Chapter

Gravity and Inertia in General Relativity

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Abstract

The relationship of gravity and inertia has been an issue in physics since Einstein, acting on an observation of Ernst Mach that rotations take place with respect to the "fixed stars", advanced the Equivalence Principle (EP). The EP is the assertion that the forces that arise in proper accelerations are indistinguishable from gravitational forces unless one checks ones circumstances in relation to distant matter in the universe (the fixed stars). By 1912, Einstein had settled on the idea that inertial phenomena, in particular, inertial forces should be a consequence of inductive gravitational effects. About 1960, five years after Einstein's death, Carl Brans pointed out that Einstein had been mistaken in his "spectator matter" argument. He inferred that the EP prohibits the gravitational induction of inertia. I argue that while Brans' argument is correct, the inference that inertia is not an inductive gravitational effect is not correct. If inertial forces are gravitationally induced, it should be possible to generate transient gravitational forces of practical levels in the laboratory. I present results of a experiment designed to produce such forces for propulsive purposes.

Keywords: gravity, inertia, general relativity, inertia as a gravitationally induced phenomenon, experimental test of inductive inertia

1. Introduction

Before Einstein's creation of general relativity theory, the conception of inertia was that captured in Newton's laws of mechanics, Newton's elaboration of the idea of inertia, first introduced by Galileo some years earlier. Inertia, properly *vis inertiae* or inert force, was taken to be an inherent property of "matter" conferred on it by its existence in absolute space, that only ceases to be inert when external forces act on matter to produce proper accelerations, rising to produce the reaction force the matter exerts on the accelerating agent to resist the impressed force. Already in Newton's day, this conception of inertia as due to absolute space was seriously called into question by, among others, Bishop Berkeley who argued that a body in an otherwise empty universe would have no inertia since there would be no other matter to refer motion of the body to. From this point of view, absolute space's action on matter is not the origin of inertia, the action of other matter in space is the cause of inertia. In the 17th and 18th, and most of the 19th centuries, Newton's view prevailed.

Berkeley's conjecture was revisited in the late 19th century by Ernst Mach, who noted that local rotation coincided with rotation relative to the "fixed stars", suggesting that local inertial frames of reference were determined by some long-range action of matter at cosmological distances. Einstein took Mach's insight to mean that in any properly constituted theory of gravity, inertia would emerge as an "inductive" gravitational effect of cosmic matter since gravity was/is the only known long-range force that might cause such effects. His first explicit attempt in this direction appeared in his, "Is There a Gravitational Effect Which is Analogous to Electrodynamic Induction?" in 1912 [1]. Einstein noted that Newtonian gravity was not sufficient to correctly encompass the induction of inertia – that is, the generation of the mass of matter by the gravitational interaction with chiefly cosmological matter – and inertial reaction forces – that is, Newton's third law forces on accelerating agents. Induction requires vector or tensor interactions. Several years later, general relativity was Einstein's theory that he was convinced accomplished this task. Indeed, that's why he called it "general relativity" because he, as we would say today, "unified" inertia and gravity by making inertia an inductive gravitational effect. Analogous to Maxwell's "unification" of electricity and magnetism in his electrodynamics.

2. Einstein's conception of gravity and inertia in general relativity

Einstein started talking about the gravitational induction of inertia as "Mach's principle" shortly after mooting general relativity. Willem de Sitter quickly pointed out that the field equations of general relativity have solutions that are plainly inconsistent with any reasonable interpretation of "Mach's principle". Einstein retreated from full-blown Mach's principle, which seemed to require action at a distance, then deemed inconsistent with the conception of field theory as articulated by Faraday. But he did not abandon the gravitational induction of inertia which is consistent with the tenets of field theory. Einstein advanced his ideas first in an address at Leiden in 1920 where he analogized his evolving view of spacetime to the "aether" of the turn of the century theory of electrodynamics. That is, spacetime is not some pre-existing void in which matter, gravity and the other forces of nature exist. It is a real, substantial entity – not a void – which *is* the gravitational field of matter sources. And then he extended his view in remarks in a series of lectures at Princeton in 1921 [2]. He calculated the action of some nearby, "spectator" matter on a test particle of unit mass (at the origin of coordinates) in the weak field limit of GR. There he found for the equations of motion of the test particle (his Equations 118):

$$\left(\frac{d}{dl}\right)[(l+\bar{\sigma})\mathbf{v}] = \nabla\sigma + \frac{\partial \mathbf{A}}{\partial l} + \nabla \times (\mathbf{A} \times \mathbf{v}), \tag{1}$$

$$\overline{\sigma} = (\kappa/8\pi)\int(\sigma/r)\ \ dV_o, \tag{2}$$

$$\mathbf{A} = (\kappa/2\pi)\int(\sigma d\mathbf{x}/dl)r^{-1}dV_o. \tag{3}$$

The second and third of these equations are the expressions for the scalar $(\bar{\sigma})$ and vector (A) potentials of the gravitational action of the spectator masses with density σ on the test particle. l is coordinate time and v is coordinate velocity of the test particle. The first equation is just Newton's second law. After writing down these equations, Einstein noted approvingly that,

> *The equations of motion, (118), show now, in fact, that.*
> *The inert mass [of the test particle of unit mass] is proportional to 1 + σ, and therefore increases when ponderable masses approach the test body.*

> *There is an inductive action of accelerated masses, of the same sign, upon the test body. This is the term dA/dl …*
> *Although these effects are inaccessible to experiment, because κ [Newton's constant of universal gravitation] is so small, nevertheless they certainly exist according to the general theory of relativity. We must see in them strong support for Mach's ideas as to the relativity of all inertial interactions. If we think these ideas consistently through to the end we must expect the* whole $g_{\mu\nu}$*-field, to be determined by the matter of the universe, and not mainly by the boundary conditions at infinity.*

Note that the small effects singled out by Einstein – the contribution of gravitational potential energy to the rest mass of the test particle and the inductive action, a force, of accelerating masses – are the two features of gravity that would supposedly account for all of inertia – origin of mass and reaction forces – in a properly constituted cosmology.

The above quote was not Einstein's last explicit word on gravity, inertia, and spacetime. In 1924, he again addressed these topics in a paper, "Concerning the Aether" [3]. In it he quickly asserted that by "aether" he did not mean the material aether of turn of the century electromagnetism. Rather, he meant a real, substantial, but not material entity that *is* spacetime, and that spacetime *is* the gravitational field of material sources. No material sources, no spacetime. Why did he make this radical break with the conception if space as a pre-existing void in which nature plays out its events in time? Arguably, this was his way of getting rid of the Minkowski and other metrics that de Sitter had shown to be anti-Machian, delimiting acceptable solutions of his field equations to those consistent with inertia as a strictly gravitational interaction. As he put it toward the end of his article:

> *The general theory of relativity rectified a mischief of classical dynamics. According to the latter, inertia and gravity appear as quite different, mutually independent phenomena, even though they both depend on the same quantity, mass. The theory of relativity resolved this problem by establishing the behavior of the electrically neutral point-mass by the law of the geodetic line, according to which inertial and gravitational effects are no longer considered as separate. In doing so, it attached characteristics to the aether [spacetime] which vary from point to point, determining the metric and the dynamical behaviour [sic.] of material points, and determined, in their turn, by physical factors, namely the distribution of mass/energy.*
>
> *That the aether of general relativity differs from those of classical mechanics and special relativity in that it is not "absolute" but determined, in its locally variable characteristics, by ponderable matter. This determination is a complete one if the universe is finite and closed …*

One may reasonably ask, if Einstein was convinced that general relativity, correctly interpreted, encompassed the gravitational induction of inertia, why today is it widely believed in the community of relativists and beyond that inertia is not gravitationally induced? That inertia is no better understood now than it was in the absolute systems of Newton and Minkowski? Carl Brans. And his "spectator matter" argument.

3. Carl Brans' "spectator matter" argument

Brans did his doctoral work at Princeton in the late 1950s. His doctoral supervisor was the noted experimentalist, Robert Dicke. After passing his qualifying exam, Dicke tasked Brans with investigating the question of as the origin of inertia in general relativity, as Dennis Sciama and others had made "Mach's principle" a central question in general relativity several years earlier. When Brans read Einstein's remarks on Machian inertia in Einstein's 1921 comments quoted above, he noted a problem. If gravitational potential energy due to nearby matter contributes to the rest masses of test particles, the Equivalence Principle is violated. This is not a minor problem. The Equivalence Principle is the bedrock of general relativity. The solution to this problem adopted in Brans' graduate school days was the imposition of a "coordinate condition". (A "gauge" solution is/was not available as general relativity is not a gauge theory).

As Brans, responding to several published papers on Mach's principle, later wrote in 1977 [4]:

> *Over the years, many and varied expressions of Mach's principle have been proposed, making it one of the most elusive concepts in physics. However, it seems clear that Einstein intended to show that locally measured inertial-mass values are gravitationally coupled to the mass distribution in the universe in his theory. For convenience I repeat the first order geodesic equations given by Einstein to support his argument:*
> *[Brans inserted here Einstein's equations displayed above.]*
>
> *… Einstein's claim is that "The inertial mass is proportional to* $(l + \overline{\sigma})$*, and therefore increases when ponderable masses approach the test body.*

Brans pointed out that having the masses of local objects, the unit mass test particle in this case, depend on their gravitational potential energies acquired by interaction with spectator matter must be wrong. Were it true, then the electric charge to mass ratios of elementary particles for example would depend on the presence of nearby matter. If this were true, gravity could be discriminated from accelerations without having to check for the presence of spectator matter by going to the window in a small lab and looking out to see if one were on Earth, or in a rocket accelerating at one "gee" in deep outer space– a violation of the Equivalence Principle. From this, Brans inferred that

> *…global, i.e., nontidal, gravitational fields are completely invisible in such local standard measurements of inertial mass, contrary to Einstein's claim… Einstein ought to have normalized his local space-time measurements to inertial frames, in which the metric has been transformed approximately to the standard Minkowski values, and for which distant-matter contributions are not present. [Emphasis added.]*

This is the "coordinate condition" required by Brans' work: that the coordinates be compatible with the assumed approximate Minkowski metric applicable in small regions of spacetime. Since the absence of gravity is presupposed for Minkowski spacetime, this amounts to the assumption that the Newtonian potential due to exterior matter in such small regions of spacetime is effectively everywhere/when equal to zero. That is, the locally measured value of the total Newtonian gravitational potential is universally zero. This certainly makes the localization of gravitational potential energy impossible in general relativity, a now widely accepted fact. And where there is effectively no gravity, there can be no gravitational induction of inertia. Accordingly, it would seem that Brans' spectator matter argument makes Machian gravitationally induced inertia incompatible with general relativity.

Carl Brans and his then graduate advisor Robert Dicke created scalar-tensor theory to redress this perceived failing of general relativity. A clear indicator of the importance of the spectator matter argument. The eventual failure of scalar-tensor gravity has left the question of the nature of inertia in a limbo that remains unresolved. Nonetheless, the spectator matter argument still stands as the key issue in the development of general relativity in the past century as it requires that any theory of inertia must satisfy the Equivalence Principle. A test that has not yet been met for any theory of gravity, general relativity included if Brans' argument is accepted as completely correct.

4. Is inertia gravitationally induced in general relativity?

The problem of the origin of inertia, that is, Mach's principle, ceased to be a topic of mainstream interest in general relativity 50 years ago. It was not forgotten by those who lived through the '50s and '60s and were parties to the debates on inertia. For example, John Wheeler, working with Ignazio Ciufolini, made it the center piece of one of his last major books on gravity: *Gravitation and Inertia* [5]. On the otherwise blank page facing the first page of chapter 1 we find, "Inertia here arises from mass there". Exactly as Einstein would have said. In the penultimate chapter (5) of the book we find, "In the next chapter we shall describe in detail dragging of inertial frames and *gravitomagnetism*, which may be thought of as a manifestation of some weak general relativistic interpretation of the Mach principle, their measurement would provide experimental foundation for this general relativistic interpretation of the origin of inertia."

Brans' spectator matter argument does not involve gravitomagnetism. But it is implicitly present in Einstein's Eqs. 118 quoted above. In the vector potential **A**. The sources of **A** are the matter density currents in the universe. As Sciama pointed out in his first paper "On the Origin of Inertia" [6], the integration over cosmic matter currents involved can be vastly simplified by noting that the important currents involved can be singled out by assuming the local accelerating body in question and (instantaneously) has velocity **v**, can be taken as at rest with the universe moving past it rigidly with velocity – **v**. This can be removed from the integration, and the remaining integral just returns the Newtonian gravitational potential for all the stuff in the universe (up to a factor of order unity). If the coefficient of the time derivative of – **v**, **a** that is, is one, then this term in the equation of motion is the inertial reaction force. So, Brans' argument does more than require that the gravitational potential energies conferred on test particles by spectator matter not influence the rest mass of the test particle. *It also demands that if inertial forces are gravitational inductive effects, the coefficient of the acceleration in the equation of motion must be one in all circumstances. This is only possible if the total Newtonian potential is a locally measured invariant equal to the square of the vacuum speed of light (which is also a locally measured invariant in general relativity).*

A series of events, too lengthy to relate in detail here, led to the rejection of the time derivative of the vector potential in the equation of motion in general relativity. This started with an article by Edward Harris in the *American Journal of Physics* in 1991 [7] where he outlined the analog of linearized weak field slow motion equations of general relativity with Maxwell's equations of electrodynamics. In this approximation, it is possible to argue that the $d\mathbf{A}/dt$ term in the field equation and equation of motion vanish by gauge invariance. Harris, of course, knowing that general relativity is not a gauge theory, allowed as how this could not be generally true. But others took this argument and tried to justify it on other terms. (See references in chapter 4 of Pfister and King. [8]) With the passing of the $d\mathbf{A}/dt$ term, so too went the question/possibility of the gravitomagnetic origin of inertial forces. This development led Herbert Pfister and Markus King, in their recent book *Inertia and Gravitation*, to remark that, "We hope to give a new synopsis of this theme [that Faraday induction that produces the time derivative of the vector potential is absent in general relativity], with this specific focus not entering most textbook presentations on the foundations of gravitation and general relativity – and in a way amend Cuifolini and Wheeler's view on gravitation and inertia ... ".

Cuifolini and Wheeler had opted for an account of inertia based on an initial spacelike hypersurface complemented by elliptic (instantaneous) constraint equations (first discussed by Wheeler and independently Lynden-Bell in the '60s) in preference to integrations over matter currents out the past light-cone to the past particle horizon. Pfister and King, having rejected the existence of Faraday induction effects in general relativity, were left no choice but the hypersurface/constraint equation approach. All of this was, wittingly or unwittingly, motivated by Carl Brans' conclusion that in sufficiently small regions of spacetime one must use the Minkowski metric which makes the local Newtonian potential vanish, eliminating gravity from external sources as an actor at that scale. That is, making the presence of cosmic matter, in Brans' word, "invisible".

Who's right? Cuifolini and Wheeler? Pfister and King? Brans? Arguably, they are, in a sense, all right, and wrong. The source of the confusion and problems regarding inertia in general relativity is the Minkowski metric – which de Sitter showed Einstein to be an acceptable formal solution of his field equations. The Minkowski metric is the metric for flat pseudo-Euclidean spacetime with gravity completely absent. That it is a solution of Einstein's field equations is not surprising. The theory is constructed on the assumption that in sufficiently small regions, spacetime is flat (and special relativity applies). As Einstein explicitly claimed, however, without gravity, there is no spacetime. This makes the Minkowski metric an unphysical pre-general relativistic idealization. Scaffolding to be removed once the construction of the theory is complete. The Maxwellian analog is the "roller bearing" mechanical aether he used to construct his equations of electrodynamics, promptly abandoned once the construction was complete. But spacetime, as a matter of observation, is essentially flat almost everywhere/when. What takes the place of Minkowski spacetime in the completed theory? Spatially flat FLRW spacetime.

5. Spatially flat cosmology and spectator matter

The one thing in the discussion of the role of inertia in general relativity that is certainly right is that Brans was absolutely correct in asserting that if spectator matter contributes to the rest masses of test particles through its contribution to the total gravitational potential by changing it (as Einstein assumed), then the Equivalence Principle is violated. This *fact*, however, does not necessitate the assumption of Minkowskian spacetime in small regions. It simply requires that the total locally measured Newtonian gravitational potential is an invariant. If the Newtonian potential is everywhere exactly the same in local measurements, that, by itself, precludes charge to mass ratios of elementary particles depending on the local gravitational environment. The question then is, does the spatially flat spacetime of FLRW cosmology have the requisite properties for a spacetime/gravitational field that yields gravitational induction of inertia?

Spatially flat, that is, curvature index "k" = 0, cosmology has several remarkable properties beyond being the transitional case between spherical, closed and hyperbolic, open geometry. The metric for k = 0 FLRW cosmology is, aside from the scale factor which multiplies the spacelike part of the metric to produce cosmic expansion, time-independent. So, a universe that starts out spatially flat, stays spatially flat throughout its history. Brans' doctoral supervisor Robert Dicke, in the '70s, identified this as a paradox, for spatial flatness should be unstable against small perturbations that should drive cosmic spacetime curvature quickly into the spherical or hyperbolic state. Why, more than 10 billion years into cosmic expansion is the universe still spatially flat? Alan Guth created inflation to address this problem.

Curvature at cosmic scale is related to energetic considerations. This is especially clear in the Newtonian analog elaboration of cosmology. (See Bernstein's *An Introduction to Cosmology*, Prentice Hall, 1995, chapter 2 for example [9]). Spatial flatness is a consequence of the balancing of gravitational potential energy and "kinetic", that is, non-gravitational energy encompassed in the Eq. $E = mc^2$. If gravitational energy exceeds non-gravitational energy, the universe is "closed" and will eventually contract after reaching some finite size. When non-gravitational energy exceeds gravitational energy, the universe is "open" and expands forever. Since gravitational and non-gravitational energies are balanced in a spatially flat universe, if we place a test particle of mass m anywhere/when in spacetime (which is the gravitational field according to Einstein), we will have:

$$m_g \phi = m_i c^2 \tag{4}$$

where the subscripts g and i identify the passive gravitational and inertial masses of the material particle. The Equivalence Principle identifies the passive gravitational and inertial masses as equal in magnitude, so:

$$\phi = c^2 \tag{5}$$

everywhere/when. The vacuum speed of light, a constant in special relativity, becomes a locally measured invariant in general relativity because, while local measurements always return the same number for c, non-local measurements may return different numbers. (Distant observers measure c in the vicinity of black holes to be much less than their locally measured value). It follows that ϕ too must be a locally measured invariant like c in spatially flat cosmology. Is spacetime spatially flat at cosmic scale? Observation answers this question in the affirmative.

ϕ/c^2 being a ratio of locally measured invariants in k = 0 cosmology does two things. First, it means that Eq. (4) can be interpreted as the assertion that inertial mass is induced by the action of gravity due to cosmic sources, as Einstein claimed to be the case – notwithstanding Brans' spectator matter argument. Indeed, since Brans' argument can be sidestepped by ϕ being a locally measured invariant (equal to c^2), his argument becomes a compelling argument **for** the gravitational induction of inertial mass. Second, $\phi/c^2 = 1$ makes inertial reaction forces an inductive gravitational effect, again, as Einstein claimed should be the case. In this case, though, the claim is complicated by the tensorial nature of gravity.

In the vector approximation based on the analogy with electrodynamics, one writes for the "gravelectric" field equation:

$$E_{grav} = -\nabla\varnothing - \frac{1}{c}\frac{\partial \boldsymbol{A}}{\partial t} \tag{6}$$

in Gaussian units. If one uses this equation for gravity, when one computes the equation of motion for a test particle, one gets ϕ/c^2 times $d\mathbf{v}/dt = \mathbf{a}$, the acceleration, from the term in the time derivative of the vector potential, and $\phi/c^2 = 1$ makes this term the inertial reaction force on the test particle. In tensor general relativity one must specify one's choice of coordinates.

6. Coordinate choice in general relativity

The most popular coordinate choice in general relativity is de Donder (harmonic) coordinates. When this choice is made, a factor of 4 appears in the term involving the analog, g_{oi}., of the vector potential, $\boldsymbol{A}$. This messes up the simple $\phi/c^2 = 1$ relationship between ϕ and c^2. But that relationship can be recovered by asserting that the gravelectric field equation be that just stated above [Eq. (6)] and only coordinates that return it are permissible. As did, for example, Braginski, Caves and Thorne [BCT] in their 1977 paper on "Laboratory experiments to test relativistic gravity" [10]. BCT, working with the coordinate choice of Misner, Thorne and Wheeler in chapter 39 of their massive *Gravitatjon* [11], worked through the details of the choice of Eq. (6) here for the potentials and metric. Their coordinate choice and gravelectric field equation determination leads to the correct Faradayan induction term in the equation of motion to account for inertial reaction forces. Edward Harris took particular note of BCT's treatment of inductive effects before adopting the gauge invariance rejection of Faradayan induction in the weak field, slow motion, but time-dependent Maxwellian analog interpretation of general relativity that led to the creation of the now fashionable sub-discipline of "gravitoelectromagnetism"; so-called GEM theory. That led in turn to the rejection of Faradayan induction effects in general relativity generally noted by Pfister and King.

The demand of explicit gravitational induction of inertial reaction forces does more than simply limit one's choice of coordinates. It can also be used in conjunction with Brans' spectator matter argument as a selection criterion for acceptable cosmologies. $k = \pm 1$ FLRW cosmologies, for example, do not conform to this criterion for in them, the gravitational and "kinetic" energies of test particles are not equal as $\phi \neq c^2$. If this is correct, then the remarkable *stability* of the $k = 0$ FLRW cosmology, remarked upon by Dicke and explained by Guth, is not a consequence of inflation. It is a consequence of the gravitational induction of inertial forces and Brans' spectator matter argument that makes ϕ a locally measured invariant equal to c^2.

What is important is that whatever one's choice in the matter is, that choice depends crucially on Carl Brans' spectator matter argument that, in turn, depends on the correctness of the Equivalence Principle – arguably the simplest expression of the *principle* of relativity for proper accelerations. If one chooses to go with Einstein regarding the role of inertia in general relativity, then Brans' argument dictates a coordinate choice like that of BCT. The question is: was Einstein right about the role of inertia in general relativity? That is a question that ultimately can only be answered by experiment.

7. Experiment and inertia

If one chooses to explain inertia as an inductive gravitational effect consistent with the EP, all one need do is impose a suitable coordinate condition. Then every real manifestation of inertia becomes, in a sense, an experimental demonstration of gravitational induction of inertia. But, in principle, all this has been known at least since BCT showed how to get the correct gravelectric field equation to account inductively for inertial forces. What we want is a novel experimental prediction that depends on the gravitational induction of inertia, a prediction that is not expected in the absence of inductive inertia. As it turns out, such a prediction exists, though it was not envisaged as a test of gravitational inertia induction when it was constructed. It is the prediction that if the local proper mass/energy density in a given region of spacetime is made to fluctuate, and that region is simultaneously subjected to a large proper acceleration to make manifest the inertial reaction gravitational field due to cosmic matter currents, that field vastly amplifies the magnitude of the masa/energy fluctuation.

This predicted rest mass fluctuation is a normally unobserved transient effect that is only obvious in special circumstances when it is sought. Those circumstances are the production of thrust in small systems, seemingly without the use of propellant, by coupling to the gravitational field of the universe through the mass fluctuations driven a stack of piezoelectric disks clamped to a brass reaction mass with an aluminum cap and screws. This is described in the precursor to this paper [12]. A device of this sort is show here in **Figure 1**. In order to maximize the oscillations of this device it is mounted using small linear ball bushings in ears on a flange on the reaction mass. The device is supported by rods in an aluminum frame as shown in **Figure 2**. A device like that in **Figure 1** has a mass of about 140 gms, whereas the support frame has a mass of about 60 gms. Work with devices like those shown in **Figures 1** and **2** commenced in the summer of 2020 using a high sensitivity torsion balance used in previous work. Large effects were produced. So large that the torsion balance was abandoned. But not until all of the various test for a genuine effect were completed.

The experiment was moved out of the balance vacuum chamber and onto a cantilever. Force generated in the device was measured by recording changes in the

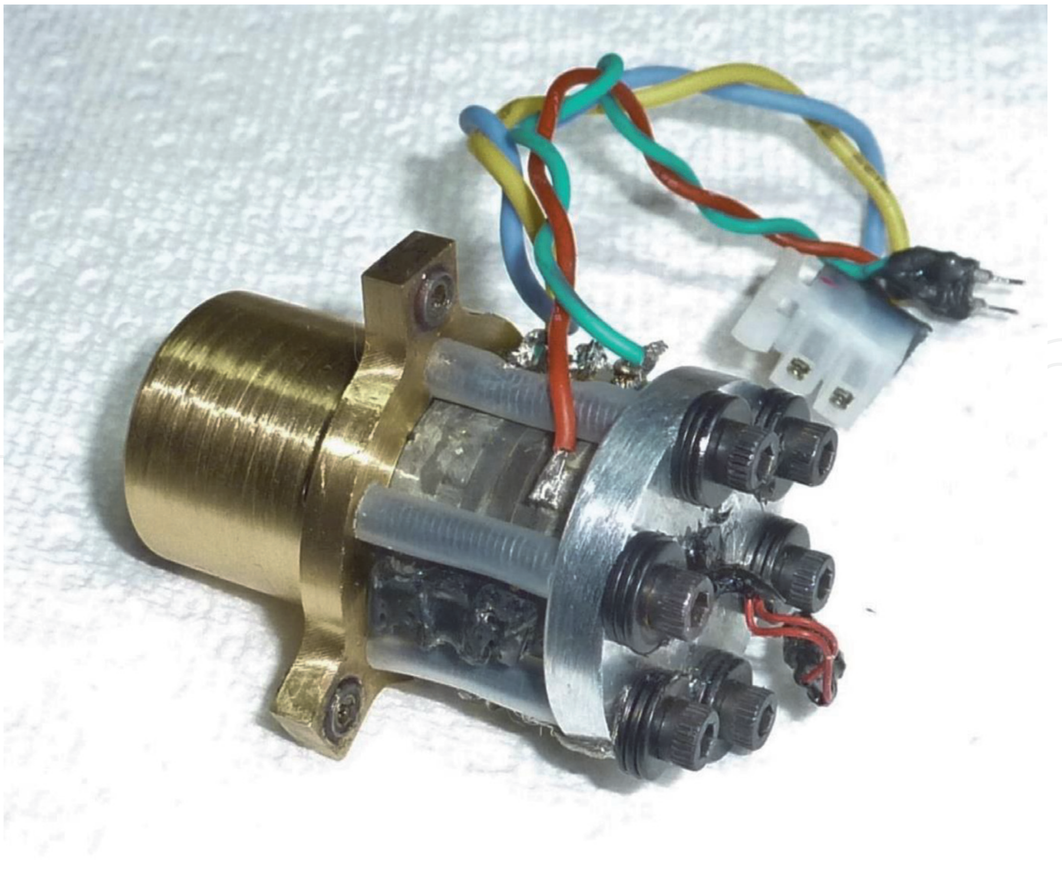

Figure 1.
A Mach effect gravity assist (MEGA) impulse engine element. Eight 19 mm diameter by 2 mm thick led zirconium titanate disks are clamped between the aluminum cap and brass reaction mass. Linear ball bushings are fitted in the "ears" on the reaction mass.

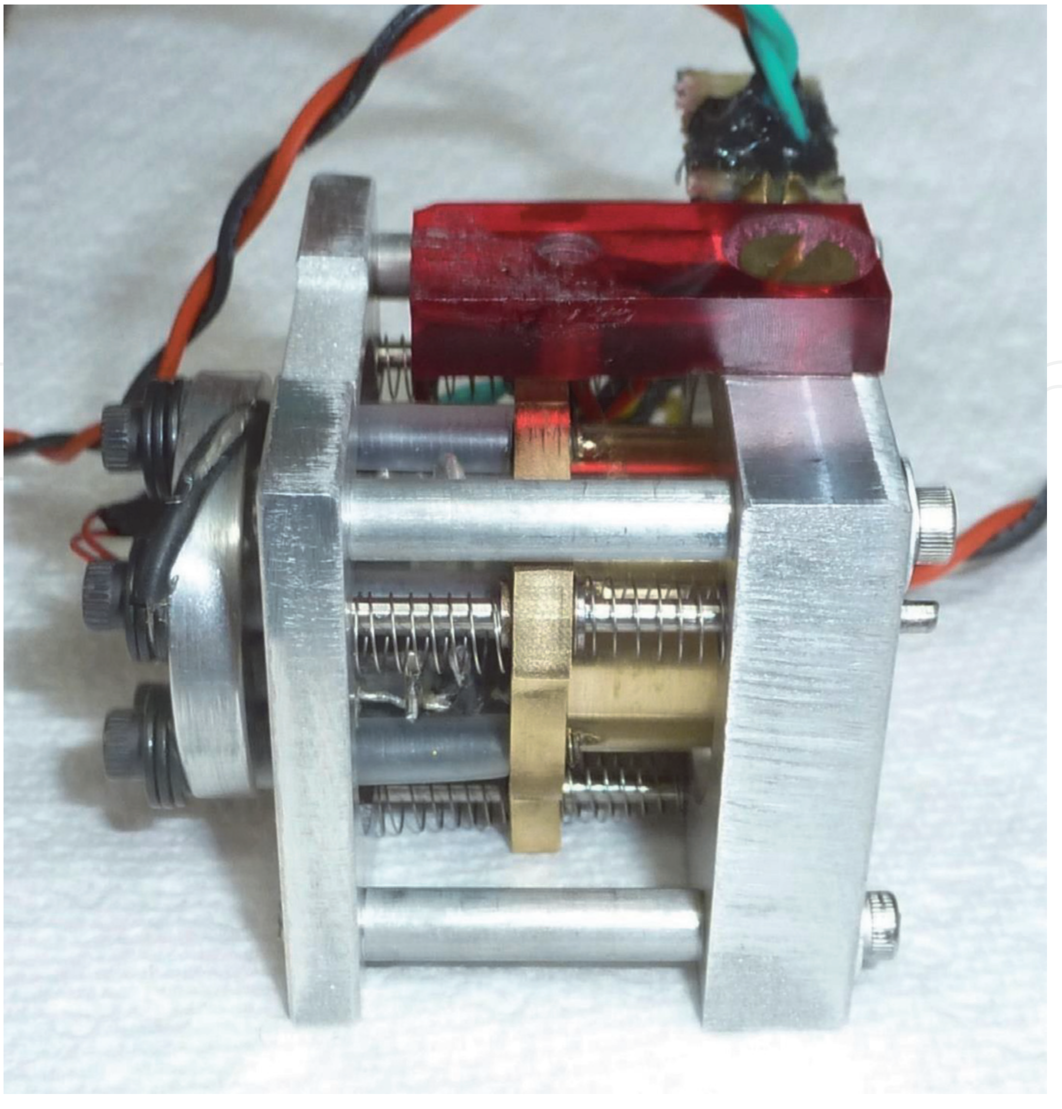

Figure 2.
*A device like that in **Figure 1** mounted on steel rods in an aluminum frame. The device is centered in the frame on the rods by very soft springs that convey very low frequency and stationary forces to the frame without communicating any high frequency vibration.*

position of the device on the rods with a Philtec position sensor. Typical results with this arrangement are reported in [12].

The chief criticism that has been advanced of the Mach effects project is that the measured thrusts were not due to any real effect. Rather, they allegedly arose from simple vibration in the systems – so-called "Newtonian vibrational artifacts". Those of us working on the project, of course, had been careful to exclude such false positives. But those determined to believe that Mach effects do not exist persisted. Indeed, they still persist, notwithstanding that we have increased the forces generated by these devices by two to three orders of magnitude [13]. And this performance increase was achieved by isolating the strong vibrations in the device from the support structure using linear ball bushings in place of a simple rubber pad in earlier design devices. To quell lingering doubts about false positives arising from vibration, my partner in this work, Hal Fearn, resuscitated an antique air track to see if an air supported "glider" could be made to move thereupon. The results, for technical reasons, were equivocal. Hal then turned to a pendulum, made with a small plastic platform suspended by three fine nylon monofilament cords with length 1.9 m from the ceiling of our lab, This has been the force detection system in use for the past several months.

It quickly became apparent that the pendulum force detection system has one great advantage over all other force detection systems. It eliminates the significant inertia of the parts of other systems, For example, the significant mass of the beam and counter masses of a torsion balance disappear. All that remains is a few tens of grams for the plastic platform and adjustment screws added to the mass of the support structure. Why is this important? Well, the answer to the Newtonian vibrational artifact hypothesis, from the outset, has been that simple vibration induced in a system by the addition of energy, but no momentum, cannot produce a steady deflection of a force detection method by the conservation of momentum. Only the generation of a real force in the system can produce a steady deflection of force detection apparatus. The counter argument to this obviously correct momentum conservation argument is that induced vibration may produce a stick–slip mechanism in the parts of the system that result in the relative motion of parts of the system, and the motion of the part of the system attached to the force detection apparatus may displace the force sensor. While a transient displacement of a force sensor may result from such action, a steady displacement cannot occur for no steady, real force is generated by this process, and the restoring force of the force detection sensor will quickly re-zero the force sensor. Surprisingly, this obviously correct counter, counter argument has fallen on at least some deaf ears.

The beauty of the pendulum force detection scheme is that with large enough forces, the stick–slip scheme of the Newtonian hypothesis can easily be discriminated from the production of a real force. This is possible because the Mach effect forces in the new devices produce forces large enough (hundreds of micronewtons) to cause displacements of the pendulum on the order of hundreds of microns. The vibrations allegedly responsible for these displacements, however, have amplitudes less than a few hundred nanometers, known by direct observation with a Polytech laser vibrometer. The only way small amplitude vibrations can produce large amplitude displacements is by a stick–slip mechanism. And momentum conservation applied to this mechanism demands that the vibrating device and the support structure move in opposite directions with equal opposite momenta to preserve the location of the center of mass of the system as no net real force is generated. Since the mass of the support structure is roughly half the mass of the device, detection of these motions is a simple matter of simultaneous measurement of the positions of the device and support structure. If especially initially they move in opposite directions, you are looking at a Newtonian artifact. If they move together in the same direction, a real force is being generated in the device.

Figure 3 shows a device mounted on our pendulum platform with the vacuum chamber and torsion balance in the background in our lab. The positions of the device and support structure are measured with two Philtech position sensors as shown in **Figure 4**. The motion of the pendulum is only very lightly damped by the leads to the thermistor in the cap of the device that records the temperature of the device. In addition to the two position and temperature measurements, the voltage and current in the power circuit were monitored. Data were acquired and displayed with three Picoscopes and a Logitec BRIO webcam that captured the motion of the device in a movie displayed along with the Picoscope outputs, the entire screen being captured with software. A typical composite display screen is shown in **Figure 5**. The record of each run consists of the display screen capture movie and the strip-chart recording of the positions, voltage and temperature of one of the Picoscopes. The display screen movie is used to calibrate the position measurements in the strip-chart for conversion to force measurements.

The strip-chart recording for this run is shown in **Figure 6**. The fuzz on the gray trace of the support structure position has been post-acquisition cleaned up by a 10 Hz low pass digital filter and the temperature trace (green) has been added. Two

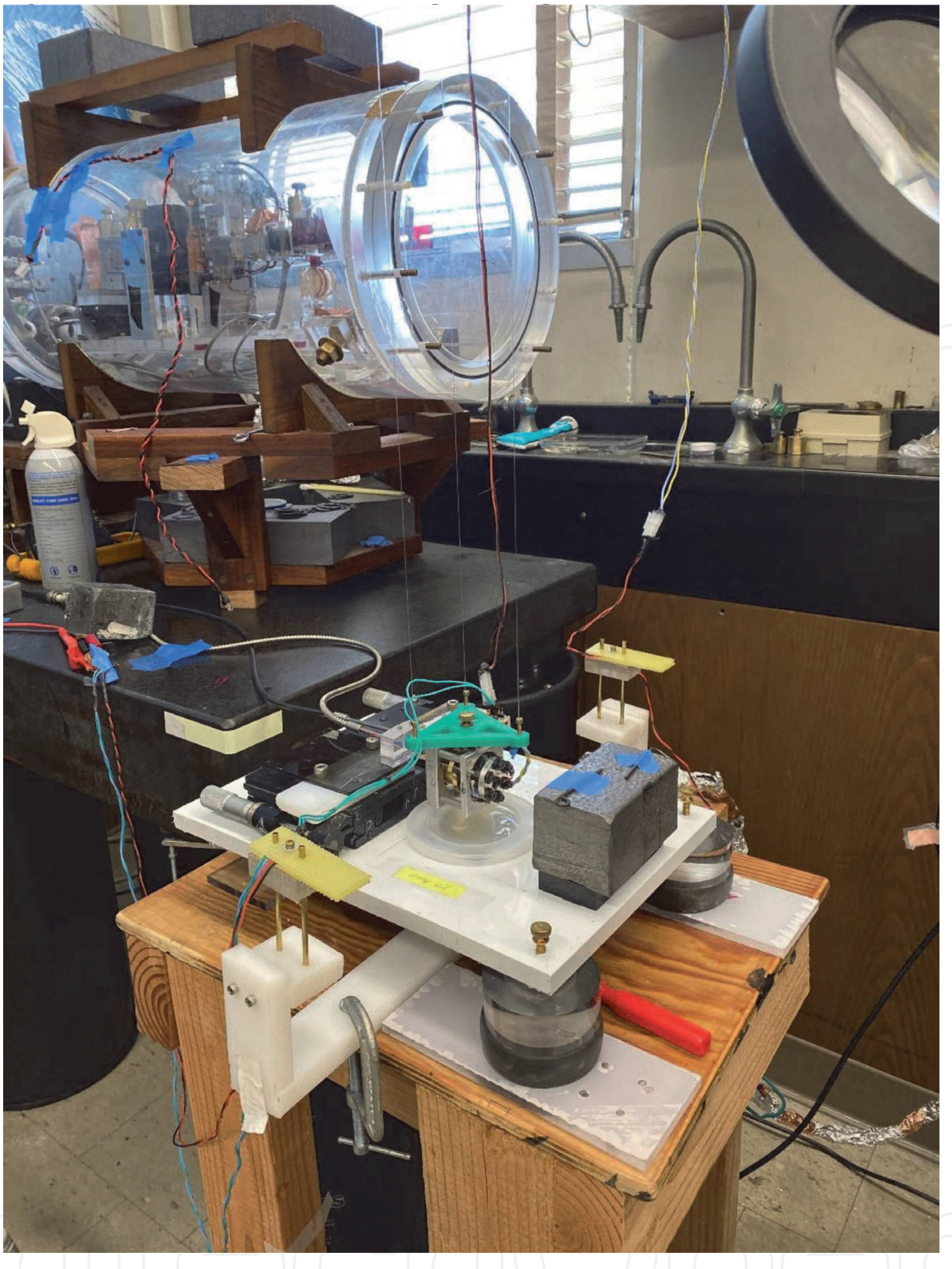

Figure 3.
The pendulum force sensing aparatus. The platform, the triangular green piece of plastic in the center of the picture, is suspended on three monofilament fibers attached at the ceiling of the room.

important inferences follow immediately from the data in **Figure 6**. First, since the two position sensors track together, it follows that a real force is generated in the MEGA impulse engine. The "level shifts" of the position traces are not consistent with "Newtonian vibrational ¯artifacts". Second, the prompt changes in the position traces at power on and off, allowing for some power on switching transient overshoot, indicate the presence of a steady force during the powered interval – as expected.

Resonances where Mach effect thrust is found, to date, have been located using the frequency sweep function of our Rigol signal generator, the present source of the single frequency sine function that drives a Carvin DCM-2000 power amplifier and 4 to 1 matching transformer. Once a suitable resonance is located, short 5 second constant frequency pulses are used to fine tune the optimal driving frequency.

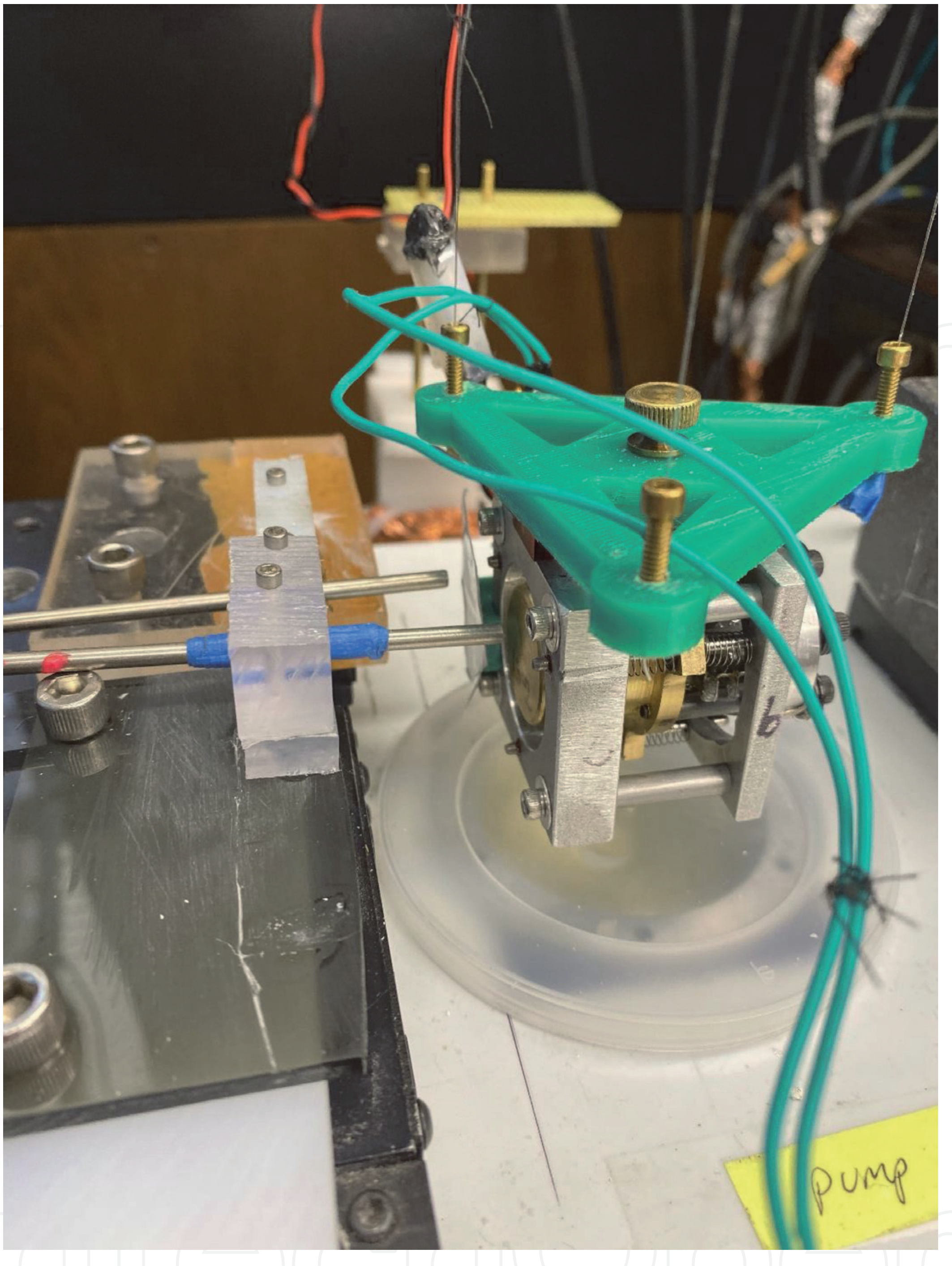

Figure 4.
The position sensors are the two steel tubes on the left attached to micrometer stages.

This procedure will soon be supplanted by the automated routine on a Piezo Drives ultrasonic drivee that also supports resonance tracking.

The force that corresponds to the power-on displacement in **Figure 6** can be computed from the length of the pendulum, 1.85 m, the mass of the "bob", 0.23 kg, and the voltage to distance scale factor determined from the run movie, where for the device trace (red) a displacement of 0.5 mm correspond to a voltage change of 4.5 volt giving 0.11 mm per volt as the scale factor. The "level shift" from the incoming trace to the switching transient at power-on is about 2 volts, so the displacement produced by turning on the force is about 0.22 mm.. The vertical force on the pendulum is mg, or 2.3 newtons. The force that produces the 0.22 mm deflection is just the sine of the deflection angle, 2.2 X 10^{-4} m divided by 1.85 m, or 1.2 X $10^{-4.}$ This multiplied times the 2.3 newton force gives 250 micronewtons, a force 250 times larger than the largest forces produced with old style Mach effect thrusters.

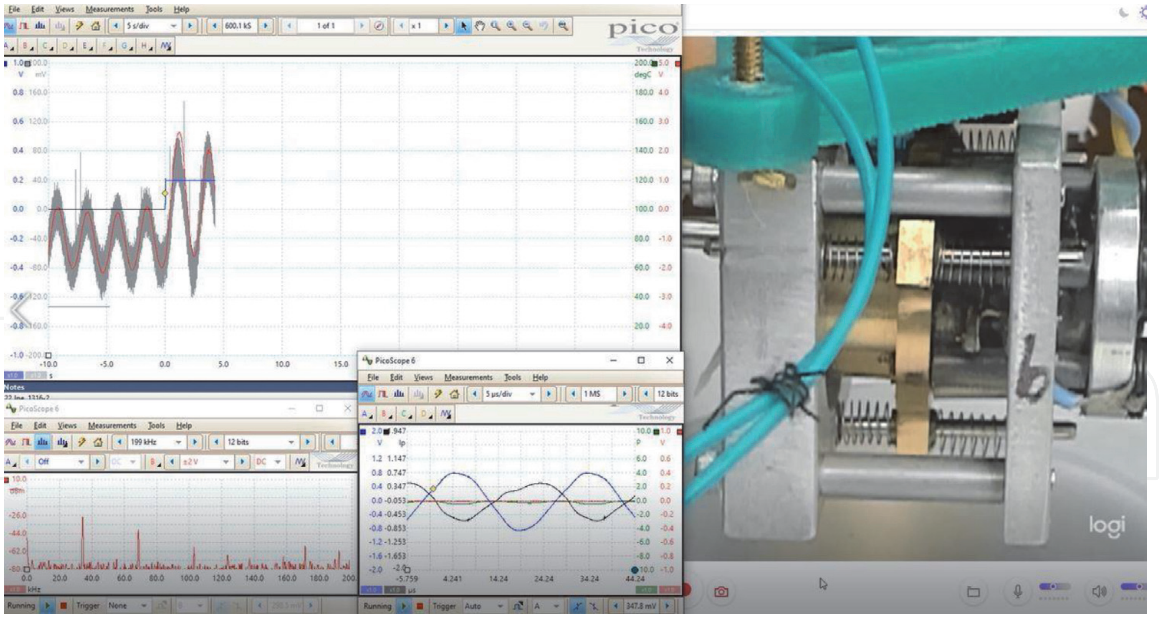

Figure 5.
A screen capture of the display screen for a run in progress. Three Picoscope displays are on the left and a movie of the device is on the right. In the upper left strip-chart recording the red and gray traces are the device and support structure positions, and the blue trace is the rectified voltage across the device. Below the strip-chart are the FFT power spectrum of the current (left, note the prominent first and second harmonics) and the waveform (right, voltage blue and current black traces) displays. The movie on the right is used to calibrate the position traces in the strip-chart so that the positions can be converted to force measurements.

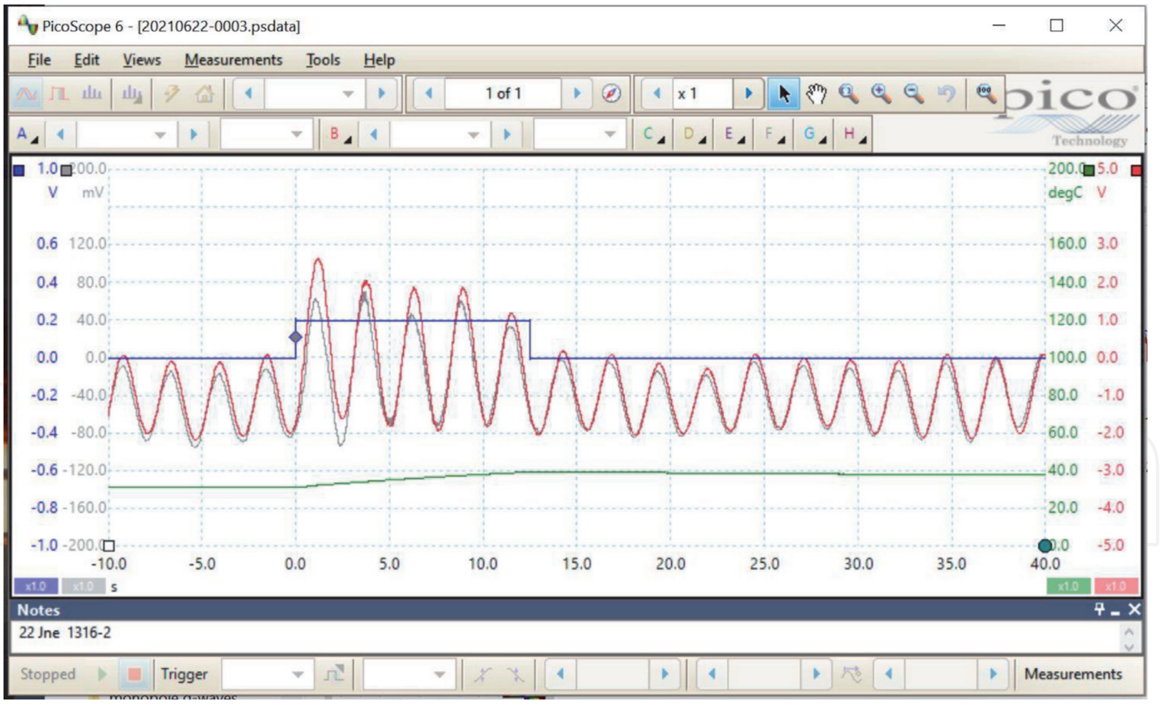

Figure 6.
The completed run. The gray trace of the support structure position has been filtered and the green temperature trace has been added.

Figure 6 (and many others like it) shows that a real force acts on the device and support structure when the device is excited. And the force continues to act as long as the device is excited. The obvious question is: can the real force responsible for the pendulum deflection in **Figure 6** be attributed to some mundane cause? The leading candidate, Newtonian vibrational artifacts [13], has already been excluded as it cannot produce a steady force and the device and support structure do not move as required by this hypothesis. The only other possibilities are coupling to the ambient air in proximity to the device and electromagnetic interactions arising from the currents and voltages present. Ambient air was excluded while working on the torsion balance before transitioning to the cantilever and then the pendulum. The operation of these devices is unaffected by operation in air at atmospheric pressure or in soft vacua of 10 milliTorr or so. Electromagnetic effects are excluded by replacing the device with a "dummy" capacitor with capacitance roughly equal to that of the devices, but without the electromechanical properties of the PZT stacks. Runs with the dummy capacitor show no signs whatsoever of any pendulum activity like that in **Figure 6** (and many others).

From the practical perspective it seems reasonable to suggest that the obvious advantages of real MEGA impulse engines will make them likely features of our future. From the physics perspective, the fact that MEGA impulse engines work constitutes experimental confirmation of Einstein's insistence on "the relativity of inertia" and the gravitational induction of inertial effects.

8. Conclusions

In the matter of the role of inertia in general relativity, we find that:

- Einstein regarded spacetime as the gravitational field. In the absence of gravity, there is no spacetime, from which it follows that

- Minkowski spacetime, used in the construction of general relativity, as it assumes the absence of gravity, is not a valid general relativistic spacetime.

- Since spacetime – the gravitational field that is – is observed to be spatially flat at cosmic scale and in sufficiently small regions, the metric that obtains where spatial flatness is the fact is that of the spatially flat FLRW cosmology.

- Carl Brans' spectator matter argument led to the rejection of Einstein's argument that inertia is gravitationally induced in general relativity.

- However, while Brans' argument is correct, the inference that it excludes gravitationally induced inertia is not correct. What Brans' argument does do is require that the total, locally measured Newtonian gravitational potential be a scalar invariant, like the vacuum speed of light, to which it is related, being the square thereof.

- This relationship between the locally measured values of the vacuum speed of light and Newtonian gravitational potential are an automatic consequence of spatially flat FLRW cosmology.

- Stipulation that inertia is to be understood as gravitationally induced in general relativity can be implemented by asserting a condition on acceptable coordinates, constraining the range of acceptable solutions of Einstein's field equations, to those that return the requisite relationship between the locally measured invariant values of the vacuum speed of light and the Newtonian gravitational potential.

In the matter of experimental confirmation of the correctness of Einstein's contention that inertia be an inductive gravitational effect, we find that:

- Extended objects capable of changing their internal energies simultaneously experience changing internal energy and proper acceleration, the action of the grav/inertial field excited by the proper acceleration amplifies the rest mass fluctuation corresponding to the changing internal energy amplifies that rest mass fluctuation. Such effects are called “Mach effects” given their dependence on inertial forces of cosmic gravitational origin first adumbrated by Ernst Mach.

- Mach effects of sufficient magnitude can be utilized for propulsion by adding a synchronous mechanical oscillation at the frequency of the Mach effect fluctuation, making a Mach effect gravity assist (MEGA) impulse engine.

- Prototype MEGA impulse engines can produce thrusts of hundreds of micronewtons and more. Straight-forward tests can eliminate mundane effects that might produce false positive results that might account for observed thrusts.

- The leading candidate for a false positive explanation of observed forces is so-called “Newtonian vibrational artifacts” induced in the device by the vibration of the lead-zirconium-titanate crystal stack in the engine.

- Using a pendulum for force detection with the current realization of the MEGA impulse engine where the engine is mounted on its support structure with rods and linear ball bushings enables a simple test of the vibrational artifact hypothesis. Conservation of momentum dictates that vibrational artifacts cannot produce a steady deflection of the pendulum. A real Mach effect force will produce a steady deflection of the pendulum.

- Vibrational artifacts, by the conservation of momentum, cause the device and its support structure to move initially in opposite directions with the system subsequently moving about zero deflection. Real force causes the device and support structure to move together with subsequent motion about a time-averaged net deflection while power is applied.

- **Figure 6** shows beyond reasonable doubt that since the device and support structure move together, a real force of about 200 micronewtons was generated by the MEGA impulse engine being tested on a pendulum that, by exclusion of mundane effects, was generated by the Mach effect.

- Further work to implement this technology is warranted.

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