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A Model of Placing Liaisons in the Two Levels of an Organization Structure of a Complete Binary Tree Minimizing Total Path Length

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1. Introduction

A pyramid organization (Takahashi, 1988) is a formal organization structure which is a hierarchical structure based on the principle of unity of command (Koontz, 1980) that every member except the top in the organization should have a single immediate superior. There exist relations only between each superior and his direct subordinates in the pyramid organization. However, it is desirable to have formed additional relations other than that between each superior and his direct subordinates in advance in case they need communication with other departments in the organization.

The pyramid organization structure can be expressed as a rooted tree, if we let nodes and edges in the rooted tree correspond to members and relations between members in the organization respectively. Then the pyramid organization structure is characterized by the number of subordinates of each member, that is, the number of children of each node and the number of levels in the organization, that is, the height of the rooted tree (Robbins, 2003; Takahara & Mesarovic, 2003). Moreover, the path between a pair of nodes in the rooted tree is equivalent to the route of communication of information between a pair of members in the organization, and adding edges to the rooted tree is equivalent to forming additional relations other than that between each superior and his direct subordinates.

We have proposed some models (Sawada, 2006; Sawada, 2009) of adding relations between members in a pyramid organization structure such that the communication of information between every member in the organization becomes the most efficient. For each model we have obtained a set of additional edges to a complete binary tree minimizing the sum of lengths of shortest paths between every pair of all nodes.

Liaisons (Gittell, 2000; Lievens & Moenaert, 2000) which have roles of coordinating different sections are also placed as a means to become effective in communication of information in an organization. However, it has not been theoretically discussed which members of an organization should form relations to the liaisons.

We have obtained an optimal set for each of the following two models of placing a liaison which forms relations to members of the same level in a pyramid organization structure which is a complete binary tree of height H : (i) a model of adding a node of liaison which gets adjacent to two nodes with the same depth (Sawada, 2008) and (ii) a model of adding a

node of liaison which gets adjacent to all nodes with the same depth (Sawada, 2007). A complete binary tree is a rooted tree in which all leaves have the same depth and all internal nodes have two children (Cormen et al., 2001). The depth of a node is the number of edges from the root to the node. Figure 1 shows an example of a complete binary tree of $H=5$. In Fig. 1 the value of N expresses the depth of each node.

Fig. 1. An example of a complete binary tree of $H=5$

The above model (ii) corresponds to the formation of additional relations between a liaison and all members in the same level. This model gives us an optimal level when we add relations to the liaison in one level of the organization structure which is a complete binary tree of height H , but this model cannot be applied to placing liaisons in two or more levels. This chapter expands the above model (ii) into the model of placing liaisons in two levels of the organization structure, which is that of adding two nodes of liaisons which get adjacent to all nodes at each depth of two depths in a complete binary tree of height H ($H = 3, 4, \dots$).

If $l_{i,j}$ ($= l_{j,i}$) denotes the path length, which is the number of edges in the shortest path from a node v_i to a node v_j ($i, j = 1, 2, \dots, 2^{H+1}-1$) in the complete binary tree of height H , then $\sum_{i < j} l_{i,j}$ is the total path length. Furthermore, if $l'_{i,j}$ denotes the path length from v_i to v_j after getting adjacent in the above model, $l_{i,j} - l'_{i,j}$ is called the shortening path length between v_i and v_j , and $\sum_{i < j} (l_{i,j} - l'_{i,j})$ is called the total shortening path length.

In Section 2 we formulate the total shortening path length when one node of liaison gets adjacent to all nodes with the same depth M ($M = 2, 3, \dots, H-1$) and the other node of liaison gets adjacent to all nodes with the same depth N ($N = M+1, M+2, \dots, H$) which is greater than M . In Section 3 we obtain an optimal pair of depth $(M, N)^*$ which maximizes the total shortening path length.

2. Formulation of Total Shortening Path Length

From formulation of shortening path length of adding a node of liaison which gets adjacent to all nodes of the same depth shown in Subsection 2.1, we formulate the total shortening path length when two nodes of liaisons which get adjacent to all nodes of each depth of two depths are added to a complete binary tree in Subsection 2.2. Since we don't consider efficiency of communication of information between the liaisons and the other members, the

total shortening path length doesn't include the shortening path length between the nodes of liaisons and nodes in a complete binary tree.

2.1 Placing a Liaison in One Level

This subsection shows formulation of the total shortening path length when one node of liaison gets adjacent to all nodes with the same depth L ($L = 2, 3, \dots, H$) in a complete binary tree of height H ($H = 2, 3, \dots$).

The sum of shortening path lengths between every pair of nodes whose depths are equal to or greater than L is given by

$$\alpha_H(L) = \{W(H-L)\}^2 2^L \sum_{i=1}^{L-1} i 2^i, \quad (1)$$

where $W(h)$ denotes the number of nodes of a complete binary tree of height h ($h=0, 1, 2, \dots$). The sum of shortening path lengths between every pair of nodes whose depths are less than L and those whose depths are equal to or greater than L is given by

$$\beta_H(L) = W(H-L) 2^{L+1} \sum_{i=1}^{L-2} (L-i-1) i 2^i, \quad (2)$$

and the sum of shortening path lengths between every pair of nodes whose depths are less than L is given by

$$\gamma(L) = 2^L \sum_{i=1}^{L-3} \sum_{j=1}^i (i-j+1) j 2^j, \quad (3)$$

where we define $\sum_{i=1}^0 \cdot = 0, \sum_{i=1}^{-1} \cdot = 0$.

From these equations, the total shortening path length $\sigma_H(L)$ is given by

$$\sigma_H(L) = \alpha_H(L) + \beta_H(L) + \gamma(L). \quad (4)$$

2.2 Placing Two Liaisons in Two Levels

This subsection shows formulation of the total shortening path length when one node of liaison gets adjacent to all nodes with the same depth M ($M = 2, 3, \dots, H-1$) and the other node of liaison gets adjacent to all nodes with the same depth N ($N = M+1, M+2, \dots, H$) which is greater than M in a complete binary tree of height H ($H = 3, 4, \dots$).

Let V_1 denote the set of nodes whose depths are less than M . Let V_2 denote the set of nodes whose depths are equal to or greater than M and are less than N . Let V_3 denote the set of nodes whose depths are equal to or greater than N .

The sum of shortening path lengths between every pair of nodes in V_3 is given by

$$A_H(N) = \alpha_H(N) \quad (5)$$

from Eq.(1). The sum of shortening path lengths between every pair of nodes in V_3 and nodes in V_1 and V_2 is given by

$$B_H(N) = \beta_H(N) \quad (6)$$

from Eq.(2). The sum of shortening path lengths between every pair of nodes in V_1 is given by

$$C(M) = \gamma(M) \quad (7)$$

from Eq.(3), and the sum of shortening path lengths between every pair of nodes in V_1 and nodes in V_2 is given by

$$D(M, N) = \beta_{N-1}(M) \quad (8)$$

from Eq.(2). The sum of shortening path lengths between every pair of nodes in V_2 is formulated as follows.

The sum of shortening path lengths between every pair of nodes in each subtree whose root is a node with depth M is given by

$$E(M, N) = \gamma(N - M)2^M \quad (9)$$

from Eq.(3). The sum of shortening path lengths between every pair of nodes in two different subtrees whose roots are nodes with depth M is given by summing up $F(M, N)$ and $G(M, N)$. $F(M, N)$ which is the sum of shortening path lengths by adding edges only between one node of liaison and all nodes with depth M is given by

$$F(M, N) = \alpha_{N-1}(M) \quad (10)$$

from Eq.(1). $G(M, N)$ which is the sum of additional shortening path lengths by adding edges between the other node of liaison and all nodes with depth N after adding edges between one node of liaison and all nodes with depth M is expressed by

$$G(M, N) = (2^M - 1) \sum_{i=1}^{N-M-2} 2^{N-i} \sum_{j=1}^{N-M-i-1} 2^{N-M-j} (N - M - i - j), \quad (11)$$

where we define $\sum_{i=1}^0 \cdot = 0, \sum_{i=1}^{-1} \cdot = 0$.

From these equations, the total shortening path length $S_H(M, N)$ is given by

$$\begin{aligned}
 S_H(M, N) &= A_H(N) + B_H(N) + C_H(N) + D(M, N) + E(M, N) + F(M, N) + G(M, N) \\
 &= \{W(H-N)\}^2 2^N \sum_{i=1}^{N-1} i 2^i + W(H-N) 2^{N+1} \sum_{i=1}^{N-2} (N-i-1) i 2^i \\
 &\quad + 2^M \sum_{i=1}^{M-3} \sum_{j=1}^i (i-j+1) j 2^j + W(N-M-1) 2^{M+1} \sum_{i=1}^{M-2} (M-i-1) i 2^i \\
 &\quad + 2^N \sum_{i=1}^{N-M-3} \sum_{j=1}^i (i-j+1) j 2^j + \{W(N-M-1)\}^2 2^M \sum_{i=1}^{M-1} i 2^i \\
 &\quad + (2^M - 1) \sum_{i=1}^{N-M-2} 2^{N-i} \sum_{j=1}^{N-M-i-1} 2^{N-M-j} (N-M-i-j).
 \end{aligned} \tag{12}$$

Since the number of nodes of a complete binary tree of height h is

$$W(h) = 2^{h+1} - 1, \tag{13}$$

$S_H(M, N)$ of Eq.(12) becomes

$$\begin{aligned}
 S_H(M, N) &= (N-2)2^{2H+2} + 2^{2H-N+3} - 2^{H+N+3} + (N+1)2^{H+3} + (N-M)2^{N+M+1} \\
 &\quad + (N-M)(N-M-3)2^N + M(M-1)2^M.
 \end{aligned} \tag{14}$$

3. An Optimal Pair of Depths

This section obtains an optimal pair of depths $(M, N)^*$ by maximizing the total shortening path length $S_H(M, N)$ of Eq.(14).

3.1 An Optimal Depth N^* for a Fixed Value of M

In this subsection, we seek $N = N^*$ which maximizes $R_{H,M}(N) = S_H(M, N)$ for a fixed value of M .

Let $\Delta R_{H,M}(N) \equiv R_{H,M}(N+1) - R_{H,M}(N)$, so that we have

$$\begin{aligned}
 \Delta R_{H,M}(N) &= (4 - 2^{-N+2}) 2^{2H} + (8 - 2^{N+3}) 2^H + (N-M+2) 2^{N+M+1} \\
 &\quad + \{(N-M)(N-M+1) - 4\} 2^N.
 \end{aligned} \tag{15}$$

for $N = M+1, M+2, \dots, H-1$. Let us define x as

$$x = 2^H, \tag{16}$$

then $\Delta R_{H,M}(N)$ in Eq.(15) becomes

$$\begin{aligned}
 T_{M,N}(x) &= (4 - 2^{-N+2}) x^2 + (8 - 2^{N+3}) x + (N-M+2) 2^{N+M+1} \\
 &\quad + \{(N-M)(N-M+1) - 4\} 2^N
 \end{aligned} \tag{17}$$

which is a quadratic function of the continuous variable x . By differentiating $T_{M,N}(x)$ in Eq.(17) with respect to x , we obtain

$$T'_{M,N}(x) = (8 - 2^{-N+3})x + 8 - 2^{N+3}. \quad (18)$$

Since $T_{M,N}(x)$ is convex downward from $4 - 2^{-N+2} > 0$, and

$$T_{M,N}(2^{N+1}) = (N-M)2^{N+M+1} + (N-M)(N-M+1)2^N + (2^M - 1)2^{N+2} > 0 \quad (19)$$

and

$$T'_{M,N}(2^{N+1}) = 2^{N+3} - 8 > 0, \quad (20)$$

we have $T_{M,N}(x) > 0$ for $x \geq 2^{N+1}$. Hence, we have $\Delta R_{H,M}(N) > 0$ for $H \geq N+1$; that is, $N = M+1, M+2, \dots, H-1$.

From the above results, the optimal depth for a fixed value of M is $N^* = H$.

3.2 An Optimal Pair of Depths $(M, N)^*$

In this subsection, we seek $(M, N) = (M, N)^*$ which maximizes $S_H(M, N)$ in Eq.(14).

Let $Q_H(M)$ denote the total shortening path length when $N = H$, so that we have

$$\begin{aligned} Q_H(M) &\equiv S_H(M, H) \\ &= (H-4)2^{2H+2} + (H-M)2^{H+M+1} + (H+2)2^{H+3} + (H-M)(H-M-3)2^H \\ &\quad + M(M-1)2^M. \end{aligned} \quad (21)$$

Let $\Delta Q_H(M) \equiv Q_H(M+1) - Q_H(M)$, so that we have

$$\Delta Q_H(M) = (H-M-2)(2^M - 1)2^{H+1} + M(M+3)2^M > 0 \quad (22)$$

for $M = 2, 3, \dots, H-2$.

From the results in Subsection 3.1 and 3.2, the optimal pair of depths is $(M, N)^* = (H-1, H)$.

Table 1 shows the optimal pairs of depths $(M, N)^*$ and the total shortening path lengths $S_H(M, N)^*$ in the case of $H=3, 4, \dots, 20$.

4. Conclusions

This study considered obtaining optimal depths of adding nodes of liaisons to a complete binary tree maximizing the total shortening path length which is the sum of shortening lengths of shortest paths between every pair of all nodes in the complete binary tree. This means to obtain optimal levels of placing liaisons to the basic type of a pyramid organization such that the communication of information between every member in the organization becomes the most efficient.

H	$(M, N)^*$	$S_H(M, N)^*$
3	(2, 3)	120
4	(3, 4)	1040
5	(4, 5)	7040
6	(5, 6)	41472
7	(6, 7)	223872
8	(7, 8)	1139456
9	(8, 9)	5563392
10	(9, 10)	26347520
11	(10, 11)	121935872
12	(11, 12)	554323968
13	(12, 13)	2484535296
14	(13, 14)	11009196032
15	(14, 15)	48325754880
16	(15, 16)	210469584896
17	(16, 17)	910568456192
18	(17, 18)	3917087244288
19	(18, 19)	16767719571456
20	(19, 20)	71468617564160

Table 1. Optimal pairs of depths $(M, N)^*$

For the model of adding a node of liaison which gets adjacent to all nodes of the same depth L to a complete binary tree of height H , we had already obtained an optimal depth $L^* = H$ in our paper (Sawada, 2007). This result shows that the most efficient way of adding relations between the liaison and all members in one level is to add relations at the lowest level, irrespective of the number of levels in the organization structure.

This chapter expanded the above model into the model of placing liaisons in two levels of the organization structure, which is that of adding two nodes of liaisons which get adjacent to all nodes at each depth of two depths M and N which is greater than M to a complete binary tree of height H . We obtained an optimal pair of depth $(M, N)^* = (H-1, H)$ which maximizes the total shortening path length. In the case of $H = 5$ illustrated with the example in Fig. 1 an optimal pair of depths is $(M, N)^* = (4, 5)$. This result means that the most efficient manner of adding relations between two liaisons and all members in each level of two levels is to add relations at the lowest level and the second lowest level, irrespective of the number of levels in the organization structure.

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