Chapter

Non-Keplerian Orbits in Dark Matter

Peter D. Morley

Abstract

This paper is concerned with the mathematical description of orbits that do not have a constant central gravitating mass. Instead, the attracting mass is a diffuse condensate, a situation which classical orbital dynamics has never encountered before. The famous Coma Cluster of Galaxies is embedded in Dark Matter. Condensed Neutrino Objects (CNO), which are stable assemblages of neutrinos and anti-neutrinos, are candidates for the Dark Matter. A CNO solution has been attained previously for the Coma Cluster, which allows mathematical modeling of galaxy orbital mechanics within Dark Matter, first reported here. For non-zero eccentricity galaxy orbits, each point along the trajectory sees a different gravitating central mass, akin to satellite orbits inside Earth. Mathematically, the galaxy orbits are non-Keplerian, spirographs.

Keywords: dark matter, coma cluster, condensed neutrino object, orbital dynamics, galaxy cluster

1. Introduction

There are two seminal observations that bracket the existence of Dark Matter, giving essential physics clues. The first is Zwicky [1] who noticed that the luminous matter in the Coma Galaxy Cluster is too small in mass to gravitationally bind the cluster. Quanitatively, the fastest bound galaxy has speed relative to the Coma center-of-mass of about 3000 km/s [2]. From the Coma Galaxy Cluster we learn:

1. The amount of (unseen) Dark Matter in the Coma Cluster vastly 'outweighs' the luminous matter.

2. We can see right through the Coma Cluster to image galaxies on the other side of the Universe, which means light is not scattered by Dark Matter: astonishingly, Dark Matter is transparent to light.

3. The gravitational potential of Dark Matter is of the same size as the Coma Galaxy Cluster dimensions.

The second seminal observation is Rubin [3] who showed that the rotational speeds of stars in spiral galaxies are too high for gravitational binding with the amount of luminous spiral galaxy mass observed. Unfortunately, here the story takes a tragic diversion, because Rubin assumed that the missing spiral Dark Matter must be in the halo of the measured galaxy itself. This mistake could be attributable to the lack of mathematical sophistication, but it has misled researchers for years. Let us discuss the situation of a spiral galaxy embedded in a Coma-like Galaxy Cluster Dark Matter potential and see the complexity of the resulting gravitational potential. The Dark Matter gravitational potential at position r inside the Dark Object is (G is the gravitational constant)

$$\Phi(\mathbf{r}) = -G\int \frac{\rho_B(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|}d^3r' \tag{1}$$

where ρ_B is the Dark Matter mass density. For Coma-like Dark Matter, this density is spherically symmetric: $\rho_B(\mathbf{r}') = \rho_B(r')$ so Eq. (1) becomes

$$\Phi(\mathbf{r}_c + \mathbf{a}) = -G\int_0^{|\mathbf{r}_c+\mathbf{a}|} \frac{dM_B(r')}{r'} \tag{2}$$

where $dM_B(r') = \rho_B(r')4\pi r'^2 dr'$, $\mathbf{r}_c$ is the radius from the origin of the Dark Matter Object to the spiral galaxy center of mass and $\mathbf{a}$ is the spiral arm vector. The embedded spiral galaxy is shown in **Figure 1**. We're interested in the difference of gravitational potential between a spiral arm and its galaxy's center of mass, $\Phi(\mathbf{a}) = \Phi(\mathbf{r}_c + \mathbf{a}) - \Phi(\mathbf{r}_c)$.

$$\Phi(\mathbf{a}) = -G\int_{|\mathbf{r}_c|}^{|\mathbf{r}_c+\mathbf{a}|} \frac{dM_B(r')}{r'} \tag{3}$$

Consider now letting a move along a spiral arm, going around 360 degrees, where this angle becomes the spiral galaxy's azimuthal angle ϕ. If the spin axis is tilted with respect to a radial, then Eq. (3) has both positive and negative values: for some ϕ: $|\mathbf{r}_c + \mathbf{a}| > |\mathbf{r}_c|$ and for 180 degrees further in ϕ: $|\mathbf{r}_c + \mathbf{a}| < |\mathbf{r}_c|$. Circumlocution means a rotating star with a fixed distance from the spiral galaxy's center will go up

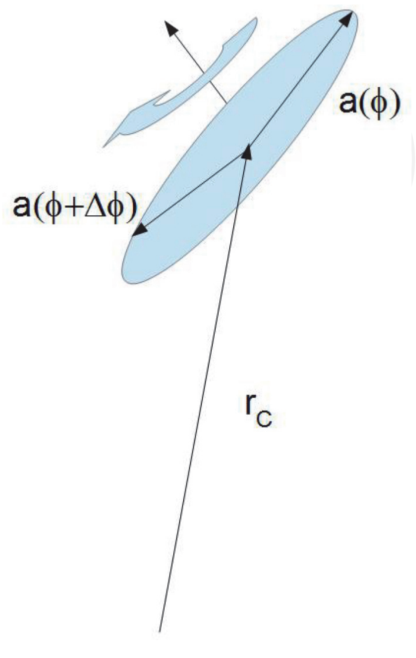

Figure 1.
Embedded spiral galaxy in a coma-like dark matter object with r_c the vector from object center to spiral center and a the spiral arm's position vector. The tilt of the spin vector s with respect to the dark matter radius vector $\mathbf{r}_c$ is the origin of the complex spiral arm star rotation speeds.

a potential hill for half its revolution and lose rotational speed. The other half of the rotation it will go down a potential hill and gain speed; the spiral arm rotational speeds become azimuthal angle dependent, Eq. (3). If however, the spin axis is parallel or antiparallel to a Dark Matter Object radial, then $|\mathbf{r}_c + \mathbf{a}|$ = constant $> |\mathbf{r}_c|$, and rotation speeds (for constant distance from center) are no longer azimuthally dependent from the Dark Matter gravitational potential. In the interesting other case that $\mathbf{a} \to 0$, very short distant scales (~ 1 Kpc), then there is no change in the Dark Matter potential ($\Phi(\mathbf{a}) \to 0$) and the star speeds are the *same near each other*. This geometric dependence of the large Coma-like Dark Matter is a significant source of confusion when the Dark Matter is attributable to a galactic halo and has led to bogus science claims [4] of satellite galaxies having enormous amounts of Dark Matter.

2. Dark matter particles as fermions

Large astronomical assemblages of matter have gravitational self-energies that will cause them to collapse. They are only stable if there is an internal pressure source. Observations of Dark Matter embedding galactic clusters reveal no Dark Matter energy generation. The Lamda-Cold-Dark-Matter cosmological model postulates that Dark Matter is cold. Therefore Maxwellian statistics are not present. Dark Matter particles are described by quantum statistics, which are either the Fermi-Dirac or Bose-Einstein distributions. However, boson stars do not exist in Nature because bosons will occupy the lowest energy state in cold matter. We come to the conclusion that Dark Matter is made up of Fermions that are in equilibrium due to degeneracy pressure. Recognizing that White Dwarfs are stable due to electron degeneracy, Neutron Stars are stable due to neutron degeneracy, we see that the size of the stable assemblage is inversely proportional to the Fermion mass. Zwicky's discovery that the Coma Cluster of galaxies is embedded in Dark Matter means that the Dark Matter Fermion particle must be incredibly small in mass, even compared to the small electron's mass.

3. Condensation of cosmological neutrinos

The additional requirement that the Dark Matter particles be the most abundent particles in the Universe identifies the condensation of cosmological neutrinos from the Big Bang as a very attractive candidate for Dark Matter. Reference [5] is the first publication that correctly evaluated the equation of state for degenerate neutrino matter, where the neutrinos and anti-neutrinos condense into stable assemblages called 'condensed neutrino objects' (CNO). Reference [6] derived the CNO mass-radius relationship

$$M(R) \simeq \frac{1.97462 \times 10^{15} M_\odot}{R^3 m_\nu^8} \tag{4}$$

where $M(R)$ is the mass of the stable CNO having radius R in units of Mpc and m_ν, neutrino mass scale (neutrinos are almost a perfect mass symmetry from neutrino mixing), is in units eV/c^2. Once m_ν is determined by the KATRIN terrestial experiment [7], Eq. (4) completely describes Dark Matter. Here, we take the opportunity to model galaxy orbital dynamics in the Coma Galaxy Cluster, using an estimated value for m_ν from reference [8].

The interesting physics of CNO is that there is no central mass. Instead, galaxies having non-zero eccentricities see a different gravitating mass at each point along their orbit. Human beings have never seen this astronomical phenomena before. It would be akin to having satellite orbits inside the Earth and it leads to non-Keplerian orbits.

4. Coma galaxy cluster CNO

When galaxies self-assemble inside CNO, they may have negligible velocities with respect to the Dark Matter or non-negligible velocities. Those galaxies having negligible velocities fall to the center of the CNO and execute simple harmonic motion (SHM), reference [5]. These galaxies then obtain their fastest speed at the center of the CNO, and when there, are the fastest galaxies embedded in the CNO that are gravitationally bound. On the other hand, galaxies which self-assemble with non-negligible velocities with respect to the CNO center of mass, execute orbital dynamics. Conservation of angular momentum prevents their appearance in the CNO center, and they never appear in the fast velocity histograms. If astronomical data is available for individual galaxies of a galaxy cluster, then picking out the fastest bound galaxy will place its location at or near the CNO center. In Ref. [8] this analysis was done to identify the CNO parameter, the neutrino Fermi Momentum (p_F) at the CNO center, associated with the Coma Cluster. The Fermi Momentum enters in the equation of hydrostatic equilibrium as $x = p_F/m_\nu c$ (called the reduced Fermi Momentum), where c is the speed of light. The different CNO in Eq. (4) have different boundary condition $x|_{R=0}$, which we denote by $x(0)$, after the neutrino mass scale is determined.

The Coma Galaxy Cluster CNO has solution [8] $x(0) = 0.010$ [1]. This Coma Cluster solution has mass $\mathcal{M}_{010}$ and radius $\mathcal{R}_{010}$

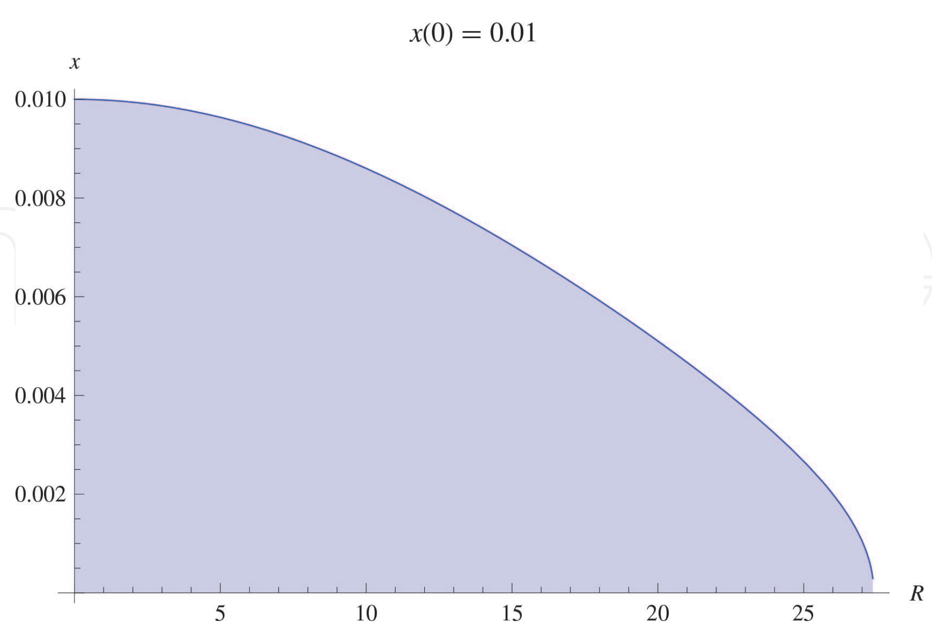

Figure 2.
Spatial variation of the reduced Fermi momentum x for the CNO having boundary value $x(0) = 0.010$. The units of length are 46128.98 pc/ m_ν^2 with m_ν in units of eV/c^2. The figure is from reference [8] and is the result of solving the equation of static equilibrium for degenerate matter.

[1] This shows just how non-relativistic these stable CNO are.

$$\mathcal{M}_{010} = \frac{9.809 \times 10^{14}}{m_\nu^2} M_\odot \tag{5}$$

$$\mathcal{R}_{010} = \frac{1.2625}{m_\nu^2} Mpc \tag{6}$$

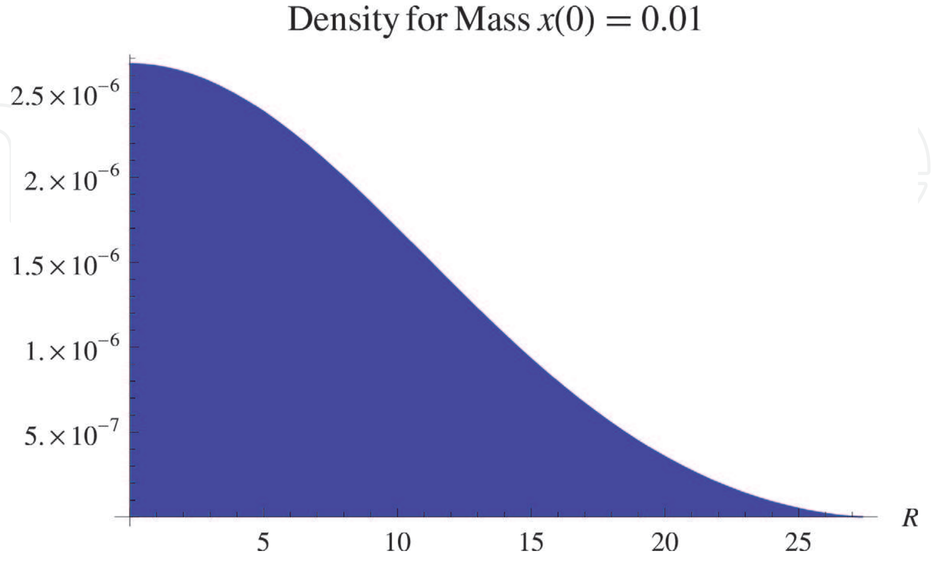

Figure 3.
CNO mass density for $x(0) = 0.010$ CNO. The units of mass density are $1.76307 \times 10^{-20} m_\nu^4$ gm/cm^3 with m_ν in units of eV/c^2 the figure is from reference [8]. Notice that the density is finite at the origin, which means there is no singularity present.

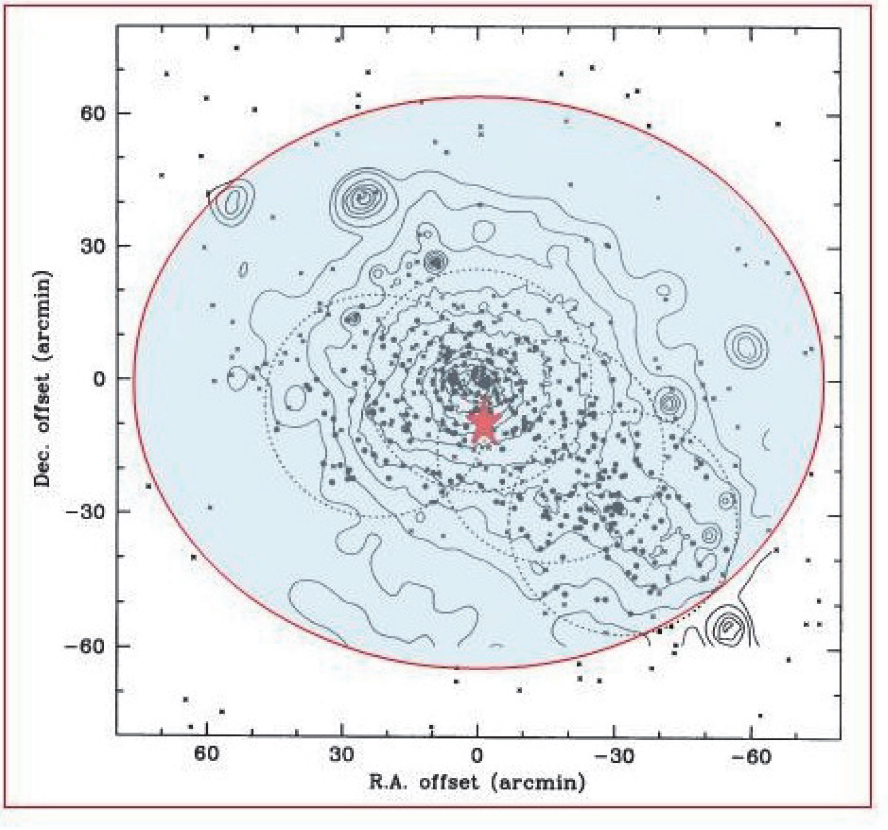

Figure 4.
Approximate location of the CNO center using fastest galaxy GMP = 3176, which has offsets R.a. +0.0579 arc-minutes and Dec −13.465 arc-minutes. The figure background is taken from reference [2], which shows the actual coma galaxy cluster and X-ray contour lines. At a distance of 101.3 Mpc, the 2.191 Mpc CNO radius translates to 74.35 arc-minutes. The composite figure is from reference [8].

where m_ν is in units of eV/c^2. In **Figure 2**, The Coma Galaxy Cluster CNO reduced Fermi momentum is plotted as a function of radial coordinate, while in **Figure 3**, the mass density is plotted.

In Ref. [8], $m_\nu = 0.759\ \ \mathrm{eV/c^2}$ was used. The center of the CNO embedding the Coma Galaxy Cluster was determined to be close to galaxy GMP [2] = 3176. The CNO Coma Galaxy Cluster solution is shown in **Figure 4**.

5. Galaxy cluster embedded in a CNO

Observationally, most or all galaxy clusters are embedded in Dark Matter, because the observable luminous matter does not produce a strong enough gravitational well to confine the experimentally observed galaxy velocities. This was the original Zwicky observation. Theoretical studies of galaxy cluster dynamical evolutions that did not understand the physics of Dark Matter reached the following conclusion (exemplified by reference [9]): galaxies collapse toward the center and virialize with Dark Matter to attain a steady-state distribution. For CNO Dark Matter, the galaxies revolve in a frictionless condensate and do not viralize with the Dark Matter at all.

5.1 Coma galaxy cluster dynamics

For the relevant neutrino mass scale of $m_\nu = 0.759\ \ \mathrm{eV/c^2}$, the Coma Dark Matter has mass 1.7 $\times 10^{15}$ M $_\odot$ with radius 2.191 Mpc. We will use this CNO to demonstrate how to mathematically compute orbits in CNO, using examples. These will be galaxies that have self-assembled with non-negligible velocities with respect to the CNO center-of-mass. The computation of the Dark Matter mass profile will allow for detailed simulations of galaxy cluster dynamics: formation of galaxies and their subsequent orbital evolution. In the section Future Work, it will be described how a simulation should be able to reproduce present measured kinetic velocity distributions.

5.2 Coma galaxy cluster orbits

The galaxies will execute orbits on a plane defined by their initial (birth) velocity components. On this 2-Dimensional plane are the r and θ polar coordinates. The radial and tangential force equations for a galaxy embedded in a CNO are (G is Newton's gravitational constant)

$$\ddot{r} - r\theta^2 = \frac{-GM_{CNO}(r)}{r^2}$$
$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right) = 0 \tag{7}$$

where $M_{CNO}(r)$ is the Dark Matter mass enclosed within radial cordinate r. For the Coma Galaxy Cluster solution of $x(0) = 0.010$ with $m_\nu = 0.759\ \ \mathrm{eV/c^2}$, this has the expression

$$M_{CNO}(r) = 5.5773407 \times 10^{17} M_\odot I(s) \tag{8}$$

where $s = r/r_{CNO},\ \ s \le 1$, with r_{CNO} = 2.1915 Mpc and $I(s)$ has a polynomial expansion

$$\begin{aligned} I(s) = &-0.00001983278760836819 \cdot s + 0.00057121308662060 88 \cdot s^2 \\ &+0.012326181706900396 \cdot s^3 + 0.030069987017380402 \cdot s^4 \\ &-0.12115460629704124 \cdot s^5 + 0.13430437610312457 \cdot s^6 \\ &-0.06587537476021942 \cdot s^7 + 0.01283085557464332 \cdot s^8 \end{aligned} \tag{9}$$

The numerical function $I(s)$ is displayed in **Figure 5**. In **Figure 6**, we show the orbital velocity components of a galaxy embedded in a CNO.

As already discussed, the reason why orbital dynamics of galaxies embedded in a CNO are different and unusual is because there is no central mass situated at the center of a CNO (see **Figure 5**). This is the first time we see non-Keplerian orbits from classical gravity. We now solve the system of Eq. (7) for interesting initial

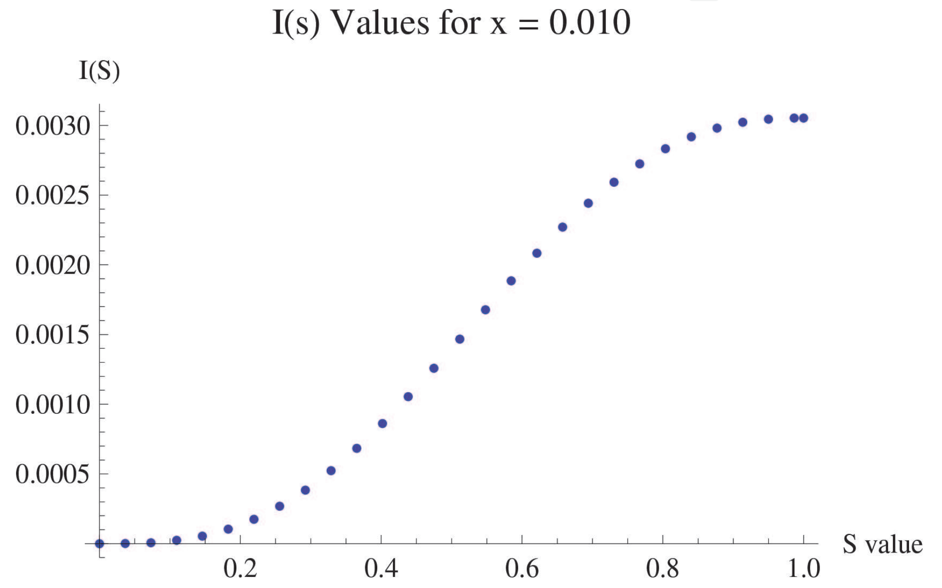

Figure 5.
Numerical evaluation of the function $I(s)$.

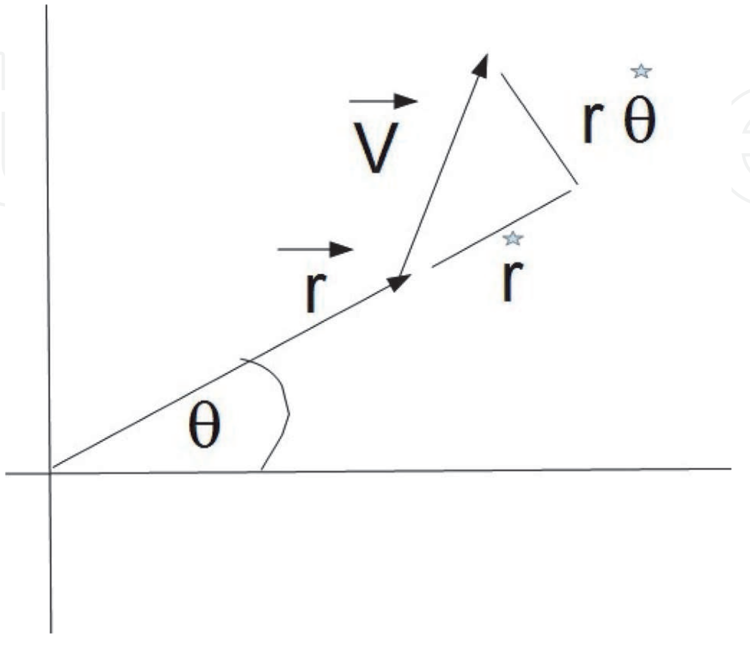

Figure 6.
2-dimensional (planar) velocity components for orbit dynamics of a galaxy embedded in dark matter CNO. The reference system is the center-of-mass of the CNO. The axis can be any perpendicular coordinates.

conditions. We will do two examples that illustrate the orbital dynamics of galaxies embedded in Dark Matter CNO.

The last equation of array Eq. (7) shows that the angular momentum per unit mass $h = r^2\dot{\theta}$ is a conserved quantity (a constant). Changing variables $r \to \tilde{s} = \frac{r_M}{r}$ where, for simplicity of notation, we put $r_{CNO} \equiv r_M$, one can show that the system Eq. (7) becomes

$$\frac{d^2\tilde{s}}{d\theta^2} + \tilde{s} = \frac{r_M G M_{CNO}(1/\tilde{s})}{h^2} \tag{10}$$

5.3 Case 1 example

The Coma Galaxy Cluster has a measured (in Coma center-of-mass frame) Gaussian distribution of speeds [2] that extends up to 3000 km/s. For our first example, we will use a small value for the initial speed of a galaxy: V_0 = 100 km/s with a velocity angle 75 degrees, starting at $\theta = 0$ degrees and located at $r_{initial} = .2r_M$. This gives the initial velocity condition (see **Figure 6**)

$$\dot{r}|_{initial} = V_0 \cos 75° \quad = \quad 25.8819\ldots \text{km/s} \tag{11}$$

$$\left(r\dot{\theta}\right)\big|_{initial} = V_0 \sin 75° \quad = \quad 96.5925\ldots \text{km/s} \tag{12}$$

For $r_{initial} = .2r_M \to \tilde{s}|_{initial} = 5$, so we have the first initial condition. We next use the chain rule of calculus

$$\dot{r} = \frac{dr}{d\theta}\dot{\theta} = \frac{h}{r^2}\frac{dr}{d\theta} \tag{13}$$

Since $\tilde{s} = r_M/r$, we have using Eq. (13)

$$\frac{d\tilde{s}}{d\theta} = -\frac{r_M}{r^2}\frac{dr}{d\theta} = -\frac{r_M}{h}\dot{r} \tag{14}$$

This gives the last initial condition

$$\left.\frac{d\tilde{s}}{d\theta}\right|_{initial} = -\frac{r_M}{h}\dot{r}_{initial} \tag{15}$$

Evaluating Eq. (15) gives

$$\left.\frac{d\tilde{s}}{d\theta}\right|_{initial} = -1.339745962 \tag{16}$$

Designating the right hand side of Eq. (10) as α, we get

$$\alpha = \frac{r_M G M_{CNO}}{h^2} = 2932920.356 \cdot I(1/\tilde{s}) \tag{17}$$

Finally, the problem to be numerically solved is reduced to

$$\frac{d^2\tilde{s}}{d\theta^2} = -\tilde{s} + 2932920.356 \cdot I(1/\tilde{s}) \tag{18}$$

$$\tilde{s}|_{initial} = 5 \tag{19}$$

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$$\left.\frac{d\tilde{s}}{d\theta}\right|_{initial} = -1.339745962 \tag{20}$$

In **Figure** 7 we give the polar graph of 4 revolutions showing that the orbit precesses without any relativistic corrections or perturbations. This is non-Keplerian behavior: a spiralgraph. If we look closely at the starting location $r|_{initial} = .2r_M$ with $\theta = 0$, we see that the galaxy does NOT extend beyond its inital radius, with the next step in polar angle showing that it is closer to the CNO center. The reason for this behavior is that the starting speed of 100 km/s at $r|_{inital} = .2r_M$ is too small for the galaxy to extend its orbit for the next discrete angle value. When we do the second example, where we change the starting speed from 100 km/s $\rightarrow$ 1000 km/s, we will see that the galaxy expands beyond $r|_{initial}$. Another aspect of **Figure** 7: it alludes to the fact that galaxies which assemble with negligible velocities undergo simple harmonic motion [5].

We now compute the period for this case. In order to do this, we have to arrange the variables such that

$$\frac{d\theta}{dt} = f(\theta) \tag{21}$$

so the period τ is

$$\tau = \int_0^{2\pi} \frac{d\theta}{f(\theta)} \tag{22}$$

This is done using h = constant = $r^2 \frac{d\theta}{dt}$. Working this out

$$\frac{d\theta}{dt} = f(\theta) = \tilde{s}^2 \left(2.856813144 \times 10^{-19}\right) \ \mathrm{s}^{-1} \tag{23}$$

This gives

$$\tau \ \text{Case1} = \int_0^{2\pi} \frac{d\theta}{\tilde{s}^2(\theta)} (110.9970809) \ \text{Gyr} \tag{24}$$

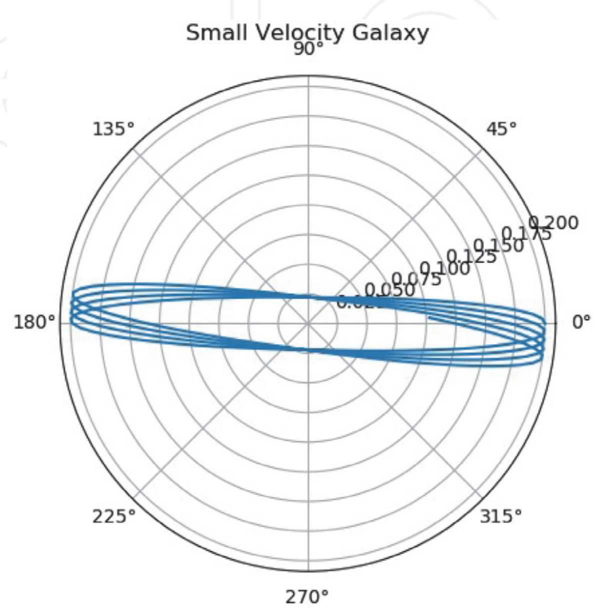

Figure 7.
Orbital mechanics for small velocity coma cluster galaxy.

Doing this final integration numerically reveals

$$\tau \ \ \text{Case1} = 3.2502 \ \ \text{Gyr} \tag{25}$$

In **Figure 8**, the speed is plotted against rotation angle, showing the near SHM of small birth velocities. Starting at 100 km/s at 0 degrees, it reaches a high ~ 870 km/s at closest approach to the center and then back again to 100 km/s on the other side, for one-half revolution.

5.4 Case 2 example

The only change from the prior case is the initial speed V_0: 100 km/s → 1000 km/s. One can show that the problem to solve becomes

$$\frac{d^2\tilde{s}}{d\theta^2} = -\tilde{s} + 29329.20356 \cdot I(1/\tilde{s}) \tag{26}$$

$$\tilde{s}|_{initial} = 5 \tag{27}$$

$$\left.\frac{d\tilde{s}}{d\theta}\right|_{initial} = -1.339745962 \tag{28}$$

In **Figure 9**, we give 4 revolutions for this case, again showing non-Keplerian behavior. Note that the starting initial conditions permit the galaxy to expand beyond $r|_{initial} = .2r_M$.

For the period of this case, one can show that the numerical integration is

$$\tau \ \ \text{Case2} = \int_0^{2\pi} \frac{d\theta}{\tilde{s}^2(\theta)} (11.09970809) \ \ \text{Gyr} \tag{29}$$

Doing this final integration numerically finds

$$\tau \ \ \text{Case2} = 3.1765 \ \ \text{Gyr} \tag{30}$$

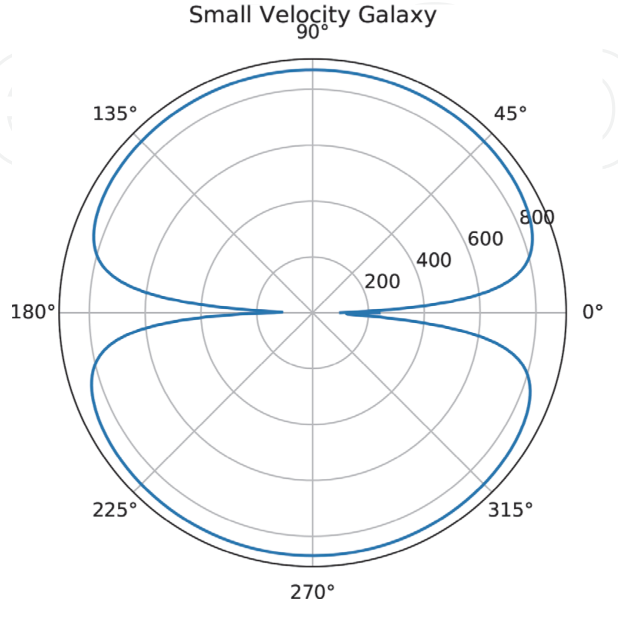

Figure 8.
The speed of the small velocity galaxy for one revolution, showing the near SHM for small initial birth velocities. See text for explanation.

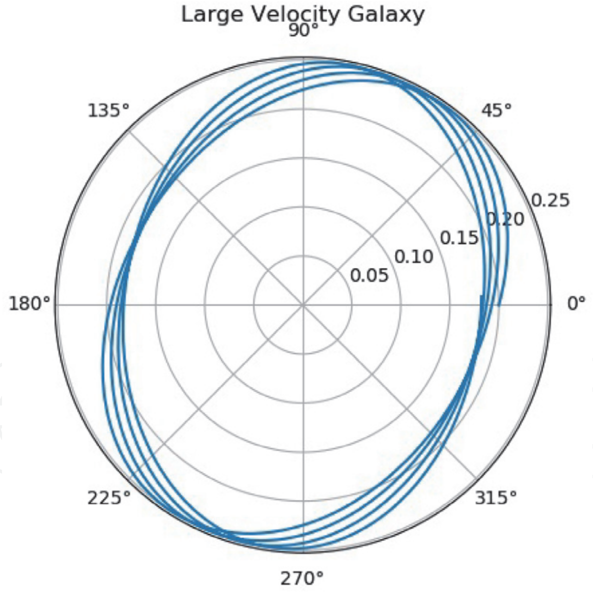

Figure 9.
Orbital mechanics for large velocity coma cluster galaxy.

6. Conclusion

The KATRIN neutrino mass experiment [7] can prove Dark Matter is the condensation of cosmological neutrinos and anti-neutrinos by obtaining a mass signal for the electron-anti-neutrino. The Planck Satellite Consortium assumes no condensation of cosmological neutrinos in their analysis of the cosmological microwave background and predicts neutrino masses too small for KATRIN to measure [8]. The identification of CNO as the Dark Matter allows mathematical modeling of embedded galaxy orbits, first reported here, using the Coma Cluster of galaxies.

7. Future work

Galaxies inside the CNO Dark Matter self-aggregate with a probability distribution at locations between $\vec{r}$ and $\vec{r} + \vec{d}r$ and with initial velocities between $\vec{v}$ and $\vec{v} + \vec{d}v$. For example, if the probability of assemblage between r and $r + dr$ is the fractional volume of available space in the CNO, then the average initial radius distance would be $.75 \cdot r_{CNO}$. The cluster galaxies then perform orbits, or collapsing toward the center. There is no equilibrium with respect to the Dark Matter, as the baryons move through a frictionless condensate. The periods have units Gyr, so a present snapshot is good for a long time.

The next step is to do a simulation of N-number of self-aggregating galaxies over an initial period of time and time-advance it to the present day. The 'birth' distribution in velocities will give rise to a predicted later time-evolved velocity distribution that can be compared to the present-day astronomically measured velocity distribution.

Because of the huge CNO Dark Matter mass, individual galaxy collisions are a small perturbation of cluster dynamics. The probability that you have cluster evaporation from many-body interactions is near zero and completely negligible: baryonic matter situated in a CNO gravitational well stays in the CNO gravitational well. For our numerical examples here, we used two galaxies at the same initial radial distance. However, the orbital planes of both galaxies could have been in any

3-dimensional orientation in 3-space, hence even the same initial radial orbits have a negligible chance of interacting.

Conflict of interest

The author states that there is no conflict of interest.

Abbreviations and notation

CNO	condensed neutrino object, stable assemblage of neutrinos and anti-neutrinos.
G	Newton's gravitational constant.
$\Phi(\mathbf{r})$	Dark Matter gravitational potential (potential energy per unit mass) at location r, Eq. (1).
$\mathbf{r}_c$	Spiral galaxy center-of-mass position inside the Dark Matter Object.
$\mathbf{a}$	spiral arm vector in spiral galaxy center of mass.
$M(R)$	mass of CNO, Eq. (4)
m_ν	neutrino mass scale, to be measured by [7].
R	radius of CNO, Eq. (4)
SHM	simple harmonic motion.
$x = p_F/m_\nu c$	reduced Fermi momentum, where c is the speed of light.
$\mathcal{M}_{010}, \mathcal{R}_{010}$	modeled mass and radius of the Coma Galaxy cluster CNO, Eqs. (5) and (6).
Mpc	mega-par-sec.
$M_\odot$	mass of the Sun.
$M_{CNO}(r)$	enclosed CNO mass at radius r, Eq. (8).
$\tilde{s} = r_M/r$,	where $r_M \equiv r_{CNO}$ is the radius of the Coma CNO.

Author details

Peter D. Morley
Blue Ridge Scientific LLC, Front Royal, VA, United States

*Address all correspondence to: peter3@uchicago.edu

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