Chapter

New Types of Dissipative Streaming Instabilities

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Abstract

Two new, previously unknown types of dissipative streaming instabilities (DSI) are substantiated. They follow from new approach, which allows solving in general form the classical problem of an initial perturbation development for streaming instabilities (SI). SI is caused by relative motion of the streams of plasma components. With an increase in level of dissipation SI transforms into a DSI. The transformation occurs because dissipation serves as a channel for energy removal for the growth of the negative energy wave of the stream. Until recently, only one type of DSI was known. Its maximal growth rate depends on the beam density n_b and the collision frequency ν in the plasma as $\sim \sqrt{n_b/\nu}$. All types of conventional beam-plasma instabilities (Cherenkov, cyclotron, etc.) transform into it. The solution of the problem of the initial perturbation development in systems with weak beam-plasma coupling leads to a new type of DSI. With an increase in the level of dissipation, the instability in these systems transforms to the new DSI. Its maximal growth rate is $\sim \sqrt{n_b}/\nu$. The second new DSI develops in beam-plasma waveguide with over-limiting current of e-beam. Its growth rate $\sim n_b/\nu$. In addition, the solutions of abovementioned problem provide much information about SI and DSI, significant part of which is unavailable by other methods.

Keywords: beam-plasma instability, dissipative instability, development of initial perturbation, growth rate, absolute/convective instability

1. Introduction

Streaming instabilities (SI) occupy a prominent place among other plasma instabilities. They are caused by a motion of some plasma components relative to others. An example is the well-known beam–plasma instability [1]. With this instability, the directed motion of a group of fast electrons passing through the background plasma excites potential oscillations with a large growth rate near the plasma frequency. Particular attention to this instability is mainly due to the idea of creating sources of powerful electromagnetic radiation on its basis. At present, these sources have many advantages over the known vacuum sources [2, 3]. One more example (we mention these two only) is the Buneman instability [4], in which plasma electrons move relative to ions.

In the overwhelming majority of investigations beam–plasma interaction is considered without any noticeable dissipation. It, actually, was assumed that the dissipation is small and cannot have any noticeable effect on the physical processes. In this case, the development of instability leads to an increase in the amplitude of electromagnetic oscillations in the plasma, as well as their energy at the expense of beam kinetic energy. In the absence of dissipation, the level of excited oscillations may be quite high, and their energy can even be comparable to the initial energy of the beam [5].

However, generally speaking, dissipation in the system (collisions between plasma particles, heating of metal surfaces due to their complex impedance, etc.) can play an essential role in plasma–beam interaction. It can become not only a decisive factor in limiting the spatial and temporal growth, determining the field amplitude and the mode structure and limits the growth rates. In addition to these properties, which are common to all systems, it is necessary to pay special attention to the unique role of dissipation in systems with a stream of charge particles: dissipation of high level does not suppress the SI completely. Strong dissipation transforms each SI to instability of other type – to dissipative streaming instability (DSI) [1]. This type of instabilities is due to the presence of the negative energy wave (NEW) in a stream of charge particles [6, 7]. In fact, dissipation serves as a channel for energy removal for excitation of this wave. This leads to instabilities of a new physical nature, to DSI. Dissipation is the cause of this instability.

The physical nature of SI is not as simple as it might seem at first glance. It takes a lot of effort[1] to understand it clearly. This is all the more so, if we are dealing with the transformation of SI into a DSI. The transformation (in general, the transformation of one type of instability into another) makes the behavior of SI in a system with dissipation especially interesting. In addition, there are other reasons that significantly increase interest in the study of problems associated with dissipation and the DSI caused by it. Some of them are as follows.

Modern high-frequency microwave electronics, both plasma and vacuum, have two basic trends of development: an increase in the frequency and power of the output radiation [2]. With increasing frequency, the thickness of the skin layer on the resonators' walls decreases. This, in turn, leads to an increase in active energy losses. Actual dissipation in the system increases.

The second trend – an increase in the power of output radiation – leads to the need to increase the beam current. The role of space charge phenomena increases also, as well as the role of the NEW. In these circumstances it becomes important to take into account all factors that also lead to the buildup of the same wave i.e. to dissipation. In a sense, dissipation becomes associated with the space charge phenomena. In addition with an increase in the beam current, the return current increases also. With account the decrease in the skin layer and the finite conductivity of metallic surfaces, this leads to an increase in the level of dissipation in the system. All this indicates that dissipation, along with the space charge of the beam plays an important role in microwave electronics. A detailed understanding of the role of all these phenomena is vital for many problems aimed at achieving high-intensity beams and their applications.

Until recently, only one DSI was known in beam–plasma interaction theory [1]. Its maximal growth rate depends on collision frequency ν in plasma and on the beam density n_b as $\sim \sqrt{n_b/\nu}$. All types of the beam–plasma instabilities (Cherenkov, cyclotron, etc.), with an increase in the level of dissipation, transform into it. This only known DSI has a number of specific features in comparison with other (no-dissipative) instabilities: relatively low level of excited oscillations, relatively small growth rate, etc. Many investigations have been devoted to its study. It was

[1] The instability of low density e-beam in plasma is a vivid example demonstrating this sense. It is discovered in 1948, experimentally proven in early sixties; however its physical meaning became finally clear in the middle of seventies (see [8]).

assumed that various phenomena in space plasma and in plasma of controlled fusion can be explained on the basis of this instability.

However, recent studies have shown that there are other DSI also [9–11]. The interaction of the stream with the background plasma critically depends on some basic parameters of the system and/or on its geometry. Their changes lead to new physics of the beam-plasma interaction and to previously unknown types of DSI. The parameters are: the level of correlation between the beam and the plasma fields and the value of the beam current.

Available methods of instability investigation do not allow getting complete information on the process of transformation of given instability into another type. Is known the most complete information on instability can be obtained by solving the problem of the evolution of fields in space and time during the development of an initial perturbation. This problem is classical in theory of instabilities [12]. Its results can clear up how the fields of given instability transform to the fields of another one along with many other accompanying details. The character of the space–time evolution of an initial perturbation is an important issue in many branches of physics. However, the results of this problem are hardly achievable. Ultimately its mathematical solution reduces to calculation of the integral with complete dispersion relation (DR) in the denominator of the integrand. For the result the DR should be specified and solved before integration. This sharply reduces generality of results. And even in the special cases, it is not always possible to carry out the integration. In [13] an approach is presented that allowed overcome difficulties and obtain analytical expression for the fields' space–time structure for all types of conventional beam-plasma instabilities. Results show that with increase in level of dissipation all types of beam-plasma instabilities transform to the only known type of DSI.

This review shows that the number of DSI is not limited by the above-mentioned DSI. Two new types of DSI are substantiated. They follow from solution of the same classical problem of initial perturbation development. One of the DSI manifests itself in the results of solving the problem in systems with weak beam-plasma coupling. Weak interaction realizes if the beam and the plasma are spatially separated by a considerable distance. Under weak coupling the beam actually is left to its own and its proper oscillation come into play. Moreover, among them is the NEW. Its interaction with plasma causes instability, the growth rate of which reaches maximum at resonance of the plasma wave with the NEW. This resonance of wave–wave type was called "Collective Cherenkov effect" [14]. An increase in the level of dissipation leads to a new DSI with the growth rate $\sim \sqrt{n_b}/\nu$. Actually the new approach to solution of the classical problem has detected this new DSI.

The second new DSI appears in results of solving of the same problem in uniform cross-section beam-plasma waveguide with over-limiting e-beam. With an increase in the beam current the fields of its space charge affects more and more on the beam-plasma interaction. This manifests itself in two ways. Along with the increasing of the role of space charge oscillations, static fields of the beam space charge set an upper limit on the beam current that can pass through a given vacuum electro-dynamical system. The limit can be overcome by plasma filling. Plasma neutralizes the space charge of the beam. Plasma-filled waveguides can transmit e-beams with a current that is several times higher than the limiting current in vacuum waveguide. The fields of overlimiting e-beam space charge changes the character of its instability. The instability of over-limiting beams is not associated with any radiation mechanism [9, 14]. Its growth rate reaches maximum at the point of exact Cherenkov resonance and depends on the beam density as $\sqrt{n_b}$ [9, 14, 15], With an increase in the level of dissipation, one more new type of DSI develops [9]. Its growth rate depends on the parameters as $\sim n_b/\nu$.

In present review special attention is paid to systems, the geometry of which is similar to geometry of plasma microwave sources. These devices are a cylindrical waveguide with thin annular plasma and spatially separated thin annular e-beam. In this geometry the new types of DSI manifest themselves also [10].

In order to dispel all possible doubts about the correctness of the results, both new DSI are also substantiated by conventional analysis of the corresponding DR. To obtain a geometry-independent result for weak beam-plasma coupling we use perturbation theory based on smallness of the coupling parameter.

2. The only known DSI and transition to it

For the beginning we shortly present rezoning, from which follow: all types of beam-plasma instabilities (Cherenkov, cyclotron, beam instability in spatially periodical structure) transform to the only known DSI with the maximal growth rate $\sim \omega_b/\sqrt{\nu}$ (ω_b is the Langmuir frequency of the beam, ν is the collision frequency in plasma). The transition takes place with an increase in the level of dissipation. This help us to reveal a criterion for identification of DSI type.

In general, the dispersion relation (DR), describing a plasma system penetrating by an electron beam can be written as

$$D(\omega, \mathbf{k}) = D_0(\omega, \mathbf{k}) + D_b(\omega, \mathbf{k}) = 0 \tag{1}$$

where $D_0(\omega, \mathbf{k}) = \operatorname{Re} D_0(\omega, \mathbf{k}) + i\operatorname{Im} D_0(\omega, \mathbf{k})$ describes the plasma (without beam), but $D_b(\omega, \mathbf{k})$ describes the beam contribution in the system dispersion

$$D_b(\omega, \mathbf{k}) = -\frac{\omega_b^2 A(\omega, \mathbf{k})}{\gamma^3 (\omega - \mathbf{k}\mathbf{V}_b - f)^2}, \tag{2}$$

ω is the frequency, $\mathbf{k}$ is the wave vector of perturbations, ω_b is Langmuir frequency of the e-beam, $\mathbf{V}_b$ is the velocity of the beam electrons (directed along z axis), $A(\omega, \mathbf{k})$ is a polynomial with respect to ω and $\mathbf{k}$, $\gamma = \left(1 - V_b^2/c^2\right)^{-1/2}$. It is assumed that $|\operatorname{Im} D_0| << |\operatorname{Re} D_0|$ and $|D_b(\omega, \mathbf{k})| << |D_0(\omega, \mathbf{k})|, f = 0$ with the Cherenkov interaction, with the cyclotron interaction $f = n\Omega/\gamma$, (Ω is the cyclotron frequency, n is the harmonic number), and $f = k_{\text{cor}} V_b$ when e-beam interacts with the periodical structure, $k_{\text{cor}} = 2\pi/l$, l is the length of spatial period.

The beam electrons interact with the proper oscillations of the system and the interaction leads to instability. Developing instability manifests itself most effectively at frequencies and wavelengths close to the proper frequencies of the system in the absence of the beam, and, at the same time, close to the beam natural frequencies. In fact, along with (1) following condition is met

$$\omega - kV_b - f = 0. \tag{3}$$

All (conventional) beam-plasma instabilities, including DSI, follow from (1)–(3). With an increase in level of dissipation all types of no-dissipative instabilities (Cherenkov, cyclotron etc) transform into the well-known DSI. If one searches the solutions of DR (1) in the form $\omega = \omega_0 + \delta$ (ω_0 satisfies (1) and (3); this case called resonance instability) he arrives to the expression

$$\delta \left(\frac{\partial D_0}{\partial \omega} \right)_{\substack{\omega = \omega_0, \\ k = k_0}} + i\operatorname{Im} D_0(\omega_0, k_0) = \frac{(\omega_b^2/\gamma^3) A(\omega_0, k_0)}{\delta^2}. \tag{4}$$

All types of no-dissipative instabilities follow the first and the right-hand side term. In this case the dissipative (second) term in (4) is small. The DSI follows from the second term (when it is greater than the first term) and the right-hand side term. The relation between the respective growth rates $\delta^{(\nu=0)}$ and $\delta^{(\nu\to\infty)}$ is

$$\delta^{(\nu\to\infty)} = \sqrt{\frac{\left\{\delta^{(\nu=0)}\right\}^3}{2\mathrm{Im}D_0}\frac{\partial D_0}{\partial\omega}} \sim \sqrt{\frac{\left\{\delta^{(\nu=0)}\right\}^3}{\nu}} \sim \sqrt{n_b/\nu} \tag{5}$$

where the frequency of collisions in plasma ν is introduced ($\mathrm{Im}D_0 \sim \nu$). The expression (5) presents relation between the growth rates of no-dissipative and dissipative instabilities. Below we use (5) and its analogs as a criterion for identification of DSI type.

3. Weak beam-plasma coupling. New type of DSI

3.1 Solution of the problem of initial perturbation development under weak beam-plasma coupling

The best way to study an instability in detail and its possible transformation to that of other type is the solving of the problem of initial perturbation development. The information obtained by other ways is insufficient and does not give any details. Here we present general (geometry independent) solution of the problem for weakly coupled beam-plasma systems.

Consider a system consisting of a mono-energetic rectilinear electron beam and cold plasma. To begin with, suppose the following: the plasma and the beam are weakly coupled (e.g. in a consequence of a sufficiently large distance between them). Let an initial perturbation arises at a point $z = 0$ (the electron beam propagates in the direction $z > 0$) at the instant $t = 0$ and the instability begins developing. Our goal is to obtain the fields' space–time distribution at an arbitrary instant $t > 0$ and investigate in detail the instability behavior by analyzing obtained expression. In the process, we interest only the longitudinal structure of the fields, i.e., their dependence on the longitudinal coordinate z and time t. The transverse structure of the fields can be obtained by expanding in terms of the system's eigenfunctions. In accordance with this, only two arguments are highlighted below: frequency and longitudinal component of the wave vector. Other arguments are irrelevant in the consideration below. To avoid overburdening the formulas, they are omitted.

In given case of weak beam-plasma coupling the instability is the result of the interaction of the beam negative energy wave (NEW) and the slowed down wave in the plasma. The interaction is of Collective Cherenkov type. We proceed from the theory of wave interaction in plasma [16]. In terms of this theory the problem of the initial perturbation evolution under instability development in non-equilibrium plasma can be considered based on the set of partial differential equations for the amplitudes of the interacting waves: beam charge density wave $E_b(z,t)$ and the slowed down electromagnetic wave $E_w(z,t)$ in the plasma

$$\left(\frac{\partial}{\partial t} + V_\mathrm{b}\frac{\partial}{\partial z}\right)E_b(z,t) - i\delta^2 E_w(z,t) = J(z,t) \tag{6}$$

$$\left(\frac{\partial}{\partial t} + V_p\frac{\partial}{\partial z} + \nu^*\right)E_w(z,t) - iE_b(z,t) = 0$$

where t is the time, z is the coordinate along the beam propagation direction, $J(z, t)$ is a function determined by the initial conditions, V_b is the directed velocity of the beam, V_p is the group velocity of the resonant wave in plasma, $V_\mathrm{b} > V_p$, ν^* describes dissipation in plasma and is proportional to the frequency of collisions in it. The meaning of the denotation δ will be cleared up below. Note, the set (6) is meaningful irrespective of the problem of development of any instability. Generally, it describes resonant interactions between two waves in unstable medium. One only condition should be satisfied: the growth rate attains maximum under Collective Cherenkov Effect. If the maximum is attained under conventional Cherenkov Effect, as for conventional beam-plasma instabilities, the interaction should be described by other set of Equations [16].

The solution of the set (6) gives the dependence of the field's amplitude on longitudinal coordinate and time under instability development. Applying the Laplace transformation with respect to time t and the Fourier transformation with respect to the spatial coordinate z, we obtain following expressions for the transform $E_w(\omega, k)$:

$$E_w(\omega, k) = \frac{J(\omega, k)}{D(\omega, \mathrm{k})}$$
$$D(\omega, k) = (\omega - kV_b)\left(\omega - kV_p + i\nu^*\right) + \delta^2 \tag{7}$$

The field's amplitude $E_w(z, t)$ can be found by inverse transformation

$$E_w(z, t) = \frac{1}{(2\pi)^2} \int\limits_{C(\omega)} d\omega \int\limits_{-\infty}^{\infty} \frac{dk\ J(\omega, k) \exp\left(-i\omega\ t + ikz\right)}{(\omega - kV_b)\left(\omega - kV_p + i\nu^*\right) + \delta^2} \tag{8}$$

where $C(\omega)$ is the contour of integration with respect to ω. For given case it is a straight line that lies in the upper half plane of the complex plane $\omega = \ \mathrm{Re}\,\omega + i\mathrm{Im}\omega$ and passes above all singularities of the integrand.

Thus, the problem has been reduced to the problem of integration in (8). It is somewhat simpler in comparison to the integral, which represents classical solution. Instead of full DR its analog stands. The analog is determined by interaction of the waves, participating in the instability development. This replacement simplifies integration. However, it remains difficult and many authors use roundabout methods carry out an expression for possible estimation of the fields behavior [17, 18]. Presented here method easily leads to the desired result i.e. to expression for space–time distribution of the fields. We merely transform the variables ω and k to another pair ω and $\omega' = \omega - kV_b$. The first integration (over ω) may be carried out by the residue method and the integration contour must be closed in the lower half-plane. The first order pole is

$$\omega = -\left(1 - V_p/V_b\right)^{-1}\left\{\delta^2/\omega' + i\nu + \omega' V_p/V_b\right\} \tag{9}$$

The second integration (over ω') cannot be carried out exactly, and we are forced to restrict ourselves to the approximate steepest descent method [19]. This method gives result in the limit of relatively large t. According to this method, the contour of integration should be deformed to pass through the saddle point in the direction of the steepest descent. The saddle point is found from the condition

$$\frac{d}{d\omega'}\left(\omega(\omega')t + i\omega' z/V_b\right) = 0 \tag{10}$$

and is equal to

$$\omega_s' = i\delta\left\{(V_b t - z)/\left(z - V_p t\right)\right\}^{1/2} \tag{11}$$

As a result we arrive to the following expression for the field's space time structure under development of the instability in spatially separated beam-plasma system

$$E_w(z,t) = -\frac{J_0}{2\sqrt{\pi}} \frac{\exp\chi_\nu^{(wk)}(z,t)}{\left(V_b - V_p\right)^{1/2}\delta^{1/2}(V_b t - z)^{1/2}} \tag{12}$$

$$\chi_\nu^{(\mathrm{wk})} = \chi_0^{(\mathrm{wk})} - \nu^* \frac{V_b t - z}{V_b - V_p}; \chi_0^{(\mathrm{wk})} = \frac{2\delta}{V_b - V_p}\sqrt{\left(z - V_p t\right)(V_b t - z)}$$

$$J_0 = J\left(\omega = \omega(\omega_s{}'), \omega' = \omega_s'\right)$$

3.2 Analysis of the instability development

The expression (12) looks very complicate. At first glance it is impossible to extract any information on the instability behavior from it. However, it turned out, the expression may be easily analyzing. Moreover, the results are obtained from scratch, i.e. they are not based on prior research. Substantial part of the information is unavailable by other way. In particular, the analysis clearly shows that with increase in level of dissipation the no-dissipative instability turns to a new type of DSI and provides detailed information on both instabilities.

The properties of the instability is determined mainly by the exponential factor

$$\exp\chi_\nu^{(wk)}(z,t) = \exp\left\{\frac{2\delta}{V_b - V_p}\sqrt{\left(z - V_p t\right)(V_b t - z)} - \nu^* \frac{V_b t - z}{V_b - V_p}\right\}, \tag{13}$$

which provides many information: the temporal and the spatial growth rates, the spread of the unstable perturbations' velocities, the nature of the instability (absolute or convective), the effect of dissipation on instability, etc.

First consider some general properties of the instability, which follow from (13).

It is easily seen that in the absence of dissipation unstable perturbations have velocities in the range from V_p to V_b. The wave packet moves in the beam propagation direction and, along with exponential growth of the fields, expands. Its length increases over time $l \sim \left(V_b - V_p\right)\ t$. The knowledge of the boundary velocities of unstable perturbations allows at once determining the nature of the instability (convective/absolute) based on the definition only, without reference to additional studies (we mean the Sturrock's laws [20]). It is clearly seen that the instability is convective in the laboratory frame and other frames moving at velocities $V > V_b$ and $V < V_p$. However, if the observer's speed is within the range $V_p < V < V_b$, then the same instability is absolute (see **Figure 1**).

Now we turn to determination of the meaning of the denotation δ in (6). For this we consider case $\nu = 0$ and find the point of the field's maximum from expression

$$\frac{\partial\chi_0^{(\mathrm{wk})}(z,t)}{\partial z} = 0 \tag{14}$$

Its root is $z_m = w_{pk}^{(\nu=0)}t$ i.e. the point of the field's maximum moves at velocity

$$w_{pk}^{(\nu=0)} = (1/2)\left(V_b + V_p\right). \tag{15}$$

In the wave theory the velocity (15) is called convective velocity. It characterizes the spatial convection of the fastest growing perturbations. (15) shows that the peak of the wave packet disposes in its middle. The packet is symmetric with respect to its peak. Substitution of z_m into the $\chi_0^{(wk)}(z,t)$ determines the field's behavior in the maximum as $E_0(z_m,t) \sim \exp(\delta\ t)$, i.e. δ represents the maximal growth rate of the instability, which develops in absence of dissipation in systems with weak beam-plasma coupling. At the point $z_m = w_{pk}^{(\nu=0)}t$ the peak forms, because here the growth rate of perturbations is maximal.

The meaning of the parameter δ may also be determined from the DR (7) only, bypassing the results of integration (12). The general expression for the group velocity $V_{gr}(\omega,k)$ obtained from DR (7) has the limit (15) under $k = 0$. The same limit (note that $\nu = 0$) leads to DR in form $\omega^2 + \delta^2 = 0$, i.e. the parameter δ is the imaginary part of complex frequency (the growth rate). In this case (absence of dissipation) the instability is due to interaction of the NEW with the plasma. To emphasize the important role of δ we add the respective indexes $\delta \equiv \delta_{NEW}^{(\nu=0)}$. Its dependence on specific parameters is found out below.

At a fixed point z the field first grows up to the value $\sim \exp\left\{\delta_{NEW}^{(\nu=0)} z/(V_b V_0)^{1/2}\right\}$ that is reached at the instant $t = z/w_a$ where

$$w_a = 2V_b V_p/\left(V_b + V_p\right). \tag{16}$$

Then the field decreases, and at the time $t \geq z/V_0$ the wave packet completely passes given point. The exponent $\delta_{NEW}^{(\nu=0)} z/(V_b V_0)^{1/2}$ is, in fact, the maximal spatial growth rate. At a given point, the field reaches its maximum at the moment when the peak has already passed it (see **Figure 1**). The reason is that perturbations moving at lower velocities reach the point for a longer time, and they have time to grow more. w_a is the velocity of the most effectively amplified perturbations.

Thus, the solution of the problem of initial perturbation development along with other detailed information, gave results of conventional initial and boundary problems. This coincidence confirms correctness of developed approach (initial assumptions, mathematics, etc.). An additional advantage of the approach is in its geometry-independence. At first glance, the presented approach seems more complicated than traditional approaches, but this complexity is only apparent.

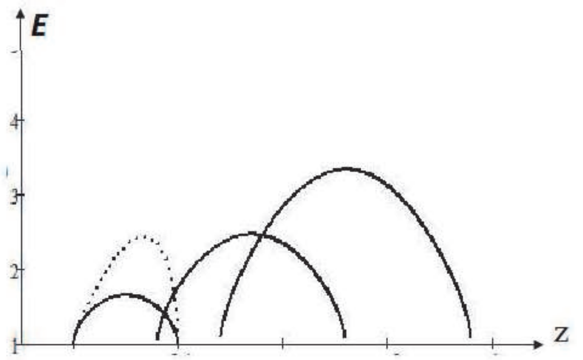

Figure 1.
Asymptotic shapes of the instability development under weak beam-plasma coupling vs. longitudinal coordinate z at instants $t_1 = 0,5/\delta_{NEW}^{\nu=0}$ $t_2 = 0,9/\delta_{NEW}^{\nu=0}$ $t_3 = 1,2/\delta_{NEW}^{\nu=0}$. *The dotted line gives the shape of the wave packet for strong beam-plasma coupling.*

3.3 The influence of dissipation. New type of DSI

Dissipation significantly influences on the presented picture of the instability development and changes it. First of all, it suppresses slow perturbations. The wave packet shortens. The threshold velocity $V_{th}^{(\mathrm{wk})}$ is determined from the condition $\chi_0^{(\mathrm{wk})} = \nu^*(V_b t - z)/(V_b - V_0)$ and is equal

$$V_{th}^{(\mathrm{wk})} = \frac{\lambda'^2 V_\mathrm{b} + V_0}{1+\lambda'^2} > V_0; \lambda' = \nu^*/\left(2\delta_{NEW}^{(\nu=0)}\right) \tag{17}$$

Only high-velocity perturbations (in the range $V_{th}^{(wk)} < v < V_b$) grow. The change in the velocity of the trailing edge shortens the packet's length and can affects the nature of instability (convective/absolute) if the frame's velocity lies in the range $V_p \le v \le V_{th}^{(wk)}$. Also, dissipation limits the growth rates of perturbations with velocity v. Substituting $z = vt$ we have for the field $E(z = vt, t) \sim \exp G(v)t$, where

$$G(v) = \frac{2\delta_{NEW}^{(\nu=0)}}{V_b - V_p}\sqrt{(V_b - v)(v\text{-}V_p)} - \nu^* \frac{V_b - v}{V_b - V_p} \tag{18}$$

As expected, the growth rates fall down. Dissipation distorts the symmetry of the induced wave packet. In presence of dissipation the dynamics of the fields can be obtained from the same Eq. (14) accounting for dissipation. It has the form

$$\left(z - w_g t\right)^2 = \lambda'^2 (V_\mathrm{b} t - z)(z - V_0 t). \tag{19}$$

The solution of (18) gives the point of the field maximum $z_{pk}^{(\nu)} = w_{pk}^{(\nu)} t$, where

$$w_{pk}^{(\nu)} = \frac{1}{2}\left\{ (V_b + V_\mathrm{p}) + \sqrt{\frac{\lambda'^2}{1+\lambda'^2}}(V_b - V_\mathrm{p}) \right\} > w_{pk}^{(\nu=0)} \tag{20}$$

This expression shows that with an increase in the level of dissipation, the peak shifts more and more to the front of the wave packet. This takes place along with the decreasing of the wave packet's length. Substitution of $w_{pk}^{(\nu)}$ into $\chi_\nu^{(\mathrm{wk})}$ gives the field value in the peak and shows the respective growth rate as the function on the level of dissipation

$$E_0\left(z = z_{pk} t, t\right) \sim \exp\left\{\delta_{NEW}^{(\nu)} t\right\}; \delta_{NEW}^{(\nu)} = \delta_{NEW}^{(\nu=0)} f\left(\lambda'^2\right); f(x) = \sqrt{1+x} - \sqrt{x} \tag{21}$$

The function $f(x)$ presents the dependence of the growth rate on the level of dissipation (see **Figures 2** and **3**). In the limit $\nu^* \to \infty$ we have $E_0 \to \sim \exp\left\{\delta_{wk}^{(\nu\to\infty)} t\right\}$, where

$$\delta_{wk}^{(\nu\to\infty)} = \left[\delta_{NEW}^{(\nu=0)}\right]^2/\nu^* \sim \sqrt{n_b}/\nu^* \tag{22}$$

As a criterion for the type of DSI this relation between the growth rates of DSI $\delta_{NEW}^{(\nu\to\infty)}$ and the growth rate of SI $\delta_{NEW}^{(\nu=0)}$ sharply differs from that for the conventional case (5). Actually the expression (22) shows that with an increase in level of dissipation in weakly coupled beam-plasma systems the instability, caused by the

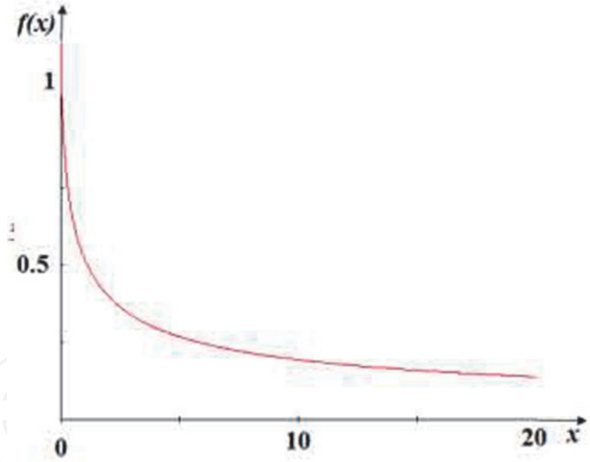

Figure 2.
The function $f(x)$ presents the dependence of the growth rate of the instability, caused by NEW excitation on the level of dissipation. Here $x = \nu/\delta_{NEW}^{\nu=0}$.

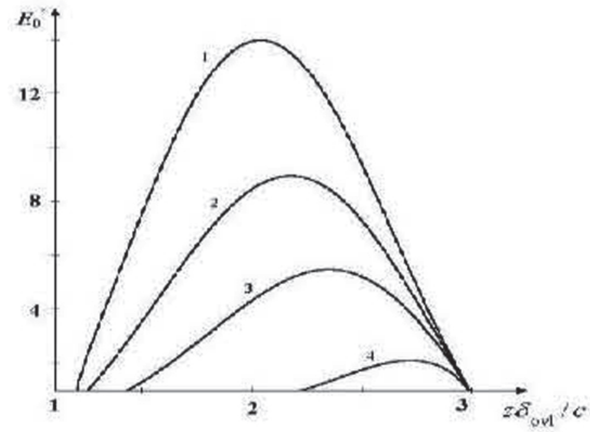

Figure 3.
Shapes of developing waveform versus longitudinal coordinate at fixed instant $3/\delta_{NEW}^{(\nu=0)}$ for various values of dissipation (parameter $k = \nu/\delta_{NEW}^{(\nu=0)}$): $k_1 = 0$, $k_2 = 1$, $k_3 = 2$, $k_4 = 4$.

beam's NEW interaction with the plasma transforms to a new type of DSI. Its characteristic peculiarity is in new, previously unknown, ***inverse proportional*** dependence of the growth rate on dissipation. Below this result is confirmed by conventional electro-dynamical analysis of the DR for weakly coupled beam-plasma system.

3.4 Substantiation of the new DSI by conventional analysis of the DR

From electro-dynamical point of view, a spatially separated beam-plasma system is nothing, but a multilayer structure. The traditional analytical consideration of such systems leads to a very cumbersome DR, which, in addition, is highly dependent on the geometry and greatly complicates with an increase in the number of layers. However, the importance of the problem and the need for its analytical investigation has led to development of specific methods. Here an approach is presented that allows avoiding abovementioned difficulties. Also, the approach has an important advantage: the procedure for obtaining the DR does not depend on specific shape/geometry. In other words, obtained results can be adapted to systems of any geometry. The approach considers the problem of weak beam-plasma interaction by perturbation theory. The small parameter, which underlies the theory, is the parameter of weak beam-plasma coupling. We briefly present here the basics of this approach accounting for dissipation [11].

Consider a system consisting of a mono-energetic rectilinear electron beam and cold plasma. To begin with, suppose the following: the plasma and the beam are weakly coupled (e.g. a consequence of a sufficiently large distance between them). We also assume their homogeneity in the cross section. The geometry of the system is not specified. It also is assumed that the beam current is less than the limiting current in the vacuum waveguide. Dissipation in the system is taken into account by the introduction the collisions in plasma. For simplicity, consideration is limited to the case of a strong external longitudinal (to the beam propagation direction) magnetic field, which prevents the transverse motion of the beam and plasma particles.

The small parameter underlying the perturbation theory is the parameter of weak coupling between the beam and the plasma (that is, the smallness of the integrals describing the overlap of beam and plasma fields). In the zero order approximation, the perturbation theory assumes independence of the beam and plasma. In the first-order approximation, the theory leads to the DR [11, 14].

$$D_p(\omega, k)D_b(\omega, k) = G\left(\kappa^4 \delta\varepsilon_p \delta\varepsilon_b\right)_{\omega=\omega_0, k=k_0} \tag{23}$$

$$D_{p,b}(\omega, k) = k^2_{\perp p,b} - \kappa^2 \delta\varepsilon_{p,b}; G << 1$$

$$\kappa^2 = k^2 - \frac{\omega^2}{c^2}; \delta\varepsilon_p = \frac{\omega_p^2}{\omega(\omega + i\nu)}; \delta\varepsilon_b = \frac{\omega_b^2}{\gamma^3(\omega - kV_b)^2},$$

ω and k are the frequency and longitudinal component of the wave vector, $\omega_{p,b}$ are Langmuir frequencies for the plasma and the beam respectively, ν is the collision frequency in the plasma, V_b is the velocity of the beam electrons, $\gamma = \left(1 - V_b^2/c^2\right)^{-1/2}$, c is speed of light, G is the coupling parameter, the point $\{\omega_0, k_0\}$ is the intersection point of the beam and the plasma dispersion curves, the values $k_{\perp p}$ and $k_{\perp b}$ play role of transverse wave numbers. Analytically, G as well as $k_{\perp p}$ and $k_{\perp b}$ are expressed through the integrals of eigenfunctions of the zero order problem [11, 14]. The integral for G represents overlap of the beam and plasma fields. It shows how far the plasma field penetrates the beam and vice versa. The specific expressions for $k_{\perp p}$, $k_{\perp b}$ and G are not essential for the subsequent presentation and are not presented here (see [11, 14]).

The expressions $D_{p,b}(\omega, k) = 0$ are the zero order DR for the plasma and the beam respectively. Their solutions are assumed to be known. The form of the DR (23) is comparatively simple. It shows the interaction of beam and plasma waves. Using (23) with small G, it is easy to describe instabilities in given system. The main result of a decrease in the beam–plasma coupling is in the increase in role of the beam NEW. Its interaction with plasma leads to instability. The spectra of slow (−) and fast (+) beam waves follow from the roots of $D_b(\omega, k) = 0$. If one searches them in form $\omega_\pm = kV_b(1 + x_\pm)$, $|x_\pm| << 1$, the roots become [11, 14].

$$x_\pm = \pm\left(\sqrt{\alpha}/\gamma\right)\left(\sqrt{\beta^4\gamma^2\alpha + 1} + \beta^2\gamma\sqrt{\alpha}\right), \tag{24}$$

where $\alpha = \omega_b^2/k_{\perp b}^2 V_b^2 \gamma^3$, $\beta = V_b/c$. The interaction of the NEW (x_-) with the plasma leads to instability. If one looks for the solutions of (23) in the form $\omega = kV_b(1 + x)$, ($|x| << 1$) it becomes [11].

$$(x + q + i\nu/kV_b)(x - x_+)(x - x_-) = G\alpha/2\gamma^4 \tag{25}$$

where $q = (2\gamma^2)^{-1}\left(k_{\perp p}^2 V_b^2 \gamma^2/\omega_p^2 - 1\right)$. Mathematically, the instability is due to corrections to the expression for the slow beam wave $x = x_- + x'$. Under collective Cherenkov resonance $q = -x_-$ [11], the equation for $x'$is

$$\left(x' + i\nu/\left(2\gamma^2 kV_b\right)\right)x' = -G\sqrt{\alpha}/\left(4\gamma^3\right) \tag{26}$$

In absence of dissipation the instability is due to NEW interaction with the plasma. Its growth rate is

$$\delta_{\mathrm{NEW}}^{(\nu=0)} = kV_b \mathrm{Im}x' = (kV_b/2\gamma)\sqrt{\left(G\sqrt{\alpha}\right)/\gamma}. \tag{27}$$

We emphasize unusual dependence on the beam density as $n_b^{1/4}$ (for strong coupling this dependence is $\sim n_b^{1/3}$). With ordinary Cherenkov resonance the system is stable. Under collective Cherenkov resonance dissipation manifested itself as an additional factor that enhances NEW growth and the instability gradually transforms to that of dissipative type. The Eq. (26) gives an expression for the growth rate as a function on level of dissipation

$$\delta(\lambda) = \delta_{\mathrm{NEW}}^{(\nu=0)} f\left(\lambda^2\right); \quad \lambda = \left(1/2\gamma^2\right)\left(\nu/\delta_{NEW}^{(\nu=0)}\right). \tag{28}$$

where $f(x)$ is the function given in (21). The dependence of the growth rate on the level of dissipation in (28) coincides to that in (21). In limit $\lambda \to 0$ (28) coincides to (27). In the opposite limit of strong dissipation $\lambda \to \infty$ (28) represents the growth rate of the new type of DSI (it also follows from (26) by neglecting the first term in brackets)

$$\delta_{\mathrm{NEW}}^{(\nu\to\infty)} = \frac{2\gamma^2\left(\delta_{\mathrm{NEW}}^{(\nu=0)}\right)^2}{\nu} = \frac{G\sqrt{\alpha}}{2\gamma}\,\frac{(kV_b)^2}{\nu} \sim \frac{\omega_b}{\nu} \tag{29}$$

We arrive to the same new type of DSI presented in (22). The expression (28) shows a gradual transition of the growth rate of no-dissipative instability caused by NEW interaction with plasma into the growth rate of new type of DSI. It develops under weak coupling and differs from the conventional DSI (with an growth rate $\sim \omega_b/\sqrt{\nu}$). In [21] the same new DSI is substantiated in a finite external magnetic field.

4. Uniform cross section beam-plasma waveguide. One more new type of DSI

4.1 Evolution of the initial perturbation in plasma waveguide with over-limiting electron e-beam

One more new DSI arises under consideration of the problem of the initial perturbation development for the instability of over-limiting beam (OEB) in uniform cross-section plasma waveguide.

Consider a cylindrical waveguide, fully filled with cold plasma. A mono-energetic relativistic electron beam penetrates it. The external longitudinal magnetic field is assumed to be strong enough to freeze transversal motion of the beam and the plasma electrons. We also assume that the beam and plasma radii coincide with the waveguide's radius and consider only the symmetrical E-modes with nonzero components E_r, E_z, and B_φ. The development of resonant instability in this system is described by the DR and resonant condition those are [1].

$$D_0(\omega,k) + D_b(\omega,k) = 0; \omega = kV_b \tag{30}$$

$$D_0 = k_\perp^2 + \kappa^2\left(1 - \frac{\omega_p^2}{\omega(\omega + i\nu)}\right); D_b = -\kappa^2\frac{\omega_b^2/\gamma^3}{(\omega - kV_b)^2}; \kappa^2 = k^2 - \frac{\omega^2}{c^2}$$

ω and k are the frequency and the longitudinal (along beam propagation direction that is z axis) wave vector, $k_\perp = \mu_{0s}/R$, R is the waveguide's radius, μ_{0s} are the roots of Bessel function J_0: $J_0(\mu_{0s}) = 0$, s = 1,2,3 … , $\omega_{p,b}$ are the Langmuir frequencies for the plasma and the beam, V_b is the beam velocity, $\gamma = \left(1 - V_b^2/c^2\right)^{-1/2}$, ν is the frequency of collisions in plasma, c is the speed of light.

The character of the beam-plasma interaction changes depending on the beam current value. If the beam current is less than the limiting current in vacuum waveguide the instability is due to induced radiation of the system eigenwaves by the beam electrons. But, if the beam is over-limiting, its instability has the same nature as the instability in medium with negative dielectric constant [9, 14, 15]. We introduce a parameter $\alpha = \omega_b^2/k_\perp^2 V_b^2\gamma^3$, which represents the beam current value and the character of beam-plasma interaction. It corresponds (correct to the factor γ^{-2}) to the ratio of the beam current to the limiting current in vacuum waveguide [14] $I_0 = mV_b^3\gamma/4e$, i.e. $\alpha = (I_b/I_0)\gamma^{-2}$ (I_b is the beam current). The values $\alpha < <\gamma^{-2}$ correspond to under-limiting beam currents $I_b < <I_0$, but the values $\gamma^{-2} < <\alpha < <1$ correspond to over-limiting beam currents. This is possible under comparatively high values of the relativistic factor γ. Here we consider development of an initial perturbation in the system, when the beam current slightly exceeds the limiting vacuum value. In this case the instability is due to a-periodical modulation of the beam density in medium with negative dielectric constant. Its growth rate attaints maximum under exact Cherenkov resonance and is equal [15].

$$\delta_{\text{ovl}}^{(\nu=0)} = \frac{\omega_b V_b}{c\sqrt{\gamma(1+\mu)}}, \quad \mu = \gamma^2\frac{k_\perp^2 V_b^2}{\omega_p^2 - k_\perp^2 V_b^2\gamma^2} \tag{31}$$

However, the resonant frequency, which is determined by the expressions (30), remains unchanged [15].

In order to show the variety of possible approaches to the solution of the problem of the initial perturbation development, in given case we solve it by other way. We turn to the set of origin equations, which describes e-beam instability in magnetized plasma waveguide

$$\begin{gathered}
\frac{\partial E_r}{\partial z} - \frac{\partial E_z}{\partial r} = -\frac{1}{c}\frac{\partial B_\varphi}{\partial t}; \quad \hat{L}\,\mathrm{v}_\mathrm{b}' = \frac{e}{m}E_z; \quad \frac{\partial \mathrm{v}_\mathrm{p}'}{\partial \mathrm{t}} = \frac{e}{m}E_z - \nu\mathrm{v}_\mathrm{p}' \\
\frac{\partial B_\varphi}{\partial z} = -\frac{1}{c}\frac{\partial E_r}{\partial t}; \quad \hat{L}n_b' = -n_0\frac{\partial \mathrm{v}_\mathrm{b}'}{\partial \mathrm{t}}; \quad \frac{\partial n_p'}{\partial t} = -n_{p0}\frac{\partial \mathrm{v}_\mathrm{p}'}{\partial \mathrm{z}} \\
\frac{1}{r}\frac{\partial}{\partial r}rB_\varphi = \frac{1}{c}\frac{\partial E_r}{\partial t} + 4\pi e\left(n_{p0}\mathrm{v}_\mathrm{p}' + n_{b0}\mathrm{v}_\mathrm{b}' + n_b{}'V_b\right); \quad \hat{L} \equiv \frac{\partial}{\partial t} + V_b\frac{\partial}{\partial z};
\end{gathered} \tag{32}$$

where t is time, z and r are the cylindrical coordinates, E_r, E_z and B_φ are the fields' components which are coupled with the beam, $\mathrm{v}'_{b,p}$ and $n'_{b,p}$ are the perturbations of velocity and density for the beam and the plasma respectively, n_0 and n_{p0} are the unperturbed densities for beam and plasma respectively. In the process, we interest only the longitudinal structure of the fields, i.e., their dependence on the longitudinal coordinate and time. The transverse structure of the fields can be obtained by expansion on series of the system's eigenfunctions. For given case those are the Bessel functions. We use the expansions

$$E_z(r,z,t) = \sum_s E_z^{(s)}(z,t) J_0(\mu_{0s} r/R), B_\phi(r,z,t) = \sum_s B_\phi^{(s)}(r,t) J_1(\mu_{1s} r/R) \qquad (33)$$

where J_0 and J_1 are the Bessel functions; μ_{os} and μ_{1s} their roots in ascending order, $J_0(\mu_{0s}) = 0, J_1(\mu_{1s}) = 0, s = 1, 2, 3, \ldots$. The quantities $v_{p,b}$ and $n_{p,b}$ should be expanded by analogy to E_z, but E_r – by analogy to B_φ. From here on we deal with the expansion coefficients and mention arguments z and t only.

The fields' growth in the linear stage reveals itself most effectively on frequencies, closely approximating to roots of the DR and, simultaneously, to kV_b (resonant instability). The conditions (30) hold. In this connection it is reasonable to assume that originated perturbations form a wave packet of following type (e.g. for $E_z^{(s)}(z,t)$):

$$E_z^{(s)}(z,t) = E_0(z,t) \exp(-i\omega_0 t + ik_0 z), \qquad (34)$$

where the carrier frequency ω_0 and wave vector k_0 satisfy the conditions (30). We also assume that the amplitude of the wave train $E_0(z,t)$ varies slowly in space and time as compared to k_0 and ω_0 that is

$$\left|\frac{\partial E_0}{\partial t}\right| < < |\omega_0 E_0| \ ; \quad \left|\frac{\partial E_0}{\partial z}\right| < < |k_0 E_0|. \qquad (35)$$

Thus, the problem of the initial pulse behavior reduces to determination of the slowly varying amplitude (SVA) $E_0(z,t)$. The equation that $E_0(z,t)$ satisfies can be derived from the set of origin Eqs. (32). The expansions (33) reduce it to a set of the equation for the amplitudes of expansions. In its turn the resulting set can be reduced to one equation for $E_0(z,t)$. We write it in form similar to the DR

$$\left(\hat{\omega} - \hat{k}V_b\right)^2 D_0\left(\hat{\omega}, \hat{k}\right) E_z^{(s)}(z,t) = \omega_b^2 \gamma^{-3} \kappa^2 E_z^{(s)}(z,t) \qquad (36)$$

where $\hat{\omega}$ and $\hat{k}$ are differential operators $\hat{\omega} \equiv i\frac{\partial}{\partial t}$; $\hat{k} \equiv -i\frac{\partial}{\partial z}$. The DR in form (30) follows from (36). To derive the equation for $E_0(z,t)$ one should expanding (36) in power series near resonant values of frequency ω_0 and wave vector k_0 by using the relations $\hat{\omega} \to \omega_0 + i\frac{\partial}{\partial t}$ and $\hat{k} \to k_0 - i\frac{\partial}{\partial z}$ with account of OEB existence condition. As a result we arrive to the following second-order partial differential equation for $E_0(z,t)$

$$\left(\frac{\partial}{\partial t} + V_b \frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial t} + V_\mathrm{p}\frac{\partial}{\partial z} + \nu'\right) E_0(z,t) = \delta_{\mathrm{ovl}}^2 E_0(z,t) \qquad (37)$$

where $\nu' = \mathrm{Im} D_0 (\partial D_0/\partial\omega)^{-1}$, $V_0 = -\left\{(\partial D_0/\partial k)(\partial D_0/\partial\omega)^{-1}\right\}_{\substack{\omega = \omega_0 \\ k = k_0}}$ and the expression for δ_{ovl} is obtained from the relation $\delta_{\mathrm{ovl}}^3 = \kappa^2 \omega_b^2 \gamma^{-3} (\partial D_0/\partial\omega)^{-1}$ accounting the condition for OEB. It is important to emphasize that this denotation (as well as V_0) is introduced for reasons of simplicity of the resulting Eq. (36) only.

The solution of (37) is, actually, known. If one returns to the set (6) and transforms it (under $J(z,t) = 0$) to one equation for $E_w(z,t)$ then the equation will completely coincide to (37). This means that we already have the solution of (36) and its analysis. It only remains to rewrite the solution (12) in new denotations and, where needed, re-interpret results. This shows that the instability in uniform cross-section beam-plasma waveguide develops in space and time in the same manner as the instability in weakly coupled beam-plasma system, and δ_{ovl} is its growth rate in limit $\nu \to 0$, that is $\delta_{\text{ovl}} \equiv \delta_{\text{ovl}}^{(\nu=0)}$. However there is a very important quantitative difference. In present case the growth rate $\delta_{\text{ovl}}^{(\nu=0)}$ depends on the beam density as $\sim n_b^{1/2}$ (for the case of weak beam-plasma coupling the dependence is $\sim n_b^{1/4}$ (see (27)). The criterion for determining the type of DSI takes the form

$$\delta_{\text{ovl}}^{(\nu\to\infty)} = \left[\delta_{\text{ovl}}^{(\nu=0)}\right]^2 / \nu' \sim \omega_b^2 / \nu' \tag{38}$$

Comparison of (38) with (22) indicates one more new type of DSI. It develops in uniform cross section beam-plasma waveguide under over-limiting beam current and high level of dissipation. Its growth rate depends on the beam density and collision frequency as $\sim n_b/\nu'$.

4.2 Substantiation of the second new DSI by conventional method

Now we substantiate the second new DSI by solving the DR (30). We look for its roots in the form $\omega = kV_b + \delta$, $\delta < < kV_b$. The DR (30) reduces to [1, 14].

$$x^3 + i\frac{\nu}{\omega_0}\frac{\omega_p^2 \mathrm{v}_0}{V_b\gamma^2\omega_\perp^2}x^2 + \frac{\alpha \mathrm{v}_0 V_b}{\gamma^2 c^2}x = \frac{\alpha}{2\gamma^4}\frac{\mathrm{v}_0}{V_b} \tag{39}$$

where $x = \delta/kV_b$, $\alpha = \omega_b^2/k_\perp^2 V_b^2\gamma^3$, $\beta = V_b/c$, $\omega_\perp^2 = k_\perp^2 V_b^2\gamma^2$, $\mathrm{v}_0 = \mu V_b/(1+\mu)$, is the group velocity of the resonant wave in the system without beam, $\mu = \gamma^2\omega_\perp^2/\omega_0^2$; $\omega_0 = \left(\omega_p^2 - \omega_\perp^2\right)^{1/2}$ is the resonant frequency of the plasma waveguide.

The solutions of (39) depend on the beam current value that is on the value of parameter α. If $\alpha < < \gamma^{-2}$ (under-limiting e-beams) one can obtain the growth rates of conventional instability under $\nu = 0$ (first and right-hand side terms) and in limit $\nu > > \delta_{\text{und}}$ i.e. DSI

$$\delta_{\text{und}} = \frac{\sqrt{3}}{2}\frac{\omega_0}{\gamma}\left(\frac{\omega_b^2}{2\omega_0^2(1+\mu)}\right)^{1/3}; \quad \delta_{\text{und}}^{(\nu)} = \frac{\omega_0^{3/2}}{2\gamma^{3/2}\omega_p}\sqrt{\frac{\omega_b}{\nu}} \tag{40}$$

If the beam current increases and become comparable or higher than the limiting vacuum current i.e. $\gamma^{-2} \le \alpha < < 1$, the physical nature of the instability changes. It becomes due to a-periodical modulation of the beam density in medium with negative dielectric constant. The distinctive peculiarity of this instability is in following: its growth rate attains maximum under exact Cherenkov resonance and is equal to (31) [9, 11, 14, 15]. If, along with the beam current, dissipation also increases the instability turns to DSI of over-limiting beam with growth rate [9].

$$\delta_{\text{ovl}}^{(\nu)} = \frac{\beta^2}{\gamma}\frac{\omega_b^2}{\omega_p^2}\frac{\omega_0^2}{\nu} \sim \frac{\omega_b^2}{\nu}. \tag{41}$$

We emphasize new dependences on ν and on the beam density. This, actually, substantiates one more new type of DSI. It develops in uniform cross section beam-plasma waveguide if the beam current is higher than the limiting vacuum current.

5. The role of the new DSI in no-uniform-cross-section beam-plasma waveguide

5.1 Statement of the problem. Dispersion relation

In this section we pay special attention to systems, the geometry of which is similar to the geometry of plasma microwave sources and possible development of the new types of DSI in such systems. The simplest theoretical model of plasma microwave generators assumes relativistic e-beam propagating along axis of a plasma filled waveguide of radius R. The beam and plasma are assumed to be completely charge and current neutralized. In the waveguide cross-section the plasma and beam are annular, with mean radii r_p and r_b. Their thicknesses Δ_p and Δ_b are much smaller, than the mean radii. Strong external longitudinal magnetic field is assumed to freeze transversal motion of beam and plasma electrons.

For theoretical study of the problem we use an approach [10], which gives result for arbitrary level of beam-plasma coupling. This condition is obligatory for obtaining comprehensive results. The DR, which follows from the approach, has a form, which clearly shows interaction of the beam and plasma waves. The approach proceeds from equation for polarization potential ψ

$$\frac{\partial}{\partial t}\left(\Delta_\perp + \hat{L}\right)\psi = -4\pi\left(J_{bz} + J_{pz}\right), \hat{L} = \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \tag{42}$$

Here $J_{bz}(\mathbf{r}_\perp, z, t) = p_b(\mathbf{r}_\perp) j_{bz}(z,t)$ and $J_{pz}(\mathbf{r}_\perp, z, t) = p_p(\mathbf{r}_\perp) j_{pz}(z,t)$ are perturbations of the longitudinal current densities in the beam and plasma. Functions $p_{b,p}(\mathbf{r}_\perp)$ describe transverse density profiles of the perturbations of the longitudinal currents in the beam and the plasma. For homogeneous beam/plasma $p_{b,p} \equiv 1$ for infinitesimal thin $p_{b,p} \sim \delta\left(r - r_{b,p}\right)$ (δ is Dirac function), $\Delta_\perp$ is the Laplace operator over transverse coordinates, z is the longitudinal coordinate, t is the time, c is the speed of light. The longitudinal electric field expresses as $E_z = \hat{L}\psi$. The equations for j_{bz} andj_{pz} are

$$\left(\frac{\partial}{\partial t} + V_b\frac{\partial}{\partial z}\right)^2 j_{bz} = \frac{\omega_b^2\gamma^{-3}}{4\pi}\frac{\partial}{\partial t}E_z; \left(\frac{\partial}{\partial t} + \nu\right) j_{pz} = \frac{\omega_p^2}{4\pi}E_z, \tag{43}$$

where $\omega_{p,b}$ are the Langmuir frequencies for plasma and beam respectively, ν is the effective collision frequency in plasma, $\gamma = \left(1 - V_b^2/c^2\right)^{-1/2}$, V_b is the velocity of beam electrons.

The DR, which follows from the statement, is still very cumbersome (of integral type). To reduce the DR to a simple algebraic form one should make following expedient for theoretical model assumption: the plasma and the beam are not just thin but infinitesimal thin. In this case the DR becomes

$$D_p(\omega,k)D_b(\omega,k) = G\kappa^4\delta\varepsilon_p\delta\varepsilon_b, \tag{44}$$

where $D_{p,b}(\omega,k) = k_{\perp p,b}^2 - \kappa^2\delta\varepsilon_{p,b}$, $\delta\varepsilon_p = \frac{\omega_p^2}{\omega(\omega+i\nu)}$, $\delta\varepsilon_b = \frac{\omega_b^2}{\gamma^3\left(\omega - kV_b^2\right)^2}$,$\kappa^2 = k^2 - \omega^2/c^2$, k is the wave vector along axis, ω is the frequency, $k_{\perp p}$ and $k_{\perp b}$ play role of the zero order transversal wave numbers for plasma and beam [11, 14].

$$k_{\perp p,b}^2 = \left\{ r_{p,b}\Delta_{p,b} I_l\left(\kappa r_{p,b}\right) \left[\frac{K_l\left(\kappa r_{p,b}\right)}{I_l\left(\kappa r_{p,b}\right)} - \frac{I_l(\kappa R)}{K_l(\kappa R)} \right] \right\}^{-1} \tag{45}$$

(I_l and K_l are modified Bessel and Mac-Donald functions, $l = 0,\ 1,\ 2\ldots$ is the azimuthal wave numbers). G is the coupling parameter. It depends on the overlap of the plasma and the beam fields and shows efficiency of their interaction

$$G = \begin{cases} \dfrac{I_l(\kappa r_b) K_l\left(\kappa r_p\right) I_l(\kappa R) - K_l(\kappa R) I_l\left(\kappa r_p\right)}{I_l\left(\kappa r_p\right) K_l(\kappa r_b) I_l(\kappa R) - K_l(\kappa R) I_l(\kappa r_b)} & r_b \le r_p \\ \dfrac{I_l\left(\kappa r_p\right) K_l(\kappa r_b) I_l(\kappa R) - K_l(\kappa R) I_l(\kappa r_b)}{I_l(\kappa r_b) K_l\left(\kappa r_p\right) I_l(\kappa R) - K_l(\kappa R) I_l\left(\kappa r_p\right)} & r_p \le r_b \end{cases} \tag{46}$$

An important property of G is: G =1 for $r_p = r_b$ and $G < 1$ in other cases. In long wavelength limit (for definiteness $l = 0$ and $r_b \le r_p$) we have $G \approx \ln\left(R/r_p\right)/\ln\left(R/r_b\right)$, but in opposite limit $G \approx \exp\left(-2\kappa\left|r_p - r_b\right|\right)$ (for arbitrary l).

5.2 Growth rates

The DR (44) determines proper oscillations of transversally no uniform beam-plasma waveguide. The changes of the physical character of beam-plasma interaction must reveal themselves on its solutions. $D_{p,b}(\omega, k) = 0$ are the DR for waveguide with thin annular plasma and e-beam respectively. The spectra of fast (+) and slow $(-)$ waves are

$$\omega_\pm = kV_b(1 + x_\pm); x_\pm = \left(\sqrt{\alpha}/\gamma\right)\left(\pm\sqrt{\beta^4\gamma^2\alpha + 1} - \beta^2\gamma\sqrt{\alpha}\right); \tag{47}$$

where $\beta = V_b/c$. The parameter $\alpha = \omega_b^2/k_{\perp b}^2 V_b^2 \gamma^3$ is familiar (see above). It determines the beam current value: $\alpha = I_b/(\gamma^2 I_0)$ (I_b is the beam current, I_0 is the limiting current in vacuum waveguide). In the limit of under-limiting beams $x_\pm \to \pm\sqrt{\alpha}/\gamma$. In opposite limit of over-limiting beam $x_+ = 1/2\beta^2\gamma^2$ and $x_- = -2\beta^2\alpha$. If one looks for solutions of (44) in form $\omega = kV_b(1 + x)$, $x < < 1$ it becomes

$$\left(x + q + i\frac{\nu}{ku}\frac{1 - 2\beta^2\gamma^2 x}{2\gamma^2}\right)(x - x_+)(x - x_-) = G\frac{\alpha}{2\gamma^4}\left(1 - 2\beta^2\gamma^2 x\right)^2, \tag{48}$$

where $q = \left(k_{\perp p}^2 u^2\gamma^2/\omega_p^2 - 1\right)/2\gamma^2$. The Eq. (48) presents sound way to study instabilities in given system. First of all, it is easily seen that in conditions of growing negative energy wave $x \approx x_-$ and collective Cherenkov resonance $q \approx -x_-$ the role of dissipation increases. For under-limiting e-beams $\alpha \le 1/\gamma^2$ and in case of strong coupling $G \sim 1$ the DR (44) leads to the well-known conventional beam instabilities of no-dissipative and dissipative type. The growth rates of these instabilities have well-known dependencies on beam density $\sim n_b^{1/3}$ and on dissipation ($\sim 1/\sqrt{\nu}$). Both for these instabilities proper oscillations of the beam are neglected. Only for explanation of the physical meaning of the DSI the conception of NEW should be invoked. However, if $G < < 1$ (weak coupling) the growing of the NEW plays dominant role. In this case the growth rate of no-dissipative instability reaches its maximum under Collective Cherenkov resonance $q = \sqrt{\alpha}/\gamma$ and is equal

$$(\mathrm{Im}\omega)_{\mathrm{und}}^{(\nu=0)} = (kV_b/2\gamma)\left(G\sqrt{\alpha}/\gamma\right)^{½}. \tag{49}$$

This expression coincides to (27). Dissipation coming into interplay transforms this instability to DSI of new type with growth rate (coincides to (29))

$$(\mathrm{Im}\omega)_{\mathrm{und}}^{(\nu\to\infty)} = G\sqrt{\alpha}(kV_b)^2/2\gamma\nu \tag{50}$$

As it should be, this is the instability discovered under consideration of the classical problem of the initial perturbation development in weakly coupled beam-plasma systems.

Of particular interest are limit of high, over-limiting currents of e-beam $\gamma^{-2} << \alpha << 1$. In this case the DR (44) takes the form

$$\left(x + q + i\frac{\nu}{ku}\,\frac{1-2\gamma^2 x}{2\gamma^2}\right)(x + 2\alpha) = -G\frac{\alpha}{\gamma^2}\left(1 - 2\gamma^2 x\right) \tag{51}$$

For $\nu = 0$ the analysis of (51) leads to following. Under single particle resonance we have either instability of negative mass type (under $G \sim 1$) with the growth rate $\mathrm{Im}\omega = ku\sqrt{\alpha}/\gamma$, or stability (under $G << 1$). But under collective Cherenkov effect $q = 2\alpha$ the growth rate of developing instabilities is

$$(\mathrm{Im}\omega)_{\mathrm{ovl}}^{(\nu=0)} = \begin{cases} \sqrt{3}kV_b\alpha & \text{for} \quad G \sim 1 \\ 2kV_b\alpha\sqrt{G} & \text{for} \quad G << 1 \end{cases} \tag{52}$$

The instability (52) under $G \sim 1$ has mixed mechanism: it is caused simultaneously (i) by a-periodical modulation of the beam density in media with negative dielectric constant and (ii) by excitation of the NEW. But the lower expression is the growth rate of instability caused only by excitation of the NEW of overlimiting e-beam. The presence of dissipation intensifies the growing of the slow beam wave. Instability turns to be of dissipative type with growth rate that again is inverse proportional to dissipation.

$$(\mathrm{Im}\omega)_{\mathrm{ovl}}^{(\nu)} = 2(ku)^2 G\alpha/\nu \sim \omega_b^2/\nu \tag{53}$$

However, the dependence on the beam density is completely different. This is the same DSI, which develops in uniform cross-section beam-plasma waveguide under over-limiting currents. Instabilities of the same type may be substantiated for finite thicknesses of the beam and plasma layers in waveguide. In this case one must use perturbation theory based on smallness of coupling coefficient.

As follows from this section, in the geometry of microwave plasma sources, the development of both new DSI is possible. Basic parameters of the both new DSI, and the conditions of their development should be taken into account upon design of the high power, high frequency plasma microwave devices.

6. Conclusion

Thus, based on very general initial assumptions, we have found out that the number of DSI in the beam-plasma interaction theory is not limited by the only previously known type. Two new, previously unknown types of DSI are presented. The new DSI reveal themselves in the analysis of solution of the problem of initial perturbation development. This problem is classical in the theory of instabilities.

The first new DSI is the dissipative instability under weak beam-plasma coupling. In absence of dissipation the instability in these systems is caused by the interaction of the beam NEW with the plasma. With an increase in the level of dissipation this instability gradually transforms to the new type of DSI. Its maximal growth rate depends on the beam Langmuir frequency ω_b and the frequency of collisions in plasma ν as ω_b/ν. This, more critical (as compared to conventional), inverse proportional dependence on ν is a result of superposition of two factors those lead to growth of the beam NEW: weak coupling and dissipation.

The second new type of DSI is dissipative instability of over-limiting e-beam in uniform cross section waveguide. With increase in the beam current, its space charge and inner degrees of freedom reveal themselves more efficiently. If the beam current becomes higher than the limiting current in vacuum waveguide then the instability mechanism changes. In uniform cross section beam-plasma waveguide the instability becomes due to a-periodical modulation of the beam density in medium with negative dielectric constant. In this case the increase in the level of dissipation leads to one more new type of DSI with the maximal growth rate $\sim \omega_b^2/\nu$.

The same types of DSI develop in systems having geometry, similar to microwave sources: cylindrical waveguide with thin annular beam and thin annular plasma. If the coupling between the beam and the plasma hollow cylinders is weak and the beam current is under-limiting the first type of DSI develops, but under over-limiting currents – the second. However, if the coupling of the beam and the plasma cylinders is strong, conventional type of DSI develops with well-known growth rate $\sim \omega_b/\sqrt{\nu}$.

Both new DSI are confirmed by conventional analysis of the respective DR.

Some words about the approach used. It has many advantages. First of all, it is based on very general initial assumptions and gives results regardless on geometry and specific parameters. The same approach is used for solving the same problem for conventional beam-plasma instabilities of all types (Cherenkov type, cyclotron type etc) [13], for the Buneman instability [22] etc. Obtained expressions for the spatial–temporal distribution of growing fields clearly show that with increase in the level of dissipation in background plasma, all these SI transform into DSI of conventional type. In addition, the analysis of obtained expressions gives much more detailed information on SI than other methods give. Part of the information on SI is not available in any other ways. The coincidence of other information to the results of conventional analysis confirms the validity of the approach (initial assumptions, mathematics etc).

Also, the presented approach shows that the DR describing the SI of given type can serve not only for solving of the initial/boundary problems and obtaining the dispersion curves. This point of view is very simplified. The approach shows that much more additional information is available from the DR. It, in fact, provides results on the initial perturbation development.

Summarizing, one can state that the presented approach can serve as an independent and very effective method for studying of any SI. There is no need to solve the problem again. One should only substitute the parameters of given instability in general expression for the field's space–time distribution. The usage of this approach instead of traditional initial/boundary problems gives complete picture of the instability development. At first glance, it might seem that this method of analyzing instabilities is more complicated. However, this complexity is only apparent. In addition, this complexity, if any, is overlapped by the completeness of the information received.

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