Chapter

An Optimal Control Approach to Portfolio Diversification on Large Cap Stocks Traded in Tokyo Stock Exchange

Muhammad Jaffar Sadiq Abdullah and Norizarina Ishak

Abstract

In this chapter, Markowitz mean-variance approach is proposed for examining the best portfolio diversification strategy within three subperiods which are during the global financial crisis (GFC), post-global financial crisis, and during the non-crisis period. In our approach, we used 10 securities from five different industries to represent a risk-mitigation parameter. In this way, the naive diversification strategy is used to serve as a comparison for the approach used. During the computation process, the correlation matrices revealed that the portfolio risk is not well diversified during non-crisis periods, meanwhile, the variance-covariance matrices indicated that volatility can be minimized during portfolio construction. On this basis, 10 efficient portfolios were constructed and the optimal portfolios were selected in each subperiods based on the risk-averse preference. Performance-wise that optimal portfolio dominated the naïve strategy throughout the three subperiods tested. All the optimal portfolios selected are yielding more returns compared to the naïve portfolio.

Keywords: naïve diversification, mean-variance, Sharpe ratio, efficient frontier, portfolio optimization

1. Introduction

Investment decision and capital allocation in the stock market is subjected to the variability of risk and return. Thus, the investors, as well as the portfolio manager, must find the best solutions to allocate the various assets efficiently. Constructing a portfolio of a stock based on the investor's risk tolerance is a difficult task. In this situation, the portfolio manager needs to have some background knowledge in the economy of the market and mathematical modeling to create a portfolio based on the market participant's appetite.

In the year 1952, Markowitz's has introduced mean-variance portfolio optimization, where the investors are called rational investors by analyzing the mean and variance to determine the value of expected return and risk preference for investors [1]. This modern portfolio theory suggested that an investor can perform diversification in allocating assets in their portfolios by concerning how much risk they are willing to bear due to the outcome uncertainty in the stock market.

On the risk-return spectrum, some of the investor's favor seeking an opportunity to invest in the large-cap stock due to its stability and safer investment during turbulent times. In this study, the Tokyo Stock Exchange (TSE) is chosen as a medium to implement modern portfolio theory since the TSE has remained the focal point for investors looking to invest in Asia's largest stock market, ranking third in the world behind the New York Stock Exchange and NASDAQ in 2018 [2]. Japan has also been recognized as a member of the G7, which includes the world's six major advanced economies: Canada, France, Italy, the United Kingdom, the United States, and Germany. However, in the last decade, Japan has undergone an economic crisis period and the emergence of China has become a threat to Japan's economy. At some point, the systematic risk that occurred might affect the investor's portfolio thus leaving the greatest risk if there is no proper plan in constructing the best portfolio strategy.

Hence, the mean-variance portfolio optimization remains widely used among investors in analyzing their portfolio investment due to its simplicity and ease of derivation [3]. Therefore, there are many previous researchers had done their study using this modern portfolio theory that was first introduced by Harry Markowitz. Kulali [1] claimed that individuals are mainly aiming to select the best diversification through these two related strategies as maximizing return or minimizing risk despite given the characteristics of the country's stock market.

So, is the optimal portfolio being always the best portfolio diversification strategy? A lot of researchers have a debate regarding the issues of the mean-variance approach versus the 1/N strategy. Few researchers such as [4–7] conducted to determine the best portfolio strategy on the various samples of the equity market. Similar results are found that naïve diversification has performed better compared to the optimization model. Ramilton [8], on the other hand, refuted DeMiguel et al. [6]'s claim that a naive portfolio is preferable to an ex-ante portfolio was preferable over the ex-ante optimal portfolio. He found out that the optimal portfolios significantly outperform the naïve portfolio strategy in terms of the Sharpe ratio. Other findings such as [9] explained that if the naïve model outperforms a more sophisticated model, it is a clear indicator that the modeling of the data generating process is not accurate enough. Other research [10–12] used the mean-variance model to optimize asset allocation, therefore, the result showed the mean-variance model outperformed the naïve diversification strategy.

The aim of this paper is briefly to construct the optimal portfolio by using by Markowitz mean-variance model and compare to the naïve diversification strategy in the context of large-cap stock in Tokyo Stock Exchange (TSE) over the three subperiods which are during the global financial crisis (GFC) (2008–2010), the post-global financial crisis (2011–2013), and the non-crisis period (2014–2018).

2. Materials and methods

This study mainly focuses on the model parameters in determining the portfolio optimization construction by using mean-variance analysis on the large-cap stocks in Tokyo Stock Exchange (TSE). Then, we determine the best portfolio diversification strategy in the three subperiods tested.

2.1 Data

The historical stock prices of the Japanese stocks are collected from the Bloomberg Terminal web page and treated as a primary source of data for this study. About 571 data of the stock prices of each stock were picked from the last trading day of every week and were then transformed to weekly return (adjusted price for Japanese Yen). The analysis based on the weekly return is conducted to avoid the non-synchronous trading effect [4].

2.1.1 Stock selection

Tokyo Stock Exchange has remained the largest stock exchange in Asia in terms of market capitalization. Thus, **Table 1** shows about 10 large-cap stocks were chosen randomly upon the stocks that listed in the TOPIX Core 30 + 70 Large and are traded in Tokyo Stock Exchange. The selections of the stocks are the companies that already been existed during the period of this study.

2.1.2 Time periods

The construction of the portfolio of the stocks will be based on 10 years. To look for the workability and superiority of the portfolio strategy in a different timeframe, the 10 years will be divided into three subperiods which are during the global financial crisis (GFC) (2008–2010), the post-global financial crisis (2011–2013), and during the non-crisis period (2014–2018).

	Stocks code	Company name	Industry
1	7201.T	Nissan Motor Co. Ltd.	Automobiles and transportation equipment
2	7267.T	Honda Motor Co. Ltd	Automobiles and transportation equipment
3	8411.T	Mizuho Financial Group, Inc.	Bank
4	8306.T	Mitsubishi UFJ Financial Group, Inc.	Bank
5	8035.T	Tokyo Electron	Electric appliances and precision instruments
6	6702.T	Fujitsu	Electric appliances and precision instruments
7	8801.T	Mitsui Fudosan	Real estate
8	8802.T	Mitsubishi Estate	Real estate
9	9342.T	Nippon Telegraph and Telephone Corporation	Information and communication
10	9433.T	KDDI Corporation	Information and communication

Table 1.
Data description for large-cap stock (TOPIX Core 30 + 70 Large).

2.2 Model framework of portfolio optimization

The rate of return determines whether the investors gain or lose money from their investment [12]. In this study, we calculate the weekly stock return for stock i at time t, as shown in the Eq. (1).

$$ri = \frac{Sit - Sit - 1}{Sit - 1} \tag{1}$$

where S_{it} is the closing price of stock i at time t. $S_{it} - 1$ be the closing price at time $t - 1$. We assume that stock i pays dividends.

We calculate the average return of stocks based on the following equation:

$$\mu i = E(ri) = \frac{1}{M}\sum_{t=1}^{M} rit \tag{2}$$

where is an average return on stock i, is a market return of stocks i at time t, M is the number of weeks. Then, García et al. [13] defines the expected return of a portfolio is the weighted average of the expected returns of individual stocks. Thus, the equation of expected return for n asset is as the following:

$$E(rp) = \sum_{i=1}^{n} wiE(ri) \tag{3}$$

where is the proportion of the funds invested in stock i, n is the number of stocks, and are the return of ith stock and the return of portfolio p, respectively.

Garcia et al. [10] consider risk as uncertainty through the variability of future returns. So, we calculate the variance of stock i on the weekly return and the index return using the historical volatility formula as in Eq. (4). The higher value in variance for an expected return, the higher the dispersion of expected returns, and the greater the risk of the investment [14].

$$\sigma_i^2 = Var(r_i) = \frac{\sum_{i=1}^{n} (r_i - \mu_i)^2}{M - 1} \tag{4}$$

where,

$Var(r_i)$ is a variance of weekly stock return,

r_i is a weekly stock return,

μ_i is an average weekly return,

M is the sample size.

Ivanovic et al. [12] were able to measure how the stocks vary together by determining the covariance of each stock. The dimensions of risk are organized in the covariance matrix which is denoted by $\Omega_{n\times n}$. This matrix contains variance in its main diagonal and covariances between all pairs of stocks.

$$\Omega_{n\times n} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \dots & \sigma_n^2 \end{pmatrix} \tag{5}$$

where,

$$\sigma_{ij} = Cov\left(r_i, r_j\right) = \frac{\sum_{i=1,j=1}^{n} (r_i - \mu_i)\left(r_j - \mu_j\right)}{n - 1} \tag{6}$$

Equation (6) can be interpreted as the sum of the distance for each value and from the mean is divided by the number of observations minus one. Then, the covariance enables us to calculate the correlation coefficient, shown as:

$$\rho = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \tag{7}$$

where σ is the standard deviation of each asset. This measure is useful to determine the degree of portfolio risk [1].

In this study, the standard deviation is used to measure the risk of the portfolio. The standard deviation of the portfolio is the most common statistical indicator of an asset's risk which measures the dispersion around the expected value [12]. By means, the higher the risk, the higher value of standard deviation. Hence, the equation for standard deviation is defined as follows:

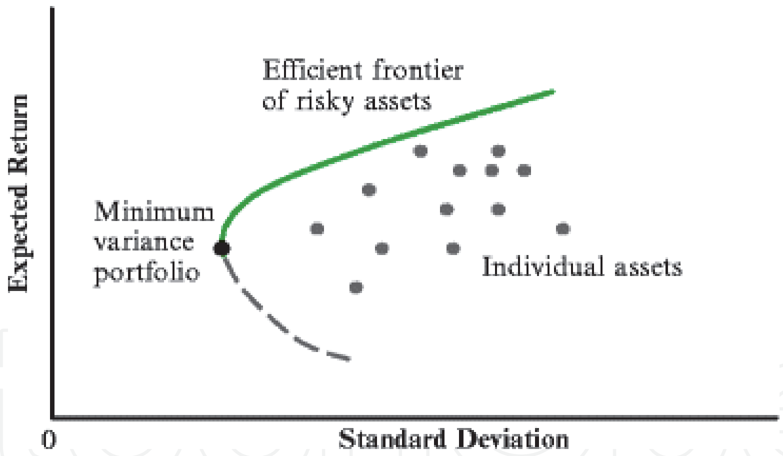

Figure 1.
Efficient frontier [15].

$$\sigma_p = \sqrt{\sum_{i=1}^{n} (w_i^2 \cdot \sigma_i^2) + 2\left(\sum_{i=1}^{n} \sum_{j=1}^{n} w_i \cdot \sigma_i \cdot w_j \cdot \sigma_j \cdot \rho\right)} \tag{8}$$

where,
σ_p is the standard deviation of a portfolio,
σ_i is the standard deviation of stocks,
w_i is the weight of stocks in a portfolio,
ρ is the correlation coefficient between stock i and j.

Based on the parameters used in the Markowitz mean-variance model, we can get different combinations of expected return and risk. Every possible asset combination is known as an attainable set that can be plotted in risk-return space, and the collection of all such possible portfolios defines a region in this space. **Figure 1** shows that the line along the upper edge of this region is known as the efficient frontier. It is defined as the best return of a portfolio with the lowest risk.

2.3 Performance evaluation ratio

In this study, the Sharpe ratio will be used to measure the portfolio performance that provides risk premium per unit of total risk, which is measured by the portfolio's standard deviation of return. The risk premium is defined as the difference between portfolio return and the risk-free rate. It can be expressed in the following equation:

$$S_p = \frac{\mu_p - R}{\sigma_p} \tag{9}$$

where R is the return on the risk-free asset.

3. Results and discussion

In this study, excel functions and data solvers are used for all calculations. The construction of portfolio optimization using the Markowitz mean-variance approach involves the following steps:

Step 1: Determining return and standard deviation of 10 stocks during three subperiods (**Tables 2–4**).

Step 2: Creating correlations matrix (**Tables 5–7**).

Step 3: Creating variance-covariance matrices during three subperiods (**Tables 8–10**).

Step 4: Calculating the volatility and return of the portfolio with equal weight (**Table 11**).

Step 5: Calculating the volatility and return of the portfolio with difference weights (**Tables 12–14** (all three tables in the Appendices).

Step 6: Creating an efficient frontier (**Figure 2A–C**).

Table 2 reflects during GFC from 2008 to 2010, most of the stock were volatile at high risk and yielding negative returns since the bearish market except Honda Motor, Tokyo Electron, and Mitsui Fudosan which annual returns vary from 0.08 to 6.24%. The risks of 10 stocks vary from 30.05 to 51.63% and Mizuho Financial Group, Inc. has the highest risk recorded during the crisis. As seen from **Table 3**, all

Stock	Average weekly return (GFC)	Weekly variance	Weekly standard deviation	Average annual return	Annual variance	Annual standard deviation
7201.T	−0.0005	0.0035	0.0596	−0.0249	0.1845	0.4296
7267.T	0.0012	0.0036	0.0601	0.0624	0.1876	0.4331
8411.T	−0.0052	0.0051	0.0716	−0.2713	0.2666	0.5163
8306.T	−0.0033	0.0040	0.0633	−0.1693	0.2086	0.4567
8035.T	0.0008	0.0041	0.0643	0.0399	0.2152	0.4639
6702.T	−0.0001	0.0031	0.0554	−0.0065	0.1594	0.3993
8801.T	0.0000	0.0044	0.0665	0.0008	0.2301	0.4796
8802.T	−0.0012	0.0042	0.0648	−0.0622	0.2186	0.4676
9342.T	−0.0013	0.0017	0.0417	−0.0684	0.0903	0.3005
9433.T	−0.0023	0.0021	0.0458	−0.1177	0.1092	0.3305

Table 2.
Risk and return of stocks during global financial crisis (GFC) (2008–2010).

Stocks	Average weekly return (post-GFC)	Weekly variance	Weekly standard deviation	Average annual return	Annual variance	Annual standard deviation
7201.T	0.0017	0.0017	0.0414	0.0882	0.0891	0.2986
7267.T	0.0027	0.0017	0.0410	0.1405	0.0875	0.2958
8411.T	0.0031	0.0014	0.0380	0.1603	0.0752	0.2743
8306.T	0.0035	0.0015	0.0393	0.1845	0.0803	0.2834
8035.T	0.0018	0.0023	0.0476	0.0931	0.1177	0.3431
6702.T	0.0009	0.0022	0.0471	0.0453	0.1153	0.3395
8801.T	0.0065	0.0024	0.0490	0.3373	0.1249	0.3533
8802.T	0.0057	0.0023	0.0477	0.2968	0.1181	0.3436
9342.T	0.0030	0.0007	0.0271	0.1552	0.0383	0.1957
9433.T	0.0073	0.0016	0.0404	0.3799	0.0849	0.2914

Table 3.
Risk and return of stocks during post-global financial crisis (2011–2013).

Stocks	Average weekly return (post-GFC)	Weekly variance	Weekly standard deviation	Average annual return	Annual variance	Annual standard deviation
7201.T	0.001	0.001	0.035	0.030	0.062	0.249
7267.T	−0.001	0.002	0.041	−0.049	0.087	0.296
8411.T	−0.001	0.001	0.038	−0.028	0.076	0.275
8306.T	0.000	0.002	0.040	0.000	0.082	0.286
8035.T	0.004	0.002	0.048	0.217	0.119	0.345
6702.T	0.002	0.002	0.048	0.101	0.121	0.349
8801.T	−0.001	0.002	0.050	−0.041	0.129	0.359
8802.T	−0.002	0.002	0.049	−0.079	0.125	0.353
9342.T	0.002	0.001	0.027	0.115	0.039	0.198
9433.T	0.001	0.002	0.040	0.071	0.084	0.290

Table 4.
Risk and return of stocks during non-crisis period (2014–2018).

	7201.T	7267.T	8411.T	8306.T	8035.T	6702.T	8801.T	8802.T	9342.T	9433.T
7201.T	1.0000									
7267.T	0.7219	1.0000								
8411.T	0.2307	0.3298	1.0000							
8306.T	0.4480	0.5110	0.4336	1.0000						
8035.T	0.6080	0.5780	0.3285	0.5550	1.0000					
6702.T	0.4944	0.5349	0.3392	0.4581	0.5457	1.0000				
8801.T	0.4380	0.4951	0.2525	0.6559	0.5903	0.3973	1.0000			
8802.T	0.4247	0.4908	0.2504	0.6595	0.6089	0.4247	0.9345	1.0000		
9342.T	0.1628	0.1932	0.6257	−0.0272	0.1526	0.0968	0.0145	0.0151	1.0000	
9433.T	0.0116	0.1050	0.3155	0.0058	0.1032	0.1025	−0.0161	−0.0193	0.5407	1.0000

Table 5.
Correlation matrix during GFC (2008–2010).

stocks are yielding positive returns ranging from 4.53 to 33.73% during the period of post-GFC. Mitsui Fudosan lead the market with the highest annual return and the lowest annual return is Fujitsu. Despite the fact that the annual returns show a difference, both stocks are significantly risky, with risk levels ranging between 35.33 and 33.95%. **Table 4** shows during the non-crisis period, about four stocks have poor performance with a negative rate of return. The remaining stocks are ranging from 0.00 to 21.7%. Tokyo Electron is dominating the market with the highest return and the Mitsubishi UFJ Financial Group, Inc. is recovering with the lowest return. The risks of each stock are varying from 19.8 to 35.3% which is dominated by the Mitsui Fudosan. The lowest risk recorded is from Nippon Telegraph and Telephone Corporation. Thus, apart from the results of the risk and return of the individual stock from the three subperiods, the variation of risks per weekly and annual basis has indicated that there is a chance of high variability of investment.

	7201.T	7267.T	8411.T	8306.T	8035.T	6702.T	8801.T	8802.T	9342.T	9433.T
7201.T	1.0000									
7267.T	0.7637	1.0000								
8411.T	0.5127	0.6103	1.0000							
8306.T	0.5722	0.6906	0.8291	1.0000						
8035.T	0.5211	0.5929	0.4321	0.4715	1.0000					
6702.T	0.5485	0.6442	0.5812	0.5465	0.4767	1.0000				
8801.T	0.4850	0.5510	0.5963	0.7078	0.4016	0.4785	1.0000			
8802.T	0.4617	0.5707	0.5644	0.6776	0.3914	0.4632	0.9088	1.0000		
9342.T	0.4512	0.4152	0.5442	0.5402	0.4066	0.4524	0.3644	0.2784	1.0000	
9433.T	0.0571	0.1661	0.1578	0.1544	−0.0024	0.0445	0.1061	0.1138	−0.0014	1.0000

Table 6.
Correlation matrix during post-GFC (2010–2013).

	7201.T	7267.T	8411.T	8306.T	8035.T	6702.T	8801.T	8802.T	9342.T	9433.T
7201.T	1.0000									
7267.T	0.7103	1.0000								
8411.T	0.7115	0.7206	1.0000							
8306.T	0.6906	0.7406	0.9125	1.0000						
8035.T	0.4873	0.4708	0.4467	0.4460	1.0000					
6702.T	0.3999	0.5016	0.4878	0.5004	0.4169	1.0000				
8801.T	0.5912	0.6267	0.6363	0.6740	0.3856	0.3891	1.0000			
8802.T	0.5492	0.6149	0.6176	0.6445	0.3753	0.3685	0.8805	1.0000		
9342.T	0.3704	0.3668	0.3275	0.3107	0.2708	0.1538	0.4520	0.4683	1.0000	
9433.T	0.3592	0.3631	0.3555	0.3454	0.2831	0.1806	0.4431	0.5035	0.6966	1.0000

Table 7.
Correlation matrix during the non-crisis period (2014–2018).

Stock	7201.T	7267.T	8411.T	8306.T	8035.T	6702.T	8801.T	8802.T	9342.T	9433.T
7201.T	0.0035	0.0026	0.0016	0.0017	0.0023	0.0016	0.0017	0.0016	0.0004	0.0007
7267.T	0.0026	0.0036	0.0020	0.0019	0.0022	0.0018	0.0020	0.0019	0.0005	0.0009
8411.T	0.0016	0.0020	0.0051	0.0039	0.0024	0.0020	0.0029	0.0027	0.0010	0.0008
8306.T	0.0017	0.0019	0.0039	0.0040	0.0023	0.0016	0.0027	0.0027	0.0008	0.0009
8035.T	0.0023	0.0022	0.0024	0.0023	0.0041	0.0019	0.0025	0.0025	0.0005	0.0010
6702.T	0.0016	0.0018	0.0020	0.0016	0.0019	0.0031	0.0014	0.0015	0.0009	0.0008
8801.T	0.0017	0.0020	0.0029	0.0027	0.0025	0.0014	0.0044	0.0041	0.0008	0.0014
8802.T	0.0016	0.0019	0.0027	0.0027	0.0025	0.0015	0.0041	0.0042	0.0008	0.0012
9342.T	0.0004	0.0005	0.0010	0.0008	0.0005	0.0009	0.0008	0.0008	0.0017	0.0012
9433.T	0.0007	0.0009	0.0008	0.0009	0.0010	0.0008	0.0014	0.0012	0.0012	0.0021

Table 8.
Variance-covariance matrix during GFC.

Stock	7201.T	7267.T	8411.T	8306.T	8035.T	6702.T	8801.T	8802.T	9342.T	9433.T
7201.T	0.0017	0.0013	0.0008	0.0009	0.0010	0.0011	0.0010	0.0009	0.0005	0.0003
7267.T	0.0013	0.0017	0.0010	0.0011	0.0012	0.0013	0.0011	0.0011	0.0005	0.0004
8411.T	0.0008	0.0010	0.0015	0.0012	0.0008	0.0010	0.0011	0.0010	0.0006	0.0005
8306.T	0.0009	0.0011	0.0012	0.0016	0.0009	0.0010	0.0014	0.0013	0.0006	0.0005
8035.T	0.0010	0.0012	0.0008	0.0009	0.0023	0.0011	0.0009	0.0009	0.0005	0.0006
6702.T	0.0011	0.0013	0.0010	0.0010	0.0011	0.0022	0.0011	0.0011	0.0006	0.0004
8801.T	0.0010	0.0011	0.0011	0.0014	0.0009	0.0011	0.0024	0.0021	0.0005	0.0005
8802.T	0.0009	0.0011	0.0010	0.0013	0.0009	0.0011	0.0021	0.0023	0.0003	0.0005
9342.T	0.0005	0.0005	0.0006	0.0006	0.0005	0.0006	0.0005	0.0003	0.0007	0.0005
9433.T	0.0003	0.0004	0.0005	0.0005	0.0006	0.0004	0.0005	0.0005	0.0005	0.0016

Table 9.
Variance-covariance matrix during post-GFC.

Stock	7201.T	7267.T	8411.T	8306.T	8035.T	6702.T	8801.T	8802.T	9342.T	9433.T
7201.T	0.0020	0.0014	0.0014	0.0018	0.0014	0.0011	0.0014	0.0012	0.0006	0.0007
7267.T	0.0014	0.0021	0.0014	0.0020	0.0013	0.0014	0.0016	0.0015	0.0006	0.0007
8411.T	0.0014	0.0014	0.0020	0.0023	0.0012	0.0013	0.0015	0.0014	0.0005	0.0007
8306.T	0.0018	0.0020	0.0023	0.0034	0.0016	0.0017	0.0021	0.0019	0.0007	0.0009
8035.T	0.0014	0.0013	0.0012	0.0016	0.0040	0.0016	0.0013	0.0012	0.0006	0.0008
6702.T	0.0011	0.0014	0.0013	0.0017	0.0016	0.0035	0.0013	0.0011	0.0004	0.0005
8801.T	0.0014	0.0016	0.0015	0.0021	0.0013	0.0013	0.0030	0.0025	0.0010	0.0011
8802.T	0.0012	0.0015	0.0014	0.0019	0.0012	0.0011	0.0025	0.0027	0.0009	0.0012
9342.T	0.0006	0.0006	0.0005	0.0007	0.0006	0.0004	0.0010	0.0009	0.0015	0.0012
9433.T	0.0007	0.0007	0.0007	0.0009	0.0008	0.0005	0.0011	0.0012	0.0012	0.0021

Table 10.
Variance-covariance matrix during non-crisis period.

Period	Portfolio return	Portfolio variance	Portfolio standard deviation
During GFC	−0.0012	0.0019	0.0435
Post-GFC	0.0036	0.0010	0.0310
Non-crisis	0.0006	0.0014	0.0372

Table 11.
Portfolio with equal weight (naïve diversification).

Correlation between assets determines the degree of portfolio risk [11]. Negative or small correlation between assets, the risk of the portfolio is low whereas the positive or large correlation between the assets, the risk of the portfolio is high. As seen in **Table 5**, only 9342.T–8306.T, 9433.T–8801.T, and 9433.T–8802.T have a negative correlation between the stocks and others are greater than zero. **Table 6** reflects that only two pairs of stocks which are 9433.T–8035.T and 9433.T–9342.T have a negative correlation and **Table 7** has all positive correlation between the stocks and not low enough. In that sense, only during the non-crisis period the portfolio constructed will not be well diversified.

When there are more than two assets, covariance can best be calculated by using matrix algebra based on Eq. (6). The covariance calculation which involved the excess return of stocks and the number of observations in each subperiod allows us to compute by using few functions in excel such as @MMULT(...) and TRANSPOSE(...). The procedure has already allowed us to determine the variance-covariance matrix as per **Tables 8–10**. The values of the variance-covariance matrices have shown that in the three subperiods tested, all the stocks are move in the same direction with a lower degree. Thus, this indicates that the volatility can be reduced during the construction of the portfolios.

The variance-covariance matrix helps in a simple way of measuring portfolio variance. At this step, the portfolio return and portfolio variance can be calculated. In this sense, the naïve allocation of weight whereby the equal proportion in the portfolio is used to calculate the portfolio return and portfolio variance. Since we have 10 stocks, each stock will have about 1/10 or 10% weight.

So, **Table 11** shows that the portfolio returns and portfolio variance of the naïve diversification strategy. Among the three subperiods, during GFC, the portfolio indicates poor performance by having a negative return of −0.12% and the highest standard deviation of 4.35%. In contrast, the portfolio return is recorded high during post-GFC which is about 0.36% with the lowest standard deviation of 3.10%.

Note that, Markowitz's mean-variance model stated that the investor must be rational and risk-averse [16]. Thus, in constructing the efficient portfolios, two conditions need to be satisfied which are maximum return for varying levels of risk and minimum risk for varying levels of expected return. In the context of three subperiods tested, this study proposed rational investors for looking at optimal portfolio with a minimum level of risk. Thus, this optimal portfolio analysis can be shown as the subject function according to the formula:

$$Min\sigma^2 \sum_{j=1}^{n} w_i w_j Cov_{ij} \tag{10}$$

where w_i and w_j are weights of stocks in the portfolio and Cov_{ij} is the covariance value between stock i and j.

Then, three main constrains in Markowitz mean-variance portfolio optimization are included for the optimization problem. The formulas are written as:

$$\sum_{i=1}^{n} w_i E(r_i)^3 \geq E* \tag{11}$$

$$\sum_{i=1}^{n} w_i = 1 \tag{12}$$

and

$$wi \geq 0; i = 1, \ldots, N \tag{13}$$

where $E(R_i)$ is the target expected return, E^* is an expected return and w_i is the weight of the stock i. The third constraint is added to restrict the short sell to happen.

Hence, Excel Solver is used to optimizing the weight by including all the constraints according to Eqs. (10)–(13). About 10 iterations were done in Solver to

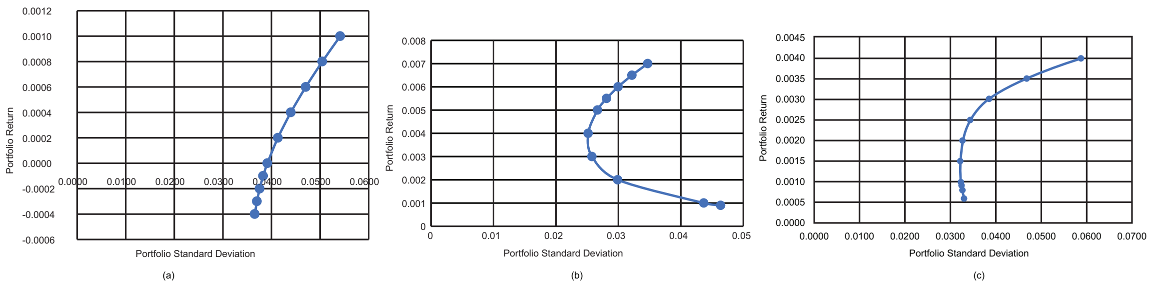

Figure 2.
(a) Efficient frontier of portfolios during GFC, (b) efficient frontier of portfolios during post-GFC, and (c) efficient frontier of portfolios during the non-crisis period.

generate 10 portfolios. The 10 portfolios that are generated produce 10 different sets of weights for the level of return which minimizes the standard deviation of the portfolio. All the results are shown in **Tables 12–14** (in Appendices) indicate the composition of stocks, portfolio return, and portfolio risk. All the set of possible expected return and risk combinations is then called attainable set [1]. The attainable set is plotted on the risk-return space as shown in **Figure 2A–C**.

Figure 2A–C demonstrate the efficient frontier with minimum variance portfolios. Ivanova et al. [11] states that given expected return, investors could choose an optimal portfolio based on risk preference since all the efficient portfolios are located over the efficient frontier curve. Many portfolios are existed on the efficient frontier curve with different risk and return combinations. Then, the optimal portfolio is selected on the efficient frontier curve in each subperiods then compare with the portfolio return and risk for naïve diversification strategy.

Table 15 reflects the analysis on the risk, return and performance of the portfolio constructed by using naïve diversification strategy and Markowitz mean-variance portfolio optimization. In the three subperiods tested, the optimal portfolio outperformed the naïve diversification strategy.

In terms of portfolio risk-return, optimal portfolios selected during global financial crisis tend to have a high standard deviation of 4.71% compared to naïve diversification which is about 4.35%, respectively. But, the portfolio with equal weight yields an abnormal return of −0.12% compared to the optimal portfolio of 0.06%. This is not a surprising result since the turbulence of economic recession contributes to the high-risk investment. During the post-GFC, again the optimal portfolio yields a higher return with 0.5% with lower risk at 2.67% compared to the naïve strategy. Furthermore, there is not much difference in standard deviation for both strategies in the non-crisis period, but the optimal portfolio still dominates with a high return at 2.5% and low risk at 3.44%.

Sharpe ratio classifies during financial crisis both strategies produce negative Sharpe ratio which indicates the portfolio's return is less than the risk-free rate. Thus, the interpretation of the negative Sharpe ratio will be put aside since it does not convey any useful meaning. For the rest two periods, the Sharpe ratio is undoubtedly the highest for the optimal portfolio compare to the naïve portfolio. This shows the reward-to-volatility ratio when investing in the optimal portfolio 6 and portfolio 7 are 0.1032 and 0.0582.

4. Conclusion

This study was aimed to help investors to plan for the best investment strategy in maximizing return with the given level of risk or minimizing risk. In this study, there are subperiods been tested for this whole study, during the global financial crisis (GFC) 2008–2010, the post-financial crisis 2011–2013 and non-financial crisis 2014–2018. Then, we followed the Markowitz mean-variance model which involved the best possible combination of expected return and risk to construct efficient portfolios in each period. The portfolio which did not contain short sale was achieved by a data solver. We made a choice to choose the optimal portfolio from this efficient set based on rational investors.

Meantime, we also constructed a portfolio with equal weight as a control parameter to determine the best diversification strategy. It was figured out during the period of study; the optimal portfolio was chosen from the efficient frontier formed by Markowitz model perform better than the portfolio with equal weight in all three subperiods. The result reiterates the previous study conducted by [1, 9, 11]. Hence, the Markowitz framework is successful since the variance-covariance and correlation played an important role in determining the optimal portfolio. So, if the Japanese and foreign investors know properly how to apply the Markowitz mean-variance model in their investment. We believe this is the best solution in many alternatives.

But, the limitation of this study is there is no out-of-sample data tested. For future research, a robust optimization approach can be considered with the out-of-sample data tested to construct the optimal portfolio. The portfolio also needs to be rebalanced again to ensure a more accurate result.

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Appendices

	Portfolio return	Portfolio standard deviation	Weight of stock									
			7201.T	7267.T	8411.T	8306.T	8035.T	6702.T	8801.T	8802.T	9342.T	9433.T
Portfolio 1	−0.0004	0.0366	0.0441	0.2204	0	0	0.1213	0.0578	0.0071	0	0.5418	0.0075
Portfolio 2	−0.0003	0.0370	0.0158	0.2558	0	0	0.1360	0.0576	0.0050	0	0.5298	0
Portfolio 3	−0.0002	0.0376	0	0.2927	0	0	0.1503	0.0552	0.0002	0	0.5017	0
Portfolio 4	−0.0001	0.0383	0	0.3268	0	0	0.1602	0.0500	0	0	0.4630	0
Portfolio 5	0.0000	0.0392	0	0.3607	0	0	0.1701	0.0450	0	0	0.4242	0
Portfolio 6	0.0002	0.0413	0	0.4285	0	0	0.1899	0.0351	0	0	0.3465	0
Portfolio 7	0.0004	0.0440	0	0.4964	0	0	0.2097	0.0252	0	0	0.2688	0
Portfolio 8	0.0006	0.0471	0	0.5642	0	0	0.2294	0.0152	0	0	0.1911	0
Portfolio 9	0.0008	0.0505	0	0.6321	0	0	0.2492	0.0053	0	0	0.1134	0
Portfolio 10	0.0010	0.0541	0	0.6988	0	0	0.2676	0	0	0	0.0335	0

Table 12.
Composition of weight and risk of different expected return portfolios during GFC period.

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	Portfolio return	Portfolio standard deviation	Weight of stock									
			7201.T	7267.T	8411.T	8306.T	8035.T	6702.T	8801.T	8802.T	9342.T	9433.T
Portfolio 1	0.0009	0.0464	0.0350	0	0	0	0	0.9650	0	0	0	0
Portfolio 2	0.0010	0.0437	0.1561	0	0	0	0	0.8439	0	0	0	0
Portfolio 3	0.0020	0.0299	0.2235	0	0	0	0.0862	0.2812	0	0	0.4091	0
Portfolio 4	0.0030	0.02588	0.1318	0	0.0182	0	0.0377	0.0341	0	0.0425	0.6933	0.0426
Portfolio 5	0.0040	0.0252	0.0710	0	0	0	0	0	0	0.1197	0.6170	0.1813
Portfolio 6	0.0050	0.0267	0	0	0	0	0	0	0.0743	0.1165	0.4765	0.3327
Portfolio 7	0.0055	0.0281	0	0	0	0	0	0	0.1656	0.0522	0.3673	0.4150
Portfolio 8	0.0060	0.03	0	0	0	0	0	0	0.2467	0	0.2555	0.4978
Portfolio 9	0.0065	0.0322	0	0	0	0	0	0	0.2842	0	0.1326	0.5832
Portfolio 10	0.0070	0.0347	0	0	0	0	0	0	0.3216	0	0.0098	0.6686

Table 13.
Composition of weight and risk of different expected return portfolios during post-GFC period.

DOI: http://dx.doi.org/10.5772/intechopen.10613

	Portfolio return	Portfolio standard deviation	Weight of stock									
			7201.T	7267.T	8411.T	8306.T	8035.T	6702.T	8801.T	8802.T	9342.T	9433.T
Portfolio 1	0.0006	0.0331	0.0579	0.1859	0.2419	0	0	0.0389	0	0.0465	0.3273	0.1016
Portfolio 2	0.0008	0.0327	0.0774	0.1554	0.2273	0	0	0.0585	0	0.0195	0.3602	0.1018
Portfolio 3	0.0009	0.0325	0.0869	0.1401	0.2198	0	0	0.0683	0	0.0062	0.3767	0.1020
Portfolio 4	0.0010	0.0323	0.0985	0.1192	0.2085	0	0	0.0795	0	0	0.3946	0.0997
Portfolio 5	0.0015	0.0322	0.1379	0.0138	0.1478	0	0.0305	0.1212	0	0	0.4687	0.0801
Portfolio 6	0.0020	0.0328	0.1324	0	0.0393	0	0.0969	0.1442	0	0	0.5373	0.0499
Portfolio 7	0.0025	0.0344	0.0331	0	0	0	0.1938	0.1506	0	0	0.6226	0
Portfolio 8	0.0030	0.0385	0	0	0	0	0.4072	0.0402	0	0	0.5525	0
Portfolio 9	0.0035	0.0468	0	0	0	0	0.6586	0	0	0	0.3414	0
Portfolio 10	0.0040	0.0587	0	0	0	0	0.9157	0	0	0	0.0843	0

Table 14.
Composition of weight and risk of different expected return portfolios during non-crisis period.

Period	Diversification strategy	Portfolio return	Portfolio standard deviation	Sharpe ratio
During GFC (2008–2009)	Naïve strategy	−0.0012	0.0435	−0.1047
	Optimal portfolio 8	0.0006	0.0471	−0.0349
Post-GFC (2011–2013)	Naïve strategy	0.0036	0.031	0.0437
	Optimal Portfolio 6	0.005	0.0267	0.1032
Non-crisis period (2014–2018)	Naïve strategy	0.0006	0.0372	0.0027
	Optimal portfolio 7	0.0025	0.0344	0.0582

Table 15.
Comparison of portfolio risk, return, and performance evaluation.

Author details

Muhammad Jaffar Sadiq Abdullah and Norizarina Ishak*
Faculty of Science and Technology, Universiti Sains Islam Malaysia, Nilai, Malaysia

*Address all correspondence to: norizarina@usim.edu.my

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