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# A Model Reference Based 2-DOF Robust Observer-Controller Design Methodology

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## 1. Introduction

As it is well known, standard feedback control is based on generating the control signal  $u$  by processing the error signal,  $e = r - y$ , that is, the difference between the reference input and the actual output. Therefore, the input to the plant is

$$u = K(r - y) \quad (1)$$

It is well known that in such a scenario the design problem has one degree of freedom (1-DOF) which may be described in terms of the stable Youla parameter (Vidyasagar, 1985). The error signal in the 1-DOF case, see figure 1, is related to the external input  $r$  and  $d$  by means of the sensitivity function  $S \doteq (1 + P_o K)^{-1}$ , i.e.,  $e = S(r - d)$ .

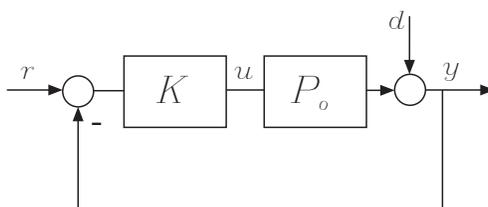


Fig. 1. Standard 1-DOF control system.

Disregarding the sign, the reference  $r$  and the disturbance  $d$  have the same effect on the error  $e$ . Therefore, if  $r$  and  $d$  vary in a similar manner the controller  $K$  can be chosen to minimize  $e$  in some sense. Otherwise, if  $r$  and  $d$  have different nature, the controller has to be chosen to provide a good trade-off between the command tracking and the disturbance rejection responses. This compromise is inherent to the nature of 1-DOF control schemes. To allow independent controller adjustments for both  $r$  and  $d$ , additional controller blocks have to be introduced into the system as in figure 2.

Two-degree-of-freedom (2-DOF) compensators are characterized by allowing a separate processing of the reference inputs and the controlled outputs and may be characterized by means of two stable Youla parameters. The 2-DOF compensators present the advantage of a complete separation between feedback and reference tracking properties (Youla & Bongiorno, 1985): the feedback properties of the controlled system are assured by a feedback

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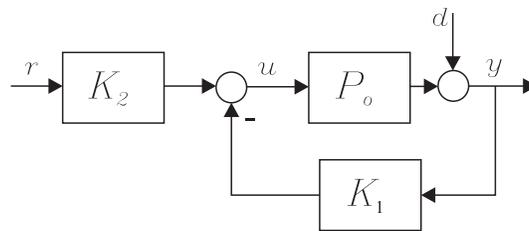


Fig. 2. Standard 2-DOF control configuration.

controller, i.e., the first degree of freedom; the reference tracking specifications are addressed by a prefilter controller, i.e., the second degree of freedom, which determines the open-loop processing of the reference commands. So, in the 2-DOF control configuration shown in figure 2 the reference  $r$  and the measurement  $y$ , enter the controller separately and are independently processed, i.e.,

$$u = K \begin{bmatrix} r \\ y \end{bmatrix} = K_2 r - K_1 y \quad (2)$$

As it is pointed out in (Vilanova & Serra, 1997), classical control approaches tend to stress the use of feedback to modify the systems' response to commands. A clear example, widely used in the literature of linear control, is the usage of reference models to specify the desired properties of the overall controlled system (Astrom & Wittenmark, 1984). What is specified through a reference model is the desired closed-loop system response. Therefore, as the system response to a command is an open-loop property and robustness properties are associated with the feedback (Safonov et al., 1981), no stability margins are guaranteed when achieving the desired closed-loop response behaviour.

A 2-DOF control configuration may be used in order to achieve a control system with both a performance specification, e.g., through a reference model, and some guaranteed stability margins. The approaches found in the literature are mainly based on optimization problems which basically represent different ways of setting the Youla parameters characterizing the controller (Vidyasagar, 1985), (Youla & Bongiorno, 1985), (Grimble, 1988), (Limebeer et al., 1993).

The approach presented in (Limebeer et al., 1993) expands the role of  $H_\infty$  optimization tools in 2-DOF system design. The 1-DOF loop-shaping design procedure (McFarlane & Glover, 1992) is extended to a 2-DOF control configuration by means of a parameterization in terms of two stable Youla parameters (Vidyasagar, 1985), (Youla & Bongiorno, 1985). A feedback controller is designed to meet robust performance requirements in a manner similar as in the 1-DOF loop-shaping design procedure and a prefilter controller is then added to the overall compensated system to force the response of the closed-loop to follow that of a specified reference model. The approach is carried out by assuming uncertainty in the normalized coprime factors of the plant (Glover & McFarlane, 1989). Such uncertainty description allows a formulation of the  $\mathcal{H}_\infty$  robust stabilization problem providing explicit formulae.

A frequency domain approach to model reference control with robustness considerations was presented in (Sun et al., 1994). The design approach consists of a nominal design part plus a modelling error compensation component to mitigate errors due to uncertainty.

However, the approach inherits the restriction to minimum-phase plants from the Model Reference Adaptive Control theory in which it is based upon.

In this chapter we present a 2-DOF control configuration based on a right coprime factorization of the plant. The presented approach, similar to that in (Pedret C. et al., 2005), is not based on setting the two Youla parameters arbitrarily, with internal stability being the only restriction. Instead,

1. An observer-based feedback control scheme is designed to guarantee robust stability. This is achieved by means of solving a constrained  $\mathcal{H}_\infty$  optimization using the right coprime factorization of the plant in an active way.
2. A prefilter controller is added to improve the open-loop processing of the robust closed-loop. This is done by assuming a reference model capturing the desired input-output relation and by solving a model matching problem for the prefilter controller to make the overall system response resemble as much as possible that of the reference model.

The chapter is organized as follows: section 2 introduces the Observer-Controller configuration used in this work within the framework of stabilizing control laws and the Youla parameterization for the stabilizing controllers. Section 3 reviews the generalized control framework and the concept of  $\mathcal{H}_\infty$  optimization based control. Section 4 displays the proposed 2-DOF control configuration and describes the two steps in which the associated design is divided. In section 5 the suggested methodology is illustrated by a simple example. Finally, Section 6 closes the chapter summarizing its content and drawing some conclusions.

## 2. Stabilizing control laws and the Observer-Controller configuration

This section is devoted to introduce the reader to the celebrated Youla parameterization, mentioned throughout the introduction. This result gives all the control laws that attain closed-loop stability in terms of two stable but otherwise free parameters. In order to do so, first a basic review of the factorization framework is given and then the Observer-Controller configuration used in this chapter is presented within the aforementioned framework. The Observer-Controller configuration constitutes the basis for the control structure presented in this work.

### 2.1 The factorization framework

A short introduction to the so-called factorization or fractional approach is provided in this section. The central idea is to factor a transfer function of a system, not necessarily stable, as a ratio of two stable transfer functions. The factorization framework will constitute the foundations for the analysis and design in subsequent sections. The treatment in this section is fairly standard and follows (Vilanova, 1996), (Vidyasagar, 1985) or (Francis, 1987).

#### 2.1.2 Coprime factorizations over $\mathcal{RH}_\infty$

A usual way of representing a scalar system is as a rational transfer function of the form

$$P_o(s) = \frac{n(s)}{m(s)} \quad (3)$$

where  $n(s)$  and  $m(s)$  are polynomials and (3) is called polynomial fraction representation of  $P_o(s)$ . Another way of representing  $P_o(s)$  is as the product of a stable transfer function and a transfer function with stable inverse, i.e.,

$$P_o(s) = N(s)M^{-1}(s) \quad (4)$$

where  $N(s), M(s) \in \mathcal{RH}_\infty$ , the set of stable and proper transfer functions.

In the Single-Input Single-Output (SISO) case, it is easy to get a fractional representation in the polynomial form (3). Let  $\delta(s)$  be a Hurwitz polynomial such that  $\deg \delta(s) = \deg m(s)$  and set

$$N(s) = \frac{n(s)}{\delta(s)} \quad M(s) = \frac{m(s)}{\delta(s)} \quad (5)$$

The factorizations to be used will be of a special type called Coprime Factorizations. Two polynomials  $n(s)$  and  $m(s)$  are said to be coprime if their greatest common divisor is 1 (no common zeros). It follows from Euclid's algorithm - see for example (Kailath, 1980) - that  $n(s)$  and  $m(s)$  are coprime iff there exists polynomials  $x(s)$  and  $y(s)$  such that the following identity is satisfied:

$$x(s)m(s) + y(s)n(s) = 1 \quad (6)$$

Note that if  $z$  is a common zero of  $n(s)$  and  $m(s)$  then  $x(z)m(z) + y(z)n(z) = 0$  and therefore  $n(s)$  and  $m(s)$  are not coprime. This concept can be readily generalized to transfer functions  $N(s), M(s), X(s), Y(s)$  in  $\mathcal{RH}_\infty$ . Two transfer functions  $M(s), N(s)$  in  $\mathcal{RH}_\infty$  are coprime when they do not share zeros in the right half plane. Then it is always possible to find  $X(s), Y(s)$  in  $\mathcal{RH}_\infty$  such that  $X(s)M(s) + Y(s)N(s) = 1$ .

When moving to the multivariable case, we also have to distinguish between right and left coprime factorizations since we lose the commutative property present in the SISO case. The following definitions tackle directly the multivariable case.

**Definition 1.** (Bezout Identity) Two stable matrix transfer functions  $N_r$  and  $M_r$  are right coprime if and only if there exist stable matrix transfer functions  $X_r$  and  $Y_r$  such that

$$\begin{bmatrix} X_r & Y_r \end{bmatrix} \begin{bmatrix} M_r \\ N_r \end{bmatrix} = X_r M_r + Y_r N_r = I \quad (7)$$

Similarly, two stable matrix transfer functions  $N_l$  and  $M_l$  are left coprime if and only if there exist stable matrix transfer functions  $X_l$  and  $Y_l$  such that

$$\begin{bmatrix} M_l & N_l \end{bmatrix} \begin{bmatrix} X_l \\ Y_l \end{bmatrix} = M_l X_l + N_l Y_l = I \quad (8)$$

The matrix transfer functions  $X_r, Y_r$  ( $X_l, Y_l$ ) belonging to  $\mathcal{RH}_\infty$  are called right (left) Bezout complements.

Now let  $P_o(s)$  be a proper real rational transfer function. Then,

**Definition 2.** A right (left) coprime factorization, abbreviated RCF (LCF), is a factorization  $P_o(s) = N_r M_r^{-1}$  ( $P_o(s) = M_l^{-1} N_l$ ), where  $N_r, M_r$  ( $N_l, M_l$ ) are right (left) coprime over  $\mathcal{RH}_\infty$ .

With the above definitions, the following theorem arises to provide right and left coprime factorizations of a system given in terms of a state-space realization. Let us suppose that

$$P_o(s) \doteq \left[ \begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] \quad (9)$$

is a minimal stabilisable and detectable state-space realization of the system  $P_o(s)$ .

Theorem 1. Define

$$\begin{bmatrix} M_r & -Y_l \\ N_r & X_l \end{bmatrix} \doteq \left[ \begin{array}{c|cc} A + BF & B & -L \\ \hline F & I & 0 \\ C + DF & -D & I \end{array} \right] \quad (10)$$

$$\begin{bmatrix} X_r & Y_r \\ -N_l & M_l \end{bmatrix} \doteq \left[ \begin{array}{c|cc} A + LC & -(B + LD) & -L \\ \hline F & I & 0 \\ C & -D & I \end{array} \right]$$

where  $F$  and  $L$  are such that  $A + BF$  and  $A + LC$  are stable. Then,  $P_o(s) = N_r(s)M_r^{-1}(s)$  ( $P_o(s) = M_l^{-1}(s)N_l(s)$ ) is a RCF (LCF).

**Proof.** The theorem is demonstrated by substituting (1.10) into equation (1.7).

Standard software packages can be used to compute appropriate  $F$  and  $L$  matrices numerically for achieving that the eigenvalues of  $A + BF$  are those in the vector

$$p_F = [p_{F_1} \cdots p_{F_n}]^T \quad (11)$$

Similarly, the eigenvalues of  $A + LC$  can be allocated in accordance to the vector

$$p_L = [p_{L_1} \cdots p_{L_n}]^T \quad (12)$$

By performing this pole placement, we are implicitly making active use of the degrees of freedom available for building coprime factorizations. Our final design of section 4 will make use of this available freedom for trying to meet all the controller specifications.

## 2.2 The Youla parameterization and the Observer-Controller configuration

A control law is said to be stabilizing if it provides internal stability to the overall closed-loop system, which means that we have Bounded-Input-Bounded-Output (BIBO) stability between every input-output pair of the resulting closed-loop arrangement. For instance, if we consider the general control law  $u = K_2 r - K_1 y$  in figure 3a internal stability amounts to being stable all the entries in the mapping  $(r, d_i, d_o) \rightarrow (u, y)$ .

Let us reconsider the standard 1-DOF control law of figure 1 in which  $u = K(r - y)$ . For this particular case, the following theorem gives a parameterization of all the stabilizing control laws.

**Theorem 2.** (1-DOF Youla parameterization) For a given plant  $P = N_r M_r^{-1}$ , let  $C_{stab}(P)$  denote the set of stabilizing 1-DOF controllers  $K_1$ , that is,

$$C_{stab}(P) \doteq \{K_1 : \text{the control law } u = K_1(r - y) \text{ is stabilizing}\}. \quad (13)$$

The set  $C_{stab}(P)$  can be parameterized by

$$C_{stab}(P) = \left\{ \frac{X_r + M_r Q_y}{Y_r - N_r Q_y} : Q_y \in \mathcal{RH}_\infty \right\} \quad (14)$$

As it was pointed out in the introduction of this chapter, the standard feedback control configuration of figure 1 lacks the possibility of offering independent processing of disturbance rejection and reference tracking. So, the controller has to be designed for providing closed-loop stability and a good trade-off between the conflictive performance objectives. For achieving this independence of open-loop and closed-loop properties, we added the extra block  $K_2$  (the prefilter) to figure 1, leading to the standard 2-DOF control scheme in figure 2. Now the control law is of the form

$$u = K_2 r - K_1 y \quad (15)$$

where  $K_1$  and  $K_2$  are to be chosen to provide closed-loop stability and meet the performance specifications. This control law is the most general stabilizing linear time invariant control law since it includes all the external inputs ( $y$  and  $r$ ) in  $u$ .

Because of the fact that two compensator blocks are needed for expressing  $u$  according to (15), 2-DOF compensators are also referred to as two-parameter compensators. It is worth emphasizing that (15) represents the most general feedback compensation scheme and that, for example, there is no *three-parameter compensator*.

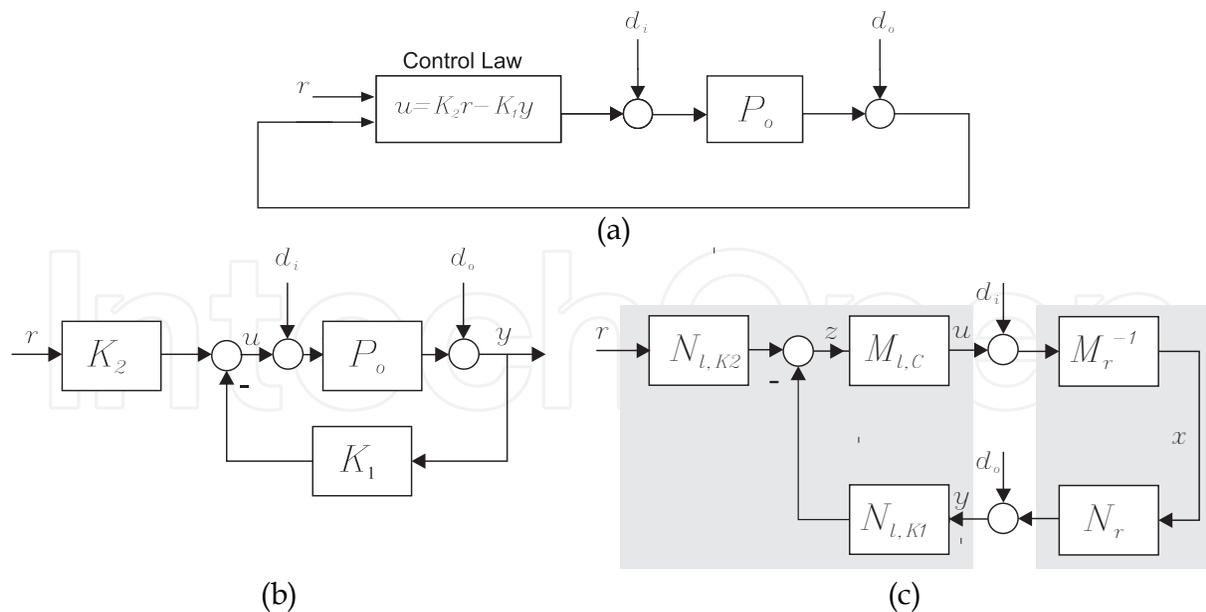


Fig. 3. (a) 2-DOF control diagram. (b) An unfeasible implementation of the 2-DOF control law  $u = K_2 r - K_1 y$ . (c) A feasible implementation of the control law  $u = K_2 r - K_1 y$ .

It is evident that if we make  $K_1 = K_2 = K$ , then we have  $u = K(r - y)$  and recover the standard 1-DOF feedback configuration (1 parameter compensator) of figure 1. Once we have designed  $K_1$  and  $K_2$ , equation (15) simply gives a control law but it says nothing about the actual implementation of it, see (Wilfred, W.K. et al., 2007). For instance, in figure 3b we can see one possible implementation of the control law given by (15) which is a direct translation of the equation into a block diagram. It should be noted that this implementation is not valid when  $K_2$  is unstable, since this block acts in an open-loop fashion and this would result in an unstable overall system, in spite of the control law being a stabilizing one. To circumvent this problem we can make use of the previously presented factorization framework and proceed as follows: define  $C = [K_1 \quad K_2]$  and let  $K_1 = M_{l,C}^{-1} N_{l,K1}$  and  $K_2 = M_{l,C}^{-1} N_{l,K2}$  such that  $(M_{l,C}, [N_{l,K1} \quad N_{l,K2}])$  is a LCF of  $C$ . Once  $C = [K_1 \quad K_2]$  has been factorized as suggested, the control action in (15) can be implemented as shown in figure 3c. In this figure the plant has been right-factored as  $N_r M_r^{-1}$ . It can be shown that the mapping  $(r, d_i, d_o) \rightarrow (z_1, z_2, u, y)$  remains stable (necessary for internal stability) if and only if so it does the mapping  $(r, d_i, d_o) \rightarrow (u, y)$ . The following theorem states when the system depicted in figure 3c is internally stable.

**Theorem 3.** The system of figure 3c is internally stable if and only if

$$R^{-1} := M_{l,C} M_r + N_{l,K2} N_r \in \mathcal{RH}_\infty, \quad R \in \mathcal{RH}_\infty \tag{16}$$

We can proceed now to announce the 2-DOF Youla Paramaterization.

**Theorem 4.** (2-DOF Youla parameterization) For a given plant  $P = N_r M_r^{-1}$ , let  $C_{stab}(P)$  denote the set of stabilizing 2-DOF controllers  $C = [K_1 \quad K_2]$ , that is,

$$C_{stab}(P) \doteq \{C = [K_1, K_2] : \text{the control law } u = K_2 r - K_1 y \text{ is stabilizing}\}. \quad (17)$$

The set  $C_{stab}(P)$  can be parameterized as follows

$$C_{stab}(P) \doteq \left\{ \left( \begin{array}{c} \frac{X_r + M_r Q_y}{Y_r - N_r Q_y}, \quad \frac{Q_r}{Y_r - N_r Q_y} \end{array} : Q_y, Q_r \in RH_\infty \right) \right\} \quad (18)$$

**Proof.** Based on theorem 2, it follows that the transfer function  $R$  will satisfy theorem 3 if and only if  $M_{l,C}^{-1} N_{l,K_1}$  equals  $(Y_r - Q_y N_r)^{-1} (X_r + Q_y M_r)$  for some  $Q_y$  in  $\mathcal{RH}_\infty$  such that  $|Y_r - Q_y N_r| \neq 0$ . Moreover,  $R$  is independent of  $N_{l,K_1}$ . This leads at once to (18).

Following with figure 3c, let us assume that we take

$$N_{l,K_1} = 1, \quad N_{l,K_2} = K_1 X_r, \quad M_{l,C} = 1 + K_1 Y_r \quad (19)$$

where  $K_1 \in \mathcal{RH}_\infty$ . Then the two-parameter compensator can be redrawn as shown in figure 4a. For reasons that will become clear later on, this particular two-parameter compensator is referred to as the Observer-Controller scheme.

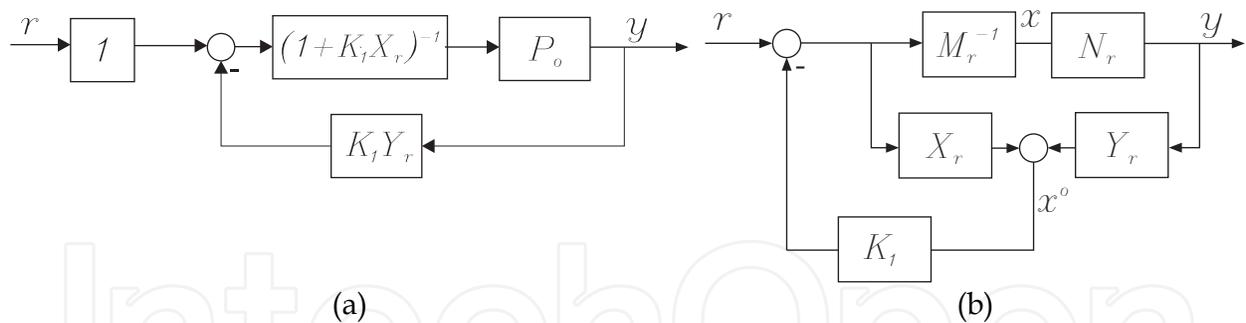


Fig. 4. (a) Observer-Controller in two blocks form. (b) Observer-Controller in three blocks form where  $P_o = N_r M_r^{-1}$  is a RCF.

Applying theorem 3 for the particular case at hand the stability condition for the system of figure 4a reduces to

$$R^{-1} = (1 + K_1 X_r) M_r + K_1 Y_r N_r = M_r + K_1 \in \mathcal{RH}_\infty, \quad R \in \mathcal{RH}_\infty \quad (20)$$

It can be verified that the relation between  $r$  and  $y$  is given by  $N_r R$ . In order to  $T_{yr}$  being stable, we have to require  $R$  to be stable. On the other hand,  $R^{-1}$  is given by  $M_r + K_1$  which

is stable having chosen  $K_1$  stable. Choosing such an  $R$  for our design the stability requirements for the overall system to be internally stable are satisfied.

It is easy to see that figure 4a can be rearranged as in figure 4b, where the plant appears in right-factored form ( $P_o = N_r M_r^{-1}$ ). Now it is straightforward to notice that the relation between  $x$  and  $x_o$  is given by

$$x^o = (X_r M_r + Y_r N_r) x = x \quad (21)$$

where the Bezout identity applies. This way, the  $X_r$  and  $Y_r$  blocks can be thought of as an observer for the fictitious signal  $x$  appearing in the middle of the RCF. So, feeding back the observation of  $x$  lets to place the close-loop eigenvalues at prescribed locations since the achieved input to output relations is given by  $y = N_r R r$  and the stable poles of both  $N_r$  and  $R$  are freely assignable. This may remind of a basic result coming from state-space control theory associated with observed state feedback: assuming a minimal realization of the plant, state feedback using observers let you change the dynamics of the plant by moving the closed-loop poles of the resulting control system to desired positions in the left half plane. Let us assume the following situation for the figure 4b

$$P = \frac{b}{a}, \quad M_r = \frac{a}{p_K}, \quad N_r = \frac{b}{p_K}, \quad X_r = \frac{n_x}{p_L}, \quad Y_r = \frac{n_y}{p_L} \quad (22)$$

Now let us take  $K_1$  to be of the form

$$K_1 = \frac{m}{p_K} \quad (23)$$

being  $m$  an arbitrary polynomial in  $s$  of degree  $n-1$ . With  $p_K$  and  $p_L$  we refer here to monic polynomials in  $s$  having as roots the entries of the vectors in (11) and (12), respectively. The dependence of  $s$  has been dropped to simplify the notation. By choosing this stable  $K_1$  the relation between the input  $r$  and the output  $y$  remains as follows

$$T_{yr} = \frac{b}{a + m} \quad (24)$$

So we have achieved a reallocation of the closed-loop poles leaving the zeros of the plant unaltered, as it happens in the context of state-space theory when one makes use of observed state feedback.

What follows is intended to fully understand the relationship between the scheme of figure 4 and conventional state-feedback controllers. For this purpose, we will remind here results

appearing in (Kailath, 1980), among others. Let us assume that the system input-output relation is given in the form

$$y = \frac{b}{a}u \quad (25)$$

One can now replace equation (25) by the following two

$$x_p = \frac{1}{a}u, \quad y = bx_p \quad (26)$$

And choose the following state variables for describing the system in the state space

$$\begin{aligned} x_p &= x_1 \\ \dot{x}_p &= x_2 = \dot{x}_1 \\ &\vdots \\ x_p^{(n-1)} &= x_n = \dot{x}_{n-1} \end{aligned} \quad (27)$$

This leads to the well-known canonical controllable form realization

$$\begin{bmatrix} \dot{x}_p \\ \ddot{x}_p \\ \vdots \\ x_p^{(n-1)} \\ x_p^{(n)} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & -a_2 & \cdots & -a_{n-1} \end{bmatrix} \begin{bmatrix} x_p \\ \dot{x}_p \\ \vdots \\ x_p^{(n-2)} \\ x_p^{(n-1)} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} u \quad (28)$$

$$y = \begin{bmatrix} b_0 - a_0 & b_1 - a_1 & \cdots & b_{n-1} - a_{n-1} \end{bmatrix} \begin{bmatrix} x_p \\ \dot{x}_p \\ \vdots \\ x_p^{(n-1)} \end{bmatrix} + u$$

The corresponding realization is shown in figure 5a.

The point is that the fictitious signal  $x_p$  can be used to determine the complete state (in the controllable canonical form realization) of the system by just deriving it  $n-1$  times. Now suppose that  $z$  and  $w$  are polynomials such that

$$za + wb = 1 \quad (29)$$

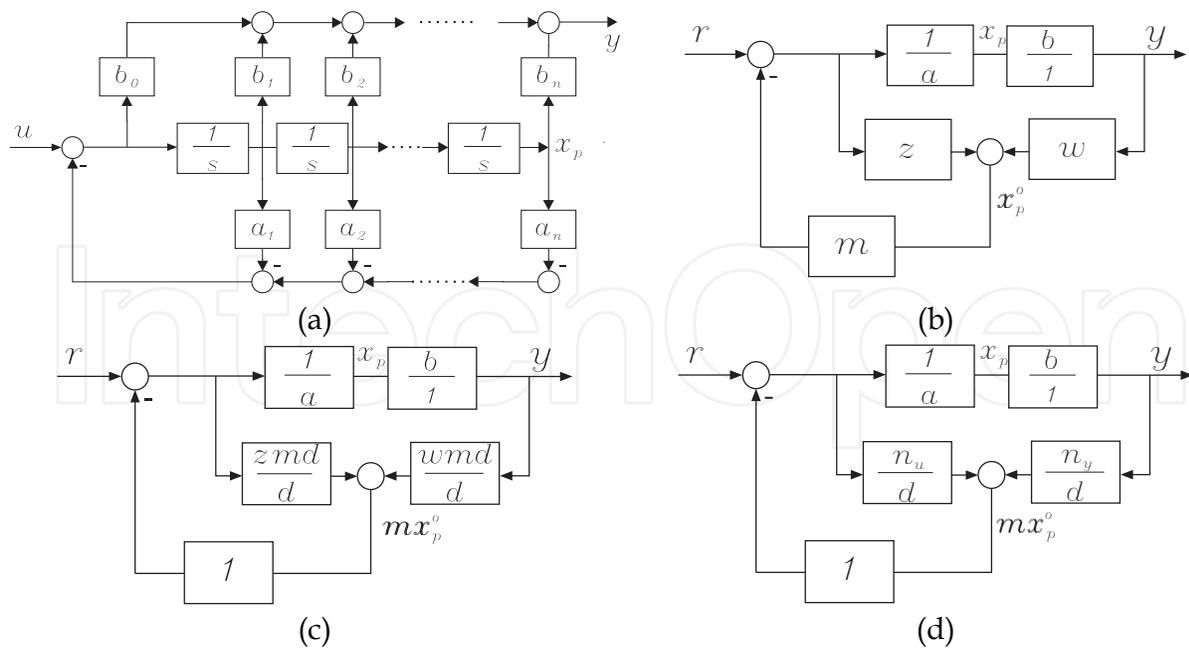


Fig. 5. (a) Controllable canonical form realization of  $\frac{b}{a}$ . (b) Unfeasible observed-based state-feedback scheme. (c) Towards a feasible observer-controller: part I. (d) Part II.

In figure 5b we can see a way of thinking of a state-feedback controller. Through  $z$  and  $w$  we observe  $x_p$  and by multiplying it by  $m$  we achieve an arbitrary linear combination of  $x_p$  and its derivatives, that is, a state feedback control law. Obviously, the scheme as such can not be implemented. But it is easy to make it realizable by introducing a  $n$ th-order polynomial (the so-called observer polynomial indeed) as in figure 5c, then  $zmd$  and  $wmd$  can be made of degree equal or less than  $n$  - see (Kailath, 1980) - without altering the state feedback gain, leading to  $n_y$  and  $n_u$  in figure 5d. So figure 5 summarises a procedure entirely based on the transfer function domain (but though at the level of polynomials) to implement a state-feedback control law. However, the scheme in figure 5d is not exactly the one we will work with.

By introducing another  $n$ th-degree stable polynomial ( $a_0$ ) figure 5c can be redrawn as in figure 6a.

By doing this we are considering that our plant is the series connection of two systems, that is  $P = P_1P_2$ , where  $P_1 = \frac{a_0}{a}$ ,  $P_2 = \frac{b}{a_0}$ . So we are considering on purpose a non-minimal realization of the plant. The series connection system is not completely controllable but completely observable. Let denote by  $A, B, C, D$  the corresponding realization matrices of  $P$  in terms of the realization matrices of  $P_1(A_1, B_1, C_1, D_1)$  and  $P_2(A_2, B_2, C_2, D_2)$  in controllable canonical form. Then we arrive at the non-minimal realization

$$P \doteq \left[ \begin{array}{c|c} A & B \\ \hline C & D \end{array} \right] = \left[ \begin{array}{cc|c} A_1 & 0 & B_1 \\ \hline B_2C_1 & A_2 & B_2D_1 \\ \hline D_2C_1 & C_2 & D_2D_1 \end{array} \right] \quad (30)$$

where the state vector for  $P$  is of the form  $x = [x_1 \ x_2]^T$ , being  $x_1$  and  $x_2$  the state vectors of  $P_1$  and  $P_2$ , respectively, in controllable canonical form. In more detail, the state matrix  $A$  of  $P$  is given by

$$A = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 & | & 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 & | & 0 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & 0 & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 & | & 0 & 0 & 0 & \dots & 0 \\ -a_0 & -a_1 & -a_2 & \dots & -a_{n-1} & | & 0 & 0 & 0 & \dots & 0 \\ \hline 0 & 0 & 0 & \dots & 0 & | & 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 0 & \dots & 0 & | & 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & | & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 & | & 0 & 0 & 0 & \dots & 1 \\ p_{k_0} - a_0 & p_{k_1} - a_1 & p_{k_2} - a_2 & \dots & p_{k_{n-1}} - a_{n-1} & | & -p_{k_0} & -p_{k_1} & -p_{k_2} & \dots & -p_{k_{n-1}} \end{bmatrix} \quad (31)$$

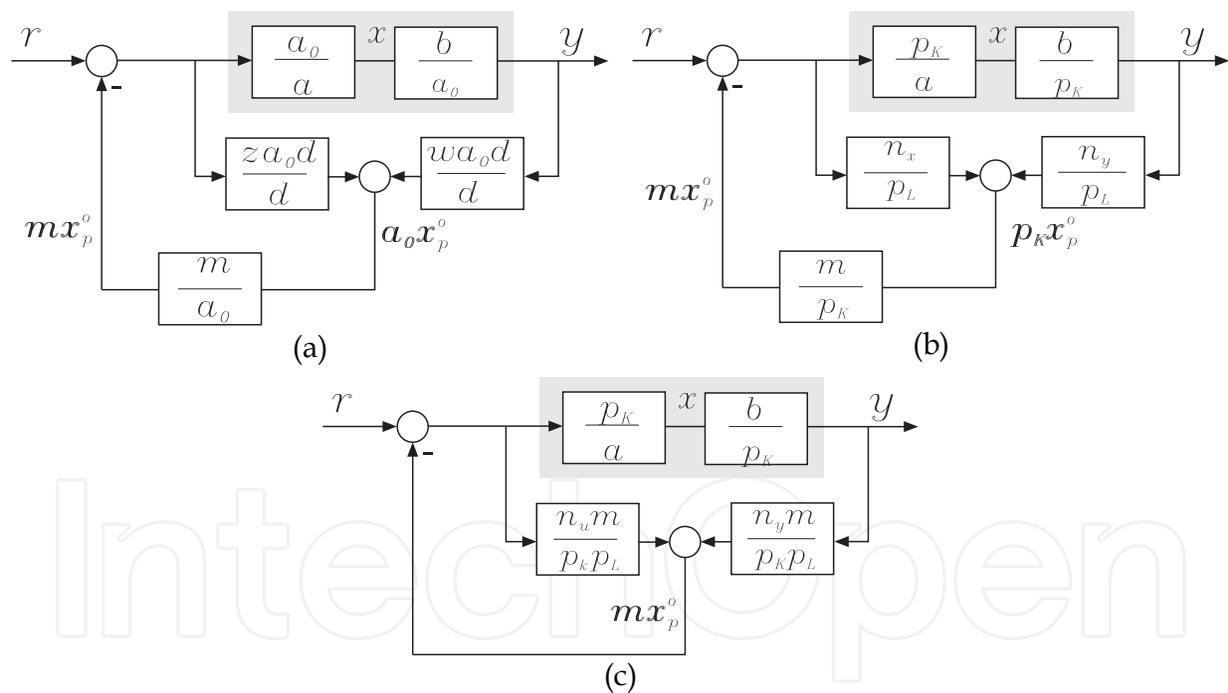


Fig. 6. (a) Non-realizable Observer-Controller configuration. (b) Realizable Observer-Controller configuration. (c) Realizable observer-controller put in the form of a standard observed state feedback (i.e., figure 5d).

Now it is straightforward to see that

$$A' = \left[ \begin{array}{c|c} A_1 & 0 \\ \hline 0 & A_2 \end{array} \right] = TAT^{-1} \quad (32)$$

where

$$T = \left[ \begin{array}{c|c} I_{n \times n} & 0_{n \times n} \\ \hline -I_{n \times n} & I_{n \times n} \end{array} \right], \quad T^{-1} = \left[ \begin{array}{c|c} I_{n \times n} & 0_{n \times n} \\ \hline I_{n \times n} & I_{n \times n} \end{array} \right] \quad (33)$$

By using this similarity state transformation the new realization matrices are given by

$$A' = \left[ \begin{array}{c|c} A_1 & 0_{n \times n} \\ \hline 0_{n \times n} & A_2 \end{array} \right], \quad B' = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 0_{n \times 1} \end{bmatrix}, \quad C' = \begin{bmatrix} c_1 \\ \vdots \\ c_n \\ \vdots \\ c_{2n} \end{bmatrix}^T, \quad D' = D \quad (34)$$

where  $c_i \neq 0 \forall i = 1 \dots 2n$ . From (34) it is evident that the controllable states are the  $n$  first states, which are obtainable through  $x_p$  and its  $n-1$  successive derivatives. Besides, it is easy to see that the similarity transformation employed does not alter the first  $n$  state components. The approach taken in this work consists of observing the  $2n$  states of the non-minimal realization (34) and consider just the  $n$  first states corresponding to the controllable part (this partial vector state of dimension  $n$  is equal to the state of  $\frac{b(s)}{a(s)}$  in controllable canonical form) for state feedback. By doing this, see figure 6b, we are introducing  $n$  extra degrees of freedom (the  $n$  roots of the Hurwitz polynomial  $p_k$ ) into the design. In figures 6b and 6c we have returned to the terminology of section 2.1 when we introduced the coprime factorizations over  $\mathcal{RH}_\infty$ , with respect to figure 6a the following identities hold:  $p_k = a_0$ ,  $p_L = d$ . The term *Observer-Controller* is used in this work to make reference to an observed-based state feedback control system designed following this approach. The method presented in section 4 uses the extra freedom which arises from using a non-minimal order observer (see figure 6c, where the observer polynomial  $p_k p_L$  has degree  $2n$ , being  $n$  the order of the plant) for trying to meet more demanding objectives.

### 3. $\mathcal{H}_\infty$ -norm optimization based robust control systems design

In this section we review the general method of formulating control problems introduced by (Doyle, 1983). Within this framework, we recall the general method for representing uncertainty for multivariable systems and determine the condition for robust stability in the presence of unstructured additive uncertainty. The presentation is fairly standard, we refer the reader to (Skogestad S., 1997) for a more detailed treatment.

#### 3.1 General control problem formulation

Within the general control configuration (Doyle, 1983) of figure 9,  $G$  is referred to as the generalized plant and  $K$  is the generalized controller. Four types of external variables are

dealt with: exogenous inputs,  $w$ , i.e., commands, disturbances and noise; exogenous outputs,  $z$ , e.g., error signals to be minimized; controller inputs,  $v$ , e.g., commands, measured plant outputs, measured disturbances; and control signals,  $u$ .

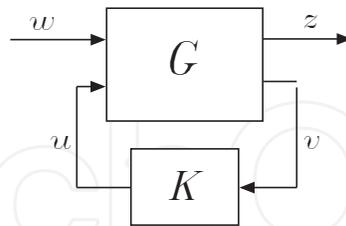


Fig. 7. Generalized plant and controller.

The controller design problem is divided into the analysis and the synthesis phases. The controller  $K$  is synthesized such that some measure, mathematically a norm, of the transfer function from  $w$  to  $z$  is minimized, e.g. the  $\mathcal{H}_\infty$  norm.

**Definition 3** ( $\mathcal{H}_\infty$ -norm) The  $\mathcal{H}_\infty$ -norm of a proper stable system  $P$  is given by

$$\|P(s)\|_\infty \doteq \sup_{\omega} \bar{\sigma}(P(j\omega)) \quad (35)$$

where  $\bar{\sigma}(P)$  denotes the largest singular value of the matrix  $P$ .

In words, the  $\mathcal{H}_\infty$ -norm of a dynamic system is the maximum amplification the system can make to the energy of the input signal in any direction. In the SISO case it is equal to the maximum value of the system's frequency response magnitude (the magnitude peak in the Bode diagram). For the general MIMO case it is equal to the system's largest singular value over all the frequencies. From this point on with every mention to a norm we would implicitly be considering the above defined  $\mathcal{H}_\infty$  norm, and no further remarks will be made.

The controller design amounts to find a  $K$  that minimizes the closed-loop norm from  $w$  to  $z$  in figure 7. For the analysis phase the designer has to make the actual system meet the form of a generalized control problem according to figure 7. Standard software packages exist that solve numerically the synthesis problem once the problem has been put in the generalized form. In order to get meaningful controller synthesis problems, frequency weights on the exogenous inputs  $w$  and outputs  $z$  are incorporated to perform the corresponding optimizations over specific frequency ranges.

Once the stabilizing controller  $K$  is synthesized, it rests to analyse the closed-loop performance that it provides. In this phase, the controller for the configuration in figure 9 is incorporated into the generalized plant  $G$  to form the system  $\mathcal{N}$ , as shown in figure 11

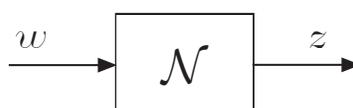


Fig. 8. Relation between  $w$  and  $z$  in the generalized control problem.

It is relatively straightforward to show that the expression for  $\mathcal{N}$  is given by

$$\mathcal{N} = G_{11} + G_{12}K(1 - G_{22}K)^{-1}G_{21} \doteq \mathcal{F}_l(G, K) \tag{36}$$

where  $\mathcal{F}_l$  denotes the lower Linear Fractional Transformation (LFT) of  $G$  and  $K$ . In order to obtain a good design for  $K$ , a precise knowledge of the plant is required. The dynamics of interest are modeled but this model may be inaccurate (this is usually the case indeed). To deal with this problem the real plant  $P$  is assumed to be unknown but belonging to a class of models built around a nominal model  $P_o$ . This set of models is characterized by a matrix  $\Delta$ , which can be either a full matrix (unstructured uncertainty) or a block diagonal matrix (structured uncertainty), that includes all possible perturbations representing uncertainty to the system. Weighting matrices  $W_1$  and  $W_2$  are usually employed to express the uncertainty in terms of normalized perturbations in such a way that  $\|\Delta\|_\infty \leq 1$ . The general control configuration in figure 9 may be extended to include model uncertainty as it is shown in figure 9

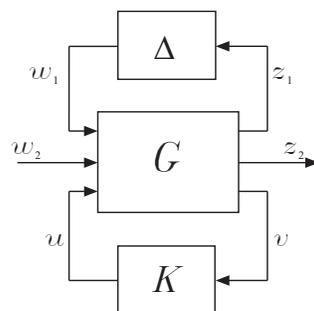


Fig. 9. Generalized control problem configuration.

The block diagram in figure 9 is used to synthesize a controller  $K$ . To transform it for analysis, the lower loop around  $G$  is closed by the controller  $K$  and it is incorporated into the generalized plant  $G$  to form the system  $\mathcal{N}$  as it is shown in figure 10. The same lower LFT is obtained as if no uncertainty was considered.

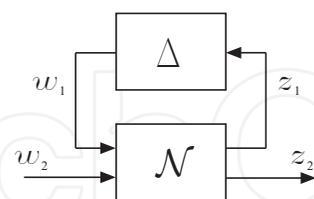


Fig. 10. Generalized block diagram for analysis in the face of uncertainty.

To evaluate the relation from  $w = [w_1 \quad w_2]^T$  to  $z = [z_1 \quad z_2]^T$  for a given controller  $K$  in the uncertain system, the upper loop around  $\mathcal{N}$  is closed with the perturbation matrix  $\Delta$ . This results in the following upper LFT:

$$\mathcal{F}_u(\mathcal{N}, \Delta) \doteq \mathcal{N}_{22} + \mathcal{N}_{21}\Delta(1 - \mathcal{N}_{11}\Delta)^{-1}\mathcal{N}_{12} \tag{37}$$

and so  $z = \mathcal{F}_u(\mathcal{N}, \Delta)w$ . To represent any control problem with uncertainty by the general control configuration it is necessary to represent each source of uncertainty by a single

perturbation block  $\Delta$ , normalized such that  $\|\Delta\|_\infty \leq 1$ . We will assume in this work that we can collect all the sources of uncertainties into a single full (unstructured) matrix  $\Delta$ .

### 3.2 Uncertainty and robustness

As already commented, an exact knowledge of the plant is never possible. Therefore, it is often assumed that the real plant, denoted by  $P$ , is unknown but belonging to a set of class models characterized somehow by  $\Delta$  and with centre  $P_o$ .

**Definition 4** (Nominal Stability) The closed-loop system of figure 9 has Nominal Stability (NS) if the controller  $K$  internally stabilizes the nominal model  $P_o$  ( $\Delta = 0$ ), i.e., the four transfer matrices  $\mathcal{N}_{11}, \mathcal{N}_{12}, \mathcal{N}_{21}, \mathcal{N}_{22}$  in the closed-loop transfer matrix  $\mathcal{N}$  shown in figure 13 are stable.

**Definition 5** (Nominal Performance) The closed-loop system of figure 12 has Nominal Performance (NP) if the performance objectives are satisfied for the nominal model  $P_o$ , i.e.,  $\|\mathcal{N}_{22}\|_\infty < 1$  in figure 10 assuming  $\Delta = 0$ .

**Definition 6** (Robust Stability) The closed-loop system has Robust Stability (RS) if the controller  $K$  internally stabilizes the closed-loop system in figure 9 ( $\mathcal{F}_u(\mathcal{N}, \Delta)$ ) for every  $\Delta$  such that  $\|\Delta\|_\infty \leq 1$ .

We will just consider in this work additive uncertainty, which mathematically is expressed as

$$\mathcal{P}_A = \{P : P = P_o + W_1 \Delta\} \quad (38)$$

Being  $w_A$  a scalar frequency weight and  $\|\Delta\|_\infty \leq 1$ . Now that we know how to describe the set of plants which our real plant is supposed to lie in the next issue is to answer the question of when a controller stabilizes all the plants belonging to this set.

**Theorem 5** (Robust Stability for unstructured uncertainty) Let us assume that we have posed our system in the form illustrated by figure 9. The overall system is robustly stable (see definition 6) iff

$$\|\mathcal{N}_{11}\|_\infty \leq 1 \quad (39)$$

where  $N$  has been defined in (36), see figure 10.

Robust stability conditions for the different uncertainty representations can be derived by posing the corresponding feedback loops as in figure 9 and then applying theorem 5, also known as the small gain theorem. See (Morari and Zafirov, 1989) for details.

## 4. The design for the proposed robust 2-DOF Observer-Controller

In this section a methodology for designing 2-DOF controllers is provided. The design is based on the Observer-Controller configuration described in section 2.2. In order to have a 2-DOF scheme a prefilter block ( $K_2$ ) has been added, leading to the general scheme shown in figure 11

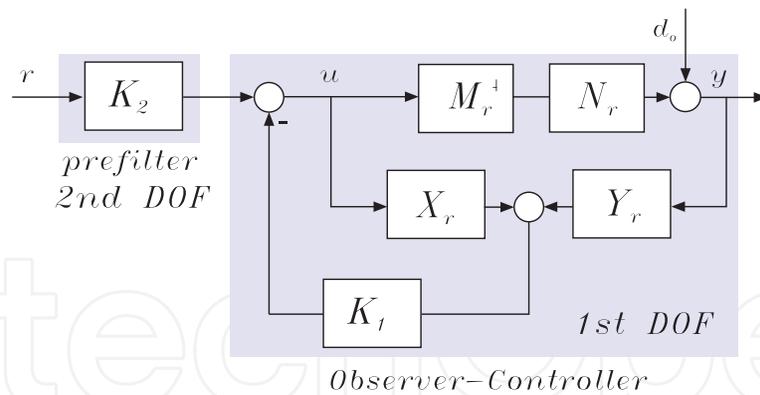


Fig. 11. The proposed 2-DOF control configuration.

The controller blocks  $X_r, Y_r, K_1$  which implicitly fix the Youla parameter  $Q_y$  of theorem 5 will be in charge of providing robust stability and good output disturbance rejection. On the other hand, the prefilter  $K_2$  (the Youla parameter  $Q_r$  of theorem 5) has to cope with the tracking properties of the system by solving a model matching problem with respect to a specified reference model which describes the desired closed-loop behaviour for the resulting controlled system.

**4.1 Step I: Design of the Observer-Controller part through direct search optimization**

In section 2.2 we characterized the Observer-Controller configuration in terms of the polynomials  $p_K, p_L$  and  $m$ . Let us assume without loss of generality that additive output uncertainty (38) is considered. In this first step of the design the objective will be to find convenient  $p_K, p_L, m$ , defining entirely  $X_r, Y_r, K_1$  in figure 12. This search will be performed in order to provide robust stability with the best possible output disturbance rejection.

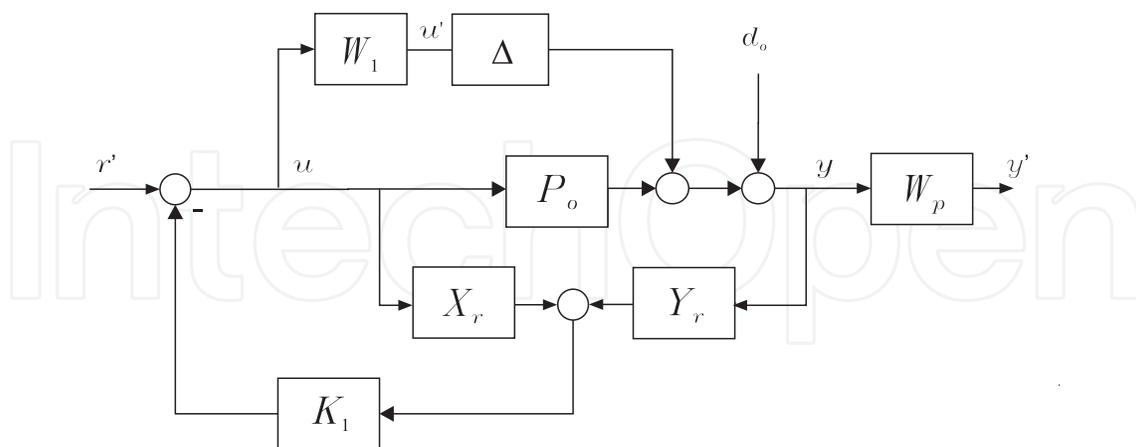


Fig. 12. Observer-Controller part with additive uncertainty.

More specifically, for the scheme in figure 12 the following relations hold

$$\begin{bmatrix} u \\ y' \end{bmatrix} = \begin{bmatrix} M_r (1 - RM_r) Y_r & M_r R \\ W_p (1 - N_r (1 - RM_r) Y_r) & N_r R \end{bmatrix} \begin{bmatrix} d_o \\ r' \end{bmatrix} \tag{40}$$

where all the terms have been defined in section 2.2. It can be proved by applying theorem 5 to figure 12 (once put in the generalized controller configuration of figure 9) that robust stability of the system in figure 12 amounts to satisfy the following inequality

$$\|T_{u'd_o}\|_{\infty} \leq 1 \quad (41)$$

The design for the Observer-Controller part reduces finally to solving the following optimization problem

$$\begin{aligned} \min_{p_K, p_L, m} & \quad \left\| W_p (1 - N_r (1 - RM_r) Y_r) \right\|_{\infty} \\ \text{subject to} & \quad \left\| W_1 (M_r (1 - RM_r) Y_r) \right\|_{\infty} \leq 1 \end{aligned} \quad (42)$$

Direct search techniques - see (Powell, M., 1998) - are suggested for solving the problem (42). Basically they consist of a method for solving optimization problems that does not require any information about the gradient of the objective function. Unlike more traditional optimization methods that use information about the gradient or higher derivatives to search for an optimal point, a direct search algorithm searches a set of points around the current point, looking for one where the value of the objective function is lower than the value at the current point. At each step, the algorithm searches a set of points, called a mesh, around the current point—the point computed at the previous step of the algorithm. The mesh is formed by adding the current point to a scalar multiple of a set of vectors called a pattern. If the pattern search algorithm finds a point in the mesh that improves the objective function at the current point, the new point becomes the current point at the next step of the algorithm. In (Henrion, D., 2006) a recent application of direct search techniques for solving a specific control problem can be consulted.

#### 4.2 Step II: Design of the prefilter controller $K_2$

In this section we tackle the second step of our design. This step is aimed at designing a prefilter controller for meeting tracking specifications given in terms of a reference model. In order to achieve this goal a model matching problem is posed as in figure 13

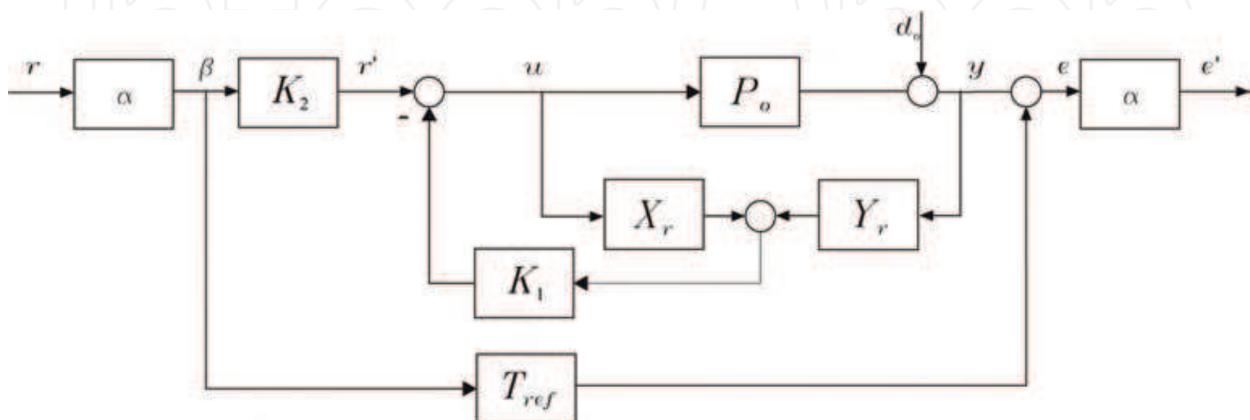


Fig. 13. Model-matching problem arrangement for the design of  $K_2$ .

where  $T_{ref}$  is the specified reference model for the closed-loop dynamics. The idea is to make use of the general control framework introduced in section 3 and design  $K_2$  so as to minimize the relation from  $r$  to  $e$  in an  $\mathcal{H}_\infty$  sense. In doing so, we also want to have certain control over the amount of control effort employed to make the close loop resemble the model reference. The parameter  $\alpha$  will precisely play the role of enabling one to arrive at a compromise between the tracking quality and the amount of energy demanded by the controller, accommodating into the design this practical consideration.

Note that in the nominal case, i.e.,  $P = P_o$ , the prefilter controller  $K_2$  sees just  $N_r R$ . Therefore, the relation from the reference to the output reads as

$$y = N_r R K_2 r \tag{43}$$

It should be noted that the  $N_r$  and  $R$  have already been fixed as a result of applying the first step of the design. To include different and independent dynamics for the step response, we have to take advantage of the second degree of freedom that  $K_2$  provides. From the overall scheme in figure 11 we can compute the transfer matrix function that relates the inputs  $[d_o \quad r]^T$ , i.e., the disturbance  $d_o$  and the command signal  $r$ , with the outputs  $[u' \quad e]^T$ , i.e., the weighted control signal  $u'$  and the weighted model matching error  $e$ .

$$\begin{bmatrix} u' \\ e \end{bmatrix} = \begin{bmatrix} -W_1(1 - M_r(X_r + RN_r Y_r)) & \alpha W_1 M_r(1 - RM_r) Y_r K_2 \\ N_r(X + RN_r Y_r) & \alpha^2(N_r R K_2 - T_{ref}) \end{bmatrix} \begin{bmatrix} d_o \\ r \end{bmatrix} \tag{44}$$

The  $\mathcal{H}_\infty$ -norm of the complete transfer matrix function (44) is minimized to find  $K_2$  without modifying neither the robust stability margins nor the disturbance rejection properties provided by the Observer Controller in the first step of the design. The 2-DOF design problem shown in figure 13 can be easily cast into the general control configuration seen in section 3. Comparing figures 12 and 13 with figure 9 we make the following pairings  $w_1 = d_o$ ,  $w_2 = r$ ,  $z_1 = u'$ ,  $z_2 = e$ ,  $v = \beta$ ,  $u = u$  and  $K = K_2$ . The augmented plant  $G$  and the controller  $K_2$  are related by the following lower LFT:

$$\mathcal{F}_l(G, K_2) \doteq G_{11} + G_{12} K_2 (1 - G_{22} K_2)^{-1} G_{21} \tag{45}$$

The corresponding partitioned generalized plant  $G$  is:

$$\begin{aligned} \begin{bmatrix} u' \\ e \\ \beta \end{bmatrix} &= \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} d_o \\ r \\ r' \end{bmatrix} \\ &= \begin{bmatrix} -W_1(1 - M_r(X_r + RN_r Y_r)) & 0 & W_1 M_r R \\ N_r(X_r + RN_r Y_r) & -\alpha^2 T_{ref} & \alpha N_r R \\ 0 & \alpha & 0 \end{bmatrix} \begin{bmatrix} d_o \\ r \\ r' \end{bmatrix} \end{aligned} \tag{46}$$

**Remark.** The reference signal  $r$  must be scaled by a constant  $W_r$  to make the closed-loop transfer function from  $r$  to the controlled output  $y$  match the desired reference model  $T_{ref}$  exactly at steady-state. This is not guaranteed by the optimization which is aimed at minimizing the  $\mathcal{H}_\infty$ -norm of the error. The required scaling is given by

$$W_r \doteq (K_2(0)N_r(0)R(0))^{-1} T_{ref}(0) \quad (47)$$

Therefore, the resulting reference controller is  $K_2 W_r$ .

### 5. Illustrative example

In this section we apply the methodology presented in section 4 for designing a 2-DOF controller according to figure 11 for the following nominal plant

$$P_o = \frac{5(s+1.3)}{(s+1)(s+2)} \quad (48)$$

The uncertainty in the model is parameterised using an additive uncertainty description as in (38) with

$$W_1 = \frac{3(s+1)}{(s+20)} \quad (49)$$

Now we initialize the optimization problem (42) with the values  $p_{k_0} = [-10 \ -10]^T$ ,  $p_{L_0} = [-20 \ -20]^T$ ,  $m_o = [1 \ 2]^T$ , where  $p_{k_0}$  ( $p_{L_0}$ ) contains the initial roots of the polynomials  $p_k$  ( $p_L$ ) and  $m_o$  the initial coefficients for the polynomial  $m$ .

For this example we have used a non-constrained direct search optimization solver and we have defined as the objective function the same that appears in the problem (42) plus a penalty that acts when the robust stability restriction is not satisfied. In order to have nearly perfect output disturbance rejection in steady state the following weight for the  $T_{y_{d_o}}$  relation has been used

$$W_p = \frac{1}{s+0.01} \quad (50)$$

this forces the direct search algorithm to provide small values for the  $T_{y_{d_o}}$  magnitude response at dc. The optimization procedure results finally in the following *optimal* controller blocks for the feedback part of the design

$$X_r = \frac{s^2 + 57.19s - 112500}{s^2 + 35.67s + 245}, \quad Y_r = \frac{2.2710s + 40190}{s^2 + 35.67s + 245}, \quad K_1 = \frac{0.05355s + 0.323}{s^2 + 24.53s + 148.2} \quad (51)$$

or in terms of  $p_k, p_L, m$  :

$$p_k = [-10.7774 \quad -13.7519]^T, p_L = [-9.2857 \quad -26.3793]^T, m = [0.0535 \quad 0.3230]^T \quad (52)$$

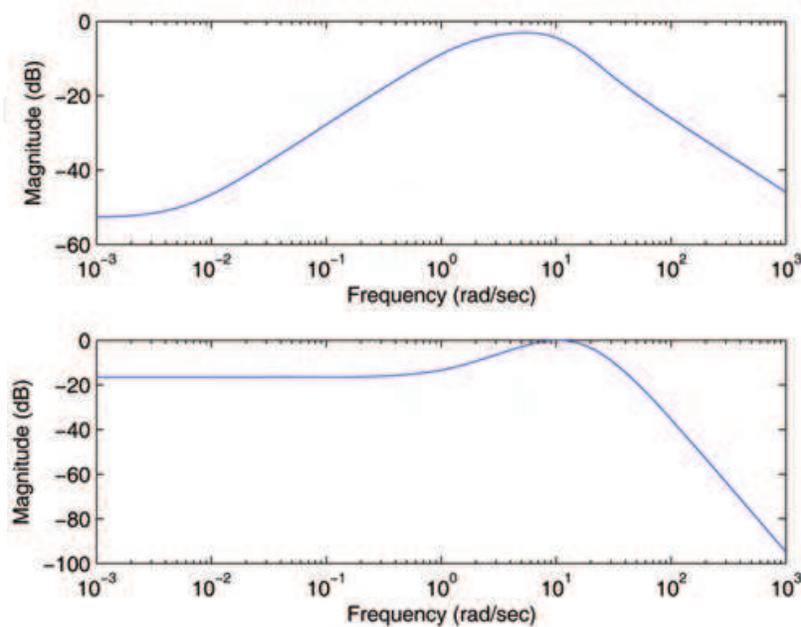


Fig. 14. Top: magnitude response of  $T_{y_{d_o}}$ . Bottom: magnitude response of  $W_1 T_{u_{d_o}}$  satisfying the robust stability condition.

In figure 14 the finally resulting  $T_{y_{d_o}}$  (output disturbance rejection) and  $W_1 T_{u_{d_o}}$  magnitude responses are shown, providing the former specially good disturbance rejection at steady state and being the  $\mathcal{H}_\infty$ -norm of the latter less than one and thus ensuring the robust stability specification.

So far we have obtained the final design for the Observer-Controller part ensuring robust stability and the best achievable output disturbance rejection in a  $\mathcal{H}_\infty$  sense. We turn now to the design of the prefilter controller  $K_2$ .

Let us assume that we are given the desired closed-loop dynamics in terms of the following reference model

$$T_{ref} = \frac{49}{(s+7)^2} \quad (53)$$

Such reference model (53) provides second order responses with time constant 1/7 seconds. The second step of the design explained in section 4 results in the prefilter block

$$K_2 = \frac{-0.006051s^3 + 11.65s^2 + 106.8s + 181.7}{s^3 + 28.65s^2 + 184.8s + 508.3} \quad (54)$$

This prefilter block has been achieved using  $\alpha = 6$ , see figure 17.

In figure 15 it is shown that the value  $\alpha = 6$  provides a tight model matching with the minimum possible control action. Larger values of the  $\alpha$  parameter do not improve significantly the model matching and cause the control action to acquire higher values.

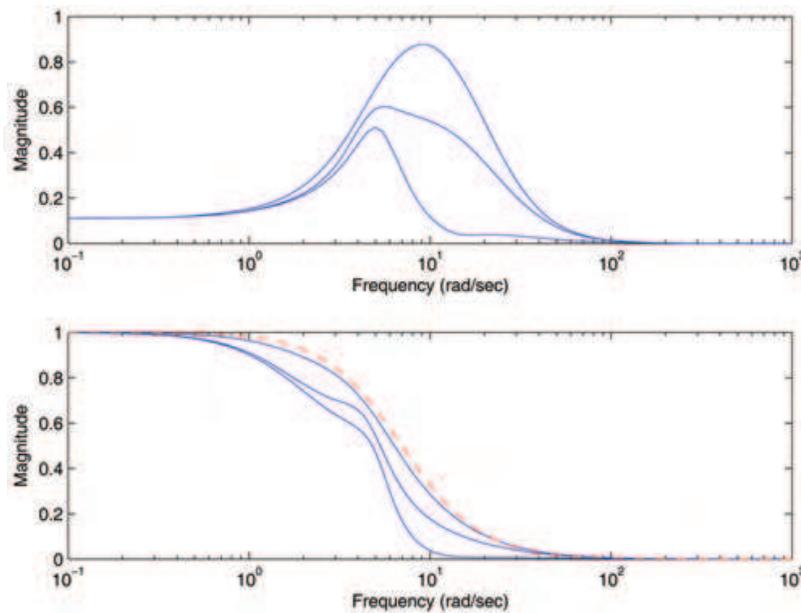


Fig. 15. From bottom to top (in solid), magnitude responses of  $T_{ur}$  (top figure) and  $T_{yr}$  (bottom figure) for  $\alpha = 1, 3, 6$ . In dashed it is shown the response of the target model  $T_{ref}$ .

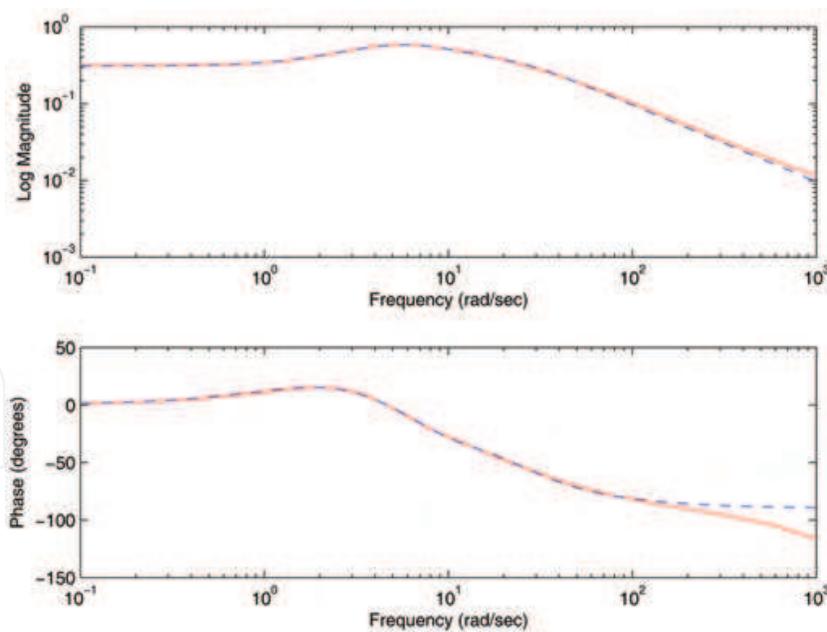


Fig. 16. Bode diagram for the original 11<sup>th</sup> order prefilter  $K_2$  (dashed) and the 3<sup>rd</sup> order prefilter  $K_2$  (solid) finally obtained by applying order reduction techniques.

The use of the  $\mathcal{H}_\infty$  optimization techniques traditionally results in very high order controllers. In this case, the resulting  $K_2$  is of order eleven. However, standard order

reduction techniques can be applied in order to reduce these orders. For this example, a model reduction based on a balanced realization and the hankel singular values - see (Skogestad S., 1997) - has been performed yielding finally a third order  $K_2$  without sacrificing any significant performance, see figure 16.

To summarize the carried out design, in figure 17 we show the closed-loop final response to a step command set-point change applied at  $t=0$  seconds and a step output disturbance applied at  $t=3$  seconds.

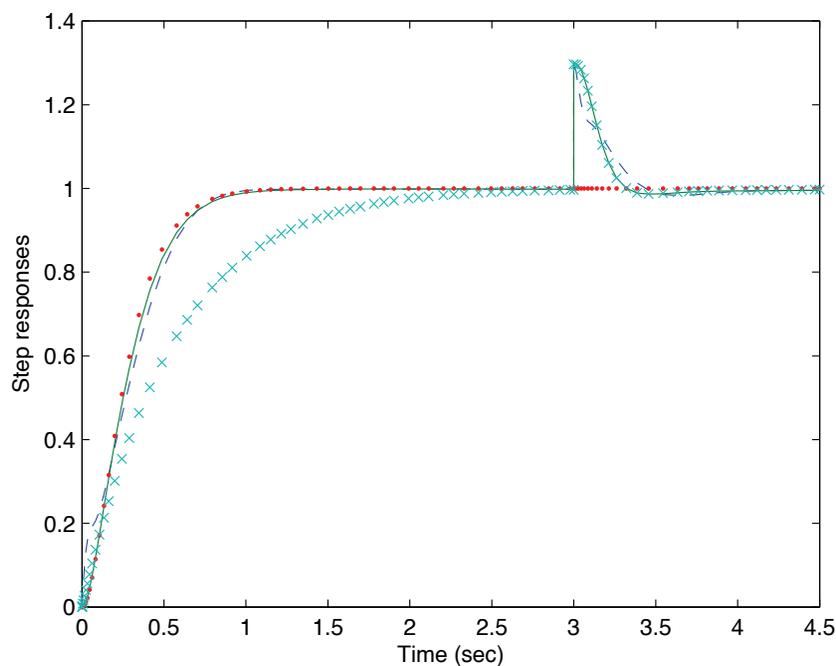


Fig. 17. Time response of the reference model  $T_{ref}$  (dotted), nominal controlled system (solid) and uncertain ( $\Delta = 0.25$  in (38)) controlled system (dashed). It is also shown the response of the nominal controlled system without making use of the prefilter controller (x-marked).

## 6. Conclusion

A new 2-DOF control configuration based on a right coprime factorization of the model of the plant has been presented. The approach has been introduced as an alternative to the commonly encountered strategy of setting the two controllers arbitrarily, with internal stability the only restriction, and parameterizing the controller in terms of the Youla parameters.

An non-minimal-observer-based state feedback control scheme has been designed first to guarantee some levels of robust stability and output disturbance rejection by solving a constrained  $\mathcal{H}_\infty$  optimization problem for the poles of the right coprime factors

$X_r, Y_r, N_r, M_r$  and the polynomial  $m$ . After that, a prefilter controller to adapt the reference command and improve the tracking properties has been designed using the generalized control framework introduced in section 3.

## 7. References

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## **New Approaches in Automation and Robotics**

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The book *New Approaches in Automation and Robotics* offers in 22 chapters a collection of recent developments in automation, robotics as well as control theory. It is dedicated to researchers in science and industry, students, and practicing engineers, who wish to update and enhance their knowledge on modern methods and innovative applications. The authors and editor of this book wish to motivate people, especially under-graduate students, to get involved with the interesting field of robotics and mechatronics. We hope that the ideas and concepts presented in this book are useful for your own work and could contribute to problem solving in similar applications as well. It is clear, however, that the wide area of automation and robotics can only be highlighted at several spots but not completely covered by a single book.

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