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Switching Control in the Presence of Constraints and Unmodeled Dynamics

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1. Introduction

Recently there has been increased research interest in the study of the hybrid dynamical systems (Sun & Ge, 2005) and (Li et al., 2005). These systems involve the interaction of discrete and continuous dynamics. Continuous variables take the values from the set of real numbers and the discrete variables take the values from finite set of symbols. The hybrid systems have the behaviour of an analog dynamic system before certain abrupt structural or operating conditions are changed. The event driven dynamics in hybrid control systems can be described using different frameworks from discrete event systems (Cassandras & Lafortune, 2008) such as timed automata, max-plus algebra or Petry nets. For dynamic systems whose component are dominantly discrete event, main tools for analysis and design are representation theory, supervisory control, computer simulation and verification. From the clasical control theory point of view, hybrid systems may be considered as a switching control between analog feedback loops. Generally, hybrid systems can achieve better performance then non-switching controllers because they can to reconfigure and reorganize their structures. For that is necessery correct coordination of discrete and analog control variables.

The mathematical model for real process, generally, has the Hammerstein-Wiener form (Crama & Atkins, 2001) and (Zhao & Chen, 2006). It means that on the input and output of the process are present nonlinear elements (actuator and sensor). Here we will consider Hammerstein model which has the input saturation as nonlinear element. That is the most frequent nonlinearity encountered in practice (Hippe, 2006). Also, unmodeled dynamics with matching condition is present. As a control strategy will be used switching control. The switched systems can be viewed as higher abstraction of hybrid systems.

The design of switching controllers having guaranted stability, known as the picewise linear LQ control (PLC), is first considered in (Wredenhagen & Belanger, 1994). The picewise linear systems are systems that have different linear dynamics in different regions of the continuous state space (Johansson, 2003). The PLC control has the associated switching surfaces in form of positively invariant sets and yields a relatively low-gain controller. In the LHG (low-and-high gain) design a low gain feedback law is first designed in such a way

Source: New Approaches in Automation and Robotics, Book edited by: Harald Aschemann, ISBN 978-3-902613-26-4, pp. 392, May 2008, I-Tech Education and Publishing, Vienna, Austria

that the actuator does not saturate in magnitude and the closed – loop system remains linear. The low gain enlarge the region in which the closed-loop system remains linear and enlarge the basin of attraction of the closed-loop systems (Lin, 1999). After that, using appropriate Lyapunov function for the closed-loop system, under this low gain feedback control law, a linear high gain feedback law is designed and added to the low gain feedback control. Combination of LHG and PLC gives the robust controllers with fast transience. The key feature of PLC/LHG controllers is that the saturation level is avoided. But, it has been recognized in references (Lin et al., 1997) and (De Dona et al., 2002) that the performance of closed-loop system can be further improved by forcing the control into saturation. Such controller increases the value of the switching regions so that each linear controller is able to act in a region where a degree of over-saturation is reached. The over-saturation means that the controller demands for input level is greater than the avaliable range.
The actuator rate saturation, also, is important problem. Namely, the phase lag associated with saturation rate has a destabilizing effect (Saberi et al., 2000). The problem is more severe when the actuator is, also, subject to magnitude saturation since small actuator output results in small stability margin even in the absence of rate saturation (Lin et al., 1997).
The problem is more complex in the presence of delay in the system. The paper (Tarbouriech & da Silva, 2000) deals with the synthesis of stabilizing controllers for linear systems with state delay and saturation controls. Performance guided hybrid LQ controller for discrete time-delay systems is considered in (Filipovic, 2005). In (Wu et al., 2007) the method for designing an output feedback law that stabilize a linear system subject to actuator saturation with large domain of attraction is considered. It is usually true that higher performance levels are associated with pushing the limits (Goodwin et al., 2005). That is motivation to operate the system on constraint bounderies. It means that problem with actuator saturation can be considered as optimisation with constraints.
In this paper the robustness of picewise linear LQ control with prescribed degree of stability using switching, low-and-high gain and over-saturation is considered. The process is described with linear uncertain dynamic system in the state space form. Structure of the uncertaintes is defined with matching conditions. By the Lyapunov stability criterion (Michel et al., 2008) it is shown that a robust PLC/LHG controller with allowed over-saturation, can exponentially stabilize linear uncertain systems with prescribed exponential rate. This approach is different in comparison with (De Dona et al., 2002) where the Riccati equation approach is used.

2. Switching controller with prescribed degree of stability

The dynamic system subject to input saturation can be described in the next form

$$\dot{x}(t) = Ax(t) + B\,\mathrm{sat}_\Delta u(t)\,,\; x(t_0) = x_0 \in X \subset R^n \tag{1}$$

Where $x \in R^n$ and $u \in R^m$. Nonlinear input function (saturation function) is defined as

$$\mathrm{sat}_\Delta : R^m \to R^m$$

$$\Delta = [\Delta_1, \cdots, \Delta_m] \,,\ \Delta_i > 0 \,,\ i = 1,2,\cdots,m$$

$$sat_\Delta(u) = \left[sat_{\Delta_1}(u_1), \cdots, sat_{\Delta_m}(u_m)\right]^T$$

$$sat_{\Delta_i}(u_i) \overset{\Delta}{=} \operatorname{sign}(u_i)\min(|u_i|, \Delta_i) \tag{2}$$

The u_i is the ith element of vector u. As in (Wredenhagen & Belanger, 1994) we take a sequence $\{\rho_i\}_{i=1}^N$ such that $\rho_1 > \rho_2 \cdots > \rho_N > 0$ and matrix $Q > 0$. Then that one can to define matrix

$$R_i = \operatorname{diag}\left[r_i^1, r_i^2, \cdots, r_i^m\right], \quad r_i^j > 0, \ \ j = 1,2,\cdots,m \tag{3}$$

Design of optimal LQ regulator with prescribed degree of stability is based on minimization of next functional

$$J(x_0, \alpha) = \int_{t_0}^{\infty} e^{2\alpha t}\left[x^T(t)Qx(t) + v^T(t)R_i v(t)\right]dt \tag{4}$$

The quantity α in (4) defines the degree of system stability for the feedback control systems. From (4) for every R_i we can get matrices P_i and K_i from equations

$$(A + \alpha I)^T P_i + P_i(A + \alpha I) - P_i B R_i^{-1} B^T P_i + Q = 0 \tag{5}$$

$$K_i = R_i^{-1} B^T P_i \tag{6}$$

where P_i is the positive solution of the algebraic Riccati equation (5) for the optimal LQ problem. Matrix K_i is gain of the controller.
The switching surfaces are ellipsoids defined by

$$E_i = E_i(P_i, \rho_i) \overset{\Delta}{=} \left\{x : x^T e^{-2\alpha t} P_i x \le \rho_i\right\}, \quad i = 1,2\cdots,N \tag{7}$$

Elements of matrix R_i are chosen, for a given q_i, to be the largest so that is satisfied

$$|v_j| = \left|\frac{1}{r_i^j} b_j^T P_i x\right| \le (1 + \beta_j)\Delta_j, \quad \forall x \in E_i(P_i, \rho_i) \,, \qquad j = 1,\cdots,m_i \tag{8}$$

where v_j is the j th element of $v = -K_i x$, b_j is the j th column of matrix B.
The ellipsoids in the sequence $\{E_i\}_{i=1}^N$ are nested, i.e.

$$E_{i+1} \subset E_i \quad , \ i = 1,2\cdots,N-1 \tag{9}$$

he state space region contained in the biggest ellipsoid can be devided into N cells $\{C_i\}_{i=1}^{N}$

$$C_i = E_i / E_{i-1} \;\; , \;\; C_N = E_N \tag{10}$$

Now, the controller based on the switching strategy is given in the next form

$$v = -K_i x \;\; , \;\; x \in C_i \;, \; i = 1,2...,N \tag{11}$$

The controller (5), (6) and (11) for $\alpha = 0$ is originally proposed in (Tarbouriech & da Silva, 2000). This controller is known as the picewise linear LQ contoller (PLC). Associated switching surfaces to PLC strategy are positively invariant sets (Blanchini, 1999) and (Blanchini & Miani, 2008) given by nested ellipsoids.

The PLC strategy which is proposed in (Lin, 1999) is low-gain controller. Such controller underutilize the available control capacity and the resulting convergence of the regulation error to zero is very slow although saturation is avoided.

Remark 1. Recent advances in miniaturizing, communication, sensing and actuation have made it feasible to envision large numbers of autonomous vehicles working cooperatively to accomplish an objective (Ren & Beard, 2008). The communication band and power constraints preclude centralized command and control. As a result a critical problem for cooperative control is to design distributed algorithms such that the group of vehicles can reach consensus on the shared information in the presence of limited and unreliable information exchange and dynamically changing interaction topologies. For the switching information exchange topologies the convergence of consensus protocol is proved (Ren & Atkins, 2007). As a controller the dwell time controller (switching controller) is used. From the stability of switched system follows that consensus can be achieved asymptotically. That is the new interesting application of hybrid systems.

In the next section will be considered switching control strategy which leads to better closed-loop performance.

3. Robust switching controller

A common feature for controllers which are described in (Hippe, 2006) is that the saturation levels are avoided. Because such controllers are low-gain controllers. It has been discovered in (De Dona et al., 2002) and (Lin et al., 1997) that the performance of closed loop system can be improved by forcing the controls into the saturation. In (De Dona et al., 2002) is shown that combination of switching, scaling and over-saturation has a superior performance then in the case where each is used seperetly.

In this section, we define a class of uncertain linear systems

$$\dot{x}(t) = [A + \Delta A\,(\omega(t))]\,x(t) + [B + \Delta B\,(\omega(t))]\,u(t) \tag{12}$$

The matrices A and B are the nominal system and input matrices respectively and $\Delta A(\omega)$ and $\Delta B(\omega)$ are uncertain matrices which depend continuosly on the uncertainty vector $\omega(t)$

$$\omega(t) \in \Omega \subset R^p \;\; , \quad t \in [0, \infty) \tag{13}$$

We will suppose that the following assumptions are satisfied

A.1) (A, B) is controllable

A.2) $\Omega \in R^p$ is a compact set

A.3) There are continuous mappings

$$D(\cdot): \omega \to R^{m\times n} \ , \ E(\cdot): \omega \to R^{m\times m}$$

$$\Delta A(\omega) = BD(\omega) \ , \ \Delta B(\omega) = BE(\omega), \forall \omega \in \Omega$$

Assumption A.3) is known as matching contition . This assumption can be relaxed using the notion of mismatching threshold (Barmish & Leitmann, 1982).
In the control law (11) a high-gain component is incorporated by multiplying the gains with a scaling factor $(1+k)$ with $k \geq 0$. The PLC control law with low-and-high gain has the form

$$u(t) = (1+k)K_i x(t) \quad , \quad x \in C_i \quad , \quad i = 1,2,..,N \tag{14}$$

where is k design parameter.
We, also, can to introduce the over-saturation index as in (De Dona et al., 2002). Let us define the function $\beta_i(t)$ as

$$\beta_i(t) = \begin{cases} \dfrac{v_i(t) - \mathrm{sat}\Delta_i(v_i(t))}{\mathrm{sat}\Delta_i(v_i(t))} & , \ v_i(t) \neq 0 \\ 0 & , \ v_i(t) = 0 \end{cases} \tag{15}$$

The over-saturation index is a constant $\overline{\beta}_i$ such that

$$\left\|\beta_i(t)\right\|_\infty \leq \overline{\beta}_i \tag{16}$$

In the case of over-saturation elements of matrix R_i are chosen to be the largest so that is satisfied

$$\left|v_{j_s}\right| = \left|\frac{1}{r_i^j} b_j^T P_i x\right| \leq \left(1+\overline{\beta}_j\right)\Delta_j \ , \ \forall x \in E_i(P_i q_i) \tag{17}$$

Now we will formulate theorem which defines the conditions under which control system which is described with relations (12) and (14) – (17) is exponentially stable.
Theorem 1. Let us suppose that the closed loop system described with (12) and (14) – (17) for wich together with assumptions A.1) - A.3), also, are satisfied

A.4) R_i is positive definite matrix

$$R_i = \begin{bmatrix} r_1^i & & D \\ & \ddots & \\ D & & r_i^m \end{bmatrix} > 0 \ , \ i = 1,2,\ldots,N$$

A.5) $\lambda_{min}\left\{Q-2D^T(\omega)B^TP_i-aK_i^TR_iK_i\right\}>0$

For precomputed gains

$$K_i \quad , \quad i=1,2..,N$$

$$a\in(0,1) \quad \text{for} \quad \|D(\omega)\|>0 \quad , \quad \|E(\omega)\|>0$$

$$a=0 \quad \text{for} \quad \|D(\omega)\|=\|E(\omega)\|=0$$

A.6) $\lambda_{min}\{E(\omega)+I\}>1-a \quad , \quad \forall\omega\in\Omega$

A.7) The allowed over-saturation for each element v_i of the control vector v is

$$[\beta_{max}]_j=\min_{i=1,2,..,N}\sqrt{1+\frac{4\lambda_{min}\left\{Q-2D^T(\omega)B^TP_i-aK_i^TR_iK_i\right\}}{\lambda_{min}\{E(\omega)+I_m\}\left(\sum_{Q=p+1}^{m} r_i^{j_l}\right)\left\|K_i^{j_s}\right\|^2}}$$

where k_i^j is the j th row of matrix K_i

Then feedback (14)-(17) exponentially stabilize system (12) with exponential speed α for $\forall\omega\in\Omega$ and $x\in E_1$.

Proof: In the control systems (12) and (14)-(17) the continuous states $x(t)$ and discrete state (relating to the switching paradigm) are present and we choose a piecewise quadratic Lyapunov function

$$V(x)=x^TP_ix\,, \qquad x\in C_i\,,\; i=1,2...,N \tag{18}$$

Using relation (12) we have

$$\begin{aligned}
\dot V(x)&=\dot x^TP_ix+x^TP_i\dot x=[(A+\Delta A(\omega))x+(B+\Delta B(\omega))]^T\cdot\\
&\cdot P_ix+x^TP[(A+\Delta A(\omega))x+(B+\Delta B(\omega))sat_\Delta(u)]=\\
&=\left[x^T(A+BD(\omega))^T+(sat_\Delta(-(1+k)K_ix))^T(B+BE(\omega))\right]^T\cdot\\
&P_ix+x^TP_i[(A+BD(\omega))x+(B+BE(\omega))sat_\Delta(-(1+k)K_ix)]=\\
&x^T\left[(A+BD(\omega))^TP_i+P_i(A+BD(\omega))\right]x+\\
&+(sat_\Delta(-(1+k)K_ix))^T(B+BE(\omega))^TP_ix+x^TP_i\cdot\\
&\cdot(B+BE(\omega))sat_\Delta(-(1+k)K_ix)
\end{aligned} \tag{19}$$

From relation (5) we have

$$A^T P_i + P_i A = P_i B R_i^{-1} B^T P_i - 2\alpha P_i - Q \tag{20}$$

Using relation (20) the first term in (19) becames

$$x^T \left[(A + BD(\omega))^T P_i + P_i (A + BD(\omega)) \right] x =$$

$$x^T \left(A^T P_i + P_i A \right) x + x^T D^T(\omega) B^T P_i x + x^T P_i D(\omega) x =$$

$$= x^T P_i B R_i^{-1} B^T P_i x - 2\alpha x^T P_i x - x^T Q x + \ + x^T D^T(\omega) B^T P_i x + x^T P_i B D(\omega) x =$$

$$= -x^T P_i B v - 2\alpha V(x) - x^T Q x + x^T D^T(\omega) B^T(\omega) \cdot P_i x + x^T P_i B D(\omega) x =$$

$$= v^T P_i v - 2\alpha V(x) - x^T Q x - x^T D^T(\omega) R_i v - v^T R_i D(\omega) x \tag{21}$$

Also, one can get

$$x^T Q x + x^T D^T(\omega) R_i v + v^T R_i D(\omega) x - a v^T R_i v - a v^T R_i v =$$

$$= x^T Q x - 2 x^T D^T(\omega) B^T P_i x - a x K_i^T R_i K_i x + a v^T R_i v =$$

$$= x^T \left(Q - 2D^T(\omega) B^T P_i - a K_i^T R_i K_i \right) x + a v^T R_i v \tag{22}$$

For a second and third terms in relation (19) we have

$$(\mathrm{sat}_\Delta(-(1+k)K_i x))^T (B + BE(\omega))^T P_i x + \ x^T P_i (B + BE(\omega)) \mathrm{sat}_\Delta(-(1+k)K_i x) =$$

$$(\mathrm{sat}_\Delta(-(1+k)K_i))^T B^T P_i x + (\mathrm{sat}_\Delta(-(1+k)K_i))^T E^T(\omega) B^T P_i x + B x^T P_i B \mathrm{sat}_\Delta(-(1+k)K_i x) +$$

$$x^T P_i B + BE(\omega) \mathrm{sat}_\Delta(-(1+k)K_i x) = -\mathrm{sat}_\Delta(-(1+k)K_i x)^T R_i v - (\mathrm{sat}_\Delta(-(1+k)K_i x))^T E^T(\omega) R_i v -$$

$$- v^T R_i \mathrm{sat}_\Delta(-(1+k)K_i x) - v^T R_i E(\omega) \mathrm{sat}_\Delta(-(1+k)K_i x) =$$

$$= -(\mathrm{sat}_\Delta(-(1+k)K_i x))^T \left[E^T(\omega) + I \right] R_i v - v^T R_i \left[E^T(\omega) + I \right] \mathrm{sat}_\Delta(-(1+k)K_i x)$$

$$= -2 v^T R_i \left[E^T(\omega) + I \right] \mathrm{sat}_\Delta(-(1+k)K_i x) \le -2\lambda_{\min}\{E(\omega) + I\} v^T R_i \mathrm{sat}_\Delta(-(1+k)k_i x)$$

$$\le -2\lambda_{\min}\{E(\omega) + I\} v^T R_i \mathrm{sat}_\Delta(-(1+k)k_i x) \tag{23}$$

From (19) and (21)-(23) and assumption A6) of Theorem follows

$$\dot{V}(x) \le (1+a) v^T R_i v - 2\alpha V(x) - x^T \left(Q - 2D^T(\omega) B^T P_i - a K_i^T R_i K_i \right) x -$$

$$- 2\lambda_{\min}\{E(\omega) + I\} v^T R_i \mathrm{sat}_\Delta(-(1+k)k_i x) \le -2\alpha V(x) + \lambda_{\min}\{E(\omega) + I\} v^T R_i v - x^T \cdot$$

$$\left(Q-2D^T(\omega)B^TP_i-aK_i^TR_iK_i\right)x-2\lambda_{min}\{E(\omega)+I\}v^TR_i sat_{\Delta}\left(-(1+k)k_ix\right)=$$

$$-2\alpha V(x)-x^T\left(Q-2D^T(\omega)B^TP_i-aK_i^TR_iK_i\right)x++\lambda_{min}\{E(\omega)+I\}\cdot$$

$$\cdot\left[\sum_{j=1}^{m} r_i^j\left|v_j\right|\left(\left|v_j\right|-2sat_{\Delta_j}\left((1+k)K_i\left|v_j\right|\right)\right)\right] \tag{24}$$

In last relation is the v_j is the j th element of $v_i=-K_ix$ i.e.

$$v_j=-\frac{1}{r_i^j}b_j^TP_ix=-k_i^jx \tag{25}$$

From relation (17) one can get

$$\left|v_j\right|\leq\left(1+\overline{\beta}\right)\Delta_j\leq\left(1+[\beta_{max}]_j\right)\quad,\quad\forall x\in C_i\subset E_i\quad,\quad j=1,2,\ldots,m \tag{26}$$

and $[\beta_{max}]_j$ is defined in the assumption A.7) of Theorem.

For the $x\neq 0$ we have two posibilites

$$\left|v_{j_s}\right|\leq 2\Delta_{j_s}\quad,\quad j_s\in S_1\quad,\quad S_1=\{j_1,j_2,\cdots,j_p\} \tag{27}$$

and

$$2\Delta_{j_s}<\left|v_{j_s}\right|\leq\left(1+[\beta_{max}]_{j_s}\right)\Delta_{j_s}\quad,\ j_s\in S_2\ ,\ S_2=\{j_{p+1},\cdots,j_m\} \tag{28}$$

whereby

$$0\leq p\leq m\quad,\quad S_1\cup S_2=\{1,2,..,m\}\quad,\quad S_1\cap S_2=\phi$$

Using argument as in (De Dona et al., 2002) one can get

$$x^Tx>\sum_{s=p+1}^{m}\frac{r_i^{j_s}}{\left(\sum_{l=p+1}^{m} r_i^{j_l}\right)}\frac{4\Delta_{j_s}^2}{\left\|k_i^{j_s}\right\|} \tag{29}$$

From (24) and (29) follows

$$\dot{V}(x)-2\alpha V(x)+\lambda_{min}\{E(\omega)+I\}\cdot\left\{\sum_{s=1}^{p} r_i^{j_s}\left|v_{j_s}\right|\left[\left|v_{j_s}\right|-2sat_{\Delta_{j_s}}\left((1+k)\left|v_{j_s}\right|\right)\right]+\sum_{s=p+1}^{m} r_i^{j_s}\cdot\right.$$

$$\cdot\left[\left|v_{j_s}\right|^2 - 2\left|v_{j_s}\right|\Delta_{j_s} - A_1\right]\Bigg\}A_1 = \frac{4\lambda_{\min}\left\{Q - 2D^T(\omega)B^T P_i - aK_i^T R_i K_i\right\}\Delta_{j_s}^2}{\lambda_{\min}\{E(\omega)+I\}\left(\sum_{l=p+1}^{m} r_i^{j_l}\right)\left\|K_i^{j_s}\right\|^2} \tag{30}$$

According to relation (27) first term in (30) is always nonpositive, i.e.

$$\left|v_{j_s}\right| - 2\,\mathrm{sat}_{\Delta_{j_s}}\left((1+k)\left|V_{j_s}\right|\right) \le 0 \quad , \quad \forall k \ge 0 \tag{31}$$

It is well known fact that quadratic polinomial

$$z^2 + p_1 z + p_2 \quad , \quad z \in R^1$$

is negative if zeros belong to the interval

$$\left(\frac{-p_1 - \sqrt{D}}{2}, \frac{-p_1 + \sqrt{D}}{2}\right) \ , \ D = p_2^2 - 4p_2$$

Using those facts it is possible to conclude that second term in relation (30) is nonpositive if

$$\left|v_{j_s}\right| \le \left(1 + \sqrt{1 + \frac{4\lambda_{\min}\left\{Q - 2D^T(\omega)B^T P_i - aK_i^T R_i K_i\right\}}{\lambda_{\min}\{E(\omega)+I\}\left(\sum_{l=p+1}^{m} r_i^{j_l}\right)\left\|K_i^{j_s}\right\|^2}}\right)\Delta_{j_s}$$

This is true because from the definition of elipsoid E_i fallows

$$\left|v_{j_s}\right| \le \left(1 + [\beta_{\max}]_{j_s}\right)\Delta_{j_s} \quad , \quad j_s \in S_1 \cup S_2 \tag{32}$$

whereby $[\beta_{\max}]_{j_s}$ is defined in the assumption A7) of Theorem.
From relation (31)-(32) follows

$$\dot{V}(x) < -2\alpha V(x) \ , \ \forall x \ne 0 \ , \ x \in C_i \ , i = 1,2,\ldots,N \tag{33}$$

It means that closed-loop system is exponentially stable. Namely

$$\left\|x(t_j)\right\| \le k_e \left\|x(t_1)\right\| \exp\{-\alpha(t_2 - t_1)\} , \forall x \ne 0 \ , \ x \in C_i \tag{34}$$

whereby

$$k_1 = \sqrt{\frac{\lambda_{\max}\{P_i\}}{\lambda_{\min}\{P_i\}}} \quad , \quad i = 1,2,\ldots,N \tag{35}$$

The trajectories in each cell C_i approach the origin with a exponential decrease in $V(x)$ along the trajectory. According with the philosophy of control strategy, the trajectories will enter the smallest ellipsoid corresponding to ρ_N. The exponential stability is assured by (33).

Remark 2. In the paper (De Dona et al., 2004) is considered the case when in the model (12) uncertainty matrix $\Delta B(w) = 0$ and the degree of prescribed stability $\alpha = 0$. Also, in that reference the Riccati equation approach is used until in this paper the Lyapunov approach is used.

Remark 3. When unmodeled dynamic is absent, i.e. in the Theorem 1

$$a = 0 \ , \ \|D(w)\| = \|E(w)\| = 0$$

the conditions A.5) and A.7) in the Theorem have the form

$$\lambda_{\min}\{Q\} > 0$$

$$[\beta_{\max}]_{j_S} = \min_{i=1,2,\ldots,N} \sqrt{1 + \frac{4\lambda_{\min}\{Q\}}{\left(\sum_{l=p+1}^{m} r_i^{j_l}\right)\left\|K_i^{j_S}\right\|^2}}$$

These assumption are identical with the assumptions in reference (De Dona et al., 2002).

4. Conclusion

In this paper the switching controller with low-and-high gain and alloved over-saturation for uncertain system is considered. The unmodeled dynamics satisfies matching conditions. Using picewise quadratic Lyapunov function it is proved the exponential stability of the closed loop system.

It would be interesting to develop the theory for output case and for the descrete-time case. Also, extremely is important application of hybrid systems in distributed coordination problems (multiple robots, spacecraft and unmanned air vehicles).

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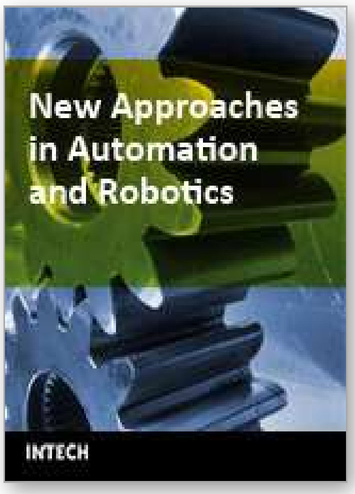

New Approaches in Automation and Robotics

Edited by Harald Aschemann

ISBN 978-3-902613-26-4
Hard cover, 392 pages
Publisher I-Tech Education and Publishing
Published online 01, May, 2008
Published in print edition May, 2008

The book New Approaches in Automation and Robotics offers in 22 chapters a collection of recent developments in automation, robotics as well as control theory. It is dedicated to researchers in science and industry, students, and practicing engineers, who wish to update and enhance their knowledge on modern methods and innovative applications. The authors and editor of this book wish to motivate people, especially under-graduate students, to get involved with the interesting field of robotics and mechatronics. We hope that the ideas and concepts presented in this book are useful for your own work and could contribute to problem solving in similar applications as well. It is clear, however, that the wide area of automation and robotics can only be highlighted at several spots but not completely covered by a single book.