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Optimal Design of Parallel Kinematics Machines with 2 Degrees of Freedom

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1. Introduction

The mechanical structure of today's machine tools is based on serial kinematics in the overwhelming majority of cases. Parallel kinematics with closed kinematics chains offer many potential benefits for machine tools but they also cause many drawbacks in the design process and higher efforts for numerical control and calibration.

The Parallel Kinematics Machine (PKM) is a new type of machine tool which was firstly showed at the 1994 International Manufacturing Technology in Chicago by two American machine tool companies, Giddings & Lewis and Ingersoll.

Parallel Kinematics Machines seem capable of answering the increase needs of industry in terms of automation. The nature of their architecture tends to reduce absolute positioning and orienting errors (Stan et al., 2006). Their closed kinematics structure allows them obtaining high structural stiffness and performing high-speed motions. The inertia of its mobile parts is reduced, since the actuators of a parallel robot are often fixed to its base and the end-effector can perform movements with higher accelerations. One drawback with respect to open-chain manipulators, though, is a typically reduced workspace and a poor ratio of working envelope to robot size.

In theory, parallel kinematics offer for example higher stiffness and at the same time higher acceleration performance than serial structures. In reality, these and other properties are highly dependent on the chosen structure, the chosen configuration for a structure and the position of the tool centre point (TCP) within the workspace. There is a strong and complex link between the type of robot's geometrical parameters and its performance. It's very difficult to choose the geometrical parameters intuitively in such a way as to optimize the performance. The configuration of parallel kinematics is more complex due to the high sensitivity to variations of design parameters. For this reason the design process is of key importance to the overall performance of a Parallel Kinematics Machines. For the optimization of Parallel Kinematics Machines an application-oriented approach is necessary. In this chapter an approach is presented that includes the definition of specific objective functions as well as an optimization algorithm. The presented algorithm provides the basis for an overall multiobjective optimization of several kinematics structures.

An important objective of this chapter is also to propose an optimization method for planar Parallel Kinematics Machines that combines performance evaluation criteria related to the following robot characteristics: workspace, design space and transmission quality index.

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Source: Parallel Manipulators, Towards New Applications, Book edited by: Huapeng Wu, ISBN 978-3-902613-40-0, pp. 506, April 2008, I-Tech Education and Publishing, Vienna, Austria

Furthermore, a genetic algorithm is proposed as the principle optimization tool. The success of this type of algorithm for parallel robots optimization has been demonstrated in various papers (Stan et al., 2006).

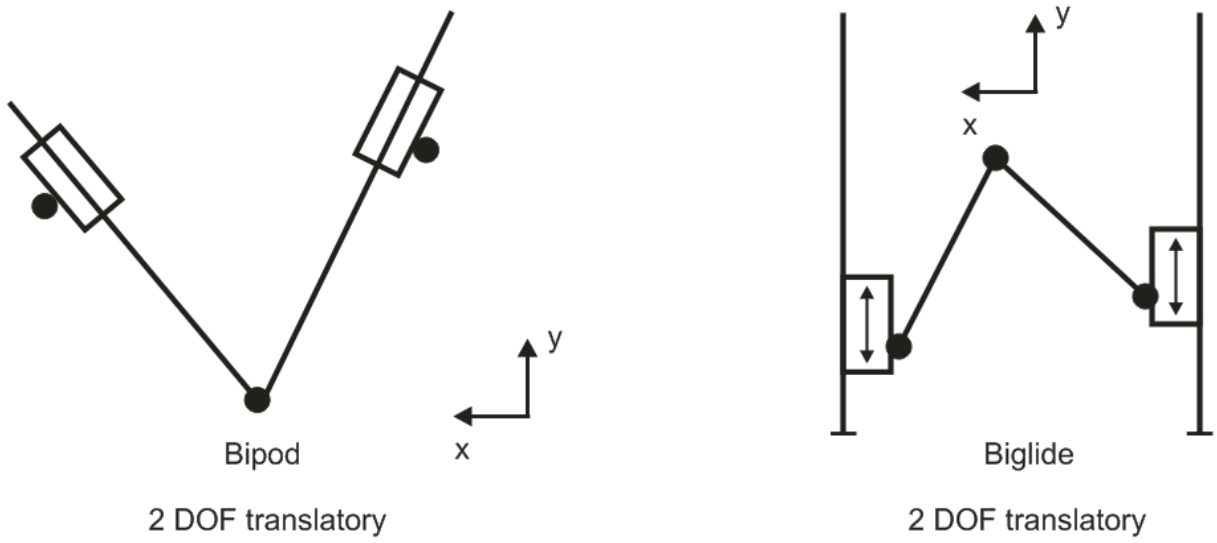

Fig. 1. Parallel kinematics for milling machines

For parallel kinematics machines with reduced number of degrees of freedom kinematics and singularity analyses can be solved to obtain algebraic expressions, which are well suited for an implementation in optimum design problems.

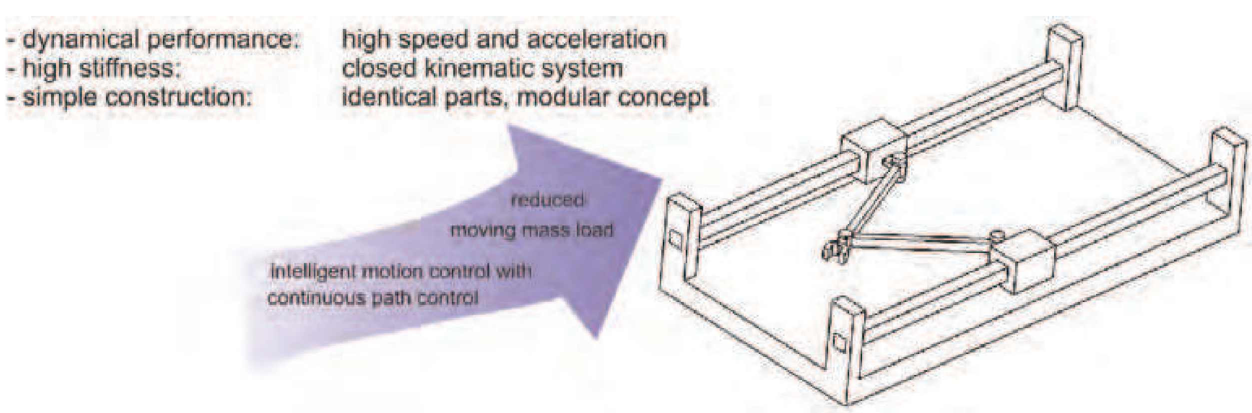

Fig. 2. Benefits of Parallel Kinematics Machines

High dynamical performance is achieved due to the low moved masses. Due to the closed kinematics the movements of parallel kinematics machines are vibration free for which the accuracy is improved. Finally, the modular concept allows a cost-effective production of the mechanical parts.

In this chapter, the optimization workspace index is defined as the measure to evaluate the performance of two degree of freedom Parallel Kinematics Machines. Another important contribution is the optimal dimensioning of the two degree-of-freedom Parallel Kinematics Machines of type Bipod and Biglide for the largest workspace using optimization based on Genetic Algorithms.

2. Objective functions used for optimization of machine tools with parallel kinematics

One of the main influential factors on the performance of a machine tool with parallel kinematics is its structural configuration. The performance of a machine tool with parallel kinematics can be evaluated by its kinematic, static and dynamic properties. Optimal design is one of the most important issues in the development of a parallel machine tool. Two issues are involved in the optimal design: performance evaluation and dimensional synthesis. The latter one is one of the most difficult issues in this field. In the optimum design process, several criteria could be involved for a design purpose, such as workspace, singularity, dexterity, accuracy, stiffness, and conditioning index.

After its choice, the next step on the machine tool with parallel kinematics design should be to establish its dimensions. Usually this dimensioning task involves the choice of a set of parameters that define the mechanical structure of the machine tool. The parameter values should be chosen in a way to optimize some performance criteria, dependent upon the foreseen application.

The optimization of machine tools with parallel kinematics can be based on the following objectives functions:

- workspace,
- the overall size of the machine tool,
- kinematic transmission of forces and velocities,
- stiffness,
- acceleration capabilities,
- dexterity,
- accuracy,
- the singular configurations,
- isotropy.

In the design process we want to determine the design parameters so that the parallel kinematics machine fulfills a set of constraints. These constraints may be extremely different but we can mention:

- workspace requirement,
- maximum accuracy over the workspace for a given accuracy of the sensors,
- maximal stiffness of the Parallel Kinematics Machines in some direction,
- minimum articular forces for a given load,
- maximum velocities or accelerations for given actuator velocities and accelerations.

Determination of the architecture and size of a mechanism is an important issue in the mechanism design. Several objectives are contradictory to each other. An optimization with only one objective runs into unusable solutions for all other objectives. Unfortunately, any change that improves one performance will usually deteriorate the other. This trade-off occurs with almost every design and this inevitable generates the problem of design optimization. Only a multiobjective approach will result in practical solutions for machine tool applications.

The classical methods of design optimization, such as iterative methods, suffer from difficulties in dealing with this problem. Firstly, optimization problems can take many iterations to converge and can be sensitive to numerical problems such as truncation and round-off error in the calculation. Secondly, most optimization problems depend on initial guesses, and identification of the global minimum is not guaranteed. Therefore, the relation between the design parameters and objective function is difficult to know, thus making it hard to obtain the most optimal design parameters of the mechanism. Also, it's rather difficult to investigate the relations between performance criteria and link lengths of all mechanisms. So, it's important to develop a useful optimization approach that can express the relations between performance criteria and link lengths.

2.1 Workspace

The workspace of a robot is defined as the set of all end-effector configurations which can be reached by some choice of joint coordinates. As the reachable locations of an end-effector are dependent on its orientation, a complete representation of the workspace should be embedded in a 6-dimensional workspace for which there is no possible graphical illustration; only subsets of the workspace may therefore be represented.

There are different types of workspaces namely constant orientation workspace, maximal workspace or reachable workspace, inclusive orientation workspace, total orientation workspace, and dextrous workspace. The constant orientation workspace is the set of locations of the moving platform that may be reached when the orientation is fixed. The maximal workspace or reachable workspace is defined as the set of locations of the end-effector that may be reached with at least one orientation of the platform. The inclusive orientation workspace is the set of locations that may be reached with at least one orientation among a set defined by ranges on the orientation parameters. The set of locations of the end-effector that may be reached with all the orientations among a set defined by ranges on the orientations on the orientation parameters constitute the total orientation workspace. The dextrous workspace is defined as the set of locations for which all orientations are possible. The dextrous workspace is a special case of the total orientation workspace, the ranges for the rotation angles (the three angles that define the orientation of the end-effector) being $[0,2\pi]$.

In the literature, various methods to determine workspace of a parallel robot have been proposed using geometric or numerical approaches. Early investigations of robot workspace were reported by (Gosselin, 1990), (Merlet, 1005), (Kumar & Waldron, 1981), (Tsai and Soni, 1981), (Gupta & Roth, 1982), (Sugimoto & Duffy, 1982), (Gupta, 1986), and (Davidson & Hunt, 1987). The consideration of joint limits in the study of the robot workspaces was presented by (Delmas & Bidard, 1995). Other works that have dealt with robot workspace are reported by (Agrawal, 1990), (Gosselin & Angeles, 1990), (Cecarelli, 1995). (Agrawal, 1991) determined the workspace of in-parallel manipulator system using a different concept namely, when a point is at its workspace boundary, it does not have a velocity component along the outward normal to the boundary. Configurations are determined in which the velocity of the end-effector satisfies this property. (Pernkopf & Husty, 2005) presented an algorithm to compute the reachable workspace of a spatial Stewart Gough-Platform with planar base and platform (SGPP) taking into account active and passive joint limits. Stan (Stan, 2003) presented a genetic algorithm approach for multi-criteria optimization of PKM (Parallel Kinematics Machines). Most of the numerical methods to determine workspace of parallel manipulators rest on the discretization of the pose parameters in order to determine the workspace boundary (Cleary & Arai, 1991), (Ferraresi et al., 1995). In the discretization approach, the workspace is covered by a regularly arranged grid in either Cartesian or polar form of nodes. Each node is then examined to see whether it belongs to the workspace. The accuracy of the boundary depends upon the sampling step that is used to create the grid.

The computation time grows exponentially with the sampling step. Hence it puts a limit on the accuracy. Moreover, problems may occur when the workspace possesses singular configurations. Other authors proposed to determine the workspace by using optimization methods (Stan, 2003). Numerical methods for determining the workspace of the parallel robots have been developed in the recent years. Exact computation of the workspace and its boundary is of significant importance because of its impact on robot design, robot placement in an environment, and robot dexterity.

Masory, who used the discretisation method (Masory & Wang, 1995), presented interesting results for the Stewart-Gough type parallel manipulator:

- The mechanical limits on the passive joints play an important role on the volume of the workspace. For ball and socket joints with given rotation ability, the volume of the workspace is maximal if the main axes of the joints have the same directions as the links when the robot is in its nominal position.
- The workspace volume is roughly proportional to the cube of the stroke of the actuators.
- The workspace volume is not very sensitive to the layout of the joints on the platforms, even though it is maximal when the two platforms have the same dimension (in this case, the robot is in a singular configuration in its nominal position).

Even though powerful three-dimensional Computer Aided Design and Dynamic Analysis software packages such as Pro/ENGINEER, IDEAS, ADAMS and Working Model 3-D are now being used, they cannot provide important visual and realistic workspace information for the proposed design of a parallel robot. In addition, there is a great need for developing methodologies and techniques that will allow fast determination of workspace of a parallel robot. A general numerical evaluation of the workspace can be deduced by formulating a suitable binary representation of a cross-section in the taskspace. A cross-section can be obtained with a suitable scan of the computed reachable positions and orientations **p**, once the forward kinematic problem has been solved to give **p** as function of the kinematic input joint variables **q**. A binary matrix P_{ij} can be defined in the cross-section plane for a crosssection of the workspace as follows: if the (i, j) grid pixel includes a reachable point, then $P_{ij} = 1$; otherwise $P_{ij} = 0$, as shown in Fig. 3. Equations (1)-(4) for determining the workspace of a robot by discretization method can be found in Ref. (Ottaviano et al., 2002).

Then is computed i and j:

$$i = \left[\frac{x + \Delta x}{x}\right] \; j = \left[\frac{y + \Delta y}{y}\right] \tag{1}$$

where i and j are computed as integer numbers. Therefore, the binary mapping for a workspace cross-section can be given as:

$$P_{ij} = \begin{cases} 0 \;\; if \;\; P_{ij} \notin W(H) \\ 1 \;\; if \;\; P_{ij} \in W(H) \end{cases} \tag{2}$$

where W(H) indicates workspace region; $\in$ stands for "belonging to" and $\notin$ is for "not belonging to".

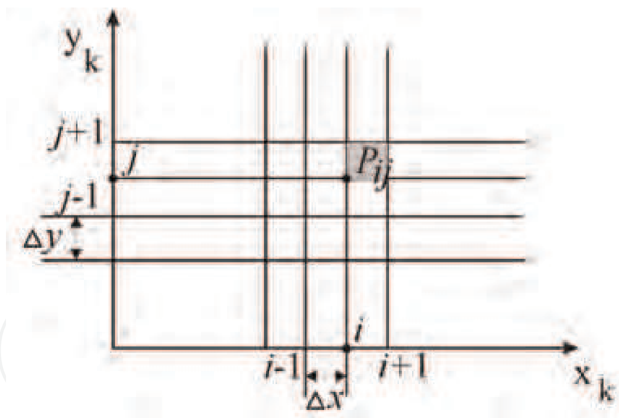

Fig. 3. The general scheme for binary representation and evaluation of robot workspace

In addition, the proposed binary representation is useful for a numerical evaluation of the position workspace by computing the sections areas A as:

$$A = \sum_{i=1}^{i_{\max}} \sum_{j=1}^{j_{\max}} \left(P_{ij} \Delta x \Delta y \right) \tag{3}$$

This numerical approximation of the workspace area has been used for the optimum design purposes.

2.2 Kinematics accuracy

The kinematics accuracy is a key factor for the design and application of the machine tools with parallel kinematics. But the research of the accuracy is still in initial stage because of the various structures and the nonlinear errors of the parallel kinematics machine tools.

To analyze the sensitiveness of the structural error is one of the directions for the research of structural accuracy. An approach was introducing a dimensionless factor of sensitiveness for every leg of the structure. Other approach includes the use of the value of Jacobian matrix as sensitivity index for the whole legs or the use of condition number of Jacobian matrix as a quantity index to describe the error sensitivity of the whole system.

2.3 Stiffness

Stiffness describes the ratio “deformation displacement to deformation force” (*static stiffness*). In case of dynamic loads this ratio (*dynamic stiffness*) depends on the exciting frequencies and comes to its most unfavorable (smallest) value at resonance (Hesselbach et al., 2003). In structural mechanics deformation displacement and deformation force are represented by vectors and the stiffness is expressed by the *stiffness matrix K*.

2.4 Singular configurations

Because singularity leads to a loss of the controllability and degradation of the natural stiffness of manipulators, the analysis of Parallel Kinematics Machines has drawn considerable attention. This property has attracted the attention of several researchers because it represents a crucial issue in the context of analysis and design. Most Parallel Kinematics Machines suffer from the presence of singular configurations in their workspace that limit the machine performances. The singular configurations (also called singularities) of a Parallel Kinematics Machine may appear inside the workspace or at its boundaries. There are two main types of singularities (Gosselin & Angeles, 1990). A configuration where a finite tool velocity requires infinite joint rates is called a serial singularity or a type 1 singularity. A configuration where the tool cannot resist any effort and in turn, becomes uncontrollable is called a parallel singularity or type 2 singularity. Parallel singularities are particularly undesirable because they cause the following problems:

- a high increase of forces in joints and links, that may damage the structure,
- a decrease of the mechanism stiffness that can lead to uncontrolled motions of the tool though actuated joints are locked.

Thus, kinematics singularities have been considered for the formulated optimum design of the Parallel Kinematics Machines.

2.5 Dexterity

Dexterity has been considered important because it is a measure of a manipulator's ability to arbitrarily change its position and orientation or to apply forces and torques in arbitrary direction. Many researchers have performed design optimization focusing on the dexterity of parallel kinematics by minimization of the condition number of the Jacobian matrix. In regards to the PKM's dexterity, the condition number ρ, given by $\rho=\sigma_{max}/\sigma_{min}$ where σ_{max} and σ_{min} are the largest and smallest singular values of the Jacobian matrix *J*.

2.6 Manipulability

The determinant of the Jacobian matrix *J*, det(*J*), is proportional to the volume of the hyper ellipsoid. The condition number represents the sphericity of the hyper ellipsoid. The manipulability measure *w*, given by $w=\sqrt{det\left(JJ^T\right)}$ was defined to describe the ability of machine tool with parallel structure to change its position and direction in its workspace.

3. Two DOF Parallel Kinematics Machines

3.1 Geometrical description of the Parallel Kinematics Machines

A planar Parallel Kinematics Machines is formed when two or more planar kinematic chains act together on a common rigid platform. The most common planar parallel architecture is composed of two R<u>P</u>R chains (Fig. 4), where the notation R<u>P</u>R denotes the planar chain made up of a revolute joint, a prismatic joint, and a second revolute joint in series. Another common architecture is <u>P</u>RRR<u>P</u> (Fig. 5). Two general planar Parallel Kinematics Machines with two degrees of freedom activated by prismatic joints are shown in Fig. 4 and Fig. 5.
There are a wide range of parallel robots that have been developed but they can be divided into two main groups:

- Type 1) Parallel Kinematics Machine with variable length struts,
- Type 2) Parallel Kinematics Machine with constant length struts.

Since mobility of these Parallel Kinematics Machines is two, two actuators are required to control these Parallel Kinematics Machines. For simplicity, the origin of the fixed base frame {B} is located at base joint A with its x-axis towards base joint B, and the origin of the moving frame {M} is located in TCP, as shown in Fig. 7. The distance between two base joints is *b*. The position of the moving frame {M} in the base frame {B} is $\mathbf{x}=(x_P, y_P)^T$ and the actuated joint variables are represented by $\mathbf{q}=(q_1, q_2)^T$.

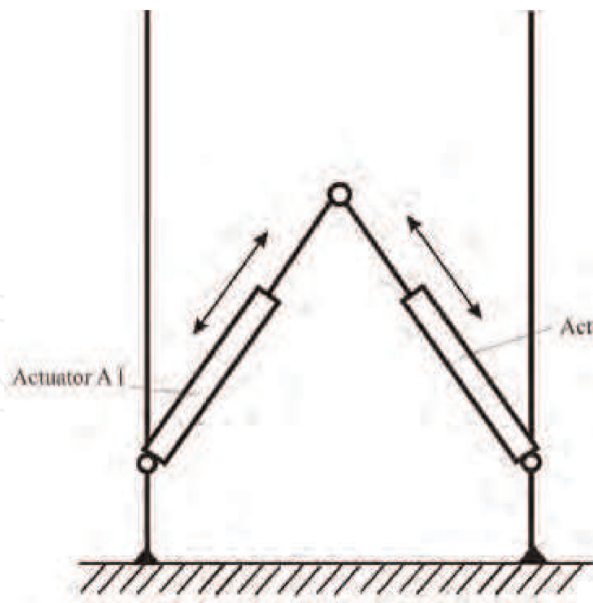

Fig. 4. Variable length struts Parallel Kinematics Machine

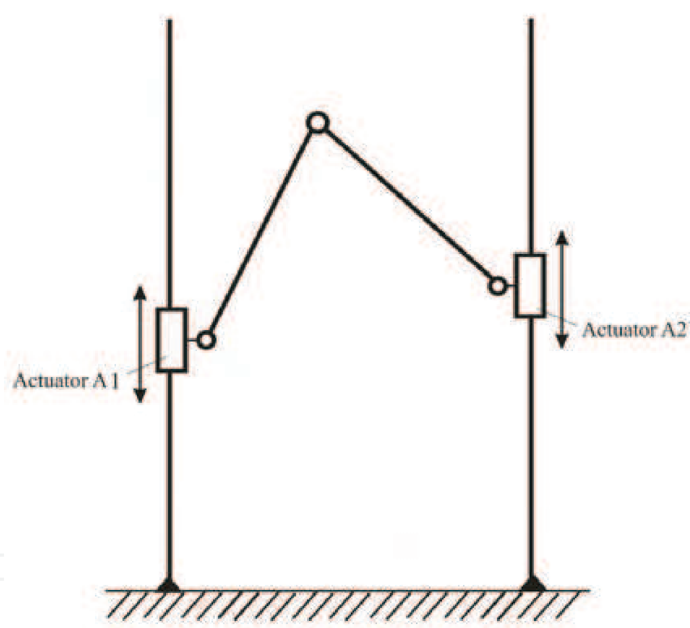

Fig. 5. Constant length struts Parallel Kinematics Machine

3.2 Kinematic analysis of the Parallel Kinematics Machines

PKM kinematics deal with the study of the PKM motion as constrained by the geometry of the links. Typically, the study of the PKMs kinematics is divided into two parts, inverse kinematics and forward (or direct) kinematics. The inverse kinematics problem involves a known pose (position and orientation) of the output platform of the PKM to a set of input joint variables that will achieve that pose. The forward kinematics problem involves the mapping from a known set of input joint variables to a pose of the moving platform that results from those given inputs. However, the inverse and forward kinematics problems of our PKMs can be described in closed form.

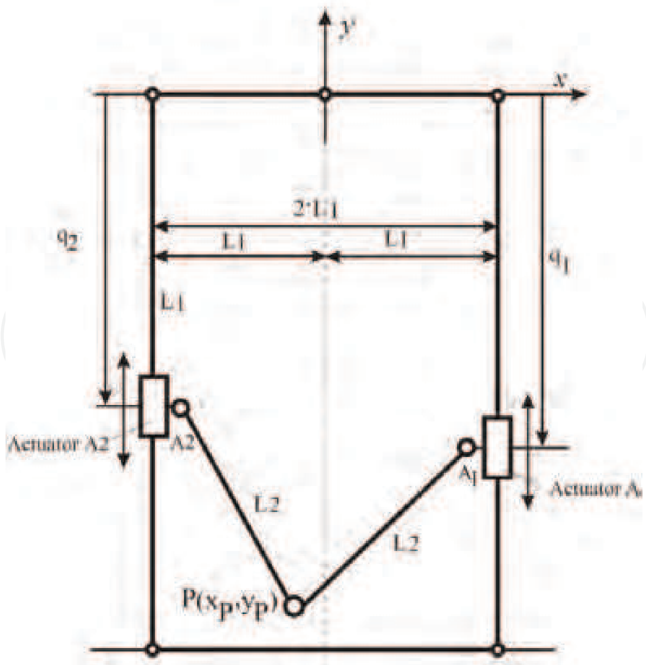

Fig. 6. The general kinematic scheme of a PRRRP Parallel Kinematics Machine

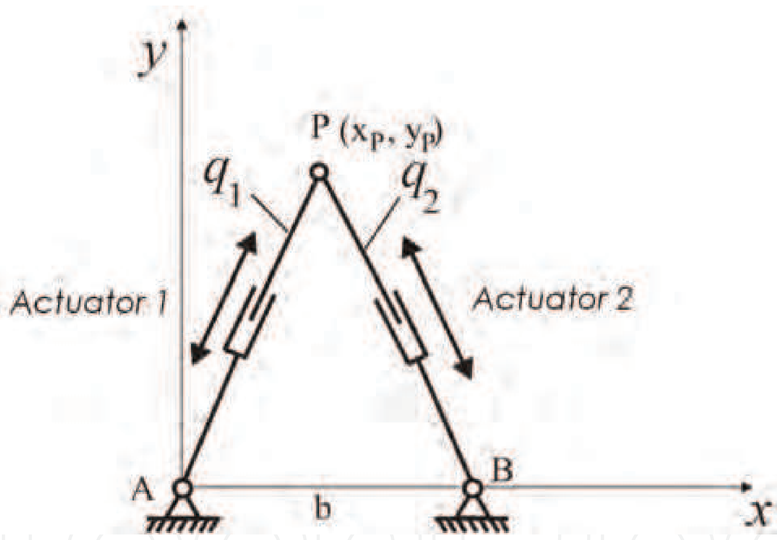

Fig. 7. The general kinematic scheme of a RPRPR Parallel Kinematics Machine

The kinematics relation between **x** and **q** of these 2 DOF Parallel Kinematics Machines can be expressed solving the following equation:

$$f(\mathbf{x}, \mathbf{q})=0 \tag{4}$$

Then the inverse kinematics problem of the PKM from Fig. 6 can be solved by writing the following equations:

$$q_1 = y_P \pm \sqrt{L_2^2 - (x_p - L_1)^2} \tag{5}$$

$$q_2 = y_P \pm \sqrt{L_2^2 - (x_p + L_1)^2}$$

Then the inverse kinematics problem of the PKM from Fig. 7 can be solved by writing the following equations:

$$q_1 = \sqrt{x_P^2 + y_P^2} \tag{6}$$

$$q_2 = \sqrt{(b - x_P)^2 + y_P^2}$$

The TCP position can be calculated by using inverted transformation, from (6), thus the direct kinematics of the PKM can be described as:

$$x_P = \frac{q_1^2 + b^2 - q_2^2}{2 \cdot b} \tag{7}$$

$$y_P = \sqrt{q_1^2 - x_P^2}$$

where the values of the x_P, y_P can be easily determined.

The forward and the inverse kinematics problems were solved under the MATLAB environment and it contains a user friendly graphical interface. The user can visualize the different solutions and the different geometric parameters of the PKM can be modified to investigate their effect on the kinematics of the PKM. This graphical user interface can be a valuable and effective tool for the workspace analysis and the kinematics of the PKM. The designer can enhance the performance of his design using the results given by the presented graphical user interface.

The Matlab-based program is written to compute the forward and inverse kinematics of the PKM with 2 degrees of freedom. It consists of several MATLAB scripts and functions used for workspace analysis and kinematics of the PKM. A friendly user interface was developed using the MATLAB-GUI (graphical user interface). Several dialog boxes guide the user through the complete process.

Fig. 8. Graphical User Interface (GUI) for solving inverse kinematics of the 2 DOF planar Parallel Kinematics Machine of type Bipod in MATLAB environment.

The user can modify the geometry of the 2 DOF PKM. The program visualizes the corresponding kinematics results with the new inputs.

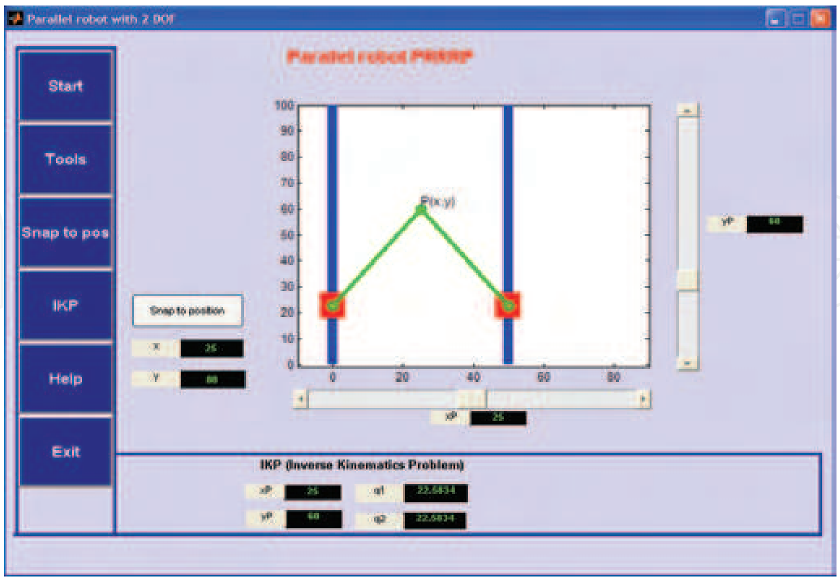

Fig. 9. Parallel Kinematics Machine configuration for X_P=25 mm Y_P=60 mm

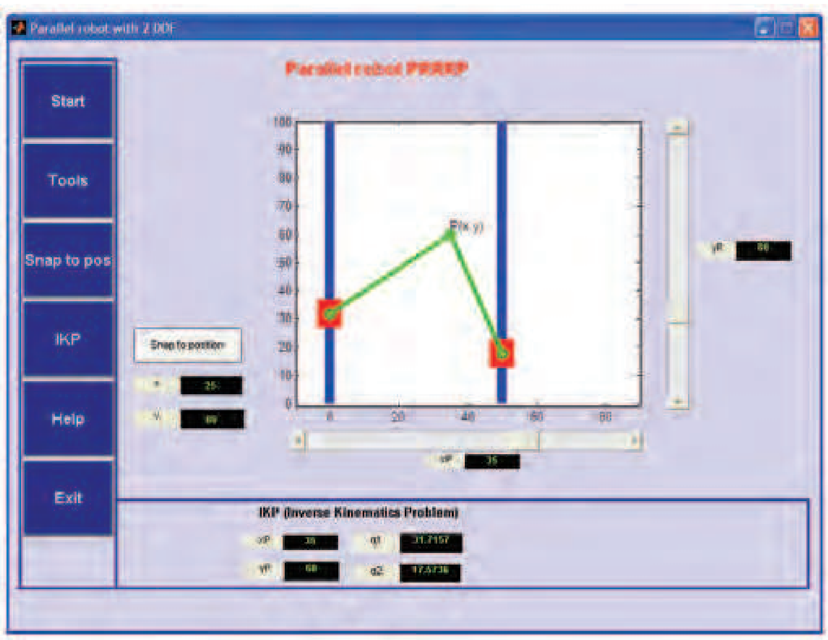

Fig. 10. Parallel Kinematics Machine configuration for X_P=35 mm Y_P=60 mm

4. Performance evaluation of Parallel Kinematics Machines

4.1 Workspace determination and optimization of the Parallel Kinematics Machines

The workspace is one of the most important kinematics properties of manipulators, even by practical viewpoint because of its impact on manipulator design and location in a workcell (Ceccarelli et al., 2005). Workspace is a significant design criterion for describing the kinematics performance of parallel robots. The planar parallel robots use area to evaluate the workspace ability. However, is hard to find a general approach for identification of the workspace boundaries of the parallel robots. This is due to the fact that there is not a closed form solution for the direct kinematics of these parallel robots. That's why instead of developing a complex algorithm for identification of the boundaries of the workspace, it's developed a general visualization method of the workspace for its analysis and its design.

A general numerical evaluation of the workspace can be deduced by formulating a suitable binary representation of a cross-section in the taskspace. Other authors proposed to determine the workspace by using optimization (Stan, 2003). A fundamental characteristic that must be taken into account in the dimensional design of robot manipulators is the area of their workspace. It is crucial to calculate the workspace and its boundaries with perfect precision, because they influence the dimensional design, the manipulator's positioning in the work environment, and its dexterity to execute tasks. Because of this, applications involving these Parallel Kinematics Machines require a detailed analysis and visualization of the workspace of these PKMs. The algorithm for visualization of workspace needs to be adaptable in nature, to configure with different dimensions of the parallel robot's links. The workspace is discretized into square and equal area sectors. A multi-task search is performed to determine the exact workspace boundary. Any singular configuration inside the workspace is found along with its position and dimensions. The area of the workspace is also computed.

The workspace is the area in the plane case where the tool centre point (TCP) can be controlled and moved continuously and unobstructed. The workspace is limited by *singularities*. At singularity poses it is not possible to establish definite relations between input and output coordinates. Such poses must be avoided by the control.

The robotics literature contains various indices of performance (Du Plessis & Snyman, 2001) (Schoenherr & Bemessen, 1998), such as the workspace index W and the general equation is given in (8). Workspace for this kind of robot may be easily generated by intersection of the enveloping surfaces and the area can be also computed.

$$W = \int_W dW \tag{8}$$

The workspace of the 2 DOF planar PKM of type Bipod is often represented as a region of the plane, which can be obtained by the reacheable points of the TCP.

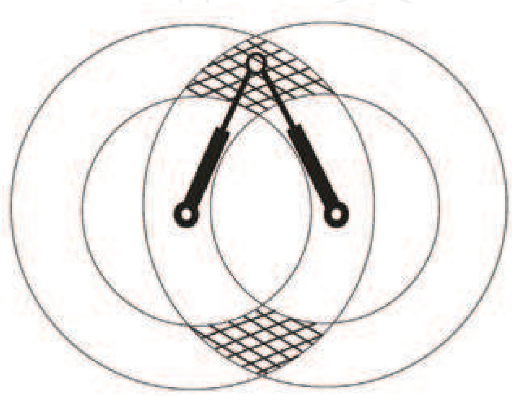

Fig. 11. The workspace is the intersection of two enveloping surface of two legs.

The following presents the main factors affecting workspace. For ease of comparison a cubic working envelope with a common contour length is used together with a machine size that

is calculated from the maximum required strut length. Other design specific factors such as the end-effector size, drive volumes have been neglected for simplification.
The working envelope to machine size using variable length struts is dependent on the following factors:

1. The length of the extended and retracted actuator (L*min*, L*max*);
2. Limitations due to the joint angle range.

The limiting effect of the joint limits is clearly illustrated in Fig. 12-13.

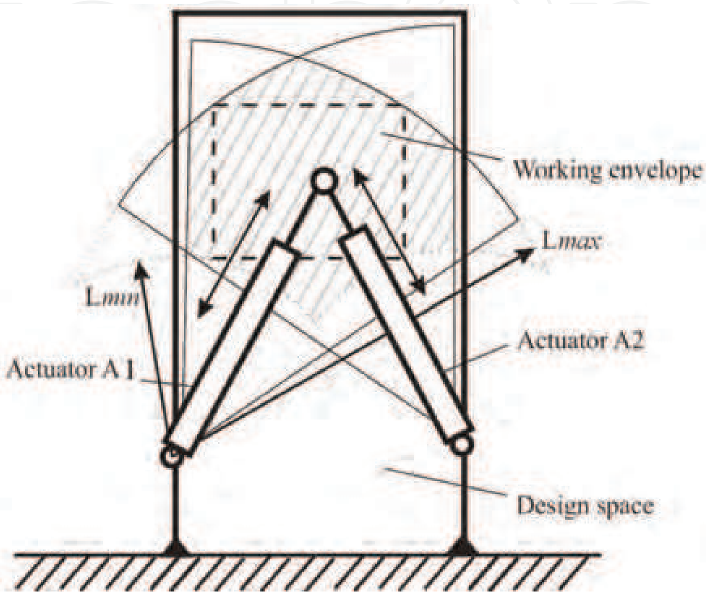

Fig. 12. Workspace of the Parallel Kinematics Machine with variable length struts

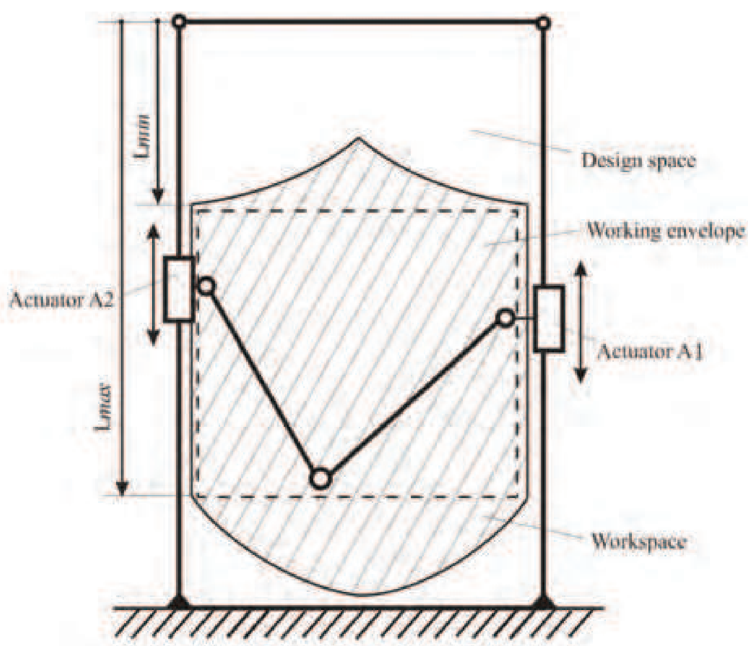

Fig. 13. Workspace of the Parallel Kinematics Machine with constant length struts

In this section, the workspace of the proposed Parallel Kinematics Machines will be discussed systematically. It's very important to analyze the area and the shape of workspace for parameters given robot in the context of industrial application. The workspace is primarily limited by the boundary of solvability of inverse kinematics. Then the workspace is limited by the reachable extent of drives and joints, occurrence of singularities and by the link and platform collisions. The PKM mechanisms PRRRP and RPRPR realize a wide workspace as well as high-speed. Analysis, visualization of workspace is an important aspect of performance analysis. A numerical algorithm to generate reachable workspace of parallel manipulators is introduced.

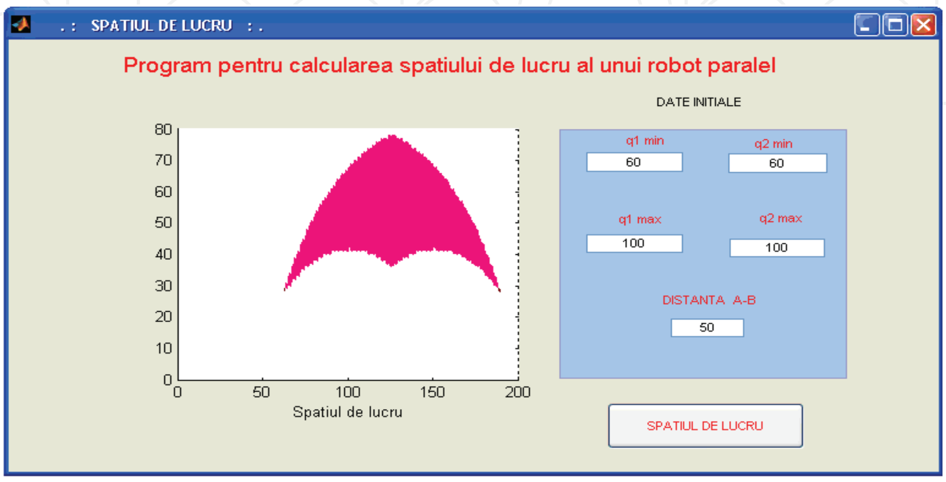

Fig. 14. The GUI for calculus of workspace for the planar 2 DOF Parallel Kinematics Machine with variable length struts

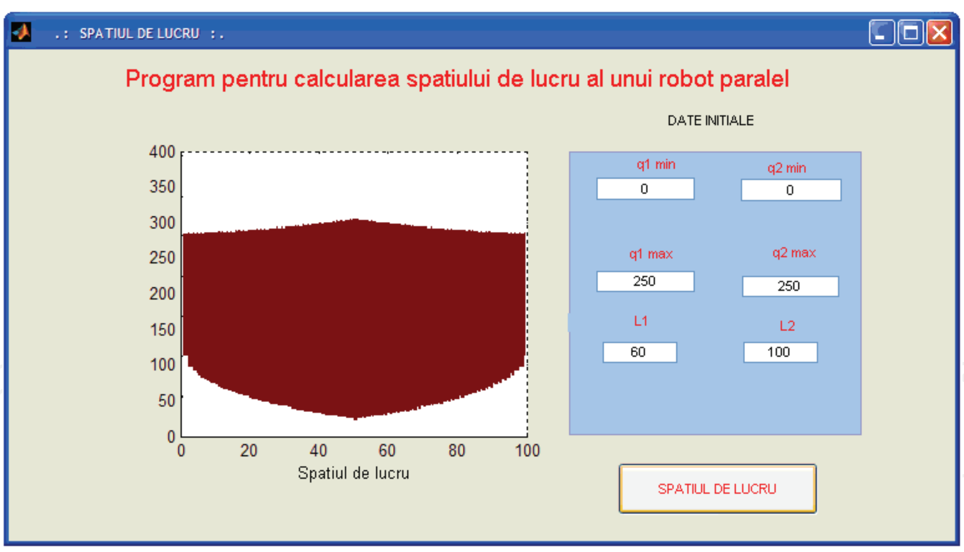

Fig. 15. The GUI for calculus of workspace for the planar 2 DOF Parallel Kinematics Machine with constant length struts

In the followings is presented the workspace analysis of 2 DOF Bipod PKM.

Case I:

Conditions:

$$q_{1min} + q_{2min} > b \text{ , } q_{1max} > b \text{ , } q_{2max} > b$$

a) for y>0

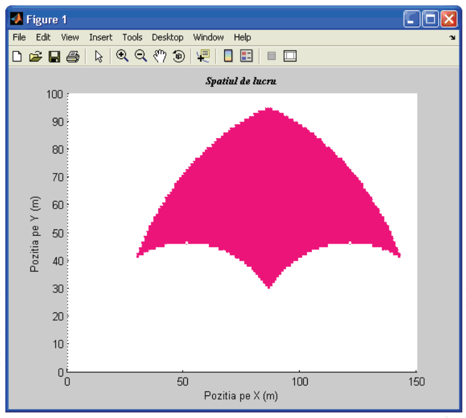

Fig. 16. The workspace of the planar 2 DOF Parallel Kinematics Machine is shown as the shading region.

b) for $-\infty < y < +\infty$ *, there exist two regions of the workspace*

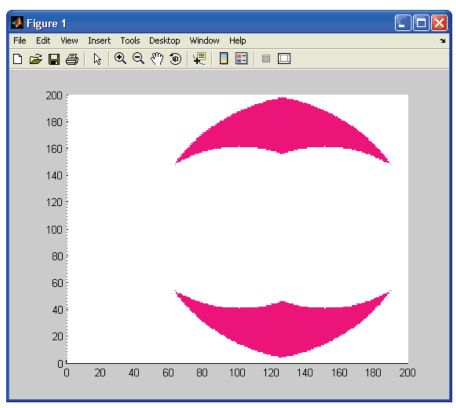

Fig. 17. The workspace of the planar 2 DOF Parallel Kinematics Machine is shown as the shading region.

Case II:

Conditions:

$$q_{1min} + q_{2min} > b \, , \; q_{1max} < b \, , \; q_{2max} < b$$

a) for y>0

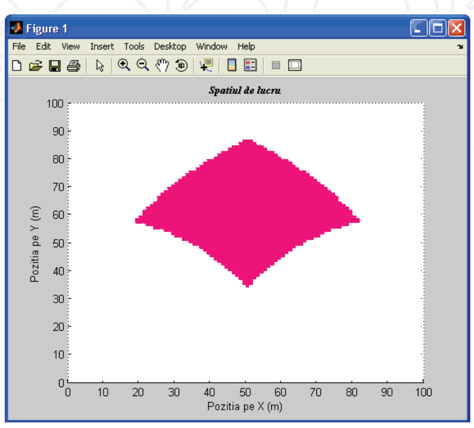

Fig. 18. The workspace of the planar 2 DOF Parallel Kinematics Machine is shown as the shading region.

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b) for $-\infty < y < +\infty$, *there exist two regions of the workspace*

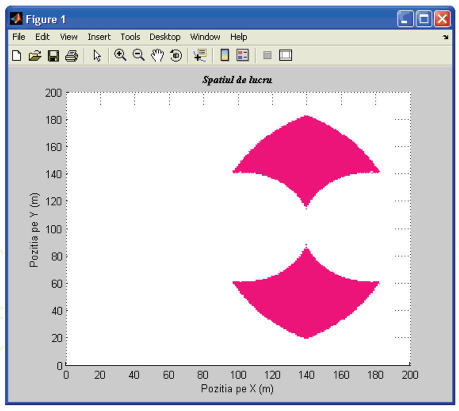


Fig. 19. The workspace of the planar 2 DOF Parallel Kinematics Machine is shown as the shading region.

Case III:

Conditions: $q_{1min} + q_{2min} < b$, $q_{1max} > b$, $q_{2max} > b$

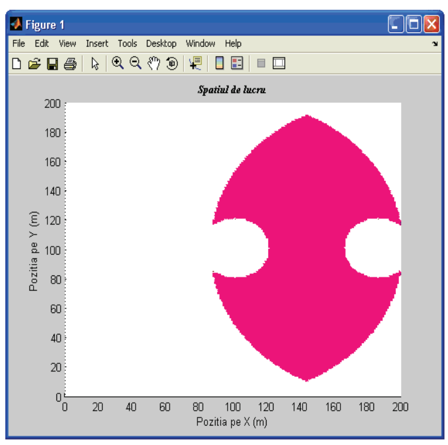


Fig. 20. The workspace of the planar 2 DOF Parallel Kinematics Machine is shown as the shading region.

Case IV:

Conditions: $q_{1min} + q_{2min} < b$, $q_{1max} < b$, $q_{2max} < b$

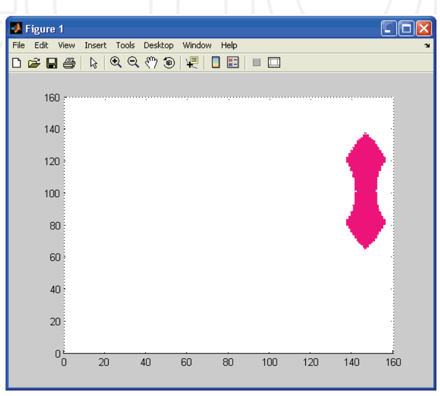


Fig. 21. The workspace of the planar 2 DOF Parallel Kinematics Machine is shown as the shading region.

Case V:

Conditions: $q_{1min} + q_{2min} < b$, $q_{1max} > b + q_{2min}$, $q_{2max} > b + q_{1min}$

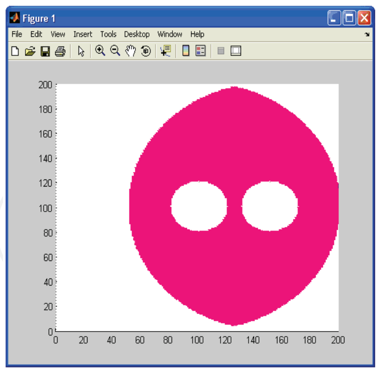


Fig. 22. The workspace of the planar 2 DOF Parallel Kinematics Machine is shown as the shading region.

Case VI:

Conditions: $q_{1min} + q_{2min} > b$, $q_{1max} > b + q_{2min}$, $q_{2max} > b + q_{1min}$

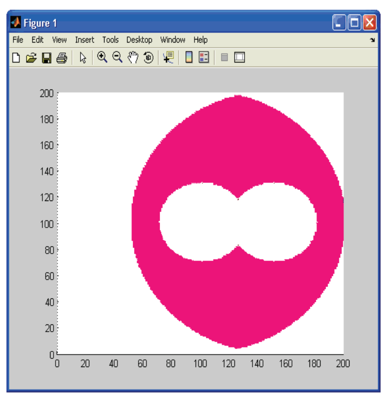


Fig. 23. The workspace of the planar 2 DOF Parallel Kinematics Machine is shown as the shading region.

Case VII:

Conditions: $q_{1min} < b$, $q_{1max} < b$, $q_{2min} < b$, $q_{2max} < b$, $q_{1min} + q_{2min} < b$, $q_{1max} + q_{2max} > b$

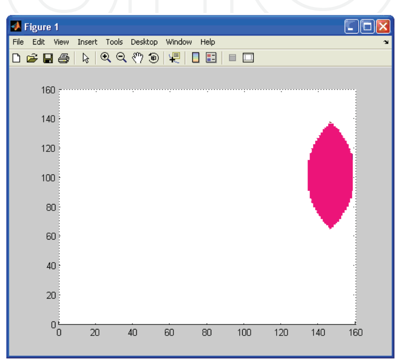
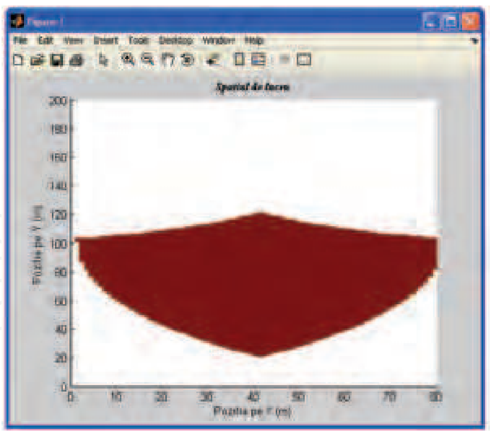
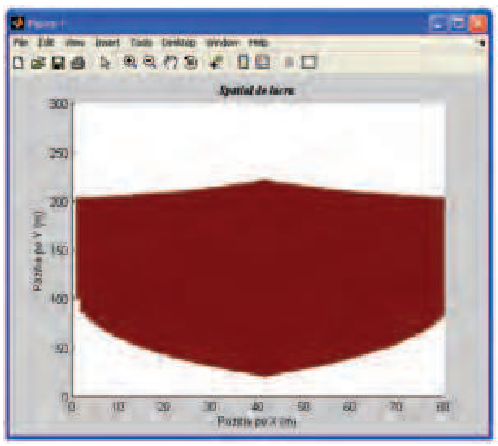


Fig. 24. The workspace of the planar 2 DOF Parallel Kinematics Machine is shown as the shading region.

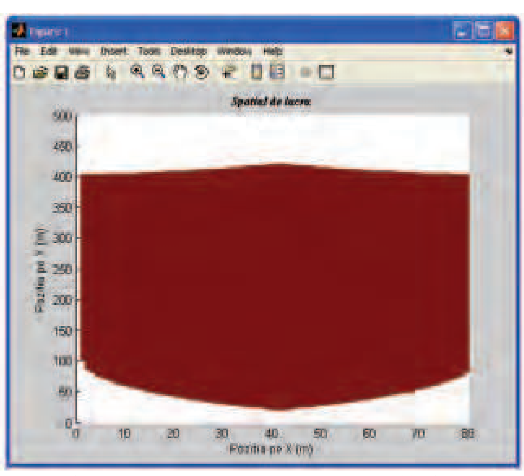
In the followings is presented the workspace analysis of 2 DOF Biglide Parallel Kinematics Machine.



a) Workspace for the planar 2 DOF Parallel Kinematics Machine, case $q_{1max} = q_{2max} = 100\ mm$



b) Workspace for the planar 2 DOF Parallel Kinematics Machine, case $q_{1max} = q_{2max} = 200\ mm$



c) Workspace for the planar 2 DOF Parallel Kinematics Machine, case $q_{1max} = q_{2max} = 400\ mm$

Fig. 25. Different regions of workspace for Biglide PKM for different lengths of stroke of actuators

4.2 Singularity analysis of the Biglide Parallel Kinematics Machine

Because singularity leads to a loss of the controllability and degradation of the natural stiffness of manipulators, the analysis of parallel manipulators has drawn considerable attention. Most parallel robots suffer from the presence of singular configurations in their workspace that limit the machine performances. Based on the forward and inverse Jacobian matrix, three cases of singularities of parallel manipulators can be obtained. Singular configurations should be avoided.

In the followings are presented the singular configurations of 2 DOF Biglide Parallel Kinematic Machine.

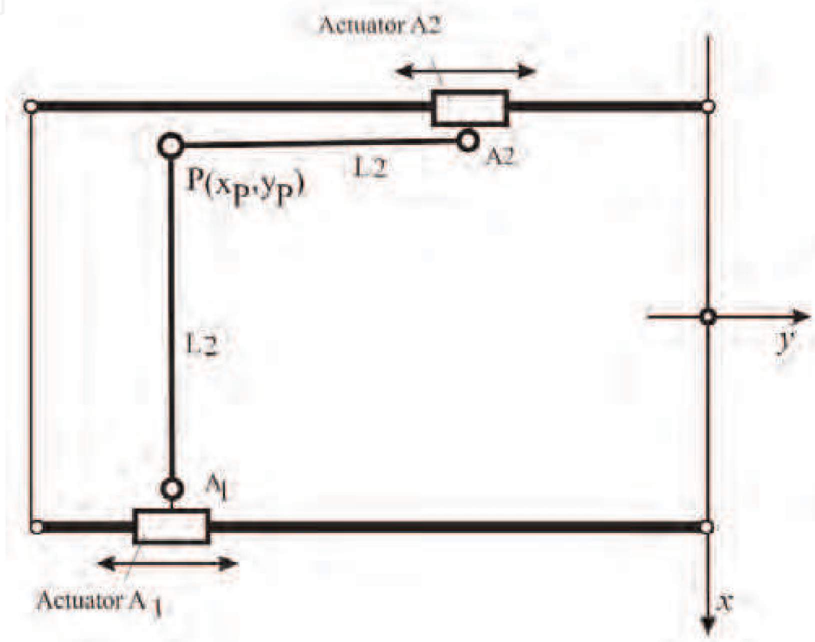


Fig. 26. Singular configuration for the planar 2 DOF Biglide Parallel Kinematic Machine

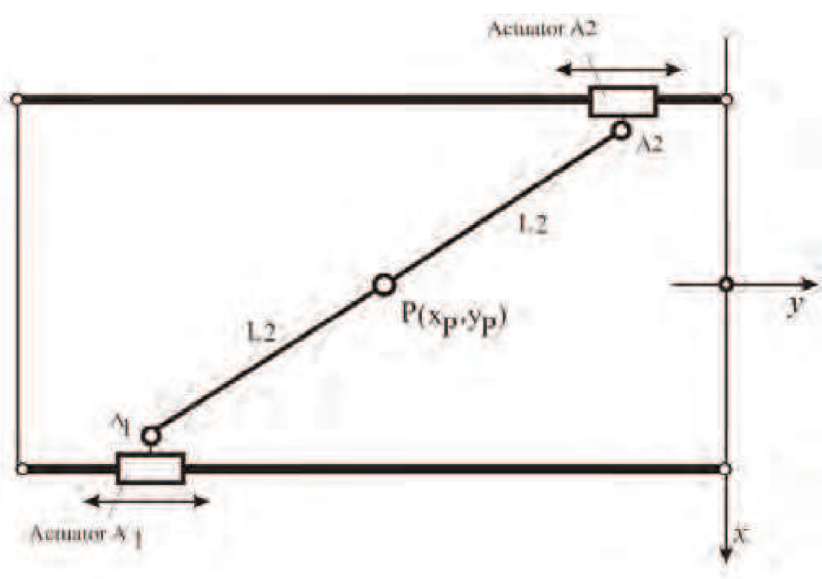


Fig. 27. Singular configuration for the planar 2 DOF Biglide Parallel Kinematic Machine

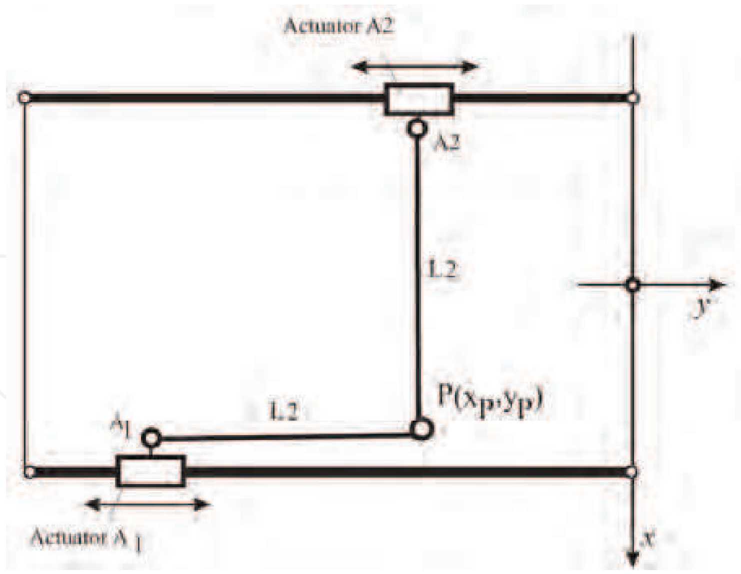


Fig. 28. Singular configuration for the planar 2 DOF Biglide Parallel Kinematic Machine

4.2 Performance evaluation

Beside workspace which is an important design criterion, transmission quality index is another important criterion. The transmission quality index couples velocity and force transmission properties of a parallel robot, i.e. power features (Hesselbach et al., 2004). Its definition runs:

$$T = \frac{\|I\|^2}{\|J\| \cdot \|J^{-1}\|} \tag{9}$$

where I is the unity matrix. T is between $0<T<1$; $T=0$ characterizes a singular pose, the optimal value is $T=1$ which at the same time stands for isotropy (Stan, 2003).

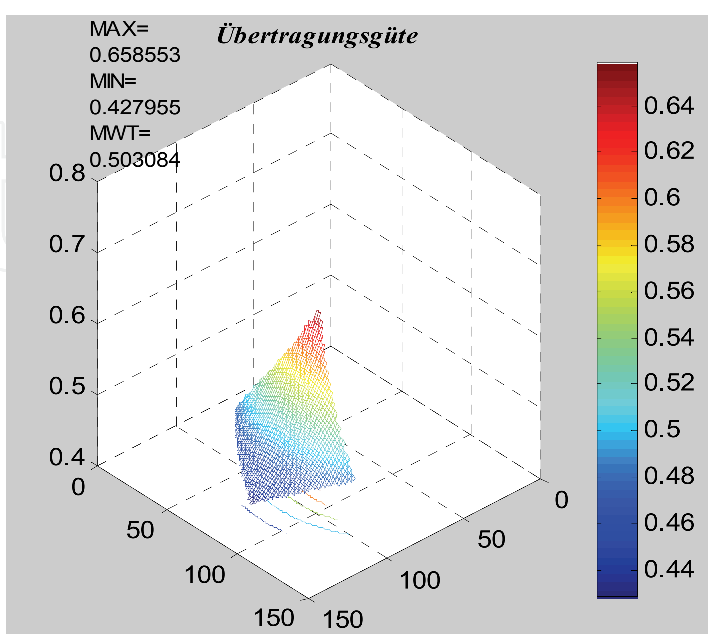


Fig. 29. Transmission quality index for RPRPR Bipod Parallel Kinematic Machine

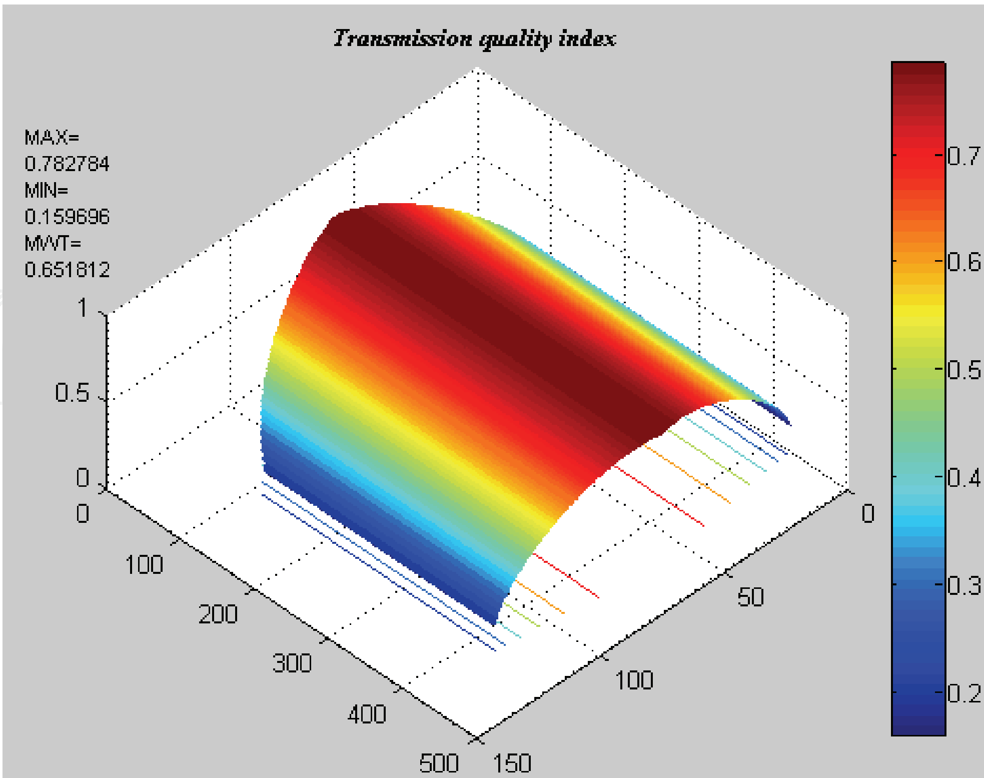


Fig. 30. Transmission quality index for PRRRP Biglide Parallel Kinematic Machine

As it can be seen from the Fig. 30, the performances of the PRRRP Biglide Parallel Kinematic Machine are constant along *y*-axis. On every *y* section of such workspace, the performance of the robot can be the same.

5. Optimal design of 2 DOF Parallel Kinematics Machines

5.1 Optimization results for RPRPR Parallel Kinematic Machine

The design of the PKM can be made based on any particular criterion. The chapter presents a genetic algorithm approach for workspace optimization of Bipod Parallel Kinematic Machine. For simplicity of the optimization calculus a symmetric design of the structure was chosen.

In order to choose the PKM's dimensions b, q_{1min}, q_{1max}, q_{2min}, q_{2max}, we need to define a performance index to be maximized. The chosen performance index is W (workspace) and T (transmission quality index).

An objective function is defined and used in optimization. It is noted as in Eq. (8), and corresponds to the optimal workspace and transmission quality index. We can formalize our design optimization problem as the following:

$$ObjFun = W + T \tag{10}$$

Optimization problem is formulated as follows: the objective is to evaluate optimal link lengths which maximize Eq. (10). The design variables or the optimization factor is the ratios of the minimum link lengths to the base link length b, and they are defined by:

$$q_{1min}/b \tag{11}$$

Constraints to the design variables are:

$$0,52<q_{1min}/b<1,35 \tag{12}$$

$$q_{1min}=q_{2min},\ q_{1max}=q_{2max},\ q_{1max}=1,6q_{1min},\ q_{2max}=1,6q_{2min} \tag{13}$$

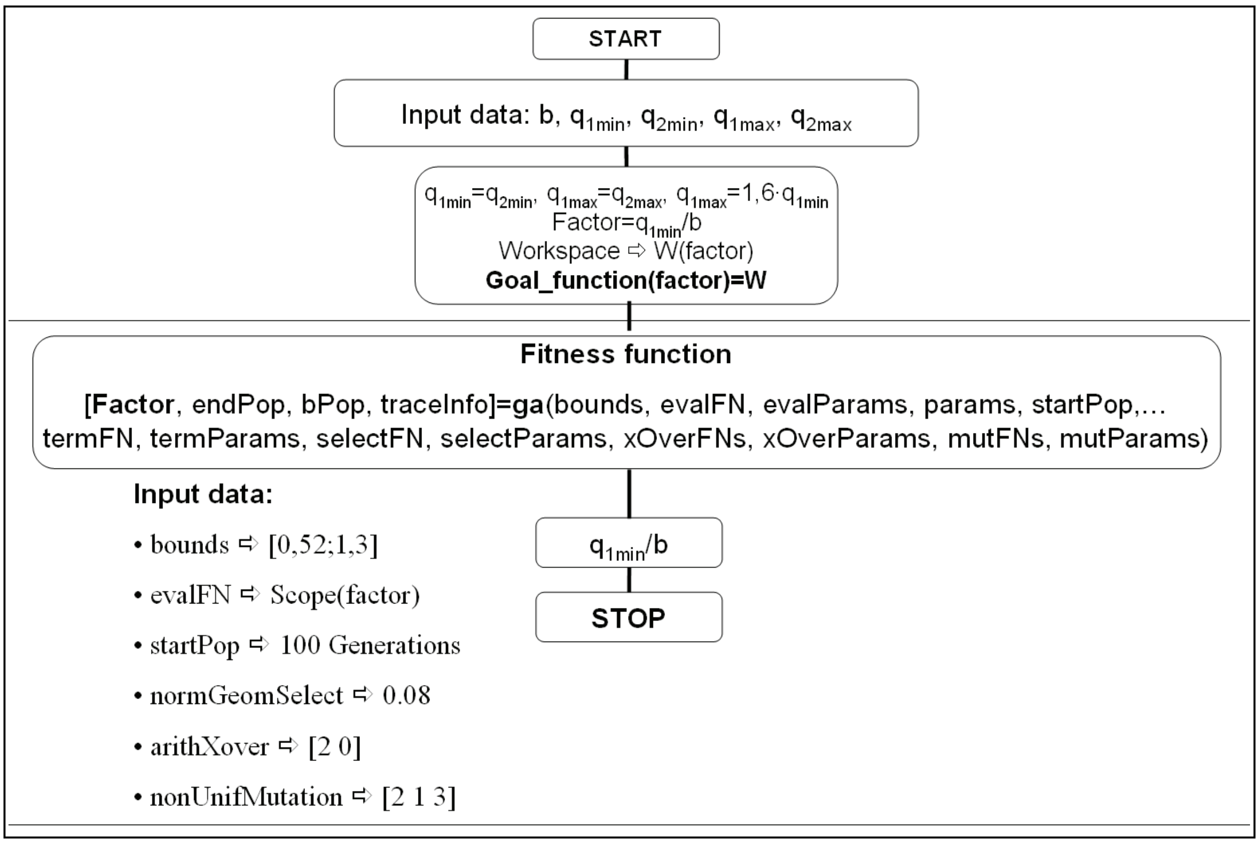


Fig. 31. Flowchart of the optimization Algorithm with GAOT (Genetic Algorithm Optimization Toolbox)

For this example the lower limit of the constraint was chosen to fulfill the condition $q_{1min}\geq b/2$ that means the minimum stroke of the actuators to have a value greater than the half of the distance between them in order to have a workspace only in the upper region. For simplicity of the optimization calculus the upper bound was chosen $q_{1min}\leq 1,35b$.
During optimization process using genetic algorithm it was used the following GA parameters, presented in Table 1.

Generations	100
Crossover rate	0.08
Mutation rate	0.005
Population	50

Table 1. GA Parameters

Researchers have used genetic algorithms, based on the evolutionary principle of natural chromosomes, in attempting to optimize the design parallel kinematics. Kirchner and Neugebaur (Kirchner & Neugebaur, 2000), emphasize that a parallel manipulator machine tool cannot be optimized by considering a single performance criterion. Also, using a genetic algorithm, they consider a multiple design criteria, such as the "velocity relationship" between the moving platform and the actuator legs, the influence of actuator leg errors on the accuracy of the moving platform, actuator forces, stiffness, as well as a singularity-free workspace.

A genetic algorithm (GA) is used because its robustness and good convergence properties. The genetic algorithms optimization approach has the clear advantage over conventional optimization approaches in that it allows a number of solutions to be examined in a single design cycle.

The traditional methods searches optimal points from point to point, and are easy to fall into local optimal point. Using a population size of 50, the GA was run for 100 generations. A list of the best 50 individuals was continually maintained during the execution of the GA, allowing the final selection of solution to be made from the best structures found by the GA over all generations.

We performed a kinematic optimization in such a way to maximize the objective function. It is noticed that optimization result for Bipod when the maximum workspace of the 2 DOF planar PKM is obtained for q_{1min}/b =1,35. The used dimensions for the 2 DOF parallel PKM were: q_{1min}=80 mm, q_{1max}=130 mm, q_{2min}=80 mm, q_{2max}=130 mm, b=60 mm. Maximum workspace of the Parallel Kinematics Machine with 2 degrees of freedom was found to be W= 4693,33 mm^2.

If an elitist GA is used, the best individual of the previous generation is kept and compared to the best individual of the new one. If the performance of the previous generation's best individual is found to be superior, it is passed on to the next generation instead of the current best individual.

There have been obtained different values of the parameter optimization (q_1/b) for different objective functions. The following table presents the results of optimization for different goal functions. W_1 and W_2 are the weight factors.

Method		GAOT Toolbox MATLAB
Goal functions	$Z=W_1 \cdot T+W_2 \cdot W$, W_1=0,7 and W_2=0,3	q_1/b = 0.92
	$Z=W1 \cdot T+W2 \cdot W$, W_1=0,3 and W_2=0,7	q_1/b= 1.13
	$Z= W_1 \cdot T$, W_1=1 and W_2=0	q_1/b=0.71
	$Z=W_2 \cdot W$, W_1=0 and W_2=1	q_1/b=1.3

Table 2. Results of Optimization for Different Goal Functions

The results show that GA can determine the architectural parameters of the robot that provide an optimized workspace. Since the workspace of a parallel robot is far from being intuitive, the method developed should be very useful as a design tool.

However, in practice, optimization of the robot geometrical parameters should not be performed only in terms of workspace maximization. Some parts of the workspace are more useful considering a specific application. Indeed, the advantage of a bigger workspace can be completely lost if it leads to new collision in parts of it which are absolutely needed in the application. However, it's not the case of the presented structure.

5.2 Optimization results for PRRRP Parallel Kinematic Machine

An objective function is defined and used in optimization. Objective function contains workspace and transmission quality index. Optimization parameter was chosen as the link length L_2. The constraints was established as $1<L_2<1.2$. After performing the optimization the following results were obtained:

Method		GAOT Toolbox MATLAB
Goal functions	$Z=W_1 \cdot T+W_2 \cdot W$, $W_1=0,7$ and $W_2=0,3$	$L_2 = 1.1$
	$Z=W_1 \cdot T+W_2 \cdot W$, $W_1=0,3$ and $W_2=0,7$	$L_2= 1.1556$
	$Z= W_1 \cdot T$, $W_1=1$ and $W_2=0$	$L_2=1$
	$Z=W_2 \cdot W$, $W_1=0$ and $W_2=1$	$L_2=1.2$

Table 3. Results of Optimization for Different Goal Functions

Based on the presented optimization methodology we can conclude that the optimum design and performance evaluation of the Parallel Kinematics Machines is the key issue for an efficient use of Parallel Kinematics Machines. This is a very complex task and in this paper was proposed a framework for the optimum design considering basic characteristics of workspace, singularities and isotropy.

6. Conclusion

The fundamental guidelines for genetic algorithm to optimal design of micro parallel robots have been introduced. It is concluded that with three basic generators selection, crossover and mutation genetic algorithm could search the optimum solution or near-optimal solution to a complex optimization problem of micro parallel robots. In the paper, design optimization is implemented with Genetic Algorithms (GA) for optimization considering transmission quality index, design space and workspace. Genetic algorithms (GA) are so far generally the best and most robust kind of evolutionary algorithms. A GA has a number of advantages. It can quickly scan a vast solution set. Bad proposals do not affect the end solution negatively as they are simply discarded. The obtained results have shown that the use of GA in such kind of optimization problem enhances the quality of the optimization outcome, providing a better and more realistic support for the decision maker.

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In recent years, parallel kinematics mechanisms have attracted a lot of attention from the academic and industrial communities due to potential applications not only as robot manipulators but also as machine tools. Generally, the criteria used to compare the performance of traditional serial robots and parallel robots are the workspace, the ratio between the payload and the robot mass, accuracy, and dynamic behaviour. In addition to the reduced coupling effect between joints, parallel robots bring the benefits of much higher payload-robot mass ratios, superior accuracy and greater stiffness; qualities which lead to better dynamic performance. The main drawback with parallel robots is the relatively small workspace. A great deal of research on parallel robots has been carried out worldwide, and a large number of parallel mechanism systems have been built for various applications, such as remote handling, machine tools, medical robots, simulators, micro-robots, and humanoid robots. This book opens a window to exceptional research and development work on parallel mechanisms contributed by authors from around the world. Through this window the reader can get a good view of current parallel robot research and applications.

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