Chapter

Adaptive Sliding Mode Control Vibrations of Structures

Leyla Fali, Khaled Zizouni, Abdelkrim Saidi, Ismail Khalil Bousserhane and Mohamed Djermane

Abstract

The sliding mode controller is one of the interesting classical nonlinear controllers in structural vibration control. From its apparition, in the middle of the twentieth century, this controller was a subject of several studies and investigations. This controller was widely used in the control of various semi-active or active devices in the civil engineering area. Nevertheless, the sliding mode controller offered a low sensitivity to the uncertainties or the system condition variations despite the presence of the Chattering defect. However, the adaptation law is one of the frequently used solutions to overcome this phenomenon offering the possibility to adapt the controller parameters according to the system variations and keeping the stability of the whole system assured. The chapter provides a sliding mode controller design reinforced by an adaptive law to control the desired state of an excited system. The performance of the adaptive controller is proved by numerical simulation results of a three-story excited structure.

Keywords: vibration control, sliding mode, adaptive law, earthquake excitation, Lyapunov stability

1. Introduction

The sliding mode controller is known as a powerful tool to control high-order complex nonlinear systems in presence of parametric uncertainty and external disturbances. The idea was initiated in the Soviet Union early in the 1930s [1, 2] after the Lyapunov stability theory apparition [3]. However, the peculiar evolution point started from the famous Emel'yanov and Barbashin works [4, 5]. Since then, the sliding mode control was a subject of several papers and works and been widely used in the various area as civil engineering [6], aircraft [7], robotic [8], energy and more other areas. After the contribution of the differential equations with discontinuous right-hand side theory established by Filippov in 1960 the sliding mode control received much more attention from researchers for wide dynamic system processes as time-varying, large-scale, infinite-dimensional or stochastic [9].

The sliding mode control decouples the dynamic motion of the whole controlled system into two components that do not depend on each other. In fact, this decomposition offered lower dimension and design simplicity to the system especially in feedback control conception. In addition to advantages presented by the sliding mode control as the insensitivity to parameter variations, complete rejection disturbances and depending on the sliding conditions the control can be combined easily to operational modes, approaches and controllers. Many interesting experimental and theoretical results are presented by combining the sliding mode control as adaptive sliding mode [10], fuzzy sliding mode [11], artificial neural network sliding mode [12] and decentralized sliding mode [13].

2. The concept of the sliding mode

The sliding mode aspect may appear in dynamical systems where the motion is presented using ordinary differential equations with discontinuous right-hand sides. Thus, the concept uses a discontinuous control signal to reform the system motions without depending on the system dynamic but the sliding parameters. This approach reduces the order of the original system equation which simplifies the mathematical modeling of the dynamic system motions. Therefore, the control output switched in high frequency between two values $\pm K$ and be subjected to discontinuities on the sliding surface in the state plane of the system to track the desired system state [14].

The linearization possibility of any mechanical system is linked to the presence of friction in the system. The force-velocity behavior depended essentially on the friction type as dry friction or fluid friction. In such a problem, the critical zone is which presented the maximum displacements. Nevertheless, in this zone the velocity value is in the neighborhood of zero with an opposite sign to the friction force. Consider the mechanical problem presented in **Figure 1** consisting of a Coulomb friction mass-spring system.

The motion equation is presented as

$$m\ddot{x} + c\dot{x} + kx = 0 \tag{1}$$

Where $\ddot{x}$, $\dot{x}$ and x are acceleration, velocity and displacement, m, c and k are the mass, friction coefficient and the spring stiffness.

In this case, the system work depends little on velocity and even if slowly moved the mass, a finite work is done in a displacement. So, even for the small velocity, the friction force existed and was defined with a finite value. Thereby, near to the zero velocity, the friction force switched to the finite limit in the two sides (positive or negative) [15].

$$c\dot{x} = \begin{cases} c_0\dot{x} & with \quad \dot{x} > 0 \\ -c_0\dot{x} & with \quad \dot{x} < 0 \end{cases} \tag{2}$$

c_0 is positive constant.

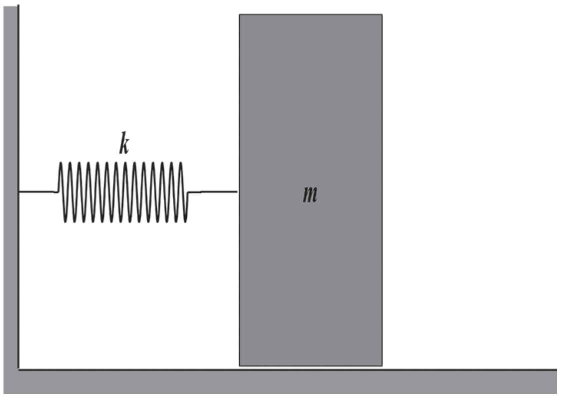

Figure 1.
Coulomb friction system.

Thus, around the state plan origin, the discontinuity is presented and the solution is unknown on the right-hand side of the differential motion equation. This situation is a frequent case in various control systems needing the differential equations with discontinuous right-hand sides theory [9].

The principle of the sliding mode approach consists of forcing the system to reach a fictive surface called the sliding surface to get the equilibrium state and keeping it switching around this surface thereafter. Hence, the first step is called the reaching phase and the second is called the sliding phase. Therefore, the trajectory in the state plane is assured by three distinct modes. The first mode is the convergence mode during which the variable to be adjusted gets the sliding surface from any initial point in the phase plane. This mode depended on the equivalent control law performance. The second mode is the smoothing mode in which the variable state reached the sliding surface and tends towards the origin of the state plane. The dynamic in this mode is characterized by the best choice of the sliding surface. The third one is the permanent regime mode characterizing the system response around the state plane origin depended on control law robustness. So, two steps to be followed are the determination of the fictive sliding surface on which the objectives of the controls are achieved then calculating the control law which ensures the state trajectory surface achievement and maintains it on this surface until reaching the state equilibrium [16].

3. Fundamental theory of the sliding mode control

The sliding mode control (SMC) is a two steps design controller in which the system motion is composed of two phases. The former step is the design of a fictive sliding surface to which the system motion must reach and hold the desired performances on it. However, the latter step is the design of the control law which drives the system motion to the sliding surface and maintains it on thereafter until reaching the equilibrium point. The sliding mode controller is a quake reacting controller. Whereas, the most advantage of this control that is insensitive to uncertainties or disturbances present in the system because the control design forces the system whatever to attain the surface prescriptions.

Let consider the general presentation of a nonlinear dynamic system as

$$\dot{x} = f(x,t) + g(x,t)u \tag{3}$$

where x is the state vector, u is the desired control, $f(x)$ and $g(x)$ are nonzero smooth uncertain functions.

The sliding surface is presented as

$$\sigma = G \cdot e \tag{4}$$

Where G is the sliding surface matrix and e is the tracking error of the system state defined as

$$e = x - x_0 \tag{5}$$

x_0 is the desired state response.

Furthermore, to push the dynamic motion to the sliding surface the following conditions must be satisfied

$$\begin{cases} \sigma(x) = 0 \\ \dot{\sigma} = \dfrac{\partial\sigma}{\partial x}\dot{x} = \dfrac{\partial\sigma}{\partial x}(\,f(x,t) + g(x,t)) = 0 \end{cases} \tag{6}$$

The solution of the equation is the equivalent control of the sliding mode given by

$$u_{eq}(x,t) = -(\sigma(x)g(x,t))^{-1}\sigma(x)f(x,t) \tag{7}$$

To assure the sliding mode existence ($\sigma = 0$) the second component of the control law have to satisfy the attractively condition verified by

$$\dot{\sigma}(x) \cdot \sigma(x) < 0 \tag{8}$$

Because of the discontinuity presented on the sliding surface when the system reaches it and the Cauchy-Lipschitz theorem of the ordinary differential equations cannot be used [17]. The solution describing the dynamic behavior in this zone is using several approaches as the Filippov approach [18] or the Utkin approach [19] or more others as [20]. The above condition results

$$u = \begin{cases} u^{+}(x,t) & if \quad \sigma > 0 \\ u^{-}(x,t) & if \quad \sigma < 0 \end{cases} \tag{9}$$

From Eqs. (8) and (9) the sliding surface function and its derivative are of reverse sign and the second part of the sliding mode controller is given by

$$u_s = K \cdot sgn\,(\sigma) \tag{10}$$

Where K is the switching control gain and $sgn\,(\sigma)$ is the sliding surface signum function expressed by

$$sgn\,(\sigma) = \begin{cases} 1 & if \quad \sigma > 0 \\ 0 & if \quad \sigma = 0 \\ -1 & if \quad \sigma < 0 \end{cases} \tag{11}$$

Finally the sliding mode control law is presented as

$$u_{SMC} = u_{eq} + u_s \tag{12}$$

4. Sliding mode adaptation

Despite the claimed robustness properties of the sliding mode control, chattering is the harmful phenomenon affecting the control stability. This phenomenon is caused by the finite frequency oscillation of the switching part of the sliding controller. The presence of chattering in sliding mode control degrades the system accuracy and leads to the stability breaking and pushing the control to the divergence. Therefore, several researches and investigations focused on the chattering suppress methods and analysis. However, most of the chattering suppress methods consist of a continuous approximation of the discontinuous in the sliding surface neighborhood. The saturation is one of the main methods used in the chattering elimination in which a thin boundary layer around the surface is introduced defined as

$$sat(\sigma) = \begin{cases} \dfrac{\sigma}{\phi} & if \quad |\sigma| < \phi \\ sgn\,(\sigma) & otherwise \end{cases} \tag{13}$$

The boundary layer attributes the solution continuity and pushes the system to converge to this bound. The size of this layer depended on the system precision and the control accuracy. Another way to overcome chattering consists of the switching gain adaptation depending on the performance control maintain.

The adaptation is the ability of the system to adjust itself to its environment. Being processed, the adaptive system compensates the performance by changing its parameters depending on the plant environment evolution. Although, all the automatic adjustments in real-time approaches are considered as adaptive approaches with which the desired performance is maintained despite the system changes in time. Nevertheless, the adaptation of the sliding mode controller attenuates both the discontinuity and the chattering problem effect by adjusting the adapted gain depending on the plant environment. Thus, the adaptive approach is proposed to the switching part of the sliding mode controller of Eq. (10) and the equivalent part of Eq. (7) is maintained.

The adaptation of the controller in sliding mode consists of modifying in real-time the limit of the sliding boundary layer. While, a large band allows the system to regain the sliding surface easily but it destabilizes the controller by the length jump of its excessive gain. On the other hand, a small band causes a difficulty for the system to regain the sliding surface but it stabilizes the controller by the short jump of its low gain. Therefore, the proposed adaptive part is written as

$$u_{as} = \hat{K} \cdot sgn\,(\sigma) \tag{14}$$

Consequently, the Eq. (12) becomes

$$u_{ASMC} = u_{eq} + u_{as} \tag{15}$$

Where u_{ASMC} is the adapted sliding mode control law, u_{as} is the adapted switching part of the adaptive control, $\hat{K}$ is the new gain of the adaptive control proposed as

$$\hat{K} = \overline{K} - \left(\overline{K} - K\right) \cdot e^{-\alpha|\sigma|} \tag{16}$$

$$\overline{K} = \lambda \cdot K \tag{17}$$

Where $\overline{K}$ the amplified control gain, K is the original control gain of the Eq. (10), α is the convergence constant and λ is the amplification constant.

5. Lyapunov stability analysis

The convergence of the proposed adaptation law is evaluated and proved using the mathematical stability analysis of Lyapunov. Wherefore, The Lyapunov candidate function is chosen as [16].

$$V = \frac{1}{2}\sigma^2 + \frac{1}{2}\hat{e}^2 \tag{18}$$

Where $\hat{e}$ is the adaptation error given by

$$\hat{e} = \hat{K} - K_d \tag{19}$$

Where K_d is the maximum value of $\hat{K}$ given by Eq. (16) with $K_d > d$ and d is the localized uncertainty related to the switching motion.

The first derivation of the candidate function can be presented as

$$\dot{V} = \sigma\dot{\sigma} + \hat{e}\ \dot{\hat{e}} \tag{20}$$

From Eqs. (17), (18) and (20) the above equation becomes

$$\dot{V} = \sigma(e\dot{e}) + \left(\hat{K} - K_d\right)\sigma \cdot sgn\,(\sigma) \tag{21}$$

$$e = K - \hat{K} = (\lambda - 1)K \cdot e^{-\alpha|\sigma|} \tag{22}$$

$$\dot{e} = -\alpha(\lambda - 1)K \cdot e^{-\alpha|\sigma|} \tag{23}$$

Thus, the Eq. (21) becomes

$$\dot{V} = \sigma\left(-\alpha(\lambda - 1)^2 K^2 \cdot e^{-2\alpha|\sigma|}\right) + \lambda\left(\hat{K} - K_d\right)\sigma \cdot sgn\,(\sigma) \tag{24}$$

$$\dot{V} = -\alpha(\lambda - 1)^2 K^2 \cdot e^{-2\alpha|\sigma|} \cdot \sigma - K_d \cdot |\sigma| \tag{25}$$

With $\lambda \geq 1$ and $0 < \alpha < 1$ the condition stability $V \cdot \dot{V}$ is verified.

6. Numerical examples

6.1 Single degree of freedom system

In order to evaluate the proposed adaptive sliding mode controller, we considered a single degree of freedom system composed of a spring-mass-damper system presented in **Figure 2**. Moreover, the system can move in the horizontal direction only and the influence of the adaptive nonlinear control responses of the vibrating system is evaluated [21]. In this example, the response of the system to a constant reference with an initial condition is presented (i.e. $x(0) = 0.11$). The parameters arbitrarily chosen of the spring-mass-damper system are: $m = 2$, $k = 1$ and $c = 0.5$.

The equilibrium force of the time-varying system is given by

$$f_I(t) + f_D(t) + f_S(t) = f_E(t) \tag{26}$$

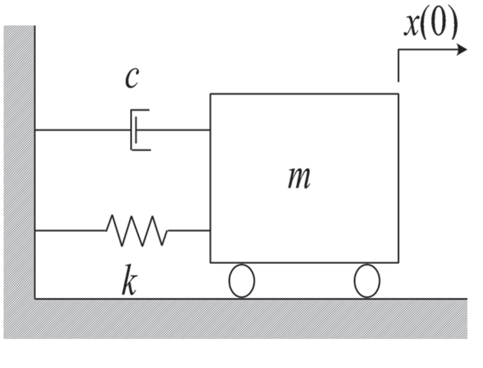

Figure 2.
Single degree of freedom example.

Where $f_I(t)$, $f_D(t)$, $f_S(t)$ and $f_E(t)$ are respectively the force of inertial, damping, spring and the external applied force given by

$$f_I(t) = m \cdot \ddot{x}(t) \tag{27}$$

$$f_D(t) = c \cdot \dot{x}(t) \tag{28}$$

$$f_S(t) = k \cdot x(t) \tag{29}$$

$$f_E(t) = k \cdot x(0) + f_c \tag{30}$$

Introducing Eqs. (27)-(30) in Eq. (26) yields

$$m \cdot \ddot{x}(t) + c \cdot \dot{x}(t) + k \cdot x(t) = k \cdot x(0) + f_c \tag{31}$$

Where m, c and k are the mass, damping and stiffness, $\ddot{x}$, $\dot{x}$ and x are acceleration, velocity and displacement, $x(0)$ and f_c are the initial displacement and the control required force.

Using Eqs. (12) and (15) to calculate respectively the classical sliding mode control and the adaptive sliding mode control forces. Thus, the two cases are simulated and compared to evaluate the robustness of the two controllers. Thereby, the numerical simulation result presented in **Figure 3** clearly shown the chattering reduction in the state plan response. This phenomenon is visibly reduced by using adaptive sliding mode control compared to the use of the classical sliding mode controller. Also, the adapted switching is clearly shown in the state plan presentation where the boundary layer thickness varied by the adaptation law depending on

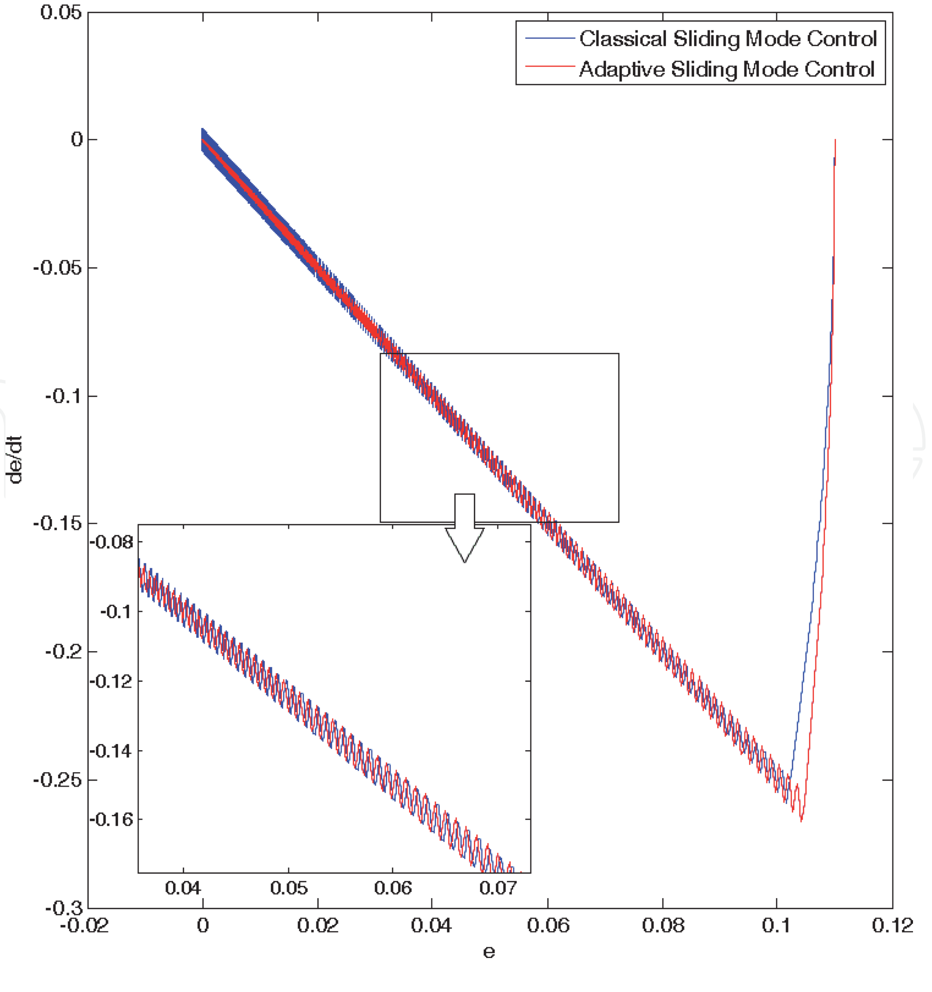

Figure 3.
Numerical simulation result of the mass-spring example.

the required performance of the plant. Besides, the **Figure 4** presented the compared numerical simulation results of the displacement responses function of time under the classical sliding mode control and the adaptive sliding mode control. Nonetheless, the displacement response of the system proves the performance of the adaptive nonlinear controller compared to the classical controller.

6.2 Multiple degree of freedom system

The previous example proved the efficiency of the used adaptive law to reinforce the control robustness. Otherwise, the applied load is a simple periodic load and the system is a simple system in which the switching output can be clearly shown in the state plan. In the present example, the three degrees of freedom system is considered under base excitation using earthquake records. This system is presented in **Figure 5** and the dynamic motion is governed by the following equation [22].

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = [M]\Lambda\ddot{x}_g + \{f_c\} \tag{32}$$

Where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices of the system, $\{\ddot{x}\}$, $\{\dot{x}\}$ and $\{x\}$ are the acceleration, velocity and displacement vectors, Λ, $\ddot{x}_g$ and $\{f_c\}$ are the load position vector, the one-dimensional earthquake vector and the control force vector. The example matrices of mass, damping and stiffness defined as [6, 23] are given respectively by

$$[M] = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = \begin{bmatrix} 98.3 & 0 & 0 \\ 0 & 98.3 & 0 \\ 0 & 0 & 98.3 \end{bmatrix} (kg) \tag{33}$$

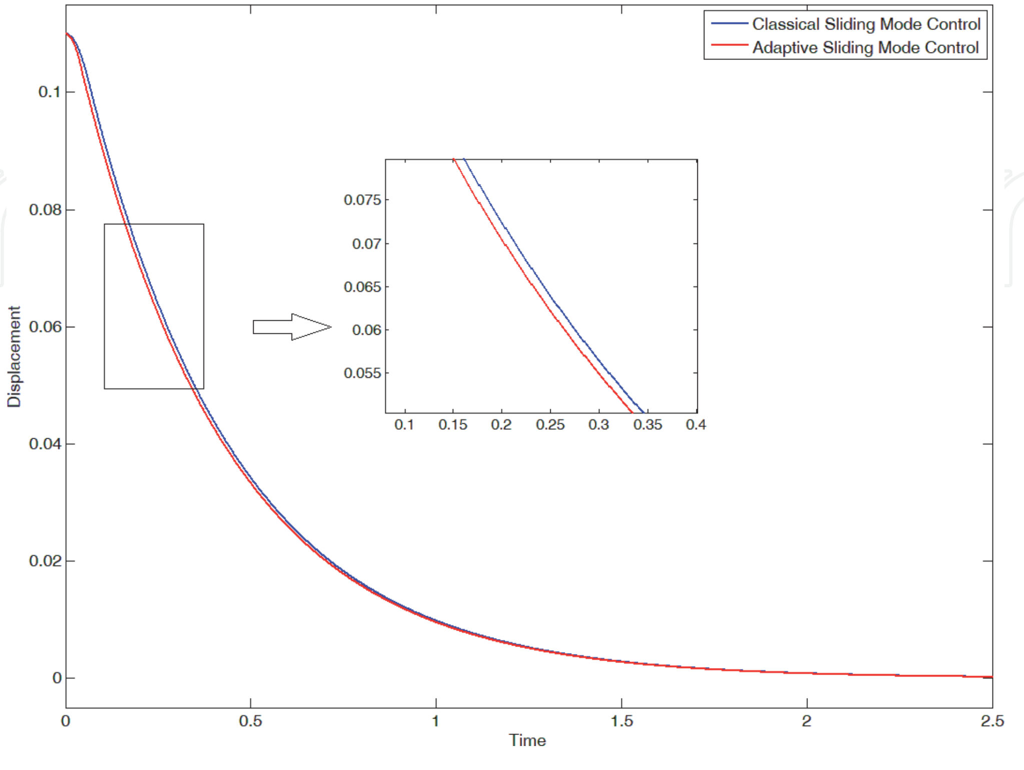

Figure 4.
Displacement responses of the mass-spring example.

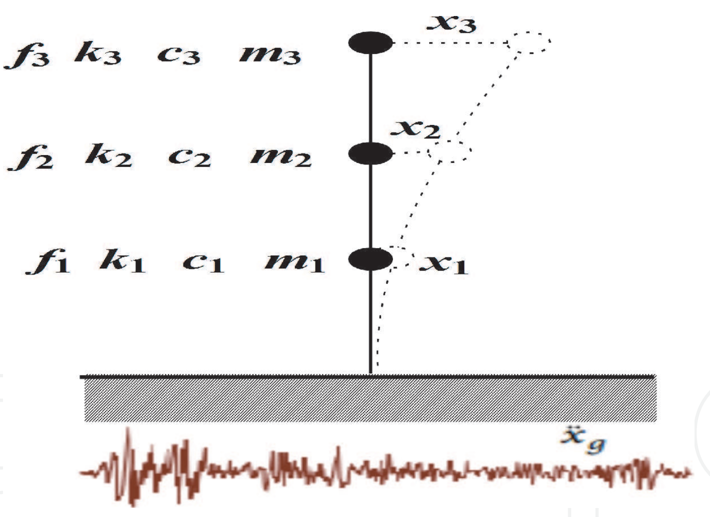

Figure 5.
Multiple degree of freedom example.

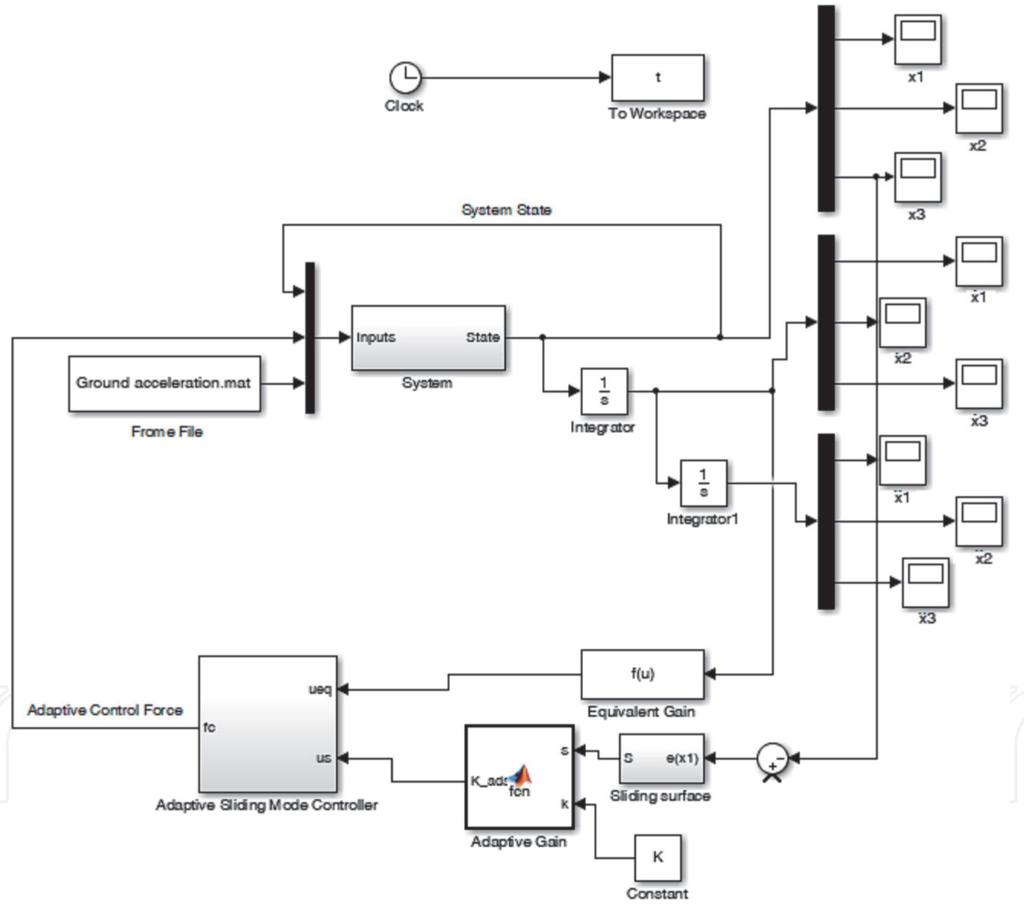

Figure 6.
Block diagram of the control system example.

$$[C] = \begin{bmatrix} c_1 + c_2 & -c_2 & 0 \\ -c_2 & c_2 + c_3 & -c_3 \\ 0 & -c_3 & c_3 \end{bmatrix} = \begin{bmatrix} 175 & -50 & 0 \\ -50 & 100 & -50 \\ 0 & -50 & 50 \end{bmatrix} (N \cdot s/m) \tag{34}$$

$$[K] = \begin{bmatrix} k_1 + k_2 & -k_2 & 0 \\ -k_2 & k_2 + k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} = 10^5 \begin{bmatrix} 12 & -6.84 & 0 \\ -6.84 & 13.7 & -6.84 \\ 0 & -6.84 & 6.84 \end{bmatrix} (N/m) \tag{35}$$

Accordingly, the control example is achieved as presented in **Figure 6** and the required control force is calculated in a closed-loop forcing the system to reach the equilibrium state.

The system is excited using the scaled time of the Tōhoku 2011 earthquake record illustrated in **Figure 7**.

Although, to prove the effectiveness of the proposed adaptive sliding mode controller to suppress the structural vibrations of the excited system the numerical simulation results of the controlled and the uncontrolled system are compared. The displacement responses of the first mass of the structure of the two cases controlled and uncontrolled are shown in **Figure 8**. However, the second and the third mass displacement responses of the compared cases of numerical simulation are presented respectively in **Figures 9** and **10**. In addition, the inter-mass drift responses of the three masses are depicted in **Figure 11** in which the numerical simulation results of the uncontrolled system are compared to those of the adaptive controlled system. The adaptation of the switching gain value function of time under the 2011 Tōhoku earthquake excitation is presented in **Figure 12**.

From **Figures 8–10** the displacement responses are clearly reduced under the earthquake excitation. The inter-mass drift responses depicted in **Figure 11** show a remarkable reduction between the two cases controlled and uncontrolled systems. Moreover, the responses of the switching gain of the proposed adaptive law illustrated in **Figure 12** show the dependence on the excitation. For example, in

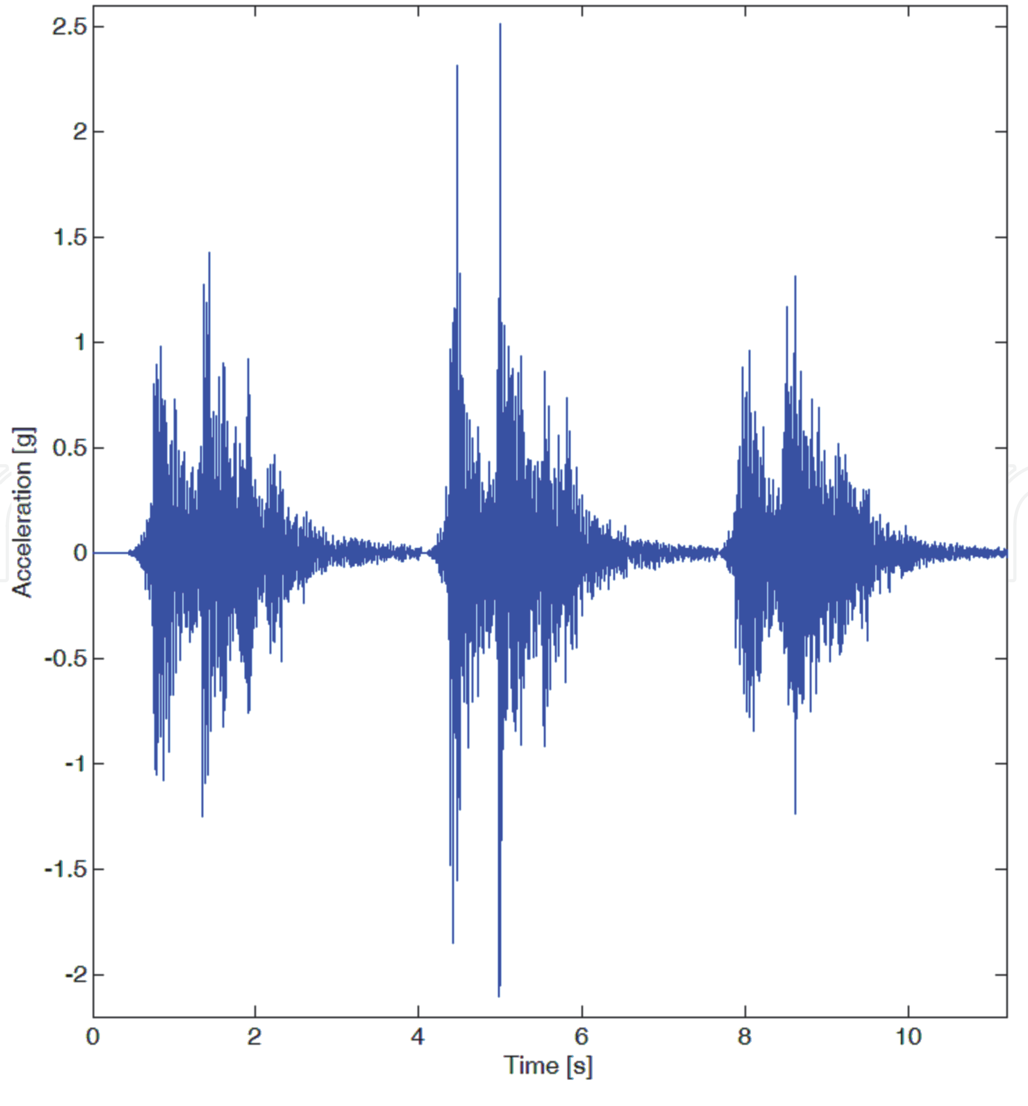

Figure 7.
The time scaled record of the 2011 Tōhoku earthquake.

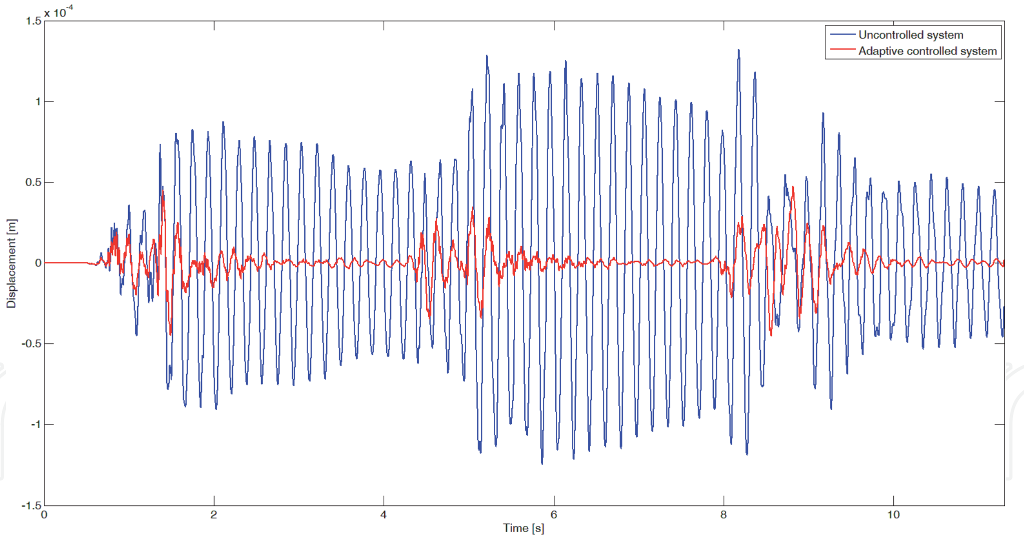

Figure 8.
The time displacement responses of the first mass under the 2011 Tōhoku earthquake.

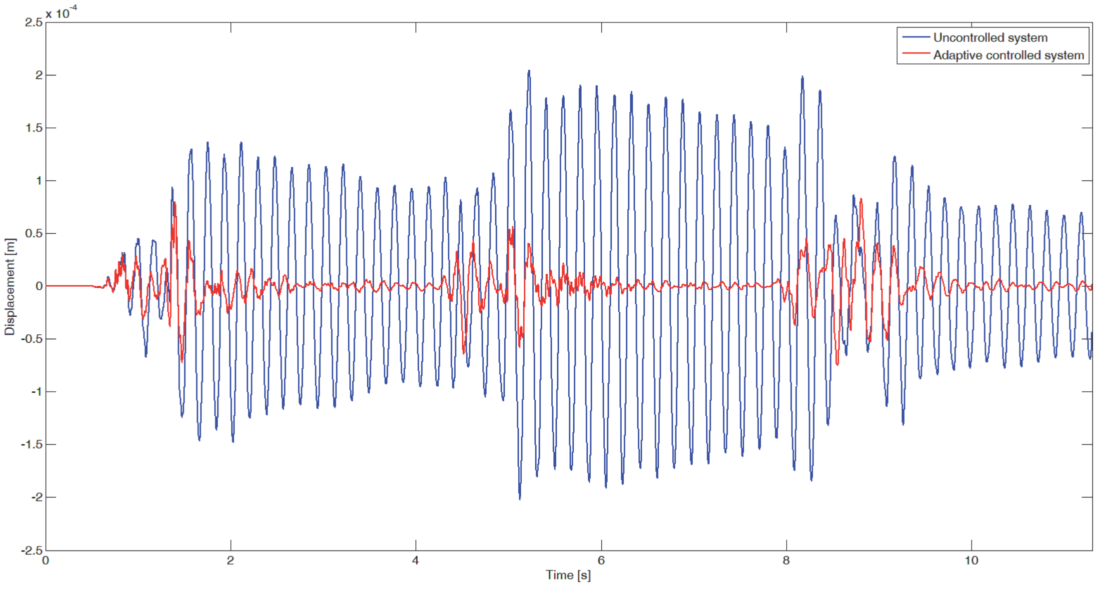

Figure 9.
The time displacement responses of the second mass under the 2011 Tōhoku earthquake.

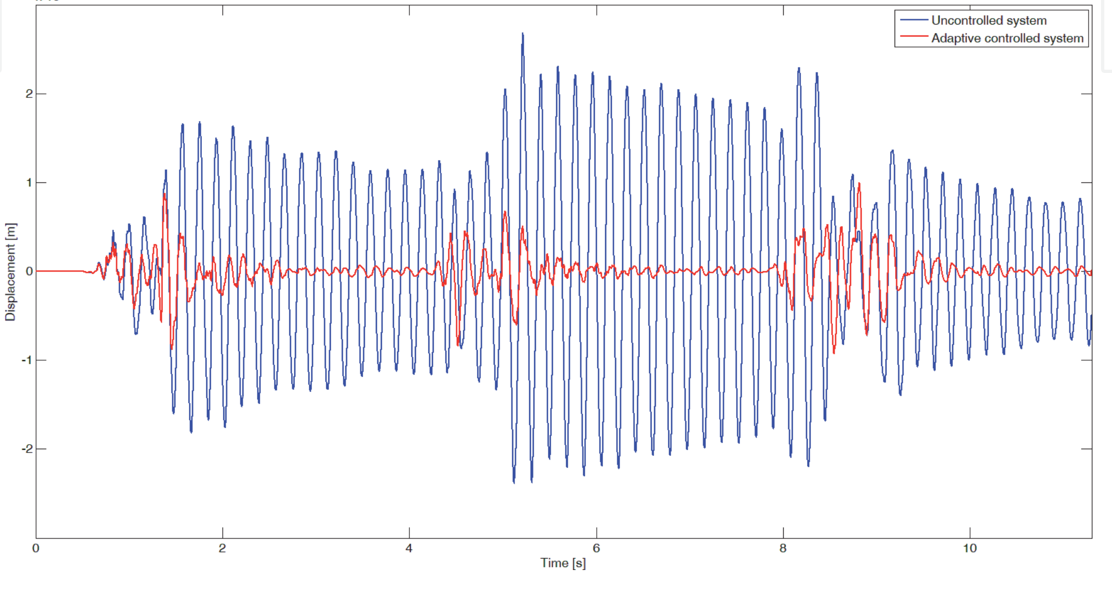

Figure 10.
The time displacement responses of the third mass under the 2011 Tōhoku earthquake.

Figure 12 between 4 and 5s where the peak seismic acceleration is located the law augmented the gain to the maximum value to track the system state better.

Over and above, the proposed adaptive control is evaluated by the result values of the system control application in the above-mentioned example. Some indexes are calculated and regrouped in **Table 1** to prove the robustness of the adaptive

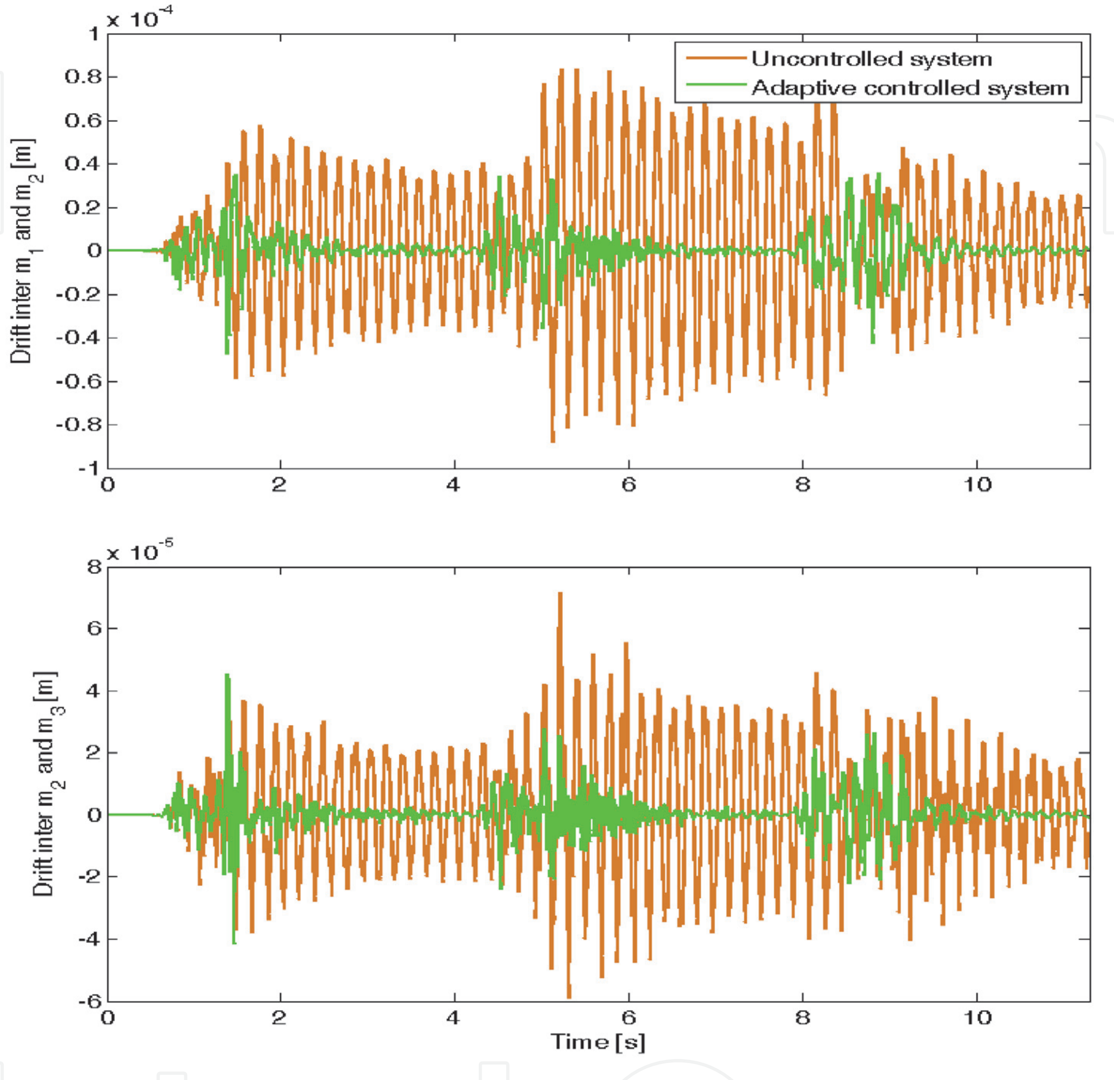

Figure 11.
The inter-mass drift responses under the 2011 Tōhoku earthquake.

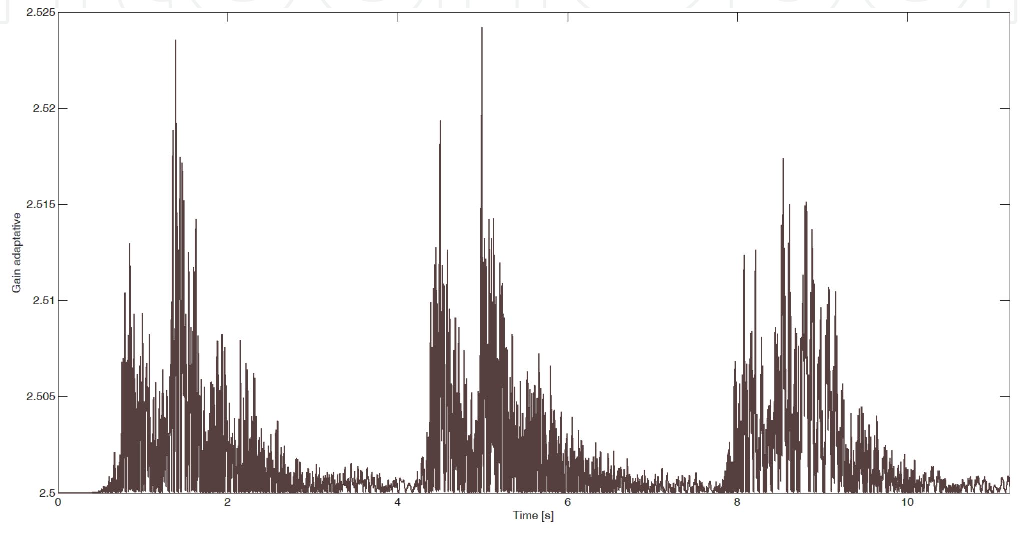

Figure 12.
The time adaptive gain variations under the 2011 Tōhoku earthquake.

Index	Formula	Mass number	Value (%)
Peak Displacement Reduction	$\lvert x_i^{max}\rvert - max\,\lvert x_i\rvert/\lvert x_i^{max}\rvert$	1	65.33
		2	58.78
		3	62.98
Peak Acceleration Reduction	$\lvert \ddot{x}_i^{max}\rvert - max\,\lvert \ddot{x}_i\rvert/\lvert \ddot{x}_i^{max}\rvert$	1	00.92
		2	02.79
		3	01.14
Peak drift Reduction	$\lvert d_i^{max}\rvert - max\,\lvert d_i\rvert/\lvert d_i^{max}\rvert$	1–2	56.63
		2–3	54.17

Table 1.
Calculated control indexes of the adaptive sliding mode control under the 2011 Tōhoku earthquake excitation.

control to attenuate the excited system vibrations. The peak displacement reduction and the peak acceleration reduction of each mass are calculated and inserted in **Table 1**. Thereby, the peak inter-mass drift reduction is also needful to evaluate the proposed adaptive controller performance.

Where the index i designed the mass number, x_i^{max} is the maximum uncontrolled mass displacement, x_i the controlled mass displacement, $\ddot{x}_i^{max}$ is the maximum uncontrolled mass acceleration, $\ddot{x}_i$ the controlled mass acceleration, d_i^{max} is the maximum uncontrolled inter-mass drift and d_i the controlled inter mass drift.

7. Conclusions

The proposed adaptive sliding mode controller robustness had been proved in the present chapter using two numerical examples. Although, the single degree of freedom example excited by a simple periodic load shown clearly the Chattering reduction as a result of the adaptive law effect. The numerical simulation results of the state plan presentation shown the switching gain adaptation value depending on the excitation effect. Moreover, the second example is a three degree of freedom system excited using an earthquake excitation to assure the presence of multiple frequencies and amplitudes. As expected, the numerical simulation results of the example prove the efficiency of the proposed adaptive controller. The peak mass displacement ratio attained 65.33%, consequently, the peak inter-mass drift is reduced by 56.63%. The peak acceleration is sparsely reduced because the adaptive control is designed to track the displacement only. In this stage, the nonlinear adaptive controller proves its effectiveness and performance in addition to the insensitivity to uncertainties or disturbances and system stability.

Nomenclature

C	Damping matrix
c	Damping or friction coefficient
c_0	Positive constant
d	Localized Uncertainty
d_i	Controlled inter-mass drift
d_i^{max}	Maximum uncontrolled inter-mass drift
e	Tracking error
$\hat{e}$	Adaptive error
f	Function
f_E	External force
f_D	Damping force

f_I	Inertial force
f_S	Spring force
f_c	Control force
G	Sliding surface matrix
g	Function
i	Mass number index
K	Switching gain, Stiffness matrix
$\hat{K}$	Adaptive switching gain
$\overline{K}$	Amplified switching gain
K_d	Maximum value of the adaptive switching gain
k	Stiffness
M	Mass matrix
m	Mass
u	The system output
u_{ASMC}	Adaptive sliding mode controller output
u_{SMC}	Sliding mode controller output
u_{eq}	Equivalent output
u_s	Switching output
u_{as}	Adaptive switching output
V	Lyapunov candidate function
x	Displacement
x_i	Controlled mass displacement
x_0	Desired response
$\dot{x}$	Velocity
$\ddot{x}$	Acceleration
$\ddot{x}_i$	Controlled mass acceleration
$\ddot{x}_i^{max}$	Maximum uncontrolled mass acceleration
σ	Sliding surface
ϕ	Boundary layer thickness
∂	Partial derivative
λ	Constant amplification
α	Convergence constant

Abbreviations

ASMC	Adaptive Sliding Mode Control
SMC	Sliding Mode Control
sat	Saturation function
sgn	Signum function

Author details

Leyla Fali[1*†], Khaled Zizouni[2†], Abdelkrim Saidi[2†], Ismail Khalil Bousserhane[2†] and Mohamed Djermane[1†]

1 FIMAS Laboratory, TAHRI Mohamed University, Bechar, Algeria

2 ArchiPEL Laboratory, TAHRI Mohamed University, Bechar, Algeria

*Address all correspondence to: fali.leyla@univ-bechar.dz

† These authors contributed equally.

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