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## Chapter

# Gravity Field Theory 

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#### Abstract

Gravity keep all things on the earth surface on the ground. Gravity method is one of the oldest geophysical methods. It is used to solve many geological problems. This method can be integrated with the other geophysical methods to prepare more accepted geophysical model. Understanding the theory and the principles concepts considered as an important step to improve the method. Chapter one attempt to discuss Newton's law, potential and attraction gravitational field, Geoid, Spheroid and geodetically figure of the earth, the gravity difference between equator and poles of the earth and some facts about gravity field.


Keywords: gravitational theory, gravitational attraction, ellipsoid and geoid, gravity variation with latitude, facts about gravity field

## 1. Introduction

The theory of gravitational method based on Newton's law expressing the force of mutual attraction between two particles in term of their masses and separation.

This law states: (that two very small particles of mass $\left(m_{1}\right)$ and $\left(m_{2}\right)$ respectively, each with dimensions very small compared with the separation ( $r$ ) of their centers of mass, will be attracted to one another with a force.

$$
\begin{equation*}
\mathrm{F}=\mathrm{G} \frac{m_{1} m_{2}}{r^{2}} \tag{1}
\end{equation*}
$$

$\mathrm{G}=$ Universal gravitational constant
$667 \times 10^{-8} \mathrm{~cm}^{3} \mathrm{~g}^{-1} \mathrm{~s}^{-2}$ in cgs system
$6.67 \times 10^{-11}\left(\mathrm{Nm}^{2} / \mathrm{Kg}^{2}\right)$ in SI system
If $\mathrm{m}_{1}, \mathrm{~m}_{2}$ in gram and $\mathrm{r}(\mathrm{cm})$
The second law of motion expressed mathematically by the following. $\mathrm{F}=\mathrm{ma}$
Where
$\mathrm{m}=$ the mass $\mathrm{a}=$ acceleration,
$r$ measured by (cm), $M_{1}, m_{2}$ measured in (gm), $F$ measured by (dyne).

## 2. Gravitational acceleration

The acceleration (a) of a mass $\left(m_{2}\right)$ due to the attraction of a mass $\left(m_{1}\right)$ a distance ( r ) away can be obtained simply by dividing the attracting force F by the mass ( $\mathrm{m}_{2}$ ).

$$
\begin{equation*}
\mathrm{F}=\frac{G m_{1} m_{2}}{R^{2}} \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{g}=\mathrm{a}=\frac{F}{m_{2}}=\frac{G m_{1}}{R^{2}} \tag{3}
\end{equation*}
$$

The acceleration is the conventional quantity used to measure the gravitational field acting any at point [1].

Is the Earth's gravitational acceleration is constant? No, it is not, that is due to the variation of mass distribution and variation of the diameters of the Earth.

In the cgs system, the dimension of acceleration is $\left(\mathrm{cm} / \mathrm{sec}^{2}\right)$. Among geophysicists this unit is referred as the (Gal) (in honor of Galileo, who Conducted pioneering research on the earth's gravity).

The gravitational acceleration at the earth surface is about ( $980 \mathrm{~cm} / \mathrm{sec}^{2}$ ) or ( 980 Gal ), but in exploration work we are likely to be measuring differences in acceleration. $10^{-7}$ of the earth's field. The unit which is more convenience in working with gravity data for geological and geodetic studies is milli-gal (milligal) or

$$
(\mathrm{mGal})=\frac{1}{1000} \mathrm{gal}=10^{-3} \mathrm{gal}
$$

This unit has come to be the common unit for expressing gravitational accelerations.

There is another unit called gravity Unit (g.u.) which is equal to 0.1 mGal . g.u. $=0.1$ milligal $=10^{-4}$ gal.
and micro- gal $=10^{-6}$ gal.

## 3. Gravitational potential

The intensity of gravitational field depends only on position, the analysis of such fields can often be simplified by using the concept of potential.

The potential at a point in a gravitational field is defined as the work required for arbitrary reference point to the point in question.

The acceleration at a distance ( $r$ ) from ( $p$ ) is

$$
\begin{equation*}
=\mathrm{Gm} / \mathrm{r}^{2} \tag{4}
\end{equation*}
$$

The work necessary to move the unit mass a distance (ds) having a component $(\mathrm{dr})$ in the direction of $(\mathrm{p})$ is

$$
\begin{equation*}
=\left(\mathrm{Gm}_{1} / \mathrm{r}^{2}\right) \mathrm{dr} \tag{5}
\end{equation*}
$$

The work (U) done in moving the mass from infinity to a point (O), (Figure 1) in the gravitational field of $\left(m_{1}\right)$ is

$$
\begin{gather*}
\mathrm{U}=\mathrm{Gm}_{1} \int^{R} \mathrm{dr} / \mathrm{r}^{2}  \tag{6}\\
\mathrm{U}=\left.\mathrm{G} m_{1} \frac{1}{r}\right|_{\alpha} ^{R}=\frac{-G m_{1}}{R}  \tag{7}\\
\mathrm{U}=\mathrm{Gm} / \mathrm{R} \tag{8}
\end{gather*}
$$

The quantity ( $\mathrm{Gm} 1 / \mathrm{R}$ ) is the gravitational potential. It is depend only on the distance ( $R$ ) from the point source $\left(m_{1}\right)$. By differentiating both sides of the above equation it can be seen that the gravitational acceleration is the derivative of the potential with respect to (r).


Figure 1.
The movement of mass unit from infinity to the point $(O)$ [1].

Any surface along with the potential is constant, so it is referred as an equipotential surface.

Sea level, for example, is an equipotential surface, even though the actual force of gravity varies along the sea surface by more than ( $0.5 \%$ ) between the equator and either of the poles.

## 4. Newton's law and large dimensions mass

When the dimensions of the source are large, it is necessary to extend the theory. The procedure is to divided the mass into many small elements, and to add the effects of each of these elements, together to measure the gravity effect on certain point.

Because force or acceleration is a vector having both magnitude and direction, it is necessary to resolve the force from each element of mess into its three components (most generally its vertical component and its north-south and east-west components in horizontal plane), before the attraction of the body at any point can be determined.

If we consider the attraction of an irregular laminar body (part of twodimensional sheet) in the xz plane at an external point (p). We first determine the x (horizontal) and Z (vertical) components of acceleration at (p) associated with this attraction.

To do this we divide the plate into ( N ) small elements of mass, each of area ( $\Delta \mathrm{s}$ ), if the density $\left(\rho_{\mathrm{n}}\right)$ is uniform within the ( $\mathrm{n}^{\text {th }}$ ) elements we express the ( x ) component of celebration at point (p) due to the attraction of this element as, (Figure 2).
laminar body, by divided the mass into small masses ( $\Delta \mathrm{s}$ ) [1].

$$
\begin{equation*}
g_{x n}=\frac{G \rho_{n} \Delta S}{r_{n}{ }^{2}} \cos \theta=\frac{G \rho_{n} \Delta S}{r_{n}{ }^{2}} \cdot \frac{X}{r_{n}}=\frac{G X}{r_{n}{ }^{3}} \rho_{n} \Delta S \tag{9}
\end{equation*}
$$



Figure 2.
The gravity determination of two dimensional irvegular.
and

$$
\begin{equation*}
g_{z n}=\frac{G \rho_{n} \Delta S}{r_{n}{ }^{2}} \sin \theta=\frac{G \rho_{n} \Delta S}{r_{n}{ }^{2}} \cdot \frac{Z}{r_{n}}=\frac{G Z}{r_{n}{ }^{3}} \rho_{n} \Delta S \tag{10}
\end{equation*}
$$

Adding the acceleration for all elements

$$
\begin{align*}
& g_{x}=G \sum_{1}^{N} \frac{X \rho_{n}}{r_{n}{ }^{3}} \cdot \Delta S  \tag{11}\\
& g_{z}=G \sum_{1}^{N} \frac{Z \rho_{n}}{r_{n}{ }^{3}} \cdot \Delta S \tag{12}
\end{align*}
$$

Where $\Delta \mathrm{S}$ is very small we can express the two components of acceleration at (p) by the respective integration.

$$
\begin{align*}
& g_{x}=G \int^{S} \frac{\rho_{x} X}{r^{3}} d s  \tag{13}\\
& g z=G \int^{S} \frac{\rho_{z} Z}{r^{3}} d s \tag{14}
\end{align*}
$$

Where S = area of body
$\mathrm{P}=$ (mass per unit area).
This case can be extended to three dimensional case for a mass per unit volume.

$$
\begin{align*}
& g_{x}=G \int^{V} \frac{\rho_{x} X}{r^{3}} d v  \tag{15}\\
& g_{y}=G \int^{V} \frac{\rho_{y} y}{r^{3}} d v  \tag{16}\\
& g_{z}=G \int^{V} \frac{\rho Z}{r^{3}} d v . \tag{17}
\end{align*}
$$

Where ( V ) is the volume of the body [1].
In gravity exploration, only the vertical component of force is measured, so that we are normally concerned only with $\left(\mathrm{g}_{\mathrm{z}}\right)$ in determining the attraction at the surface of a buried body.

One of the most important properties of Potential, that it is satisfy Laplace equation anywhere outside the effective gravity mass.

$$
\begin{equation*}
\nabla^{2} \mathrm{~A}=\frac{\partial^{2} A}{\partial x^{2}}+\frac{\partial^{2} A}{\partial y^{2}}+\frac{\partial^{2} A}{\partial z^{2}}=O \tag{18}
\end{equation*}
$$

Laplace equation caused the Ambiguity in gravity and magnetic fields.

## 5. Gravitational attraction

Newton's law states that in case of two masses $\left(\mathrm{m}_{1}\right)$ and $\left(\mathrm{m}_{1}\right)$ separated by a distance ( $r$ ) between them, then these two masses attract each other's by force ( F ).

$$
\begin{equation*}
F=\frac{G m_{1} m_{2}}{r^{2}} \tag{19}
\end{equation*}
$$

$\mathrm{G}=$ Universal gravitational constant
If The mass of the earth (M), and the radius of the earth (r), then the weight (w) of a body with a mass ( m ) on the surface of the earth equal to:

$$
\begin{equation*}
W=m g=\frac{G M m}{r^{2}} \tag{20}
\end{equation*}
$$

The above equation used only for non-rotating earth.

$$
\begin{equation*}
g=\frac{G M}{r^{2}} \tag{21}
\end{equation*}
$$

(g) usually expressed as a weight of a unit mass or earth attraction.

The gravity attraction of the earth varied from point to point on the earth surface, due to that the radius of the earth (r) (which is not constant everywhere on the earth) in addition to the centrifugal force create duo to the rotation of the earth, (Figure 3).

## Centrifugal force



Figure 3.
The variation of centrifugal force value with latitude.

The gravity attraction (g) can be measured with high accuracy and the size of the earth can be determined using the astronomical geodetic studies. And as the universal gravitational constant (G) is known, the earth mass can calculated easily.

The gravity attraction can be measured using a pendulum method with accuracy of 5 ppm , while the relative variation in gravity can be measured using special high accurate gravity meter instruments (composed of springs system) with an accuracy of $10^{-9}$.

The following equation used on assumption of non-rotated earth

$$
\begin{equation*}
F=\frac{G M m}{r^{2}} \tag{22}
\end{equation*}
$$

But the earth is a rotated body. It is rotate around the long axis. This rotation created a centrifugal force (a) varied relatively with the variation of the radius of the circular shape plotted by a rotated points on the earth surface.

$$
\begin{equation*}
a=r \omega^{2} \tag{23}
\end{equation*}
$$

Where
$\omega$ is the angular velocity
T is the period

$$
\begin{equation*}
\omega=\frac{2 \lambda}{T} \tag{24}
\end{equation*}
$$

Then

$$
\begin{equation*}
\alpha=4 \pi^{2} r / T^{2} \tag{25}
\end{equation*}
$$

So
$\alpha$ value at the equator is 3.4 gal

$$
\begin{equation*}
a=3.4 \mathrm{~cm} / \mathrm{sec}^{2}=\frac{g}{289} \tag{26}
\end{equation*}
$$

For the other latitude circle of the earth which symbolized by $\varnothing$, the radius (r) replaced by

$$
r \cos \varnothing
$$

then the angular acceleration become

$$
\alpha \cos \varnothing
$$

This acceleration have two components:
The vertical component is

$$
\alpha \cos ^{2} \varnothing
$$

This component reach its maximum value at the equator ( 3.4 gal ), while its value at the pole is zero which is the minimum value.

The horizontal component is

$$
\alpha \cos \varnothing \sin \emptyset
$$

The horizontal component reach its maximum value ( 0.5 gal ) at the latitude $\varnothing=45^{\circ}$, while it reach its minimum value (zero) at the equator and poles.

The total gravity force which is determine the weight of anybody on the earth during a free fall represented as a resultant of the gravitational attraction of the earth and the centrifugal force at certain point on the earth.

Tenth thousands of gravity measurements on the earth surface indicate that g values at poles is greater than $g$ values at equator by about $(1 / 189)$ of the total gravity value. Where the centrifugal force caused a difference between the equator and poles by about ( $1 / 289$ ), while the increase in radius at the equator by ( 21 km .) more than pole caused a variation by about ( $1 / 547$ ).
$1 / 289=0.003460207$ Centrifugal force effect.
$1 / 547=0.00182815$ Earth radius difference effect
$!/ 189=1 / 289+1 / 547$
$1 / 189=0.0052884$ The difference in gravity between the equator and poles relative to the total gravity value.

## 6. Gravitational theory

The first approximation of the shape of the earth is the sphere.
The second approximation of the earth is the oblate spheroid.
For theoretically studies and for simple applications it is possible to use the horizontal surface (level surface) or (equipotential surface everywhere, which is perpendicular on the plumb line (force line).

The problem of determine the shape of the earth is actually is the problem of determine the shape of equipotential surface.

The gravitational field include infinity equipotential surfaces.
These equipotential surfaces are not intersect at all.
The scientists deals to considered the sea level as the reference equipotential surface in gravitational studies and call this surface geoid after Listing 1873.

The equipotential surface not necessarily coincide with the equal gravity surface. So the sea level considered approximately a surface of equipotential gravitational surface (because it is perpendicular on the gravity force at every point).

## 7. Clairaut theorem

At 1743 the French mathematical scientist Clairaut found a mathematical expression to represent the relation between the gravity measurements on the earth surface and the shape of the earth.

Supposing that.
$\mathrm{a}=$ maximum radius of oblate spheroid.
$\mathrm{b}=$ minimum radius of oblate spheroid.
$a$ and $b$ are the major semi axes of an oblate spheroid of revolution. and write

$$
\begin{gather*}
\mathrm{b}=\mathrm{a}(1-\mathrm{f})  \tag{27}\\
\mathrm{f}=(\mathrm{a}-\mathrm{b}) / \mathrm{a} \tag{28}
\end{gather*}
$$

F = oblateness or ellipticity of the spheroid for spherical body

$$
\begin{equation*}
\left(\left(\mathrm{x}^{2}+\mathrm{y}^{2}\right) / \mathrm{a}^{2}\right)+\left(\mathrm{z}^{2} /\left(\mathrm{a}^{2}(1-2 \mathrm{f})\right)\right)=1 \tag{29}
\end{equation*}
$$

That is

$$
\begin{equation*}
x^{2}+y^{2}+z^{2}(1+2 f)=a^{2} \tag{30}
\end{equation*}
$$

if
is the colatitude angle $\theta$
the above equation written as

$$
\begin{equation*}
\mathrm{r}^{2}\left(1+2 \mathrm{f}_{\left.\operatorname{Cos}^{2} \theta\right)=\mathrm{a}^{2}}\right. \tag{31}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{a}^{2} / \mathrm{r}^{2}=\left(1+2 \mathrm{f} \operatorname{Cos}^{2} \Theta\right) \tag{32}
\end{equation*}
$$

Considering the earth as an oblate spheroid, rotate around its axis and the variation in gravity measurements on the earth, Clairaut obtained the following theorem:-

$$
\begin{equation*}
g=g_{e}\left[1+\left(\frac{5}{2} C-f\right) \sin ^{2} \Phi\right] \tag{33}
\end{equation*}
$$

Where
$\mathrm{g}=$ Theoretical gravity value at any point on the earth according to its latitude angle.
$g_{e}=$ gravity at the equator.
$\Phi=$ latitude angle

$$
\begin{equation*}
C=\omega^{2} a / g_{e}=(\text { Centrifugal force } / \text { Attraction force }) \text { at equator } \tag{34}
\end{equation*}
$$

(where latitude angle equal zero)
The variable of $\sin ^{2} \Phi$ which is
(5c/2) - f
Represent a gravitational flattening ( $\beta$ )
Which is written in other expression as following:

$$
\begin{equation*}
\beta=\frac{g_{p}-g_{e}}{g_{e}} \tag{35}
\end{equation*}
$$

$\mathrm{g}_{\mathrm{e}}=$ Gravity at equator
$\mathrm{g}_{\mathrm{p}}=$ Gravity at pole
The Clairaut's theorem also written in other expression:

$$
\begin{equation*}
\mathrm{g}_{\circ}=\mathrm{g}_{\mathrm{e}}\left(1+\beta \sin ^{2} \Phi\right) \tag{36}
\end{equation*}
$$

The value of the factor $\beta$ can be obtained from measuring a lot of absolute gravity values at different locations at the earth surface. The slope of the best fit line of the gravity value versus the latitude angle represent the factor $\beta$ multiplied by the gravity at the equator $g_{e}$.

$$
\begin{equation*}
\text { Considering } f=1 / 297 \tag{37}
\end{equation*}
$$

The determination of gravity values at the sea level over the world led to obtained the following equation:

$$
\begin{equation*}
\mathrm{G}=978.049\left(1+0.0052884 \sin ^{2} \Phi-0.0000059 \sin ^{2} 2 \Phi\right) \tag{38}
\end{equation*}
$$

Grand and West, [2].

This equation called the international gravity formula, or the 1939's equation, where the gravity value at equator ( $\mathrm{g}_{\mathrm{e}}$ ) is ( 978.049 gal ), which calculated statistically from measurements on the earth surface. The value of the factor of $\left(\sin ^{2} \Phi\right)$ represent the flattening factor effect, while the value of the factor of $\left(\sin ^{2} 2 \Phi\right)$ represent a correction value to fit the earth shape with rotated spheroid body shape. These two factors depends on the shape of the earth and speed of rotation of the earth.

Depending on the principles of gravity anomalies the earth seems as triaxle ellipsoid. The long axis of the earth lying $10^{\circ}$ west of Greenwich and the difference between radiuses of the earth may be within $150 \pm 58$ meters, which is considered very low variation relative to the average radius of the earth therefore it is neglected in most cases.

There is another formulas such as
Helmert, 1901 formula

$$
\begin{equation*}
\mathrm{g}_{\mathrm{o}}=978.030\left(1+0.005302 \sin ^{2} \Phi-0.000007 \sin ^{2} 2 \Phi\right) \tag{39}
\end{equation*}
$$

the radius of the according to this equation are:
$\mathrm{a}=6378200 \mathrm{~m}$.
$b=6356818 \mathrm{~m}$
and

$$
\begin{equation*}
f=1 / 298.2 \tag{40}
\end{equation*}
$$

This formula used in old gravity measurements.
Other formula used in 1917 in the United States of America using the following formula:

$$
\begin{equation*}
\mathrm{g}_{\mathrm{o}}=978.039\left(1+0.005294 \sin ^{2} \Phi-0.000007 \sin ^{2} 2 \Phi\right) \tag{41}
\end{equation*}
$$

The above formula is obtained depending on 216 gravity stations in USA, 42 in Canada, 17 in Europe, and 73 gravity stations in India. The flattening value of this formula (1/297.4).

The international gravity formula 1967 which is adopted by the Geodetic Reference System (GRS-1967) with different factors values according to new observations [3]

$$
\begin{equation*}
\mathrm{g}_{\circ}=978.03185\left(1+0.0053024 \sin ^{2} \Phi-0.0000059 \sin ^{2} 2 \Phi\right) \tag{42}
\end{equation*}
$$

This formula used to remove the variation in gravity with latitude (latitude correction). This formulae consider the earth as rotating ellipsoid without geologic or topographic complexities.

The GRS 1980 formula is

$$
\begin{equation*}
\mathrm{g}_{\circ}=978.0327\left(1+0.0053024 \sin ^{2} \Phi-0.0000058 \sin ^{2} \Phi\right) \tag{43}
\end{equation*}
$$

The gravity survey for small sites for examples in case of engineering studies the latitude correction approximated using the a correction factor $0.813 \sin 2 \Phi \mathrm{mgal} /$ km in the north- south direction.

## 8. Geocentric latitude

The geocentric latitude represent the angle between a line at any point at the earth surface and passing through the center of the earth and the plane of the equator.

## 9. Geodetic latitude ( $\Phi$ )

The geodetic latitude is the geographical latitude. The geodetic is the angle between normal line on the geoid (which is approximately the earth shape) at any point on the earth and the plane of equator (which approximately the earth shape). The geodetic latitude is differ from geocentric latitude because of elliptical shape of the earth.

The geodetic latitude - geocentric latitude $=11.7 \sin 2 \Phi$ in minutes of arc. The maximum value difference between them is about 21.5 km . at latitude $45^{\circ}$.

## 10. Geoid

It is an gravitational equipotential surface. The vertical gravity component is perpendicular on this surface on all its points. This surface is approximately equal to the sea surface in seas and oceans. In land area it is considered as the extension of sea level below the continents.

## 11. The normal spheroid, ellipsoid and geoid

The earth considered as an ideal spheroid or normal spheroid in case of considering the earth as a completely liquid, means without any lateral density change. The direction of gravity attraction in such case is perpendicular on the earth everywhere and passing through the earth center. But the increase of radius of the earth at equator from that at poles by about 21 km make the ellipsoid shape is better approximation for earth than spheroid shape, (Figure 4).

Also, the earth in fact is not uniform and the density change laterally at least in the crust and upper most part of the mantle of the earth. The actual surface of the earth can be represent by the geoid which is equal to sea level and its extension in the land. The geoid surface may be up or down the ellipsoid surface depending on the distribution of density in the earth or the topographic changes, (Figure 5). The evidences on the difference between the geoid and ellipsoid obtained from the observation of the deflection of plumb line. This deflection measured during the geodetic and astronomical measurements, where the plump line deflect toward the excess masses in the continent, while it is deflected away from an area of mass deficiency as in oceans area.

The difference between geoid and ellipsoid surfaces small relative to the radius of the earth. For example it is about 40 meters in Rocky Mountains.


Figure 4.
The spheroid and ellipsoid relationship [4].


Figure 5.
The ellipsoid and geoid relationship [4].

## 12. Line of forces

The gravity field can be described by lines. These lines usually perpendicular on the equipotential surface. When the body isotropic, these lines is vertically on the surface. These lines coincide with the force direction of the gravity field. If these lines are converging to each other that means an access of mass or positive anomaly, the diverging of these lines indicate a deficiency in mass or negative anomaly.

## 13. Component of force

The three basic principle directions of force are:

$$
\begin{aligned}
& \mathrm{Fx} \rightarrow \mathrm{x} \\
& \mathrm{Fy} \rightarrow \mathrm{y} \\
& \mathrm{Fz} \rightarrow \mathrm{z}
\end{aligned}
$$

The equation of force lines are

$$
\begin{equation*}
\frac{\partial x}{F x}=\frac{\partial y}{F y}=\frac{\partial Z}{F z} \tag{44}
\end{equation*}
$$

The force lines which pass a unit area (ds) vertically on the surface of the mass expressed as a Gravity Field Intensity


All points in the space outside the attractive mass in the potential gravity field characterized its subject to Laplace equation. The fact is the reason of ambiguity in gravity field.

$$
\begin{equation*}
\nabla^{2} A=\frac{\partial^{2} A}{\partial x^{2}}+\frac{\partial^{2} A}{\partial y^{2}}+\frac{\partial^{2} A}{\partial z^{2}}=0 \tag{46}
\end{equation*}
$$

Where A is the potential field which is function of point position according to (x), (y), (z)

## 14. Gradient of potential

The gradient of potential define as the force divided by the mass

$$
\begin{equation*}
\text { gradient of potential }=\frac{- \text { Force }}{\text { mass }} \tag{47}
\end{equation*}
$$

If $i, j, k$ are vector units, and the gravity field is $U$

$$
\begin{equation*}
g=\left[\vec{i} \frac{d u}{d x}+\vec{j} \frac{\partial u}{\partial x}+\vec{k} \frac{\partial u}{\partial z}\right] \tag{48}
\end{equation*}
$$

$\nabla=$ del $\Rightarrow$ Gradient operator

$$
\begin{gather*}
g=\vec{\nabla} U \\
\vec{\nabla}=\left[\vec{i} \frac{\partial}{\partial x}+\vec{j} \frac{\partial}{\partial y}+\vec{k} \frac{\partial}{\partial z}\right] \tag{49}
\end{gather*}
$$

Gauss's law:- The total gravitation flux through any closed surface is equal to $(-4 \chi \mathrm{G})$ times the mass enclosed by the surface.

$$
\begin{equation*}
\Phi=-4 \chi \mathrm{GM} \tag{50}
\end{equation*}
$$

## 15. Gauss's law for force flux

The surface integral of the normal component of intensely of the gravitational field gives the flux through the closed surface.

$$
\begin{equation*}
\vec{\Phi}=\int \vec{g} \cdot \overrightarrow{d s} \tag{51}
\end{equation*}
$$

$\overrightarrow{d s}=$ vector normal to the surface (ds) and magnitude equaled to the area (ds).
The integral over the whole surface gives Gauss's law for total mass enclosed by the surface.
(The surface integral $\Rightarrow$ volume integral) divergences theorem

$$
\begin{gather*}
\vec{\Phi}=\int_{5} \vec{g} \cdot \overrightarrow{d s} \Rightarrow \int_{v} \vec{\nabla} \cdot \vec{g} \cdot d v  \tag{52}\\
\vec{\nabla} \cdot \vec{g}=\frac{\partial g x}{\partial x}+\frac{\partial g_{y}}{\partial y}+\frac{\partial g_{z}}{\partial z}  \tag{53}\\
\therefore \phi=\int_{v} \vec{\nabla} \cdot \vec{g} \cdot d v=-\int_{v} \vec{\nabla} \nabla U d v=-4 \chi G M \tag{54}
\end{gather*}
$$

$$
\begin{gather*}
=\int_{v} \overrightarrow{\nabla^{2}} U \cdot d v=+4 \chi G M  \tag{55}\\
=\int_{v} \overrightarrow{\nabla^{2}} U \cdot d v=+4 \chi G \int_{v} \sigma d v \tag{56}
\end{gather*}
$$

Where $\sigma=$ The density of mass distribution in the volume V
Poisson's equation

$$
\begin{equation*}
\vec{\nabla} \cdot \vec{\nabla} U=4 \chi G \sigma \tag{57}
\end{equation*}
$$

Laplace equation of the potential (U) of the gravitational field is equal to a constant times the density of the distribution matter in the field.

## 16. Laplace theorem

For a point Located outside attracting masses. The sum of second order derivation of the attraction potential along the axes of orthogonal coordinate is

$$
\begin{equation*}
\frac{\partial^{2} u}{\partial x^{2}}+\frac{\partial^{2} u_{y}}{\partial y^{2}}+\frac{\partial^{2} u}{\partial z^{2}}=0 \tag{58}
\end{equation*}
$$

But when the point being attracted lies inside the attracting mass the Laplace operator becomes

$$
\begin{equation*}
\Phi=-4 \pi \mathrm{G} \sigma \tag{59}
\end{equation*}
$$

Simply we can be considered that Laplace equation is a special application of Poisson's equation, where the mass density is zero, This case found outside the attracting mass (outside the earth surface).

## 17. Forces of gravity

In addition to the attraction force of the earth mass, there is another force which effect the rotated earth called (Centrifugal Force). This force created due to the continuous rotation of the earth around its axis. The force related to the radius of rotation and the square of the angular velocity:-

Therefore
The total gravity force $=$ Attraction Force + Centrifugal Force (FC)

$$
\begin{equation*}
F c_{p}=r \omega^{2} \operatorname{COS} \theta \tag{60}
\end{equation*}
$$

$\mathrm{P}=$ point at the earth surface.
$\theta=$ latitude
Centrifugal force
Fc $p \propto \mathrm{r} \omega^{2}$

$$
\begin{equation*}
F c p=m r \omega^{2} \tag{61}
\end{equation*}
$$

Where $\mathrm{m}=$ mass
$r=$ radius of rotation
$\omega=$ angular velocity

$$
\begin{equation*}
\therefore-g=-G \int \frac{1}{r^{2}} d m+m r \omega^{2} \tag{62}
\end{equation*}
$$

The Centrifugal force contribute the total gravity effect on the earth surface by about $10^{-7}-10^{-8} \mathrm{gal}$. The force have its maximum value at the equator of the earth and reach zero value at the poles of the earth.

The absolute gravity value at the equator is about 978 gal, while its value at poles about 983 gal, The difference in gravity value between the equator and pole is range between ( 5.17 gal ) or ( 5170 mgal ).

## 18. The difference in gravity between equator and pole

1.The acceleration of centrifugal force act outward (away from the earth) in other word opposite of gravity attraction. The centrifugal force reach maximum value at equator and its minimum value, which equal zero at poles. The factor create a difference in gravity between the equator and poles of about 3.39 gal. So this factor reduce the gravity at equator by 3.39 gal.
Therefore the gravity seems more at poles by $\mathbf{+ 3 . 3 9}$ gal than at equator.
2. The Poles are nearer to the center of the earth than the equator by about 21 km . This factor will increase the gravity at poles than at equator by about +6.63 gal.
3.The mass-shape of the earth (increase of earth radius at equator) will cause an increase of gravity attraction at the equator than that at the pole due to an increase of mass, by about 4.85 gal. Therefore the gravity at poles seems lower than at equator by -4.85 gal .

Finally the summation of gravity value difference indicated that the pole gravity is more at poles than the equator by about +5.17 gal . According to the following equation

The gravity at poles $=+3.39+6.63-4.85=+5.17$ gal (The excess in gravity value at the pole relative to the equator) (63).

## 19. Some facts about gravity field

1. Gravity method involves measurement a field of force in the earth that is neither generated by the observer nor influence by anything he does.
2. The field of gravitational or magnetic prospecting is a composite of contributions can be individually resolved only in special care.
3. In gravity measurements, the quantity actually observed is not the earths true gravitational attraction but its variations from point to another.
4. The instruments are designed to measure difference in gravity rather than its actual magnitude.
5. The variation of gravity depend only upon lateral changes in density of earth materials in the vicinity of the measuring point, [5].
6. The density is a physical property that changes significantly from one rock type to another. Knowledge of the distribution of this property within the ground would give information, of great potential value about the subsurface geology.
7. Density is the source of a potential field which is intrinsic to the body and acts at a distance from it. The strength of gravitational field of a body is in proportion to its density.
8. All materials in the earth influence gravity, but because of the inverse square law of behavior of rocks that lie close to the point of observation will have a much greater effect than those farther away.
9. The bulk of the gravitational pull of the earth (i.e. the weight of a unit mass), however, has little to do with the rocks of the earth's crust. It is caused by the enormous of the mantle and core, and since these are regular in shape and smooth varying in density so the earth's gravitational field is, in the main regular and smoothly varying also.
10. Only about three parts in one thousand $(0.3 \%)$ of $(\mathrm{g})$ are due to the material contained within the earth's crust, and of this small amount roughly $(15 \%) 0.05 \%$ of $(\mathrm{g})$ is accounted for by the uppermost ( 5 km ) of rock (that region of the crust generally being the base of geological phenomena). Changes in the densities of rocks within this region will produce variations in (g) which generally do not exceed ( $0.01 \%$ ) of absolute gravity value anywhere.
11. The geological structures contribute very little to the earth gravity, but the importance of that small contribution lies in the fact that it has a point to-point variation which can be mapped.
12. To produce meaningful maps, two imperatives must be fulfilled these are:
A. The measuring apparatus must be sufficiently sensitive to detect the effect of geology on (g).
B. Effective methods must be used to compensate the data for all sources of variation other than the local geology. These corrections will includes chiefly the effect of changing elevation and of crustal heterogeneity on abroad scale.
13. When data are finally reduced to a form meaningful in terms of local geology, they must be interpreted.

The interpretation of gravity field is done mainly in two parts:
First solution is sought to the (inverse) potential problem, which Consists in deducing the shape of the source from its potential field these solutions are never unique.

Second, The solutions deduced from (potential field theory) are interpreted in geological terms. This requires some knowledge of the factors which determines the densities of rocks.

## 20. Some selected useful terms of gravity field modified

1.Density contrast:- The density difference between the gravity source and the host rocks [6]. In case of excess of mass it is positive, While it is negative in case of mass deficiency.
2. Ellipticity:- The ratio of the major to minor axes of an ellipse.
3.Eötvös unit:- A unit of gravitational gradient which is equal to $10^{-6} \mathrm{mgal} / \mathrm{cm}$.
4. Graticule: - A template for graphically calculating of gravity.
5. Gravimeter: - An instrument for measuring variation in gravitational attraction (gravity meter).
6. Hilbert - Transform technique:- A technique for determining the phase of a minimum- phase function from its power spectrum.
7. Inverse problem: The problem of gaining knowledge of the physical features of a disturbing body by making observations of its effects, (finding the model from the observed data).
8. Direct (Forward) (Normal) Problem: Calculating the possible suspected values from a given model.
9. Invers square law: A potential field surrounding a unit element has a magnitude inversely as the square of the distance from the element (in gravity case the element is the massand the field is equal to $\mathrm{Gm} / \mathrm{r}^{2}$ ).
10. Sensitivity: The least change in a quantity which a detector is able to perceive, (An instrument can have excellent sensitivity and yet poor accuracy).
11. Accuracy: The degree of freedom from error (the total error compared to the true value).
12. Readability: The least discernible change in a readout device, which can be readily estimated.
13. Precision: The repeatability of an instrument (measured by the mean deviation of set of readings from the average value).
14. Repeatability: The maximum deviation from the average of corresponding data taken from repeated tests under ideal conditions.
15. Tidal effect: Variation of gravity observation due to the distribution of the earth resulting from attraction of the moon and sun.

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