Chapter

Taxation and Redistribution against Inequality: A Mathematical Model

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Abstract

Reducing inequality is a tremendously important sustainable development goal. Albeit providing stylised frames for modelling, also mathematics can contribute to understanding and explaining the emergence of collective patterns in complex socio-economic systems. It can then effectively help to identify actions and measures to be taken and support policy-makers towards adoption of conceivable welfare measures aimed at halting the growth of inequality. Based on these assumptions, we here discuss some variants of a mathematical "micro-to-macro" model for the dynamics of taxation and redistribution processes in a closed trading market society. The model has an exploratory character resulting from possible tuning of various parameters involved: through its analysis, one can foresee the consequences on the long-run income distributions of different fiscal policies and differently weighted welfare policies, interventions, and subsidy provision, as well as the impact of the extent of tax evasion. In short, the model shows that in the long term redistributive policy results in a lower level of economic inequality in society.

Keywords: taxation and redistribution, welfare, income distribution, economic inequality, mathematical models

1. Introduction

Income disparities and economic inequality are all but recent problems. They are, however, becoming worryingly and increasingly significant (for a few references in this regard see e.g. [1–7]). In fact, reducing inequality within and between countries is one of the sustainable development goals included in the 2030 Agenda of by the United Nations General Assembly. Certainly, such an objective is, as least in the first instance, a main responsibility and competence of economists and policy-makers. Nonetheless, mathematics can provide some hints and contributions towards it.

The goal of this chapter is to provide a brief illustration of the role that mathematics can play in this regard. By way of example, we will recall an elementary (first approximation) model first proposed by Bertotti in [8], together with some variants and extensions of it, developed and investigated in a series of other studies by Bertotti et al. [9–13].

The approach of this model can be seen as a contribution attempt in the spirit of complexity economics. This paradigm, which began to take shape in the late 1980s, looks at the economy as an evolving system, not in equilibrium, and puts emphasis on the process through which structures and patterns emerge from the micro-interactions (see e.g. [14–16]). Similarly, the perspective of the model here discussed is to put the interactions among heterogeneous individuals at the very heart of the question. These interactions lead to self-organised collective features and macro-observables, which emerge from the system as a whole.

It should be noted here that during the last two/three decades a research line has been developing, not only but mostly among the physicist's community, which addresses socio-economic questions and phenomena using ideas, methods and tools, which have their roots in statistical mechanics and the kinetic theory of gases. An explanation for that comes from the existing analogy, for example, between complex systems composed by a number of individuals who interact exchanging money with each other (may be participating in a financial market) and physical systems consisting of a huge number of particles (atoms or molecules), which interact with each other undergoing collisions.

A variety of tools and techniques, including for example Boltzmann type equations, Fokker-Planck type equations, Ising type models and agent-based simulations, have been adapted and employed in this connection. See for example references [17–21] and the survey papers [22, 23] that offer an interesting historical perspective and also contain extensive bibliographies.

The model developed in the paper [8] and then further generalised and explored in subsequent work [9–13] differs to a great extent from those we know belonging to the mentioned literature strand. The motivation behind [8] was precisely to understand how the taxation process and diverse fiscal systems could affect the income distribution of a population. Aiming at modelling a fiscal system with taxes on personal income levied at a finite number of progressive rates, with high-income earners expected to pay more than low-income earners, as in the case of the Italian IRPEF (Imposta sul Reddito delle Persone Fisiche), the most natural approach seemed to be one dividing a population into a finite number of income classes. This is the reason behind the construction of a discrete framework, suitable for the formulation of the model, which will be briefly recalled in the next section. Albeit expressed using a system of ordinary differential equations (ODE)—as many as the income classes in the society—the model incorporates stochastic and probabilistic components. In a nutshell, the ODE system governs the evolution in time of the income distribution, generated by a whole of money exchanges expressing binary individual interactions, and a whole of withdrawals and earnings of the individuals, due to taxation and redistribution. Specifically, each differential equation in the system describes the variation in time of the fraction of individuals belonging to a certain class. As we will see in the next section, the framework allows possible tuning of various parameters (the frequencies with which the interactions are supposed to occur, the probability that in an encounter between two individuals the one who pays is one or the other, the tax rates or other) that give it an exploratory character.

The study of the dynamics of the model, supported by some analytical results and, inevitably, largely pursued through numerical simulations (performed using Mathematica software [24]), focuses on the asymptotic behaviour of the system. What all simulations suggest is that after a sufficiently long time the solutions of the equations reach a stationary state corresponding to an income distribution, which depends on the total wealth and the interaction parameters, but not on the specific initial distribution. This stationary state represents in fact a macro-observable feature. At the micro-individual level, the economic exchanges continue to take place and the situation is a non-equilibrium one.

The interest is to find and compare one with the other different shapes of the asymptotic income distributions corresponding to different fiscal policies. In this connection, the model shows that over time redistributive policy leads to a reduction of economic inequality.

The rest of the chapter is organised as follows:

In Section 2, we recall the framework of the original model.

In Section 3, we revisit some insights and extensions of the model discussed in previous work. The issues include the following:

- the occurrence of fat tails of the asymptotic income distribution in cases with a high number of classes;
- the existence of a very good fit of certain asymptotic income distributions with cases (characterised by suitable parameters) of the κ-generalised distribution introduced by Kaniadakis [25] and then analysed in connection with real data in the works [26, 27];
- the incorporation in the model of an additional welfare form;
- the analysis of the negative correlation between economic inequality and social mobility predicted by the model;
- the effect of tax evasion.

In Section 4, a novel application of the model is developed, which shows the impact of different fiscal policies.

Section 5 contains concluding considerations.

2. General framework

Referring the reader to the paper [8] for further explanations and details on the mechanism behind the formulation of the proposed framework, we recall here that, if n denotes the number of income classes of a population, characterised by their average incomes $r_1 < r_2 < \ldots < \ r_n$, and $x_i(t)$, with $x_i : \mathbf{R} \to [0, +\infty)$ for $i = 1, 2, \ldots, n$ denotes the fraction at the time t of individuals belonging to the i-th class (with the normalisation $\sum_{i=1}^{n} x_i = 1$), the variation in time of the quantities $x_i(t)$ may be thought to obey a system of differential equations of the form

$$dx_i dt = \sum_{h=1}^{n} \sum_{k=1}^{n} \left(C_{hk}^{i} + T_{[hk]}^{i}(x) \right) x_h x_k - x_i \sum_{k=1}^{n} x_k, \qquad i = 1, 2, \ldots, n. \tag{1}$$

The coefficients C_{hk}^{i}'s and the continuous functions $T_{[hk]}^{i}$'s in (Eq. (1)) incorporate the instructions for the variation of the fraction of individuals (i.e. the movement of individuals) from one class to another. They keep into account "impoverishment and enrichment" due both to direct money exchanges taking place between pairs of individuals and to the (small) withdrawals and earnings of each individual, due to taxation and redistribution, processes that are here considered as occurring in correspondence to each transaction. More precisely, $C_{hk}^{i} \in [0, +\infty)$ expresses the probability density that an individual of the h-th class will belong to the i-th class after a direct interaction with an individual of the k-th class. Accordingly, the identity $\sum_{i=1}^{n} C_{hk}^{i} = 1$ has to be satisfied for any fixed h and k;

- $T^i_{[hk]} : \mathbf{R}^n \to \mathbf{R}$ expresses the variation density in the i-th class due to an interaction between an individual of the h-th class with an individual of the k-th class. The functions $T^i_{[hk]}$ are required to satisfy $\sum_{i=1}^n T^i_{[hk]}(x) = 0$ for any fixed h, k and $x \in \mathbf{R}^n$.

Specific expressions for these quantities have to be carefully calibrated if we want, as is the case in the model at hand, to treat a case in which the total amount of money is constant. Towards this, let

- S denote a fixed minimum amount of money that individuals may exchange;
- $p_{h,k}$ (for $h, k = 1, 2, \dots, n$) denote the probability that in an interaction between an individual of the h-th class with an individual of the k-th class, the one who pays is the former one. In principle, no interaction may occur between individuals of two classes, and thus, the $p_{h,k}$ are required to satisfy $0 \le p_{h,k} \le 1$ and $p_{h,k} + p_{k,h} \le 1$;
- $0 \le \tau_1 \le \tau_2 \le \dots \le \tau_n \le 1$ denote the tax rates relative to the n income classes.

The ratio for the definition of C^i_{hk} and $T^i_{[hk]}$ is that when an individual of the h-th class pays a quantity S to an individual of the k-th class, this one in turn has to pay a tax $S\tau_k$. The government, for its part, redistributes to the entire population the revenue collected by all taxes and this one in particular (this redistribution may be interpreted as public expenditure in health, education, security and defence, transports and so on). From a practical standpoint, the effect of a payment of S from an h-individual to a k-individual can be thought, bypassing the government, as the same of a payment of $S(1-\tau_k)$ from the h-individual to the k-individual and payment of $S\tau_k$ from the h-individual to the entire population.

Skipping here some technical details, we recall that the expressions proposed in paper [8] for C^i_{hk} and $T^i_{[hk]}$ are as follows: each C^i_{hk} can be written as $C^i_{hk} = a^i_{hk} + b^i_{hk}$, where the only nonzero elements a^i_{hk} are $a^i_{ij} = 1$ for $i, j = 1, 2, \dots, n$ and the only possibly nonzero elements b^i_{hk} are those of the form

$$\begin{aligned} b^i_{i+1,k} &= p_{i+1,k} S \frac{1-\tau_k}{r_{i+1} - r_i}, \\ b^i_{i,k} &= -p_{k,i} S \frac{1-\tau_i}{r_{i+1} - r_i} - p_{i,k} S \frac{1-\tau_k}{r_i - r_{i-1}}, \\ b^i_{i-1,k} &= p_{k,i-1} S \frac{1-\tau_{i-1}}{r_i - r_{i-1}}, \end{aligned} \tag{2}$$

whereas $T^i_{[hk]}(x) = U^i_{[hk]}(x) + V^i_{[hk]}(x)$, with

$$U^i_{[hk]}(x) = \frac{p_{h,k} S \tau_k}{\sum_{j=1}^n x_j} \left(\frac{x_{i-1}}{r_i - r_{i-1}} - \frac{x_i}{r_{i+1} - r_i} \right), \tag{3}$$

and

$$V^i_{[hk]}(x) = p_{h,k} S \tau_k \left(\frac{\delta_{h,i+1}}{r_h - r_i} - \frac{\delta_{h,i}}{r_h - r_{i-1}} \right) \frac{\sum_{j=1}^{n-1} x_j}{\sum_{j=1}^n x_j}. \tag{4}$$

In particular, $U^i_{[hk]}(x)$ keeps track of the advancement from a class to the subsequent one, due to the benefit of tax revenue redistribution and $V^i_{[hk]}(x)$ of the retrocession from a class to the preceding one, due to the payment of some tax. The symbol $\delta_{h,k}$ denotes the *Kronecker delta* and all expressions are to be thought as present only for meaningful values of the indices.

Well-posedness of the equation system (Eq. (1)) is proved in [8]: in correspondence to any initial condition $x_0 = (x_{01}, \dots, x_{0n})$ with $x_{0i} \geq 0$ for all $i = 1, 2, \dots, n$ and $\sum_{i=1}^{n} x_{0i} = 1$, a unique solution $x(t) = (x_1(t), \dots, x_n(t))$ of (Eq. (1)), satisfying $x(0) = x_0$, exists, defined for all $t \in [0, +\infty)$, and such that for all $t \geq 0$, both $x_i(t) \geq 0$ for $i = 1, 2, \dots, n$ and $\sum_{i=1}^{n} x_i(t) = 1$ hold true. Hence, the solutions of (Eq. (1)) are distribution functions. Also, the expressions of $U^i_{[hk]}(x)$ and $V^i_{[hk]}(x)$ above simplify becoming linear in the variables x_j and the right-hand sides of (Eq. (1)) turn out to be polynomials of degree three.

A second result proved in [8] is that the scalar function $\mu(x) = \sum_{i=1}^{n} r_i x_i$, expressing the global income (total amount of money) and, due to the population normalisation, also the mean income, is a first integral for the system (Eq. (1)).

Also, the following empirical fact (not analytically proved) is recognised to be true according to a large number of numerical simulations. If the parameters in the model are fixed, for any fixed value of the global income μ, a unique asymptotic stationary solution of (Eq. (1)) exists to which all solutions $x(t) = (x_1(t), \dots, x_n(t))$ satisfying $x(0) = x_0$ with $\mu(x_0) = \mu$ (i.e. all solutions evolving from initial conditions which share the same value μ of the global income) tend as $t \to \infty$.

As already emphasised, a great freedom remains for the choice of various parameters, namely the average incomes $r_1, r_2, \dots, r_n$, the tax rates $\tau_1, \tau_2, \dots, \tau_n$, and the $p_{h,k}$ for $h, k = 1, 2, \dots, n$. Different cases were already considered in [8].

3. Properties of the model and its variants

The first result of interest from a socio-economic point of view, which is discussed in [8] is that for fixed parameters $r_1, r_2, \dots, r_n$ and $p_{h,k}$ $(h, k = 1, 2, \dots, n)$ and fixed growth laws of the tax rates $0 \leq \tau_1 \leq \tau_2 \leq \dots \leq \tau_n \leq 1$, the effect of an increase of the difference $\tau_n - \tau_1$ between the maximum and the minimum tax rate in correspondence to the stationary income distribution is an increase of the fraction of individuals belonging to the middle classes, accompanied by a decrease of the fraction of individuals belonging to the poorest and the richest classes. We remark that only five income classes were considered in [8], the motivation being that the number of different tax rates generally foreseen in real world is similarly small (in Italy the number of the IRPEF tax rates relative to different income ranges is exactly five).

To try and see whether the model allows to obtain long-time stationary income distributions with shapes exhibiting fat tails as it occurs in real world, a larger number of classes in the model were considered in the work by Bertotti et al. [9]. Various choices of the parameters were evaluated. The purpose was to deal with cases as realistic as possible, and initial distributions of the population were chosen with a majority of individuals in lower-income classes and only a minority in higher income classes. In this way, stationary income distributions with Pareto-like behaviour were found.

Among other aspects to be explored, the curiosity remained to see whether one can find an analytic expression of a distribution, to which the stationary solutions of the model suit. A focus of the paper [10] by Bertotti et al. is on the search for such an analytic expression. Several parameter choices as well as various distributions proposed in the literature are considered in that paper. What is found is that an excellent fitting can be obtained between distributions arising from numerical simulations of the model and the κ-generalised distribution proposed by Kaniadakis in [25]. And it is worth pointing out that, in turn, the κ-generalised distribution has proved to greatly perform when considered in connection with empirical data: for example, its agreement with data on personal income of Germany, Italy and the United Kingdom is discussed by Kaniadakis et al. in [26] and that one with data on personal income of Australia and the United States by Kaniadakis et al. in [27].

In real life, welfare policies provide benefits, in particular to the lowest income classes, in connection with health care, education, home, to help improve living conditions. To simulate a policy of this kind, a modified version of the model is treated in the paper by Bertotti et al. [11], where also the contribution of what can be considered as a welfare form is incorporated. This is achieved through some weights that differently measure the amount of tax revenue redistributed among classes. In the same paper, also a comparison is established between different ways to fight economic inequality. A specific result therein obtained is that, at least under certain hypotheses, inequality reduction is more efficiently reached by a policy of reduction of the welfare and subsidies for the rich classes than by an enlargement of the tax rate difference $\tau_n - \tau_1$ aimed at taxing rich people much more than poor ones.

A further issue on which the model was tested relates to social mobility. Empirical data relative to several countries show the general existence of a negative correlation between economic inequality and mobility (a reference for that being e.g. the article by Corak [1]). This relevant topic is dealt with in the paper by Bertotti et al. [12]. Certainly, in the model at hand, one cannot distinguish different generations. Nonetheless, some indicators are introduced, useful to quantify mobility, which is meant here as a probability for individuals of a given class to climb up [respectively, down] the income ladder and pass to an upper [respectively, lower] class. Without entering technical details, we emphasise that a negative correlation between economic inequality and upward mobility turns out to be in fact a feature of the model.

Finally, the question of tax evasion, occurring as a matter of fact in a stronger or weaker form in several countries, can be and was investigated in the context of the model under consideration. In the work by Bertotti et al. [13], for instance, also the co-existence of different evasion levels among individuals was postulated and its consequences were explored. In particular, it was shown there that, besides leading to a reduction in tax revenue, the evasion misbehaviour too contributes to an increase of economic inequality.

4. The impact of different fiscal policies towards economic inequality

To give a further illustration of the impact of different fiscal policies on the shape of income distribution and economic inequality as suggested by the model, we develop in this section a novel application.

To solve numerically the differential equations, we have to fix the parameters that are so far free. We choose for example

- the number of income classes in which the population is divided to be equal to $n = 15$,
- the unitary amount of money that can be exchanged in each transaction to be given by $S = 1$,

- the average incomes of the classes to be linearly growing according to

$$r_j = 25j, \tag{5}$$

- the tax rates relative to the different income classes to be of the form

$$\tau_j = \tau_{min} + \frac{j-1}{n-1}(\tau_{max} - \tau_{min}), \tag{6}$$

for $j = 1, \dots, n$, with τ_{min} rand τ_{max} respectively denoting the minimum and maximum tax rate.

Finally,

- with the purpose to define reasonable heterogeneous transaction and payment probabilities, we assume the coefficients $p_{h,k}$ to be given by

$$p_{h,k} = \min\{r_h, r_k\}/4r_n, \tag{7}$$

except for the terms

$$\begin{aligned} &p_{j,j} = r_j/2r_n \quad \text{for} \quad j = 2, \dots, n-1, \\ &p_{h,1} = r_1/2r_n \quad \text{for} \quad h = 2, \dots, n, \\ &p_{n,k} = r_k/2r_n \quad \text{for} \quad k = 1, \dots, n-1, \\ &p_{1,k} = 0 \quad \text{for} \quad k = 1, \dots, n, \\ &p_{h,n} = 0 \quad \text{for} \quad h = 1, \dots, n. \end{aligned} \tag{8}$$

Such a choice stands for the belief that poorer individuals usually spend and earn less than richer ones. The requirements for the coefficients with $h, k = 1$ or n are of a technical nature, due to constraints on the extreme classes.

According to the empirical result recalled at the end of Section 2, for a specific given model (i.e. once parameters are fixed), the solutions of (Eq. (1)) evolving from all initial conditions x_0 with the same global income tend to a same asymptotic equilibrium.

The application we are going to discuss here includes four steps and is constructed as follows:

- Step (i): Starting from a quasi-random initial condition x_0 (the only requirement for realism being that the majority of individuals occupy the lowest income classes), we assume that in the closed society at hand no taxation exists. Accordingly, we put $\tau_{min} = 0$ and $\tau_{max} = 0$. Making to evolve the equations (Eq. (1)) for a sufficiently long time, we obtain an "asymptotic" stationary solution corresponding to a first income distribution, which is displayed in Panel (i) in **Figure 1**.

- Step (ii): We postulate at this point the introduction of a taxation system that provides the same tax rate for each income class. Towards this and to fix ideas, we choose $\tau_{min} = \tau_{max} = 20\%$. Then, we take the asymptotic stationary solution of step (i) as the initial condition, and we make the equations (Eq. (1)) (which are of course different from those in the previous step) evolve. After a sufficiently long time, a new "asymptotic" stationary solution is reached, which represents a second income distribution. It is that one displayed in Panel (ii) in **Figure 1**.

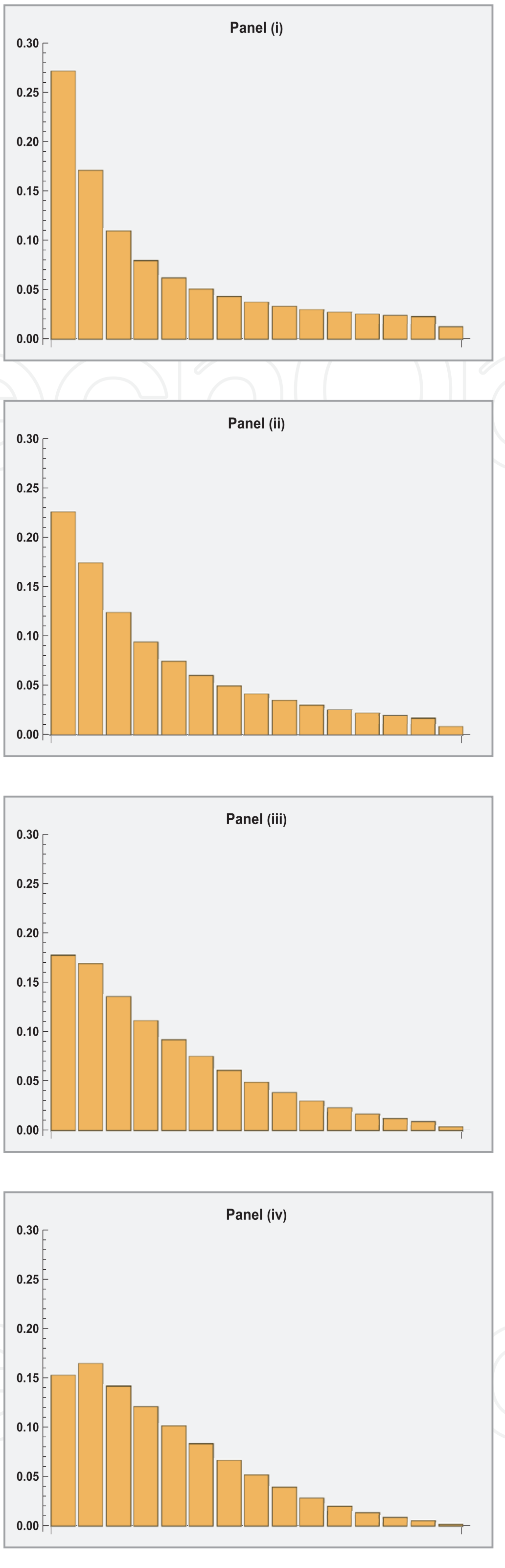

Figure 1.
The four panels display the stationary income distributions in correspondence to the same given global income for the four different fiscal policies described in steps (i), (ii), (iii), and (iv). Even a simple look provides evidence of the fact that in passing from each panel to the next one the fraction of individuals in the poorest as well as in the richest class decreases while increasing in the intermediate classes. Correspondingly, economic inequality decreases.

- Step (iii): To simulate the implementation of a more targeted fiscal policy, we now introduce another change amounting to the choice of a progressive taxation. Equivalently, we fix different tax rates, lower for low-income earners and higher for high-income earners. Specifically, we choose here $\tau_{min} = 20\%$ and $\tau_{max} = 50\%$. The "asymptotic" stationary solution obtained in correspondence to an initial condition coinciding with the asymptotic stationary solution of step (ii) is displayed in Panel (iii) in **Figure 1**.

- Step (iv): As a further focused fiscal policy, we also incorporate in the taxation algorithm what can be thought of as an addition of welfare provision. From a technical point of view, this can be achieved through the introduction of suitable weights in the terms $U^i_{[hk]}(x)$ and $V^i_{[hk]}(x)$ in system (Eq. (1)). Such weights allow to differently measure the portion of redistributed tax revenue to individuals of different income classes. A formula able to realise this is given in [11], and we refer to that paper for further details. What is of interest here is the final "asymptotic" income distribution relative to the equations, which include this modification and to an initial condition coinciding with the asymptotic stationary solution of step (iii). This income distribution is shown in Panel (iv) in **Figure 1**.

Already a simple look at the panels in **Figure 1** provides evidence of the fact that the effect in the long run of each of the different fiscal policies adopted throughout the steps (i), (ii), (iii), and (iv) is to modify the income distribution over the population so as to lower the number of individuals in the poorest as well as in the richest classes, simultaneously increasing this number in the intermediate classes. Also, an alternative, unified representation of the four stationary income distributions corresponding to the four different taxation system fiscal policies (i), (ii), (iii), and (iv) is given in **Figure 2**. Lastly, in **Figure 3** the evolution in time of the fraction of individuals in the 15 income classes is displayed. Once again, together with others, one may notice that the fractions of individuals that are initially the largest and the smallest (fractions to which the poorest and the richest individuals belong) are both non-increasing in time.

We emphasise that economic inequality decreases in passing from the income distribution displayed in Panel (i) of **Figure 1** to the income distributions in Panel (ii). The same holds true in passing from the distribution in Panel (ii) to that one in Panel (iii), and from the distribution in Panel (ii) to that one in Panel (iv).

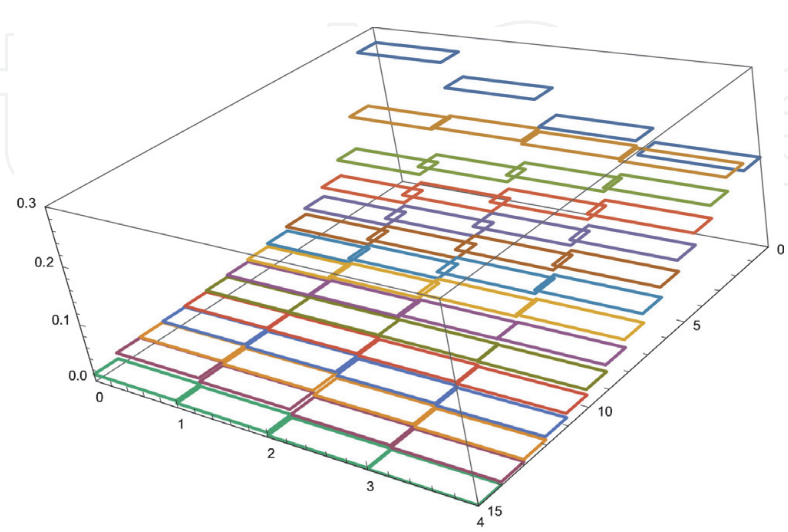

Figure 2.
An alternative representation of the stationary income distributions in correspondence to a given global income for the four different taxation systems fiscal policies described in steps (i), (ii), (iii), and (iv). One clearly notices that the fraction of the poorest and the fraction of the richest individuals decrease when passing from the distribution for step (i) (corresponding to the strip [0,1]) to the distribution for step (iv) (corresponding to the strip [3, 4]).

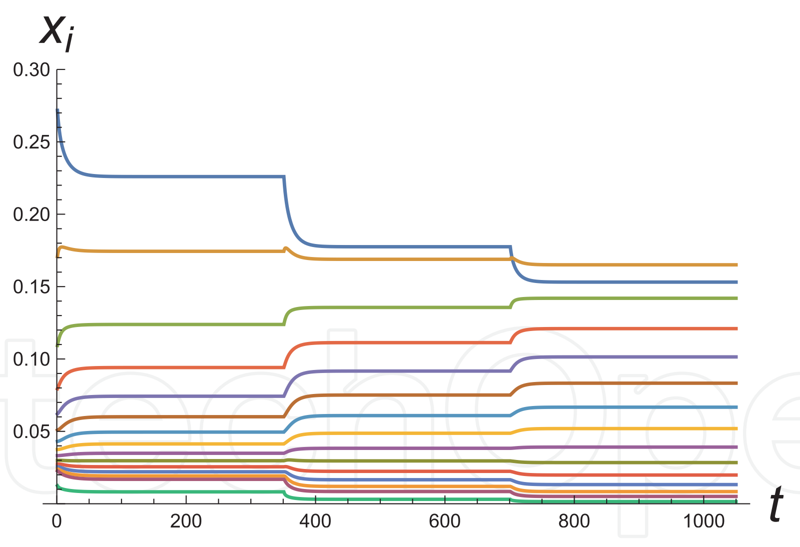

Figure 3.
The evolution in time of the fraction of individuals in the 15 income classes for the model with fiscal policies as in steps (ii), (iii), and (iv). One may notice, in particular, that the fractions of individuals which are initially (in the stationary distribution reached in absence of taxes) the largest and the smallest—fractions to which the poorest and the richest individuals belong—are both non-increasing in time.

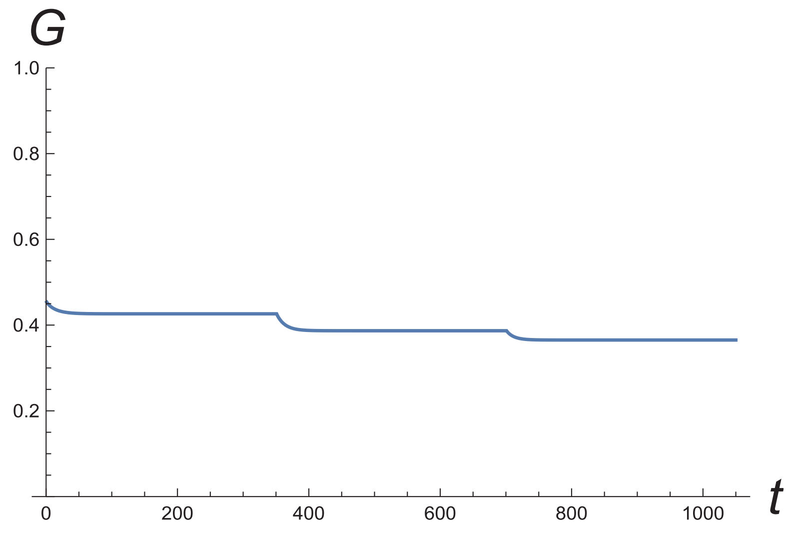

Figure 4.
The graph of the Gini coefficient as a function of time in correspondence to the application in the sequence of the three different fiscal policies adopted throughout the steps (ii), (iii), and (iv).

A quantitative measure of economic inequality is given by the Gini coefficient G (named after the Italian statistician and economist C. Gini who introduced it in the early twentieth century, see [28]), whose definition we recall next: if the Lorenz curve expresses on the y-axis, the cumulative percentage of the total income of a population earned by the bottom percentage of individuals (represented, in turn, on the x-axis), denote A_1 the area between the Lorenz curve of the distribution at hand and the line of perfect equality $y = x$, characterising a uniform distribution; also, denote A_2 the total area under the line of perfect equality. The Gini coefficient is defined as the ratio A_1/A_2 and takes values in the interval $[0, 1]$. The extreme values 0 and 1 of G respectively represent complete equality and complete inequality.

The Gini coefficients relative to the income distributions in the Panels (i), (ii), (iii), and (iv) in **Figure 1** are

$$G = 0.453551, \quad G = 0.426308, \quad G = 0.386833, \quad G = 0.365182 \tag{9}$$

respectively.

Time t	G for (ii)[a]	G for (iii)[b]	G for (iv)[c]
500	0.4474	0.4170	0.3812
1000	0.4430	0.4102	0.3771
2500	0.4349	0.3978	0.3703
5000	0.4292	0.3899	0.3665
10,000	0.4266	0.3871	0.3653
25,000	0.4263	0.3868	0.3652
55,000	0.4263	0.3868	0.3652

[a]*refers the solution at time t of the equation system with coefficients as in step (ii),*
[b]*refers to the solution at time t of the equation system with coefficients as in step (iii),*
[c]*refers to the solution at time t of the equation system with coefficients as in step (iv).*

Table 1.
In this table, the Gini coefficients G relative to the income distributions are evaluated in correspondence of a finite number of times for the model systems described in steps (ii), (iii), and (iv). One sees here that each of the solutions G decreases. Accordingly, economic inequality is decreasing for each of the three models (ii), (iii), and (iv), models characterised respectively by the existence of a taxation system with a unique tax rate, the existence of a progressive taxation system with different tax rates, the existence of a taxation system integrated by welfare.

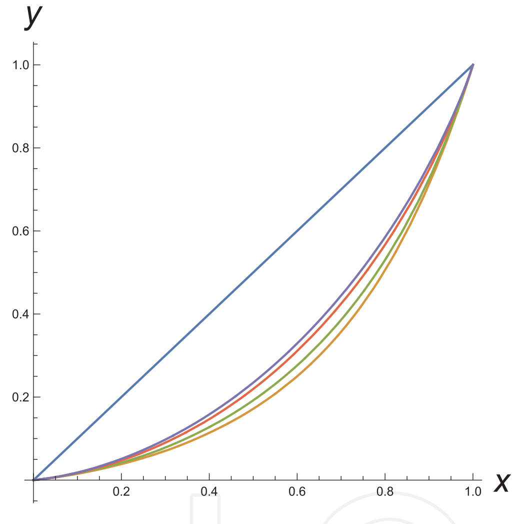

Figure 5.
The Lorenz curves corresponding to the stationary income distributions reached at the end of steps (i), (ii), (ii), and (iv). The variable on the x-axis denotes the bottom percentage of individuals and the variable on the y-axis is the cumulative percentage of the total income earned by the corresponding percentage of individuals. The Lorenz curves referring to the final distribution relative to steps (i), (ii), (ii), and (iv) are ordered from the lowest to the highest. Accordingly, in passing from step (i) to step (ii) to step (iii) to step (iv) the Gini coefficient decreases.

It is also worth noting that the Gini coefficient decreases along with the solutions of the equation systems relative to the three models defined in steps (ii), (iii), and (iv). In particular, in **Table 1** some values of G are reported, relative to the income distributions in a finite number of instants during the evolution of the three dynamical systems. An overall picture of this behaviour is contained in **Figure 4**: there, the graph of the Gini coefficient as a function of time is shown, in correspondence to the application in the sequence of the three different fiscal policies adopted throughout the steps (ii), (iii), and (iv). Lastly, **Figure 5** displays the Lorenz curves corresponding to the stationary income distributions reached at the end of steps (i), (ii), (ii), and (iv).

5. Conclusions

In this chapter, we have revisited, also discussing a novel application of it, a mathematical "micro-to-macro" model suitable for the study of the aggregate formation of the income distribution in a closed market society out of a whole of economic interactions including taxation and redistribution. The model, originally proposed by Bertotti in [8], was further developed and analysed in various papers by Bertotti et al. [9–13]. We have shown that it can be adapted to analyse issues related to economic inequality. In particular, the model identifies in redistributive policy a driver towards economic inequality reduction. The theme is complex and requires a broad spectrum of skills, knowledge, real data, ideas. The model encompasses (as is inevitable) great simplifications and probably a naive approach, and cannot offer magical solutions to the problems it addresses. Nonetheless, thanks to the considerable flexibility it enjoys and to its ability to make predictions in the presence of different conditions and policies, it could hopefully contribute to providing some insight towards forecasting of possible outcomes and behaviours, in this way serving as an inspiration and source of suggestions for policy-makers.

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