Optimization of Industrial Production of Feed Blends as Multiple Criteria Programming Problem

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1. Introduction

Industrial production of feed blends involves mixing of single ingredients that are simultaneously used for feeding of various kinds and categories of domestic animals. The optimal mix of ingredients in industrial production of livestock feed has to be satisfactory in terms of quality and cost considering the need for particular sorts of feed and their seasonal availability.

While the feed has to meet the nutritional requirements of livestock to allow maximal weight gain its industrial production has to be economical, which can only be ensured by an optimal blending of ingredients.

Optimization of industrial production of feed blends in terms of their nutritive value and in terms of economic criteria can be carried out by application of mathematical optimization methods. The benefit of application of these methods is fast and efficient solution for the optimal combination of ingredients considering the nutritive needs of animals and constraints in their availability. Unlike the standard problem of feed where the requirements for basic nutrients have to be met at minimised costs and which is solved mainly by linear programming, the authors introduce also the goals of meal quality where different requirements of decision makers are modelled by multiple-criteria linear programming.

The lack of appropriate methodology to solve the problem of optimizing industrial production of livestock feed blends without investing much effort of analysts and with minimal engagement of decision maker is the essential issue of this research. This problem requires defining of feed blend optimization methodology by using some of the existing multicriteria programming methods. Defined in this way the problem requires building up a general multicriteria programming model for optimization of feed blends, selecting optimization criteria, choosing the method to solve the concrete model, and choosing the method to determine the weights of criteria functions. Consequently the basic aims of this work are:

(1) to point on the concrete example that optimization of feed blend production is essentially a multi-criteria problem, (2) to develop a multi-criteria programming model which allows optimization of feed blend production for different kinds and categories of

livestock, (3) to apply a multi-criteria programming method in solving the problem of the concrete pig farming company, (4) to analyse the obtained solutions and point to the strengths and weaknesses of the formed model as well as to the possibilities of further research in this field.

2. Criteria and optimization model of feed blend production

When determining criteria for optimization of feed blend production the factors to be considered are:
- the cost of blend preparation
- the nutrition needs of animals for which the blends are prepared
- the blend quality

It would be ideal if the costs were minimal, the nutrition needs completely satisfied, and the blend quality maximal. Consequently, the criteria for optimization of feed blend production are:
1. Cost expressed in monetary units,
2. Nutrients (in percentage) needed for the maximal weight gain,
3. Nutrients (in percentage) affecting the quality of the blend, and thus also the weight gain of the animal for which the feed is prepared.

The problem of determining the optimal production of feed blends emerges if the needs for certain sorts and quantities of blends are given but the quantity (availability) of some ingredients is limited. This problem arises when the farmers prepare the feed blends in their own plants, but it may also appear if the feed blend is produced to be marketed.
Consequently, the problem of determining the optimal feed blend production for particular kinds and categories of livestock is the multi-criteria programming problem. If we want to solve the problem by multiple criteria programming methods, we have to start from the following:
- Criteria for determination of the optimal feed blend are given.
- The feed blends have to meet the needs for nutrients of the given animal kinds and categories.
- Particular feed sorts and quantities are available that can be used as blend components.

Let us introduce the following marks:
f_j - criteria functions ($j = 1, ..., p$),
b_{kq} - need for a nutrient of k kind in the blend unit q, ($k = 1, ..., m; q = 1, ... , s$),
n - number of available sorts of feed (ingredients),
c_{ijq} - i coefficient of j criterion function for q blend, ($i = 1, ...,n;\ j = 1, ..., p; q = 1, ... ,s$),
x_{iq} - quantity of i ingredient in q blend, ($i = 1, ..., n; q=1, ... ,s$),
a_{ikq} - quantity of k nutrient per unit of i ingredient in the q blend ($i = 1, ..., n; k = 1, ..., m; q = 1, ... , s$),
d_i - available quantity of i ingredient, ($i = 1, ..., n$),
a_q – the needed quantity of q blend, ($q = 1, ... , s$).
Now we can form a multiple criteria linear programming model for optimization of feed blend production:

$$\min \left[\sum_{i=1}^{n}\sum_{q=1}^{s} c_{i1q}x_{iq},\ldots,\sum_{i=1}^{n}\sum_{q=1}^{s} c_{ipq}x_{iq} \right] \tag{1}$$

s.t.

$$\sum_{i=1}^{n} a_{ikq}x_{iq} \geq a_q b_{kq}\,, \quad (k = 1,\ldots, m;\ q=1,\ \ldots\ ,\ s) \tag{2}$$

$$\sum_{i=1}^{n} x_{iq} = a_q\,, \quad (q = 1,\ \ldots\ ,\ s) \tag{3}$$

$$\sum_{q=1}^{s} x_{iq} \leq d_i \quad (i = 1,\ \ldots\ ,\ n) \tag{4}$$

$$x_{iq} \geq 0\,, \quad (i = 1,\ldots,n;\ q = 1,\ \ldots\ ,\ s). \tag{5}$$

Naturally, besides the constraints of minimal and maximal requirements for particular nutrients (b_{kq}) other constraints are also possible, for instance the requirement to limit one ingredient to a certain quantity. It is also frequently required that a particular sort of feed is not included in quantities too large or too small. Of course, all the requirements will depend on the kind of animal and suggestions of nutritionists. The obtained model is a multiple criteria linear programming model and can be solved by appropriate multiple criteria linear programming methods.

3. Multicriteria linear programming

3.1. The concept of multicriteria programming

Multicriteria programming is a complex process of determining non-dominated solutions from the set of possible solutions and determining the preferred solution from the set of non-dominated solutions. The basic stages of multicriteria programming are:

1. defining of system goals and determining ways to achieve them
2. mathematical description of the system and defining the way to evaluate criteria functions
3. application of the existing multicriteria programming methods
4. final decision making
5. if the final decision is not accepted, new information is arranged and the procedure repeated from the 2nd stage onward by repeated task definition (Perić, 2008).

These stages also appear in mono-criterion programming but they are not emphasized because programming here usually involves determining the optimal solution, which corresponds to the third stage of multicriteria programming.

To solve the multicriteria programming models a large number of methods have been developed in the last thirty years. These methods are based on the optimum concept established by the Italian economist V. Pareto in 1896.

The concept of Pareto Optimum was introduced into operational research in 1951 in the pioneering work of Koopmans (1951). A more general approach seen as the problem of vector function maximization over a limited set of constraints was stated by Kuhn and Tucker in 1951. We also have to note the work of Markowitz (1959), who introduced the concept of efficient set.

3.2. Multicriteria programming model

Multicriteria programming model is a programming model with two or more criteria functions on a set of possible solutions. The mathematical form of this model can be expressed as:

$$\begin{aligned} &(\max)\ \underline{f} = \left[f_1(\underline{x}), \cdots, f_k(\underline{x}) \right], (k \geq 2) \\ &\text{s.t.} \\ &g_i(\underline{x}) \leq 0,\ i = 1, \cdots, m, \end{aligned} \tag{6}$$

or in vector form:

$$\begin{aligned} &(\max)\ \underline{f}(\underline{x}) \\ &\text{s.t.} \\ &\underline{g}(\underline{x}) \leq \underline{0}, \end{aligned} \tag{7}$$

where $\underline{x}$ is n-dimensional vector.

From the expression (6) it can be seen that the multi-criteria programming (MP) model contains k criteria functions that are to be maximized (if the model contains functions that are to be minimized it is enough to multiply them by (-1)), m constraints and n variables. If in the model all the functions $f_j(\underline{x})$ and $g_i(\underline{x})$ are linear, then we are dealing with a multicriteria linear programming model. If, however, any of these functions is non-linear then we are dealing with a non-linear programming model.

The result obtained by solving a multicriteria programming model is one or more non-dominated (efficient, Pareto optimal) solutions. The non-dominated solution obtained in this way in which all criteria functions are maximized is defined as follows:

$\underline{x}^*$ is non-dominated solution of the multicriteria programming model if there is no other allowable $\underline{x}$ such that $f_j(\underline{x}) \geq f_j(\underline{x}^*)$, implying that $f_j(\underline{x}) \geq f_j(\underline{x}^*)$ for all $j = 1, \cdots, k$, with strict inequality for at least one j.

In literature there is a large number of methods for solving multicriteria linear programming models to obtain one or several non-dominated solutions.

In order to optimize industrial production of feed blends with minimal participation of the decision maker in the process of obtaining a compromise non-dominated solution we decide to use the weight coefficient method. This method provides a non-dominated solution in the extreme point of convex polyhedron. The weight coefficient method requires a priori determination of weights for all the criteria functions. It is, however, not realistic to expect that the decision maker will be able to determine in advance the weights of criteria functions. It is therefore recommended to determine weight coefficients by applying an appropriate method on the payoff table of marginal solutions with simultaneous correction of the obtained solutions by information on the subjective importance of criteria functions obtained from the decision maker.

The preconditions for successful solving of the feed blend optimization problem are:

1. an adequate technological support in terms of analysis of ingredients to be included in the blend
2. an adequate analysis of the supply market for the blend ingredients
3. an adequate analysis of the sales market for the produced blends.

On the basis of this information it is possible to optimize the industrial production of feed blends applying the multicriteria programming methods.

3.3. Weight coefficient method

In all the methods using the utility function the MP model looks as:

$$
\begin{aligned}
&(\max)\, U(f_1, f_2, \cdots, f_k) = U(\underline{f}) \\
&\text{s.t.} \\
&g_i(\underline{x}) \le 0, \quad i = 1, \cdots, m,
\end{aligned}
\tag{8}
$$

where $U(\underline{f})$ is a multicriteria utility function. Consequently, utility function methods require the $U(\underline{f})$ to be known a priori to the MP model solving. Utility function has to reflect the decision maker's preferences. Nevertheless, it is difficult to determine $U(\underline{f})$, even for the simple problems.

What makes the utility function method significant? If $U(\underline{f})$ is correctly estimated, the decision maker will obtain the best solution. It will not only be most beneficial for the decision maker, but also, according to the non-domination theorem (Perić, 2008), it will also be non-dominated. However, an important difficulty of this method is that it requires the decision maker to express his preferences in terms of each single criteria function in advance (prior to any preliminary model solving).

Utility function $U(\underline{f})$ may take several forms. Their common characteristic is the assumption that the decision maker's utility function is separable in terms of criteria functions (Fishburn, 1974). The assumptions underlying the utility function methods are:

(1) if $\underline{f}^1 = \left[f_1^1, f_2^1, \cdots, f_k^1\right]$, and $\underline{f}^2 = \left[f_1^2, f_2^2, \cdots, f_k^2\right]$ are two vectors of $\underline{f}$, then $\underline{f}^1$ is better than $\underline{f}^2$, if $U(\underline{f}^1) > U(\underline{f}^2)$, and

(2) $U(\underline{f}) = U_1(f_1) + U_2(f_2) + \cdots + U_k(f_k)$, where U_j are utility functions for each f_j, $j = 1, \cdots, k$.

For additive utility function methods, the MP model is given in the following form:

$$
\begin{aligned}
&(\max)\, U = \sum_{j=1}^{k} U_j(f_j) \\
&\text{s.t.} \\
&g_i(\underline{x}) \le 0, \quad i = 1, \cdots, m.
\end{aligned}
\tag{9}
$$

The most frequently used form of this model in multicriteria problems is the use of weights w_j to show the importance of each criteria function. Such a model is given in the following form:

$$
\begin{aligned}
&(\max) \sum_{j=1}^{k} w_j f_j(\underline{x}) \\
&\text{s.t.} \\
&g_i(\underline{x}) \le 0, \quad i = 1, \cdots, m.
\end{aligned}
\tag{10}
$$

In order to meet the additivity assumption and criteria function linear utility assumption, as they are almost always expressed in different measure units, we propose a certain modification of this method. Namely, reducing the criteria functions value to the dimensionless space $[0,1]$ will satisfy the above assumptions. To do this we will use the linearization method. Then the model (10) gets the following form:

$$
\begin{aligned}
& (\max) \sum_{j=1}^{k} w_j f_j(\underline{x}) / f_j^* \\
\text{s.t.} \quad & \\
& g_i(\underline{x}) \leq 0 \qquad i = 1, \cdots, m,
\end{aligned}
\tag{11}
$$

where f_j^* are maximal values of criteria functions.

Model (11) is a linear programming model and to solve it we can use any method for solving linear programming models.

To determine the weights w_j several procedures have been developed and the most frequently used ones are the following two:

(1) The criteria functions from the MP model are shown in the square matrix $\underline{B}$ of dimensions $k \times k$ whose components denote the relations between the criteria functions in the following way:

$$
b_{ij} = \begin{cases} 1 & \text{if } f_i(\underline{x}) \text{ is of equal importance as } f_j(\underline{x}), \\ 2 & \text{if } f_i(\underline{x}) \text{ is more important than } f_j(\underline{x}), \\ 4 & \text{if } f_i(\underline{x}) \text{ is much more important than } f_j(\underline{x}), \\ 0 & \text{in other cases.} \end{cases}
$$

Thus the following table is obtained:

	f_1	f_2	$\cdots$	f_k	$\sum b_{ij}$
f_1	1	b_{12}	$\cdots$	b_{1k}	δ_1
f_2	b_{21}	1	$\cdots$	b_{2k}	δ_2
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
f_k	b_{k1}	b_{k2}	$\cdots$	1	δ_k

Table 1. Relations between criteria functions

The weight coefficients w_j are determined by the following formula:

$$
w_j = \frac{\delta_j}{\sum_{j=1}^{k} \delta_j} \qquad (j = 1, \cdots, k). \tag{12}
$$

This way of determining weight coefficients is applied in the cases when it is possible to compare all the criteria functions.

(2) Use of marginal solutions payoff table:

	f_1	f_2	$\cdots$	f_k
$\underline{x}_1^*$	f_1^*	f_1^2	$\cdots$	f_1^k
$\underline{x}_2^*$	f_2^1	f_2^*	$\cdots$	f_2^k
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\underline{x}_k^*$	f_k^1	f_k^2	$\cdots$	f_k^*
Weight coefficients	w_1	w_2	$\cdots$	w_k

Table 2. Marginal solutions payoff table

This table consists of elements that show the values of single marginal solutions to criteria functions, while the elements on the main diagonal represent the ideal vector.

Analyzing the distances between the single values in columns and the corresponding elements of the ideal vector we can see that two cases are possible:

a. If the distances are small we take that the weight of the given criteria function is small, as for this function several marginal solutions have approximately similar values so that this function is given small weight coefficients;

b. If the distances are large, the given criteria function has a grater value so it is given a greater weight coefficient.

When we know the data from the payoff table, i.e. the values of all the alternatives (compromise solutions) for each criterion, we may use the entropy method to estimate the weights of criteria functions.

Entropy has become an important concept in social sciences, physics, and particularly in information theory. It is used for measuring the expected content of information in a message. In information theory entropy represents a criterion for the extent of uncertainty represented by a discreet distribution of probability p_j, where a wide distribution means a higher extent of uncertainty than the narrow one. This uncertainty measure was defined by Shannon (see Shannon, 1993) as:

$$S(p_1, p_2, \cdots\cdots, p_k) = -l\sum_{j=1}^{k} p_j \ln\ p_j \tag{13}$$

where l is a positive constant. This formula is called probability distribution entropy p_j. The terms entropy and uncertainty are therefore often considered synonymous. When all the p_j are equal to each other, i.e. $p_j = 1/k$, the expression $S(p_1, p_2, \cdots\cdots, p_k)$ acquires its maximal value which is equal to $l \cdot \ln k$. Namely,

$$S(\frac{1}{k}, \frac{1}{k}, \cdots\cdots, \frac{1}{k}) = -l\sum_{j=1}^{k} \frac{1}{k} \cdot \ln \frac{1}{k} = -l \cdot k \cdot \frac{1}{k} \cdot (-\ln k) = l \cdot \ln k\ .$$

Consequently, entropy is maximal when all the evaluations are equal to each other.

The payoff table for the set of alternatives contains a certain quantity of information and

entropy can be used as a tool for criteria evaluation. The entropy concept is particularly useful in investigating contrasts between data sets. For example, a criterion may not provide much information useful for comparison of alternatives if all the alternatives by this criterion have similar values. Moreover if all the values by that criterion are equal, the criterion can be eliminated from further consideration. In such a case entropy calculated on the basis of these values will be great, and the criterion will be given a smaller weight.
We will now consider a multicriteria decision making problem in which we have m alternatives and k criteria. The evaluation of the i alternative by the j criterion p_{ij} contains some extent of information that can be measured by the entropy value. However, the significance of p_{ij} is determined by different evaluations of all the alternatives by the j criterion, therefore we will consider the evaluation of the i alternative by j criterion as a part (percentage) of the total sum of values obtained by all the alternatives by the j criterion $(\sum_{i=1}^{m} x_{ij})$. Consequently, the valuation of the i alternative by the j criterion is defined as:

$$p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}, \ \forall \text{ i, j}, \tag{14}$$

where x_{ij} are the elements of the payoff table.
Entropy E_j of the evaluation set of all the alternatives by j criterion (since we are esuring the weight of the j criterion) is then:

$$E_j = -l \sum_{i=1}^{m} p_{ij} \ln\ p_{ij}, \tag{15}$$

where l is the constant $l = \frac{1}{\ln m}$ ensuring $0 \le E_j \le 1$.

The degree of diversification d_j of the information obtained by evaluations by j criterion can be defined as

$$d_j = 1 - E_j, \forall j. \tag{16}$$

If the decision maker has no reason to prefer one criterion to another, the principle of insufficient reasons suggests equal weights for all the criteria. Considering the entropy as well the best set of weights, instead of equal weights, is then given as:

$$w_j = \frac{d_j}{\sum_{j=1}^{k} d_j}, \forall j \quad .(17)$$

If, however, the decision maker a priori has subjectively chosen weights λ_j, this can be used as additional information for determining weights. The final weights then are:

$$w_j^{0} = \frac{\lambda_j w_j}{\sum_{j=1}^{k} \lambda_j w_j}, \forall j \tag{18}$$

4. Solving the problem of optimization of feed blend production

4.1. Setting the problem

We will now establish the multiple-criteria linear programming (MLP) model to optimize feed blend production in a company dealing with pig farming and preparation of feed blend, and then we will solve the obtained model by application of multi-criteria linear programming, in the first step separately optimizing each chosen criteria function.

The subsequent breakdown gives the data necessary for optimization of production of 5 different sorts of feed blend for pigs: PS-1 pigs weighing up to 15 kg, PS-2 pigs weighing 15-25 kg, PS-3 pigs weighing 25-60 kg, PS-4 pigs weighing 60-100 kg, and PS-5 pigs weighing over 100 kg. For the coming period of one month the required quantities are: 10000 kg of blend PS-1 (a_1), 20000 kg of blend PS-2 (a_2), 30000 kg of blend PS-3 (a_3), 50000 kg of blend PS-4 (a_4) and 20000 kg of blend PS-5 (a_5). The optimal production is to meet the requirements for minimal and maximal quantities of nutrients in the blends. Besides, it is necessary to meet the requirement for the necessary quantities of the feed blends for this period considering the constraint in terms of ingredients availability. Determination of minimal and maximal shares of nutrients in the feed blend is based on scientific research (see e.g. Perić, 2008. It has to be noted that due to the varying quality of ingredients prior to the preparation of feed blends it is necessary to analyse the content of nutrients in all the available ingredients. All the data are shown in the Tables 3, 4, and 5.

The ingredients for pig feed blends, their purchasing price per unit, the percentage of nutrients, water and raw protein per unit, as well as the available quantity are shown in the Table 3. The total cost has to be minimised, the total share of nutrients in the blend has to be maximised, the share of water in the optimal meal has to be minimized, while the total share of raw protein has to be maximised.

The choice of criteria functions for optimization of feed blend production is carried out in cooperation with the nutritionists.

Sorts of feed		Price – c_{i1} (min)	Nutrients – c_{i2} (max)	Water – c_{i3} (min)	Raw protein c_{i4} (max)	Available quantity in kg - d_i
H1	Barley	1.75	70	11	11.5	30000
H2	Maize	1.75	80	12	8.9	50000
H3	Lucerne	1.65	32	6.9	17.0	20000
H4	Powdered milk	6	86	8.4	33.0	10000
H5	Fish meal	9	69	9	61.0	20000
H6	Soya	2.7	92	10	38.0	2000
H7	Soya hulls	3.5	79	11	42.0	2000
H8	Dried whey	9	78	6	12.0	5000
H9	Rape pellets	1.8	66	8.0	36.0	20000
H10	Wheat	1.8	79	12	13.5	70000
H11	Rye	1.8	75	11.4	12.6	40000
H12	Millet	3.5	65	10.0	11.0	40000
H13	Sunfl. pellets	1.8	68	7.0	42.0	20000

Table 3. Sorts of feed

Nutrients		Constraint type	Min or max req. PS-1 (b_{k1})	Min or max req. PS-2 (b_{k2})	Min or max req. PS-3 (b_{k3})	Min or max req. PS-4 (b_{k4})	Min or max req. PS-5 (b_{k5})
E1	Raw protein %	≥	20	18	16	14	13
E2	Pulp %	≤	4	6	7	7	9
E3	Calcium–Ca %	≤	1.2	1.1	0.9	0.8	1
E4	Phosporus–P %	≥	0.6	0.5	0.5	0.5	0.50
E5	Ash %	≤	8	8	8	8	8
E6	Metionin %	≥	0.70	0.55	0.45	0.40	0.25
E7	Lizin %	≥	1.30	1.00	0.70	0.60	0.50
E8	Triptofan %	≥	0.18	0.15	0.11	0.11	0.11
E9	Treonin %	≥	0.69	0.56	0.44	0.41	0.40
E10	Izoleucin %	≥	0.69	0.56	0.44	0.41	0.41
E11	Histidin %	≥	0.25	0.20	0.16	0.15	0.15
E12	Valin %	≥	0.69	0.56	0.44	0.41	0.41
E13	Leucin %	≥	0.83	0.68	0.52	0.48	0.48
E14	Arginin %	≥	0.28	0.23	0.18	0.16	0.16
E15	Fenkalanin %	≥	0.69	0.56	0.44	0.41	0.41
E16	Copper mg	≥	6	6	0	0	20
E17	Iodine mg	≥	0.20	0.20	0.20	0.20	0.20
E18	Iron mg	≥	80	80	0	0	80
E19	Mangan mg	≥	20	20	20	20	30
E20	Sedan mg	≥	0.10	0.10	0.10	0.10	0.10

Table 4. Needs for nutrients

The nutrients that have to be contained in the feed blend (in compliance with the requirements of nutritionists) are shown in the Table 4. In addition to that the table shows the nutritionists' requirements, for particular nutrients in the special blends prepared for five different categories of pigs. Some of the nutrients are given in terms of minimal and some in terms of maximal requirements.

The Table 5 is a nutrition matrix and its elements a_{ik} are the contents of particular nutrient in the feed unit.

	H1	H2	H3	H4	H5	H6	H7	H8	H9	H10	H11	H12	H13
E1	11.5	8.9	17.0	33	61	38	42	12	36	13.5	12.6	11.0	42
E2	5.0	2.9	24.0	0.0	1.0	5.0	6.5	0.0	13.2	3.0	2.8	10.5	13.0
E3	0.08	0.01	1.3	1.25	7.0	0.25	0.2	0.87	0.6	0.05	0.08	0.1	0.4
E4	0.42	0.25	0.23	1.0	3.5	0.59	0.6	0.79	0.93	0.41	0.3	0.35	1.0
E5	2.5	1.5	9.6	8.0	24	4.6	6.0	9.7	7.2	2.0	1.45	4.0	7.7
E6	0.18	0.17	0.28	0.98	1.65	0.54	0.6	0.2	0.67	0.25	0.16	0.2	1.5
E7	0.53	0.22	0.73	2.6	4.3	2.4	2.7	1.1	2.12	0.4	0.4	0.4	1.7
E8	0.17	0.09	0.45	0.45	0.7	0.52	0.65	0.2	0.46	0.18	0.14	0.18	0.5
E9	0.36	0.34	0.75	1.75	2.6	1.69	1.7	0.8	1.6	0.35	0.36	0.28	1.5
E10	0.42	0.37	0.84	2.1	3.1	2.18	2.8	0.9	1.41	0.69	0.53	0.53	2.1
E11	0.23	0.19	0.35	0.86	1.93	1.01	1.1	0.2	0.95	0.17	0.27	0.18	1.0
E12	0.62	0.42	1.04	2.38	3.25	2.02	2.2	0.7	1.81	0.69	0.62	0.62	2.3
E13	0.8	1.0	1.3	3.3	4.5	2.8	3.8	1.2	2.6	1.0	0.7	0.9	2.6
E14	0.5	0.52	0.75	1.1	4.2	2.8	3.2	0.4	2.04	0.6	0.5	0.8	3.5
E15	0.62	0.44	0.91	1.58	2.8	2.1	2.1	0.4	1.41	0.78	0.62	0.62	2.2
E16	6	3.2	0	11.7	8	17	0	4	6	6	15	10	0
E17	0.05	0	0	0.9	5.2	0	0	0	0.6	0.04	0.05	0.02	0
E18	80	20	0	3	635	90	0	0	160	50	65	40	0
E19	25	4.9	0	2	73	35	0	4.2	53	30	80	10	23
E20	0.15	0.04	0	0.12	4.3	0	0	0	1	0.06	0.10	0.07	0

Table 5. Nutrition matrix (a_{ik})

The data in the Table 5 are applied in optimization of feed blend production and are the same for each sort of feed blend ($a_{ik1} = a_{ik2} = \ldots = a_{ik5}$).

4.2. Multiple criteria linear programming model for optimization of feed blend production

Considering the data given in the above tables and the requirements of the decision maker a multicriteria linear programming model is formed in which the function of total production costs is minimized as well as the function of the total share of water in the feed blends, while the functions of the total share of nutrients and the total share of protein in the feed blends are maximized. Thus the following MLP model is formed:

Function of total costs

$$(\min) f_1 = \sum_{i=1}^{13} \sum_{q=1}^{5} c_{i1q} x_{iq} \tag{19}$$

Function of total nutrients

$$(\max) f_2 = \sum_{i=1}^{13} \sum_{q=1}^{5} c_{i2q} x_{iq} \tag{20}$$

Function of total share of water

$$(\min) f_3 = \sum_{i=1}^{13}\sum_{q=1}^{5} c_{i3q} x_{iq} \tag{21}$$

Function of total raw protein

$$(\max) f_4 = \sum_{i=1}^{13}\sum_{q=1}^{5} c_{i4q} x_{iq} \tag{22}$$

s.t.

Technological constraints

$$\sum_{i=1}^{13} a_{ikq} x_{iq} \geq a_q b_{kq} \quad (k = 1, ..., 20;\ q = 1, ... ,5) \tag{23}$$

Ingredients availability constraint

$$\sum_{q=1}^{5} x_{iq} \leq d_i \ (i = 1, ..., 13) \tag{24}$$

Necessary quantities of feed blends

$$\sum_{i=1}^{13} x_{iq} = a_q \ (q = 1, ..., 5) \tag{25}$$

Non-negativity conditions

$$x_{iq} \geq 0 \ (i = 1, ..., 13;\ q = 1, ... ,5) \tag{26}$$

It means that model has 65 variables and 118 constraints and this model is first solved by optimization of each of the four criteria functions on the given set of constraints. In this way we obtain marginal solutions and the payoff table of marginal solutions. The values of variables for the obtained marginal solutions are shown in the following tables:

	Minimization of total cost				
	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}
Barley	0	0	9878.172	8077.07	12044.76
Maize	0	0	0	0	0
Lucerne	0	0	0	2912.37	3235.943
Powdered milk	1750.432	0	0	0	0
Fish meal	797.34	660.2048	158.3151	679.5731	311.4196
Soya	0	176.9375	0	0	0
Soya hulls	0	0	0	0	0
Dried whey	0	0	0	0	0
Rape pellets	0	2059.813	7134.743	7972.742	2832.702
Wheat	5767.513	0	10050.18	0	0
Rye	0	12850.39	0	25574.43	1575.178
Millet	0	0	0	0	0
Sunflower pellets	1684.617	4252.652	2778.591	4783.815	0

Table 6. Marginal solution - min f_1

	Maximization of nutrients in the blend				
	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}
Barley	0	0	0	0	0
Maize	659.6186	6701.568	12720.87	28822.86	1095.092
Lucerne	0	0	0	0	0
Powdered milk	4345	4611.989	1043.365	0	0
Fish meal	903.6412	2263.838	3455.857	5475.34	639.2281
Soya	0	0	2000	0	0
Soya hulls	0	0	2000	0	0
Dried whey	0	0	0	0	0
Rape pellets	0	0	0	0	0
Wheat	3999.23	6422.605	8669.662	13989.52	18265.68
Rye	0	0	0	0	0
Millet	0	0	0	0	0
Sunflower pellets	92.8638	0	110.25	1712.282	0

Table 7. Marginal solution - max f_2

	Minimization of water per ingredient unit				
	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}
Barley	0	7305.556	0	8849.842	0
Maize	0	0	0	0	0
Lucerne	0	1320.187	0	5478.875	0
Powdered milk	1306.265	5171.465	0	2066.237	1456.033
Fish meal	1315.917	1574.477	2610.556	2726.9	1817.3
Soya	0	0	0	2000	0
Soya hulls	0	0	0	0	0
Dried whey	0	765.3964	0	4234.604	0
Rape pellets	0	0	17567.21	7432.788	0
Wheat	0	0	0	0	0
Rye	5610.544	0	14822.23	15717.82	3849.409
Millet	0	0	0	0	0
Sunflower pellets	1767.274	3862.528	0	1492.94	12877.26

Table 8. Marginal solution - min f_3

	Maximization of raw protein per ingredient unit				
	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}
Barley	0	0	0	0	0
Maize	0	0	0	0	0
Lucerne	0	0	0	0	0
Powdered milk	3778.667	5537.011	0	0	684.323
Fish meal	850.6279	1607.562	2775.259	4924.480	1899.625
Soya	1903.325	96.6747	0	0	0
Soya hulls	0	0	0	0	2000.000
Dried whey	0	0	0	0	0
Rape pellets	0	1951.010	9418.059	5954.299	2676.633
Wheat	1544.320	4834.492	14858.050	39121.220	3584.363
Rye	0	0	0	0	0
Millet	0	0	0	0	0
Sunflower pellets	1923.060	5973.251	2948.632	0	9155.057

Table 9. Marginal solution – max f_4

	f_1 - min	f_2 - max	f_3 - min	f_4 - max
x_1^*	257858 (100.00% of f_1^*)	9131010 (88.93% of f_2^*)	1312668 (108.93% of f_3^*)	2528896 (69% of f_4^*)
x_2^*	370412.9 (143.65% of f_1^*)	10267552 (100.00% of f_2^*)	1470209 (122.01% of f_3^*)	2724500 (75% of f_4^*)
x_3^*	384296.8 (149.03% of f_1^*)	9155563 (89.17% of f_2^*)	1205029 (100.00% of f_3^*)	2718333 (74% of f_4^*)
x_4^*	368014.4 (143% of f_1^*)	9765424 (95% of f_2^*)	1301827 (108% of f_3^*)	3648734 (100% of f_4^*)

Table 10. Payoff table

The conflict of criteria functions is evident from the payoff table: optimization of the model by one of the given criteria functions results in an inadequate value of the other two criteria functions. This points to the fact that the model is a multicriteria model and that it is necessary to find a compromise non-dominated solution according to the decision maker's preferences.

To solve this model we can use numerous multicriteria linear programming methods. To find a compromise solution we will here use the weight coefficient method. One may wonder why the weight coefficient method is appropriate for solution of the feed blend problem. The reason for that is that the application of this method eventually leads to better quality of company operation even though the costs for feed blend recipe are not minimal, as for example maximisation of nutrients contributes to the better nutritive value of the feed blend resulting in better weight gain or in lower feed blend quantity necessary to achieve the same weight gain. Also, lower share of water results in better performance through the feed quality (it is less perishable) and the lower quantity of feed necessary to obtain the required weight gain. Besides, the weight coefficient method is easy to use and the decision makers have a high degree of trust in it and the compromise solution obtained by its use. In their opinion this method maximizes the total utility function (Perić, 2008).

4.3. Solving the model by weight coefficient method

Applying this method on the data given in the Tables 3, 4 and 5 the following model is solved:

$$\begin{aligned}(\min) f &= w_1^0 f_1(\underline{x}) / f_1^* - w_2^0 f_2(\underline{x}) / f_2^* + w_3^0 f_3(\underline{x}) / f_3^* - w_4^0 f_4(\underline{x}) / f_4^* = \\ &= \frac{w_1^0}{f_1^*}\sum_{1=1}^{13}\sum_{q=1}^{5} c_{i1q}x_{iq} - \frac{w_2^0}{f_2^*}\sum_{1=1}^{13}\sum_{q=1}^{5} c_{i2q}x_{iq} + \frac{w_3^0}{f_3^*}\sum_{1=1}^{13}\sum_{q=1}^{5} c_{i3q}x_{iq} - \frac{w_4^0}{f_4^*}\sum_{1=1}^{13}\sum_{q=1}^{5} c_{i4q}x_{iq}\end{aligned}$$

s.t. (27)

$$\underline{x} \in X,$$

where $\underline{x} \in X$ represents a set of model constraints (from 23 to 26), $w_1^0, w_2^0, w_3^0, w_4^0$ represent the weight coefficients of the model, while $c_{i1q}, c_{i2q}, c_{i3q}$ and c_{i4q} represent the corresponding coefficients of criteria functions.
To solve the model it is first necessary to determine the weight coefficients. We will determine them by the entropy method.

From the payoff table we will firstly calculate $\sum_{i=1}^{m} x_{ij}$, and then we will calculate the normalized values of the decision matrix $p_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}}, \quad \forall\, i, j$. The calculation results are shown in the following table:

	f_1 - min	f_2 - max	f_3 - min	f_4 - max
x_1^*	257858 (100.00% of f_1^*)	9131010 (88.93% of f_2^*)	1312668 (108.93% of f_3^*)	2528896 (69% of f_4^*)
x_2^*	370412.9 (143.65% of f_1^*)	10267552 (100.00% of f_2^*)	1470209 (122.01% of f_3^*)	2724500 (75% of f_4^*)
x_3^*	384296.8 (149.03% of f_1^*)	9155563 (89.17% of f_2^*)	1205029 (100.00% of f_3^*)	2718333 (74% of f_4^*)
x_4^*	368014.4 (143% of f_1^*)	9765424 (95% of f_2^*)	1301827 (108% of f_3^*)	3648734 (100% of f_4^*)
$\sum_{i=1}^{4} x_{ij}$	1380582.10	38319549	5289733	11620463
p_{1j}	0.186775	0.238286	0.248154	0.217624
p_{2j}	0.268302	0.267946	0.277936	0.234457
p_{3j}	0.278359	0.238927	0.227805	0.233926
p_{4j}	0.266565	0.254842	0.246104	0.313992

Table 11. Original and normalized values of the payoff table

We also have to calculate $E_j = -l \sum_{i=1}^{m} p_{ij} \ln\, p_{ij}$, where $l = \frac{1}{\ln m}$.

$\ln m = \ln 4 = 1.386294$, so that $l = \frac{1}{\ln m} = \frac{1}{1.386294} = 0.721348$.

$E_1 = -0.72134752 \cdot (0.186775 \cdot \ln 0.186775 + 0.268302 \cdot \ln 0.268302 + 0.278359 \cdot \ln 0.278359 +$
$+ 0.266565 \cdot \ln 0.266565) = 0.991696$,
$E_2 = 0.999132$, $E_3 = 0.998154$, $E_4 = 0.992221$.
Further calculations are shown in the following table:

	f_1	f_2	f_3	f_4	
E_j	0.991696	0.999132	0.998154	0.992221	
$d_j = 1 - E_j$	0.008304	0.000868	0.001846	0.007779	$\sum_{j=1}^{4} d_j = 0.018797$
$w_j = \frac{d_j}{\sum_{j=1}^{4} d_j}$	0.441773	0.046178	0.098207	0.413843	
λ_j	0.40	0.20	0.10	0.30	
$\lambda_j w_j$	0.176709	0.009236	0.009821	0.124153	$\sum_{j=1}^{4} \lambda_j w_j = 0.319919$
$w_j^0 = \frac{\lambda_j w_j}{\sum_{j=1}^{4} \lambda_j w_j}$	0.552355	0.028870	0.030698	0.388076	

Table 12. Calculated weight coefficients

Solving the model by this method we obtain the following compromise solution:

	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}
Barley	0	0	0	0	10000.000
Maize	0	8321.443	0	12811.220	3867.340
Lucerne	0	1792.594	798.2579	6906.411	502.738
Powdered milk	2177.182	1988.912	0	833.907	0
Fish meal	716.710	1853.135	89.373	1370.655	908.057
Soya	0	1000.0000	0	0	0
Soya hulls	0	0	0	1000.000	0
Dried whey	0	0	0	0	0
Rape pellets	0	0	7884.614	2115.386	0
Wheat	0	0	13678.830	21321.170	0
Rye	5205.502	1551.856	5146.733	0	3095.909
Millet	0	1499.819	0	0	1562.145
Sunflower pellets	1900.607	1992.241	2402.083	3641.259	63.811

Table 13. Compromise solution

	f_1 - min	f_2 - max	f_3 - min	f_4 - max
x^{com1}	295108.300 (114.40% of f_1^*)	9390745 (91.46% of f_2^*)	1358061 (112.70% of f_3^*)	2528896 (69.31% of f_4^*)

Table 14. Compromise solution: function values

The decision maker may accept this solution as the preferred one. On the other hand, if he does not accept this compromise non-dominated solution, a wider set of non-dominated compromise solutions has to be formed. This can be carried out solving the model (27) by varying weight coefficients. In our case the decision maker was not satisfied with the value of the function f_4^*. Varying the subjectively selected criteria function weights λ_1 and λ_4 by 0.05 (λ_1 was reduced by 0.05 and λ_4 was increased by 0.05) leaving the subjectively selected λ_2 and λ_3 unaltered we obtained two compromise solutions which in criteria function values are insignificantly different from the solution x^{com1}. The values of the subjectively selected weights, weight coefficients, and criteria functions for the insignificantly altered compromise solutions are shown in the following table:

	f_1	f_2	f_3	f_4
λ_j	0.35	0.20	0.10	0.35
w_j^{01}	0.485431	0.028996	0.030833	0.454740
f_j^{com01}	295108.30	9390745	1358061	2528893
λ_j	0.30	0.20	0.10	0.40
w_j^{01}	0.417916	0.029124	0.030969	0.521991
f_j^{com02}	295108.60	9390752	1358062	2528897

Table 15. Values of the subjectively chosen weights, weight coefficients and criteria functions

For $\lambda_1 = 0.25$, $\lambda_2 = 0.20$, $\lambda_3 = 0.10$ i $\lambda_4 = 0.45$ we obtained the following compromise solution:

	x_{i1}	x_{i2}	x_{i3}	x_{i4}	x_{i5}
Barley	0	0	1256.199	8743.801	0
Maize	0	0	0	25000.000	0
Lucerne	0	1568.448	2538.671	0	5892.881
Powdered milk	2177.182	822.818	0	0	0
Fish meal	716.710	1306.830	2656.165	3655.435	1664.861
Soya	0	113.109	886.891	0	0
Soya hulls	0	0	0	1000.000	0
Dried whey	0	0	0	0	0
Rape pellets	0	0	6638.633	3361.367	0
Wheat	0	5152.585	16023.440	3485.665	10338.310
Rye	5205.502	7690.550	0	0	2103.949
Millet	0	0	0	0	0
Sunflower pellets	1900.607	3345.661	0	4753.732	0

Table 16. Compromise solution – values of variables

	f_1 - min	f_2 - max	f_3 - min	f_4 - max
λ_j	0.25	0.20	0.10	0.45
w_j^{04}	0.349803	0.029253	0.031106	0.589838
x^{com4}	317950 (123.30% of f_1^*)	9369000 (91.25% of f_2^*)	1356200 (112.55% of f_3^*)	2738000 (75.04% of f_4^*)

Table 17. Compromise solution: function values

The testing of the first compromise solution (x^{com1}) shows low sensitivity to changes in the decision maker's subjectively chosen weights. Namely, the increase in value of the subjectively chosen weight λ_1 from 0.40 to 0.65 with reduction in value of the subjectively chosen weight λ_4 from 0.30 to 0.05 keeping the values of the subjectively chosen weights $\lambda_2 = 0.20$ and $\lambda_3 = 0.10$ does not lead to any significant alteration in criteria function values. Consequently, with the constant values $\lambda_2 = 0.20$ and $\lambda_3 = 0.10$ criteria function values will not be significantly changed in the following intervals: $\lambda_1 \in \left[0.30, 0.65\right]$ i $\lambda_4 \in \left[0.40, 0.05\right]$.

Here we will briefly analyse the compromise solution x^{com4} accepted by the decision maker as the preferred solution. The analysis of the obtained compromise solution shows that:

(1) the compromise solution contains eleven ingredients except dried whey and millet,

(2) to produce blends PS-1 and PS-5 it is necessary to use four different ingredients, for blends PS-2 and PS-4 seven ingredients are needed, while for blends PS-3 six ingredients are needed.

(3) to produce the given feed blend quantity the available ingredients x_1, ,x_2, x_3, x_5, x_6, x_7, x_9, and x_{13} are used up completely, while the ingredient x_4 is used by 60%, and the ingredients x_8 and x_{12} are not used,

(4) the number of used ingredients is increased by introduction of additional constraints that limit the share of a particular ingredient in the blend.

Consequently, the analysis of marginal solutions and the obtained compromise solution shows the complexity of the feed blend optimization problem in the case when there are constraints in available ingredients. From the obtained compromise solution an optimal recipe can be calculated for each of the five blends. This is done dividing the obtained values x_{iq} by the given blend quantity to be produced a_q.

We have here presented the possibility of application of only one multicriteria programming method. The research is to be continued to test the applicability of other multicriteria programming methods according to the adopted criteria. It is also necessary to investigate the possibility of incorporating the proposed model and methods into decision making support system designed for optimization of industrial production of feed blends.

5. Conclusion

Based on the above we can make some general conclusions:

(a) Production of feed blends is essentially a complex multicriteria problem.

(b) To solve the problem of feed blend optimisation an appropriate multiple criteria linear programming model is to be formed and one of the multiple criteria linear programming methods is to be applied to solve the obtained model.

(c) Application of the linear programming model and methods requires investigation of criteria for optimization of feed blends, analysis of ingredients used in industrial production of feed blends, as well as continuous monitoring of its supply market and sale market. The success of optimization will depend on the quality of this information.
(d) Solving the concrete problem of industrial production of feed blends by the weight coefficient method reveals the high degree of applicability of this method. However, since different methods start from different assumptions and in most cases give different compromise solutions, it is necessary to investigate the applicability of different multicriteria programming methods in solving the model of feed blend production.
(e) Further research will have to investigate the possibility of designing a decision making support system in industrial production of feed blends that will include the proposed model and the most suitable method for its solving.

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