5

Dynamic analysis of activity of e-learning system users

R. Mosdorf, J. Kilon and B. Ignatowska
Higher School of Finance and Management in Białystok
Ciepla Street 40, Bialystok, Poland

1. Introduction

One of the key problems that should be taken into consideration when constructing, developing and evaluating a computer system is the identification of the dynamical characteristics of the interaction between a computer system and its users. The user's activities modify different properties of the computer system. The analysis of the changes in time of such properties can be useful for understanding the dynamics of interaction between users and computer systems.

The Learning Management System (LMS) records activity of many users: students, teachers and administration workers. Each student using the LMS can be treated as dynamical system whose activity is caused by LMS, but system activity results from activity of teachers and university administration. Each of the mentioned users can be treated as dynamical system as well. It means that e-learning system is a virtual place where thousands of dynamical systems (users) communicate each others. These communication leads to changes of users' activities.

The users (human) behavior is nonlinear (Sulis et al., 1995), therefore we can say that LMS is a virtual platform of interaction of nonlinear dynamical systems (Ignatowska et al., 2005; Ignatowska et al., 2008; Mosdorf et al., 2006).

The trajectories of nonlinear dynamical system in the phase space form objects called strange attractors of the structure resembling the fractal (Schuster, 1993; Ott, 1993). The analysis of strange attractor gives us information about the properties of dynamical system such as system complexity and its stability. The application of Takens theorem (Schuster, 1993) - in connection with possibilities of modern measuring and computing techniques - enables an analysis of dynamics of non linear processes by analysis of single time series. In nonlinear analysis the reconstruction of attractor in certain embedding dimension has been carried out using the stroboscope coordination. In this method subsequent co-ordinates of attractor points have been calculated basing on the subsequent samples, between which the distance is equal to time delay τ. The time delay is a multiplication of time between the samples. In the nonlinear analysis the following time series is analyzed.

$$\{ x_n \} = \{ x(t), x(t+\tau), \ldots, x(t+n\cdot\tau) \} \tag{1}$$

where x is a measure quantity

In case of LMS system the number of logs of each user is a one of the measure of his activity, therefore in the chapter the dynamics of changes in time of logs to LMS have been analyzed. The identification of behavior of nonlinear dynamic systems must be done step by step. The process of analysis cannot be automated and subsequent steps of analysis require the detailed interpretation of calculation results (Awrejcewicz et al., 2003). The following phases of analysis have been presented in the chapter:

- The frequency analysis (Fourier and wavelet analysis),
- Long range dependence analysis,
- Attractor reconstruction,
- Recurrence plot analysis,
- Calculation of attractor dimension and larger Lyapunov exponent.

Because e-learning system focus activities of many users therefore the problem of synchronization of users and system can be analyzed (Mosdorf et al., 2007). In e-learning system the synchronization between students and teachers can be treated as a measure of effectiveness of education process. The synchronization analysis allows us to better understand the correlation between activities of different groups of e-learning system users. The third part of the chapter will contain the analysis of synchronization between e-learning system users.

2. The nonlinear analysis of e-learning system users

2.1 Data collection and preliminary statistical analysis

The university LMS is used by different groups of users like:

- students,
- teachers,
- administrative workers,
- users who search for information about the university.

For the evaluation of the activity of e-learning system we need to know the dynamic characteristic of different kinds of system users' behaviours. We would like to answer the question which methods are useful to identify the behaviours of different groups of users.

In academic year 2002/2003 the Internet LMS was applied in University of Finance and Management in Bialystok in order to support the management of university administration and the learning process. The system is integrated with the university website and consists of such modules like: E-student, Candidate, Teacher and Dean's Office (Daniewicz et al., 2005a; Daniewicz et al., 2005b). Teachers use their module to input syllabuses, marks, reports and materials for students. The E-students module is integrated with Moodle e-learning platform. At present the LMS system contains registered accounts of about 6 500 students, 480 teachers and 30 administrative users. There are also about 1500 syllabuses in the system.

In the chapter the analysis of three data sets has been presented.

- The first set is the time series of number of requests of LMS system per day.
- The second one is the time series of number of requests of LMS system per every two hours.
- The third one is the time series of number of requests of Moodle e-learning platform per hour.

Preliminary statistical analysis allows us to find out the similarity between the analyzed system and the stochastic system as well as the correlation between the samples. Usually the analysis starts from the calculation of the basic statistical coefficients and comparing the obtained characteristics with standard distribution (Gajek et al., 2000; Awrejcewicz et al., 2003).

2.1.1 Set number one

The analyzed data comes from two sources: logs into the Recto system (the system database) and logs into the web server, which were processed by the program Analog 5.91beta1 (Ignatowska et al., 2005). In Figure1 the logs time series have been presented.

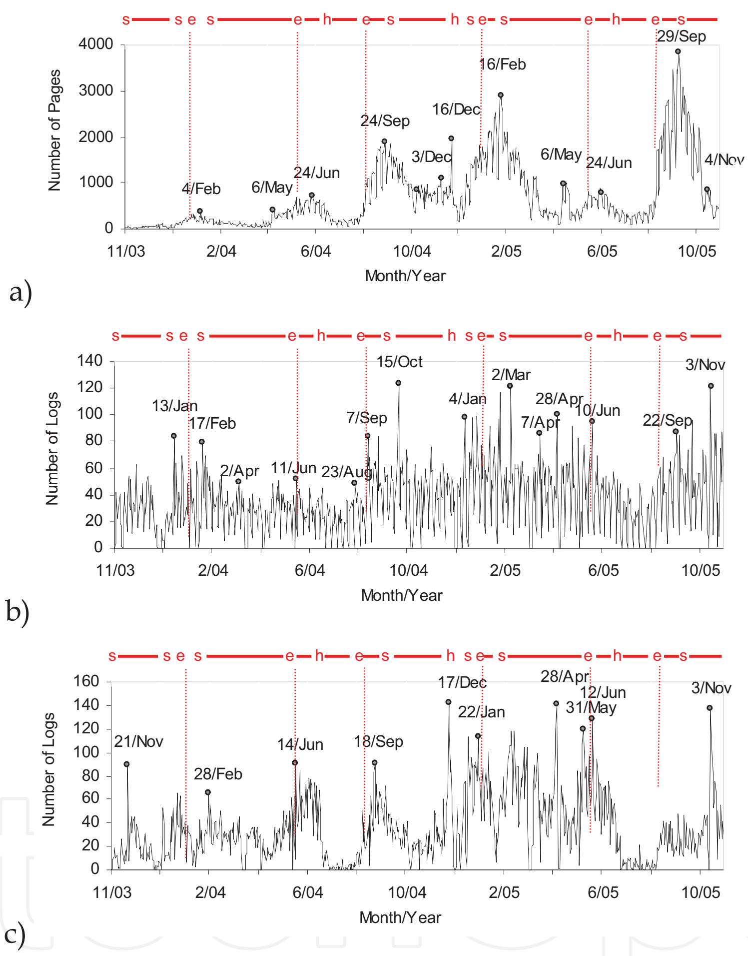


Fig. 1. The logs into: www pages, administration module and teacher module of Recto system: a) logs into the web server; processed by the program Analog 5.91beta1 (Mosdorf et al., 2006), b) logs of the university administration into the Recto system (database), c) logs of teachers into the Recto system (database), e – exams, h – holidays, s – semester

2.1.2 Set number two

The second set of data is the time series of number of requests of LMS system per every two hours. In Figure 2 the time series of LMS system logs in two hours long sampling interval,

normalized to (0;1) range have been presented. Data has been collected in two intervals (Ignatowska et al., 2008):
– Learning time (7 January 2005 – 13 March 2005) – number of samples: 792,
– Holiday (1 July 2005 – 1 September 2005) – number of samples: 756.

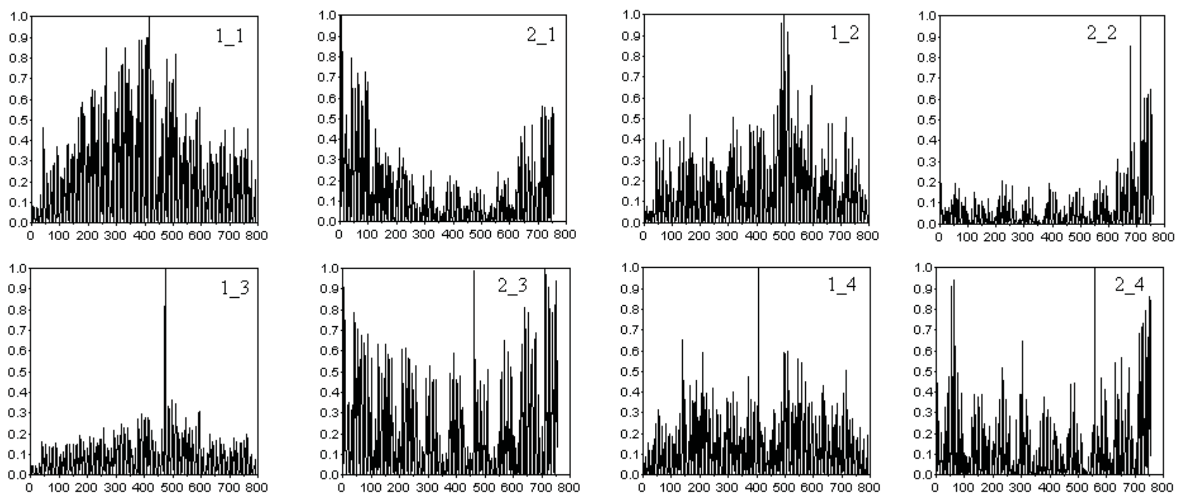


Fig.2. The changes of the number of requests of LMS system www site in two hours long sampling interval normalized to (0;1) period. 1_1. The logs into LMS system in the learning time. 2_1. The logs into LMS system during holidays. 1_2. The logs into Internet university information board in the learning time. 2_2. The logs into Internet university information board during holidays. 1_3. The logs into main university web site in the learning time. 2_3. The logs into main university web site during holidays. 1_4. The logs into download system in the learning time. 2_4. The logs into download system during holidays. (Ignatowska et al., 2008).

2.1.3 Set number three

The E-students module of LMS is integrated with Moodle e-learning platform. The Moodle e-learning platform was implemented in academic year 2008/2009, is used as a virtual learning environment for 10 e-learning courses, attended by approx. 250 students of the university. There are 16 academic teachers who are course designers responsible for the learning process. The courses provided via Moodle platform were designed to be interactive, with virtual support of the learning process by the teachers via forums, chats etc. All of the courses contain not only the learning content (i. e. reading material, films, presentations etc.) but also tests (quizzes), group work environment (such as wiki). Therefore studying via e-learning platform requires regular attendance in the courses, both for the students and teachers.
In Figure 3 the logs into the Moodle platform have been presented. The data from Moodle platform were sampled using 1-hour long interval.

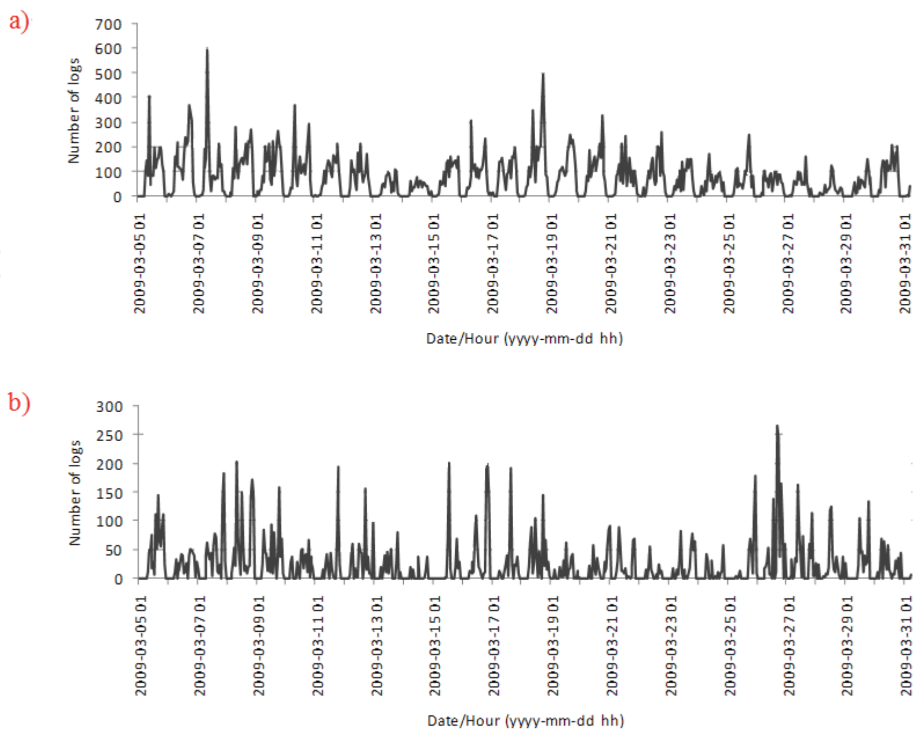


Fig. 3. The activity of Moodle e-learning platform users (1 hour sampling interval): a) logs of students, b) logs of course designers and teachers.

2.2 The frequency analysis

The Fourier transformation allows us to represent time-domain data in the frequency domain. The Fourier power spectrum answers the question which frequencies contain the signal power. The answer has the form of a distribution of power values as a function of frequency. In the frequency domain, this is the square of Fourier transformation magnitude. The power spectrum can be computed for the entire signal at once or for segments of the time signal. For the measurement data in the form of discrete series $\{x_n\}$ the Fourier transformation has a following form: (Gajek et al., 2000)

$$F_k = \sum_{n=0}^{N-1} x_n e^{-j\frac{2\pi}{N}kl} \tag{2}$$

In this case the power spectrum is defined as $|F_k|^2$.

The power spectrums of time series of logs (set number one) are shown in Figure 4. The results of frequency analysis presented in Figure 4 allow us to distinguish limited number of dominant frequencies. Five dominant frequencies have been found out.

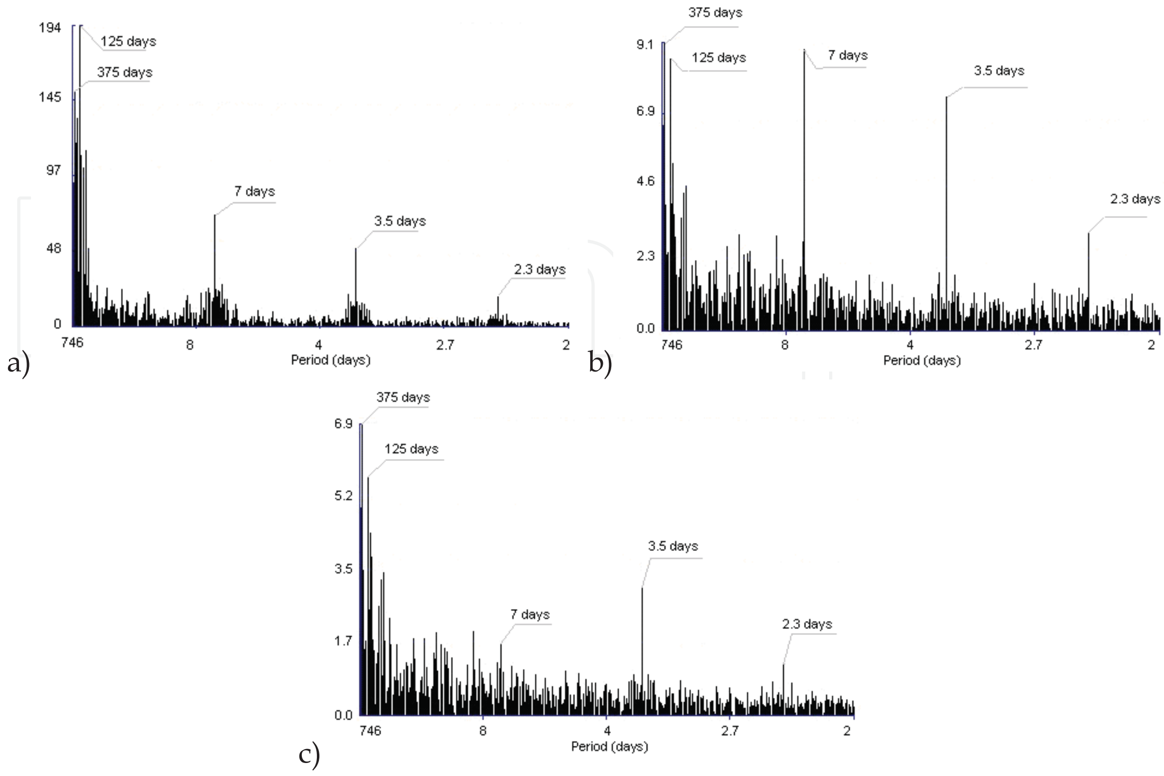


Fig. 4. The power spectrum of LMS system logs time series (set number one): a) logs into the web server, b) logs of the university administration, c) logs of teachers. (Mosdorf et al., 2006).

The example of changes of number of logs into www pages has been shown in Figure 5, where the observed periods changes of the number of visited web pages have been schematically shown.

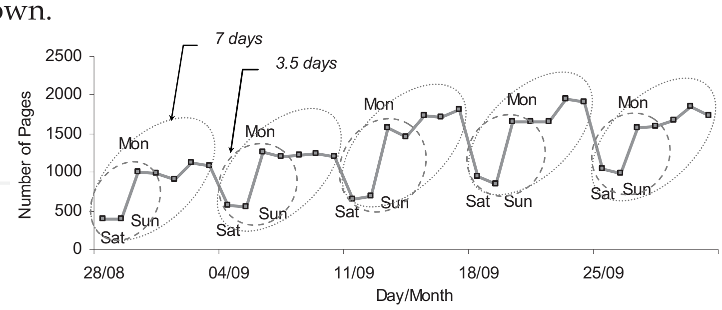
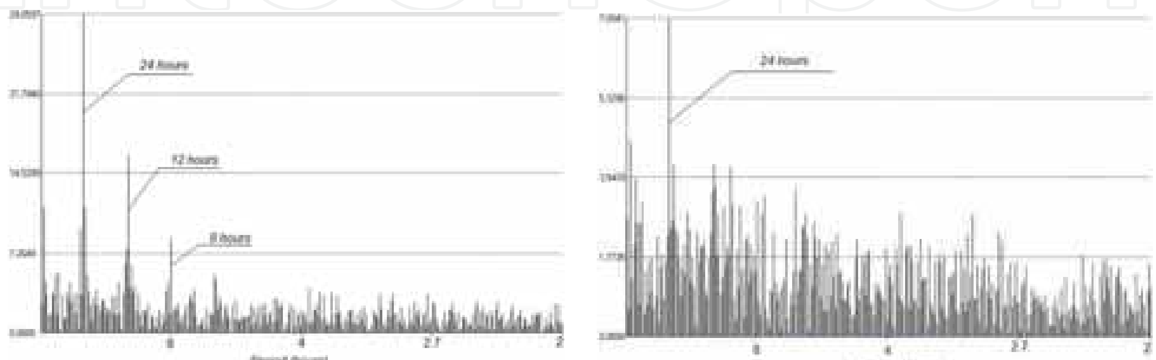


Fig. 5. The examples of periodic changes of daily www logs (Recto system) (Mosdorf et al., 2006).

In all data series we can identify two dominant low frequencies connected with time periods equal to 375 and 125 days. It seems that these changes are connected with yearly and semestrally activity of system users. The rest of identified dominate frequencies with time periods equal to 7, 3.5, 2.3 days appear in analyzed data series with different intensity. These frequencies can be clearly identified in the data series describing the activity of university administration (Figure 4b). The 7 days cycle seems to correspond with the weekly cycle of administration work, but remain 3.5 and 2.3 days cycles are connected with changing of logs

numbers within the weeks. The examples of such changes are shown in Figure 5 where the decrease of number of logs within subsequent weekend can be observed. The changes characterized by time periods equal to 7, 3.5 and 2.3 days are clearly visible also within logs into the www pages (Figure 4a), whereas the 7 days cycle disappears in teachers' logs data series. That can be driven by the fact that university teachers don't work as regular as administration. A lot of them have lectures on Sunday, Saturday, every fortnight or once a month. The teachers like to cumulate their hours and come to have some time to carry out their research, prepare themselves to lectures and so on.
In Figure 6 the Fourier power spectrum of data from set number three has been presented.



a) b)

Fig. 6. Fourier power spectrum for Moodle e-learning system users' activity time series:
a) logs of students, b) logs of teachers

The analysis of activity of the users (1-hour sampling interval) allows us to identify the dominant frequencies that stand for 24, 12 and 8 hours long cycles. The existence of observed cycles can be described by the normal daily routine of users. Teachers usually visit platform at least once a day at their own specified time – to check or update the courses. Students, who have more activities to fulfill usually visit platform more than once a day.

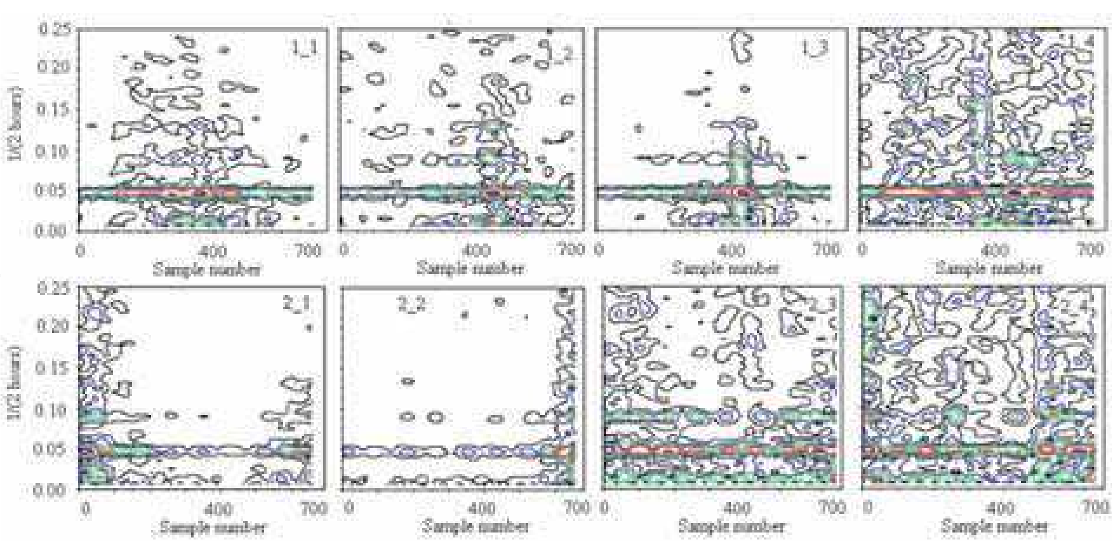


Fig. 7. The spectrograms of the number of requests of LMS system www site in two hours long sampling interval. The ordering of the pictures is described in Figure2. (Ignatowska et al., 2008).

In Figure 7 the spectrograms of the number of requests of LMS system www site in two hours long sampling interval time series (set number two) have been presented.
As we can see the bands equal for 24, 12 and 8 hours long changes can be identified. The results obtained in the analysis allows us to conclude that all of the times series being investigated have various characteristics: the way the frequencies appear and disappear is different for each dataset. In case of the logs to LMS system during the semester there are many different frequencies visible, but during the holiday period only the dominant cycle of 24 hours can be observed. The similar result can be seen when looking on the frequency of the university web information board. In other words it can be concluded that during the summer vacations period the users log into Recto system, as well as read the information from the University information board in 24 hours long cycle, whereas during the teaching term the cycles of 8 and 12 hours become important. In case of the main university web-site the summer vacation period seems to be more noisy than the teaching period, where the cycles of 24, 12 and 8 hours are clearly visible. The data obtained from the files download module of the LMS system seem to have the same complicated structure both for the teaching and vacation period.
The Fourier power spectrum do not allow us to identify the frequencies changes in time. This problem can be analyzed using the windowed Fourier transformation but this is an inaccurate method for localizing time–frequency. The wavelet analysis is free from this inaccuracy, it is a tool for analyzing the localized variations of power spectrum within the time series x_n, with equal time spacing δt.
The continuous wavelet transformation of a discrete sequence x_n is defined as the convolution of x_n with a scaled wavelet Ψ (Białosiewicz, 2000; Torrence et al., 1998a; Torrence et al., 1998b):

$$W(t,s) = \sum_{t'=0}^{N-1} x_{t'} \Psi^* \left[\frac{(t'-t)\delta t}{s} \right] \tag{3}$$

where (*) indicates the complex conjugate.

Because the wavelet function $\Psi_o(\eta)$ is in general complex, the wavelet transformation $W(t,s)$ is also complex. The wavelet power spectrum is defined as: $|W(t,s)|^2$. During the analysis the Morlet wavelet has been used as the based wavelet. The Morlet wavelet has a form (Białosiewicz, 2000; Torrence et al., 1998a; Torrence et al., 1998b):

$$\Psi_o(\eta) = \pi^{-1/4} e^{i\omega_o \eta} e^{-\eta^2/2} \tag{4}$$

where: ω_o - nondimensional frequency, in the paper it is equal to 6.

In Eq. 3 the parameter s assigns the frequency whereas the parameter t identifies the time around which the assigned frequencies are investigated. The wavelet power spectrum is presented in the form of three dimensional map, where the horizontal axis shows the values of parameter t, while the vertical axis shows the values of parameter s (frequency). The values of wavelet power spectrum $|W(t,s)|^2$ are presented as an altitude. The wavelet

power spectrum allows us to observe the changes in time of each frequency power (Białosiewicz, 2000; Torrence et al., 1998a).
The wavelet power spectrum of LMS system logs time series is shown in Figure 8. The wavelet analysis allows us to identify the time periods in which we can observe the changes of log numbers with frequency identified in Fourier analysis. The 7 days cycle of changes of number of university administration logs (Figure 8b) is present in the whole analyzed time. The 3.5 days cycle appears in the whole first year but can be identified only in separated periods in second year. It happens close to terms of exams. The wavelet analysis of the teachers' activity (Figure 8c) shows that the appearance of 3.5 days cycle is correlated with the time of exams. Incidentally this cycle appears during the time of semester. The cycle of 7 days is not intensive and appears incidentally which shows the irregular usage of system by teachers. The intensity of appearance of 7 days cycle is similar with the intensity of 14 days cycle, which seems to be connected with the teachers' work during external study. The disappearance of the cycles shorter than 20 days can be observed during the periods of summer students' holidays. In the power spectrum of logs into www pages (Figure 8a) the 7 days cycle is visible too. The intensity of appearance of such cycle increases during the students' examination time and disappears during the summer holiday and during semesters. The increase in intensity of appearance of 7 days cycle is accompanied by the increase in intensity of 3.5 and 2.3 cycles. It seems that this process is connected with administration workers and teachers' activities. Activities of other users cause the disappearance of 3.5 and 2.3 days cycles.

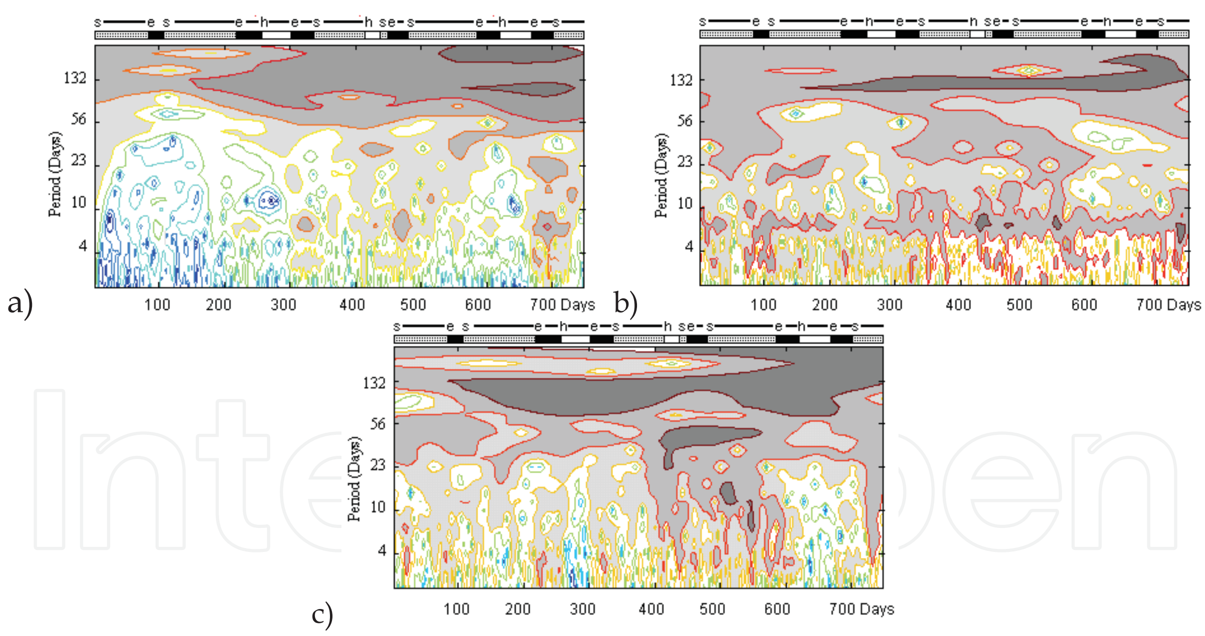


Fig. 8. The wavelet power spectrum of Recto system logs series. a) logs into the web server, b) logs of the university administration, c) logs of teachers, (e – exams, h – holidays, s – semester). The grey area indicates the local maximum (Mosdorf et al., 2006). The calculations have been carried out using computer program published in (Białosiewicz, 2000; Torrence et al., 1998a). (Wavelet software was provided by C. Torrence and G. Compo, and is available at URL: http://paos.colorado.edu/research/wavelets/).

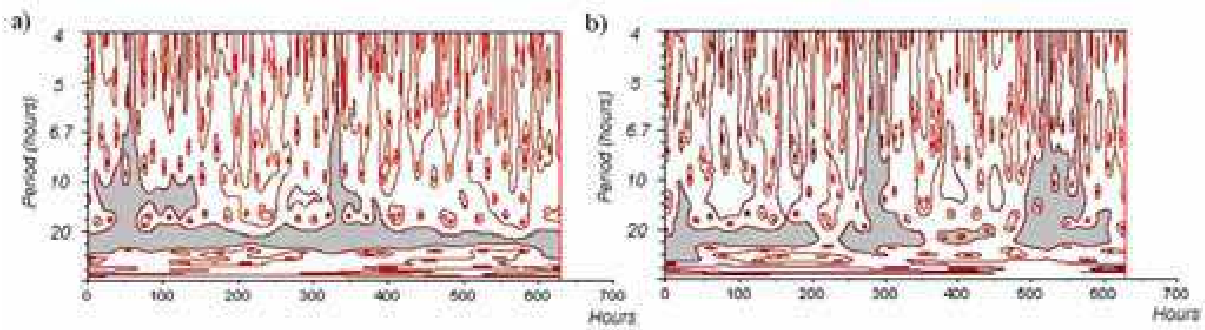


Fig. 9. The wavelet power spectrum of Moodle platform logs series: a) logs of students, b) logs of logs of teachers

The wavelet power spectrum of Moodle platform logs time series (data set number three) is shown in Figure 9. We can conclude that 24-hours long cycle of users activity is present during the analyzed period. When comparing the wavelet power spectrums of both main groups of platform users it can be observed that students seem to work more regularly than the teachers. The 24-hours cycle for the students can be identified in the whole analyzed period, whereas for the teachers it is not as regular (there are some periods where the cycle disappears). The other cycles of shorter length (i. e. 4–12 hours long) can be seen mostly in approx. 300 hours long interval. What is more, when comparing the activity of the students (Figure 9a) to the teachers' logs (Figure 9b) we can see that there is a strict correlation between the appearance of shorter cycles of activities in both groups. The excessive teacher's activity seems to appear at about 24 to 48 hours before the observed higher students' activity. The answer for that phenomena seems to lay in the organizational background of the studies and the e-learning classes schedule as well. Both for students and teachers the excessive activity is observed during (or just before) the weekend classes of extra-mural studies. Such classes are conducted every two weeks, that gives the period of about 300 hours between the accumulation of cycles (see Figure 9a and 9b).

2.3 Long-Range Dependence

A time-series has long-range dependence if it has correlations which persists over all time scales. The long-range dependence is characterized by the parameter H. The Hurst parameter in range $(1/2;\ 1)$ indicates the presence of long-range dependence. There are a number of different statistics which can be used to estimate the Hurst parameter. For time series X_t which has a finite mean (weakly-stationary time series) the autocorrelation function is given by (Clegg et al., 2005; Cajueiro et al., 2005)

$$\rho(k) = \frac{E[(X_t - \mu)(X_{t+k} - \mu)]}{\sigma^2} \tag{5}$$

where $E[X_t]$ is the expectation of X_t, μ is the mean and σ^2 is the variance.

From the formal point of view we can say that the time-series X_t is said to be long range dependent if $\sum_{k=-\infty}^{\infty}\rho(k)$ diverges. For estimation of changes in time the function of $\rho(k)$ the following function is used (Clegg et al., 2005; Cajueiro et al., 2005)

$$\rho(k) \approx C_{\rho} k^{-\alpha} \tag{6}$$

where $C_{\rho} > 0$ and $a = (0;\ 1)$, the symbol ~ mean asymptotically equal to. The parameter a is related to the Hurst parameter (Clegg et al., 2005; Cajueiro et al., 2005)

$$H = (2-\alpha)/2 \tag{7}$$

Similar definition can be formulated in frequency domain. The spectral density $f(\lambda)$ of a function with autocorrelation function $\rho(k)$ and variance σ^2 can be defined as

$$f(\lambda) = \frac{\sigma^2}{2\pi}\sum_{k=-\infty}^{k=\infty}\rho(k)e^{ik\lambda} \tag{8}$$

where λ is the frequency, σ^2 is the variance and $i=\sqrt{-1}$.

The spectral density can be estimated by function (Clegg et al., 2005; Cajueiro et al., 2005)

$$f(\lambda) \approx C_f |\lambda|^{-\beta} \text{ as } \lambda \to 0, \tag{9}$$

for some $C_f > 0$ and some real $\beta \in (0;\ 1)$.
The parameter β is related to the Hurst parameter by (Clegg et al., 2005; Cajueiro et al., 2005)

$$H = (1+\beta)/2 \tag{10}$$

In Figure 10 and the autocorrelation function of the users activity time series (data set number three) obtained from e-learning system is shown. In case of periodic function of users activity (presented function of Moodle students' activity in 1-hour sampling interval, Figure 10a) each sample in the interval of multiplication of τ takes the same (or similar) value. Because of this the local maximum of autocorrelation function seems to appear periodically. In case of not periodic data the increase of τ cause that subsequent values $x_{i+\tau}$ and x_i are not correlated and subsequent elements in the sum (5) will contain the large and small elements. As a result there is a decrease of value of sum (5) in comparison with situation when there are two large elements in the sum. In both cases the $\sum_{k=-\infty}^{k=\infty}\rho(k)$ diverges (Figure 10.). It means that the analyzed data are long range dependent.

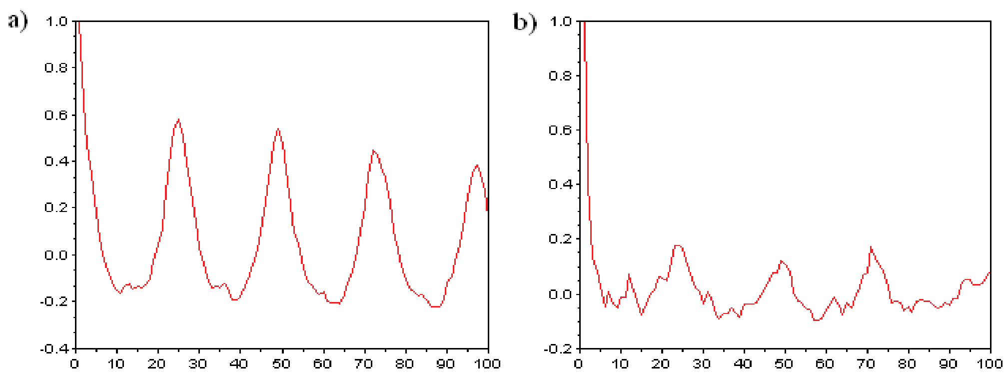


Fig. 10. Autocorrelation function for Moodle e-learning system users' activity: a) logs of students b) logs of teachers

2.3.1 R/S statistic

For the stationary time series $\{x_1, x_2, x_3, \ldots x_\tau\}$ the *R/S* statistic is given by (Cajueiro et al., 2005):

$$\left(\frac{R}{S}\right)_\tau = \frac{1}{s_\tau}\left[\max_{1\le t\le\tau}\sum_{k=1}^{t}(x_k - \bar{x}_\tau) - \min_{1\le t\le\tau}\sum_{k=1}^{t}(x_k - \bar{x}_\tau)\right] \tag{11}$$

where s_τ is a standard deviation (Cajueiro et al., 2005)

$$s_\tau = \left[\frac{1}{\tau}\sum_{t=1}^{\tau}(x_\tau - \bar{x}_\tau)^2\right]^{1/2} \tag{12}$$

and $\bar{x}_\tau$ denote the mean value.

Hurst found that the rescaled range *R/S*, for many records in time is very well described by the following empirical relation (Cajueiro et al., 2005)

$$\left(\frac{R}{S}\right)_\tau = \left(\frac{\tau}{2}\right)^H \tag{13}$$

By means of the *R/S* analysis, the Hurst exponent may be evaluated by plotting the data $(R/S)_\tau$ versus τ in a log–log plot and measuring the slope of the straight line. The slope of tangent to 1n(R/S) in the function 1n(N) gives the value of H exponent.

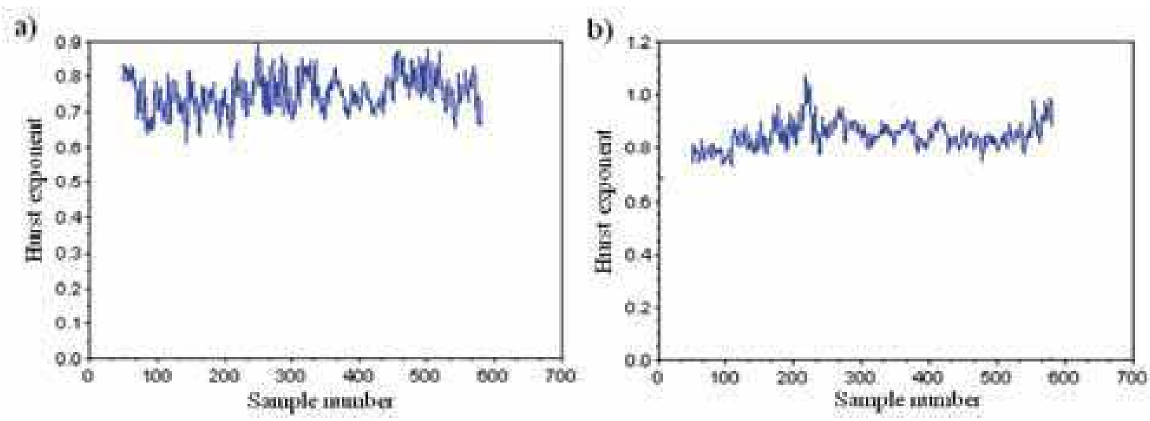


Fig. 11. Hurst exponent estimation for Moodle platform users' activity data using R/S method: a) logs of students, b) logs of teachers

In Figure 11 the estimation of Hurst exponent using R/S method for the data obtained from Moodle platform (data set number three) have been presented. During the analyzed period the change of H value can be observed. In most cases the H value varies from 0.6 to 0.8 and never takes the value of 0.5. It indicates that the Moodle system users' behaviour is not random. It is rather closer to deterministic chaotic process with long-range memory effects.

2.4 Non linear analysis

2.4.1 Attractor reconstruction

The trajectories of the chaotic system in the phase space do not form any single geometrical object such as circle or torus, but form objects called strange attractors of the structure resembling the one of a fractal (Schuster, 1993). Non linear analysis starts from attractor reconstruction. Reconstruction of attractor in certain embedding dimension has been carried out using the stroboscope coordination. In this method subsequent coordinates of attractor points are calculated basing on the subsequent samples distant of time delay τ. The time delay is multiplication of time between the samples. For the measured data in the form of time series: $\{x_n\} = \{x_1, x_2, ..., x_n\}$ the way of calculation of subsequent coordinates of points of attractor is shown in Figure12.

Embedding dimension (subsequent coordinates)

Point number	1	2	3	4	5	6	
1	x_1	$x_{1+\tau}$	$x_{1+2\tau}$	$x_{1+3\tau}$	$x_{1+4\tau}$	$x_{1+5\tau}$	...
2	x_2	$x_{2+\tau}$	$x_{2+2\tau}$	$x_{2+3\tau}$	$x_{2+4\tau}$	$x_{2+5\tau}$	...
3	x_3	$x_{3+\tau}$	$x_{3+2\tau}$	$x_{3+3\tau}$	$x_{3+4\tau}$	$x_{3+5\tau}$	...
4	x_4	$x_{4+\tau}$	$x_{4+2\tau}$	$x_{4+3\tau}$	$x_{4+4\tau}$	$x_{4+5\tau}$	...
5	x_5	$x_{5+\tau}$	$x_{5+2\tau}$	$x_{5+3\tau}$	$x_{5+4\tau}$	$x_{5+5\tau}$	...
6	x_6	$x_{6+\tau}$	$x_{6+2\tau}$	$x_{6+3\tau}$	$x_{6+4\tau}$	$x_{6+5\tau}$	...
...	...	...	...	...	...	...	...

Fig. 12. Time delay algorithm of calculation of attractor coordinates.

The image of the attractor in n-dimensional space depends upon time-delay τ. When the time-delay is too small, the attractor gets flattened, that makes further analysis of its structure impossible. The selection of time-delay value is of great significance in the analysis of the attractor properties. Therefore the analysis of the experimental data is initiated by determining the time-delay. For that purpose the autocorrelation function is calculated. Autocorrelation function allows identification of correlation between the subsequent samples. In case of chaotic data the value of autocorrelation function rapidly decrease when τ increase. Value of the time-delay τ is determined from the condition $C(\tau) \approx 0.5 * C(0)$ (Schuster, 1993).
Attractor reconstruction from data set number two are presented in Figure 13. In Figure 14 the reconstruction of the attractor from www logs series has been presented. In Figure 15 the reconstruction of the attractor from Moodle logs time series has been presented.
The method of calculation of the mutual information can be also used to determine proper delay coordinates for reconstructed attractors. In this case two time series are considered, $X(t)$ and $X(t+\tau)$. As τ is increased, $I(\tau)$ decreases, then usually rises again. τ for which I obtain the first minimum is a proper value of τ.

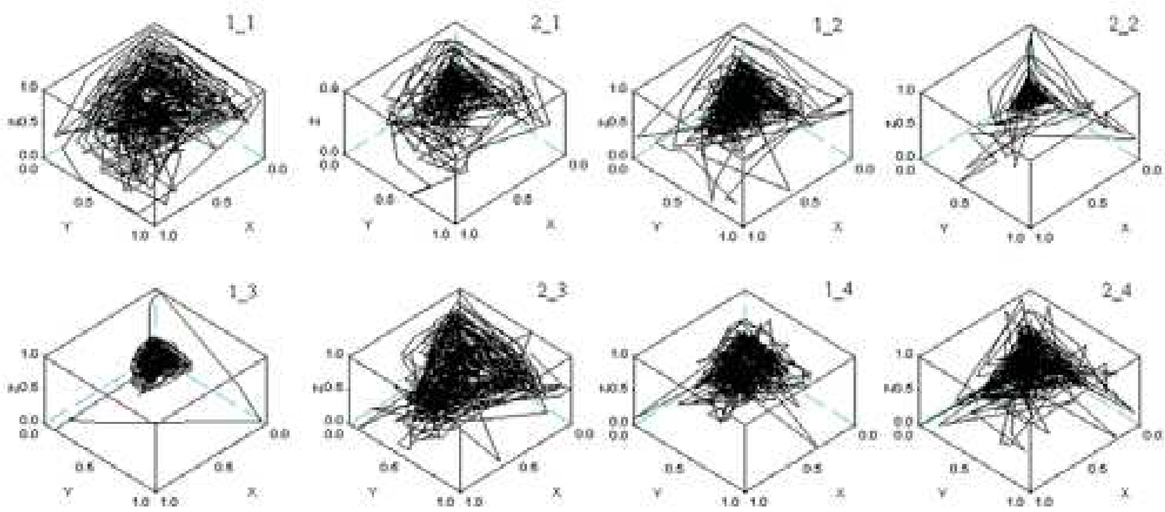


Fig. 13. Attractor reconstruction from data set number two. The ordering of the pictures is described in Figure 2. (Ignatowska et al., 2008)

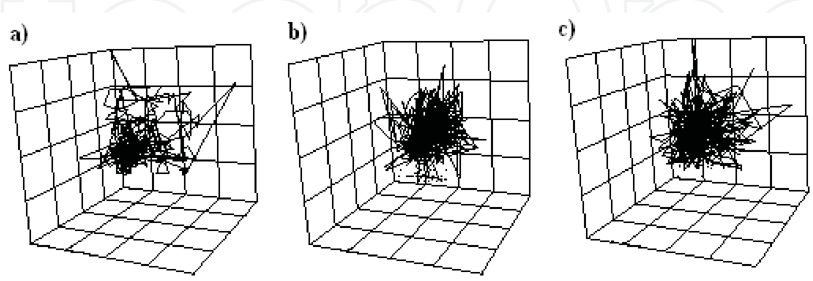


Fig. 14. Attractor reconstruction from time series of data set number one: a) the students, b) the university administration, c) the teachers; for time delay equal to 7 days. (Mosdorf et al., 2007)

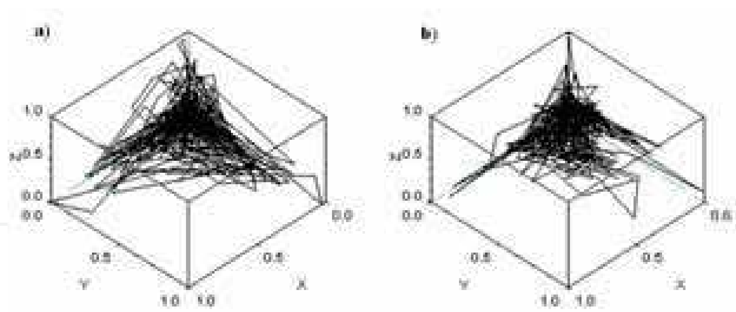


Fig. 15. Attractor reconstruction from data set number three. Moodle platform users' activity. a) logs of teachers, b) logs of students

The mutual information of two discrete random variables X and Y can be defined as (Marwan et al., 2007):

$$I(X,Y)=\sum_{y\in Y}\sum_{x\in X}p(x,y)\log\left(\frac{p(x,y)}{p_1(x)p_2(y)}\right) \tag{14}$$

where $p(x,y)$ is the joint probability distribution function of X and Y, and $p_1(x)$ and $p_2(y)$ are the marginal probability distribution functions of X and Y.
The mutual information is nonnegative, is equal to zero if X and Y are independent random variables.
The examples of mutual information calculated for data from the third data set are presented in Figure 16.

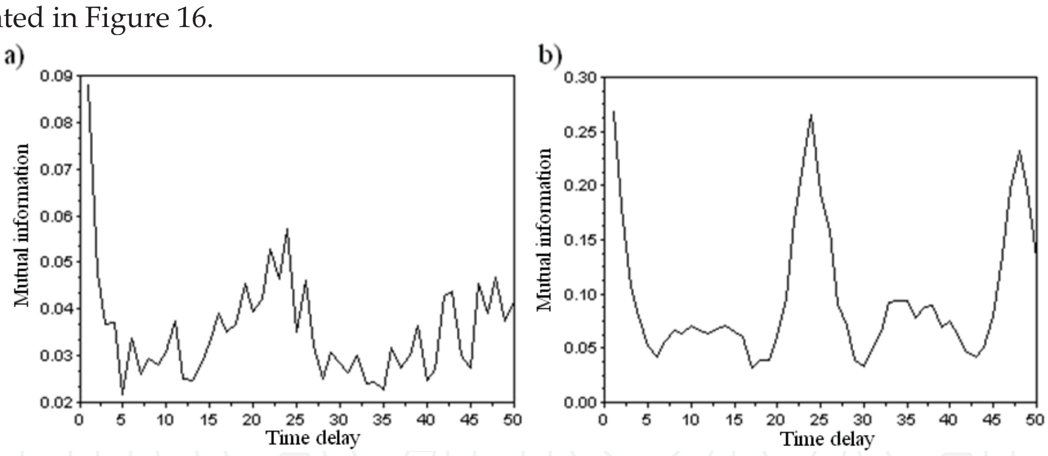


Fig. 16. The mutual information versus time delay for Moodle e-learning system users' activity time series (data set three): a) logs of students, b) logs of teachers

In Figure 17 the mutual information calculated for data from data set has been shown.

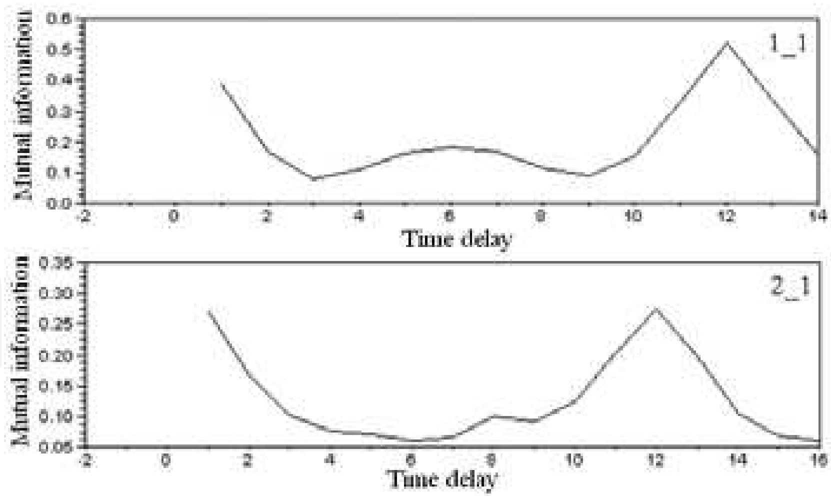


Fig. 17. The mutual information versus time delay for the number of requests of LMS system www site in two hours long sampling interval. The ordering of the pictures is described in Figure2.

The alternative method of time-delay τ calculation is used for determining of Hurst exponent. The slope of tangent to ln(*R/S*) in the function ln(*N*) gives the value of *H* exponent. If *N* number contains too many measured points the process resembles the random motion (the long-term memory - the memory between succeeding intervals disappears). In this case the slope of a curve changes. For the stochastic signals *H*=0.5. Border point *N** between area where the *H* > 0.5 and area where *H* = 0.5 corresponds with the boundary of the natural period of a physical system. *N** quantity enables determining of time-delay τ necessary for attractor reconstruction; τ is calculated from the relation: $\tau = N^{*}/d$.

2.4.2 Largest Lyapunov exponent and correlation dimension

The important characteristic of attractor is the largest Lyapunov exponent. In this case on the attractor immersed in *D* dimensional space two points have been selected. The distance between these two points $L(t_j)$ is at least one orbiting period. After the passage of some evolution time the distance of the selected points has been calculated again and denoted as $L(t_{j+1})$. The largest Lyapunov exponent has been calculated according to formula (Schuster, 1993; Wolf et al., 1985):

$$\lambda_1 = \frac{1}{t} \sum_{j=1}^{m} \log_2 \frac{L(t_{j+1})}{L(t_j)} \tag{15}$$

where: *m* - number of point pairs examined, *t* - time of evolution

The largest Lyapunov exponent allows us to calculate time period (*1/L*) of long time memory in the system in which the process of stability loss occurs. The largest Lyapunov exponent can be determined when such characteristics of attractor as fractal dimension, average orbiting time and time-delay are known. For a long time series, the results of calculation of λ_1 approach stable value, being an estimation of the largest value of Lyapunov

exponent. At least 10^n measuring points and 10^{n-1} orbiting periods are required to determine the largest Lyapunov exponent. The calculation of the largest Lyapunov exponent is possible only if the fractal dimension of the attractor is known. To determine the fractal dimension of the attractor, value of time-delay (the quantity necessary for the reconstruction of the attractor) is required (Wolf et al., 1985).
One of the essential characteristics of fractals is their dimension. For experimental data correlation dimension D_2 based on the Grassberger – Procaccia method is calculated with using the following method (Schuster, 1993; Grassberger et al., 1985; Pawelzik et al., 1987; Parker et al., 1987):

$$D_2 = \lim_{l \to 0} \frac{1}{\ln r} \ln \sum_i p_i^2 \tag{16}$$

where:

$$\sum_i p_i^2 \approx \lim_{N \to \infty} \frac{1}{N^2} \sum_{i,j} \Theta\left(r - \left|\vec{x}_i - \vec{x}_j\right|\right) = C_2(r) \tag{17}$$

$\Theta\ (x)$ Heaviside's step function.

C_2 correlation integral is a measure of probability of finding the two points spacing r. During the calculation neighborhood of all points is examined. This algorithm can be accelerated in this case the neighborhood only the certain randomly selected points has been considered. In this case the correlation integral has a form (Schuster, 1993; Parker et al., 1987):

$$C_2(r) = \lim_{N \to \infty} \frac{1}{N_{ref}} \sum_{j=1}^{N_{ref}} \frac{1}{N} \sum_{i=1}^{N} \Theta\left(r - \left|\vec{x}_i - \vec{x}_j\right|\right) \tag{18}$$

Such defined correlation integral fulfill the following formula (Schuster, 1993):

$$C_2(r) \approx r^{D_2} \tag{19}$$

where: D_2 correlation integral.

Equation 19 can be explained as follows: probability of finding the points of attractor in cubic contains the whole attractor is equal 1 and probability of finding the points of attractor in cubic of size r in D_2 dimension space is proportional to cubic volume.
Using the log function to both size of Equation 19 we can obtain:

$$\log[C(r)] \approx \log\left(r^{D_2}\right) = D_2 \log(r) \tag{20}$$

Than slope of regression line thought linear part of curve $\log[C(r)]$ against $\log(r)$ define the value of correlation dimension of D_2. Correlation dimension is lower border of Hausdorff dimension (Schuster, 1993; Pawelzik et al., 1987).

In Table 1 the results of calculations of correlation dimension and the largest Lyapunov exponent for data set number two have been presented. Obtained results show that visiting the main University internet site during the holidays is more complex phenomenon than visiting it during the period of learning time. The result is consistent with the one obtained from the frequency analysis. The schemas of using downloading file system seem to be the same during holidays and during the time of learning.

The logs into:	Correlation dimension		Largest Lyapunov exponent [1/(2 hours)]	
	Learning time	Holiday	Learning time	Holiday
LMS system	3	2.7	0.15	0.12
Internet university information board	3.2	2.8	0.13	0.08
Main university web site	2.59	33	0.09	0.13
Download system	3.76	3.59	0.14	0.16

Table 1. Correlation dimension and the largest Lyapunov exponent for data from data set number two (Mosdorf et al., 2007)

2.4.3 Example of nonlinear analysis

For identification of dynamic properties of different group of LMS users the nonlinear methods of data analysis have been used. The data presented in Figure 1 have been analysed.

The identification of nature (deterministic chaos, periodic or stochastic) of analyzed data can be estimated with using the Hurst exponent. For the signals of stochastic character H=0.5. The border point N^* between area, where the H>0.5 and area, where H=0.5 corresponds with the boundary of the natural period of analyzed system.

For experimental data correlation dimension D_2 based on the Grassberger – Procaccia method (Fermat et al., 1999) and largest Lyapunov exponent based on the Wolf (Schuster, 1993) algorithm can be calculated. Correlation dimension is the measure of number of degree of freedom of the system but the largest Lyapunov exponent (L) identifies the nature of data. A positive value of largest Lyapunov exponent indicates that in the considered system the deterministic chaos appears. The time of stability loss in the system can be estimated by value of $1/L$.

In the Table 2 the results of non-linear analysis have been presented. The identification of correlation between the time series of e-learning server requests can be calculated with using the correlation coefficient (C). When $|C|$ is close to 1, the analyzed time series are correlated. When the large and low values in both series appear at the same time, then C>0; but when large values in first series meet low values in other series, then C<0. When C is close to zero, then the time series are not correlated.

	Pages	Teachers	Administration	Unit
Time of autocorrelation disappearance	35	47	31	days
Hurst Exponent	0.93	0.812	0.839	
Border point N^*	140	130	130	days
Correlation Dimension	2.25	5.25	5.5	
Largest Lyapunov Exponent	0.105	0.0685	0.0698	1 bit/days
Time of stability loss	9.5	14.6	14.3	days

Table 2. Results of nonlinear analysis (Mosdorf et al., 2006)

Results of calculation (presented in the Table 2) of time periods, in which the autocorrelation disappears, show that the shortest period appears in time series of administration logs and is equal to 31 days. This value corresponds with monthly cycle of Dean Office working time. We can observe the similar length of time period of autocorrelation disappearance in the time series of www logs. In this case the time period is equal to 35 days. The largest time period appears in time series of teachers' logs and is equal to 47 days. The obtained values of Hurst exponent and largest Lyapunov exponent show that all analyzed series have the character of deterministic chaos. The largest value of Hurst exponent has been obtained for time series of www logs. The natural periods of all analyzed series obtained from R/S analysis are similar to semester.

The correlation dimension obtained for attractors reconstructed from analyzed time series shows that the attractors reconstructed from administration workers and teachers' logs have the most complex structure. In case of www logs the obtained correlation dimension equal to 2.25 suggests that changes of number of www logs can be modelled by low dimensional model.

The analyses of largest Lyapunov exponent allow us to estimate the time interval in which the system remains stable. In the result of analyses we may conclude that changes of www logs are the most unstable process. In this case the number of www logs can be predicted within one week. In case of administration workers and teachers' logs the number of logs can be predicted within two weeks.

2.5 Recurrence plot

Recurrence plot (RP) visualize the recurrence of states x_i in a phase space. The RP enables us to investigate the recurrence of state in m-dimensional phase. The recurrence of a state at time *i* at a different time *j* is marked within black dots in the plot, where both axes are time axes. From the formal point of view the RP can be expressed as (Marwan et al., 2007):

$$R_{i,j} = \Theta\left(\varepsilon_i - \left\|x_i - x_j\right\|\right), x \in \Re^m, i, j - 1 \ldots N \tag{21}$$

where N is the number of considered states x_i, ε_i is a threshold distance, $||\cdot||$ a norm and $\Theta(\cdot)$ the Heaviside function.

Homogeneous RPs are typical of stationary systems in which relaxation times are short in comparison with the time of system investigation. Oscillating systems have RPs with diagonal oriented, periodic recurrent structures. For quasi-periodic systems, the distances between the diagonal lines are different. The drift is caused by systems with slowly varying parameters which cause changes of brightens the RP's upper-left and lower-right corners. Abrupt changes in the dynamics as well as extreme events cause white areas or bands in the RP.

Single, isolated recurrence points can occur if states are rare, if they do not persist for any time or if they fluctuate heavily. However, they are not a unique sign of chance or noise (for example in maps). A diagonal line occurs when a segment of the trajectory runs parallel to another segment. The distance between trajectories is less than ε. The length of this diagonal line is determined by the duration of this phenomenon. A vertical (horizontal) line indicates a time in which a state does not change or changes very slowly.

In Figure 18 the recurrence plots for data from data set number two have been presented.

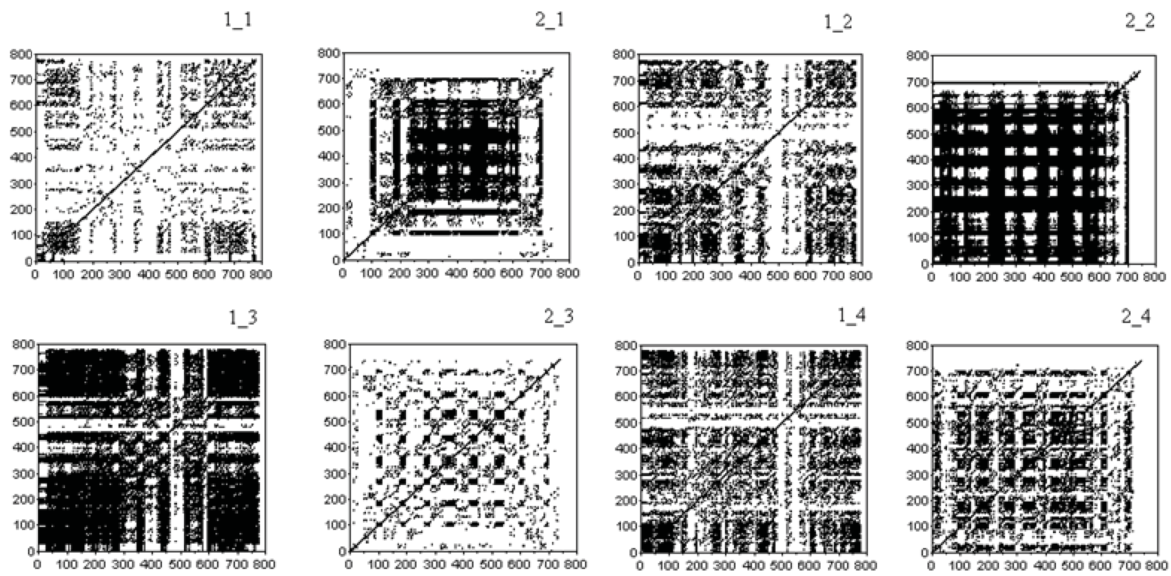


Fig. 18. The recurrence plots for data from data set number two. The ordering of the pictures is described in Figure 2. (Ignatowska et al., 2008)

The obtained results show that visiting the main University internet site during the holidays is more complex phenomenon than visiting it during the period of learning time. The visiting of module E-student (LMS) and Internet university information board is more complex during the learning time in comparison with holiday. The schemas of using downloading file system seem to be the same during holidays and during the time of learning.

In Figure 19 the recurrence plots for data from data set number three have been presented.

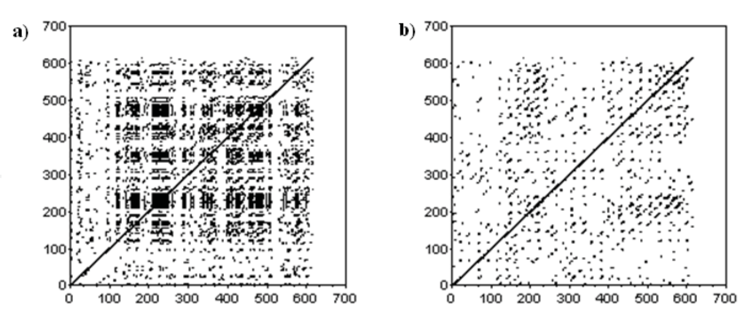


Fig. 19. The recurrence plots for data from data set number three. Moodle platform users' activity: a) logs of teachers, b) logs of students

The results obtained for data set number three show that using the university's Moodle platform by the teachers is more complex phenomenon than the students' activity. What is more, the teachers' behaviour seem to be similar to the main university's web-site users logging schema (Figure 19 a vs. Figure 18 2_3) during the learning time.

3. On frequency synchronization of e-learning web system users

3.1 Introduction

The dynamics of changes in time of logs to: administration module, teacher module and student module has been analyzed. The correlation, Fourier and wavelet analyses have been used to identification of the data nature. Basing on the wavelet analysis it has been proposed the new criterion of evaluation of frequency synchronization. The modified wavelet power spectrum has been used to identify the frequency synchronization of user groups of e-learning system.

The changes in time of daily logs of: administration workers, teachers and students have been analyzed. The data analyzed in the chapter have been presented in Figure 1. It has been observed that the number of students logs significantly increases during the exam sessions and decreases during classes and holidays. The activity of teachers and university administration significantly decreases only during holidays.

The correlation coefficient between time series under consideration has been calculated according to the following formula (Gajek et al., 2000):

$$C_{M,S} = \frac{Cov(x_{i,M}, x_{i,S})}{\sigma_{x_{i,M}} \sigma_{x_{i,S}}} \tag{22}$$

where $x_{i,M}$, $x_{i,S}$ are the time changes of number of logs; $\sigma_{x_{i,M}}$, $\sigma_{x_{i,S}}$ are the standard deviations of time series $x_{i,M}$ and $x_{i,S}$, i is a sample number.

When $|C_i|$ is close to 1, time series $x_{i,M}$, $x_{i,S}$ are correlated. When the large values in both series appear at the same time, then $C_i > 0$; but when large values in first series meet low values in other series, then $C_i < 0$. When C_i is close to zero, then the time series $x_{i,M}$, $x_{i,S}$ are not correlated.

Because the value of correlation coefficient is low therefore we can conclude that data analyzed in the paper is low correlated. It means that more sophisticated methods are necessary to identify the relations between the time series under consideration.

The correlation values between considered time series have been presented in Table 3.

C_i	Students	Administration	Teachers
Students	-	0.20	0.34
Administration	0.20	-	0.31
Teachers	0.34	0.31	-

Table 3. The correlation coefficient between: students, university administration and teachers time series. (Mosdorf et al., 2007)

The power spectrum of time series of logs is shown in Figure 4. The results of frequency analysis presented in Figure 4 allow us to distinguish the limited number of dominant frequencies. The identified dominant frequencies are 3.5 and 7 days for students and administration workers and 3.5 and 30 days for teachers. We can state that in all data series one dominant frequency corresponds to data period equal to 3,5 days. The wavelet power spectrums of logs time series are shown in Figure 8.

3.2 General concept of frequency synchronization analysis

The LMS system records activity of many users: students, teachers and administration workers. Each student using the LMS system can be treated as dynamical system whose activity is caused by LMS system, but system activity results from activity of teachers and university administration. Each of the mentioned users can be treated as dynamical system as well. It means that e-learning system is a virtual place where thousands of dynamical systems (users) communicate each others. These communication leads to changes of users' activities therefore we can say that system users interact each others. Number of logs of each user is a measure of his activity. The analysis of logs of user groups (students, teachers and administration) shows that changes of daily numbers of logs of user groups can be treated as a result of deterministic chaos phenomena (Mosdorf et al., 2007; Ignatowska et al., 2005; Mosdorf et al., 2006). In Figure 14 it has been shown the attractor reconstruction from time series of logs of the students, logs of the university administration and logs of teachers. The obtained results presented in Figure 14 confirm that time series under consideration are created by deterministic chaos phenomena. Therefore we can say that LMS system is a virtual platform of interaction of three chaotic dynamical systems.
In the chaos synchronization the trajectories of synchronized systems become similar in spite of synchronized systems being different. In the process of synchronization two different chaotic systems interact. In the theory of synchronization one of them is called master and the other one – slave. The measure of synchronization is a synchronization error defined by (Fermat et al., 1999; Li-Qun Chen, 2004):

$$x_i = x_{i,M} - x_{i,S} \tag{23}$$

where $x_{i,M}$ describes the time changes of master system and $x_{i,S}$ describes time changes of slave system.

In the paper (Li-Qun Chen, 2004) the following types of synchronization have been defined: complete synchronization, partial synchronization, practical synchronization, exact synchronization, almost synchronization. Two chaotic systems are *exactly synchronized* if the synchronization error exponentially reaches the zero. This implies that at a finite time $x_{i,M} = x_{i,S}$. Two chaotic systems are *practically synchronized* if the synchronization error is close to zero. This implies that for all time $t >= t^*$ the trajectories of the slave system are close to the master trajectories, i.e., $x_{i,M} \approx x_{i,S}$. It is said that two chaotic systems are *completely synchronized* if and only if all states of both master and slave systems are practically or exactly synchronized. Two chaotic systems are *partially synchronized* if, at least, one of the states of the synchronization error system is either practically or exactly synchronized and only if, at least, one of the states of the synchronization error system is neither practically nor exactly synchronized. Two chaotic systems are *almost synchronized* if and only if the both master and slave systems display oscillations with the same phase and different amplitude (Li-Qun Chen, 2004).
One of types of synchronization of chaotic system is the frequency synchronization (Fermat et al., 1999). It has been originally defined for periodic behavior of a dynamical system. Recently, the frequency synchronization between two chaotic systems has been discussed.

The key issue to extend frequency synchronization to chaotic systems is to calculate the frequency of a chaotic system (Fermat et al., 1999).
The wavelet power spectrum allows us to observe changes in time of all frequencies. Therefore we can use the wavelet power spectrum to identify the time correlation between frequencies. For that purpose the following coefficient has been proposed:

$$C_f(s) = \frac{Cov\left[\left|W^M(t,s)\right|^2, \left|W^S(t,s)\right|^2\right]}{\sigma_{\left|W^M\right|^2}\sigma_{\left|W^S\right|^2}} \tag{24}$$

where $|W^M(t,s)|^2$ and $|W^S(t,s)|^2$ are time series of wavelet power for the given value of parameter s for master and slave systems; $\sigma_{\left|W^M\right|^2}$, $\sigma_{\left|W^S\right|^2}$ are the standard deviations of time changes of wavelet power spectrum for the given value of parameter s for master and slave systems.

The coefficient C_f, is a measure of frequency synchronization in interacting systems. When C_f is close to 1, then the considered frequency appears in both time series in the same moment of time. When C_f is close to -1, then the considered frequency appears in one time series and disappears in other time series at the same moment. In Figure 20 it has been shown the C_f changes against time periods.
The wavelet power spectrum shows the changes in time of the dominant frequencies. The dynamical systems interaction occurs only on selected frequencies identified by frequency correlation coefficient (24). To indicate the frequencies that are important for synchronization, each wavelet power spectrum has been multiplied by coefficient $C_f(s)$. Finally we obtain the modified wavelet power spectrum in the following form:

$$\left|W_M(t,s)\right|^2 = C_f(s)\cdot\left|W(t,s)\right|^2 \tag{25}$$

The modified wavelet power spectrum of students' logs as the results of multiplication of it by students-teachers frequency correlation C_f has been shown in Figure 21a. It identifies the frequency and the time periods when the correlation between students and teachers appears.

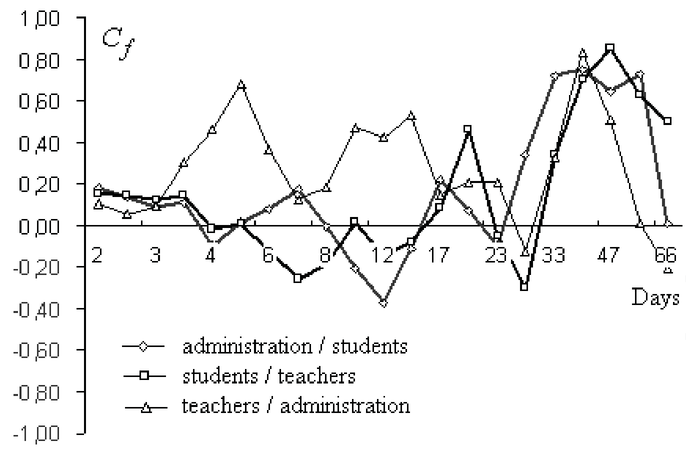


Fig. 20. The coefficient of frequency correlation of time changes of wavelet power spectrum for three user groups

The positive correlation appears for 20 days time period. The negative value of frequency correlation for 12 and 7 days time periods appears in the same time periods – close the exam sessions. Such negative correlation is caused by the fact that in exam sessions the regularity of weekly work disappears for stationary students and in the same time part-time students lost the regularity of 2 weeks work. In other words we can say that students use the system more often near exam sessions and teachers use the system more often near the end of the month.

In Figure 21b the modified wavelet spectrum of administration logs multiplied by administration-students frequency correlation C_f has been shown. Figure 21b has been prepared for selected frequencies of administration activities and it identifies periods when the correlation between administration and students can be observed. The positive correlation appears for time periods of 7 and 12 days. The correlation of 7 days is high, it appears in the time of classes during semesters and exam sessions. This correlation disappears in holidays. The correlation of 12 days appears in the longer periods during classes and it appears before and during exam session in September. The negative correlation between frequency of logins of students and administration has not been observed. The frequency of 7 days is strongly connected with administration activity. The other two groups of system users under consideration do not work in such systematic way.

In Figure 21c the administration logs wavelet spectrum multiplied by administration-teachers C_f has been shown. Figure 21c identifies time periods when the correlation (of 5-7 and 14 days) between administration and teachers appears. The maximum value of correlation of 14 days has been reached in the middle of classes during the semesters and at the end of holidays. The correlation of 7 days has the lowest value during holidays. The minimum value has been observed during summer holidays, which is connected with disappearance of teachers' activity during this time.

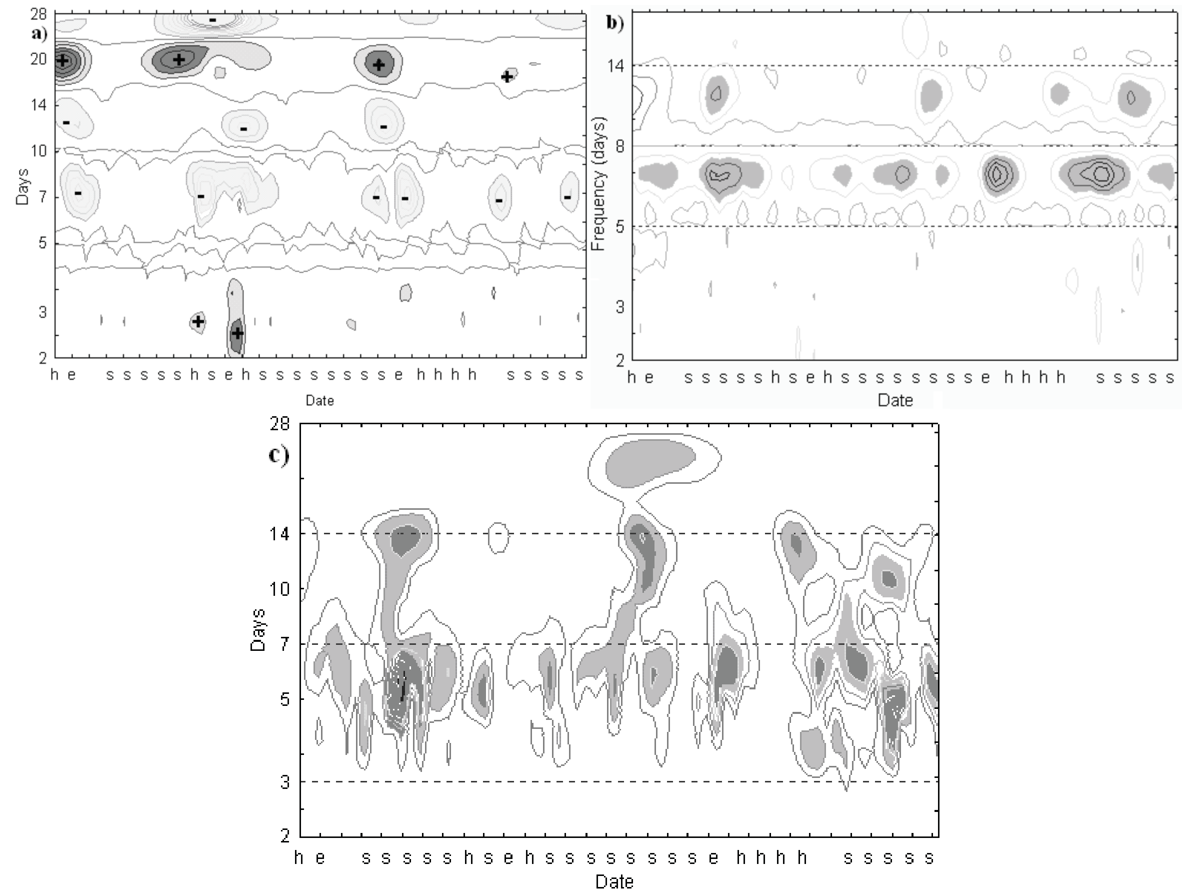


Fig. 21. Modified wavelet spectrum. a) students' logs wavelet spectrum multiplied by students-teachers C_f b) administration' logs wavelet spectrum multiplied by administration-students C_f c) administration logs wavelet spectrum multiplied by administration-teachers C_f .

4. Conclusion

The Learning Management System (LMS) records activity of many users. The behaviours of students, teachers and administration workers usually are nonlinear. Finally the e-learning system is a virtual place where nonlinear users communicate each others. These communication leads to changes of users' activities and LMS properties. Therefore we can say that LMS with all users is a large nonlinear dynamical system. The analysis of dynamical properties of e-learning system can be helpful during the construction and development of Learning Management System and its evaluation. Using the nonlinear methods can be also helpful in detection of anomalies in e-learning system.

In the chapter some selected data from e-learning system have been analyzed:

- the time series of number of requests of LMS system per day;
- the time series of number of requests of LMS system per every two hours;
- the time series of number of requests of Moodle e-learning platform per hour.

Due to finding of many nonstationary behaviours of e-learning systems the obtained results have a preliminary character.

The results of analysis presented in the chapter show that:

– The obtained values of Hurst exponent and largest Lyapunov exponent prove that all analyzed series have the character of deterministic chaos.
– The attractor reconstruction technique confirms that time series under consideration are created by deterministic chaos phenomena.
– The Fourier analysis allows us to identify the dominant frequencies of the system.
– It has been found out that wavelet analysis can be useful method for identification of the e-learning activity.
– We came to conclusion that e-learning process generates the sophisticated chaotic attractor from changes of number of logs to the web server.
– The modified wavelet power spectrum analysis allows us to identify the changes in time of correlations between different user groups.

Analyses carried out in the chapter have shown that non-linear methods are useful in analysing e-learning system dynamics. The modified wavelet power spectrum analysis proposed in the chapter 3 allows us to better understand, in comparison with using the classical wavelet power spectrum, the correlation between different groups of web system users. This kind of analyses identifies changes in time of correlations between different user groups.

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E-Learning is a vast and complex research topic that poses many challenges in every aspect: educational and pedagogical strategies and techniques and the tools for achieving them; usability, accessibility and user interface design; knowledge sharing and collaborative environments; technologies, architectures, and protocols; user activity monitoring, assessment and evaluation; experiences, case studies and more. This book's authors come from all over the world; their ideas, studies, findings and experiences are a valuable contribution to enriching our knowledge in the field of eLearning. The book is divided into three sections. The first covers architectures and environments for eLearning, while the second part presents research on user interaction and technologies for building usable eLearning environments, which are the basis for realizing educational and pedagogical aims, and the final last part illustrates applications, laboratories, and experiences.

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Phone: +385 (51) 770 447
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InTech China
Unit 405, Office Block, Hotel Equatorial Shanghai
No.65, Yan An Road (West), Shanghai, 200040, China
中国上海市延安西路65号上海国际贵都大饭店办公楼405单元
Phone: +86-21-62489820
Fax: +86-21-62489821