}(\vec{r}',\vec{r}'') = G_m^{advanced}(\vec{r}',\vec{r}'') \tag{26}$$

The Weyl representation of the retarded Green's function is given by

$$\begin{aligned} G_m^{retarded}(\vec{r}',\vec{r}'') &= \frac{j}{2\pi}\iint_{\mathbb{R}^2}e^{j[u(x'-x'')+v(y'-y'')]}\frac{e^{-jq_m|z'-z''|}}{q_m}\,du\,dv \\ q_m &= \begin{cases}(k_m^2-u^2-v^2)^{1/2} & \text{if } k_m^2 \geq u^2+v^2 \\ -j(u^2+v^2-k_m^2)^{1/2} & \text{if } k_m^2 \leq u^2+v^2\end{cases} \end{aligned} \tag{27}$$

Therefore, the invariance under the change (26) is equivalent to

$$q_m \longrightarrow \begin{cases} -q_m & \text{if } q_m \in \mathbb{R} \\ q_m & \text{if } q_m \in \mathbb{I}\end{cases} \tag{28}$$

or, more formally, $q_m \to -q_m^*$. However, the damped cylindrical waves given by imaginary values of q_m have been neglected and therefore the invariance holds under the transformation

$$q_m \to -q_m$$

This symmetry permits the calculation of (24) by using

$$\Big\langle \psi(q_m) \Big\rangle = \frac{1}{2}\Big(\Big\langle \psi(q_m) \Big\rangle + \Big\langle \psi(-q_m) \Big\rangle \Big)$$

where ψ is any of the functions in (24) to be averaged. Thus, there are two averages to be computed, namely,

$$\begin{aligned}\Big\langle e^{-jk_{sz}(z'-z''')}\, e^{jk_z(z''-z''')}\, \hat{F}^m_{qp}(\vec{k}^i,\vec{k}^s,\vec{g}_m)\, e^{jq_m|z'-z''|} \Big\rangle \\ = \Big\langle e^{-jk_{sz}(z'-z''')}\, e^{jk_z(z''-z''')}\, \frac{1}{2}\Big[\hat{F}^m_{qp}(\vec{k}^i,\vec{k}^s,u,v,\Phi_{z'z''}\, q_m)\, e^{jq_m|z'-z''|} \\ + \hat{F}^m_{qp}(\vec{k}^i,\vec{k}^s,u,v,-\Phi_{z'z''}\, q_m)\, e^{-jq_m|z'-z''|} \Big] \Big\rangle \end{aligned} \tag{29}$$

and

$$\begin{aligned}\Big\langle e^{-jk_{sz}z'}\, e^{jk_z z''}\, \hat{F}^m_{qp}(\vec{k}^i,\vec{k}^s,\vec{g}_m)\, e^{jq_m|z'-z''|} \Big\rangle \\ = \Big\langle e^{-jk_{sz}z'}\, e^{jk_z z''}\, \frac{1}{2}\Big[\hat{F}^m_{qp}(\vec{k}^i,\vec{k}^s,u,v,\Phi_{z'z''}\, q_m)\, e^{jq_m|z'-z''|} \\ + \hat{F}^m_{qp}(\vec{k}^i,\vec{k}^s,u,v,-\Phi_{z'z''}\, q_m)\, e^{-jq_m|z'-z''|} \Big] \Big\rangle \end{aligned} \tag{30}$$

There are two types of addends in these averages: terms dependent on $\Phi_{z'z'''}\, q_m$ and terms dependent on q_m^2 or completely independent of q_m. Only the former are functions of the space coordinates through $\Phi_{z'z''}$. Therefore, we have to compute the following quantities

$$\begin{aligned}\Big\langle e^{-j[k_{sz}(z'-z''')-k_z(z''-z''')]}\, e^{jq_m|z'-z''|} \Big\rangle \\ = \Big\langle e^{-j[k_{sz}(z'-z''')-k_z(z''-z''')]}\, \cos(q_m|z'-z''|) \Big\rangle \\ = \frac{1}{2}\Big(\Big\langle e^{-j[k_{sz}(z'-z''')-k_z(z''-z''')]}\, e^{jq_m(z'-z'')} \Big\rangle \\ + \Big\langle e^{-j[k_{sz}(z'-z''')-k_z(z''-z''')]}\, e^{-jq_m(z'-z'')} \Big\rangle \Big) \end{aligned} \tag{31a}$$

and similarly

$$\begin{aligned}\Big\langle e^{-j(k_{sz}z'-k_z z'')}\, e^{jq_m|z'-z''|} \Big\rangle = \frac{1}{2}\Big(\Big\langle e^{-j(k_{sz}z'-k_z z'')}\, e^{jq_m(z'-z'')} \Big\rangle \\ + \Big\langle e^{-j(k_{sz}z'-k_z z'')}\, e^{-jq_m(z'-z'')} \Big\rangle \Big) \end{aligned} \tag{31b}$$

$$\begin{aligned}\Big\langle e^{-j[k_{sz}(z'-z''')-k_z(z''-z''')]}\, e^{jq_m|z'-z''|}\, \Phi_{z'z''}\, q_m \Big\rangle \\ = \frac{q_m}{2}\Big(\Big\langle e^{-j[k_{sz}(z'-z''')-k_z(z''-z''')]}\, e^{jq_m(z'-z'')} \Big\rangle \\ - \Big\langle e^{-j[k_{sz}(z'-z''')-k_z(z''-z''')]}\, e^{-jq_m(z'-z'')} \Big\rangle \Big) \end{aligned} \tag{31c}$$

$$\left\langle e^{-j(k_{sz}z'-k_z z'')}\, e^{jq_m|z'-z''|}\, \Phi_{z'z''}\, q_m \right\rangle = \left\langle e^{-j(k_{sz}z'-k_z z'')}\, \Phi_{z'z''}\, jq_m \sin(q_m|z'-z''|) \right\rangle - \left\langle e^{-j(k_{sz}z'-k_z z'')}\, e^{-jq_m(z'-z'')} \right\rangle \Big) \tag{31d}$$

Hence, we compute again the averages

$$\left\langle e^{-j[k_{sz}(z'-z''')-k_z(z''-z''')]}\, e^{jq_m|z'-z''|} \right\rangle = \frac{1}{2}\left(e^{jw_1}\, e^{-\sigma_{w_1}^2} + e^{jw_2}\, e^{-\sigma_{w_2}^2} \right) \tag{32a}$$

$$\left\langle e^{-j(k_{sz}z'-k_z z'')}\, e^{jq_m|z'-z''|} \right\rangle = \frac{1}{2}\left(e^{jw_3}\, e^{-\sigma_{w_3}^2} + e^{jw_4}\, e^{-\sigma_{w_4}^2} \right) \tag{32b}$$

$$\left\langle e^{-j[k_{sz}(z'-z''')-k_z(z''-z''')]}\, e^{jq_m|z'-z''|}\, \Phi_{z'z''}\, q_m \right\rangle = \frac{q_m}{2}\left(e^{jw_1}\, e^{-\sigma_{w_1}^2} - e^{jw_2}\, e^{-\sigma_{w_2}^2} \right) \tag{32c}$$

$$\left\langle e^{-j(k_{sz}z'-k_z z'')}\, e^{jq_m|z'-z''|}\, \Phi_{z'z''}\, q_m \right\rangle = \frac{q_m}{2}\left(e^{jw_3}\, e^{-\sigma_{w_3}^2} - e^{jw_4}\, e^{-\sigma_{w_4}^2} \right) \tag{32d}$$

where

$$\begin{aligned} w_1 &= \omega_1(k_{sz},k_z,q_m) \\ w_2 &= \omega_1(k_{sz},k_z,-q_m) \\ w_3 &= \omega_2(k_{sz},k_z,q_m) \\ w_4 &= \omega_2(k_{sz},k_z,-q_m) \\ \omega_1(k_{sz},k_z,q_m) &= -(k_{sz}-q_m)\bar{z}' + (k_z-q_m)\bar{z}'' + (k_{sz}-k_z)\bar{z}''' \\ \omega_2(k_{sz},k_z,q_m) &= -(k_{sz}-q_m)\bar{z}' + (k_z-q_m)\bar{z}'' \end{aligned} \tag{33}$$

and

$$\begin{aligned} \sigma_{w_1} &= \sigma_{\omega_1}(k_{sz},k_z,q_m) \\ \sigma_{w_2} &= \sigma_{\omega_1}(k_{sz},k_z,-q_m) \\ \sigma_{w_3} &= \sigma_{\omega_2}(k_{sz},k_z,q_m) \\ \sigma_{w_4} &= \sigma_{\omega_2}(k_{sz},k_z,-q_m) \\ \sigma_{\omega_1}(k_{sz},k_z,q_m) &= \sigma[k_{sz}^2 + k_z^2 + q_m^2 - (k_{sz}+k_z)q_m - k_z k_{sz} \\ &\quad -(k_{sz}-q_m)(k_z-q_m)\rho(z',z'') \\ &\quad +(k_{sz}-q_m)(k_z-k_{sz})\rho(z',z''') \\ &\quad -(k_z-q_m)(k_z-k_{sz})\rho(z'',z''')] \\ \sigma_{\omega_2}(k_{sz},k_z,q_m) &= \sigma[k_{sz}^2 + k_z^2 + 2q_m^2 - 2(k_{sz}+k_z)q_m \\ &\quad -2(k_{sz}-q_m)(k_z-q_m)\rho(z',z'')]/2 \end{aligned} \tag{34}$$

Putting all these results together and defining new spatial coordinates $\xi = x' - x'''$, $\eta = y' -$

y''', $\xi' = x'' - x'''$ and $\eta' = y'' - y'''$, we can rewrite (24) as follows

$$
\begin{aligned}
S_{qp}^{dkc} = \frac{\left|KE_o\right|^2}{16\pi^2}\mathrm{Re}\{1/\eta_1\}\sum_{m=1,2}\sum_{r=-1,1}\mathrm{Re}\Bigg\{ & \hat{f}_{qp}^{*}\int_{\mathbb{R}^2} du\,dv\int d\xi\,d\eta\,d\xi'd\eta' \\
& \cdot e^{j[u(\xi-\xi')+v(\eta-\eta')]}\, e^{-j[k_{sx}\xi+k_{sy}\eta]}\, e^{j[k_x\xi'+k_y\eta']} \\
& \cdot D_2(\xi,\eta,\xi',\eta';k_{sz},k_z,r\,q_m)\hat{F}_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{l}_m^r) \\
& \cdot e^{-\sigma^2[k_{sz}^2+k_z^2+q_m^2-(k_{sz}+k_z)r\,q_m-k_{sz}k_z-(k_{sz}-r\,q_m)(k_z-r\,q_m)\rho_{12}]} \\
& \cdot \left(e^{-\sigma^2[(k_{sz}-r\,q_m)(k_z-k_{sz})\rho_{13}-(k_z-r\,q_m)(k_z-k_{sz})\rho_{23}]}-1\right)\Bigg\}
\end{aligned}
\tag{35}
$$

with

$$
D_2(\xi,\eta,\xi',\eta';k_{sz},k_z,r\,q_m) = \int dx'''dy'''\, e^{-j[(k_{sz}-r\,q_m)\bar{z}'-(k_z-r\,q_m)\bar{z}''-(k_{sz}-k_z)\bar{z}''']} \tag{36}
$$

and

$$
\begin{aligned}
z' &= z(x''' + \xi, y''' + \eta) & \rho_{12} &= \rho(\xi - \xi', \eta - \eta') \\
z'' &= z(x''' + \xi', y''' + \eta') & \rho_{13} &= \rho(\xi, \eta) \\
z''' &= z(x''', y''') & \rho_{23} &= \rho(\xi', \eta')
\end{aligned}
$$

3.2.3 Complementary Incoherent Power

Finally, the diffuse scattered power for the complementary term is

$$
\begin{aligned}
S_{qp}^{dc} =& \frac{1}{2}\mathrm{Re}\{1/\eta_1\}\left\{\left\langle E_{qp}^{sc}E_{qp}^{sc*}\right\rangle - \left\langle E_{qp}^{sc}\right\rangle\left\langle E_{qp}^{sc*}\right\rangle\right\} \\
=& \frac{|KE_o|^2}{2^7\pi^4}\mathrm{Re}\{1/\eta_1\}\sum_{m,n=1,2}\Bigg\{\int_{\mathbb{R}^4} du\,dv\,du'\,dv'\int_{S^4} dx'dy'dx''dy''dx'''dy'''dx'^{\nu}dy'^{\nu} \\
& \cdot e^{j[u(x'-x'')-u'(x'''-x'^{\nu})+v(y'-y'')-v'(y'''-y'^{\nu})]}\, e^{-j[k_{sx}(x'-x''')+k_{sy}(y'-y''')]} \\
& \cdot e^{j[k_x(x''-x'^{\nu})+k_y(y''-y'^{\nu})]}\Big[\Big\langle e^{-jk_{sz}(z'-z''')}\, e^{jk_z(z''-z'^{\nu})}\, e^{jq_m|z'-z''|} \\
& \cdot e^{-jq_n'|z'''-z'^{\nu}|}\hat{F}_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{g}_m)\,\hat{F}_{qp}^{n*}(\vec{k}^i,\vec{k}^s,\vec{g}_n')\Big\rangle \\
& - \Big\langle e^{-j(k_{sz}z'+k_zz'')}\, e^{jq_m|z'-z''|}\hat{F}_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{g}_m)\Big\rangle \\
& \Big\langle e^{j(k_{sz}z'''-k_zz'^{\nu})}e^{-jq_n'|z'''-z'^{\nu}|}\hat{F}_{qp}^{n*}(\vec{k}^i,\vec{k}^s,\vec{g}_n')\Big\rangle\Big]\Bigg\}
\end{aligned}
\tag{37}
$$

Applying the same arguments used to calculate the averages relevant for the cross term power, we obtain the following relations

$$
\begin{aligned}
&\left\langle e^{-jk_{sz}(z'-z''')}\, e^{jk_z(z''-z'^{\nu})}\, e^{jq_m|z'-z''|}\, e^{-jq_n'|z'''-z'^{\nu}|}(\Phi_{z'z''}\,q_m)^{\alpha}(\Phi_{z'''z'^{\nu}}\,q_n')^{\beta}\right\rangle \\
&\qquad = \frac{q_m^{\alpha}q_n^{\beta}}{4}\left(e^{j\omega_1}\,e^{-\sigma_{\omega_1}} + (-1)^{\alpha}e^{j\omega_2}\,e^{-\sigma_{\omega_2}} + (-1)^{\beta}e^{j\omega_3}\,e^{-\sigma_{\omega_3}} + (-1)^{\alpha+\beta}e^{j\omega_4}\,e^{-\sigma_{\omega_4}}\right)
\end{aligned}
\tag{38}
$$

where $\alpha, \beta = 0,1$ and the other coefficients are compactly given by

$$\begin{aligned}
\varpi_1 &= \pi(k_{sz}, k_z, q_m, q_n') \\
\varpi_2 &= \pi(k_{sz}, k_z, -q_m, q_n') \\
\varpi_3 &= \pi(k_{sz}, k_z, q_m, -q_n') \\
\varpi_4 &= \pi(k_{sz}, k_z, -q_m, -q_n') \\
\sigma_{\varpi_1} &= \sigma_\pi(k_{sz}, k_z, q_m, q_n') \\
\sigma_{\varpi_2} &= \sigma_\pi(k_{sz}, k_z, -q_m, q_n') \\
\sigma_{\varpi_3} &= \sigma_\pi(k_{sz}, k_z, q_m, -q_n') \\
\sigma_{\varpi_4} &= \sigma_\pi(k_{sz}, k_z, -q_m, -q_n')
\end{aligned} \tag{39}$$

by including the general functions π and σ_π in the form

$$\begin{aligned}
\pi(k_{sz}, k_z, q_m, q_n') &= -(k_{sz} - q_m)\bar{z}' + (k_z - q_m)\bar{z}'' + (k_{sz} - q_n')\bar{z}''' - (k_z - q_n')\bar{z}'^{\nu} \\
\sigma_\pi(k_{sz}, k_z, q_m, q_n') &= \sigma^2[k_{sz}^2 + k_z^2 + q_m^2 + q_n'^2 - (k_{sz} + k_z)(q_m + q_n') \\
&\quad - (k_{sz} - q_m)(k_z - q_m)\rho(z', z'') - (k_{sz} - q_m)(k_{sz} - q_n')\rho(z', z''') \\
&\quad + (k_{sz} - q_m)(k_z - q_n')\rho(z', z'^{\nu}) + (k_z - q_m)(k_{sz} - q_n')\rho(z'', z''') \\
&\quad - (k_z - q_m)(k_z - q_n')\rho(z'', z'^{\nu}) - (k_{sz} - q_n')(k_z - q_n')\rho(z''', z'^{\nu})]
\end{aligned} \tag{40}$$

Upon substituting (38) into (37) we find that

$$\begin{aligned}
S_{qp}^{dc} = \frac{|KE_o|^2}{2^9\pi^4}\mathrm{Re}\{1/\eta_1\} \sum_{m,n=1,2} \sum_{r,r'=-1,1} \Bigg\{ & \int_{\mathbb{R}^4} du\, dv\, du'\, dv' \int d\xi\, d\eta\, d\xi'\, d\eta'\, d\tau\, d\kappa \\
& e^{j[u(\xi+\tau-\xi')-u'\tau+v(\eta+\kappa-\eta')-v'\kappa]}\, e^{-j(k_{sx}\xi+k_{sy}\eta)}\, e^{j(k_x\xi'+k_y\eta')} \\
& D_3(\xi, \eta, \xi', \eta', \tau, \kappa; k_{sz}, k_z, rq_m, r'q_n') \\
& \hat{F}_{qp}^m(\vec{k}^i, \vec{k}^s, \vec{l}_m^r)\hat{F}_{qp}^{n*}(\vec{k}^i, \vec{k}^s, \vec{l}_n'^{r'}) \\
& e^{-\sigma^2[k_{sz}^2 + k_z^2 + q_m^2 + q_n'^2 - (k_{sz}+k_z)(rq_m + r'q_n')]} \\
& e^{-\sigma^2[(k_{sz} - rq_m)(rq_m - k_z)\rho_{12} + (k_{sz} - r'q_n')(r'q_n' - k_z)\rho_{34}]} \\
& \Big(e^{-\sigma^2[(k_{sz} - rq_m)(r'q_n' - k_{sz})\rho_{13} + (k_{sz} - rq_m)(k_z - r'q_n')\rho_{14}]} \\
& e^{-\sigma^2[(k_z - rq_m)(k_{sz} - r'q_n')\rho_{23} + (k_z - rq_m)(r'q_n' - k_z)\rho_{24}]} - 1 \Big) \Bigg\}
\end{aligned} \tag{41}$$

where $\xi = x' - x'''$, $\eta = y' - y'''$, $\xi' = x'' - x'^{\nu}$, $\eta' = y'' - y'^{\nu}$, $\tau = x''' - x'^{\nu}$ and $\kappa = y''' - y'^{\nu}$, the function D_3

$$\begin{aligned}
& D_3(\xi, \eta, \xi', \eta', \tau, \kappa; k_{sz}, k_z, rq_m, r'q_n') \\
& \qquad = \int dx'^{\nu} dy'^{\nu} e^{-j[(k_{sz} - q_m)\bar{z}' - (k_z - q_m)\bar{z}'' - (k_{sz} - q_n')\bar{z}''' + (k_z - q_n')\bar{z}'^{\nu}]}
\end{aligned} \tag{42}$$

and

$$z' = z(x'^{\nu} + \xi + \tau, y'^{\nu} + \eta + \kappa) \qquad \rho_{12} = \rho(\xi + \tau - \xi', \eta + \kappa - \eta')$$
$$z'' = z(x'^{\nu} + \xi', y'^{\nu} + \eta') \qquad \rho_{13} = \rho(\xi, \eta)$$
$$z''' = z(x'^{\nu} + \tau, y'^{\nu} + \kappa) \qquad \rho_{14} = \rho(\xi + \tau, \eta + \kappa)$$
$$z'^{\nu} = z(x'^{\nu}, y'^{\nu}) \qquad \rho_{23} = \rho(\xi' - \tau, \eta' - \kappa)$$
$$\rho_{24} = \rho(\xi', \eta')$$
$$\rho_{34} = \rho(\tau, \kappa)$$

3.3 Bistatic Scattering Coefficient for the Scattered Field

The *radar cross section* of a particle producing isotropic scattering is defined as the ratio between the scattered and incident power densities, S^{scat} and S^{inc} multiplied by the area of the spherical surface centred at the particle and with a radius R equal to the distance between the particle and the observation point

$$\sigma \equiv \frac{4\pi R^2 S^{scat}}{S^{inc}} \tag{43}$$

Next, we define the *radar scattering cross section* of a finite scatterer in a given direction as the cross section of a particle which would scatter isotropically the same power density in any direction, should it be illuminated by the same incident power density.
For the case of a scattering surface, it is adequate to define the *differential scattering coefficient* as the average value of the scattering cross section per unit area, namely,

$$\sigma^o \equiv \frac{4\pi R^2 S^{scat}}{A S^{inc}} \tag{44}$$

where A denotes the area of the surface. Usually, the term "radar scattering cross section" is shortened to "radar cross section", whereas "differential scattering coefficient" is referred to as "scattering coefficient".
Both radar cross section and scattering coefficient can be either monostatic or bistatic, when the observation point is located at the site from where the incident field is transmitted or elsewhere, respectively. Thus, the bistatic scattering coefficient associated to the coherent and diffuse fields scattered by a random rough surface are given by

$$(\sigma^o)^c_{qp} = \frac{8\pi R^2}{A\,\mathrm{Re}\{1/\eta_1\}\,E_o^2} S^c_{qp} \tag{45a}$$

$$(\sigma^o)^d_{qp} = \frac{8\pi R^2}{A\,\mathrm{Re}\{1/\eta_1\}\,E_o^2} \left(S^{dk}_{qp} + S^{dkc}_{qp} + S^{dc}_{qp}\right) \tag{45b}$$

where the power densities S^c_{qp}, S^{dk}_{qp}, S^{dkc}_{qp} and S^{dc}_{qp} have been calculated in previous sections.

4. Formulation of the IEM2M Model for Topographical Surfaces

The scattering coefficient in (45) is described in terms of the integrals included in S^c_{qp}, S^{dk}_{qp}, S^{dkc}_{qp} and S^{dc}_{qp}. The coherently scattered power calculated in (3.1) is the final form proposed here. However, the integrals corresponding to the diffuse power can be manipulated further. A distinction is drawn then between surfaces with small or moderate rms height normalized

to wave number, $k\sigma$, and surfaces with larger values for $k\sigma$. Thus, a forward scattering model is defined by Taylor expansion of the exponentials in the corresponding integrands. This is done for each scattering coefficient term in the next subsections.

4.1 Scattering Model for Surfaces with Small or Moderate Heights

When the product of the rms height of the surface by the wave number has a small or moderate value, the argument of the exponential functions in (22), (35) and (41) will also have a small value. It is then useful to write the exponential functions in the form of a Taylor series.

4.1.1 Kirchhoff Term

The exponential function in (22) involving the correlation between the heights of the two scattering centres $\vec{r}\,'$ and $\vec{r}\,''$ can be expanded as

$$e^{p_z^2\sigma^2\rho(\xi,\eta)} = \sum_{n=0}^{\infty} \frac{[\sigma^2 p_z^2 \rho(\xi,\eta)]^n}{n!} \tag{46}$$

Consequently, the Kirchhoff term (22) of the scattering coefficient takes on the form

$$(\sigma^o)_{qp}^{dk} = \frac{1}{2}k_1^2 |\hat{f}_{qp}|^2 e^{-\sigma^2(k_{sz}-k_z)^2} \sum_{n=1}^{\infty} \frac{(\sigma^2(k_{sz}-k_z)^2)^n}{n!} W_1^{(n)}(k_{sx}-k_x, k_{sy}-k_y) \tag{47}$$

where

$$W_1^{(n)}(k_{sx}-k_x, k_{sy}-k_y) = \frac{1}{2\pi A}\int d\xi\, d\eta\, \rho^n(\xi,\eta)\, e^{-j[(k_{sx}-k_x)\xi+(k_{sy}-k_y)\eta]} D_1(\xi,\eta,k_{sz}-k_z) \tag{48}$$

4.1.2 Cross Term

The exponential functions in (24) can be expanded in the form

$$\begin{aligned} & e^{\sigma^2[(k_{sz}-rq_m)(k_z-rq_m)\rho(z',z'')]}\Big(e^{-\sigma^2[(k_{sz}-rq_m)(k_z-k_{sz})\rho(z',z''')]} \\ & \cdot e^{\sigma^2[(k_z-rq_m)(k_z-k_{sz})\rho(z'',z''')]} - 1\Big) \\ & \qquad = \sum_{i=0}^{\infty} \frac{[\sigma^2(k_{sz}-rq_m)(k_z-rq_m)\rho(z',z'')]^i}{i!} \\ & \qquad \Big[\sum_{n=0}^{\infty} \frac{[-\sigma^2(k_{sz}-rq_m)(k_z-k_{sz})\rho(z',z''')]^n}{n!} \\ & \qquad \sum_{l=0}^{\infty} \frac{[\sigma^2(k_z-rq_m)(k_z-k_{sz})\rho(z'',z''')]^l}{l!} - 1\Big] \end{aligned} \tag{49}$$

The interactions of second order can be described as specular reflections and Snell's refractions. Second-order scattering events can occur connecting points within the correlation length or distant from each other. When the interacting point sources are within the correlation length, we will have either $k_{sz} \simeq q_m$, for $r = 1$, or $k_z \simeq -q_m$, for $r = -1$, and the first exponential function in (49) will have a negligible argument, provided that σ is not large. When those points are distant, the correlation function ρ will be very small. Thus, the first

summation in (49) can be approximated by unity for surfaces with small or moderate rms height

$$e^{\sigma^2[(k_{sz}-rq_m)(k_z-rq_m)\rho(z',z'')]} \simeq 1 \tag{50}$$

and hence

$$\begin{aligned} & e^{\sigma^2[(k_{sz}-rq_m)(k_z-rq_m)\rho(z',z'')]}\Big(e^{-\sigma^2[(k_{sz}-rq_m)(k_z-k_{sz})\rho(z',z''')]} \\ & \cdot e^{\sigma^2[(k_z-rq_m)(k_z-k_{sz})\rho(z'',z''')]} - 1\Big) \\ & \qquad \simeq \sum_{n=1}^{\infty} \frac{[-\sigma^2(k_{sz}-rq_m)(k_z-k_{sz})\rho(z',z''')]^n}{n!} \\ & \qquad + \sum_{l=1}^{\infty} \frac{[\sigma^2(k_z-rq_m)(k_z-k_{sz})\rho(z'',z''')]^l}{l!} \\ & \qquad + \sum_{n=1}^{\infty} \frac{[-\sigma^2(k_{sz}-rq_m)(k_z-k_{sz})\rho(z',z''')]^n}{n!} \\ & \qquad\qquad \cdot \sum_{l=1}^{\infty} \frac{[\sigma^2(k_z-rq_m)(k_z-k_{sz})\rho(z'',z''')]^l}{l!} \end{aligned} \tag{51}$$

This yields

$$\begin{aligned} (\sigma^o)_{qp}^{dkc} = \frac{k_1^2}{8\pi} \sum_{m=1,2} \sum_{r=-1,1} \mathrm{Re}\Bigg\{ & \hat{f}_{qp}^{*} e^{-\sigma^2[k_{sz}^2+k_z^2-k_{sz}k_z]} \\ & \int_{\mathbb{R}^2} du\, dv\, \hat{F}_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{l}_m^r)\, e^{-\sigma^2[q_m^2-(k_{sz}+k_z)rq_m]} \\ & \cdot \Bigg[\sum_{n=1}^{\infty} \frac{[-\sigma^2(k_{sz}-rq_m)(k_z-k_{sz})]^n}{n!} W_2^{n,0}(\vec{l}_m^r;\vec{k}^s,\vec{k}^i) \\ & + \sum_{l=1}^{\infty} \frac{[\sigma^2(k_z-rq_m)(k_z-k_{sz})]^l}{l!} W_2^{0,l}(\vec{l}_m^r;\vec{k}^s,\vec{k}^i) \\ & + \sum_{n=1}^{\infty} \frac{[-\sigma^2(k_{sz}-rq_m)(k_z-k_{sz})]^n}{n!} \\ & \sum_{l=1}^{\infty} \frac{[\sigma^2(k_z-rq_m)(k_z-k_{sz})]^l}{l!} W_2^{n,l}(\vec{l}_m^r;\vec{k}^s,\vec{k}^i) \Bigg] \Bigg\} \end{aligned} \tag{52}$$

where

$$\begin{aligned} W_2^{(\alpha,\beta)}(u,v,w;\vec{k}^s,\vec{k}^i) = & \\ & \frac{1}{(2\pi)^2 A} \int d\xi\, d\eta\, d\xi'\, d\eta'\, e^{j[(u-k_{sx})\xi+(v-k_{sy})\eta-(u-k_x)\xi'-(v-k_y)\eta']} \\ & \cdot D_2(\xi,\eta,\xi',\eta',k_{sz},k_z,w)\rho^{\alpha}(\xi,\eta)\rho^{\beta}(\xi',\eta') \end{aligned} \tag{53}$$

4.1.3 Complementary Term

The complementary term of the scattering coefficient involves the evaluation of an integral containing the following expression

$$
\begin{aligned}
&e^{-\sigma^2[(k_{sz}-r\,q_m)(r\,q_m-k_z)\rho_{12}+(k_{sz}-r'\,q'_n)(r'\,q'_n-k_z)\rho_{34}]}\Big(e^{-\sigma^2[(k_{sz}-r\,q_m)(r'\,q'_n-k_{sz})\rho_{13}]}\\
&e^{-\sigma^2[(k_{sz}-r\,q_m)(k_z-r'\,q'_n)\rho_{14}+(k_z-r\,q_m)(k_{sz}-r'\,q'_n)\rho_{23}+(k_z-r\,q_m)(r'\,q'_n-k_z)\rho_{24}]}-1\Big)\\
&\quad=\sum_{i=0}^{\infty}\frac{[-\sigma^2(k_{sz}-r\,q_m)(r\,q_m-k_z)\rho_{12}]^i}{i!}\sum_{j=0}^{\infty}\frac{[-\sigma^2(k_{sz}-r'\,q'_n)(r'\,q'_n-k_z)\rho_{34}]^j}{j!}\\
&\quad\Bigg[\sum_{h=0}^{\infty}\frac{[-\sigma^2(k_{sz}-r\,q_m)(r'\,q'_n-k_{sz})\rho_{13}]^h}{h!}\sum_{l=0}^{\infty}\frac{[-\sigma^2(k_{sz}-r\,q_m)(k_z-r'\,q'_n)\rho_{14}]^l}{l!}\\
&\quad\sum_{n=0}^{\infty}\frac{[-\sigma^2(k_z-r\,q_m)(k_{sz}-r'\,q'_n)\rho_{23}]^n}{n!}\sum_{t=0}^{\infty}\frac{[-\sigma^2(k_z-r\,q_m)(r'\,q'_n-k_z)\rho_{24}]^t}{t!}-1\Bigg] \qquad (54)
\end{aligned}
$$

As explained in the previous subsection, the correlation between points producing effective second-order scattering is negligible. These points are represented in the summation above by the pairs 1 and 2 on the one hand and by 3 and 4 on the other. Thus, the first two summations containing ρ_{12} and ρ_{34} can be approximated by unity. Further, all the products between summations of the form $\sum_1^\infty$ containing ρ_{13} and ρ_{14} are negligible. This is so because significant correlation between points 1 and both points 3 and 4 would generally imply a significant correlation between 3 and 4. The same reasoning applies to products with ρ_{13} and ρ_{23}, ρ_{23} and ρ_{24} or ρ_{14} and ρ_{24}. Thereby,

$$
\begin{aligned}
&e^{-\sigma^2[(k_{sz}-r\,q_m)(r\,q_m-k_z)\rho_{12}+(k_{sz}-r'\,q'_n)(r'\,q'_n-k_z)\rho_{34}]}\Big(e^{-\sigma^2[(k_{sz}-r\,q_m)(r'\,q'_n-k_{sz})\rho_{13}]}\\
&e^{-\sigma^2[(k_{sz}-r\,q_m)(k_z-r'\,q'_n)\rho_{14}+(k_z-r\,q_m)(k_{sz}-r'\,q'_n)\rho_{23}+(k_z-r\,q_m)(r'\,q'_n-k_z)\rho_{24}]}-1\Big)\\
&\quad\simeq\sum_{h=1}^{\infty}\frac{[-\sigma^2(k_{sz}-r\,q_m)(r'\,q'_n-k_{sz})\rho_{13}]^h}{h!}+\sum_{l=1}^{\infty}\frac{[-\sigma^2(k_{sz}-r\,q_m)(k_z-r'\,q'_n)\rho_{14}]^l}{l!}\\
&\quad+\sum_{n=1}^{\infty}\frac{[-\sigma^2(k_z-r\,q_m)(k_{sz}-r'\,q'_n)\rho_{23}]^n}{n!}+\sum_{t=1}^{\infty}\frac{[-\sigma^2(k_z-r\,q_m)(r'\,q'_n-k_z)\rho_{24}]^t}{t!}\\
&\quad+\sum_{h=1}^{\infty}\frac{[-\sigma^2(k_{sz}-r\,q_m)(r'\,q'_n-k_{sz})\rho_{13}]^h}{h!}\sum_{t=1}^{\infty}\frac{[-\sigma^2(k_z-r\,q_m)(r'\,q'_n-k_z)\rho_{24}]^t}{t!}\\
&\quad+\sum_{l=1}^{\infty}\frac{[-\sigma^2(k_{sz}-r\,q_m)(k_z-r'\,q'_n)\rho_{14}]^l}{l!}\sum_{n=1}^{\infty}\frac{[-\sigma^2(k_z-r\,q_m)(k_{sz}-r'\,q'_n)\rho_{23}]^n}{n!} \qquad (55)
\end{aligned}
$$

Introducing this approximation, (41) becomes

$$
\begin{aligned}
(\sigma^o)_{qp}^{dc} = \frac{k_1^2}{2^7\pi^2} \sum_{m,n=1,2} \sum_{r,r'=-1,1} \Bigg\{ & e^{-\sigma^2(k_{sz}^2+k_z^2)} \int_{\mathbb{R}^4} du\, dv\, du'\, dv' \\
& \hat{F}_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{l}_m^{\,r})\, \hat{F}_{qp}^{n\,*}(\vec{k}^i,\vec{k}^s,\vec{l}_n^{\,\prime r'}) \\
& e^{-\sigma^2[q_m^2+q_n'^2-(k_{sz}+k_z)(r\,q_m+r'\,q_n')]} \\
& \Bigg[\sum_{h=1}^{\infty} \frac{[-\sigma^2(k_{sz}-r\,q_m)(r'\,q_n'-k_{sz})]^h}{h!} W_3^{h,0,0,0}(\vec{l}_m^{\,r},\vec{l}_n^{\,\prime r'};\vec{k}^s,\vec{k}^i) \\
& + \sum_{l=1}^{\infty} \frac{[-\sigma^2(k_{sz}-r\,q_m)(k_z-r'\,q_n')]^l}{l!} W_3^{0,m,0,0}(\vec{l}_m^{\,r},\vec{l}_n^{\,\prime r'};\vec{k}^s,\vec{k}^i) \\
& + \sum_{n=1}^{\infty} \frac{[-\sigma^2(k_z-r\,q_m)(k_{sz}-r'\,q_n')]^n}{n!} W_3^{0,0,n,0}(\vec{l}_m^{\,r},\vec{l}_n^{\,\prime r'};\vec{k}^s,\vec{k}^i) \\
& + \sum_{t=1}^{\infty} \frac{[-\sigma^2(k_z-r\,q_m)(r'\,q_n'-k_z)]^t}{t!} W_3^{0,0,0,t}(\vec{l}_m^{\,r},\vec{l}_n^{\,\prime r'};\vec{k}^s,\vec{k}^i) \\
& + \sum_{h=1}^{\infty} \frac{[-\sigma^2(k_{sz}-r\,q_m)(r'\,q_n'-k_{sz})]^h}{h!} \\
& \sum_{t=1}^{\infty} \frac{[-\sigma^2(k_z-r\,q_m)(r'\,q_n'-k_z)]^t}{t!} W_3^{h,0,0,t}(\vec{l}_m^{\,r},\vec{l}_n^{\,\prime r'};\vec{k}^s,\vec{k}^i) \\
& + \sum_{l=1}^{\infty} \frac{[-\sigma^2(k_{sz}-r\,q_m)(k_z-r'\,q_n')]^l}{l!} \\
& \sum_{n=1}^{\infty} \frac{[-\sigma^2(k_z-r\,q_m)(k_{sz}-r'\,q_n')]^n}{n!} W_3^{0,m,n,0}(\vec{l}_m^{\,r},\vec{l}_n^{\,\prime r'};\vec{k}^s,\vec{k}^i) \Bigg] \Bigg\}
\end{aligned}
\tag{56}
$$

where

$$
\begin{aligned}
& W_3^{(h,l,n,t)}(u,v,w,u',v',w';\vec{k}^s,\vec{k}^i) \\
& \quad = \frac{1}{(2\pi)^3 A} \int d\xi\, d\eta\, d\xi'\, d\eta'\, d\tau\, d\kappa\, e^{j[(u-k_{sx})\xi-(u-k_x)\xi'+(v-k_{sy})\eta-(v-k_y)\eta']} \\
& \quad e^{j[(u-u')\tau+(v-v')\kappa]}\, D_3(\xi,\eta,\xi',\eta',\tau,\kappa;k_{sz},k_z,w,w')\, \rho^h(\xi,\eta) \\
& \quad \rho^l(\xi+\tau,\eta+\kappa)\rho^n(\xi'-\tau,\eta'-\kappa)\, \rho^t(\xi',\eta')
\end{aligned}
\tag{57}
$$

4.2 Scattering Model for Surfaces with Large Heights

Although a series of the type given in (47) is convergent for any value of the argument, it is only practical to compute it when the argument is not large. Thus, the summations describing the scattering coefficient for the diffuse field in the previous section are not practical for large rms height. Besides, it was assumed that, on the whole, the correlation between points producing second-order scattering was negligible and, as will be shown below, this is not the case for surfaces with large rms height.

4.2.1 Kirchhoff Term

Let us reconsider first the Kirchhoff term in the form given in Subsection 3.2.1

$$S_{qp}^{dk} = \frac{1}{4\pi A} k_1^2 \hat{f}_{qp}^2 \iint d\xi d\eta \, e^{-j[(k_{sx}-k_x)\xi+(k_{sy}-k_y)\eta]} \\ (e^{-(k_{sz}-k_z)^2\sigma^2(1-\rho(\xi,\eta))} - e^{-(k_{sz}-k_z)^2\sigma^2}) \, D_1(\xi,\eta;k_{sz}-k_z) \quad (58)$$

Large values for $k_1\sigma$ give rise to very negative arguments in the exponentials of (58). As a matter of fact the coherent term subtracted in this equation is negligible and the additive exponential is significant only when the correlation function is near unity. It is then possible to perform a Taylor expansion of the correlation function about the origin to obtain

$$\begin{aligned} 1-\rho(\xi,\eta) &\simeq \frac{1}{2}|\rho_{\xi\xi}(0)|\,\xi^2 + \frac{1}{2}|\rho_{\eta\eta}(0)|\,\eta^2 + |\rho_{\xi\eta}(0)|\,\xi\,\eta \\ &\equiv \frac{1}{2}|\rho^o_{\xi\xi}|\,\xi^2 + \frac{1}{2}|\rho^o_{\eta\eta}|\,\eta^2 + |\rho^o_{\xi\eta}|\,\xi\,\eta \end{aligned} \quad (59)$$

were the subscripts in ρ denote partial derivatives and the superscript o denotes that the correlation function is evaluated at the origin. Likewise, we expand the function $D_1(\xi,\eta;k)$ about the origin

$$\begin{aligned} D_1(\xi,\eta;k) &\simeq D_1(0,0;k) + D_{1,\xi}(0,0;k)\xi + D_{1,\eta}(0,0;k)\eta \\ &\quad + \frac{1}{2}D_{1,\xi,\xi}(0,0;k)\xi^2 + \frac{1}{2}D_{1,\eta,\eta}(0,0;k)\eta^2 + D_{1,\eta,\xi}(0,0;k)\eta\xi \\ &\equiv D_1^o(k) + D^o_{1,\xi}(k)\xi + D^o_{1,\eta}(k)\eta \\ &\quad + \frac{1}{2}D^o_{1,\xi,\xi}(k)\xi^2 + \frac{1}{2}D^o_{1,\eta,\eta}(k)\eta^2 + D^o_{1,\eta,\xi}(k)\eta\xi \end{aligned} \quad (60)$$

Upon replacing (59) and (60) in (58), we arrive at

$$\begin{aligned} (\sigma^o)_{qp}^{dk} = &\frac{1}{4\pi A} k_1^2 \hat{f}_{qp}^2 \iint d\xi d\eta \, e^{-j[(k_{sx}-k_x)\xi+(k_{sy}-k_y)\eta]} \\ &\exp\left[-(k_{sz}-k_z)^2\sigma^2(\frac{1}{2}|\rho^o_{\xi\xi}|\xi^2 + \frac{1}{2}|\rho^o_{\eta\eta}|\eta^2 + |\rho^o_{\xi\eta}|\xi\eta)\right] \\ &[D_1^o(k_{sz}-k_z) + D^o_{1,\xi}(k_{sz}-k_z)\,\xi + D^o_{1,\eta}(k_{sz}-k_z)\,\eta \\ &+ \frac{1}{2}D^o_{1,\xi,\xi}(k_{sz}-k_z)\,\xi^2 + \frac{1}{2}D^o_{1,\eta,\eta}(k_{sz}-k_z)\,\eta^2 + D^o_{1,\eta,\xi}(k_{sz}-k_z)\,\eta\xi] \end{aligned} \quad (61)$$

where the subtraction of the coherent term has been disregarded.
The following integral identity will be used

$$\begin{aligned} &\iint_{-\infty}^{\infty} dx\,dy\, e^{-(ax^2+by^2+2cxy)}\,(A+Bx+Cx^2+Dy+Ey^2+Fxy)\, e^{-j(k_x x+k_y y)} \\ &\qquad = \frac{\pi}{4(ab-c^2)^{(5/2)}} \exp\left\{-\frac{k_x^2 b - 2ck_xk_y + k_y^2 a}{4(ab-c^2)}\right\} \\ &\qquad [A\,\alpha_A(a,b,c) + B\,\alpha_B(a,b,c,k_x,k_y) + C\,\alpha_C(a,b,c,k_x,k_y) \\ &\qquad\quad + D\,\alpha_D(a,b,c,k_x,k_y) + E\,\alpha_E(a,b,c,k_x,k_y) + F\,\alpha_F(a,b,c,k_x,k_y)] \end{aligned} \quad (62)$$

where

$$\begin{aligned}
\alpha_A(a,b,c) &= 4\,(ab-c^2)^2 \\
\alpha_B(a,b,c,k_x,k_y) &= -2j\,(ab-c^2)(bk_x-ck_y) \\
\alpha_C(a,b,c,k_x,k_y) &= 2b\,(ab-c^2)-(bk_x-ck_y)^2 \\
\alpha_D(a,b,c,k_x,k_y) &= -2j\,(ab-c^2)(ak_y-ck_x) \\
\alpha_E(a,b,c,k_x,k_y) &= 2a\,(ab-c^2)-(ak_y-ck_x)^2 \\
\alpha_F(a,b,c,k_x,k_y) &= -2c\,(ab-c^2)+(ck_x-ak_y)(bk_x-ck_y)
\end{aligned} \tag{63}$$

Therefore, (61) results in

$$(\sigma^o)^{dk}_{qp} = \frac{2k_1^2\,\hat{f}^2_{qp}\,\mathcal{I}^k(\vec{p})}{p_z^{10}\sigma^{10}[|\rho^o_{\xi,\xi}||\rho^o_{\eta,\eta}|-|\rho^o_{\xi,\eta}|^2]^{5/2}A}\exp\left\{-\frac{p_x^2|\rho^o_{\eta,\eta}|-2p_xp_y|\rho^o_{\xi,\eta}|+p_y^2|\rho^o_{\xi,\xi}|}{2\,p_z^2\sigma^2(|\rho^o_{\xi,\xi}||\rho^o_{\eta,\eta}|-|\rho^o_{\xi,\eta}|^2)}\right\} \tag{64}$$

where

$$\begin{aligned}
\mathcal{I}^k(\vec{p}) =& D_1^o(p_z)\,\tilde{\alpha}_A + D^o_{1,\xi}(p_z)\,\tilde{\alpha}_B + \frac{1}{2}D^o_{1,\xi,\xi}(p_z)\,\tilde{\alpha}_C \\
&+ D^o_{1,\eta}(p_z)\,\tilde{\alpha}_D + \frac{1}{2}D^o_{1,\eta,\eta}(p_z)\,\tilde{\alpha}_E + D^o_{1,\eta,\xi}(p_z)\,\tilde{\alpha}_F
\end{aligned} \tag{65}$$

with $\vec{p}=\vec{k}^s-\vec{k}^i$, and

$$\begin{aligned}
\tilde{\alpha}_A &= \alpha_A(\kappa_1|\rho^o_{\xi,\xi}|,\kappa_1|\rho^o_{\eta,\eta}|,\kappa_1|\rho^o_{\xi,\eta}|) \\
\tilde{\alpha}_\zeta &= \alpha_\zeta(\kappa_1|\rho^o_{\xi,\xi}|,\kappa_1|\rho^o_{\eta,\eta}|,\kappa_1|\rho^o_{\xi,\eta}|,p_x,p_y) \qquad \zeta = B,C,D,E,F \\
\kappa_1 &= p_z^2\sigma^2/2
\end{aligned} \tag{66}$$

The expression obtained in (61) is the result obtained from classic geometric optics, multiplied by a factor of correction due to the deterministic component of the surface.

4.2.2 Cross Term

From Subsection 3.2.2 we get

$$\begin{aligned}
(\sigma^o)^{dkc}_{qp} = \frac{k_1^2}{2^5\pi^3 A}\sum_{m=1,2}\sum_{r=-1,1}\mathrm{Re}\Big\{&\hat{f}^*_{qp}\int_{\mathbb{R}^2}du\,dv\int d\xi\,d\eta\,d\xi'\,d\eta' \\
&\cdot e^{j[u(\xi-\xi')+v(\eta-\eta')]}\,e^{-j[k_{sx}\xi+k_{sy}\eta]}\,e^{j[k_x\xi'+k_y\eta']} \\
&\cdot D_2(\xi,\eta,\xi',\eta';k_{sz},k_z,rq_m)\hat{F}^m_{qp}(\vec{k}^i,\vec{k}^s,\vec{l}^r_m) \\
&\cdot e^{-\sigma^2[(k_{sz}-rq_m)(k_z-rq_m)(1-\rho_{12})]}\Big[e^{-\sigma^2[(k_{sz}-rq_m)(k_{sz}-k_z)(1-\rho_{13})]} \\
&\cdot e^{-\sigma^2[(rq_m-k_z)(k_{sz}-k_z)(1-\rho_{23})]}-e^{-\sigma^2(k_{sz}-k_z)^2}\Big]\Big\}
\end{aligned} \tag{67}$$

Some simplifications are applicable but, before introducing them, some remarks are in order. As in Paragraph4.1.2, the approach is seeing the interactions of second order as specular reflections and Snell's refractionsAlso, surface integration is taken over two regions for each

correlation function, the region where points are close in terms of the correlation length and the region where points are distant from each other. The correlation function ρ_{12} links the points that are connected by second-order scattering events, and the functions ρ_{13} and ρ_{23} relate a point acting as a secondary wave source of second order and a point with a first-order scattering role. The second type of functions are present due to the fact that the cross term described by (67) is an interference between first and second-order scattering in the calculation of the scattered power.

The situation is more complicated now than in (58) as the sign of the arguments in the exponential functions depends on the value of $k_{sz} - r q_m$ and $k_z - r q_m$. We observe the following

1. The coherent component in (67) can be written as

$$e^{-\sigma^2[(k_{sz}-r q_m)(k_z-r q_m)(1-\rho_{12})]}\, e^{-\sigma^2(k_{sz}-k_z)^2} = e^{-\sigma^2[k_{sz}^2+k_z^2+q_m^2-(k_{sz}+k_z)r q_m-k_{sz}k_z-(k_{sz}-r q_m)(k_z-r q_m)\rho_{12}]} \tag{68}$$

   The second exponential at the l.s. of (68) has a large negative argument for large $k\sigma$ values. Therefore, the coherent term will be very small except, perhaps, when the argument of the first exponential at the l.s. of (68) has a positive argument. For this to happen, we need either $k_{sz} - q_m > 0$ when $r = 1$ or $k_z + q_m < 0$ when $r = -1$. In both cases, according to the argument of the exponential at the r.s. of (68), the product of the two exponential functions with different signs in their argument is negligible. Thereby, the coherent component subtracted in (67) is not significant, as we should expect from a surface with large rms height.

2. For the incoherent term, and according to the aforementioned distinction between the two areas of integration for each correlation function, we note that

   (a) If the three correlation functions ρ_{12}, ρ_{13} and ρ_{23} are all very small, the exponential functions yield

$$\begin{aligned} e^{-\sigma^2(k_{sz}-r q_m)(k_z-r q_m)}\, e^{-\sigma^2(k_{sz}-r q_m)(k_{sz}-k_z)}\, e^{-\sigma^2(r q_m-k_z)(k_{sz}-k_z)} &= e^{-\sigma^2(k_{sz}-r q_m)^2}\, e^{-\sigma^2(r q_m-k_z)(k_{sz}-k_z)} \\ &= e^{-\sigma^2(k_z-r q_m)^2}\, e^{-\sigma^2(k_{sz}-r q_m)(k_{sz}-k_z)} \\ &= e^{-\sigma^2(k_{sz}-k_z)^2}\, e^{-\sigma^2(k_{sz}-r q_m)(k_z-r q_m)} \end{aligned} \tag{69}$$

   From (69), it is clear that the product of the three exponential functions is negligible no matter the sign of $k_{sz} - r q_m$ and $k_z - r q_m$.

   (b) If two correlation functions are very small and the other one is close to unity, then we obtain similar identities to (69). For instance, provided that $\rho_{12} \simeq 1$, the product of exponentials is written as

$$e^{-\sigma^2(k_{sz}-k_z)^2}\, e^{-\sigma^2(k_{sz}-r q_m)(k_z-r q_m)(1-\rho_{12})} \simeq e^{-\sigma^2(k_{sz}-k_z)^2} \tag{70}$$

   and can be neglected. The same holds for either of the other two correlation functions.

(c) The region of the integration domain where two correlation functions are close to unity and the other is negligible can be regarded as having measure zero. For example, if $\rho_{12} \simeq 1$ and $\rho_{13} \simeq 1$, we expect $\rho_{23} \simeq 1$, that is, if the pair of points $(1,2)$ and $(1,3)$ are highly correlated, then the pair $(2,3)$ is generally expected to be highly correlated, too.

(d) When the three correlation functions are all close to unity, the exponentials can have moderate or small arguments and therefore they do contribute to the integral. Thereupon, the most significant region of the integration domain corresponds to small values of ξ, η, ξ' and η' and ρ_{12}, ρ_{13} and ρ_{23} can be Taylor expanded about the origin. To see the order of approximation to be taken for each correlation function we investigate their physical meaning. The exponential function containing ρ_{12} represents the interference between the sources located at points 1 and 2, which are the secondary wave sources involved in a second-order scattering event. On the other hand, ρ_{13} and ρ_{23} represent the interference between one of those second-order sources on the surface and the source located at point 3, which is a first-order - or Kirchhoff - secondary wave source. As the Kirchhoff field is expected to be of a higher magnitude than the complementary field, we expand ρ_{13} and ρ_{23} about the origin up to second order and ρ_{12} only up to first order.

According to these remarks, the product of exponential functions in (67) can be replaced by

$$\begin{aligned} & e^{-\sigma^2[(k_{sz}-r\,q_m)(k_z-r\,q_m)(1-\rho_{12})]} \\ & \left[e^{-\sigma^2[(k_{sz}-r\,q_m)(k_{sz}-k_z)(1-\rho_{13})]}\, e^{-\sigma^2[(r\,q_m-k_z)(k_{sz}-k_z)(1-\rho_{23})]} - e^{-\sigma^2(k_{sz}-k_z)^2}\right] \\ & \qquad \simeq e^{-\frac{1}{2}\sigma^2(k_{sz}-r\,q_m)(k_{sz}-k_z)[|\rho^o_{\xi\xi}|\,\xi^2+|\rho^o_{\eta\eta}|\,\eta^2+2|\rho^o_{\xi\eta}|\,\xi\eta]} \\ & \qquad\quad e^{-\frac{1}{2}\sigma^2(r\,q_m-k_z)(k_{sz}-k_z)[|\rho^o_{\xi\xi}|\,\xi'^2+|\rho^o_{\eta\eta}|\,\eta'^2+2|\rho^o_{\xi\eta}|\,\xi'\eta']} \end{aligned} \tag{71}$$

However, it is important to note that this replacement is only possible when the arguments of the exponential functions at the r.s. of (71) are negative. Therefore, if $(k_{sz} - r\,q_m)$ or $(r\,q_m - k_z)$ are not positive, the substitution is not possible and $\exp\{-\sigma^2[(k_{sz} - r\,q_m)(k_z - r\,q_m)(1-\rho_{12})]\}$ cannot be discarded. The assumption here is to consider that the reflections and refractions involved in second-order scattering are unlikely to produce first deviations where the modulus of the z-component of the wave vector increases, such that $r\,q_m < k_z$, or second deviations where it decreases, such that $k_{sz} < r\,q_m$. Thus the integration domain in u and v, Γ_r, will be constrained to the following conditions

$$\Gamma_r : \begin{cases} q_m < |k_z| & \text{if} \quad r = -1 \\ q_m < k_{sz} & \text{if}\, r = 1 \end{cases} \tag{72}$$

We expand also D_2 in (36) about the origin

$$\begin{aligned} D_2(\xi,\eta,\xi',\eta';k,k',k'') = & D_2^o(k,k',k'') + \sum_{\beta=\xi,\eta,\xi',\eta'} D^o_{2,\beta}(k,k',k'')\beta \\ & + \frac{1}{2}\sum_{\beta,\gamma=\xi,\eta,\xi',\eta'} D^o_{2,\beta,\gamma}(k,k',k'')\beta\gamma \end{aligned} \tag{73}$$

where the subscripts denote partial derivatives and the superscript o in D_2 means that this function or its derivatives have been evaluated at the origin.
By making use of (62), we obtain that (67) can be written as

$$\begin{aligned}(\sigma^o)_{qp}^{dkc} = \frac{2k_1^2}{\sigma^{20}\pi A}\sum_{m=1,2}\sum_{r=-1,1}\mathrm{Re}\Bigg\{\hat{f}_{qp}^{*}\int_{\Gamma_r} du\,dv \\ \frac{1}{p_{sz}^{(r)5}\,p_{iz}^{(r)5}\,p_z^{10}(|\rho_{\xi,\xi}^o||\rho_{\eta,\eta}^o| - |\rho_{\xi,\eta}^o|^2)^5}\,\mathcal{I}^{kc}(\vec{k}^i,\vec{k}^s,\vec{l}_m^r) \\ \hat{F}_{qp}^m(\vec{k}^i,\vec{k}^s,\vec{l}_m^r) \\ \cdot\exp\left\{-\frac{p_{sx}^2|\rho_{\eta,\eta}^o| - 2p_{sx}\,p_{sy}|\rho_{\xi,\eta}^o| + p_{sy}^2|\rho_{\xi,\xi}^o|}{2\sigma^2 p_{sz}^{(r)}\,p_z(|\rho_{\xi,\xi}^o||\rho_{\eta,\eta}^o| - |\rho_{\xi,\eta}^o|^2)}\right\} \\ \cdot\exp\left\{-\frac{p_{ix}^2|\rho_{\eta,\eta}^o| - 2p_{ix}\,p_{iy}|\rho_{\xi,\eta}^o| + p_{iy}^2|\rho_{\xi,\xi}^o|}{2\sigma^2 p_{iz}^{(r)}\,p_z(|\rho_{\xi,\xi}^o||\rho_{\eta,\eta}^o| - |\rho_{\xi,\eta}^o|^2)}\right\}\Bigg\}\end{aligned} \tag{74}$$

where

$$\mathcal{I}^{kc}(\vec{k}^s,\vec{k}^i,u,v,r\,q_m) = \hat{\alpha}^t\,D_2\,\hat{\alpha}' \tag{75}$$

with

$$\hat{\alpha} = \begin{bmatrix}\hat{\alpha}_A\\ \hat{\alpha}_B\\ \hat{\alpha}_C\\ \hat{\alpha}_D\\ \hat{\alpha}_E\\ \hat{\alpha}_F\end{bmatrix} \qquad \hat{\alpha}' = \begin{bmatrix}\hat{\alpha}'_A\\ \hat{\alpha}'_B\\ \hat{\alpha}'_C\\ \hat{\alpha}'_D\\ \hat{\alpha}'_E\\ \hat{\alpha}'_F\end{bmatrix}$$

$$D = \begin{bmatrix} D_2^o & D_{2,\xi'}^o & D_{2,\xi',\xi'}^o/2 & D_{2,\eta'}^o & D_{2,\eta',\eta'}^o/2 & D_{2,\xi',\eta'}^o \\ D_{2,\xi}^o & D_{2,\xi,\xi'}^o & 0 & D_{2,\xi,\eta'}^o & 0 & 0 \\ D_{2,\xi,\xi}^o/2 & 0 & 0 & 0 & 0 & 0 \\ D_{2,\eta}^o & D_{2,\xi',\eta}^o & 0 & D_{2,\eta,\eta'}^o & 0 & 0 \\ D_{2,\eta,\eta}^o/2 & 0 & 0 & 0 & 0 & 0 \\ D_{2,\xi,\eta}^o & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{76}$$

and

$$\begin{aligned}\hat{\alpha}_A &= \alpha_A(\kappa_2^{(r)}|\rho_{\xi,\xi}^o|,\kappa_2^{(r)}|\rho_{\eta,\eta}^o|,\kappa_2^{(r)}|\rho_{\xi,\eta}^o|)\\ \hat{\alpha}'_A &= \alpha_A(\kappa_3^{(r)}|\rho_{\xi,\xi}^o|,\kappa_3^{(r)}|\rho_{\eta,\eta}^o|,\kappa_3^{(r)}|\rho_{\xi,\eta}^o|)\\ \hat{\alpha}_\zeta &= \alpha_\zeta(\kappa_2^{(r)}|\rho_{\xi,\xi}^o|,\kappa_2^{(r)}|\rho_{\eta,\eta}^o|,\kappa_2^{(r)}|\rho_{\xi,\eta}^o|,p_{sx},p_{sy})\\ \hat{\alpha}'_\zeta &= \alpha_\zeta(\kappa_3^{(r)}|\rho_{\xi,\xi}^o|,\kappa_3^{(r)}|\rho_{\eta,\eta}^o|,\kappa_3^{(r)}|\rho_{\xi,\eta}^o|,p_{ix},p_{iy})\\ \zeta &= B,C,D,E,F\\ \kappa_2^{(r)} &= p_{sz}^{(r)}\,p_z\sigma^2/2\\ \kappa_3^{(r)} &= p_{iz}^{(r)}\,p_z\sigma^2/2\end{aligned} \tag{77}$$

$$
\begin{aligned}
p_{sx} &= k_{sx} - u & p_{sy} &= k_{sy} - v & p_{sz}^{(r)} &= k_{sz} - r\,q_m \\
p_{ix} &= u - k_x & p_{iy} &= v - k_y & p_{iz}^{(r)} &= r\,q_m - k_z \\
\vec{p} &= \vec{k}^s - \vec{k}^i & & & & \qquad (78)
\end{aligned}
$$

The notation has been simplified for the matrix D, where all the elements are evaluated at $(k_{sz}, k_z, r\,q_m)$. The modulation due to the topography of the surface is contained in the function $\mathcal{I}^{kc}(\vec{k}^s, \vec{k}^i, u, v, r\,q_m)$.

4.2.3 Complementary Term

Recalling the results of Subsection 3.2.3 for the complementary term of the diffuse scattered power, we can write

$$
\begin{aligned}
S_{qp}^{dc} = \frac{k_1^2}{2^{10}\pi^3 A} \sum_{m,n=1,2} \sum_{r,r'=-1,1} \Bigg\{ & \int_{\mathbb{R}^4} du\,dv\,du'\,dv' \int d\xi\,d\eta\,d\xi'\,d\eta'\,d\tau\,d\kappa \\
& e^{j[u(\xi+\tau-\xi')-u'\tau+v(\eta+\kappa-\eta')-v'\kappa]}\, e^{-j(k_{sx}\xi+k_{sy}\eta)}\, e^{j(k_x\xi'+k_y\eta')} \\
& D_3(\xi,\eta,\xi',\eta',\tau,\kappa;k_{sz},k_z,rq_m,r'q_n') \\
& \hat{F}_{qp}^{m}(\vec{k}^i,\vec{k}^s,\vec{l}_m^{\,r})\, \hat{F}_{qp}^{n*}(\vec{k}^i,\vec{k}^s,\vec{l}_n^{\,\prime r'}) \\
& e^{-\sigma^2[(k_{sz}-r\,q_m)(k_z-r\,q_m)(1-\rho_{12})+(k_{sz}-r'\,q_n')(k_z-r'\,q_n')(1-\rho_{34})]} \\
& \Big[e^{-\sigma^2[(k_{sz}-r\,q_m)(k_{sz}-r'\,q_n')(1-\rho_{13})+(k_{sz}-r\,q_m)(r'\,q_n'-k_z)(1-\rho_{14})]} \\
& e^{-\sigma^2[(k_z-r\,q_m)(r'\,q_n'-k_{sz})(1-\rho_{23})+(k_z-r\,q_m)(k_z-r'\,q_n')(1-\rho_{24})]} \\
& - e^{-\sigma^2(k_{sz}^2+k_z^2)} \Big] \Bigg\} \qquad (79)
\end{aligned}
$$

Although the higher dimensionality in (79) makes this integral more complex than (67) the same principles used to simplify the exponential functions apply in both cases. Yet, instead of repeating the same reasoning as in the previous paragraph, we will try to make use of the physics already found there. In (67) we had the interference between the first-order component and the second-order components of the incoherently scattered field. We found that the most meaningful contribution comes from the interference between the first-order secondary sources located near the second-order secondary sources, which are in turn close to one another. This means that the waves transmitted by these secondary sources interfere more constructively when the sources are near each other, as we might expect from a rough surface with high rms height and small or moderate correlation length. Furthermore, the coherently scattered power for such a surface is negligible. Assuming that this is also the case for the complementary term of the scattered power, where the interference occurs between second-order secondary waves only, we will get significant contribution for the integral over small values of $\xi, \eta, \xi', \eta', \tau$ and κ. As we did for the cross term, the order of the Taylor series for the correlation functions is different for each function. We assume that the most significant interferences occur between the secondary sources which do not belong to the same second-order scattering event. Thus, ρ_{14}, ρ_{23} and ρ_{24} are approximated at second order, whereas ρ_{12} and ρ_{34} are approximated at first order. The correlation function ρ_{13} describes the interference between the secondary sources of the outgoing field and are also approximated only at first

order. Then, we obtain the following approximation

$$\begin{aligned}
&e^{-\sigma^2[(k_{sz}-r\,q_m)(k_z-r\,q_m)(1-\rho_{12})+(k_{sz}-r'\,q'_n)(k_z-r'\,q'_n)(1-\rho_{34})]}\\
&\Big[e^{-\sigma^2[(k_{sz}-r\,q_m)(k_{sz}-r'\,q'_n)(1-\rho_{13})+(k_{sz}-r\,q_m)(r'\,q'_n-k_z)(1-\rho_{14})]}\\
&e^{-\sigma^2[(k_z-r\,q_m)(r'\,q'_n-k_{sz})(1-\rho_{23})+(k_z-r\,q_m)(k_z-r'\,q'_n)(1-\rho_{24})]}\\
&-e^{-\sigma^2(k_{sz}^2+k_z^2)}\Big]\\
&\simeq e^{-\frac{1}{2}\sigma^2(k_{sz}-r\,q_m)(r'\,q'_n-k_z)[|\rho^o_{\xi\xi}|\,(\xi+\tau)^2+|\rho^o_{\eta\eta}|\,(\eta+\kappa)^2+2|\rho^o_{\xi\eta}|\,(\xi+\tau)(\eta+\kappa)]}\\
&\quad e^{-\frac{1}{2}\sigma^2(k_z-r\,q_m)(r'\,q'_n-k_{sz})[|\rho^o_{\xi\xi}|\,(\xi'-\tau)^2+|\rho^o_{\eta\eta}|\,(\eta'-\kappa)^2+2|\rho^o_{\xi\eta}|\,(\xi'-\tau)(\eta'-\kappa)]}\\
&\quad e^{-\frac{1}{2}\sigma^2(k_z-r\,q_m)(k_z-r'\,q'_n)[|\rho^o_{\xi\xi}|\,\xi'^2+|\rho^o_{\eta\eta}|\,\eta'^2+2|\rho^o_{\xi\eta}|\,\xi'\eta']}
\end{aligned} \tag{80}$$

Similar comments to those made after (71) are in order. Thus, (80) is to be used under the constrains of $(k_{sz}-r\,q_m)>0$, $(r\,q_m-k_z)>0$, $(k_{sz}-r'\,q'_n)>0$, and $(r'\,q'_n-k_z)>0$. The substitution (80) is then introduced into (79) with the domain of integration for (u,v,u',v') restricted to $\Gamma_r\times\Gamma'_{r'}$

$$\Gamma_r:\begin{cases} q_m<|k_z| & \text{if}\quad r=-1\\ q_m<k_{sz} & \text{if}\, r=1\end{cases}\qquad \Gamma'_{r'}:\begin{cases} q'_n<|k_z| & \text{if}\quad r=-1\\ q'_n<k_{sz} & \text{if}\, r=1\end{cases} \tag{81}$$

It is now convenient to redefine the integration coordinates as follows

$$\begin{aligned} \xi''&=\xi+\tau & \eta''&=\eta+\kappa\\ \xi'''&=\xi'-\tau & \eta'''&=\eta'-\kappa \end{aligned} \tag{82}$$

Accordingly, the modulation function D_3 is reformulated as $\hat{D}_3$

$$\begin{aligned}
&\hat{D}_3(\xi',\eta',\xi'',\eta'',\xi''',\eta''';k_{sz},k_z,rq_m,r'q'_n)\\
&\qquad\equiv D_3(\xi''+\xi'''-\xi',\eta''+\eta'''-\eta',\xi',\eta',\xi'-\xi''',\eta'-\eta''';k_{sz},k_z,rq_m,r'q'_n)
\end{aligned} \tag{83}$$

and then the following Taylor series is carried out as

$$\begin{aligned}
\hat{D}_3(\xi',\eta',\xi'',\eta'',\xi''',\eta''';k_{sz},k_z,rq_m,r'q'_n)&=\hat{D}^o_3(k,k',k'',k''')\\
&+\sum_{\beta=\xi',\eta',\xi'',\eta'',\xi''',\eta'''}\hat{D}^o_{3,\beta}(k,k',k'',k''')\,\beta\\
&+\frac{1}{2}\sum_{\beta,\gamma=\xi',\eta',\xi'',\eta'',\xi''',\eta'''}\hat{D}^o_{3,\beta,\gamma}(k,k',k'',k''')\,\beta\,\gamma
\end{aligned} \tag{84}$$

The spatial coordinates can be integrated in (79) with the help of (62) to produce

$$
\begin{aligned}
(\sigma^o)^{dc}_{qp} = \frac{k_1^2}{2\sigma^{30} A} \sum_{m,n=1,2} \sum_{r,r'=-1,1} \Bigg\{ \int_{\mathbb{R}^4} du\, dv\, du'\, dv' \\
\frac{1}{p_{sz}^{(r)5}\, p_{sz}^{\prime(r)5}\, p_{iz}^{(r)10}\, p_{iz}^{\prime(r)10} (|\rho^o_{\xi,\xi}||\rho^o_{\eta,\eta}| - |\rho^o_{\xi,\eta}|^2)^{15/2}} \\
\mathcal{I}^c(\vec{k}^s, \vec{k}^i, \vec{l}^r_m, \vec{l}^{\prime r'}_n) \\
\hat{F}^m_{qp}(\vec{k}^i, \vec{k}^s, \vec{l}^r_m)\, \hat{F}^{n*}_{qp}(\vec{k}^i, \vec{k}^s, \vec{l}^{\prime r'}_n) \\
\cdot \exp\left\{ - \frac{(p'_{ix} - p_{sx})^2 |\rho^o_{\eta,\eta}| - 2(p'_{ix} - p_{sx})\,(p'_{iy} - p_{sy}) |\rho^o_{\xi,\eta}| + (p'_{iy} - p_{sy})^2 |\rho^o_{\xi,\xi}|}{2\sigma^2 p_{iz}^{(r)}\, p_{iz}^{\prime(r)} (|\rho^o_{\xi,\xi}||\rho^o_{\eta,\eta}| - |\rho^o_{\xi,\eta}|^2)} \right\} \\
\cdot \exp\left\{ - \frac{p_{sx}^2 |\rho^o_{\eta,\eta}| - 2 p_{sx}\, p_{sy} |\rho^o_{\xi,\eta}| + p_{sy}^2 |\rho^o_{\xi,\xi}|}{2\sigma^2 p_{sz}^{(r)}\, p_{iz}^{\prime(r)} (|\rho^o_{\xi,\xi}||\rho^o_{\eta,\eta}| - |\rho^o_{\xi,\eta}|^2)} \right\} \\
\cdot \exp\left\{ - \frac{p_{sx}^{\prime 2} |\rho^o_{\eta,\eta}| - 2 p'_{sx}\, p'_{sy} |\rho^o_{\xi,\eta}| + p_{sy}^{\prime 2} |\rho^o_{\xi,\xi}|}{2\sigma^2 p_{iz}^{(r)}\, p_{sz}^{\prime(r)} (|\rho^o_{\xi,\xi}||\rho^o_{\eta,\eta}| - |\rho^o_{\xi,\eta}|^2)} \right\} \Bigg\}
\end{aligned}
\tag{85}
$$

where

$$
\begin{aligned}
p_{sx} &= k_{sx} - u & p_{sy} &= k_{sy} - v & p_{sz}^{(r)} &= k_{sz} - r\, q_m \\
p_{ix} &= u - k_x & p_{iy} &= v - k_y & p_{iz}^{(r)} &= r\, q_m - k_z \\
p'_{sx} &= k_{sx} - u' & p'_{sy} &= k_{sy} - v' & p_{sz}^{\prime(r)} &= k_{sz} - r'\, q'_n \\
p'_{ix} &= u' - k_x & p'_{iy} &= v' - k_y & p_{iz}^{\prime(r)} &= r'\, q'_n - k_z
\end{aligned}
\tag{86}
$$

and $\mathcal{I}^c$ is the modulation factor due to the topography. It is given in terms of a tensorial product

$$
\mathcal{I}^c(\vec{k}^s, \vec{k}^i, u, v, r q_m, u', v', r' q'_n) = \sum_{i,j,k=1}^{6} \hat{D}_3^{i,j,k}\, \breve{\alpha}_i\, \breve{\alpha}'_j\, \breve{\alpha}''_k \tag{87}
$$

where the tensor $\hat{D}_3^{i,j,k}$ is defined by the partial derivatives of D_3 on $\xi', \eta', \xi'', \eta'', \xi'''$ and η''' in the following manner: a) the first superscript denotes the degree of derivation on the pair of variables (ξ', η'); thus, 1 refers to no derivation, 2 and 3 refer to the first and second derivative on ξ', 4 and 5 to the first and second derivative on η', and 6 to the cross derivative on ξ' and η'; b) the second and third superscripts have equivalent meanings for the pairs of variables (ξ'', η'') and (ξ''', η'''), respectively; c) all the tensor elements corresponding to derivatives of order higher than 2 are set to zero. The vectors $\breve{\alpha}$, $\breve{\alpha}'$ and $\breve{\alpha}''$ are given by

$$
\breve{\alpha} = \begin{bmatrix} \breve{\alpha}_A \\ \breve{\alpha}_B \\ \breve{\alpha}_C \\ \breve{\alpha}_D \\ \breve{\alpha}_E \\ \breve{\alpha}_F \end{bmatrix} \qquad \breve{\alpha}' = \begin{bmatrix} \breve{\alpha}'_A \\ \breve{\alpha}'_B \\ \breve{\alpha}'_C \\ \breve{\alpha}'_D \\ \breve{\alpha}'_E \\ \breve{\alpha}'_F \end{bmatrix} \qquad \breve{\alpha}'' = \begin{bmatrix} \breve{\alpha}''_A \\ \breve{\alpha}''_B \\ \breve{\alpha}''_C \\ \breve{\alpha}''_D \\ \breve{\alpha}''_E \\ \breve{\alpha}''_F \end{bmatrix}
$$

$$
\begin{aligned}
\breve{\alpha}_A &= \alpha_A(\kappa_4^{(r)}|\rho^o_{\xi,\xi}|, \kappa_4^{(r)}|\rho^o_{\eta,\eta}|, \kappa_4^{(r)}|\rho^o_{\xi,\eta}|) \\
\breve{\alpha}'_A &= \alpha_A(\kappa_5^{(r)}|\rho^o_{\xi,\xi}|, \kappa_5^{(r)}|\rho^o_{\eta,\eta}|, \kappa_5^{(r)}|\rho^o_{\xi,\eta}|) \\
\breve{\alpha}''_A &= \alpha_A(\kappa_6^{(r)}|\rho^o_{\xi,\xi}|, \kappa_6^{(r)}|\rho^o_{\eta,\eta}|, \kappa_6^{(r)}|\rho^o_{\xi,\eta}|) \\
\breve{\alpha}_\zeta &= \alpha_\zeta(\kappa_4^{(r)}|\rho^o_{\xi,\xi}|, \kappa_4^{(r)}|\rho^o_{\eta,\eta}|, \kappa_4^{(r)}|\rho^o_{\xi,\eta}|, p'_{ix} - p_{sx}, p'_{iy} - p_{sy}) \\
\breve{\alpha}'_\zeta &= \alpha_\zeta(\kappa_5^{(r)}|\rho^o_{\xi,\xi}|, \kappa_5^{(r)}|\rho^o_{\eta,\eta}|, \kappa_5^{(r)}|\rho^o_{\xi,\eta}|, p_{sx}, p_{sy}) \\
\breve{\alpha}''_\zeta &= \alpha_\zeta(\kappa_6^{(r)}|\rho^o_{\xi,\xi}|, \kappa_6^{(r)}|\rho^o_{\eta,\eta}|, \kappa_6^{(r)}|\rho^o_{\xi,\eta}|, p'_{sx}, p'_{sy}) \\
&\qquad \zeta = B, C, D, E, F \\
\kappa_4^{(r)} &= p_{iz}^{(r)} p_{iz}'^{(r)} \sigma^2/2 \\
\kappa_5^{(r)} &= p_{sz}^{(r)} p_{iz}'^{(r)} \sigma^2/2 \\
\kappa_6^{(r)} &= p_{iz}^{(r)} p_{sz}'^{(r)} \sigma^2/2
\end{aligned}
\tag{88}
$$

Effect of Geometrical Shadowing in Random Rough Surfaces The derivation of the far-zone scattered field with IEM2M is a second-order approach based on the Kirchhoff surface fields. As already mentioned, this causes the field components of the model to be approximations to the exact first and second-order scattered field components. One of the corrections that can be made to improve these approximations is to include the shadowing effects that are not considered in the Kirchhoff surface fields, on which the whole derivation is based. The Kirchhoff approximation fails to take account of the different states of illumination under the incident field. These states range from full illumination to complete shadowing by other parts of the surface, as well as regimes of semishadowing caused by diffraction. The replacement of semishadowed regions by sharply edged illuminated and shadowed regions is made by assuming ray paths instead of waves. This approximation is referred to as geometrical shadowing and is the type of shadowing that will be considered here.

The first well known attempt to include geometrical shadowing effects in rough surfaces was made by Beckmann Beckmann (1965). However, Brockelman and Hagfors' results Brockelman & Hagfors (1966) obtained by Monte-Carlo simulation proved to be in great disagreement with Beckmann's predictions. Two different shadowing functions were introduced by Wagner Wagner (1966) and shortly afterwards by Smith Smith (1967). Hardin Hardin (1971) extended the theory to allow the source to be at a finite height above the surface, making a special case of Wagner's theory when the source is at an infinite height. Bass and Fuks also investigated rough surface shadowing in Bass & Fuks (1979). We will follow the recent study by Kapp and Brown Kapp & Brown (1994), based on Ricciardi and Sato's work on first passage time problems for Gaussian processes Ricciardi & Sato (1983; 1986), which shows how Wagner's shadowing function can be obtained in a more rigourous way than in Wagner (1966). The theory is extended here to include topographical surfaces. Bass and Fuks' considerations Bass & Fuks (1979) regarding the shadowing phenomenon in the framework of perturbation series are also considered.

5. Shadowing: Formulation for First Crossing Problems

The random rough surface $z = \zeta(x,y)$ which serves as the target of our scattering experiment is assumed to be represented by a Gaussian distribution. Thus, for any set of n pairs

$(x_1,y_1),(x_2,y_2),\ldots,(x_n,y_n)$, the joint pdf $f_{z_1,z_2,\ldots,z_n}$ of the points $z_i = z(x_i,y_i)$ on the surface is given by

$$f_{z_1,z_2,\ldots,z_n}(z_1,z_2,\ldots,z_n) = \frac{1}{(2\pi)^{\frac{n}{2}}\,\sigma^n\,D^{\frac{n}{2}}}\,\exp\left\{-\frac{1}{2D\sigma^2}\sum_{i,k=1}^{n} D_{ik}(z_i-\bar{z}_i)(z_k-\bar{z}_k)\right\} \tag{89}$$

where

$$D = \det(\rho_{ik})\,,\ \rho_{ik} = \sigma^{-2}E\{z_i z_k\}$$

and D_{ik} is the cofactor of the element ρ_{ik} in the determinant D. Each point on the surface is assumed to have a distinct mean value but all are described by a single variance σ^2.
Let S be a ray impinging upon the surface at point $\vec{r}_0 = (x_0,y_0,z_0)$ given by

$$\begin{cases} z_S &= z_0 + a(x-x_0) \\ y_S &= y_0 \end{cases} \tag{90}$$

with an angle $\theta = \operatorname{arccot} a$ over the normal, where the ray is chosen to lie in the $y = y_0$ plane for convenience. The z coordinate of the ray will be written as $z_S(\vec{r}_0,x)$ in what follows. We define $g(\zeta,S,x;x_0,y_0|z_0)\,dx\,dy_0$ as the probability that ζ will cross the incoming ray S in the interval $(x,x+dx)\times(y_0,y_0+dy_0)$ but not in the segment $(x_0,x)\times(y_0,y_0+dy_0)$, with $x>x_0$, given that the height at (x_0,y_0) is z_0. The function $g(\zeta,S,x;x_0,y_0|\zeta(x_0,y_0)=z_0)$ can be written as

$$\begin{aligned} g(\zeta,S,x;x_0,y_0|\zeta(x_0,y_0)=z_0)\,dx\,dy_0 = \Pr[&\zeta \text{ crosses S from below in} \\ &(x,x+dx)\times(y_0,y_0+dy_0)|\zeta(x_0,y_0)=z_0] \\ &-\int_{x_0}^{x} dx_1 \Pr[\zeta \text{ crosses S from below in} \\ &(x,x+dx)\times(y_0,y_0+dy_0) \text{ and in} \\ &(x_1,x_1+dx_1)\times(y_0,y_0+dy_0)] \\ &\text{but not in} x' : x' \in (x,x_1)|\zeta(x_0,y_0)=z_0] \end{aligned} \tag{91}$$

Thereby, $g(\zeta,S,x|\zeta(x_0,y_0)=z_0)$ can be found by iterating (91) to obtain the following infinite series

$$\begin{aligned} g(\zeta,S,x;x_0,y_0|z_0) = {} & w_1(x;x_0,y_0|\zeta(x_0,y_0)=z_0) \\ & -\int_{x_0}^{x} dx_1\, w_2(x,x_1;x_0,y_0|\zeta(x_0,y_0)=z_0) \\ & +\int_{x_0}^{x} dx_1 \int_{x_0}^{x_1} dx_2\, w_3(x,x_1,x_2;x_0,y_0|\zeta(x_0,y_0)=z_0) - \ldots \\ & +(-1)^n \int_{x_0}^{x} dx_1 \int_{x_0}^{x_1} dx_2 \cdots \int_{x_0}^{x_{n-1}} dx_n \\ & \quad w_{n+1}(x,x_1,\ldots,x_n;x_0,y_0|\zeta(x_0,y_0)=z_0) \\ & +\ldots \end{aligned} \tag{92}$$

where $w_i(x,x_1,\ldots,x_{i-1};x_0,y_0|\zeta(x_0,y_0)=z_0)\,dx\,dx_1\cdots dx_{i-1}\,dy_0$ is the joint probability that the ray ζ crosses S i times from below ("up-crossing"), specifically in the intervals $(x,x+dx)\times$

(y_0, y_0+dy_0), $(x_1, x_1+dx_1)\times(y_0, y_0+dy_0)$, …, $(x_{i-1}, x_{i-1}+dx_{i-1})\times(y_0, y_0+dy_0)$, given $\zeta(x_0,y_0)=z_0$. These pdf's can be written as

$$w_i(x_1,\ldots,x_i;x_0,y_0|\zeta(x_0,y_0)=z_0)=\int_a^\infty dz_1'\cdots\int_a^\infty dz_i'\prod_{j=1}^{i}(z_j'-a)$$
$$f_{i,i}[z_S(\vec{r}_0,x_1),\ldots,z_S(\vec{r}_0,x_i);z_1',\ldots,z_i'|\zeta(x_0,y_0)=z_0] \tag{93}$$

with $f_{i,i}[z_S(\vec{r}_0,x_1),z_S(\vec{r}_0,x_2),\ldots,z_S(\vec{r}_0,x_i);z_1',z_2',\ldots,z_i'|\zeta(x_0,y_0)=z_0]$ being the joint pdf of $\zeta(x_k,y_0)=z_S(\vec{r}_0,x_k)$ for $k=1,\ldots,i$, conditional upon $\zeta(x_0,y_0)=z_0$. Ricciardi and Sato obtained a similar infinite series for $g(\zeta,S,x;x_0,y_0|\zeta(x_0,y_0)=z_0)$ in Ricciardi & Sato (1983; 1986).

6. Shadowing Function

We will introduce two assumptions:

i. the heights and slopes at the shadowing points are uncorrelated with the height of the shadowed points, and
ii. the shadowing points are uncorrelated with each other.

Under these approximations, the joint pdf in (93) satisfies

$$\begin{aligned}f_{i,i}[z_S(\vec{r}_0,x_1),z_S(\vec{r}_0,x_2),\ldots,z_S(\vec{r}_0,x_i);z_1',z_2',\ldots,z_i'|\zeta(x_0,y_0)=z_0]\\ =\left(\frac{1}{2\pi\sigma\sigma'}\right)^i\prod_{k=1}^{i}e^{-\frac{(z_S(\vec{r}_0,x_k)-\bar{z}(x_k,y_0))^2}{2\sigma^2}}e^{-\frac{(z_k'-\bar{z}'(x_k,y_0))^2}{2\sigma'^2}}\\ =f_{1,1}[z_S(\vec{r}_0,x_1);z_1']\,f_{1,1}[z_S(\vec{r}_0,x_2);z_2']\cdots f_{1,1}[z_S(\vec{r}_0,x_i);z_i']\end{aligned} \tag{94}$$

The probability density function that a point on the surface at (x_0,y_0) will not be shadowed when the surface is illuminated by a plane wave of propagation vector $\hat{k}$ is

$$W(\hat{k};x_0,y_0)=\int_{-\infty}^{\infty}dz_0\,W(\hat{k};x_0,y_0|\zeta(x_0,y_0)=z_0)\,p(\zeta(x_0,y_0)=z_0) \tag{95}$$

where

$$W(\hat{k};x_0,y_0|\zeta(x_0,y_0)=z_0)=1-\int_{x_0}^{\infty}dx\,g(\zeta,S,x;x_0,y_0|\zeta(x_0,y_0)=z_0) \tag{96}$$

Combining (92), (93) and (94) with (96), we obtain

$$W(\hat{k};x_0,y_0|\zeta(x_0,y_0)=z_0)=e^{-G_\infty(x_0,y_0;z_0;\hat{k})} \tag{97}$$

with

$$\begin{aligned}G_\infty(x_0,y_0;z_0;\hat{k})=\frac{1}{2\sqrt{2}\sqrt{\pi}\sigma}\int_0^\infty dx\,e^{-\frac{(z_S(z_0,x)-\bar{z}(x,y_0))^2}{2\sigma^2}}\\ \left[\sigma'\sqrt{\frac{2}{\pi}}e^{-\frac{(a-\bar{z}'(x,y_0))}{2\sigma'^2}}-(a-\bar{z}'(x,y_0))\,\mathrm{erfc}\left(\frac{a-\bar{z}'(x,y_0)}{\sqrt{2}\sigma'}\right)\right]\end{aligned} \tag{98}$$

where $\bar{z}(x,y)$ represents the mean height at (x,y), $\bar{z}(x,y)$ is the mean slope at this point and σ'^2 is the variance of the slope. Therefore,

$$W(\hat{k};x_0,y_0)=\int_{-\infty}^{\infty}dz_0\,p(\zeta(x_0,y_0)=z_0)\,e^{-G_\infty(x_0,y_0;z_0;\hat{k})}=\left\langle e^{-G_\infty(x_0,y_0;z_0;\hat{k})}\right\rangle_{z_0} \tag{99}$$

where $\langle \quad \rangle_{z_0}$ denotes an average over z_0 values. Obtaining a 3-D shadowing function from (99) is immediate. Thus,

$$\mathcal{S}(\hat{k}^i) = W(\hat{k}) \equiv \left\langle e^{-G_\infty(x_0,y_0;z_0;\hat{k})} \right\rangle_{(x_0,y_0;z_0)} \tag{100}$$

where $\langle \quad \rangle_{(x_0,y_0;z_0)}$ denotes an average over z_0 as well as x_0 and y_0. The pdf for the variables x_0 and y_0 is a uniform distribution over a finite, topographical surface. Yet, it is important to note that we have assumed a surface with infinite dimensions in (96) and hence also in obtaining (97). However, it is possible to assume that the border effects due to a finite surface are negligible so (100) can be derived from (99) in order to compute the shadowing.

7. Bistatic Shadowing

The problem of shadowing is present both in the directions of incidence and scattering. Expressions for the relevant shadowing functions in first and second-order scattering events are derived in this section.

7.1 Single Scattering

To introduce a bistatic shadowing function for first-order scattering, let us first consider an incident ray S_i and a reflected or scattered ray S_s crossing a point (x_0,y_0,z_0) on the surface with angles θ_i and θ_s over the normal and slopes $a_i = \cot\theta_i$ and $a_s = \cot\theta_s$. Likewise, k_i and k_s represent the propagation vectors of the plane waves along the incident and reflected ray directions. The probability $W(A,B)$ that the surface will not cross either S_i (event A) or S_s (event B) anywhere equals the product of the probability that it will not cross S_i, W_A, and the conditional probability that it will not cross S_s given that it does not cross S_i, $W(B|A)$. Within a solid angle "pencil" or neighbourhood around the ray S_1 and up to some distance or radius from (x_0,y_0,z_0), the event B is correlated to event A and $W(B|A) \neq W(B)$ in general.
Both this radius and the width of the pencil are proportional to the correlation length of the surface. In the surfaces we are considering there are two correlation lengths, namely, the one corresponding to the deterministic component which shapes the surface as topographical and the one corresponding to the random component. The former is larger than the latter. The correlation between the statistical events A and B is only due to the random component of the correlation. Therefore, the scope of the statistical interference of A and B is small at the scale of the whole surface. Hence, we can approximate $W(B|A) = W(B)$ for cases other than backscattering and write

$$\begin{aligned} &W(\hat{k}^i,\hat{k}^s;x_0,y_0|\zeta(x_0,y_0)=z_0) \\ &\qquad = W(\hat{k}^i;x_0,y_0|\zeta(x_0,y_0)=z_0)\, W(\hat{k}^s;x_0,y_0|\zeta(x_0,y_0)=z_0) \end{aligned} \tag{101}$$

where $W(\hat{k}^i,\hat{k}^s;x_0,y_0|\zeta(x_0,y_0)=z_0)$ is a more rigorous notation for $W(A,B)$. Hence, the following bistatic shadowing function is found

$$\mathcal{S}(\hat{k}^i,\hat{k}^s) = \left\langle e^{-[G_\infty(x_0,y_0;z_0;\hat{k}^i)+G_\infty(x_0,y_0;z_0;\hat{k}^s)]} \right\rangle_{(z_0;x_0,y_0)} \tag{102}$$

For the case of backscattering, $W(B|A) = 1$ and

$$\mathcal{S}(\hat{k}^i,\hat{k}^s) = \mathcal{S}(\hat{k}^i) \tag{103}$$

7.2 Second Order Scattering

For the case of second-order scattering we will apply, in a reiterative fashion, the result given in (102) for bistatic shadowing. However, there are some remarks to be made. First, we differentiate between those intermediate plane waves propagating through the medium below the surface, $|\vec{l}_2\rangle$, and those propagating through the incidence medium, $|\vec{l}_1\rangle$. Then it is necessary to consider that the intermediate plane waves travel both upwards and downwards. We present shadowing functions for all these four combined cases.

Let us consider the scattering event of a second-order deflection where the intermediate plane wave propagates upwards within the incidence medium. The bistatic shadowing function $\mathcal{S}(\hat{k}^i,\hat{l}_1^+)$ defined in (102) represents the fraction of the surface which scatters the incident power outwards. Therefore, $1-\mathcal{S}(\hat{k}^i,\hat{l}_1^+)$ is the fraction of the scattered power that is once more intercepted by the surface. In the same fashion, only the fraction $\mathcal{S}(\hat{l}_1^+,\hat{k}^s)$ of the surface rescatters the power into the $\hat{k}^s$ direction. Hence, the second order shadowing function for "reflected" intermediate waves can be written as

$$\mathcal{S}_1(\hat{k}^i,\hat{l}_1^+,\hat{k}^s)=[1-\mathcal{S}(\hat{k}^i,\hat{l}_1^+)]\,\mathcal{S}(\hat{l}_1^+,\hat{k}^s) \tag{104}$$

Likewise, we obtain

$$\mathcal{S}_1(\hat{k}^i,\hat{l}_1^-,\hat{k}^s)=\mathcal{S}(\hat{k}^i,\hat{l}_1^-)\,\mathcal{S}(\hat{l}_1^-,\hat{k}^s) \tag{105}$$

as $\mathcal{S}(\hat{k}^i,\hat{l}_1^-)$ is the fraction of the first-order scattered power that will impinge again upon the surface.

When the intermediate wave planes propagate through the medium below the surface, the same principles as above apply. The only difference is that the computation of the bistatic shadowing functions have to be made with the surface equation $z=\zeta(x,y)$ replaced by $z=-\zeta(x,y)$. If we denote such shadowing functions as $\mathcal{S}'(\hat{k}^i,\hat{l}_2^+)$ and $\mathcal{S}'(\hat{l}_2^+,\hat{k}^s)$, the second-order shadowing for "refracted" intermediate waves is given by

$$\mathcal{S}_2(\hat{k}^i,\hat{l}_2^+,\hat{k}^s)=\mathcal{S}'(\hat{k}^i,\hat{l}_2^+)\,\mathcal{S}'(\hat{l}_2^+,\hat{k}^s) \tag{106}$$

$$\mathcal{S}_2(\hat{k}^i,\hat{l}_2^-,\hat{k}^s)=[1-\mathcal{S}'(\hat{k}^i,\hat{l}_2^-)]\,\mathcal{S}'(\hat{l}_2^-,\hat{k}^s) \tag{107}$$

Acknowledgements

This chapter is dedicated to the memory of Tanos (Tony) Elfouhaily, an extraordinary man whose early departure left our scientific community utterly desolated.

Appendix A.The C Coefficients.

As stated before, all the expressions for f_{qp} and F_{qp} are given in Alvarez-Perez (2001), namely, in its equations (A1) to (A5) for f_{qp} and (B3) to (B10) for F_{qp}. Equations (A5) were criticized as a result of a misinterpretation: they do not imply any relationship for the fields but only for the scattering coefficient as effective reflection coefficients, yet they do guarantee reciprocity.

However, the C coefficients are written there in a very general manner which requires a great deal of work by the implementer. Here these coefficients are worked out and incorporate the

first power of the $\pm(k_{sz}+k_z)$, $(k_{sz}\mp k_z^{(2)})$ and $(\pm k_{sz}^{(2)}-k_z)$ included in the i_n's.

$$
\begin{aligned}
C_1(k_x,k_y,-k_z) &= -C_1(k_{sx},k_{sy},-k_{sz}) = k_1\cos\phi_s(\cos\theta_s-\cos\theta)\\
C_1(k_x,k_y,\pm k_z^{(2)}) &= \cos\phi_s(k_1\cos\theta_s \mp \sqrt{k_2^2-k_1^2\sin^2\theta})\\
C_1(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= \cos\phi_s(k_1\cos\theta \pm \sqrt{k_2^2-k_1^2\sin^2\theta_s})
\end{aligned}
$$

$$
\begin{aligned}
C_2(k_x,k_y,-k_z) &= k_1^2\cos\theta(\cos\phi_s-\cos\phi_s\cos\theta\cos\theta_s-\sin\theta\sin\theta_s)\\
C_2(k_{sx},k_{sy},-k_{sz}) &= k_1^2\cos\theta_s(\cos\phi_s-\cos\phi_s\cos\theta\cos\theta_s-\sin\theta\sin\theta_s)\\
C_2(k_x,k_y,\pm k_z^{(2)}) &= \cos\theta\,[\cos\phi_s(k_2^2 \mp k_1\cos\theta_s\sqrt{k_2^2-k_1^2\sin^2\theta}) - k_1^2\sin\theta\sin\theta_s]\\
C_2(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= \pm k_1\sin\theta\sin\theta_s\sqrt{k_2^2-k_1^2\sin^2\theta_s}\\
&\quad -\cos\phi_s[\cos\theta(k_2^2-k_1^2\sin^2\theta_s)\pm k_1\sqrt{k_2^2-k_1^2\sin^2\theta_s}]
\end{aligned}
$$

$$
\begin{aligned}
C_3(k_x,k_y,-k_z) &= -k_1^2\sin\theta(\cos\phi_s\cos\theta_s\sin\theta-\cos\theta\sin\theta_s)\\
C_3(k_{sx},k_{sy},-k_{sz}) &= -k_1^2\sin\theta_s(\cos\phi_s\cos\theta\sin\theta_s-\cos\theta_s\sin\theta)\\
C_3(k_x,k_y,\pm k_z^{(2)}) &= -k_1\sin\theta(k_1\cos\phi_s\cos\theta_s\sin\theta \mp \sqrt{k_2^2-k_1^2\sin^2\theta}\sin\theta_s)\\
C_3(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= -k_1\sin\theta_s(k_1\cos\phi_s\cos\theta\sin\theta_s \pm \sqrt{k_2^2-k_1^2\sin^2\theta_s}\sin\theta)
\end{aligned}
$$

$$
\begin{aligned}
C_4(k_x,k_y,-k_z) &= -k_1\cos\theta[\cos\phi_s(\cos\theta\cos\theta_s-1)+\sin\theta\sin\theta_s]\\
C_4(k_{sx},k_{sy},-k_{sz}) &= -k_1\cos\theta_s[\cos\phi_s(\cos\theta\cos\theta_s-1)+\sin\theta\sin\theta_s]\\
C_4(k_x,k_y,\pm k_z^{(2)}) &= \cos\theta[\cos\phi_s\cos\theta_s(k_1\cos\theta_s \mp \sqrt{k_2^2-k_1^2\sin^2\theta})\\
&\quad -k_1\sin\theta_s(\sin\theta-\cos\phi_s\sin\theta_s)]\\
C_4(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= \cos\theta_s[\cos\phi_s(k_1 \pm \cos\theta\sqrt{k_2^2-k_1^2\sin^2\theta_s}) - k_1\sin\theta\sin\theta_s]
\end{aligned}
$$

$$
\begin{aligned}
C_5(k_x,k_y,-k_z) &= C_2(k_x,k_y,-k_z)\\
C_5(k_{sx},k_{sy},-k_{sz}) &= C_2(k_{sx},k_{sy},-k_{sz})\\
C_5(k_x,k_y,\pm k_z^{(2)}) &= \pm\sqrt{k_2^2-k_1^2\sin^2\theta}[\cos\phi_s\cos\theta_s(k_1\cos\theta_s \mp \sqrt{k_2^2-k_1^2\sin^2\theta})\\
&\quad -k_1\sin\theta_s(\sin\theta-\cos\phi_s\sin\theta_s)]\\
C_5(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= \cos\theta_s[\cos\phi_s(k_2^2 \pm k_1\cos\theta\sqrt{k_2^2-k_1^2\sin^2\theta_s}) - k_1^2\sin\theta\sin\theta_s]
\end{aligned}
$$

$$
C_6(k_x,k_y,-k_z) = C_6(k_{sx},k_{sy},-k_{sz}) = C_6(k_x,k_y,\pm k_z^{(2)}) = C_6(k_{sx},k_{sy},\pm k_{sz}^{(2)}) = 0
$$

$$
\begin{aligned}
C_7(k_x,k_y,-k_z) &= -k_1\sin\phi_s(\cos\theta\cos\theta_s-1)\\
C_7(k_{sx},k_{sy},-k_{sz}) &= k_1\cos\theta_s\sin\phi_s(\cos\theta-\cos\theta_s)\\
C_7(k_x,k_y,\pm k_z^{(2)}) &= \sin\phi_s[\cos\theta_s(k_1\cos\theta_s\mp\sqrt{k_2^2-k_1^2\sin^2\theta})+k_1\sin^2\theta_s]\\
C_7(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= \cos\theta_s\sin\phi_s(k_1\cos\theta\pm\sqrt{k_2^2-k_1^2\sin^2\theta_s})
\end{aligned}
$$

$$
\begin{aligned}
C_8(k_x,k_y,-k_z) &= k_1^2\cos\theta\sin\phi_s(\cos\theta_s-\cos\theta)\\
C_8(k_{sx},k_{sy},-k_{sz}) &= k_1^2\cos\theta_s\sin\phi_s(\cos\theta_s-\cos\theta)\\
C_8(k_x,k_y,\pm k_z^{(2)}) &= \cos\theta\sin\phi_s(k_2^2\cos\theta_s\mp k_1\sqrt{k_2^2-k_1^2\sin^2\theta})\\
C_8(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= -\cos\theta_s\sin\phi_s(k_2^2\cos\theta\pm k_1\sqrt{k_2^2-k_1^2\sin^2\theta_s})
\end{aligned}
$$

$$
\begin{aligned}
C_9(k_x,k_y,-k_z) &= -k_1^2\sin\phi_s\sin^2\theta(\cos^2\theta_s+\sin\theta\sin\theta_s))\\
C_9(k_{sx},k_{sy},-k_{sz}) &= 0\\
C_9(k_x,k_y,\pm k_z^{(2)}) &= C_9(k_x,k_y,-k_z)\\
C_9(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= 0
\end{aligned}
$$

$$
\begin{aligned}
C_{10}(k_x,k_y,-k_z) &= k_1\cos\theta\sin\phi_s(\cos\theta-\cos\theta_s)\\
C_{10}(k_{sx},k_{sy},-k_{sz}) &= k_1\sin\phi_s(\cos\theta\cos\theta_s-1)\\
C_{10}(k_x,k_y,\pm k_z^{(2)}) &= -\cos\theta\sin\phi_s(k_1\cos\theta_s\mp\sqrt{k_2^2-k_1^2\sin^2\theta})\\
C_{10}(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= -\sin\phi_s(k_1\pm\cos\theta\sqrt{k_2^2-k_1^2\sin^2\theta_s})
\end{aligned}
$$

$$
\begin{aligned}
C_{11}(k_x,k_y,-k_z) &= -C_8(k_x,k_y,-k_z)\\
C_{11}(k_{sx},k_{sy},-k_{sz}) &= -C_8(k_{sx},k_{sy},-k_{sz})\\
C_{11}(k_x,k_y,\pm k_z^{(2)}) &= \mp\sqrt{k_2^2-k_1^2\sin^2\theta}\sin\phi_s(k_1\cos\theta_s\mp\sqrt{k_2^2-k_1^2\sin^2\theta})\\
C_{11}(k_{sx},k_{sy},\pm k_{sz}^{(2)}) &= \mp\sqrt{k_2^2-k_1^2\sin^2\theta_s}\sin\phi_s(k_1\cos\theta\pm\sqrt{k_2^2-k_1^2\sin^2\theta_s})
\end{aligned}
$$

$$
\begin{aligned}
C_{12}(k_x,k_y,-k_z) &= C_{12}(k_x,k_y,\pm k_z^{(2)}) = 0\\
C_{12}(k_{sx},k_{sy},-k_{sz}) &= C_{12}(k_{sx},k_{sy},\pm k_{sz}^{(2)}) = -k_1^2\sin\phi_s\sin^2\theta_s
\end{aligned}
$$

With these expression it is straightforward to prove that the SPM limit for the most general case of bistatic scattering is reached when we take (1) to first order in σ^2. Probably the formal character of the C's as given in Alvarez-Perez (2001) has precluded other authors to properly implement the model, as it is the case in Fung et al. (2002) or Du (2008), where incorrect IEM2M results were provided. A Mathematica version of the code is available from the author upon request.

8. References

Alvarez-Perez, J. L. (2001). An extension of the IEM/IEMM surface scattering model, *Waves Random Media* **11**: 307–29.

Bass, F. G. & Fuks, I. M. (1979). *Wave Scattering from Statistically Rough Surfaces*, Pergamon Press.

Beckmann, P. (1965). Shadowing of rough surfaces, *IEEE Transactions on Antennas and Propagation* **13**: 384–388.

Brockelman, R. A. & Hagfors, T. (1966). Note on the effect of shadowing on the backscattering of waves from random rough surfaces, *IEEE Transactions on Antennas Propagation* **14**(5): 621–629.

Chen, K., Wu, T., Tsang, L., Li, Q., Shi, J. & Fung, A. (2003). Emission of rough surfaces calculated by the integral equation method with comparison to three-dimensional moment method simulations, *IEEE Transactions on Geoscience and Remote Sensing* **41**(1): 90–101.

Chen, K., Wu, T., Tsay, M. & Fung, A. (2000). A note on the multiple scattering in an IEM model, *IEEE Transactions on Geoscience and Remote Sensing* **38**(1): 249–256.

Du, Y. (2008). A new bistatic model for electromagnetic scattering from randomly rough surfaces, *Waves in Random and Complex Media* **18**(1): 109–128.

Elfouhaily, T. M. & Guerin, C.-A. (2004). A critical survey of approximate scattering wave theories from random rough surfaces, *Waves in Random Media* **14**(4): R1–R40.

Fung, A. & Chen, K. (2004). An update on the IEM surface backscattering model, *IEEE Geoscience and Remote Sensing Letters* **1**(2): 75–77.

Fung, A. K. (1994). *Microwave scattering and emission models and their applications*, Artech House.

Fung, A., Liu, W., Chen, K. & Tsay, M. (2002). An improved IEM model for bistatic scattering from rough surfaces, *Journal of Electromagnetic Waves and Applications* **16**(5): 689–70.

Fung, A. & Pan, G. W. (1986). An integral equation model for rough surface scattering, *Proccedings of the International Symposium on Multiple Scattering of Wave in Random Media and Random Surface (Pennsylvania State University Press)* pp. 701–714.

Hardin, J. C. (1971). Theoretical analysis of rough surface shadowing from point source radiation, *Journal of the Acoustical Society of America* **52**: 227–233.

Hsieh, C.-Y., Fung, A. K., Nesti, G., Sieber, A. J. & Coppo, P. (1997). A further study of the IEM surface scattering model, *IEEE Transactions on Geoscience and Remote Sensing* **35**(4): 901–909.

Kapp, D. A. & Brown, G. S. (1994). Effect of correlation between shadowing and shadowed points in rough surface scattering, *IEEE Transactions on Antennas and Propagation* **42**(8): 11554–1160.

Poggio, A. J. & Miller, E. K. (1973). *Integral Equation Solution of three dimensional scattering problems (in Computer Tecniques for Electromagnetics, ed. R. Mittra)*, Pergamon.

Ricciardi, L. M. & Sato, S. (1983). A note on first passage time problems for Gaussian processes and varying boundaries, *IEEE Transactions on Information Theory* **29**(3): 454–457.

Ricciardi, L. M. & Sato, S. (1986). On the evaluation of first passage time densities for Gaussian processes, *Signal Processing* **11**: 339–357.

Smith, B. G. (1967). Geometrical shadowing of a random rough surface, *IEEE Transactions on Antennas and Propagation* **15**(5): 668–671.

Wagner, R. J. (1966). Shadowing of randomly surfaces, *Journal of the Acoustical Society of America* **41**(1): 138–147.

Wu, T. & Chen, K. (2004). A reapraisal of the validity of the IEM model for backscattering from rough surfaces, *IEEE Transactions on Geoscience and Remote Sensing* **42**(4): 743–53.
Wu, T., Chen, K., Shi, J., Lee, H.-W. & Fung, A. (2008). A study of an AIEM model for bistatic scattering from randomly rough surfaces, *IEEE Transactions on Geoscience and Remote Sensing* **46**(9): 2584–98.

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Remote Sensing is collecting and interpreting information on targets without being in physical contact with the objects. Aircraft, satellites ...etc are the major platforms for remote sensing observations. Unlike electrical, magnetic and gravity surveys that measure force fields, remote sensing technology is commonly referred to methods that employ electromagnetic energy as radio waves, light and heat as the means of detecting and measuring target characteristics. Geoscience is a study of nature world from the core of the earth, to the depths of oceans and to the outer space. This branch of study can help mitigate volcanic eruptions, floods, landslides ... etc terrible human life disaster and help develop ground water, mineral ores, fossil fuels and construction materials. Also, it studies physical, chemical reactions to understand the distribution of the nature resources. Therefore, the geoscience encompass earth, atmospheric, oceanography, pedology, petrology, mineralogy, hydrology and geology. This book covers latest and futuristic developments in remote sensing novel theory and applications by numerous scholars, researchers and experts. It is organized into 26 excellent chapters which include optical and infrared modeling, microwave scattering propagation, forests and vegetation, soils, ocean temperature, geographic information , object classification, data mining, image processing, passive optical sensor, multispectral and hyperspectral sensing, lidar, radiometer instruments, calibration, active microwave and SAR processing. Last but not the least, this book presented chapters that highlight frontier works in remote sensing information processing. I am very pleased to have leaders in the field to prepare and contribute their most current research and development work. Although no attempt is made to cover every topic in remote sensing and geoscience, these entire 26 remote sensing technology chapters shall give readers a good insight. All topics listed are equal important and significant.

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