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## Acceleration Analysis of 3-RPS Parallel Manipulators by Means of Screw Theory

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### 1. Introduction

According to the notation proposed by the International Federation for the Theory of Mechanisms and Machines IFToMM (Ionescu, 2003); a parallel manipulator is a mechanism where the motion of the end-effector, namely the moving or movable platform, is controlled by means of at least two kinematic chains. If each kinematic chain, also known popularly as limb or leg, has a single active joint, then the mechanism is called a fully-parallel mechanism, in which clearly the nominal degree of freedom equates the number of limbs. Tire-testing machines (Gough & Whitehall, 1962) and flight simulators (Stewart, 1965), appear to be the first transcendental applications of these complex mechanisms. Parallel manipulators, and in general mechanisms with parallel kinematic architectures, due to benefits --over their serial counterparts-- such as higher stiffness and accuracy, have found interesting applications such as walking machines, pointing devices, multi-axis machine tools, micro manipulators, and so on. The pioneering contributions of Gough and Stewart, mainly the theoretical paper of Stewart (1965), influenced strongly the development of parallel manipulators giving birth to an intensive research field. In that way, recently several parallel mechanisms for industrial purposes have been constructed using the, now, classical hexapod as a base mechanism: Octahedral Hexapod HOH-600 (Ingersoll), HEXAPODE CMW 300 (CMW), Cosmo Center PM-600 (Okuma), F-200i (FANUC) and so on. On the other hand one cannot ignore that this kind of parallel kinematic structures have a limited and complex-shaped workspace. Furthermore, their rotation and position capabilities are highly coupled and therefore the control and calibration of them are rather complicated.

It is well known that many industrial applications do not require the six degrees of freedom of a parallel manipulator. Thus in order to simplify the kinematics, mechanical assembly and control of parallel manipulators, an interesting trend is the development of the so called defective parallel manipulators, in other words, spatial parallel manipulators with fewer than six degrees of freedom. Special mention deserves the Delta robot, invented by Clavel (1991); which proved that parallel robotic manipulators are an excellent option for industrial applications where the accuracy and stiffness are fundamental characteristics. Consider for instance that the Adept Quattro robot, an application of the Delta robot, developed by Francois Pierrot in collaboration with Fatronik (Int. patent appl. WO/2006/087399), has a

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2.0 kilograms payload capacity and can execute 4 cycles per second. The Adept Quattro robot is considered at this moment the industry's fastest pick-and-place robot.

Defective parallel manipulators can be classified in two main groups: Purely translational (Romdhane et al, 2002; Parenti-Castelli et al, 2000; Carricato & Parenti-Castelli, 2003; Di Gregorio & Parenti-Castelli, 2002; Ji & Wu, 2003; Kong & Gosselin, 2004a; Kong & Gosselin, 2002) or purely spherical (Alizade et al, 1994; Di Gregorio, 2002; Gosselin & Angeles, 1989; Kong & Gosselin, 2004b; Liu & Gao 2000). A third class is composed by parallel manipulators in which the moving platform can undergo mixed motions (Parenti-Castelli & Innocenti, 1992; Gallardo-Alvarado et al, 2006; Gallardo-Alvarado et al, 2007). The 3-RPS, Revolute + Prismatic +Spherical, parallel manipulator belongs to the last class and is perhaps the most studied type of defective parallel manipulator.

The 3-RPS parallel manipulator was introduced by Hunt (1983) and has been the motive of an exhaustive research field where a great number of contributions, approaching a wide range of topics, kinematic and dynamic analyses, synthesis, singularity analysis, extensions to hyper-redundant manipulators, etc; have been reported in the literature, see for instance Lee & Shah (1987), Kim & Tsai (2003), Liu & Cheng (2004), Lu & Leinonen (2005). In particular, screw theory has been proved to be an efficient mathematical resource for determining the kinematic characteristics of 3-RPS parallel manipulators, see for instance Fang & Huang (1997), Huang and his co-workers (1996, 2000, 2001, 2002); including the instantaneous motion analysis of the mechanism at the level of velocity analysis (Agrawal, 1991).

This paper addresses the kinematics of 3-RPS parallel manipulators, including position, velocity and acceleration analyses. Firstly the forward position analysis is carried out in analytic form solution using the Sylvester dialytic elimination method. Secondly the velocity and acceleration analyses are approached by means of the theory of screws. To this end, the velocity and reduced acceleration states of the moving platform, with respect to the fixed platform, are written in screw form through each one of the limbs of the mechanism. Finally, the systematic application of the Klein form to these expressions allows obtaining simple and compact expressions for computing the velocity and acceleration analyses. A case study is included.

## 2. Description of the mechanism

A 3-RPS parallel manipulator, see Fig. 1, is a mechanism where the moving platform is connected to the fixed platform by means of three extendible limbs. Each limb is composed by a lower body and an upper body connected each other by means of an active prismatic joint. The moving platform is connected at the upper bodies via three distinct spherical joints while the lower bodies are connected to the fixed platform by means of three distinct revolute joints.

An effective general formula for determining the degrees of freedom of closed chains still in our days is an open problem. An exhaustive review of formulae addressing this topic is reported in Gogu (2005). Regarding to the existing methods of computation, these formulae are valid under specific conducted considerations. For the parallel manipulator at hand, the mobility is determined using the well-known Kutzbach-Grübler formula

$$F = 6(n - j - 1) + \sum_{i=1}^j f_i \quad (1)$$

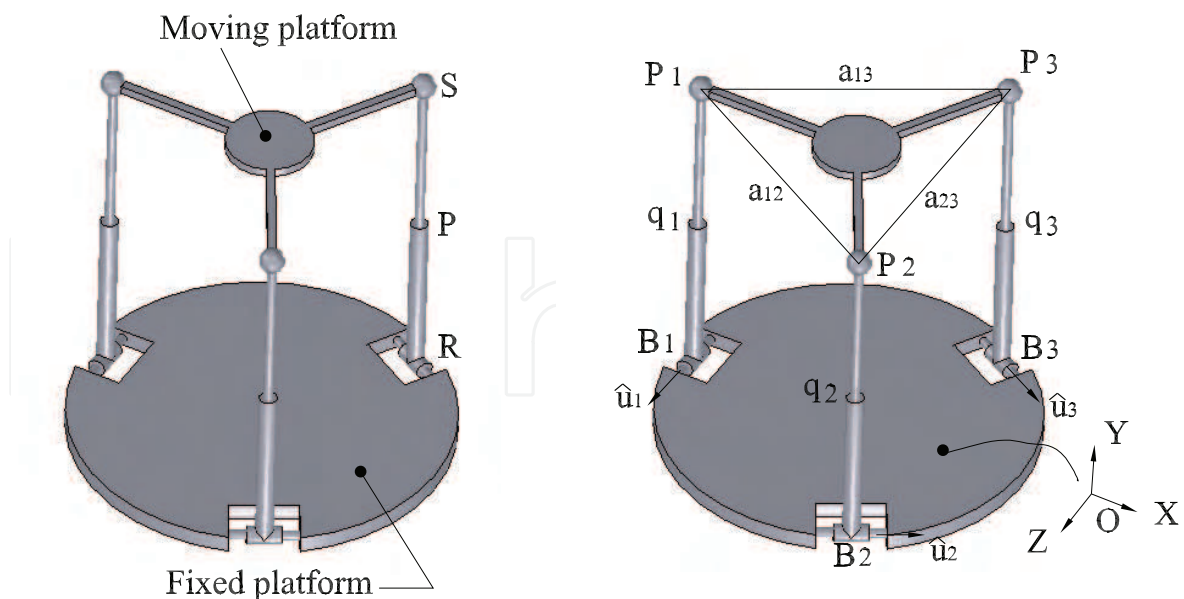


Fig. 1. The 3-RPS parallel manipulator and its geometric scheme.

Where  $n$  is the number of links,  $j$  is the number of kinematic pairs and  $f_i$  is the number of freedoms of the  $i$ -th pair. Thus, taking into account that for the mechanism at hand  $n=8$ ,  $j=9$  and  $\sum_{i=1}^j f_i = 15$ ; then the degrees of freedom of it are equal to 3, an expected result.

## 2. Position analysis

In this section the forward finite kinematics of the 3-RPS parallel manipulator is approached using analytic procedures. The inverse position analysis is considered here a trivial task and therefore it is omitted.

The geometric scheme of the spatial mechanism is shown in the right side of Fig. 1. Accordingly with this figure;  $B_i$ ,  $q_i$  and  $P_i$  denotes, respectively, the nominal position of the revolute joint, the length of the limb and the center of the spherical joint in the same limb. While  $u_i$  denotes the direction of the axis associated to the revolute joint. On the other hand  $a_{mn}$  represents the distance between the centers of two spherical joints.

In this work, the forward position analysis of the 3-RPS parallel manipulator consists of finding the pose, position and orientation, of the moving platform with respect to the fixed platform given the three limb lengths or generalized coordinates  $q_i$  of the parallel manipulator. To this end, it is necessary to compute the coordinates of the three spherical joints expressed in the reference frame XYZ.

When the limbs of the parallel manipulator are locked, the mechanism becomes a 3-RS structure. In order to simplify the analysis, the reference frame XYZ, attached at the fixed platform, is chosen in such a way that the points  $B_i$  lie on the XZ plane. Under this consideration the axes of the revolute joints are coplanar and three constraints are imposed by these joints as follows

$$(P_i - B_i) \bullet u_i = 0 \quad i \in \{1,2,3\} \quad (2)$$

where the dot denotes the usual inner product operation of the three dimensional vectorial algebra. It is worth to mention that expressions (2) were not considered, in the form derived, by Tsai (1999), and therefore the analysis reported in that contribution requires a particular arrangement of the positions of the revolute joints over the fixed platform accordingly to the reference frame XYZ. Furthermore, clearly expressions (2) are applicable not only to tangential 3-RPS parallel manipulators, like the mechanism of Fig. 1, but also to the so-called concurrent 3-RPS parallel manipulators.

On the other hand, clearly the limb lengths are restricted to

$$(P_i - B_i) \bullet (P_i - B_i) = q_i^2 \quad i \in \{1,2,3\} \quad (3)$$

Finally, three compatibility constraints can be obtained as follows

$$\begin{aligned} (P_2 - P_3) \bullet (P_2 - P_3) &= a_{23}^2 \\ (P_1 - P_3) \bullet (P_1 - P_3) &= a_{13}^2 \\ (P_1 - P_2) \bullet (P_1 - P_2) &= a_{12}^2 \end{aligned} \quad (4)$$

Expressions (2)-(4) form a system of nine equations in the nine unknowns  $\{X_1, Y_1, Z_1, X_2, Y_2, Z_2, X_3, Y_3, Z_3\}$ . In what follows, expressions (2-4) are reduced systematically into a highly non linear system of three equations in three unknowns. Afterwards, a sixteenth-order polynomial in one unknown is derived using the Sylvester dialytic elimination method.

It follows from Eqs. (2) that

$$X_i = f(Z_i) \quad i \in \{1,2,3\} \quad (5)$$

On the other hand with the substitution of (5) into expressions (3), the reduction of terms leads to

$$Y_i^2 = p_i \quad i \in \{1,2,3\} \quad (6)$$

where  $p_i$  are second-degree polynomials in  $Z_i$ . Finally, the substitution of Eqs. (6) into Eqs. (4) results in the following highly non-linear system of three equations in the three unknowns  $Z_1, Z_2$  and  $Z_3$

$$\begin{aligned} c_1 Z_2^2 + c_2 Z_3^2 + c_3 Z_2^2 Z_3 + c_4 Z_2 Z_3^2 + c_5 Z_2 Z_3 + c_6 Z_2 + c_7 Z_3 + c_8 &= 0 \\ d_1 Z_1^2 + d_2 Z_3^2 + d_3 Z_1^2 Z_3 + d_4 Z_1 Z_3^2 + d_5 Z_1 Z_3 + d_6 Z_1 + d_7 Z_3 + d_8 &= 0 \\ e_1 Z_1^2 + e_2 Z_2^2 + e_3 Z_1^2 Z_2 + e_4 Z_1 Z_2^2 + e_5 Z_1 Z_2 + e_6 Z_1 + e_7 Z_2 + e_8 &= 0 \end{aligned} \quad (7)$$

therein  $c, d$  and  $e$  are coefficients that are calculated accordingly to the parameters and generalized coordinates, namely the length limbs of the parallel manipulator.

Expressions (7) are similar to those introduced in Tsai (1999); however their derivation is simpler due to the inclusion, in this contribution, of Eqs. (2).

Please note that only two of the unknowns are present in each one of Eqs. (7) and therefore their solutions appear to be an easy task. For example,  $Z_2$  and  $Z_3$  can be obtained as functions of  $Z_1$  from the last two quadratic equations; afterwards the substitution of these variables into the first quadratic yields a highly non-linear equation in  $Z_1$ . The handling of such an expression is a formidable and impractical task. Thus, an appropriated strategy is required for solving the system of equations at hand. Some options are

- A numerical technique such as the Newton-Raphson method. It is an effective option, however only one and imperfect solution can be computed, and there are not guarantee that all the solutions will be calculated.
- Using computer algebra like Maple®. An absolutely viable option that guarantee the computation of all the possible solutions.
- The application of the Sylvester dialytic elimination method. An elegant option that allows to compute all the possible solutions.

In this contribution the last option was selected and in what follows the results will be presented.

With the purpose to eliminate  $Z_3$ , the first two quadratics of (7) are rewritten as follows

$$\begin{aligned} p_1 Z_3^2 + p_2 Z_3 + p_3 &= 0 \\ p_4 Z_3^2 + p_5 Z_3 + p_6 &= 0 \end{aligned} \quad (8)$$

where  $p_1$ ,  $p_2$  and  $p_3$  are second-degree polynomials in  $Z_2$  while  $p_4$ ,  $p_5$  and  $p_6$  are second-degree polynomials in  $Z_1$ . After a few operations, the term  $Z_3$  is eliminated from (8). With this action, two linear equations in two unknowns, the variable  $Z_3$  and the scalar 1, are obtained. Casting in matrix form such expressions it follows that

$$M_1 \begin{bmatrix} Z_3 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (9)$$

where

$$M_1 = \begin{bmatrix} p_1 p_5 - p_2 p_4 & p_1 p_6 - p_3 p_4 \\ p_3 p_4 - p_1 p_6 & p_3 p_5 - p_2 p_6 \end{bmatrix}$$

It is evident that expression (9) is valid if, and only if,  $\det(M_1) = 0$ . Thus clearly one can obtain

$$p_7 Z_2^4 + p_8 Z_2^3 + p_9 Z_2^2 + p_{10} Z_2 + p_{11} = 0 \quad (10)$$



where  $p_7, p_8, p_9, p_{10}$  and  $p_{11}$  are fourth-degree polynomials in  $Z_1$ ; and the first step of the Sylvester dialytic elimination method finishes with the computation of this eliminant.

Please note that Eq. (10) and the last quadratic of Eqs. (7) represents a non-linear system of two equations in the unknowns  $Z_1$  and  $Z_2$ , and in what follows it is reduced into an univariate polynomial equation. As an initial step, that last quadratic of (7) is rewritten as

$$p_{12}Z_2^2 + p_{13}Z_2 + p_{14} = 0, \quad (11)$$

where  $p_{12}, p_{13}$  and  $p_{14}$  are second-degree polynomials in  $Z_1$ . It is very tempting to assume that the non-linear system of two equations formed by (10) and (11) can be easily solved obtaining first  $Z_2$  in terms of  $Z_1$  from Eq. (11) and later substituting it into Eq. (10). However, when one realize this apparent evident action with the aid of computer algebra, an excessively long expression is derived, and its handling is a hazardous task. Thus, the application of the Sylvester dialytic elimination method is a more viable option.

In order to avoid extraneous roots, it is strongly advisable the deduction of a minimum of linear equations. For example, the term  $Z_2^4$  is eliminated multiplying Eq. (10) by  $p_{12}$  and Eq. (11) by  $p_7Z_2^2$ . The subtraction of the obtained expressions leads to

$$(p_{13}p_7 - p_{12}p_8)Z_2^3 + (p_{14}p_7 - p_{12}p_9)Z_2^2 - p_{12}p_{10}Z_2 - p_{12}p_{11} = 0. \quad (12)$$

Expressions (11) and (12) can be considered as a linear system of two equations in the four unknowns  $Z_2^3, Z_2^2, Z_2$  and 1. Therefore it is necessary the search of two additional linear equations.

An equation is easily obtained multiplying Eq. (11) by  $Z_2$

$$p_{12}Z_2^3 + p_{13}Z_2^2 + p_{14}Z_2 = 0. \quad (13)$$

The search of the fourth equation is more elusive, for details the reader is referred to Tsai (1999). To this end, multiply Eq. (10) by  $(p_{12}Z_2 + p_{13})$  and Eq. (11) by  $(p_7Z_2^3 + p_8Z_2^2)$ . The subtraction of the resulting expressions leads to

$$\begin{aligned} (p_{12}p_9 - p_7p_{14})Z_2^3 + (p_{12}p_{10} + p_{13}p_9 - p_9p_{14})Z_2^2 \\ + (p_{12}p_{11} + p_{13}p_{10})Z_2 + p_{13}p_{11} = 0 \end{aligned} \quad (14)$$

Casting in matrix form expressions (11)-(14) it follows that

$$M_2 \begin{bmatrix} Z_2^3 \\ Z_2^2 \\ Z_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (15)$$

where

$$M_2 = \begin{bmatrix} 0 & P_{12} & P_{13} & P_{14} \\ P_{13}P_7 - P_{12}P_8 & P_{14}P_7 - P_{12}P_9 & -P_{12}P_{10} & -P_{12}P_{11} \\ P_{12} & P_{13} & P_{14} & 0 \\ P_{12}P_9 - P_7P_{14} & P_{12}P_{10} + P_{13}P_9 - P_8P_{14} & P_{12}P_{11} + P_{13}P_{10} & P_{13}P_{11} \end{bmatrix}$$

Clearly expression (15) is valid if, and only if,  $\det(M_2) = 0$ . Therefore, this eliminant yields a sixteenth-order polynomial in the unknown  $Z_1$ .

It is worth to mention that expressions (10) and (11) have the same structure of those derived by Innocenti & Parenti-Castelli (1990) for solving the forward position analysis of the Stewart platform mechanism. However, this work differs from that contribution in that, while in this contribution the application of the Sylvester Dialytic elimination method finishes with the computation of the determinant of a 4x4 matrix, the contribution of Innocenti & Parenti-Castelli (1990), a more general method than the presented in this section, finishes with the computation of the determinant of a 6x6 matrix.

Once  $Z_1$  is calculated,  $Z_2$  and  $Z_3$  are calculated, respectively, from expressions (11) and the second quadratic of (8) while the remaining components of the coordinates,  $X_i$  and  $Y_i$ , are computed directly from expressions (5) and (6), respectively. It is important to mention that in order to determine the feasible values of the coordinates of the points  $P_i$ , the signs of the corresponding discriminants of  $Z_2$ ,  $Z_3$  and  $Y_i$  must be taken into proper account. Of course, one can ignore this last recommendation if the non-linear system (3) is solved by means of computer algebra like Maple©.

Finally, once the coordinates of the centers of the spherical joints are calculated, the well-known  $4 \times 4$  transformation matrix  $T$  results in

$$T = \begin{bmatrix} R & r_{C/O} \\ 0_{1 \times 3} & 1 \end{bmatrix}, \quad (16)$$

where,  $r_{C/O} = (P_1 + P_2 + P_3)/3$  is the geometric center of the moving platform, and  $R$  is the rotation matrix.

### 3. Velocity analysis

In this section the velocity analysis of the 3-RPS parallel manipulator is carried out using the theory of screws which is isomorphic to the Lie algebra  $e(3)$ . This section applies well known screw theory; for readers unfamiliar with this mathematical resource, some appropriated references are provided at the end of this work (Sugimoto, 1987; Rico and Duffy, 1996; Rico et al, 1999).



The mechanism under study is a spatial mechanism, and therefore the kinematic analysis requires a six-dimensional Lie algebra. In order to satisfy the dimension of the subspace spanned by the screw system generated in each limb, the 3-RPS parallel manipulator can be modelled as a 3-R<sup>\*</sup>RPS parallel manipulator, see Huang and Wang (2000), in which the revolute joints R<sup>\*</sup> are fictitious kinematic pairs. In this contribution, see Fig. 2, each limb is modelled as a Cylindrical + Prismatic + Spherical kinematic chain, CPS for brevity. It is straightforward to demonstrate that this option is simpler than the proposed in Huang and Wang (2000). Naturally, this model requires that the joint rate associated to the translational displacement of the cylindrical joint be equal to zero.

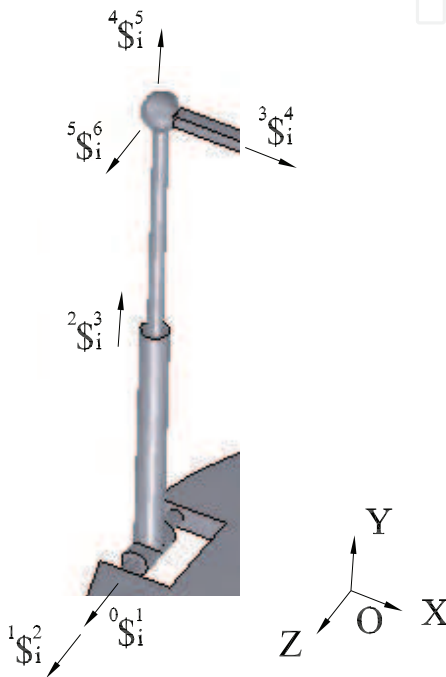


Fig. 2. A limb with its infinitesimal screws

Let  $\omega = (\omega_X, \omega_Y, \omega_Z)$  be the angular velocity of the moving platform, with respect to the fixed platform, and let  $V_O = (V_{OX}, V_{OY}, V_{OZ})$  be the translational velocity of the point  $O$ , see Fig. 2; where both three-dimensional vectors are expressed in the reference frame  $XYZ$ . Then, the velocity state  $V_O = [\omega \ v_O]$ , also known as the twist about a screw, of the moving platform with respect to the fixed platform, can be written, see Sugimoto (1987), through the  $j$ -th limb as follows

$$\sum_{i=0}^5 {}^i \omega_{i+1}^j {}^i \$j^{i+1} = V_O \quad j \in \{1, 2, 3\}, \quad (17)$$

where, the joint rate  ${}_2\omega_3^j = \dot{q}_j$  is the active joint associated to the prismatic joint in the  $j$ -th limb, while  ${}_0\omega_1^j = 0$  is the joint rate of the prismatic joint associated to the cylindrical joint. With these considerations in mind, the inverse and forward velocity analyses of the mechanism under study are easily solved using the theory of screws.

The inverse velocity analysis consists of finding the joint rate velocities of the parallel manipulator, given the velocity state of the moving platform with respect to the fixed platform. Accordingly to expression (17), this analysis is solved by means of the expression

$$\Omega_j = J_j^{-1} V_O. \quad (18)$$

Therein

- $J_j = \begin{bmatrix} {}^0\$_j^1 & {}^1\$_j^2 & {}^2\$_j^3 & {}^3\$_j^4 & {}^4\$_j^5 & {}^5\$_j^6 \end{bmatrix}$  is the Jacobian of the  $j$ -th limb, and
- $\Omega_j = \begin{bmatrix} {}_0\omega_1^j & {}_1\omega_2^j & {}_2\omega_3^j & {}_3\omega_4^j & {}_4\omega_5^j & {}_5\omega_6^j \end{bmatrix}^T$  is the matrix of joint velocity rates of the  $j$ -th limb.

On the other hand, the forward velocity analysis consists of finding the velocity state of the moving platform, with respect to the fixed platform, given the active joint rates  $\dot{q}_j$ . In this analysis the Klein form of the Lie algebra  $e(3)$  plays a central role.

Given two elements  $\$ _1 = [s_1 \ s_{O1}]$  and  $\$ _2 = [s_2 \ s_{O2}]$  of the Lie algebra  $e(3)$ , the Klein form,  $\{*,*\}$ , is defined as follows

$$\{\$ _1, \$ _2\} = s_1 \bullet s_{O2} + s_2 \bullet s_{O1}. \quad (19)$$

Furthermore, it is said that the screws  $\$ _1$  and  $\$ _2$  are reciprocal if  $\{\$ _1, \$ _2\} = 0$ .

Please note that the screw  ${}^4\$ _i^5$  is reciprocal to all the screws associated to the revolute joints in the same limb. Thus, applying the Klein form of the screw  ${}^4\$ _i^5$  to both sides of expression (17), the reduction of terms leads to

$$\{V_O, {}^4\$ _i^5\} = \dot{q}_i \quad i \in \{1,2,3\}. \quad (20)$$

Following this trend, choosing the screw  ${}^5\$ _i^6$  as the *cancellator screw* it follows that

$$\{V_O, {}^5\$ _i^6\} = 0 \quad i \in \{1,2,3\}. \quad (21)$$

Casting in a matrix-vector form expression (20) and (21), the velocity state of the moving platform is calculated from the expression

$$J^T \Delta V_O = \dot{Q}, \quad (22)$$

wherein

- $J = \begin{bmatrix} {}^4\$_1^5 & {}^4\$_2^5 & {}^4\$_3^5 & {}^5\$_1^6 & {}^5\$_2^6 & {}^5\$_3^6 \end{bmatrix}$  is the Jacobian of the parallel manipulator,
- $\Delta = \begin{bmatrix} 0_{3 \times 3} & I_3 \\ I_3 & 0_{3 \times 3} \end{bmatrix}$  is an operator of polarity, and
- $\dot{Q} = [\dot{q}_1 \quad \dot{q}_2 \quad \dot{q}_3 \quad 0 \quad 0 \quad 0]^T$ .

Finally, once the angular velocity of the moving platform and the translational velocity of the point O are obtained, respectively, as the primal part and the dual part of the velocity state  $V_O = [\omega \quad v_O]$ , the translational velocity of the center of the moving platform, vector  $v_C$ , is calculated using classical kinematics. Indeed

$$v_C = v_O + \omega \times r_{C/O}. \quad (23)$$

Naturally, in order to apply Eq. (22) it is imperative that the Jacobian J be invertible. Otherwise, the parallel manipulator is at a singular configuration, with regards to Eq. (18).

#### 4. Acceleration analysis

Following the trend of Section 3, in this section the acceleration analysis of the parallel manipulator is carried out by means of the theory of screws.

Let  $\dot{\omega} = (\dot{\omega}_X, \dot{\omega}_Y, \dot{\omega}_Z)$  be the angular acceleration of the moving platform, with respect to the fixed platform, and let  $a_O = (a_{OX}, a_{OY}, a_{OZ})$  be the translational acceleration of the point O; where both three-dimensional vectors are expressed in the reference frame XYZ. Then the reduced acceleration state  $A_O = [\dot{\omega} \quad a_O - \omega \times v_O]$ , or accelerator for brevity, of the moving platform with respect to the fixed platform can be written, for details see Rico & Duffy (1996), through each one of the limbs as follows

$$\sum_{i=0}^5 {}^i\dot{\omega}_{i+1}^j {}^i\$_j^{i+1} + \$_{Lie-j} = A_O \quad j \in \{1, 2, 3\}, \quad (24)$$

where  $\$_{Lie-j}$  is the *Lie screw* of the j-th limb, which is calculated as follows

$$\$_{Lie-j} = \sum_{k=0}^4 \left[ {}^k\omega_{k+1}^j {}^k\$_j^{k+1} \quad \sum_{r=k+1}^5 {}^r\omega_{r+1}^j {}^r\$_j^{r+1} \right],$$

and the brackets  $[* \quad *]$  denote the Lie product.

Equation (24) is the basis of the inverse and forward acceleration analyses.

The inverse acceleration analysis, or in other words the computation of the joint acceleration rates of the parallel manipulator given the accelerator of the moving platform with respect to the fixed platform, can be calculated, accordingly to expression (24), as follows

$$\dot{\Omega}_j = J_j^{-1} (A_O - \$_{Lie-j}), \quad (25)$$

where  $\dot{\Omega}_j = [\dot{\omega}_1^j \quad \dot{\omega}_2^j \quad \dot{\omega}_3^j \quad \dot{\omega}_4^j \quad \dot{\omega}_5^j \quad \dot{\omega}_6^j]^T$  is the matrix of joint acceleration rates.

On the other hand, the forward acceleration analysis, or in other words the computation of the accelerator of the moving platform with respect to the fixed platform given the active joint rate accelerations  $\ddot{q}_j$  of the parallel manipulator; is carried out, applying the Klein form of the reciprocal screws to Eq. (24), using the expression

$$J^T \Delta A_O = \ddot{Q}, \quad (26)$$

where

$$\ddot{Q} = \begin{bmatrix} \ddot{q}_1 + \left\{ {}^4 \$^5_1, \$_{Lie-1} \right\} \\ \ddot{q}_2 + \left\{ {}^4 \$^5_2, \$_{Lie-2} \right\} \\ \ddot{q}_3 + \left\{ {}^4 \$^5_3, \$_{Lie-3} \right\} \\ \left\{ {}^5 \$^6_1, \$_{Lie-1} \right\} \\ \left\{ {}^5 \$^6_2, \$_{Lie-2} \right\} \\ \left\{ {}^5 \$^6_3, \$_{Lie-3} \right\} \end{bmatrix}$$

Once the accelerator  $A_O = [\dot{\omega} \quad a_O - \omega \times v_O]$  is calculated, the angular acceleration of the moving platform is obtained as the primal part of  $A_O$ , whereas the translational acceleration of the point O is calculated upon the dual part of the accelerator. With these vectors, the translational acceleration of the center of the moving platform, vector  $a_C$ , is computed using classical kinematics. Indeed

$$a_C = a_O + \dot{\omega} \times r_{C/O} + \omega \times (\omega \times r_{C/O}). \quad (27)$$

Finally, it is interesting to mention that Eq. (26) does not require the values of the passive joint acceleration rates of the parallel manipulator.

## 5. Case study. Numerical example

In order to exemplify the proposed methodology of kinematic analysis, in this section a numerical example, using SI units, is solved with the aid of computer codes.

The parameters and generalized coordinates of the example are provided in Table 1.

$$B_1 = (.1246762518, 0, .4842063942)$$

$$B_2 = (.3569969122, 0, -.3500759985)$$

$$B_3 = (-.4816731640, 0, -.1341303959)$$

$$u_1 = (.9684127885, 0, -.2493525036)$$

$$u_2 = (-.7001519970, 0, -.7139938243)$$

$$u_3 = (-.2682607918, 0, .9633463279)$$

$$a_{12} = a_{13} = a_{23} = \sqrt{3} / 2$$

$$q_1 = -0.5\sin^2(t)\cos(t)$$

$$q_2 = 0.35\sin[\sin(t)\cos(t)]$$

$$q_3 = -0.35\sin(t)\cos[\sin(t/2)]$$

$$0 \leq t \leq 2\pi$$

Table 1. Parameters and instantaneous length of each limb of the parallel manipulator

According with the data provided in Table 1, at the time  $t=0$  the sixteenth polynomial in  $Z_1$  results in

$$\begin{aligned} & 490873788e09 + .627748325e10Z_1 + .246379238e11Z_1^2 - .82281001e10Z_1^3 - \\ & .281160758e12Z_1^4 - .444113311e12Z_1^5 + .964036155e12Z_1^6 + .2739680775e13Z_1^7 - \\ & .108993550e13Z_1^8 - .672039554e13Z_1^9 + .3921344e11Z_1^{10} + .786657045e13Z_1^{11} - \\ & .64783709e12Z_1^{12} - .373666459e13Z_1^{13} + .195532604e13Z_1^{14} - .3787349072e12Z_1^{15} + \\ & .261153294e11Z_1^{16} = 0. \end{aligned}$$

The solution of this univariate polynomial equation, in combination with expressions (5) and (6), yields the 16 solutions of the forward position analysis, which are listed in Table 2. Taking solution 3 of Table 2 as the initial configuration of the parallel manipulator, the most representative numerical results obtained for the forward velocity and acceleration analyses are shown in Fig. 3.

Solution	P <sub>1</sub>	P <sub>2</sub>	P <sub>3</sub>
1,2	(-.086,± 0.307, - .335)	(.432,±.994, - .424)	(-.364,±1.093, - .101)
3,4	(.121,± .899, .471)	(.361, ± .999, - .354)	(-.468,±1.099,-.130)
5,6	(.161, ± .888, .625)	(.236,±.985, - .231)	(.544,±.273, .151)
7,8	(-.099,±.054, - .385)	(-.091,±.778, .089)	(.558,±.209, .155)
9,10	(.193,±.857,.749)	(-.321,±.312, .314)	(.528,±.333,.147)
11,12	(.182,±.869,.709)	(-.326,±.287,.320)	(-.185,±1.056,-.051)
13,14	(-.104,±.194i,-.407)	(-.628,±.950i, .615)	(.578,±.004i,.160)
15,16	(-.104,±.195i,-.407)	(-.657,±1.009i,.644)	(.578,±.004i, .160)

Table 2. The sixteen solution of the forward position analysis

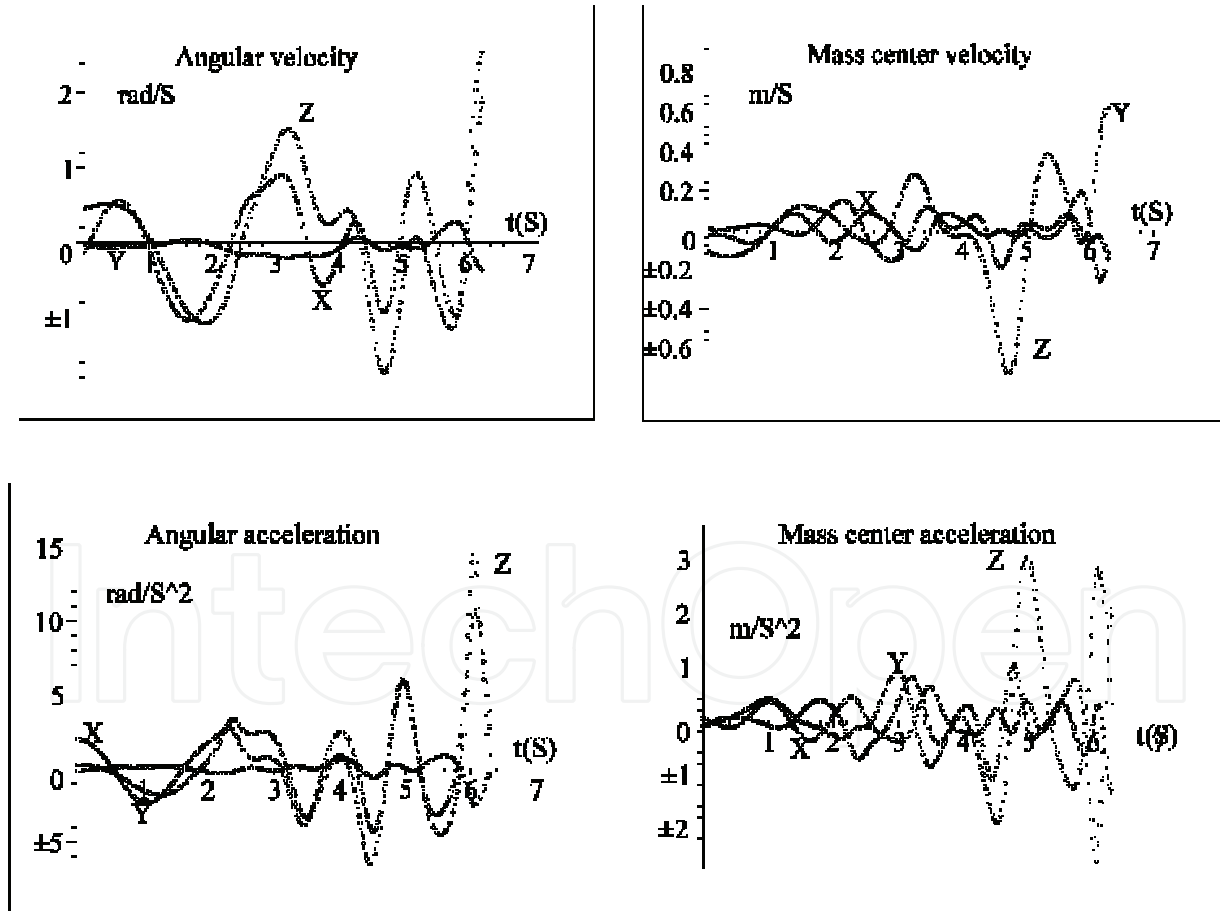


Fig. 3. Forward kinematics of the numerical example using screw theory

Furthermore, the numerical results obtained via screw theory are verified with the help of special software like ADAMS©. A summary of these numerical results is reported in Fig. 4.



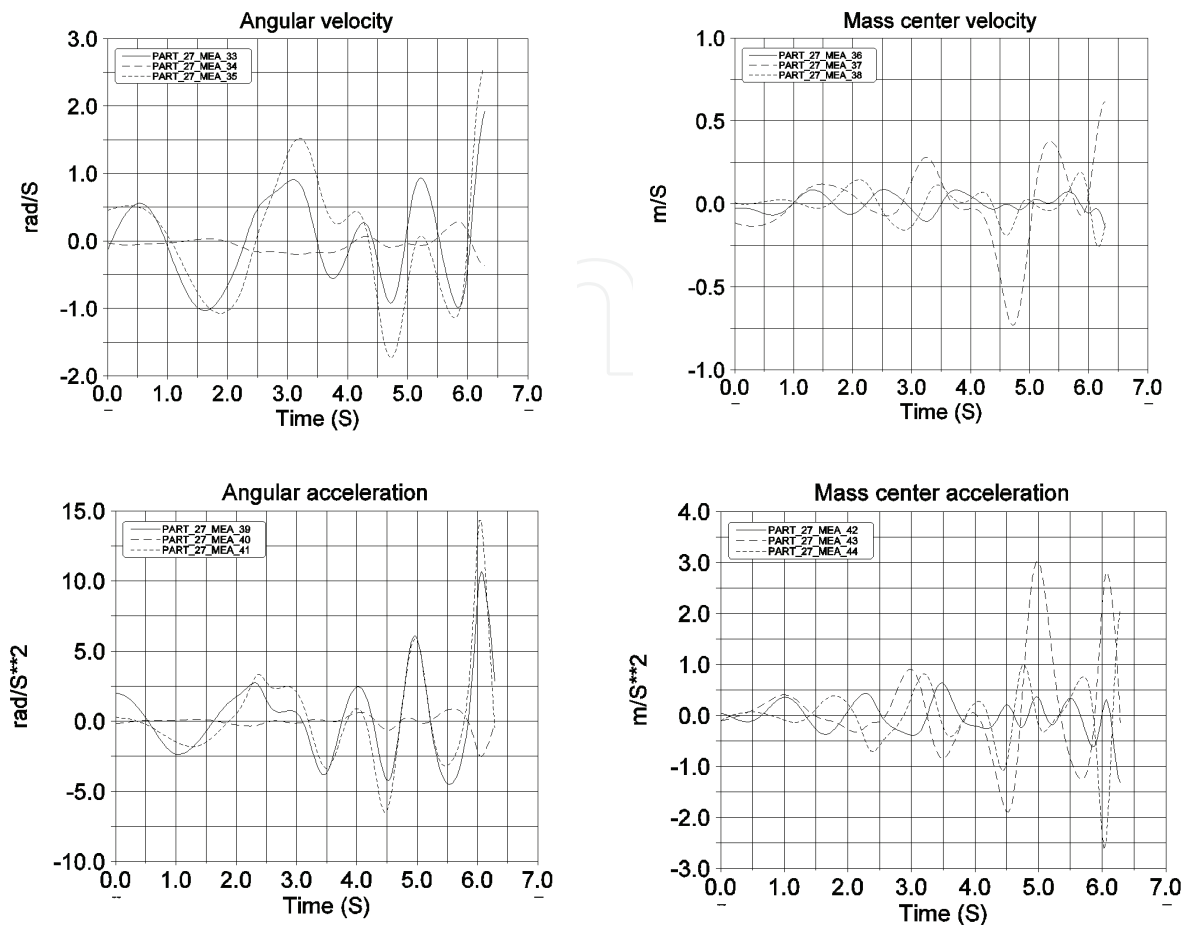


Fig. 4. Forward kinematics of the numerical example using ADAMS©

Finally, please note how the results obtained via the theory of screws are in excellent agreement with those obtained using ADAMS©.

## 6. Conclusions

In this work the kinematics, including the acceleration analysis, of 3-RPS parallel manipulators has been successfully approached by means of screw theory. Firstly, the forward position analysis was carried out using recursively the Sylvester dialytic elimination method, such a procedure yields a 16-th polynomial expression in one unknown, and therefore all the possible solutions of this initial analysis are systematically calculated. Afterwards, the velocity and acceleration analyses are addressed using screw theory. To this end, the velocity and reduced acceleration states of the moving platform, with respect to the fixed platform are written in screw form through each one of the three limbs of the manipulator. Simple and compact expressions were derived in this contribution for solving the forward kinematics of the spatial mechanism by taking advantage of the concept of reciprocal screws via the Klein form of the Lie algebra  $e(3)$ . The obtained expressions are simple, compact and can be easily translated into computer codes. Finally, in order to exemplify the versatility of the chosen methodology, a case study was included in this work.

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## **Parallel Manipulators, New Developments**

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Parallel manipulators are characterized as having closed-loop kinematic chains. Compared to serial manipulators, which have open-ended structure, parallel manipulators have many advantages in terms of accuracy, rigidity and ability to manipulate heavy loads. Therefore, they have been getting many attentions in astronomy to flight simulators and especially in machine-tool industries. The aim of this book is to provide an overview of the state-of-art, to present new ideas, original results and practical experiences in parallel manipulators. This book mainly introduces advanced kinematic and dynamic analysis methods and cutting edge control technologies for parallel manipulators. Even though this book only contains several samples of research activities on parallel manipulators, I believe this book can give an idea to the reader about what has been done in the field recently, and what kind of open problems are in this area.

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