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Fault Diagnosis on Electric Power Systems based on Petri Net Approach¹

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1. Introduction

Fault diagnosis is a process that identifies and locates faults occurrence in systems using their inputs, outputs, and their structures. In the context of electric power systems fault diagnosis is realized using the voltages and currents measurements at each node and the physical connections. In this process, there are two conventional important problems that must be solved: 1) how to determine whether a system is diagnosable and 2) how to design a diagnoser. The problem of fault diagnosis has been addressed through various approaches and application methods, for example based on artificial intelligence techniques, and based on Discrete Event Systems. In the last approach, Finite Automata and Petri nets have been mainly used as modelling formalism.

Although the Finite Automata are suitable for Discrete Event Systems, its application is limited to small systems, since the models should explicitly taken in account all the possible states of the system, resulting in very large models which are very difficult to work with them. In order to cope with the state explosion problem, the structural models, represented as vector Discrete Event Systems or Petri nets allow a compact representation of a Discrete Event Systems, avoiding a large set of states as Finite Automata does. Also, with this formalism, the discrete event systems analysis can be realized using linear algebra methods. Using the advantage of these features, fault diagnosis has been addressed using Petri nets.

Several methods have been proposed recently, for example in (Sheng-Luen et al., 2003) and (Genc & Lafortune, 2003) the Petri nets approach is used, but the diagnosability test has been based on the reachability graph; thus the method is limited to small size systems (Sampath et al., 1995) and (Sampath et al., 1996). In (Lefebvre & Delherm, 2007) a diagnoser based on the Petri nets paths and causality relationships is proposed to determine the presence of faults in a system. In (Hadjicostis & Verghese, 1999a), (Hadjicostis & Verghese, 1999b) and (Hadjicostis & Verghese, 2000) Petri nets model is used in order to introduce

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redundancy into the system that allow to detect and isolate faults. Unfortunately, all those solutions have the state explosion problem.

In order to cope with the state explosion problem, in (Ramírez-Treviño et al., 2004), (Ramírez-Treviño et al., 2007) and (Ruiz-Beltrán et al., 2007) techniques based on structural characterizations for determining the diagnosability property are proposed; in (Ruiz-Beltrán et al., 2007) is extended the characterizations and the diagnoser structures of (Ramírez-Treviño et al., 2004) and (Ramírez-Treviño et al., 2007), but that result is limited for binary Interpreted Petri nets. In these approaches, the diagnosability property is tested as a linear programming problem with polynomial complexity. In (Sheng-Luen et al., 2003), (Ulerich & Prowers, 1998), and (Yang et al., 2004) some applications of fault diagnosis using Petri nets are proposed, but they do not consider the diagnosability analysis of the system, which do not guarantee that a fault can be detected in a finite number of events when this fault occur. However, this condition is fulfilled in (Ramírez-Treviño et al., 2004), (Ramírez-Treviño et al., 2007), and (Ruiz-Beltrán et al., 2007).

Recently, the model-based approach for fault diagnosis in Discrete Event Systems is widely been worked by the research community of Discrete Event Systems. Several works on the matter use Petri nets for performing the fault diagnosis in electric power systems (Hadjicostis & Verghese, 1999a), (Hadjicostis & Verghese, 2000), (Ren et al., 2004), and (Ren & Zengqiang, 2006). In (Hadjicostis & Verghese, 1999a) and (Hadjicostis & Verghese, 2000) the authors propose coding theory techniques for detecting and locating failures in Discrete Event Systems in the field of electric power systems, they build a monitoring Petri net, which operates concurrently with the power network. However, for some failures, their techniques can not give a right explanation, and they do not determine how to construct the monitoring matrix. In (Ren et al., 2004), (Ren & Zengqiang, 2006) and (Proth et al., 1993) the same method is further studied. All possible failures are analyzed and remapped to the embedded Petri net models and a method of how to construct the monitoring matrix to encode the former Petri net model is proposed. However, the previous works do not determine the diagnosability of the electrical systems, i.e., it can not be determined off-line if a fault can be detected and located in a finite number of steps.

Based on the voltages and currents measurements and its digital processing with a relay to maintain the operation of the power electric system; in this chapter we will address: 1) how the operational behaviour of power electrical system with Interpreted Petri Net is represented, where the Interpreted Petri Net captures the normal and fault condition of the power electrical system. 2) How it is determined whether the model of the power electric system with Interpreted Petri Net is diagnosable and 3) how it is designed a diagnoser that has the same operative characteristics (security and velocity) that a relay in a real case. Thus, the results in (Ramírez-Treviño et al., 2004), (Ramírez-Treviño et al., 2007), (Ruiz-Beltrán et al., 2007) and (Santoyo-Sanchez et al., 2008) will be extended.

This chapter will be as follows. Section 2 will introduce fault features in electric power systems, how the electric power systems are modelled, and how the fault diagnosis is executed into the context of electric power systems. In the next section will be introduced a review Petri net notation and concepts used in the chapter. Section 4 will introduce the diagnostic from the Petri net point of view; in this case it is proposed a necessary condition to determine if a power electrical system is diagnosable. Moreover, it will be presented a method for designing a diagnoser, and how the diagnostic can be done. Next, in section 5

the results will be illustrated with a case study. Finally, in section 6 conclusions and future research will be given.

2. Fault modeling conditions in electric power systems

The analysis of the power electrical systems involve three stages namely, pre-fault condition (steady-state operation), fault condition (transient condition) and post-fault condition (how the network arrives to a new steady-state operation condition). If the relay is the equipment to protect the entire system, so it is necessary to know the exact relay behaviour under each stage. The tests of the relay performance are doing by off-line simulations; this reason renders necessary the modelling of the network components, the digital processing of the electrical signals and the procedure to take a logical decision: order a trip in a fault condition, do nothing in a safe condition. This decision is processed sample by sample. The relay behaviour, the used models to simulate the power electrical system network and the signal processing are described in order to clarify why and how Petri Nets are used to increment or to give redundancy to the protection schemes.

2.1 Relay behavior

The distance elements are the most widely used to protect transmission lines. Usually, they shared a communication channel to implement pilot schemes which increases the overall performance of the line protection. Each distance element has a boundary of the zone of protection in the locations of the currents transformers. The other boundary is extended in the forward direction and is determined by measurements of the system quantities that may vary with generation and configuration schemes. One of the most important design considerations of relaying is security. It is a measure of the relay's ability to avoid operation for all other conditions for which tripping is not desired. It is hard to achieve, and almost infinite variety of tests would be needed to simulate all possible conditions to which a relay may be exposed. Another important design consideration is protection velocity. The fault clearance must be as fast as possible for conditions within the transmission line. For distance elements is not a problem in their primary zone, but it is used to be a delay within an extended backup zone. This is shown in figure 1.

Fig. 1. Relay protection zones

Figure 2a shows the functional diagram of the distance relay. The process begins when the voltages and currents flow to the relay through the potential transformers and currents transformer installed in the power electric system network. These signals are passed through the relay's internal transducers, and then the signals are filtered before the analogical-digital conversion. The analogical filtering has the function of limit the bandwidth, so it is possible to use a low-frequency rate sampling without introduce a big error due to the aliasing phenomenon. The next step is the use of a digital filter to eliminate the superior harmonics and the no-periodic exponential component still remaining; the useful filters are the finite impulse response ones like the Fourier or cosine types. Phasors of fundamental frequency (60 Hz) are computed by the discrete convolution between the digitalized, voltages (V) and currents (I), measured samples and the coefficients of the digital filter. It is demonstrated that independently of the frequency used rate in the analogical-digital conversion, the most convenient window length of the digital filter is equal to one period of fundamental frequency which means a delay of the trip, to fulfill the window with the fault signal, of one cycle the normal rate of relays is of 32 samples by cycle. The adjust parameters, to model short-circuits, are based in phasors which are used to represent the steady-state operation. Under ideal conditions, the estimate impedance is equal to the line section from the relay to the fault location. Faults provoke transient operation state in the network, so the impedance estimation is affected by the transient state components (fault resistance), this affects the relay response related with the fault detection in the limit zone operation (zone discrimination); for this reason the first zone of the relay is adjust to 80-90% of the protected line. Figure 2b shows the stages of the distance relay procedure and the normal consuming time each stage. Times are normalized with the fundamental frequency. In this figure, it could be notice that the execution of the digital filtering to compute the phasors of fundamental frequency is the most demanding time in the process of the trip signal. For typical filters the delay is one cycle of 60 Hz, the corresponding time is around 16.66 milliseconds. The delay of the digital filter is then a limitation that could not be avoided.

Fig. 2.a) Relay signal processing, b) Relay consuming times

The algorithms of the relay use both, phasors and the modal transformation described in section 2.2 of this document, here called symmetrical components; with this information the relay elaborate its response, the instantaneous one and the off-line one. The first involves the fault detection and the fault zone discrimination. The second is referred to the fault classification and localization.

- The relay detect a fault in its zone protection by comparing an adjust parameter with a function related with the voltages and currents phasors.
- The capacity to discriminate faults in front or behind near to the fault zone, is difficult because the estimate voltage phasor is almost zero; so, the effective and useful method is the polarization of the relay with the positive sequence network of the symmetrical components.
- The fault classification is performing by the symmetrical components as it is shown in table 1 described in section 2.2 of this document.
- The fault localization is computed by comparing the voltage-current relation with the total impedance of the line, so it is expressed in % of the line length.

National Electrical System is characterized by the large distance between the generation plants and the consuming points (charge). For that reason, the National Electrical System is divided into eight strategic located regional centers for operating and coordinating the production and distribution of the electric energy. Figure 3 shows the limits of each control area, also it is showed in this figure the electrical connectivity of the western control area which will be taken like an example, in this work, to implement the develop methodology. The fundamental criteria of reliability and operative security must be satisfied, so it is essential that must exists a good coordination among the operative entities in all operative stages. In this context these systems of energy generation have diverse exploration braches of study, among them, the maintenance area that as important factor as for costs, where innovating techniques of fault diagnosis for a later plan of reliable maintenance are required.

Fig. 3. Hierarchical level of control areas from Mexico and western area

2.2 Modeling the electrical system

The electrical systems are composed by generators that are interconnected through transmission lines to loads, sensors, capacitors, switches, substations, and transformers with the objective to produce and to distribute electrical energy. In order to simulate and analyze the electrical system behavior, the minimum models of elements of the system are: A) lines, B) sources and C) charges; which are brief described below. Figure 4 shows a piece of three phase line connected with a three phase source and a single charge each phase.

Fig. 4. Three phase line configuration

A) LINES

A rigorous analysis of the transmission line with losses is obtained by the Maxwell equations after some simplifications concerning to the electromagnetic field distribution. These simplifications conducted to the quasi-transversal electromagnetic fields. So, the line electromagnetic response of a transmission line is modeled by the Transmission Line Telegrapher's Equations as:

$$-\frac{\partial \mathbf{V}(x,t)}{\partial x} = \mathbf{R}(x)\mathbf{I}(x,t) + \mathbf{L}(x)\frac{\partial \mathbf{I}(x,t)}{\partial t} \quad (1)$$

$$-\frac{\partial \mathbf{I}(x,t)}{\partial x} = \mathbf{G}(x)\mathbf{V}(x,t) + \mathbf{C}(x)\frac{\partial \mathbf{V}(x,t)}{\partial t} \quad (2)$$

where $\mathbf{L}(x)$, $\mathbf{R}(x)$, $\mathbf{C}(x)$ and $\mathbf{G}(x)$ are the line per unit-length inductance, resistance, capacitance and conductance matrices, respectively.

The solution of these equations in frequency domain proceeds as follows; one has firstly the solution of the voltage and current like function of time and space as:

$$\mathbf{V}(x,t) = \mathbf{V}'(x,t)e^{j(\omega t + \phi)} \quad \mathbf{I}(x,t) = \mathbf{I}'(x,t)e^{j(\omega t + \phi)} \quad (3)$$

If one has, $\mathbf{V}(x) = \mathbf{V}'(x)e^{j\phi}$ and $\mathbf{I}(x) = \mathbf{I}'(x)e^{j\phi}$ then;

$$\mathbf{V}(x,t) = \text{Re}(\mathbf{V}(x)e^{j\omega t}) \quad \mathbf{I}(x,t) = \text{Re}(\mathbf{I}(x)e^{j\omega t}) \quad (4)$$

Substituting equation (4) into equation (1) and (2) yields to:

$$\frac{\partial (\text{Re}(\mathbf{V}(x)e^{j\omega t}))}{\partial x} + \mathbf{L}(x)\frac{\partial (\text{Re}(\mathbf{I}(x)e^{j\omega t}))}{\partial t} + \mathbf{R}(x)(\text{Re}(\mathbf{I}(x)e^{j\omega t})) = 0 \quad (5)$$

$$\frac{\partial (\text{Re}(\mathbf{I}(x)e^{j\omega t}))}{\partial x} + \mathbf{C}(x)\frac{\partial (\text{Re}(\mathbf{V}(x)e^{j\omega t}))}{\partial t} + \mathbf{G}(x)(\text{Re}(\mathbf{V}(x)e^{j\omega t})) = 0 \quad (6)$$

Solving these equations, one has:

$$\operatorname{Re}\left(e^{j\omega t} \frac{\partial \mathbf{V}(x)}{\partial x}\right) + \operatorname{Re}\left(\mathbf{L}(x)\mathbf{I}(x) \frac{\partial(e^{j\omega t})}{\partial t}\right) + \operatorname{Re}(\mathbf{R}(x)\mathbf{I}(x)e^{j\omega t}) = 0 \quad (7)$$

$$\operatorname{Re}\left(e^{j\omega t} \frac{\partial \mathbf{I}(x)}{\partial x}\right) + \operatorname{Re}\left(\mathbf{C}(x)\mathbf{V}(x) \frac{\partial(e^{j\omega t})}{\partial t}\right) + \operatorname{Re}(\mathbf{G}(x)\mathbf{V}(x)e^{j\omega t}) = 0 \quad (8)$$

Re-arranging equation (7) and (8) one obtains:

$$\operatorname{Re}\left(\left(\frac{\partial \mathbf{V}(x)}{\partial x} + \mathbf{L}\mathbf{I}(x)j\omega + \mathbf{R}\mathbf{I}(x)\right)e^{j\omega t}\right) = 0 \quad (9)$$

$$\operatorname{Re}\left(\left(\frac{\partial \mathbf{I}(x)}{\partial x} + \mathbf{C}\mathbf{V}(x)j\omega + \mathbf{G}\mathbf{V}(x)\right)e^{j\omega t}\right) = 0 \quad (10)$$

So, the real part is:

$$-\frac{\partial \mathbf{V}(x)}{\partial x} = (j\omega \mathbf{L} + \mathbf{R})\mathbf{I}(x) \quad (11)$$

$$-\frac{\partial \mathbf{I}(x)}{\partial x} = (j\omega \mathbf{C} + \mathbf{G})\mathbf{V}(x) \quad (12)$$

If $\mathbf{Z} = (j\omega \mathbf{L} + \mathbf{R})$ and $\mathbf{Y} = (j\omega \mathbf{C} + \mathbf{G})$, finally one obtains:

$$-\frac{\partial \mathbf{V}(x)}{\partial x} = \mathbf{Z}\mathbf{I}(x) \quad (13)$$

$$-\frac{\partial \mathbf{I}(x)}{\partial x} = \mathbf{Y}\mathbf{V}(x) \quad (14)$$

where $\mathbf{Z}$ represents the impedance series matrix, $\mathbf{Y}$ represents the admittance in derivation, both in per unit length and calculated like a linear function of the frequency ($j\omega$).

Treating the line like a simple circuit means neglecting the traveling time by taking into account the line length in the phase angle of each node (phase representation), one obtains the following equation (from equation (13)):

$$\mathbf{V}_\phi = \mathbf{Z}\mathbf{I}_\phi \quad (15)$$

For a geometric configuration of a common three phase horizontal line, if the line is constructed with the same material and has equal radius; additionally if it is positioned at a same height respect to ground and by assuming some simplifications like: equal coupling between conductors and homogeneous earth resistivity, it will conduce to the special case which has the following symmetry:

$$\mathbf{Z} = \begin{bmatrix} Z_s & Z_m & Z_m \\ Z_m & Z_s & Z_m \\ Z_m & Z_m & Z_s \end{bmatrix} \quad (16)$$

This work makes special emphasis on the uses of the diagonalization methodology proposed in (Naredo et. al.,1987) to handle three phase systems like single ones. Using a modal transformation, for both voltage and current, one has:

$$\mathbf{V}_\phi = \mathbf{M}\mathbf{V}_m \quad \mathbf{I}_\phi = \mathbf{M}\mathbf{I}_m \quad (17)$$

where $\mathbf{V}_m$ and $\mathbf{I}_m$ are called modal voltages and currents respectively and $\mathbf{M}$ is the modal transformation matrix.

By substituting equation (17) into equation (15), one obtains:

$$\mathbf{M}\mathbf{V}_m = \mathbf{Z}\mathbf{M}\mathbf{I}_m \quad (18)$$

This process yields to:

$$\mathbf{V}_m = \mathbf{M}^{-1}\mathbf{Z}\mathbf{M}\mathbf{I}_m \quad (19)$$

It is demonstrated (Naredo 1992) (Strang 1998) (Greenspan D. & Casulli V., 1988) that the product $\mathbf{M}^{-1}\mathbf{Z}\mathbf{M} = \Lambda$ lead to a diagonal matrix when it is used the self modes (eigenvectors of $\mathbf{Z}$), which is the required condition to treat the coupled three phase system like an uncoupled one. By using symmetrical components to diagonalizable this equation, which means by using

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 \\ a & a^2 & 1 \\ a^2 & a & 1 \end{bmatrix} \text{ and } \mathbf{M}^{-1} = \frac{1}{3} \begin{bmatrix} 1 & a^2 & a \\ 1 & a & a^2 \\ 1 & 1 & 1 \end{bmatrix} \quad (20)$$

then equation (19) is as follows:

$$\begin{bmatrix} V_m^+ \\ V_m^- \\ V_m^0 \end{bmatrix} = \begin{bmatrix} Z^+ & 0 & 0 \\ 0 & Z^- & 0 \\ 0 & 0 & Z^0 \end{bmatrix} \begin{bmatrix} I_m^+ \\ I_m^- \\ I_m^0 \end{bmatrix} \quad (21)$$

where $Z^+ = Z_g - Z_m$, $Z^- = Z_g - Z_m$ and $Z^0 = Z_g + 2Z_m$. Like these, the line coupled model could be solved like an uncoupled one, as:

$$V_m^+ = Z^+ I_m^+, \quad V_m^- = Z^- I_m^-, \quad \text{and} \quad V_m^0 = Z^0 I_m^0 \quad (22)$$

B) SOURCES

The steady state solution yields to the following case, for example, if it is placed an ideal voltage source of 60 Hz, 115 kV line to line, in one end; modal voltages are (by definition $V_{ab} = \sqrt{3}V_m \angle 30^\circ$), so one has:

$$V_{LL} = \begin{cases} 115kV \angle 0^\circ \\ 115kV \angle 120^\circ \\ 115kV \angle 240^\circ \end{cases} \rightarrow V_{LN} = \begin{cases} 66.4 \angle -30^\circ \\ 66.4 \angle 90^\circ \\ 66.4 \angle 210^\circ \end{cases} \quad (23)$$

The relation $\mathbf{V}_m = [\mathbf{M}^{-1}\mathbf{V}_\phi^{\text{initial}}]$ gives the modal components, so by using Symmetrical Components one obtains:

$$\begin{bmatrix} V_m^+ \\ V_m^- \\ V_m^0 \end{bmatrix} = \begin{bmatrix} \text{Voltage} \angle -30^\circ \\ 0 \\ 0 \end{bmatrix} \quad (24)$$

So, using equation (24), the complete representation for a three phase network in steady state is solved with the positive sequence network as:

$$V_m^+ = Z_m^+ I_m^+ \quad (25)$$

C) CHARGES

The charges are represented like single impedances connected to the network, which means; by definition the charge is uncoupled so it does not need to be treated by applying the modal transformation.

FINAL MODEL FOR DIAGNOSE PROCEDURE

Using the line and the source modal transformations, the three phase coupled network is represented like three single uncoupled lines like it is shown in figure 5. By simple analysis one could note that in a steady state condition the three phase coupled system could be fully represented by the positive sequence network, so, by applying the inverse modal transformation to the solution of this network one obtains the solution of the three phase coupled one. In a fault condition the voltages and currents in the nearby node will have values not only for the positive sequence one, so this procedure could give enough information to diagnose the fault condition on the system and the kind of the fault. This is the normal procedure to diagnose and characterize a fault in a power electric system. So, based on the information of the symmetrical components the faults are detected and classified.

In figure 5, the dashed lines of the sources means that for negative and zero sequences the value is zero in steady state operation. In case of unsymmetrical charge; if the asymmetry is due a natural over charge, the voltage-current relationship (denoted as V and I respectively) in phase domain is healthy, so the relay does not process the asymmetry like a fault condition.

By example, in a three phase faults one does not has negative sequence current, nether zero sequence current. For single faults there are equal currents in the sequence networks, etc. Table 1 presents these and other features of the relay input signal when these are processed with symmetrical components.

Kind of Fault	Characteristics of V	Characteristics of I
Three phase	$V(p) \neq 0, V(n) = V(0) = 0$	$I(p) \neq 0, I(n) = I(0) = 0$
Two phase	$V(p) \neq V(n) \neq 0, V(0)=0$	$I(p) = I(n), I(0) = 0$
Two phase-ground	$V(p) \neq V(n) \neq V(0) \neq 0$	$I(p) \neq I(n) \neq I(0)$
Single phase	$V(p) \neq V(n) \neq V(0) \neq 0$	$I(p) = I(n) = I(0)$

Table 1. Features of the relay input signals in symmetrical components.

2.3 Fault electrical features

Under the point of view of the relay, the power electrical system have three stages called; pre-fault condition, fault condition (transient condition) and post-fault condition. In the next paragraphs each one will be briefly described.

Fig. 5. Model of the three phase line configuration like three single lines.

Pre-fault condition.- Each energy control center takes information from the power electrical network (sub-stations) in real time, so it is possible to know and to supervise the operation limits. The operator engineer takes actions to maintain the steady state condition; that means, frequency, voltage limits, maximum power flows in each line, etc. Under this steady-state condition, the entire power electric system works close to their nominal frequency (60 Hz). This value is achievable by automatic action of the automatic generation control, which creates a balance between generated and consumed active power (Fink *et. al.* 1985), (IEEE Std C37.1 – 1994).

The relation between the voltage phasor and the current phasor is linear, so if the voltage is constant (operative condition) and the current grow up from its minimum value to its maximum depending of the demanding energy, the voltage-current relation each phase is out of the fault characteristic zone, which is defined with the relation resistance-inductance of the line. Figure 6 shows the characteristic zones of relay and the trajectory of a typical fault. This trajectory is out of the fault zones in the steady state operation, and it is maintain out of the fault zones for this operative condition.

Fault condition.- This is a transient condition phenomena because the protection devices acts as fast as possible to disconnect the faulted element. This is the main purpose of a relay and it is also part of the topic of the present work. This stage involves a number of actions beginning with topological changes, automatically produced by action of local protections. The fault clearing requires the use of substation measurements, relay coordination, and communication systems. This stage ends when protections stop disconnecting those elements that were affected by the fault or situated inside the protection zone. Nowadays during a transient state is common the use of load shedding, blocking schemes, and dedicated controls. Most of these schemes can take action in milliseconds, so these schemes become the first automatic corrective action (Madani *et. al.* 2004), (Guzman *et. al.* 2006). After that, like an off line procedure means after the logical decision of the relay (tripping decision), the relay classifies, locate, typifies and disconnect the fault. In a fault condition, the voltage-current relation is not linear because the inclusion of the fault resistance in the path of the energy, so it is beginning a trajectory to the fault zone. The inclusion into the fault zone is not instantaneously because the computation of the phasor is due with a cycle and all the samples but one are healthy. Sample by sample the data window to compute the phasor is fully with fault data, this defines a fault trajectory (L-R-t trajectory). Figure 6 shows the behavior of this trajectory for a typical fault.

Fig.6. Characteristic zones of a relay

A fault classify algorithm identifies the kind of fault being based in the information of the modal transformation. By example, in a three phase faults one does not has negative sequence current, nether zero sequence current. For single faults there are equal currents in the sequence networks, etc. Note that table 1 presents these and other features of the relay input signal.

Physically, the line could present very complex faults, although protection is develop only to four groups of faults: a) three phase faults, b) two phase faults, c) two phase faults to ground and d) single faults. Figure 7 represents the faults statistical of one year in the Mexican Western Transmission Network. The RELIEVE was consulted to extract the events of the period with a confirmed fault. This system confirms that 69% were single faults, 19% two phase faults and 3% three phase faults.

Fig. 7. Percent of faults in the MWTN in 2006

Post-fault condition.- Once the faulted elements are disconnected, the entire network arrives to a different steady state operation condition. The operator engineer takes corrective actions to reach the normal operation condition. Normally, he leads the network to the new steady state without losing more electrical elements, which means, he maintains the network as healthy as possible. This state can remain for several seconds or minutes until to reach another steady state normal condition, in which, there exist a balance between charge and generation.

2.4 Diagnosis Example

For illustrate the diagnoser behavior in power electrical systems consider the power electrical network of the figure 8. In a steady state operation, I^{abc} measurements in each relay don't overpass the adjust pick up. In this specific point of operation the entire network is healthy, that mean, the electrical system is working normally. Figure 8 shows a simple network in a steady state condition, this figure shows how the power flows from generator 1 and 2 to the charge.

Fig. 8. Steady state operation

While the information of energy control center from the power electrical network (sub-stations) is into the operation limits, the operation of the electrical network is mantanied. When a fault occurs, it is established a trajectory from the generators to the fault point, this is shown in figure 9. In this case, the fault is in the closer line to the generator 2. Analyzing the fault from the point of view of the generator 1, there are four relays that detect the fault in from of them. All relays process the signal as it is shown in figure 2 (figure with the operative times), then each determine is fault zone (figure 6 has the relay circles). Like a resulting of this process, each relay determines its tripping time. The closer relay to the fault detects the fault at the first zone so acts instantaneously, the next in the second one so acts with a delay and the next in the third one so act with an additional delay, the last one (and so on in case of more relay) see the fault out of the protection zone for which they were programmed. There are some relays that detects the fault on behind, so these relays don't act. From generator 2, there is only one relay which determines the fault location at the first zone, so this relay acts instantaneously.

Fig. 9. Fault in the electrical system

Each relay into the protection zone 1, 2, 3 and 4 makes the digital process as illustrated in the figure 9 So, each relay into the protection zone 1 see the fault instantaneously, while that the others relays has a delay. In the case of the protection zone 2 the additional delay is one, into the protection zone 3 the additional delay is two, and in the protection zone 4 is considered as out of operation zone. Then all relays from the figure 1 see the fault, and the power electrical network is in pre-fault state.

The next step is analyzing the voltages and currents signals of the relay into the protection zone 1. These signals are passed through the relay internal transducers, and then the signals are filtered before the analogical-digital conversion. Later a digital filter is used to eliminate the superior harmonics and the no-periodic exponential component still remaining; the useful filters are the finite impulse response ones like the Fourier or cosine types. Remember that to fulfill the window with the fault signal, of one cycle the normal rate of relays is of 32 samples by cycle.

If the fault signal is complete, i.e. it is detected 32 samples in a cycle, and then power electrical network change of the pre-fault to fault state. In other case, when the fault signal is not complete, i.e. it is not detected 32 samples in a cycle, power electrical network change from the pre-fault to normal state.

For this example is considered that the fault signal is complete, then as next step the fault classify algorithm based in the information of the modal transformation identifies the kind of fault. Finally, the electrical components conected to the relay in protection zone 1 are desconnected.

Based on the voltages and currents measurements and its digital processing with a relay to maintain the operation of the power electric system; in the next section we will propos represent with an Interpreted Petri Net the operational behaviour each component for he network in steady state (normal condition), this Interpreted Petri Net will represent the diagnoser. Under the point of view of the relay, the power electrical system have three stages called; pre-fault condition, fault condition (transient condition) and post-fault condition, the Interpreted Petri Net model for the power electrical system includes some

transitions that captures these three stages. Also other real condition of the electrical systems is that a fault is detected with a delay of one cycle due to digital filter (32 samples), thus the pre – faulty transition is fired when a faulty condition appear. On the other hand the reset transition fires when is detected that the system operates in normal conditions and it is necessary reset the pre – faulty condition, so the system change from pre- fault state to normal state.

3. Petri nets and Interpreted Petri nets

This section presents a review the main concepts of the Petri net (PN) and Interpreted Petri Net (IPN) formalism used in this chapter. An interested reader can consult (Ramírez-Treviño et al., 2004), (Desel et al., 2005), (Santoyo et al., 2001) and (Ramírez-Treviño et al., 2003) for more details.

3.1 Petri Nets

Definition 1: A PN system is a pair (N, M_0) where $N = (P, T, I, O)$ is a bipartite digraph which specifies the net structure and $M_0 : P \rightarrow Z^+$ is the initial marking. Each element of N is defined as follows $P = \{p_1, p_2, \dots, p_n\}$ is a finite set of places; $T = \{t_1, t_2, \dots, t_m\}$ is a finite set of transitions; $I : P \times T \rightarrow Z^+$ and $O : P \times T \rightarrow Z^+$ are functions representing the weighted arcs going from places to transitions and from transitions to places, respectively. The initial marking of PN M_0 is a function that assigns to each place of N a non-negative number of tokens, depicted as black dots inside the places.

A PN structure N can be represented by its incidence matrix $C = [c_{i,j}]_{n \times m}$, where $c_{i,j} = O(p_i, t_j) - I(p_i, t_j)$. The sets $\bullet t_j = \{p_i \mid I(p_i, t_j) \neq 0\}$ and $t_j \bullet = \{p_i \mid O(p_i, t_j) \neq 0\}$ are the set of input and output places of a transition t_j respectively, which are denominated predecessors and successors of t_j respectively. Analogously, the sets of input and output transitions of a place p_i are $\bullet p_i = \{t_j \mid I(p_i, t_j) \neq 0\}$ and $p_i \bullet = \{t_j \mid O(p_i, t_j) \neq 0\}$ respectively. In a PN system, a self-loop is a relation where $c_{i,j} = O(p_i, t_j) - I(p_i, t_j) = 0$ and $O(p_i, t_j) \neq 0, I(p_i, t_j) \neq 0$.

The marking at the k -th instant is often represented by a vector $M_k = [M_k(p_1) \ M_k(p_2) \ \dots \ M_k(p_n)]^T$. Hereafter, a marking M can be represented by a list $M = [1^{M(p_1)}, 2^{M(p_2)}, \dots, i^{M(p_i)}, \dots, n^{M(p_n)}]$ where i -th item is omitted if $M(p_i) = 0$ and exponents $M(p_i) = 1$ are also omitted. For example, a marking $M = [2 \ 0 \ 1 \ 1]^T$ can be represented by the list $M = [1^2, 3, 4]$.

A transition t_j is enabled at marking M_k if $\forall p_i \in P, M_k(p_i) \geq I(p_i, t_j)$; when an enabled transition t_j is fired, then a new marking M_{k+1} is reached. This new marking is computed as $M_{k+1} = M_k + C v_k$, where v_k is an m -entry firing vector whit $v_k(j) = 1$ when t_j is fired once and $v_k(i) = 0$ if $i \neq j$ and t_j is not fired, v_k is called Parikh vector; the equation

$M_{k+1} = M_k + Cv_k$ is called the PN state equation. The set of enabled transitions at marking M_k is $E(M_k) = \{t \mid \forall p \in P, M_k(p) \geq I(p,t)\}$.

A firing sequence of an PN system (N, M_0) is a transition sequence $\sigma = t_i t_j \dots t_k$ such as $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \dots \xrightarrow{t_k} M_k$. The firing language of (N, M_0) is the set $L(N, M_0) = \{\sigma \mid \sigma = t_i t_j \dots t_k \wedge M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \dots \xrightarrow{t_k} M_k\}$, while the Parikh vector $\bar{\sigma}: T \rightarrow (Z^+)^m$ of σ maps every $t \in T$ to the number of occurrences of t in σ . The fact of reaching M_k from M_0 by firing an enabled sequence σ is denoted by $M_0 \xrightarrow{\sigma} M_k$. The set of all reachable markings from M_0 is $R(N, M_0) = \{M_k \mid M_0 \xrightarrow{\sigma} M_k \text{ and } \sigma \in L(N, M_0)\}$ and it is called reachability set.

Example 1: Consider the PN of Figure 10.a. The net consists of 8 places $P = \{p_1, p_2, \dots, p_8\}$ and 5 transitions $T = \{t_1, t_2, \dots, t_5\}$. The incidence matrix is illustrated in the figure 10.b.

Fig. 10. a) Petri Net System A, b) Incidence matrix of Petri Net System A.

The sets of input and output places of t_1 are $\bullet t_1 = \{p_1\}$ and $t_1 \bullet = \{p_2, p_3\}$ respectively. The initial marking is $M_0 = [0 \ 2 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1]^T$ or $M_0 = [2^2, 4, 5, 7, 8]$. The set of enabled transitions at M_0 is $E(M_0) = \{t_3, t_4\}$. When transition t_3 fires the net reaches the marking $M_1 = [5, 6, 7^2, 8]$. ■

3.2 Interpreted Petri Nets

This chapter uses Interpreted Petri Nets (IPN), an extension to PN (Meda M. E., et al., 1998). This extension consists in assigning input and output signals to PN models. Formally IPN are defined as follows.

Definition 2: An Interpreted Petri Net (IPN) system is a 6-tuple $Q = (N', \Sigma, \Phi, \lambda, \Psi, \phi)$ where $N' = (N, M_0)$ is a PN system; $\Sigma = \{\alpha_1, \alpha_2, \dots, \alpha_r\}$ is the input alphabet, where α_i is an input symbol; $\Phi = \{\delta_1, \delta_2, \dots, \delta_s\}$ is the output alphabet of the net, where δ_i is an output symbol;

$\lambda : T \rightarrow \Sigma \cup \{\varepsilon\}$ is a function that assigns an input symbol to each transition of the net, with the following constraint: $\forall t_j, t_k \in T, j \neq k$, if $\forall p_i, I(p_i, t_j) = I(p_i, t_k) \neq 0$ and both $\lambda(t_j) \neq \varepsilon, \lambda(t_k) \neq \varepsilon$, then $\lambda(t_j) \neq \lambda(t_k)$. In this case, ε represents an internal system event. $\Psi : P \rightarrow \Phi \cup \{\varepsilon\}$ is a labelling function of places that assigns an output symbol or the null event ε to each place of the net as follows: $\Psi(p_i) = \delta_k$ if p_i represents a output signal, in otherwise $\Psi(p_i) = \varepsilon$. In this case $P_m = \{p_i | \Psi(p_i) \neq \varepsilon\}$, $p_i \in P_m$ is called measurable place and $q = |P_m|$ is the number of measured places. Finally, $\varphi : R(N, M_0) \rightarrow (Z^+)^q$ is a function that associates an output vector to every reachable marking of the net as follows: $\varphi(M_k) = M_k|_{P_m}$, where $M_k|_{P_m}$ is the projection of M_k over P_m i.e. if $M_k = [M_k(p_1) \ M_k(p_2) \ \dots \ M_k(p_n)]^T$ and $P_m = \{p_i, p_j, \dots, p_h\}$ then $M_k|_{P_m} = [M_k(p_i) \ M_k(p_j) \ \dots \ M_k(p_h)]^T$:

Notice that function φ is linear and can be represented as a matrix $\varphi = [\varphi_{ij}]_{q \times n}$, where each row $\varphi(k, \bullet)$ of this matrix is an elementary vector where $\varphi(k, i) = 1$ if place p_i is the k -th measured place and otherwise $\varphi(k, i) = 0$ and it is called non-measured.

The transition input alphabet Σ of an IPN can be thought as actuator signals attached to the transitions of the net. Similarly, the output alphabet can be thought as sensor signals attached to places. In this context, it is possible to distinguish between controllable and uncontrollable transitions, and between measured and non-measured places of the net as established in the following definitions.

Definition 3. If $\lambda(t_i) \neq \varepsilon$ then transition t_i is said to be *controlled*, otherwise *uncontrolled*. T_c and T_u are the sets of controlled and uncontrolled transitions, respectively.

Definition 4. A place $p_i \in P$ is said to be *measured* if the i -th column vector of φ is not null, i.e. $\varphi(\bullet, i) \neq 0$; otherwise p_i is *non-measured*. Thus, the set $P_m = \{p | \exists j \in 1, 2, \dots, r \text{ such that } \varphi(\bullet, j) \neq 0\}$ is the set of measured places and $P_{nm} = P \setminus P_m$ is the set of non-measured places.

In this chapter, a measured place is depicted as a unfilled circle, while a non-measured place is depicted as a filled circle. Similarly, non-manipulated transitions are depicted by filled bars and manipulated transitions are depicted by unfilled bars. Also, (Q, M_0) will be used instead of $Q = (N', \Sigma, \Phi, \lambda, \Psi, \varphi)$ to emphasize the fact that there is an initial marking in an IPN.

Example 2: Consider the IPN shown in Figure 11a. The input and output alphabets are $\Sigma = \{a, b\}$ and $\Phi = \{\delta_1, \delta_2, \delta_3\}$ respectively. Functions Ψ and λ are given by:

i	1	2	3	4	5	6	7	8
$\Psi(p_i)$	δ_1	ε	ε	ε	δ_2	δ_3	ε	ε

k	1	2	3	4	5
$\lambda(t_k)$	a	ε	ε	ε	b

Fig. 11. a) Interpreted Petri Net System B, b) Incidence matrix of Interpreted Petri Net System B.

Thus, the controlled transitions are $T_c = \{t_1, t_5\}$ and the uncontrolled ones are $T_u = \{t_2, t_3, t_4\}$. The measured places are $P_m = \{p_1, p_5, p_6\}$ and the non-measured are $P_{nm} = \{p_2, p_3, p_4, p_7, p_8\}$. In this case, the output function is the matrix:

$$\varphi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \quad (26)$$

The initial output is $y_0 = \varphi(M_0) = [0 \ 1 \ 0]^T$. ■

Similarly to a PN, in an IPN system, a transition t_j is enabled at marking M_k if $\forall p_i \in P, M_k(p_i) \geq I(p_i, t_j)$; however t_j has fire conditions. When t_j is enabled and t_j is controllable for that t_j fire, it is necessary that the input signal $\lambda(t_j) \neq \varepsilon$ must be given as input. Otherwise when enabled transition t_j is uncontrollable, then it can be fired. In both cases, when a transition t_j is fired a new marking $M_{k+1} = M_k + C(\bullet, t_j)$ is reached and the output symbol $y_{k+1} = \varphi(M_{k+1})$ is observed.

The following definitions relate the input and output symbol sequences with the firing sequences and the corresponding generated marking sequences. These concepts are useful in the study of the diagnosability property since they are relating to the observed input and output information as the system evolves.

Definition 5. A firing sequence of an IPN (Q, M_0) is a transition sequence $\sigma = t_i t_j \dots t_k \dots$ such as $M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \dots M_w \xrightarrow{t_k} \dots$. The firing language of (Q, M_0) is the set $L(Q, M_0) = \{\sigma \mid \sigma = t_i t_j \dots t_k \wedge M_0 \xrightarrow{t_i} M_1 \xrightarrow{t_j} \dots M_w \xrightarrow{t_k} \dots\}$.

Definition 6. A sequence of input-output symbols of an IPN (Q, M_0) is a sequence $w = (\alpha_0, y_0)(\alpha_1, y_1) \dots (\alpha_k, y_k)$, where $\alpha_i \in \Sigma \cup \{\varepsilon\}$ is the current input of (Q, M_0) when the output changes from y_i to y_{i+1} . It is assumed that $\alpha_0 = \varepsilon$, $y_0 = \varphi(M_0)$ and (α_{i+1}, y_{i+1})

belongs to w if the pair (α_i, y_i) belongs to the sequence and $y_{i+1} \neq y_i$ and $\neg \exists y_j$ different from y_i , y_{i+1} occurring after y_i and before y_{i+1} . α_{i+1} are the current input of the IPN when the output changes from the sequence when is the first output change since the occurrence of y_i . The set of all input-output sequences is denoted by $\Lambda(Q, M_0) = \{w | w \text{ is an input-output sequence}\}$, the set of input-output sequences of length greater or equal than to k is $\Lambda^k(Q, M_0) = \{w | w \in \Lambda(Q, M_0) \text{ and } |w| \geq k\}$.

Definition 7. Let (Q, M_0) be an IPN and $w = (\alpha_0, y_0)(\alpha_1, y_1) \dots (\alpha_k, y_k)$ be an input-output sequence. The set of all possible firing transitions sequences that could generate w is defined as $\Omega(w) = \{\sigma = t_i t_j \dots t_k | \sigma \in L(Q, M_0) \text{ and } \lambda(t_i) \lambda(t_j) \dots \lambda(t_k) = \alpha_0 \alpha_1 \dots \alpha_k\}$. However, the transition sequence whose firing actually generates w is denoted by σ_w . Notice that $\sigma_w \in \Omega(w)$.

Definition 8. The set of all input-output sequences leading to an ending marking in the IPN is $\Lambda_B(Q, M_0) = \{w \in \Lambda(Q, M_0) | \exists \sigma \in \Omega(w) \text{ such that } M_0 \xrightarrow{\sigma} M_j \text{ and if } t_j \in E(M_j) \text{ then } C(\bullet, j) = \emptyset\}$. An ending marking is a reachable marking that has no enabled transitions or enables only self-loop transitions.

Definition 9. Let (Q, M_0) be an IPN and $w = (\alpha_0, y_0)(\alpha_1, y_1) \dots (\alpha_k, y_k)$ be an input-output sequence. The set of all possible marking sequences corresponding to w is $S_w = \{M_i M_j \dots M_k | M_k \xrightarrow{t_x} M_j \xrightarrow{t_y} \dots \xrightarrow{t_z} M_k \text{ and } \sigma = t_x t_y \dots t_z \in \Omega(w) \text{ and } y_0 = \phi(M_i), y_1 = \phi(M_j), \dots, y_k = \phi(M_k)\}$.

Example 3: Consider the IPN shown in Figure 11.a where $P_m = \{p_1, p_5, p_6\}$ and $T_c = \{t_1, t_5\}$, its reachability graph and its outputs is showed in the figure 12. The following languages can be obtained:

$$L(Q, M_0) = \{t_4, t_4 t_3, t_4 t_3 t_5, t_4 t_3 t_5 t_1, \dots\},$$

$$\Lambda(Q, M_0) = \{(\epsilon, [0 \ 1 \ 0]^T), (\epsilon, [0 \ 1 \ 0]^T)(\epsilon, [0 \ 0 \ 0]^T), (\epsilon, [0 \ 1 \ 0]^T)(\epsilon, [0 \ 0 \ 0]^T), (\epsilon, [0 \ 0 \ 1]^T), \dots\},$$

$$\Lambda^2(Q, M_0) = \{(\epsilon, [0 \ 1 \ 0]^T)(\epsilon, [0 \ 0 \ 0]^T), (\epsilon, [0 \ 1 \ 0]^T)(\epsilon, [0 \ 0 \ 0]^T)(\epsilon, [0 \ 0 \ 1]^T), \dots\}$$

If $w = (\epsilon, [0 \ 1 \ 0]^T)(\epsilon, [0 \ 0 \ 0]^T)$ then $\Omega(w) = \{t_4\}$ and $S_w = \{M_0 M_2, M_6 M_7\}$. ■

4. Diagnosability and Diagnosable IPN systems

Diagnosability is the property of detecting and locating the fault occurrence in the system through the available information of the system inputs, outputs and structure. This section reviews the diagnosability definition and a characterization of diagnosable IPN. The reader can consult (Aguirre-Salas & Santoyo-Sanchez, 2009), (Ramírez-Treviño et al., 2007), (Ruiz-Beltrán et al., 2007) and (Santoyo-Sanchez et al., 2008) for more details.

Fig. 12. Reachability graph of the Interpreted Petri Net System B and its outputs.

4.1 Concepts

Definition 10: An IPN given by (Q, M_0) is *input-output diagnosable* if there exist an integer $k < \infty$ such that $\forall w \in \Lambda^k(Q, M_f) \cup \Lambda_B(Q, M_f)$ it holds that the information provides by w and (Q, M_0) suffices to distinguishable any marking $M_f \in F$ from any other $M_k \in R(Q, M_0)$, by the firing of the transition sequence σ_w where F is the set of fault markings. A *fault marking* is a reachable marking that indicates that the system has reached fault conditions.

This definition implies that, for any fault marking of the Interpreted Petri net, its marking becomes distinguish after a finite number of k events. From this definition in order to achieve diagnosability in an IPN, it is necessary to detect the occurrence of each system event, i.e. the firing of each transition from M_0 to M_f . This leads to the following concepts: sequence-detectability (the possibility of determinate the occurrence of each firing sequence), event-detectability (the possibility of determine the firing of each transition after its occurrence and before another transition occurrence) and marking detectability (the possibility of determinate the system state after the observation of a finite sequence of input-output symbols). The reader can consult (Aguirre-Salas & Santoyo-Sanchez 2009) for more details.

Definition 11. An IPN given by (Q, M_0) is *sequence-detectable* if there is an integer $k < \infty$ such that $\forall w \in \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0)$ it holds that the information provided by w and (Q, M_0) suffices to uniquely determine σ_w .

Sequence-detectability implies the knowledge of all firing sequences of an IPN. In others words, there is a function $\Delta_s : \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0) \times (Q, M_0) \rightarrow L(Q, M_0)$ where $\Delta_s(w, (Q, M_0)) = \sigma_w$. The problem of determining whether or not a system is sequence-detectable has a high computational complexity. However, the following definition provides conditions that reduce the computational complexity.

Definition 12. An IPN given by (Q, M_0) is *event detectable* if the firing of any transition $t_k \in T$ at a marking $M_k \in R(Q, M_0)$ can be uniquely determined through the information provided by the input symbol $\alpha = \lambda(t_k)$ and the output signals $\phi(M_k)$, where $\phi C(\bullet, k)$ is the column of ϕC corresponding to transition t_k .

Note that this definition implies that, all events can be detected and distinguishable, i.e. their firing can be detected and distinguishable from each other after its occurrence and before another event occurs. Thus, an event-detectable IPN system is also sequence-detectable. Event detectable has a structural characterization captured in the next lemma that can be tested in a polynomial time.

Lemma 1: An IPN given by (Q, M_0) is *event detectable* if only if $\forall i \in [1, 2, 3, \dots, m], \phi C(\bullet, i) \neq \vec{0}$ and $\forall j \neq k \in [1, 2, 3, \dots, m]$ such $\phi C(\bullet, j) = \phi C(\bullet, k)$, then $\lambda(t_j) \neq \lambda(t_k)$.

Proof. You can find the proof in (Ramírez-Treviño et al., 2003). ■

Observe that an IPN given by (Q, M_0) where it has transitions $t_k, t_j \in T$ with $\lambda(t_k) = \lambda(t_j)$ and $\phi C(\bullet, k) = \phi C(\bullet, j) \neq 0$ the firing of t_k, t_j cannot be distinguishable, in this case t_k, t_j are called *indistinguishable*.

Definition 13. An IPN given by (Q, M_0) is *marking-detectable* if there is an integer $k < \infty$ such that $\forall w \in \Lambda^k(Q, M_0)$ it holds that the information provided by w and (Q, M_0) suffices to uniquely determine the marking M_i reached by firing σ_w .

In other works, there is a function $\Delta_M : \Lambda^k(Q, M_0) \cup \Lambda_B(Q, M_0) \times (Q, M_0) \rightarrow R(Q, M_0)$ where $\Delta_M(w, (Q, M_0)) = M_i$, $\Delta_s(w, (Q, M_0)) = \sigma_w$ and $M_0 \xrightarrow{\sigma_w} M_i$.

Example 4: Consider the IPN shown in figure 11.a where $P_m = \{p_1, p_5, p_6\}$ and $T_c = \{t_1, t_5\}$. Its incidence matrix shown in the figure 11.b and its output function is the matrix of the equation (26). In this case, ϕC is the matrix:

$$\phi C = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & -1 \end{bmatrix} \quad (27)$$

Note that the IPN system in the figure 11.a is event detectable by the lemma 1. Then the fire of all events in this IPN can be detected and distinguishable after its occurrence and before another event occurs. To illustrate as the fire of any event in IPN system is detected consider that the information provide by the IPN system is:

$$\text{input } u_1 = \varepsilon; \text{ output } \begin{cases} y_0 = [0 & 1 & 0]^T \\ y_1 = [0 & 0 & 0]^T \end{cases} \quad (28)$$

In this case, $\Delta\varphi = y_1 - y_0 = [0 \ -1 \ 0]^T$ and $\varphi C(\bullet, 4)$ is the column of φC corresponding to transition t_4 . Then the fire of the transition t_4 is detected through the information of the input, outputs and structure. Also $\sigma_w = \lambda(t_4)$ where $w = (\varepsilon, [0 \ 1 \ 0]^T)(\varepsilon, [0 \ 0 \ 0]^T)$, and the IPN system in the figure 11.a is sequence-detectable. Note that also it IPN system is marking-detectable in 2-steps. Finally, consider that set of fault marking is $F = \{M_2, M_7\}$ (see the figure 12) in this case the information provided by w and (Q, M_0) suffices to uniquely determine the marking $M_i \in F$ reached by firing σ_w , then the IPN system shown in the figure 11.a is input-output diagnosable in 2-steps. ■

4.2 Characterization

Sequence-detectability and marking-detectability are both necessary and sufficient conditions for input-output diagnosability, for provide a sufficient condition of input-output diagnosable with a reduced computational complexity is considered a class of IPN defined as follows.

Definition 14. A subnet of an IPN given by (Q, M_0) with $Q = (N, \Sigma, \Phi, \lambda, \Psi, \varphi)$ and $N = (P, T, I, O)$ is a net $(\tilde{Q}, \tilde{M}_0)$ such that $\tilde{P} \subseteq P$, $\tilde{T} \subseteq T$, $\tilde{\Sigma} \subseteq \Sigma$, $\tilde{\Phi} \subseteq \Phi$ and functions $\tilde{I}, \tilde{O}$ are restrictions of I and O over $\tilde{P} \times \tilde{T}$ respectively, $\tilde{\lambda} : \tilde{T} \rightarrow \tilde{\Sigma} \cup \{\varepsilon\}$, $\tilde{\Psi} : \tilde{P} \rightarrow \tilde{\Phi} \cup \{\varepsilon\}$ and $\tilde{\varphi} : R(\tilde{Q}, \tilde{M}_0) \rightarrow (Z^+)^q$.

Definition 15. The set of fault transitions of an IPN given by (Q, M_0) with $Q = (N, \Sigma, \Phi, \lambda, \Psi, \varphi)$ and $N = (P, T, I, O)$ is the set $T_f = \{t_k \mid \varphi O(\bullet, t_k) = \bar{0} \text{ and } \lambda(t_k) = \varepsilon\}$.

Definition 16. Let (Q, M_0) with $Q = (N, \Sigma, \Phi, \lambda, \Psi, \varphi)$, $N = (P, T, I, O)$ an IPN where $T_f \neq \{\}$. The subnet induced by T_f is (Q_{Tf}, M_0^{Tf}) with $N_{[Tf]} = (P_{Tf}, T_{Tf}, I_{Tf}, O_{Tf})$ where $T_{Tf} = T_f$, $P_{Tf} = \bigcup_{i=1}^k \bullet t_i$, $\bullet \in T_f$ and I_{Tf} , O_{Tf} are restrictions of I and O over $P_{Tf} \times T_{Tf}$ respectively, $\Sigma_{Tf} = \{\}$, $\Phi_{Tf} \subseteq \Phi$ and functions $\lambda_{Tf} : T_{Tf} \rightarrow \varepsilon$, $\Psi_{Tf} : P_{Tf} \rightarrow \Phi_{Tf} \cup \{\varepsilon\}$ and $\tilde{\varphi} : R(Q_{Tf}, M_0^{Tf}) \rightarrow (Z^+)^q$. This subnet is named *fault model*.

Definition 17. Let (Q, M_0) with $Q = (N, \Sigma, \Phi, \lambda, \Psi, \varphi)$, $N = (P, T, I, O)$ an IPN where $T_f \neq \{\}$. The subnet of normal behavior is (Q^N, M_0^N) with $Q^N \subseteq Q$ where $T_N = T - T_f$, $P_N = P - \{\bigcup_{i=1}^k \bullet t_i \mid \bullet \in T_f\}$, $\Sigma_N = \Sigma$, $\Phi_N \subseteq \Phi$ and function I_N , O_N are restrictions of I and O over $P_N \times T_N$ respectively. In this case $\lambda_N : T_N \rightarrow \Sigma \cup \{\varepsilon\}$, $\Psi_N : P_N \rightarrow \Phi_N \cup \{\varepsilon\}$ and $\varphi_N : R(Q^N, M_0^N) \rightarrow (Z^+)^q$. This subnet is named *diagnoser model*.

Example 5: Consider the IPN shown in Figure 13a. The input and output alphabets are $\Sigma = \{a, b\}$ and $\Phi = \{\delta_1, \delta_2, \delta_3\}$ respectively. Functions Ψ , λ , C and φ are given by:

i	1	2	3
$\Psi(p_i)$	δ_1	δ_2	δ_3

k	1	2	3
$\lambda(t_k)$	a	b	ε

$$C = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \varphi = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (29)$$

Fig. 13. a) Interpreted Petri Net System C, b) A subnet of Interpreted Petri Net System C, c) Subnet induced by T_f .

Thus, the controlled transitions are $T_c = \{t_1, t_2\}$ and the uncontrolled ones are $T_u = \{t_3\}$. The measured places are $P_m = \{p_1, p_2, p_3\}$ and the non-measured are $P_{nm} = \{\}$. In this case, the figure 13.b is a subnet of normal behavior Interpreted Petri Net system of the figure 13.a with functions Ψ_N, λ_N, C_N and ϕ_N are given by:

i	1	2
$\Psi_N(p_i)$	δ_1	δ_2

k	1	2
$\lambda_N(t_k)$	a	b

$C_N = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$\phi_N = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(30)

The subnet induced by $T_f = \{t_3\}$ is showed in the figure 13.c where functions $\Psi_{Tf}, \lambda_{Tf}, C_{Tf}$ and ϕ_{Tf} are given by:

i	1	3
$\Psi_{Tf}(p_i)$	δ_1	δ_3

k	3
$\lambda_{Tf}(t_k)$	ε

$\phi_{Tf} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(31)

In this chapter to emphasize the fact that the IPN system captures the normal and fault behaviour (illustrated as the IPN systems of the figures 13.b and 13.c respectively) the next definition is proposed.

Definition 18. An IPN system to diagnose given by (Q^D, M_0^D) is a net where $T_f \neq \{\}$ and the funtions I, O and C are:

$I = \begin{bmatrix} I_N & I_{Tf} \\ \bar{0} & \end{bmatrix}; O = \begin{bmatrix} O_N & O_{Tf} \\ \bar{0} & \end{bmatrix}$

and

$C = \begin{bmatrix} C_N & C_{Tf} \\ \bar{0} & \end{bmatrix}$

(32)

where I_N, O_N, C_N are restrictions of I and O over $P_N \times T_N$, i.e. over the subnet indiced by normal behavior (Q^N, M_0^N) ; and I_{Tf}, O_{Tf}, C_{Tf} are restrictions of I and O over $P_{Tf} \times T_{Tf}$, i.e. over the subnet induced by fault behavior $T_f ((Q_{Tf}, M_0^{Tf}))$.

Note that the incidence matrix PN system 13.a is:

$C = \left[\begin{array}{cc|c} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right]$

with

$C_N = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

and

$C_{Tf} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

(33)

The characterization of input-output diagnosable is based on the idea from the IPN system captures the normal and fault behaviour (illustrated as the IPN systems of the figures 13.b and 13.c respectively. Note that in the figure 13.c non-manipulated transition marking a faulty place. Then it is possible to know when a faulty place is marked and to determinate which is the marked place. This idea is formalized in the next theorem.

Theorem 1: Let (Q^D, M_0^D) be an IPN, live, strongly connected and event detectable with $Q = (N, \Sigma, \Phi, \lambda, \Psi, \phi)$ and $N = (P, T, I, O)$. Let $\{X_1, \dots, X_r\}$ be the set of all T-invariants of (Q^D, M_0^D) . Let (Q^N, M_0^N) be subnet induced by T_f . If $\forall p_i \in P_{T_f}$, where $p_i \in \bullet t_j$ and $t_j \in T_{T_f}$ (i.e. p_i are predecessors of any fault transition) the following conditions hold:

1. $\forall r, \exists j \quad X_r(j) \geq 1$, where $t_j \in (p_i) - T_{T_f}$;
2. $\forall t_k \in (p_i) - T_{T_f}, \bullet(t_k) = \{(p_i)\}$ and $\lambda(t_k) \neq \varepsilon$.

Then the IPN (Q^D, M_0^D) is input-output diagnosable.

Proof. You can find the proof in (Ramírez-Treviño et al., 2007). ■

4.3 Diagnoser Design

The issue with detection and localization of faults consist in identify the abnormal behavior in the systems and locate the root cause or resources that are working in a wrong way. Diagnoser design proposed in (Santoyo-Sanchez et al., 2008) is shows in the figure 14, this scheme consist of six components

1. *Diagnoser model*, which is an IPN, denoted by (Q^N, M_0^N) that represents the normal behavior of the system.
2. *System model*, it is an IPN denoted by (Q^D, M_0^D) which contains the normal and abnormal behavior from the system, where the diagnoser IPN is embedded into the IPN system.
3. *Firing events detector block*, which detect and determine which transitions was fired into the system.
4. *Error block*, it is an Error IPN defined between (Q^D, M_0^D) and (Q^N, M_0^N) , which compares the behavior between both IPN systems.
5. *Detecting Fault Marking algorithm*, it detects and locates fault through the Error IPN, also indicating faulty state.
6. *Diagnoser Fault algorithm*, it indicate the component fault and specify the kind of fault that occur.

Fig. 14. Scheme for proposed diagnoser.

The diagnose process is based on the idea of that the system behavior is modeled as IPN, which contains the normal and fault behavior. When a transition fires, due to system IPN is event-detectable then it is possible to determine its fires (with the firing events detect block). Moreover, when the system does not fire fault transitions the output of both models (system and diagnoser) is equal, i.e. the system behavior only includes the fire of normal transitions. In the other case, when a fault transition fires in the system, its fire is detected but this transition is not include into the diagnoser model, then the output of both models (system and diagnoser) is not equal. In this case a fault is detected, and the next steps are to locate the fault, indicate the component fault and specify the kind of fault. To illustrate the general diagnose process consider the next example.

Example 6: Consider the IPN shown in figure 13a. as the system model, and the IPN shown in figure 13.b as the diagnoser model. Both system the input and output alphabets are $\Sigma = \{a, b\}$ and $\Phi = \{\delta_1, \delta_2, \delta_3\}$ respectively. And its functions Ψ , λ , C and φ are given by the equations (29) and (30) respectively. In this case, φC and $\varphi_N C_N$ are the matrix:

$$\varphi C = \begin{bmatrix} 1 & -1 & -1 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad \varphi_N C_N = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad (34)$$

Note that the IPN system in the figure 13.a is event detectable by the lemma 1. Assume that from M_0 and M_0^N the information provide by the IPN system is:

$$\text{input } u_1 = b ; \text{ output } \begin{cases} y_0 = [1 & 0]^T \\ y_1 = [0 & 1]^T \end{cases} \text{ and input}_N u_1 = b ; \text{ output}_N \begin{cases} y_0^N = [1 & 0]^T \\ y_1^N = [0 & 1]^T \end{cases} \quad (35)$$

In this case, $\Delta\phi = y_1 - y_0 = [1 \quad -1]^T$ and $\phi C(\bullet, 2)$ is the column of ϕC corresponding to transition t_2 . Also its output

error between both systems is $\Delta y = y_1^N - y_1 = [0 \quad 0]^T$. In this case the system does not fire fault transitions. Assume that

from M_0 and M_0^N the information provide by the IPN system is:

$$\text{input } u_1 = \varepsilon ; \text{ output } \begin{cases} y_0 = [1 & 0]^T \\ y_1 = [0 & 0]^T \end{cases} \text{ and input}_N u_1 = \varepsilon ; \text{ output}_N \begin{cases} y_0^N = [1 & 0]^T \\ y_1^N = [1 & 0]^T \end{cases} \quad (36)$$

In this case, $\Delta\phi = y_1 - y_0 = [-1 \quad 0]^T$ and $\phi C(\bullet, 3)$ is the column of ϕC corresponding to transition t_3 . And its output error between both systems is $\Delta y = y_1^N - y_1 = [1 \quad 0]^T$. In this case a fault transition fires in the system, and a fault is detected, i.e. an error different to zero.

In general, the error concept among systems is computed as differences among their outputs. In the observer and controller design when the error is zero then the system reach a required behavior (De Jesús & Ramírez-Treviño, 2001). In the context of diagnoser design to localize the fault transition and the place of fault is used an error structure introduced in (De Jesús & Ramírez-Treviño, 2001) and (Santoyo-Sanchez et. al, 2008), which is presented in the next definition.

Definition 19 Let (Q^D, M_0^D) and IPN where (Q^N, M_0^N) and (Q_{Tf}, M_0^{Tf}) are its subnets IPN diagnoser model and IPN fault model respectively. Structure Error between (Q^N, M_0^N) and (Q^D, M_0^D) is defined as (N^E, M_0^E) where $N^E = (P^E, T^E, I_E, O_E)$ with $P^E = P^D$, $T^E = T^D$, $I_E : P^E \times T^E \rightarrow Z^+$, $O_E : P^E \times T^E \rightarrow Z^+$ and $C_E : P^E \times T^E \rightarrow Z^+$ defined as:

$$I_E = \begin{bmatrix} I_N & -I_N & -I_{Tf} \\ \bar{0} & \bar{0} & \bar{0} \end{bmatrix}; O_E = \begin{bmatrix} O_E & -O_E & -O_{Tf} \\ \bar{0} & \bar{0} & \bar{0} \end{bmatrix} \text{ and } C_E = \begin{bmatrix} C_N & -C_N & -C_{Tf} \\ \bar{0} & \bar{0} & \bar{0} \end{bmatrix} \quad (37)$$

The initial marking of IPN $M_0^E : P^E \rightarrow Z$ defined as:

$$M_0^E = \begin{bmatrix} M_0^N \\ \bar{0} \end{bmatrix} - \begin{bmatrix} M_0^N \\ M_0^{Tf} \end{bmatrix} \quad (38)$$

The marking at the k -th instant is:

$$M_k^E = \begin{bmatrix} M_k^N \\ \bar{0} \end{bmatrix} - \begin{bmatrix} M_k^N \\ M_k^{Tf} \end{bmatrix} \quad (39)$$

A transition $t_j \in T^N$ is enabled at marking M_k^E if $\forall p_i \in P^N$, $M_k^E(p_i) \geq I^N(p_i, t_j)$; while that a transition $t_j \in T^D$ is enabled at marking M_k^E if $\forall p_i \in P^D$, $M_k^E(p_i) < I^D(p_i, t_j)$. When a transition t_j is fired, then a new marking M_{k+1} is reached. This new marking is computed as:

$$M_{k+1}^E = M_k^E + \begin{bmatrix} C^N & -C^N & -C^F \\ \vec{0} & \vec{0} & \vec{0} \end{bmatrix} \begin{bmatrix} \sigma_k^N \\ \gamma_k^D \\ \beta_k^F \end{bmatrix} \quad (40)$$

where σ_k^N is an m -entry firing vector of structure (Q^N, M_0^N) , γ_k^N is an m -entry firing vector of structure (Q^D, M_0^D) and β_k^F is an m -entry firing vector of structure (Q_{Tf}, M_0^{Tf}) .

Example 7: Consider the IPNs shown in Figure 13.a, 13.b and 13.c note that this IPN are (Q^D, M_0^D) , (Q^N, M_0^N) and (Q_{Tf}, M_0^{Tf}) respectively. Structure Error between (Q^N, M_0^N) and (Q^D, M_0^D) is:

$$M_0^E = \begin{bmatrix} M_0^N \\ \vec{0} \end{bmatrix} - \begin{bmatrix} M_0^N \\ M_0^{Tf} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}; \quad C = \left[\begin{array}{cc|cc|c} -1 & 1 & 1 & -1 & 1 \\ 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{array} \right] \quad \varphi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (41)$$

which is showed in the figure 15. In this error marking there not are enabled transitions. Assume that from M_0 and M_0^N the information provide by the IPN system is:

$$\text{input } u_1 = \varepsilon; \text{output } \begin{cases} y_0 = [1 & 0]^T \\ y_1 = [0 & 0]^T \end{cases} \text{ and input}_N \quad u_1 = \varepsilon; \text{output}_N \quad \begin{cases} y_0^N = [1 & 0]^T \\ y_1^N = [1 & 0]^T \end{cases} \quad (42)$$

In this case, $\Delta\varphi = y_1 - y_0 = [-1 \ 0]^T$ and $\varphi C(\bullet, 3)$ is the column of φC corresponding to transition t_3 . And its output

error between both systems is $\Delta y = y_1^N - y_1 = [1 \ 0]^T$. In this case an error is detected and the error marking is:

$$M_1^E = \begin{bmatrix} M_1^N \\ \vec{0} \end{bmatrix} - \begin{bmatrix} M_1^N \\ M_1^{Tf} \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \quad (43)$$

Fig. 15. Representation of the structure error model.

In this marking, the enabled transitions are $t_2 \in T^N$ and $t_3 \in T_{Tf}$, if t_2 fires into the structure error the new marking M_{k+1} is reached. This new marking is computed as:

$$M_2^E = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + C[0 \ 1 \ 0 \ 0 \ 0]^T = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix} \quad (44)$$

In the case of that t_3 fires into the structure error the marking:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + C[0 \ 0 \ 0 \ 0 \ 1]^T \quad (45)$$

Then from M_1^E to reach the error zero, it is necessary fires the fault transition t_3 and $p_3 = \bullet t_3$ is the fault place. ■

The following theorem characterizes the diagnosis based on structure error models.

4.4 Diagnose Fault

Theorem 2: Let (Q^D, Q^N) be a pair system-diagnoser with the state equations (46) and (47) respectively. Where the diagnoser is based on the idea that the diagnoser IPN (Q^N, M_0^N) is embedded into the IPN system (Q^D, M_0^D) and each faulty transition is non-manipulable (see the figure 13.a).

$$Q^D = \begin{cases} M_{k+1}^D &= M_k^D + C^D \sigma_k^D \\ y_k^D &= \varphi_N(M_k^D) \end{cases} \quad (46)$$

$$Q^N = \begin{cases} M_{k+1}^N &= M_k^N + C^N \sigma_k^N \\ y_k^N &= \varphi_N(M_k^N) \end{cases} \quad (47)$$

If (Q^D, M_0^D) is input-output diagnosable (Ramírez-Treviño et al., 2007) and the structure Error between (Q^N, M_0^N) and (Q^D, M_0^D) is defined as (N^E, M_0^E) then it is possible to detect and isolate a fault into the system (Q^D, M_0^D) .

Proof. You can find the proof in (Santoyo-Sanchez et al., 2008). ■

Based on the theorem 2 the following algorithm is presented to detect and isolate error marking.

Algorithm 1: Detecting and isolate error marking.

Inputs: The IPN model of the pair system-diagnoser.

Outputs: The error marking M_k^E , faulty place p_{Fi} and faulty transition.

Procedure:

1. Define the structure error.
2. When $M_k^E \neq \vec{0}$ then:

$\varphi_z(M_k) = [3 \ 2 \ 1 \ 0]^T$ then relay 1(into the line 2) is in the third protection zone, relay 2 (into the line 3) is in the second protection zone, and finlay the relay 3 (into the line 4) is in the first protection zone. ■

When a fault occurs, the marking is analized to determine each fault zone (figure 6). Like a resulting of this process, each relay determines its tripping time. The relays acts instantaneously when the fault location at the first zone. Then the protection devices acts as fast as possible to disconnect the faulted element, the element into the first zone. The next algorithm captures this idea.

Algorithm 2. Diagnosis Fault.

Inputs: The IPN model of system, the structure error, error marking M_k^E , faulty place p_F and faulty transition .

Outputs: Faulty component $Comp_F$, the sets $Pr\ oteccD_{Comp_F}$ and $Pr\ oteccI_{Comp_F}$.

Procedure:

When $M_k^E \neq \emptyset$ do:

1. Compute the protection zone using φ_z .
 2. Define $Comp_F = \{\forall c_i \mid p_i \in (t_F) \bullet \text{ and } p_i \text{ is the place used to represent the system element } c_i \text{ and } c_i \text{ into the protection zone 1}\}$.
 3. Define the Protection behavior.
 - 3.1 $Pr\ oteccD_{Comp_F} = \{\forall c_i \mid p_i \in M(p_i) = 1, \text{ where } p_i \text{ is the place used to represent the disconnection of the system element } c_i \text{ by the fault in } Comp_F \}$.
 - 3.2 $Pr\ oteccI_{Comp_F} = \{\forall c_i \mid p_i \in M(p_i) \Rightarrow 1, \text{ where } p_i \text{ is the place use to describe how the electrical system distribute power electrical and } c_i \text{ indicates the electrical element that is stressed due to fault into } Comp_F \}$.
 4. Diagnostic. Return the sets $Comp_F$, $Pr\ oteccD_{Comp_F}$ and $Pr\ oteccI_{Comp_F}$.
-

The algorithm 2 indicates fault and disconnection of elements by the protections occurrence in the electrical system; thus it is possible to distinguish between fault elements and consequences of the faults. In this case the IPN model and the algorithms (1 and 2) are desingned for a line, due to the line has three phases the model is generated for each phase (sequence positive, negative and zero). Finally using the information of the table 1 is specify the kind of fault.

Fig. 16. a) IPN system for power electrical network of the figure 8. b)IPN diagnoser from the power electrical network of the figure 8.

5. Application Example

For illustrate the diagnoser behavior consider the figure 16.a and 16.b which are the pair system (Q^D, M_0^D) -diagnoser (Q^N, M_0^N) for the power electrical network of the figure 8 and the power flow from the generator 1, φ_Z defined in the equation (49), φ_N is a identity matrix of 31×31 and $\varphi_D = [\varphi_N \ D]$ where D is matrix of 31×4 with zero. Consider that a fault occurs in LU_5 (see the figure 9), when the window fault is completed all the really front the marking in the system is $M_k^D = [1,3,5,10,15,20,25,27,28,29,30,31,32,33,34,35]$ while that in the diagnoser is $M_k^N = [1,3,5,9^{32},10,14^{32},15,19^{32},20,24^{32},25,27,28,29,30,31]$, then its output error between both systems is $\Delta y = y_k^N - y_k^D = [0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 32 \ 0 \ 0 \ 0 \ 0 \ 32 \ 0 \ 0 \ 0 \ 0 \ 32 \ 0 \ 0 \ 0 \ 0 \ 32 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$

in this case all fault transitions fire in the system, and faults are detected, i.e. an error different to zero. Marking error of structure error between (Q^N, M_0^N) and (Q^D, M_0^D) in the k -th instant is: $M_k^E = [0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 32\ 0\ 0\ 0\ 0\ 32\ 0\ 0\ 0\ 0\ 32\ 0\ 0\ 0\ 0\ 32\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ -1\ -1\ -1\ -1]^T$. Using the algorithm 1, it is obtained $p_F = \{p_{32}, p_{33}, p_{34}, p_{35}\}$, $t_F = \{t_{157}, t_{158}, t_{159}, t_{160}\}$. The diagnosis algorithm (algorithm 2) computes the protection zone as $\phi_z(M_k^D) = [4\ 3\ 2\ 1]^T$ (illustrated in the figure 9), $Comp_F = \{LU_5\}$, i.e. the faulty component is the transmission line 5 because p_{35} is used to represent a part of behavior of LU_5 into the first protection zone. $ProteccD_{Comp_F} = \{\}$; i.e. all others electrical elements are connected without fault. Moreover no one other component of the electrical system is disconnecting as consequence of the fault. $ProteccI_{Comp_F} = \{BU_4\}$, i.e. the bus (BU) is stressed due to the fault in LU_5 . Note that electrical element stressed is easy to computed, because the faulty in $Comp_F = \{LU_5\}$ produces that transition t_{155} cannot be fired, and the predecessor and successors of $\bullet t_{155} \bullet = \{p_{20}, p_{22}, p_{30}, p_{31}\}$, where p_{20} and p_{22} represent the normal operation of Bus (BU_4) and line LU_5 respectively; p_{30} is the place into DU_1 such that the tokens will be accumulated into it, and p_{31} is the place into DU_1 such that the tokens are decreased. Thus using the structure of the IPN it is possible to anticipate a future fault in the electrical component stressed. Thus the kind fault depends of the characteristic of V and I for each phase (sequence positive, negative and zero). If the fault is maintain then eventually the algorithm 2 compute:

$ProteccD_{Comp_F} = \{BU_4, LU_5, LU_4, BU_3\}$; i.e. electrical elements disconnected by protection, in this case by the previous relay. And $ProteccI_{Comp_F} = \{LU_3, BU_2\}$; i.e. the next protections operations will activate. ■

6. Conclusions

If the most important design consideration of relaying is security then is really important and reasonable to have a support methodology to assure the relay operation. Security involves the ability to avoid operations for which tripping is not desired because in core the electrical system is designed to maintain the electric service. In this sense IPN is an adequate methodology to watch over the correct operation of the entire power system network, IPN diagnoses sample by sample is the electric system is working in a steady state condition, therefore IPN adds redundancy to the relay. IPN do not substitute the relay, IPN increases security in the system operation which is the main goal of relaying.

We have been proposed a diagnosis scheme allowing detecting and locating faults of electrical systems modeled as IPN. Since the electrical elements can fault simultaneously, the system IPN contain relations denoted as DU representing the electrical flow. The method that is proposed for diagnosis consists in two algorithms. In the first algorithm is detected and located the error marking through of the structure error. While the second algorithm classified the set of faulty systems in power systems in order to estimate the origin of the fault and consequences of the fault. The diagnoser function was illustrated with a study case. The proposed is based on the voltages and currents measurements and its digital processing with a relay to maintain the operation of the power electric system.

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Petri Nets are graphical and mathematical tool used in many different science domains. Their characteristic features are the intuitive graphical modeling language and advanced formal analysis method. The concurrence of performed actions is the natural phenomenon due to which Petri Nets are perceived as mathematical tool for modeling concurrent systems. The nets whose model was extended with the time model can be applied in modeling real-time systems. Petri Nets were introduced in the doctoral dissertation by K.A. Petri, titled „Kommunikation mit Automaten“ and published in 1962 by University of Bonn. During more than 40 years of development of this theory, many different classes were formed and the scope of applications was extended. Depending on particular needs, the net definition was changed and adjusted to the considered problem. The unusual “flexibility” of this theory makes it possible to introduce all these modifications. Owing to varied currently known net classes, it is relatively easy to find a proper class for the specific application. The present monograph shows the whole spectrum of Petri Nets applications, from classic applications (to which the theory is specially dedicated) like computer science and control systems, through fault diagnosis, manufacturing, power systems, traffic systems, transport and down to Web applications. At the same time, the publication describes the diversity of investigations performed with use of Petri Nets in science centers all over the world.

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