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Bilinear Time Series in Signal Analysis

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1. Introduction

Time series models (named also output or signal models), considered in this chapter, are functions of accessible process outputs, observed as a set of uniformly sampled data, which are one and the only information on the process itself. They are mainly applied in signals modeling and prediction. Stochastic time series models, i.e. models that use white noise series as a part of the model, have been used in signal analysis since the sixties of the XX century. Time series modeling consists in fitting a function *f(·)* into a given data set y_i.

The *f (·)* is a function of previous data y_{i-j}, for *j=1,…, J* and an innovation series w_{i-k}, for *k=1,…,K*. In general, stochastic time series model has the following form:

$$y_i = f(y_{i-j}, w_{i-k}) + w_i \tag{1}$$

The innovation series w_i use to be assumed either a white noise series e_i, or series of model errors $\varepsilon_i = y_i - \hat{y}_i$. The function *f(·)* may be either linear or nonlinear. Though real processes use to be non-linear and non-Gaussian, they are often modeled as linear ARMA (Box, 1983), (Yaffee, 2000).

$$y_i = \frac{C(D)}{A(D)} e_i = \sum_{k=0}^{dC} c_k e_{i-k} + \sum_{j=1}^{dA} a_j y_{i-j} \tag{2}$$

where:

e_i - Gaussian white noise series,

D - time delay operator: $D^k y_i = y_{i-k}$.

Theory of Gaussian linear time series models, including stability and invertibility conditions, as well as analysis and identification methods, are well established, e.g. (Box, 1983). However, asymmetrical time series or time series that are characterized by data anomalies cannot be modeled as linear. There are a great number of possible nonlinear structures of the function f*(·),* but the most common one is a nonlinear polynomial structure (3). The model of such form is named nonlinear ARMA model (NARMA).

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$$y_i = w_i + \sum_{k=1}^{K} c_k w_{i-k} + \sum_{k_1=1}^{K_1}\sum_{k_2=1}^{K_2} c_{k_1,k_2} w_{i-k_1} w_{i-k_2} + \sum_{k_1=1}^{K_1}\sum_{k_2=1}^{K_2}\sum_{k_3=1}^{K_3} c_{k_1,k_2,k_3} w_{i-k_1} w_{i-k_2} w_{i-k_3} + \ldots$$
$$+ \sum_{j=1}^{J} a_j y_{i-j} + \sum_{j_1=1}^{J_1}\sum_{j_2=1}^{J_2} a_{j_1,j_2} y_{i-j_1} y_{i-j_2} + \sum_{j_1=1}^{J_1}\sum_{j_2=1}^{J_2}\sum_{j_3=1}^{J_3} a_{j_1,j_2,j_3} y_{i-j_1} y_{i-j_2} y_{i-j_3} + \ldots \quad (3)$$

Bilinear ARMA models are a subset of the class of NARMA models, and are described by the following equation:

$$y_i = w_i + \sum_{k=1}^{K} c_k w_{i-k} + \sum_{k=1}^{K}\sum_{l=1}^{L} \beta_{k,l} w_{i-k} y_{i-l} + \sum_{j=1}^{J} a_j y_{i-j} \,. \quad (4)$$

Nonlinear time series analysis, particularly – establishing stability and invertibility conditions, is in general much more complex than analysis of the linear ones. Therefore, only the particular model structures are being analyzed in practice. In 1978, Granger and Andersen derived some interesting properties of the bilinear model with the simplest structure (Granger & Andersen, 1978)

$$y_i = e_i + \beta_{11} e_{i-1} y_{i-1}, \quad (5)$$

where e_i is an independent white noise sequence with zero mean and the variance $m_e^{(2)}$. Since then, simple bilinear models have been also investigated by Martins (Martins, 1997), (Martins, 1999), Berlin Wu (Berlin Wu, 1995), Tong (Tong 1993), Granger and Terasvirta (Granger & Terasvirta, 1993). Opinion on the usefulness of bilinear series vary from a skeptic one "*Using economic data, bilinear models have not been found to be very relevant*", (Tong 1993) to an enthusiastic "*The bilinear model has been used successfully to model time series that have been traditionally difficult to fit with classical linear time series methods*" (Martins, 1999). The aim of the paper is to assume an attitude towards the above statements, especially in the field of technological and medical processes. In the chapter, elementary bilinear model *EB(k,l)*:

$$y_i = w_i + \beta_{kl} w_{i-k} y_{i-l}\,, \quad (6)$$

where $k \le l$, is considered and then applied in signal analysis.

The chapter is organized in the following way:

Section 2 is dedicated to elementary bilinear processes. Analytical relations between process moments and process parameters are presented for diagonal and sub-diagonal elementary bilinear processes. In general, they are valid under assumption that inaccessible process input is uncorrelated and symmetrically distributed.

In *Section 3*, methods of parameters' estimation for elementary bilinear models are presented. Identification algorithms for simple and generalized methods of moments for elementary bilinear models are formulated.

Section 4 is dedicated to application of elementary bilinear models in simulation and prediction. A hybrid linear-bilinear model is introduced and, on its basis, a bilinear minimum-variance prediction algorithm is derived, for model residuum represented by diagonal and sub-diagonal elementary bilinear model.

In *Section 5*, the most important results are summarized.

2. Bilinear time series models

Large amount of dynamical systems may be described with set of conservation equations in the following form:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) + \sum_{k=1}^{m} \mathbf{N}_k u_k(t)\mathbf{x}(t), \tag{7}$$

where the last term creates the bilinear part of the equation. Bilinear equations are the natural way of description of a number of chemical technological processes like decantation, distillation, and extraction, as well as biomedical systems, e.g. (Mohler, 1999), (Nise, 2000). Though the nature of many processes is bilinear, identification of the model (7) can be difficult, at least because some of the state or input variables may be immeasurable. This is the case of many biological or biomedical processes. Often, the discrete set of the output observation $\{y_i\}$, for $i=1,\ldots,n$, is the only information on the considered process. In such cases bilinear time series model (8), which explains relation between the set of the output data only, may be considered.

$$A(z^{-1})y_i = C(z^{-1})e_i + \sum_{k=1}^{K}\sum_{l=1}^{L} \beta_{kl} e_{i-k} y_{i-l} \tag{8}$$

Bilinear time series models have been mentioned in control engineering since early seventieth. Schetzen Theorem (Schetzen, 1980) states, that any stable time variant process may be modeled as time invariant bilinear time series. General structure of bilinear time series model (8) is complex enough to make its analysis very difficult. Therefore, in practise the particular model structures are being analysed.

Stochastic processes are completely characterized by their probabilistic structure, i.e. probability or probability density $p(y)$ (e.g. Therrien, 1992). However, in practice, probabilistic structure of a considered system is unknown and, therefore, the system analysis is performed on the ground of its statistical moments. The moments for any stochastic process with any probabilistic density $p(y)$ are expressed as:

$$M_y^{(r)} = E\{y_i^{\,r}\} \tag{9}$$

where E is an operator of expected value :

$$E\{y\} = \sum_x yp(y) = \mu\ . \tag{10}$$

Central moments are:

$$M_y^{'(r)} = E\{(y_i - \mu)^r\} \tag{11}$$

When the structure of particular bilinear model is simple, the moments and the central moments may be analytically calculated based on the process equation, and the moments' definitions (9), (11). Elementary bilinear time series models, considered in this chapter, in dependence on their structures, are classified as sub diagonal or diagonal.

2.1 Sub diagonal elementary bilinear time series EB(k,l)

When the structure *k,l* of elementary bilinear time series model *EB(k,l)* satisfy relation $k < l$, the model (12) is named sub diagonal.

$$y_i = e_i + \beta_{kl} e_{i-k} y_{i-l}, \tag{12}$$

The model is characterized by two parameters, β_{kl} and m_e^2, related to each other. It may be proven, (e.g. Tong, 1993) that the model (12) is stable when $|\beta_{kl}^2 m_e^{(2)}| < 1$, and is invertible when $|\beta_{kl}^2 m_e^{(2)}| < 0.5$. Time series invertibility means that for a stable time series

$$y_i = f(e_i, e_{i-1}, ..., e_{i-k}, y_{i-1}, ..., y_{i-l}) \tag{13}$$

operation of inversion

$$e_i = f(e_{i-1}, ..., e_{i-k}, y_i, y_{i-1}, ..., y_{i-l}) \tag{14}$$

is stable. The moments and the central moments of *EB(k,l)* may be analytically calculated based on the process equation (12), and the moments' definitions (9), (11). Relations between moments and parameters are given in the table 1. The variance $M_y^{(2)}(0)$ of *EB(k,l)* is bounded when:

$$|\beta_{kl}^2 m_e^{(2)}| < 1. \tag{15}$$

The fourth moment $M_y^{(4)}(0,0,0)$ of *EB(k,l)* is bounded when:

$$|\beta_{kl}^4 m_e^{(4)}| < 1 \tag{16}$$

Irrespective of the probabilistic density of e_i, sub diagonal *EB(k,l)* is non-Gaussian and uncorrelated. Gaussian equivalent of the sub diagonal *EB(k,l)* with a bounded variance is a Gaussian white noise with the first and the second moments the same as the respective moments of the *EB(k,l).* Comparison of an *EB(2,4)* time series and its Gaussian equivalent is shown in the Fig.(1).

Moments	Formulae
$M_y^{(1)}$	0
$M_y^{(2)}(0)$	$\frac{m_e^{(2)}}{1-\beta_{kl}^2 m_e^{(2)}}$
$M_y^{(2)}(m>0)$	0
$M_y^{(3)}(0,0)$	0
$M_y^{(3)}(l_1 \neq k, l_2 \neq l)$	0
$M_y^{(3)}(k,l)$	$\beta_{kl} m_e^{(2)} M_y^{(2)}(0)$
$M_y^{(4)}(0,0,0)$	$\frac{m_e^{(4)} + 6\beta_{kl}^2 (m_e^{(2)})^2 M_y^{(2)}(0)}{1-\beta_{kl}^4 m_e^{(4)}}$

Table 1. Relation between moments and *EB(k,l)* parameters

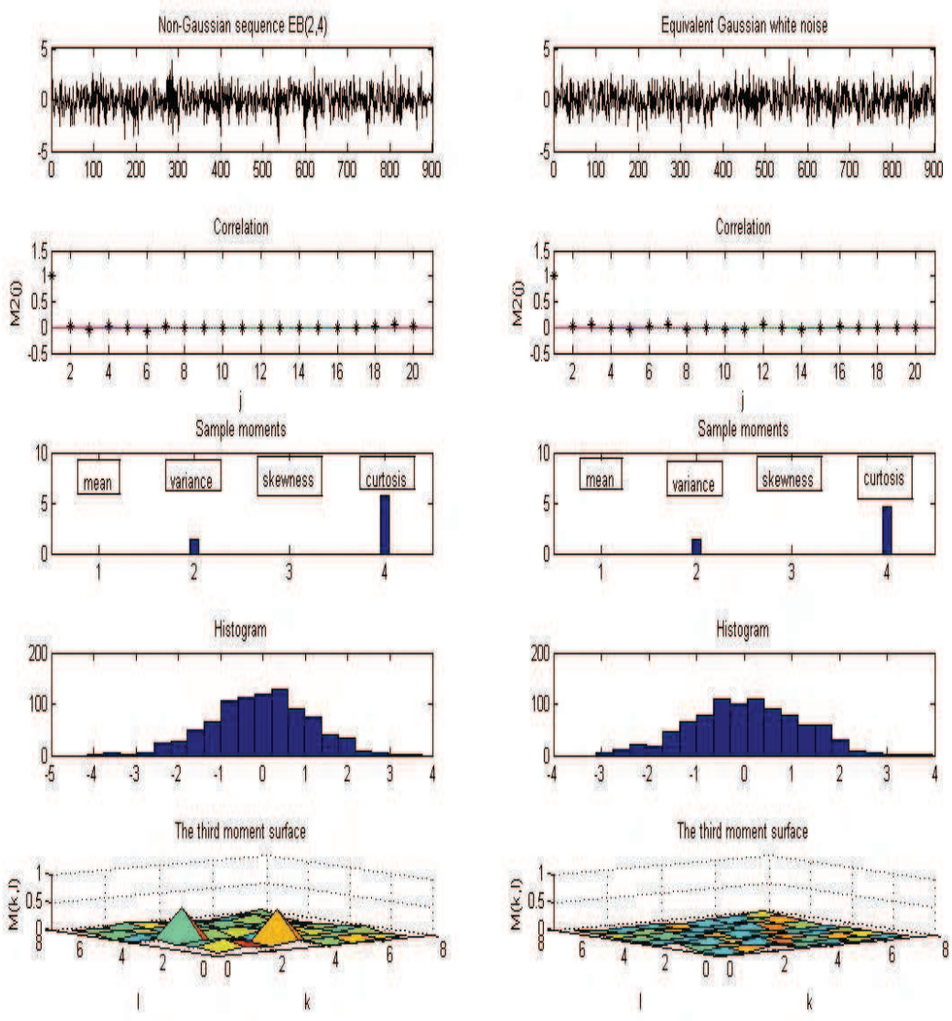

Fig. 1. Comparison of the estimated moments of EB(2,4) and an equivalent white noise

2.2 Diagonal elementary bilinear time series EB(k,k)

Elementary diagonal bilinear time series model, *EB(k,k)* has the following structure:

$$y_i = e_i + \beta_{kk} e_{i-k} y_{i-k}. \tag{17}$$

Properties of the model depend on two parameters, β_{kk} and $m_e^{(2)}$, related to each other. Stability and invertibility conditions for *EB(k,k)* are the same as for sub diagonal *EB(k,l)* time series model. Having known the process equation (17) and the moments' definitions (9) and (11), moments and central moments of the *EB(k,k)*may be analytically calculated as functions of model parameters. Though *EB(k,l)* and *EB(k,l)* with respect to model equation are similar to each other, their statistical characteristics are significantly different. Relation between

succeeding moments and model parameters are given in the table 2. An example of a single realization of EB*(5,5)* series as well as its sampled moments is shown in the the Fig. 2.

Moments	Formulae
$M_y^{(1)}$	$\beta_{kk} m_e^{(2)}$
$M_y^{(2)}(0)$	$\frac{m_e^{(2)} + \beta_{kk}^2 (m_e^{(4)} - (m_e^{(2)})^2)}{1 - \beta_{kk}^2 m_e^{(2)}}$
$M_y^{(2)}(m \neq k)$	$\beta_{kk}^2 \left(m_e^{(2)}\right)^2$
$M_y^{(2)}(k)$	$2\beta_{kk}^2 \left(m_e^{(2)}\right)^2$
$M_y^{(3)}(0,0)$	$3\beta_{kk}^2 (m_e^{(2)})^2 + \beta_{kk}^3 \frac{m_e^{(6)} - \beta_{kk}^2 m_e^{(2)} m_e^{(6)} + 3\beta_{kk}^2 (m_e^{(4)})^2)}{1 - \beta_{kk}^2 m_e^{(2)}}$
$M_y^{(3)}(k,l<k)$	$2\beta_{kk}^3 \left(m_e^{(2)}\right)^3$
$M_y^{(3)}(k,k)$	$\beta_{kk} m_e^{(4)} + \frac{3\beta_{kk}^3 m_e^{(2)} m_e^{(4)}}{1 - \beta_{kk}^2 m_e^{(2)}}$
$M_y^{(3)}(k,l>k)$	$2\beta_{kk}^3 \left(m_e^{(2)}\right)^3$
$M_y^{(3)}(k,2k)$	$4\beta_{kk}^3 \left(m_e^{(2)}\right)^3$

Table 2. Relations between moments and *EB(k,k)* parameters

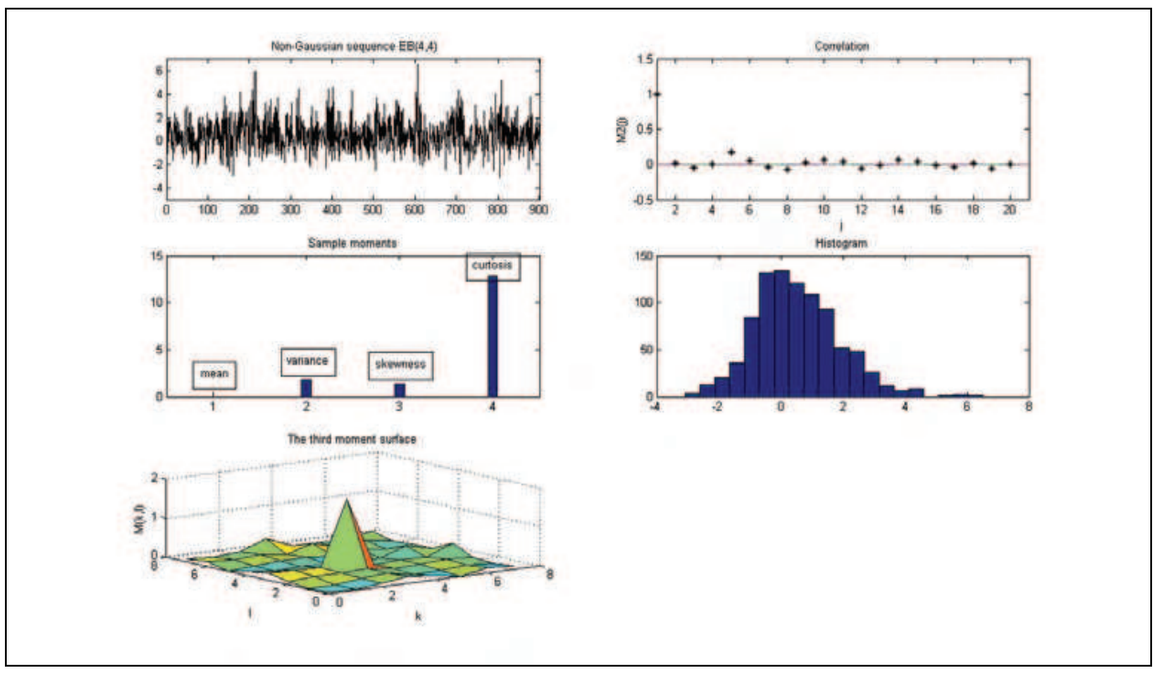

Fig. 2. EB(5,5) sequence and its characteristics

Diagonal *EB(k,k)* time series $\{y_i\}$ has a non-zero mean value, equal to $M_y^{(1)}$. Deviation from the mean $z_i = y_i - M_y^{(2)}$ is a non-Gaussian time series. A Gaussian equivalent of z_i is a *MA(k)* series:

$$z_i = w_i + c_k w_{i-k} \tag{18}$$

where w_i is a Gaussian white noise series. Values of c_k and $m_w^{(2)}$ can be calculated from the set of equations (19):

$$\begin{gathered} \frac{m_e^{(2)}(1+\beta_{kk}^2 m_e^{(2)} + \beta_{kk}^4 (m_e^{(2)})^2)}{1-\beta_{kk}^2 m_e^{(2)}} = m_w^{(2)}(1+c_k^2) \\ \beta_{kk}^2 (m_e^{(2)})^2 = c_k m_w^{(2)} \end{gathered} \tag{19}$$

3. Identification of EB(k,l) models

Under assumption that the model *EB(k,l)* is identifiable, and that the model structure is known, methods of estimation of the model parameters are similar to the methods of estimation of linear model parameters. The similarity stems from that the bilinear model structure, though nonlinear in e_i and y_i, is linear in parameter β_{kl}. A number of estimation methods originate from minimization of a squared prediction error (20). Three of them, which are frequently applied in estimation of bilinear model parameters, will be discussed in the section 3.1.

$$\varepsilon_i = y_i - \hat{y}_{i|i-1} \tag{20}$$

Moments' methods are an alternative way of parameters' estimation. Model parameters are calculated on the base of estimated stochastic moments (Tang & Mohler, 1988). Moments' methods are seldom applied, because hardly ever analytical formulae connecting moments and model's parameters are known. For elementary bilinear time series models the formulae were derived, (see table 1, table 2) and therefore, method of moments and generalized method of moments, discussed in section 3.2, may be implemented to estimate elementary bilinear models parameters.

3.1 Methods originated from minimization of the squared prediction error

Methods that originate from minimization of the squared prediction error (20) calculate model parameters by optimization of a criterion $J(\varepsilon_i^2)$, being a function of the squared prediction error. In this section the following methods are discussed:

- minimization of sum of squares of prediction error,
- maximum likelihood,
- repeated residuum.

a) Minimization of the sum of squares of prediction error

Minimization of the sum of squares of prediction error is one of the simplest and the most frequently used methods for time series model identification. Unfortunately, the method is sensitive to any anomaly in data set applied in model identification (Dai & Sinha, 1989). Generally, filtration of the large data deviation from the normal or common course of time

series (removing outliers) precedes the model identification. However, filtration cannot be applied to the bilinear time series, for which sudden and unexpected peaks of data follows from the bilinear process nature, and should not be removed from the data set used for identification. Therefore, the basic *LS* algorithm cannot be applied to elementary bilinear model identification and should be replaced by a modified LS algorithm, resistant to anomalies. Dai and Sinha proposed robust recursive version (RLS) of LS algorithm, where β_{kl} parameter of the model *EB(k,l)* is calculated in the following way:

$$\begin{aligned} b_{kl,i} &= b_{kl,i-1} + k_i\left(y_i - \Phi_i b_{kl,i-1}\right) \\ k_i &= \frac{P_{i\text{-}1}\Phi_i}{\alpha_i + \Phi_i^2 p_{i-1}} \\ P_i &= \frac{1}{\alpha_i}\left(P_{i\text{-}1} - \frac{P_{i-1}^2\Phi_i^2}{\alpha_i + \Phi_i^2 p_{i-1}}\right) \end{aligned} \tag{21}$$

where:

- $b_{kl,i}$ -- evaluation of model parameter β_{kl} calculated in *i-th* iteration,
- $\Phi_i = \hat{w}_{i-k} y_{i-l}$ -- generalized input,
- $\hat{w}_i = y_i - \Phi_i b_{kl,i-1}$ -- one step ahead prediction error,
- α_i -- coefficient that depends upon the prediction error as follows:

$$\alpha_i = \begin{cases} \dfrac{sign(\hat{w}_i) y_{tresh}}{\hat{w}_i} & \text{for } |\hat{w}_i| > y_{tresh} \\ 1 & \text{for } |\hat{w}_i| \le y_{tresh} \end{cases}$$

y_{tresh} -- a threshold value

b) Maximum likelihood

Maximum likelihood method was first applied to bilinear model identification by Priestley (Priestley, 1980) then Subba (Subba, 1981), and others e.g. (Brunner & Hess, 1995). In this method, elementary bilinear model *EB(k,l)* is represented as a function of two parameters $y_{\text{model}}(b_{kl}, y_{i-k})$:

$$y_{\text{model}} = b_{kl} y_{i-k} w_{i-l} \tag{22}$$

where w_i is an innovation series, equivalent to the model errors:

$$w_i = y_i - y_{\text{model}}(b_{kl}, y_{i\text{-}k}). \tag{23}$$

Likelihood is defined as:

$$L = L(b_{kl}, m_w^{(2)}) = \prod_{i=1}^{N} f(b_{kl}, m_w^{(2)}; w_i) \tag{24}$$

Maximization of L is equivalent to minimization of $-l=-ln(L)$:

$$-l(b_{kl}, m_w^{(2)}) = -\sum_{i=1}^{N} \ln(f(b_{kl}, m_w^{(2)}; w_i)) \tag{25}$$

Assuming that w_i is a Gaussian series with the mean value equal to zero, and the variance equal to $m_w^{(2)}$, negative logarithm likelihood *-ln(L)* is:

$$-\ln(L) = -l(w_N, w_{N-1}, ..., w_1 \mid b_{kl}, m_w^{(2)}) = \frac{N}{2}\ln(2\pi m_w^{(2)}) + \sum_{i=1}^{N} \frac{w_i^2}{2m_w^{(2)}}. \tag{26}$$

Having assumed initial values $b_{kl,0}$ and $m_{w0}^{(2)}$, parameters b_{kl} and $m_w^{(2)}$ are calculated by minimization of (26). Solution is obtained iteratively, using e.g. Newton-Raphson method. Essential difficulty lies in the fact that w_i is immeasurable and, in each iteration, should be calculated as:

$$w_i = y_i - b_{kl,i-1} w_{i-k} y_{i-l} \tag{27}$$

Obtained estimates of *EB(k,l)* parameters are asymptotically unbiased if w_i is Gaussian (Kramer & Rosenblatt, 1993). For other distributions, Gaussian approximation of the probability density function $f(y_i - y_{\text{model}}(b_{kl}, y_{i-k}))$ causes that the estimated parameters are biased.

c) Repeated residuum method

Alternative estimation method, named repeated residuum method, is proposed in (Priestley, 1980). Implemented to identification of elementary bilinear models, the method may be presented as the following sequence of steps:

1. Model *EB(k,l)* is expressed as:

$$y_i = w_i(1 + b_{kl} y_{i-l} D^k) \tag{28}$$

or equivalently:

$$w_i = \frac{y_i}{1 + b_{kl} y_{i-l} D^k} \tag{29}$$

2. Assuming b_{kl} small, the (29) may be approximated by:

$$w_i = (1 - b_{kl} y_{i-l} D^k) y_i = y_i - b_{kl} y_{i-l} y_{i-k}. \tag{30}$$

Presuming w_i is an identification error, an initial estimate $b_{kl,0}$ of the parameter b_{kl} can be evaluated from the (30), with the use of e.g. LS method.

3. Next, starting from $b_{kl,0}$ and $w_0 = 0$, succeeding w_i can be calculated iteratively:

$$w_i = y_i - b_{kl,0} w_{i-k} y_{i-l} \quad \text{for } i = k, k+1, ..., N. \tag{31}$$

4. Having known y_i and w_i for *i=k,...N*, an improved estimate b_{kl} that minimizes the following sum of squared errors (32) may be calculated.

$$V(b_{kl}) = \sum_{i=k}^{N} (y_i - b_{kl} w_{i-k} y_{i-l})^2. \tag{32}$$

5. The steps 3 and 4 are repeated until the estimate achieves an established value.

3.2 Moments method

With respect to the group of methods that originate from the minimization of the squared prediction error, a precise forms of estimation algorithms can be formulated. On the contrary, for moments method a general idea may be characterized only, and the details depend on a model type and a model structure. Moments method *MM* consists of two stages:

Stage 1: Under the assumption that the model structure is the same as the process structure, moments and central moments $M_y^{(r)}$ are presented as a function of process parameters Θ:

$$M_y^{(r)} = f(\Theta) \tag{33}$$

If it is possible, the moments are chosen such that the set of equations (33) has an unique solution.

Stage 2: In (33) the moments $M_y^{(r)}$ are replaced with their evaluation $\hat{M}_y^{(r)}$, estimated on the base of available data set y_i.

$$\hat{M}_y^{(r)} = f(\Theta) \tag{34}$$

The set of equations (34) is then solved according to the parameters Θ. Taking into consideration particular relation between moments and parameters for elementary bilinear models, *MM* estimation algorithm in a simple and a generalized version can be proposed.

MM – simple version

It is assumed that w_i is a stochastic series, symmetrical distributed around zero, and that the even moments $m_w^{(2r)}$ satisfy the following relations:

$$m_w^{(2r)} = k_{2r}(m_w^{(2)})^r \quad \text{for } r = 1,2,3... \tag{35}$$

Identification of *EB(k,l)* consists of identification of the model structure *(k,l),* and estimation of the parameters b_{kl} and $m_w^{(2)}$. Identification algorithm is presented below as the sequence of steps:

1. Data analysis:
   a. On the base of data set $\{y_i\}$ for *i=1,...,N,* estimate the following moments:
   $\hat{M}_y^{(1)}$; $\hat{M}_y^{(2)}(m)$ for $m = 0,1,2...$; $\hat{M}_y^{(3)}(l_1,l_2)$ for $l_1,l_2 = 0,1,2...$; $\hat{M}_y^{(4)}(0,0,0)$
   b. Find the values of $l_1 \neq 0$ and $l_2 \neq 0$ ($l_1 \leq l_2$), for which the absolute value of the third moment $\left|\hat{M}_y^{(3)}(l_1,l_2)\right|$ is maximal.
2. Structure identification:
   a. If $l_1 = k$, $l_2 = l$ then subdiagonal model *EB(k,l)* should be chosen.
   b. If $l_1 = k$, $l_2 = k$ then diagonal model *EB(k,k)* should be chosen
3. Checking system identifiability condition:
   If the model *EB(k,l)* was chosen, than:
   a. Calculate an index

$$W_3 = \frac{(\hat{M}_y^{(3)}(k,l))^2}{(\hat{M}_y^{(2)}(0))^3} \tag{36}$$

b. If W_3 <0.25 it is impossible to find a bilinear model *EB(k,l)* that has the same statistical characteristics as the considered process. Nonlinear identification procedure should be stopped. In such case either linear model may be assumed, or another non-linear model should be proposed.

If the model *EB(k,k)* was chosen, than:

a. Calculate an index

$$W_4 = \frac{\hat{M}_y^{(3)}(k,k)}{\hat{M}_y^{(2)}(0)\sqrt{\hat{M}_y^{(2)}(k)}} \tag{37}$$

b. If $\left|W_4 - \frac{3}{\sqrt{2}}\right| < \varepsilon$, where ε is an assumed accuracy, then the model input may be assumed Gaussian.

i. Calculate an index

$$W_5 = \frac{\hat{M}_y^{(3)}(k,k)\hat{M}_y^{(2)}(k)}{\hat{M}_y^{(2)}(0)\hat{M}_y^{(3)}(0,0)} \tag{38}$$

ii. If $W_5 < 0.23$, than the model *EB (k,k)* with the Gaussian input may be applied. If not than linear model *MA(k)* should be taken into account.

c. If $\left|W_4 - \frac{3}{\sqrt{2}}\right| \geq \varepsilon$ than the model input w_i cannot be Gaussian.

4. Estimation of model parameters :

a. When the model *EB(k,l)* was chosen in the step 2:

i. Find the solutions x_1, x_2 of the equation:

$$W_3 = x(1-x), \tag{39}$$

where $x = b_{kl}^2 m_w^{(2)}$

ii. For each of the solutions x_1, x_2 calculate the model parameters from the following equations:

$$\begin{aligned} m_w^{(2)} &= \hat{M}_y^{(2)}(0)(1-x), \\ b_{kl}^2 &= \frac{x}{m_w^{(2)}} \end{aligned} \tag{40}$$

iii. In general, the model *EB(k,l)* is not parametric identifiable, i.e. there is no unique solution of the equation (39) and (40). Decision on the the final model parameters should be taken in dependance on model's destination. Models applied for control and prediction should be stable and invertible. Models used for simulation should be stable but do not have to be invertible.

b. When in the step 2 the model *EB (k,k)* is chosen*:*

i. If $\left|W_4 - \frac{3}{\sqrt{2}}\right| \geq \varepsilon$ then $x=\frac{k_4 - W_4\sqrt{2}}{W_4\sqrt{2}(k_4-1)-2k_4}$, where:

$$\text{for } k_4<3: \quad \frac{k_4\sqrt{2}}{2} < W_4 < \frac{3\sqrt{2}}{2} \ ,$$

$$\text{for } k_4>3: \quad \frac{3\sqrt{2}}{2} < W_4 < \frac{k_4\sqrt{2}}{2} \ .$$

ii. If $W_4 \approx \frac{3}{\sqrt{2}}$, i.e. w_i is Gaussian, then the folloving equation have to be solved:

$$W_5 = \frac{6x(1-x)}{3+2x+22x\,\hat{}\,2}$$

Because the model EB(k,k) with the Gaussian input is not parametric identifiable, the final model should be chosen according to its destination, taking into account the same circustances as in the paragraph a) -iii.

MM -- generalized version:

Generalized moments method (GMM) (Gourieroux et al., 1996) (Bond et al., 2001), (Faff & Gray 2006), is a numerical method in which model parameters are calculated by minimization of the following index:

$$I = \sum_{j=1}^{J} f_k(y_i,\Theta)^2, \tag{41}$$

where:

Θ -- vector of parameters,

$f_j(y_i,\Theta)$ -- a function of data $y(i)$ and parameters Θ, for which:

$$E\{y_i,\Theta_0\} = 0 \quad \text{when} \quad \Theta=\Theta_0 \tag{42}$$

Θ_0 -- vector of parameters minimizing the index *I*.

Function $f_j(y_i,\Theta)$ for *j=1,2,...,J* is defined as a difference between analytical moment $M_y^{(k)}(\Theta)$ dependant upon the parameters Θ, and the evaluation $\hat{M}_y^{(k)}$ calculated on the base of y_i for *i=1,...,N*. The number *J* of considered moments depends on the model being identified.

Identification of the subdiagonal, elementary bilinear model *EB(k,l)* makes use of the four moments. Functions f_j, for *j=1,...,4* are defined in the following way:

$$f_1(y_i,\Theta) = M_y^{(2)}(0) - \hat{M}_y^{(2)}(0)$$
$$f_2(y_i,\Theta) = M_y^{(3)}(k,l) - \hat{M}_y^{(3)}(k,l)$$
$$f_3(y_i,\Theta) = M_y^{(4)}(0,0,0) - \hat{M}_y^{(4)}(0,0,0)$$
$$f_4(y_i,\Theta) = m_w^{(2)} - \hat{m}_w^{(2)}$$

Diagonal model EB(k,k) is identified on the base of three moments. The functions f_j for $j=1,...,6$ are:

$$f_1(y_i,\Theta) = M_y^{(1)} - \hat{M}_y^{(1)}$$
$$f_2(y_i,\Theta) = M_y^{(2)}(0) - \hat{M}_y^{(2)}(0)$$
$$f_3(y_i,\Theta) = M_y^{(2)}(k) - \hat{M}_y^{(2)}(k)$$
$$f_4(y_i,\Theta) = M_y^{(3)}(0,0) - \hat{M}_y^{(3)}(0,0)$$
$$f_5(y_i,\Theta) = M_y^{(3)}(k,k) - \hat{M}_y^{(3)}(k,k)$$
$$f_6(y_i,\Theta) = m_w^{(2)} - \hat{m}_w^{(2)}$$

For elementary bilinear models vector of parameters contains two elements: $m_w^{(2)}$ and b_{kl}. The parameters are calculated by minimization of the index (41), using e.g. nonlinear least squares method. It is assumed that starting point $\Theta_0 = \left[b_{kl,0}, m_{w0}^{(2)}\right]$ is a solution obtained with the use of the simple method of moments. Minimum of the index I may be searched assuming that the parameters b_{kl} and $m_w^{(2)}$ are constrained. The constrains result from the following attributes:

– The variance $m_w^{(2)}$ of the model input should be positive and less than the output variance, hence:

$$0 < m_w^{(2)} < m_y^{(2)}, \tag{43}$$

– The model should be stable, hence:

$$b_{kl}^2 m_w^{(2)} < 1 \tag{44}$$

3.3 Examples

The methods discussed above were applied to elementary bilinear time series identification under the following conditions:

1. Elementary diagonal and subdiagonal time series were identified.
2. Distribution of the white noise w_i was assumed:
   - Gaussian,
   - even

   with the zero mean and the variance $m_w^{(2)} = 1$.
3. All considered processes were invertible, i.e. the parameters satisfied the following condition: $b_{kl}^2 m_w^{(2)} < 0.5$ (Tong, 1993).
4. Identification was performed for 200 different realizations of the time series consisted of 1000 data.
5. For generalized moments method:

– Minimization of the performance index was carried out with the constrains:

$$\frac{-0.5}{m_y^{(2)}} < \hat{b}_{kl} < \frac{0.5}{m_y^{(2)}}$$
$$0 < m_w^{(2)} < m_y^{(2)}$$

– Starting point was calculated using simple moments method.

Result of conducted investigation may be summarized as follows:

1. Not every invertible elementary bilinear process is identifiable.
2. Correct identification results were obtained for processes, for which $\beta_{kl} \leq 0.4$, what is equivalent to: $\beta_{kl}^2 m_w^{(2)} \leq 0.16$.
3. When $\beta_{kl}^2 m_w^{(2)} > 0.16$ number of process realization, for which elementary bilinear model cannot be identified grows with the growth of $\beta_{kl}^2 m_w^{(2)}$.
4. When $\beta_{kl} \leq 0.4$ all the tested methods give the expected values of identified parameters b_{kl} equal to the truth values β_{kl}.
5. Generalized moments method is somewhat better than other considered methods, because the variances of the estimated parameters are the smallest.
6. For the processes with Gaussian excitation the variances of the identified parameters are greater than for the processes with even distribution of the input signal.

4. Application of EB(k,l) in signal modelling and prediction

Elementary bilinear time series models, which statistical attributes as well as methods of identification have been presented in the previous sections, are fit to modelling a limited class of signals only. However, an idea of using *EB(k,l)* models as a part of a hybrid linear-bilinear model, let to widen the class of signals, for which improving accuracy of modelling and prediction become possible.

4.1 Hybrid linear-bilinear model

Idea of a hybrid linear-bilinear (HLB) model is presented in the Fig. 3. Elementary bilinear model *EB(k,l)*, for which is assumed that $k \leq l$, and $e(i)$ is an independent white noise series, is applied as a part of the HLB. For *k<l* HLB model may be considered as linear autoregressive model stimulated by *EB(k,l)* series. The hybrid model consists of two parts:

– linear, that is built on the original data series $y(i)$:

$$y_i^L = -\sum_{j=1}^{dA} a_j y_{i-j} \tag{45}$$

– nonlinear that is built on the residuum w_i:

$$w_i = y_i - y_i^L \tag{46}$$

Residuum w_i is described in the following way:

$$w_i = \eta_i - \overline{\eta} \tag{47}$$

where:

$$\overline{\eta} = \begin{cases} \beta_{kk} m_e^{(2)} & \text{for } EB(k,k) \text{ model} \\ 0 & \text{for } EB(k,l) \text{ model} \end{cases} \tag{48}$$

and η_i is described by the elementary bilinear model *EB(k,l)* or *EB(k,k):*

$$\eta_i = \begin{cases} e_i + \beta_{kk} e_{i-k} \eta_{i-k} & \text{for } EB(k,k) \text{ model} \\ e_i + \beta_{kl} e_{i-k} \eta_{i-l} & \text{for } EB(k,l) \text{ model} \end{cases} \tag{49}$$

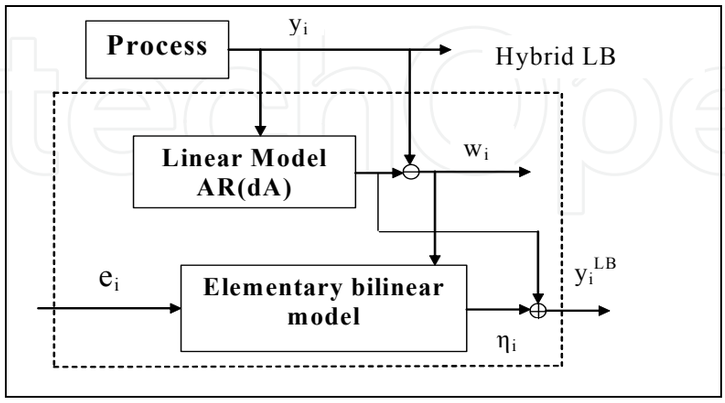

Fig. 3. Hybrid Linear-Bilinear model

The output of the H*LB* model is the following sum:

$$y_i^{LB} = y_i^L + \eta_i \tag{50}$$

Identification of the HLB model is done in three stages.

1. First stage -- data pre-processing -- is optional. If the original data set *{x(i)}* contain linear trends, they are removed according to:

$$z_i = x_i - x_{i-1} \tag{51}$$

If it is necessary, obtained data set z_i may be transformed. One of possible data transformation is:

$$y_i = \frac{z_i - \overline{z}}{\mathrm{var}(z)} \tag{52}$$

2. The second stage -- linear model AR(dA) (53) is identified.

$$A(z^{-1}) y_i = w_i \tag{53}$$

From the experience follows, that the *AR(dA)* models satisfying the coincidence condition:

$$r_j a_j \geq 0 \quad \text{for } j = 1,...,dA \tag{54}$$

where:

$$r_j = \frac{1}{N-j} \sum_{i=1}^{N-j} y_i y_{i-j} \tag{55}$$

are not only parsimonious but also have the better predictive properties than the AR*(dA)* models with the full rank.

3. The third stage -- elementary bilinear time series model is identified for residuum w_i in a way discussed in section 3.

4.2 Prediction

Time series models are mainly applied for signals' prediction. In this section, a prediction algorithm derived on the base of HLB model is presented. As it was discussed in the section 2, elementary bilinear models *EB(k,l)* and *EB(k,k)* have different statistical attributes. Therefore, prediction algorithms, though based on the same *HLB* model, have to be designed separately for residuum represented as *EB(k,l)* and *EB(k,k).* Minimum variance prediction algorithms have roots in the following theorems.

Theorem 1.

If y_i is a non-Gaussian stochastic time series described by the hybrid model HLB: $A(D)y_i = \eta_i$, where:

– *residuum η_i is represented as a sub diagonal model EB(k,l) and k<l:*

$$\eta_i = w_i + b_{kl} w_{i-k} \eta_{i-l},$$

– *w_i is an independent white noise series with the variance $m_w^{(2)}$,*

then the h-step prediction according to the algorithm:

$$\hat{y}_{i+h|i} = G(D)y_i + \beta_{kl} F(D)\left(\varepsilon^{\eta}_{i+h-k}\eta_{i+h-l}\right) \tag{56}$$

where:

$$\varepsilon_i^{\eta} = \eta_i - \hat{\eta}_{i|i-h},$$

$$\hat{\eta}_{i|i-h} = b_{kl}\varepsilon^{\eta}_{i+h-k}\eta_{i+h-l},$$

gives the prediction error $\varepsilon_i^y = F(D)w_i$ with the minimal possible variation:

$$E\left\{\varepsilon_i^y\right\}^2 = m_w^{(2)}\left(1 + \sum_{i=1}^{h-1} f_i^2\right) \tag{57}$$

In the above equations D – states for a nonlinear delay operator defined as follows:

$$D^k(y_i) = y_{i-k}$$

$$D^k(y_i x_i) = y_{i-k}x_{i-k}$$

$$D^k(y_i)x_i = y_{i-k}x_i,$$

A(D), F(D), G(D) – are polynomials in D with degrees *dA, h-1, dA-1* respectively. The polynomials are related to each other so to satisfy the following equation:

$$1 = A(D)F(D) + D^h G(D) \tag{58}$$

When residuum is a diagonal *EB(k,k)* process, the following theorem is formulated.

Theorem 2.

If yi is a non-Gaussian stochastic time series described by the hybrid model HLB: $A(D)y_i = z_i$, *where residuum* z_i *may be presented as:*

$$z_i = \eta_i - \overline{\eta},$$

$$\eta_i = w_i + \beta_{kk} w_{i-k}\eta_{i-k},$$

$$\overline{\eta} = \beta_{kk} m_w^{(2)},$$

then the h-step prediction according to the algorithm:

$$\hat{y}_{i+h|i} = G(D)y_i + \beta_{kk}F(D)\left(\varepsilon^{\eta}_{i+h-k} z_i\right) + z_{i+h-k} + \overline{\eta} + \beta_{kk}F(D)\left(\varepsilon^{\eta}_{i+h-k}\overline{\eta}\right) + F(D)\overline{\eta} \tag{59}$$

where:

$$\varepsilon_i^{\eta} = \eta_i - \hat{\eta}_{i|i-h},$$

$$\hat{\eta}_{i|i-h} = b_{kk}\varepsilon^{\eta}_{i+h-k}\eta_{i+h-k},$$

gives the prediction error: $\varepsilon_i^{y} = F(D)w_i$ *with the minimal possible variation:*

$$E\left\{\varepsilon_i^{y}\right\}^2 = m_w^{(2)}\left(1 + \sum_{i=1}^{h-1} f_i^2\right)$$

Delay operator *D* and the polynomials *A(D), F(D), G(D)* are defined in the same way as in the Theorem 1.

4.3 Prediction strategy

Prediction strategy means a way of data processing that should be applied to the original data series to obtain the accepted prediction. In this section MV -HLB prediction strategy is formulated. The strategy has the form of an algorithm built of a sequence of the following steps:

1. The original set of data y_i, *i=1,…,N* is divided into two sets:
   - training set, for $i = 1,...,N_{train}$, that is used for model identification,
   - testing set, for $i = 1,...,N_{test}$, on which the prediction algorithm is tested.
2. On the training set, parameters of a linear *AR(dA)* model:

$$y_i = -a_1 y_{i-1} - a_2 y_{i-2} - ... - a_{dA} y_{i-dA} \tag{60}$$

   are estimated. For further consideration, such models that satisfy coincidence condition (54) are accepted only.
3. On the training set the residuum is calculated according to the equation:

$$\eta_i = y_i + a_1 y_{i-1} + a_2 y_{i-2} + ... + a_{dA} y_{i-dA} \tag{61}$$

4. In the following steps *4-7* identification procedures described in details in section 3 are realized.
5. The first, the second, the third and the fourth moments of the residuum η_i are estimated.
6. Identifiability criterion for *EB(k,l)* process is checked for the series of residuum. If fitting elementary bilinear model is possible, one can continue in the step 7. If not, one should move to the step 12.
7. The structure *(k,l)* of the *EB(k,l)* model is established on the base of the third moment for residuum.
8. The values of β_{kl} and $m_w^{(2)}$ are calculated using e.g. one of the moments' methods.
9. For the assumed prediction horizon h and the estimated polynomial $A(D)$ the diophantine equation (58) is solved, and the parameters f_k, $k=1,\ldots,h-1$ of the polynomial $F(D)$ as well as the parameters g_j, $j=1,\ldots,dA-1$ of the polynomial $G(D)$ are calculated. Then, if the prediction horizon $h \leq \min(k,l)$, prediction algorithm is designed either on the base of the Theorem 1 -- for the *EB(k,l)* model of the residuum, or on the base of the Theorem 2 -- for the *EB(k,k)* model of the residuum.
10. The designed prediction algorithm is tested on the testing set. STOP.
11. If $h > \min(k,l)$ then move to the step 12.
12. Design linear prediction algorithm e.g. [1], [4]: $\hat{y}_{i+h|i} = G(D)y_i$
13. Test it on the training set. STOP.

The above prediction strategy was tested for simulated and real world time series. In the next section, the strategy is applied to series of sunspot numbers and MVB prediction is compared with the non-linear prediction performed using the benchmark SETAR model, proposed by Tong (Tong, 1993).

4.4 Sunspot number prediction

Sunspots events have been observed and analysed for more than 2000 years.

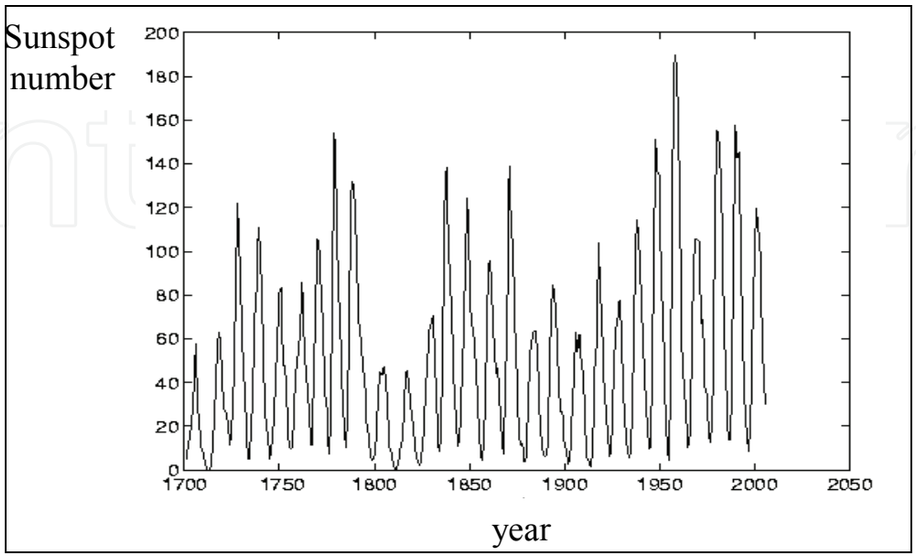

Fig. 4. Sunspot events

The earliest recorded date of a sunspot event was 10 May 28 BC. The solar cycle was first noted in 1843 by the German pharmaceutical chemist and astronomer, Samuel Heinrich Schwabe as a result of 17 years of daily observations. The nature of solar cycle, presented in the Fig. 4 characterized by a number of sunspots that periodically occurs, remains a mystery to date. Consequently, the only feasible method to predict future sunspot number is time series modeling and time series prediction. Linear prediction do not give acceptable results hence, the efforts are made to improve the prediction using nonlinear models and nonlinear methods. Tong (Tong, 1993) has fitted a threshold autoregressive (SETAR) model to the sunspot numbers of the period 1700-1979:

$$Y_i = \begin{cases} 1.92 + 0.84Y_{i-1} + 0.07Y_{i-2} - 0.32Y_{i-3} + 0.15Y_{i-4} - 0.20Y_{i-5} \\ -0.00Y_{i-6} + 0.19Y_{i-7} - 0.27Y_{i-8} + 0.21Y_{i-9} + 0.01Y_{i-10} + e_i^1 \quad when \;\; Y_{i-8} \le 11.93 \\ \\ 4.27 + 1.44Y_{i-1} - 0.84Y_{i-2} - 0.06Y_{i-3} + e_i^2 \quad when \;\; Y_{i-8} > 11.93 \end{cases} \tag{62}$$

The real data were transformed in the following way:

$$Y_i = 2(\sqrt{1 + y_i} - 1) \tag{63}$$

where y_i is the sunspot number in the year *1699+i*.

Based on the model (62) prediction for the period 1980-2005 was derived, and used as a benchmark for comparison with the prediction, performed in the way discussed in the paper. The *HLB* model (64) was then fitted to the sunspot numbers, coming from the same period 1700-1979, under the assumption that the linear part of the *HLB* model satisfies the coincidence condition.

$$\begin{aligned} Y_i &= 0.81Y_{i-1} + 0.21Y_{i-8} + \eta_i \\ \eta_i &= e_i + 0.02\eta_{i-7}e_{i-7} \end{aligned} \tag{64}$$

The Y_i is a variable transformed in the same way as in the Tong's model (62), and the variance of residuum is $\mathrm{var}(\eta) = 8.13$.

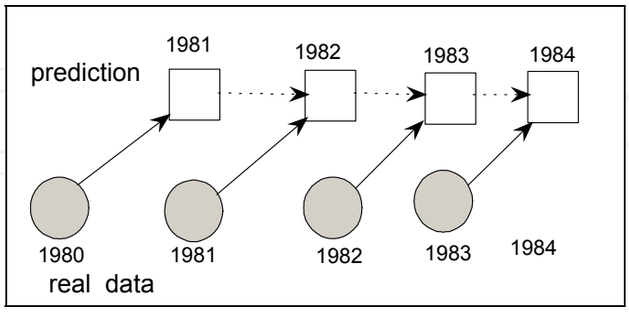

Fig. 5. Scheme of prediction calculation

Sunspot events prediction for the period 1981—2005 was performed according to the scheme showed in the Fig. 5. One step ahead prediction $\hat{y}_{i+1|i}$ calculated at time *i* depends on the previous data and the previous predictions. Prediction algorithm has the form specified in Theorem 2.For the data transformed according to (63) predictions obtained based on Tong's model and the *HLB* model are compared in the Fig. 6.

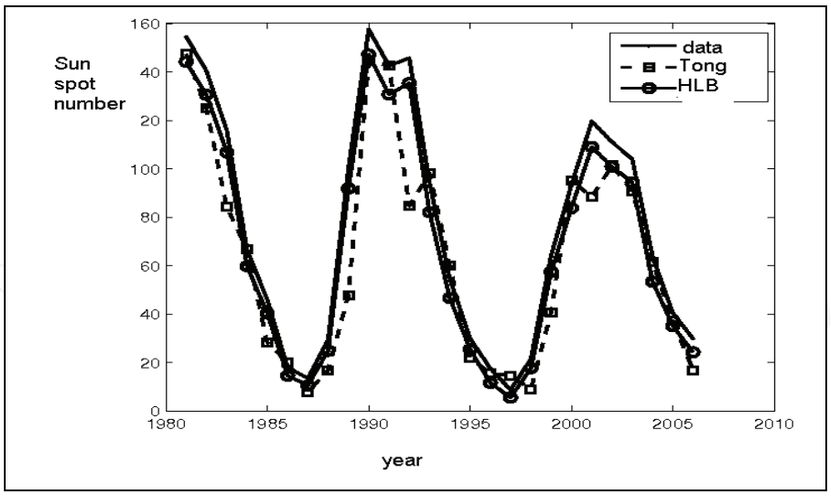

Fig. 6. Prediction for the period 1981-2005 based on Tong's and *HLB* models

The *HLB* prediction is evidently more precise than the one derived on the base of the Tong's model. Sum of squares of the Tong's prediction errors was:

$$S_T = 1.07 \times 10^4 ,$$

while sum of squares of the *HLB* prediction errors was:

$$S_{MLB} = 1.70 \times 10^3$$

Data transformation (63) is not natural for minimum variance prediction. Therefore, *HLB* model was once more identified, for the data transformed in the following way:

$$Y_i = \frac{y_i - \overline{y}}{\mathrm{var}(y)} . \tag{65}$$

This time the following *HLB* model was identified:

$$\begin{aligned} Y_i &= 0.80Y_{i-1} - 0.29Y_{i-7} + 0.52Y_{i-8} + \eta_i \\ \eta_i &= e_i + 0.08\eta_{i-3}e_{i-3} \end{aligned} \tag{66}$$

and variance of the residuum $\mathrm{var}(\eta) = 0.24$. Prediction algorithm was built on the base of model (66) in a way specified in Theorem 2. The sum of squares of the *HLB* prediction errors was this time:

$$S_{MLB} = 30 ,$$

hence, higher quality of the *HLB* prediction was obtained this time than previously. Fig. 7 illustrates prediction for the period 1981-2005, obtained on the base of Tong's model (62), built on the data transformed according to (63), and on the base of *HLB* model (66).

Tong (Tong, 1993) after discussion with Sir David Cox, one of the greatest statisticians in XX century, defined genuine prediction, as the prediction of data that are entirely not known at the stage of prediction establishing. The idea is illustrated in the following scheme, and is known also as a multi-step prediction.

In 1979, genuine prediction of sun spot numbers was established for years 1980—1983 on the base of Tong, and *HLB* models. Sums of squares of the prediction errors were equal to 347 and 342, respectively. The results are showed in the Fig. 9.

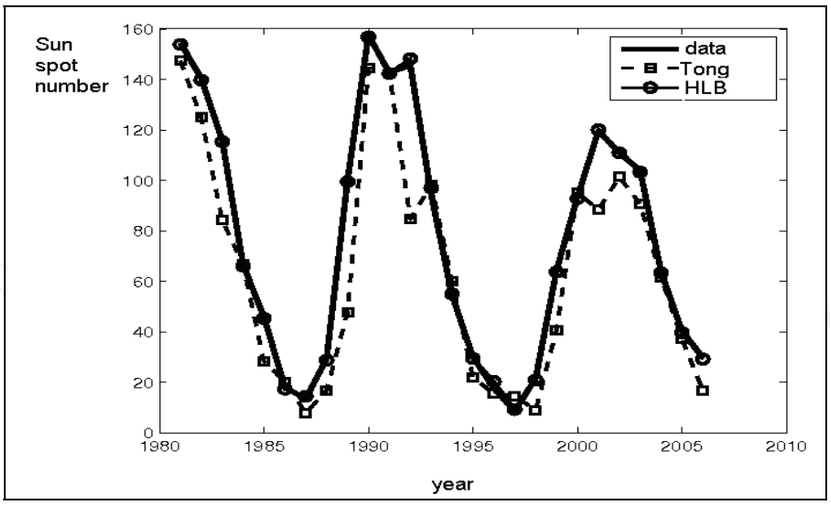

Fig. 7. Prediction for the period 1981-2005 based on Tong's and *HLB* models.

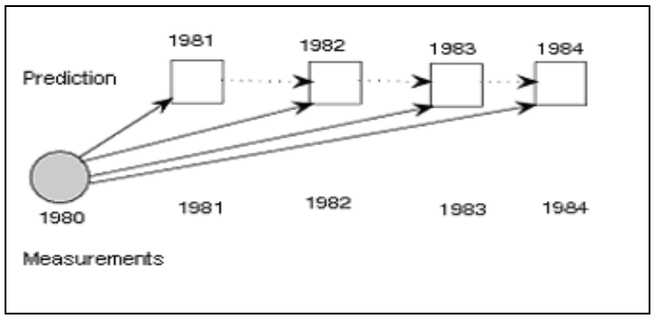

Fig. 8. Illustration of genuine prediction

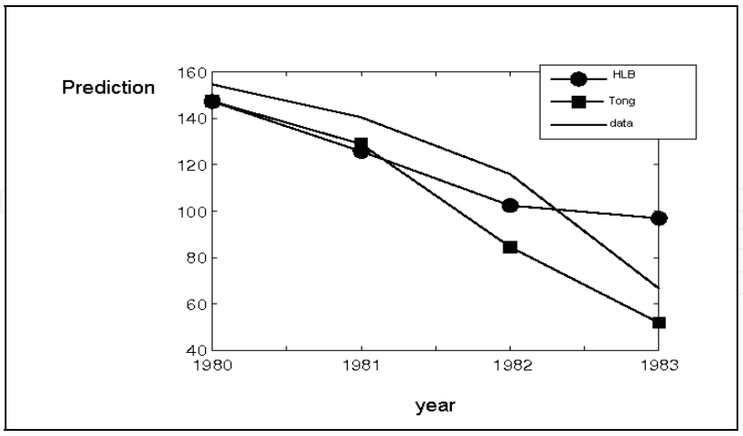

Fig. 9. Genuine prediction for the period 1980-84

5. Resume

In the chapter, a new method of time series analysis, by means of elementary bilinear time series models was proposed. To this aim a new, hybrid linear – elementary bilinear model

structure was suggested. The main virtue of the model is that it can be easily identified. Identification should be performed for the linear and the non-linear part of the model separately. Non-linear part of the model is applied for residuum, and has elementary bilinear structure. Model parameters may be estimated using one of the moments' methods, because relations between moments and parameters of elementary bilinear time series models are known.

Based on HLB model, minimum-variance bilinear prediction algorithm was proposed, and the prediction strategy was defined. The proposed prediction strategy was than applied to one of the best-known benchmark – sunspot number prediction. Prediction efficiency obtained with the use of *HLB* model, and bilinear prediction algorithm, in the way described in the paper, occurred much better than the efficiency obtained on the base of SETAR model, proposed by Tong.

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