Chapter

Quarks Mixing in Chiral Symmetries

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Abstract

We discuss a subject of the quarks mixing in $SU_4 * SU_4$ and $SU_6 * SU_6$ symmetries trying to calculate the quarks mixing angles and the complex phase responsible for the CP non-conservation on the basis of the Gell-Mann Oakes Renner model. Assuming symmetry breaking in a limit of exact sub-symmetries for simultaneous quarks rotations in both electric charge sub-spaces we can estimate all mention above parameters. A perfect agreement of the experimental value of the Cabibbo angles with a sum of simultaneous quarks mixing angles in doublets (u,c) and (d,s) in the $SU_4 * SU_4$ symmetry suggests that a quarks mixing is realized in a maximal allowed range. The same assumption used for the $SU_6 * SU_6$ and a simultaneous maximal allowed quarks mixing in both electric charge triplets (u,c,t) and (d,s,b) gives a perfect agreement with the experimental value of the Cabibbo angle and estimation on the angles Θ_2 and Θ_3 as well as a bond for the complex phase δ.

Keywords: quarks mixing, chiral symmetries, Cabibbo angle, Kobayashi-Maskawa mixing matrix, symmetry breaking

1. Introduction

1.1 Quarks mixing in chiral $SU_n * SU_n$ broken symmetry in the limit of exact $SU_k * SU_k$ symmetry

The hierarchy of chiral symmetry breaking [1–3] has been investigated since seventies of the previous century [4–8]. The symmetry breaking and mixing of quarks are connected with the rotation of quark currents and Hamiltonian densities. The determination of the rotation angle becomes an important problem. For the first time the procedure of chiral symmetry breaking, based on the Gell-Mann, Oakes, Renner (GMOR) model [9] has been used in $SU_3 * SU_3$ symmetry in the limit of exact $SU_2 * SU_2$ symmetry [4] to determine the value of the Cabibbo angle [10]. The transformation of rotation is connected with the seventh generator of the SU_3 group. After the charmed particles have been discovered the $SU_3 * SU_3$ symmetry is no longer adequate to describe the strong interactions. The $SU_4 * SU_4$ symmetry introduced earlier [11] to explain the behavior of charged and neutral currents becomes quite satisfactory model describing the hadron world. The problem of determining the Cabibbo angle in $SU_4 * SU_4$ symmetry has arisen. It is considered in [5–6] and the method of calculating the Cabibbo angle in $SU_4 * SU_4$ symmetry is described in [7]. It is known that the formula describing the rotation angle is not changed if the symmetry is extended. This is not unexpected because the Cabibbo angle is connected with the mixing of the d and s quarks and the rotation is performed around the seventh axis in SU_3 subspace too.

The problem of chiral $SU_4 * SU_4$ symmetry breaking in the limit of exact $SU_2 * SU_2$ symmetry is considered in [6]. Symmetry breaking is connected with the transformation of rotation around the tenth axis in SU_4 space. The rotation angle is determined in [7].

The other variant of the $SU_4 * SU_4$ symmetry breaking in the limit of the exact $SU_3 * SU_3$ symmetry is described in [8]. It is connected with the rotation around the fourteenth axis in SU_4 space. In this paper we introduce the general method of rotation angle description in the broken $SU_n * SU_n$ symmetry. The chiral $SU_n * SU_n$ symmetry is broken according to the GMOR model. In the first step we introduce the Hamiltonian density breaking $SU_n * SU_n$ symmetry but invariant under $SU_k * SU_k$ symmetry. In the second step we introduce quark mixing and the resulting exact symmetry is $SU_{k-1} * SU_{k-1}$ The particular investigation of cases like the above is not necessary.

The generalized GMOR model is used. It is assumed that by enlargement to a higher symmetry the new quantum numbers are the charges (as for example: electric charge, strangeness, charm but not isospin). Then the $SU_n * SU_n$ symmetry breaking Hamiltonian density can be written as a linear combination of diagonal operators u^i.

$$H_E = \sum_{j=1}^{n} c_{j^2-1} u^{j^2-1} \tag{1}$$

where the scalar densities $u^i = \overline{q}\lambda^i q$ and pseudo-scalar densities $v^i = i\overline{q}\lambda^i\gamma_5 q$ satisfy the equal-time commutation rules

$$\begin{aligned} \left[Q^i, u^j\right] &= if_{ijk}u^k \qquad & \left[Q^i, v^j\right] &= if_{ijk}v^k \\ \left[\overline{Q}^i, u^j\right] &= id_{ijk}v^k \qquad & \left[\overline{Q}^i, v^j\right] &= -id_{ijk}u^k \end{aligned} \tag{2}$$

where f_{ijk} are the structure constants, d_{ijk} - symmetric generators of the $SU_n * SU_n$ group. If the $SU_k * SU_k$ symmetry is exact then

$$\partial^\mu V_\mu^i = \partial^\mu A_\mu^i = 0 \qquad \left(i = 1, 2, \ldots, k^2 - 1\right) \tag{3}$$

In the GMOR model the divergences of currents can be calculated as follows

$$\partial^\mu V_\mu^i = i\left[H_E, Q^i\right] \qquad \partial^\mu A_\mu^i = i\left[H_E, \overline{Q}^i\right] \tag{4}$$

We require that the $SU_k * SU_k$ symmetry be exact, then the following constraints are obeyed

$$\begin{gathered} c_{j^2-1} = 0 \qquad\qquad (j = 2, \ldots, k) \\ \sqrt{\frac{2}{n}}c_0 + \sum_{j=k+1}^{n} \sqrt{\frac{2}{j(j+1)}}c_{j^2-1} = 0 \end{gathered} \tag{5}$$

The symmetry breaking Hamiltonian density can be written as follows

$$H_E = c_0\left(u^0 - \sqrt{n-1}u^{n^2-1}\right) + \sum_{j=k+1}^{n-1} c_{j^2-1}\left(u^{j^2-1} - \sqrt{\frac{n(n-1)}{j(j-1)}}u^{n^2-1}\right) \tag{6}$$

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Using the standard representation of λ matrices one obtains

$$u^0 = \sqrt{\frac{2}{n}} \sum_{j=1}^{n} \overline{q}_j q_j \tag{7}$$

$$u^{j^2-1} = \sqrt{\frac{2}{j(j-1)}} \left(\sum_{l=1}^{j-1} \overline{q}_l q_l - (j-1) \overline{q}_j q_j \right) \tag{8}$$

$$u^0 - \sqrt{n-1}\ u^{n^2-1} = \sqrt{2n}\ \overline{q}_n q_n \tag{9}$$

$$u^{j^2-1} - \sqrt{\frac{n(n-1)}{j(j-1)}} u^{n^2-1} = \sqrt{\frac{2}{j(j-1)}} \left((n-1)\ \overline{q}_n q_n - j\ \overline{q}_j q_j - \sum_{l=j+1}^{n-1} \overline{q}_l q_l \right) \tag{10}$$

Let us note that the term $\overline{q}_k q_k$ does not exist in Eq. 11.

$$\begin{aligned} H_E = & \left(\sqrt{2n} c_0 + (n-1) \sum_{j=k+1}^{n-1} c_{j^2-1} \sqrt{\frac{2}{j(j-1)}} \overline{q}_n q_n \right) \\ & - \sum_{j=k+1}^{n-1} c_{j^2-1} \sqrt{\frac{2}{j(j-1)}} \left(j\ \overline{q}_j q_j + \sum_{l=j+1}^{n-1} \overline{q}_l q_l \right) \end{aligned} \tag{11}$$

The chiral $SU_n * SU_n$ symmetry with the exact $SU_k * SU_k$ sub-symmetry is broken by the rotation of the $SU_k * SU_k$ invariant Hamiltonian density around the axis with the index $m = (n-1)^2 + 2k - 1$.

$$H_{SB} = e^{-2i\alpha Q^m}\ H_E\ e^{2i\alpha Q^m} \tag{12}$$

Only the quarks q_k and q_n are mixed. The $SU_k * SU_k$ symmetry is no longer exact. Only the term $\overline{q}_n q_n$ is rotated under transformation (12), because there is no $\overline{q}_k q_k$ term in the Hamiltonian density (11).

$$e^{-2i\alpha Q^m} H_E e^{2i\alpha Q^m} = \overline{q}_n q_n - \left(\overline{q}_n q_n - \overline{q}_k q_k \right)\ \sin^2\alpha - \frac{1}{2} \left(\overline{q}_k q_n + \overline{q}_n q_k \right)\ \sin 2\alpha \tag{13}$$

The above consideration is limited to processes not having the change of the quantum number N connected with the SU_n symmetry. So in the broken Hamiltonian density $H_{SB(\Delta N=0)}$ the terms $\overline{q}_n q_k$ and $\overline{q}_k q_n$ do not appear. The broken Hamiltonian density is a linear combination of the diagonal operators u^i only.

$$H_{SB(\Delta N=0)} = H_E + A \sum_{j=k}^{n-1} \left(\sqrt{\frac{j+1}{2j}} u^{(j+1)^2-1} - \sqrt{\frac{j-1}{2j}} u^{j^2-1} \right) \sin^2\alpha \tag{14}$$

where

$$A = \sqrt{2n}\ c_0 + (n-1) \sum_{j=k+1}^{n-1} c_{j^2-1}\ \sqrt{\frac{2}{j(j-1)}} \tag{15}$$

In more detail Eq. (14) is given as follows:

$$H_{SB(\Delta N=0)} = c_0 \ u^0 - A\sqrt{\frac{k-1}{2k}}\sin^2\alpha \ u^{k^2-1} + \sum_{j=k+1}^{n-1} u^{j^2-1}\left(c_{j^2-1} + A \ \ \sin^2\alpha \ \sqrt{\frac{1}{2j(j-1)}}\right) +$$
$$-u^{n^2-1}\left(c_0 \ \sqrt{n-1} \ + \sum_{j=k+1}^{n-1} c_{j^2-1} \ \sqrt{\frac{n(n-1)}{j(j-1)}}\right) - A \ u^{n^2-1} \ \sqrt{\frac{n}{2(n-1)}} \ \sin^2 \ \alpha \tag{16}$$

If the $SU_k * SU_k$ symmetry is exact then the pseudo-scalar mesons corresponding to the indices $\left(j = 1, \ldots, k^2 - 1\right)$ are massless [9]. After the $SU_k * SU_k$ has been broken, the $SU_{k-1} * SU_{k-1}$ symmetry is still exact, because the operator Q^m does not mix the quarks $q_1, \ldots, q_{k-1}$ neither with themselves nor with other quarks. The mesons corresponding to the indices $\left(j = 1, \ldots, (k-1)^2 - 1\right)$ after symmetry breaking are still massless, while the mesons corresponding to the indices $\left(j = (k-1)^2, \ldots k^2 - 1\right)$ belong to the massive multiplet (k)[1] The masses of mesons are determined in the GMOR model. Before the $SU_n * SU_n$ symmetry is broken the masses are described by the coefficients $c_0, \ldots, c_{n^2-1}$ from Eq. (1). After the symmetry has been broken the new factors $c'_0, \ldots, c'_{n^2-1}$ are obtained as the coefficients standing by the operators u^i in the broken Hamiltonian density (16) [7].

$$c'_0 = c_0 \tag{17}$$
$$c'_{k^2-1} = A\sqrt{\frac{k-1}{2k}}\sin^2\alpha$$
$$c'_{j^2-1} = c_{j^2-1} + A\sin^2\alpha\sqrt{\frac{1}{2j(j-1)}} \quad (k<j<n)$$
$$c'_{n^2-1} = -c_0 \ \sqrt{n-1} - \sum_{j=k+1}^{n-1} c_{j^2-1} \ \sqrt{\frac{n(n-1)}{j(j-1)}} \ + A \ \sqrt{\frac{n}{2(n-1)}} \ \sin^2\alpha$$

The masses of the mesons are determined as follows [12].

$$m_a^2 J_a^2 \delta_{ab} = \sqrt{\frac{2}{n}}\left(c_0 \ d_{0ab} + \sum_{j=k+1}^{n} c'_{j^2-1} d_{(j^2-1)ab}\right) < u^0 >_0 \tag{18}$$

The relation between the indices a, b, j and meson states is described, for example, for the $SU_4 * SU_4$ symmetry in [12, 13]. For $a = b = k^2 - l$, the mass of the (k) meson is given as follows

$$m_{(k)}^2 f_{(k)}^2 = \sqrt{\frac{2}{n}}\left(\sqrt{\frac{2}{n}}c_0 \ + \sum_{j=k+1}^{n} d_{(j^2-1)ab} c'_{j^2-1}\right) < u^0 >_0 = \frac{1}{k} \ \sqrt{\frac{2}{n}} \ A \ \sin^2\alpha \ < u^0 >_0 \tag{19}$$

For $a = b = m = (n-1)^2 + 2k - 1$, the mass of the (n) meson is given by

$$m_{(n)}^2 f_{(n)}^2 = \sqrt{\frac{2}{n}}\left(\sqrt{\frac{2}{n}}c_0 \ + \sum_{j=k+1}^{n} d_{(j^2-1)mm} \ c'_{j^2-1}\right) < u^0 >_0 \tag{20}$$

Because

$$d_{(j^2-1)mm} = \sqrt{\frac{1}{2j(j-1)}} \quad (j < n) \tag{21}$$

$$d_{(n^2-1)mm} = \frac{2-n}{\sqrt{2n(n-1)}} \tag{22}$$

$$m^2_{(n)} f^2_{(n)} = \sqrt{\frac{1}{2n}}\, A\left(1 - \left(1 - \frac{1}{k}\right)\sin^2\alpha\right) < u^0 >_0 \tag{23}$$

In formulas (19) and (23) to determine the masses of (k) and (n) mesons one has (n-k + 3) unknown quantities with which to deal ($< u^0 >_0, c_0,$ $c_{(k+1)^2-1}, \ \dots, c_{n^2-1}, \ \sin\alpha$). Nevertheless the angle a is determined by the masses and decay constants of two pseudo-scalar mesons (k) and (n) only.

$$\sin^2\alpha = \frac{k \ m^2_{(k)} f^2_{(k)}}{2 \ m^2_{(n)} f^2_{(n)} + (k-1) \ m^2_{(k)} f^2_{(k)}} \tag{24}$$

All the cases of symmetry breaking considered in [4–8] can be described by formula (24). Let us give simple examples: a) for k = 2, n = 3 a is the original Cabibbo angle Θ associated with rotation around the seventh axis in SU_3 subspace [4–7].

$$\sin^2\Theta = \frac{2 \ m^2_\pi f^2_\pi}{2 \ m^2_K f^2_K + \ m^2_\pi f^2_\pi} \tag{25}$$

b) for k = 2, n = 4 and rotation around the tenth axis [6–7] one obtains

$$\sin^2\alpha = \frac{2 \ m^2_\pi f^2_\pi}{2 \ m^2_D f^2_D + \ m^2_\pi f^2_\pi} \tag{26}$$

c) for k = 3, n = 4 and rotation around the fourteenth axis [7–8] one obtains

$$\sin^2\alpha = \frac{3 \ m^2_K f^2_K}{2\left(m^2_D f^2_D + \ m^2_K f^2_K\right)} \tag{27}$$

In general the determination of the rotation angle (24) in $SU_n * SU_n$ symmetry is possible only if the new quantum numbers introduced by a transition to the higher symmetry are scalars of the charge type (additiv). So the Hamiltonian density (1) can be constructed as a linear combination of the diagonal operators u^i; only $\left(i = j^2 - 1, j = 1, \dots, n\right)$. The method of determining the rotation angle, discussing and interpreting the symmetry breaking is described in more detail in [7] on $SU_4 * SU_4$ symmetry as an example.

2. Quarks mixing and the Cabibbo angle in the $SU_4 * SU_4$ broken symmetry

It is known that the Cabibbo angle has been introduced into SU_3 symmetry to explain the suppression of processes in which strangeness is not conserved [4]. The

Cabibbo angle is connected with the mixing of d and s quarks for weak interactions of hadrons. Its value, calculated by Oakes, does not contradict the experimental data. Before the charmed particles were discovered Glashow, Iliopoulos and Maiani [11] have suggested the generalization of a strong interaction symmetry to SU_4 [6]. The charged weak current is then given as follows

$$J_\mu = \overline{q}\ \gamma_\mu\ (1-\gamma_5)\ A\ q \tag{28}$$

where

$$A = \begin{pmatrix} 0 & 0 & \cos\Theta & \sin\Theta \\ 0 & 0 & -\sin\Theta & \cos\Theta \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{29}$$

The current (28) can be expressed in another form

$$J_\mu = (\overline{u}, \overline{c})\ \gamma_\mu\ (1-\gamma_5)\begin{pmatrix} \cos\Theta & \sin\Theta \\ -\sin\Theta & \cos\Theta \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix} \tag{30}$$

so, quark mixing is described by an orthogonal matrix. On the grounds of Eq. (30) we cannot come to a conclusion about quarks in which the doublets are mixed. If the matrix A is generalized to the following form

$$A = \begin{pmatrix} 0 & 0 & \cos\Theta & \sin\Theta \\ 0 & 0 & -\sin\Theta & \cos\Theta \\ \cos\phi & \sin\phi & 0 & 0 \\ -\sin\phi & \cos\phi & 0 & 0 \end{pmatrix} \tag{31}$$

the quarks in the doublets (u, c) and (d, s) are mixed independently. The zeros in Eq. (31) are associated with the fact that the neutral currents which change the strangeness and/or charm are not observed. So, the current (28) can be given in the following form

$$J_\mu = (\overline{u}, \overline{c})\ \gamma_\mu\ (1-\gamma_5)\begin{pmatrix} \cos(\Theta+\phi) & \sin(\Theta+\phi) \\ -\sin(\Theta+\phi) & \cos(\Theta+\phi) \end{pmatrix}\begin{pmatrix} d \\ s \end{pmatrix} \tag{32}$$

If the currents only are taken into consideration we cannot solve the problem if the quarks are mixed in one or both doublets. This is not unexpected because the currents are built as a bi-linear combination of quark states and the angles Θ and ϕ, can always be substituted the effective angle $(\Theta+\phi)$. To solve the problem the Gell-Mann, Oakes, Renner (GMOR) model [9] will be used.

The charged weak current in SU_3 symmetry can be written as follows

$$J_\mu(\Theta) = \cos\Theta\ \left(J_\mu^1 + i\ J_\mu^2\right) + \sin\Theta\ \left(J_\mu^4 + i\ J_\mu^5\right) \qquad \Theta - \text{Cabibbo angle} \tag{33}$$

$$J_\mu = \overline{q}\ \gamma_\mu\ (1-\gamma_5)\lambda^k\ q \qquad q = \begin{pmatrix} u \\ d \\ s \\ c \end{pmatrix} \tag{34}$$

The current (33) can be obtained from the isospin component of the current $\left(J_\mu^1 + i\ J_\mu^2\right)$ by rotation through an angle 2Θ about the seventh axis in SU_3 space according to

$$J_\mu(\Theta) = e^{-2i\Theta F^7}\left(J_\mu^1 + i\ J_\mu^2\right)e^{2i\Theta F^7} \tag{35}$$

where

$$F^k = \int d^3x\ q^+(x)\ \frac{\lambda^k}{2}\ q(x) \tag{36}$$

The charged weak current in SU_4 symmetry (30) can be expressed in the following form

$$\begin{aligned} J_\mu(\Theta) = {} & \cos\Theta\ \left(J_\mu^1 + i\ J_\mu^2\right) + \sin\Theta\ \left(J_\mu^4 + i\ J_\mu^5\right) - \sin\Theta\ \left(J_\mu^{11} - i\ J_\mu^{12}\right) \\ & + \cos\Theta\ \left(J_\mu^{13} - i\ J_\mu^{14}\right) \end{aligned} \tag{37}$$

The current (37) can be obtained by rotation of the components $\Delta S = \Delta C$ through an angle 2Θ about the seventh axis in SU_4 space

$$J_\mu(\Theta) = e^{-2i\Theta F^7}\left(J_\mu^1 + i\ J_\mu^2 + J_\mu^{13} - i\ J_\mu^{14}\right)\ e^{2i\Theta F^7} \tag{38}$$

The transformation (38) changes the strangeness but not the charm because

$$\left[F^7, q_1\right] = \left[F^7, q_4\right] = 0 \tag{39}$$

The transformation (38) is connected with the mixing of d and s quarks (as in the case of SU_3 symmetry). In the SU_4 symmetry the mixing in electric charge subspace +2/3 can be taken into consideration. This is not possible in the SU_3 symmetry where only one state with the +2/3 charge exists. The possibility of expressing the current (37) by the transformation which changes charm but not strangeness should exist. The transformation has been described by Ebrahim in [6].

$$J_\mu(\phi) = e^{-2i\phi F^{10}}\left(J_\mu^1 + i\ J_\mu^2 + J_\mu^{13} - i\ J_\mu^{14}\right)\ e^{2i\phi F^{10}} \tag{40}$$

$$\left[F^{10}, q_2\right] = \left[F^{10}, q_3\right] = 0 \tag{41}$$

The transformation (40) is connected with the mixing of u and c quarks. The fact that there exist two transformations giving the current (37) but connected with different generators of the SU_4 group changing strangeness or charm respectively suggests that independent mixing in both doublets is possible. It is known that the Cabibbo angle is connected with strangeness non-conservation in weak interactions. The formula describing the value of the Cabibbo angle has been obtained by Oakes [4] in the procedure of symmetry breaking. Namely the $SU_3 * SU_3$ symmetry in the limit of the exact $SU_2 * SU_2$ symmetry is broken. The $SU_2 * SU_2$ subsymmetry is no longer exact. The symmetry is broken by the rotation of the $SU_2 * SU_2$ invariant Hamiltonian density through angle 2Θ about the seventh axis. Then the pion becomes massive. The symmetry breaking is connected with the mixing of d and s quarks. The rotation angle Θ, as a measure of symmetry violation, is a function of the mass and the decay constant of the pion and of the mass and the decay constant of the kaon as well (it is connected with the mixing of the strange quark and the strangeness non-conservation). If the breaking of the chiral $SU_2 * SU_2$ symmetry, the mass of the pion, the Cabibbo angle as well as a strangeness and charm non-conservation have a common origin then it seems that as a result of $SU_4 * SU_4$ symmetry breaking in the limit of the exact $SU_2 * SU_2$ sub-symmetry by the rotation of the $SU_2 * SU_2$ invariant Hamiltonian density through an angle 2ϕ about the tenth axis the angle ϕ connected with the mixing of u and c quarks as a measure of a symmetry violation should be a function of the mass and decay constant of the pion (breaking of the $SU_2 * SU_2$ symmetry) and a function of the mass and decay constant of a charmed meson (charm non-conservation). The cases of the separate and then simultaneously mixing of quarks in the sub-spaces of electric charge will be considered below.

If the electromagnetic mass splitting of u-d quarks is neglected the Hamiltonian density breaking the chiral $SU_4 * SU_4$ symmetry is given in the form

$$H = c_0 u^0 + c_8 u^8 + c_{15} u^{15} \tag{42}$$

where c_0, c_8, c_{15} are constants, u^a (a = 0, 1, ..., 15) are the scalar components of the $(\overline{4}, 4) + (4, \overline{4})$ representation of the chiral $SU_4 * SU_4$ group. On the grounds of the GMOR model the following relation for masses of the pseudo-scalar mesons can be obtained [12].

$$\begin{aligned} i \ <0|\left[\overline{Q}^a, \overline{D}^b\right]|0> &= \delta^{ab} f_a^2 m_a^2 + \int \frac{dq^2}{q^2} \rho^{ab} = \\ &= <u^0>_0 \left(\frac{c_0}{2}\delta^{ab} + \frac{c_8}{\sqrt{2}} d_{a8b} + \frac{c_{15}}{\sqrt{2}} d_{a15b}\right) + \\ &+ <u^8>_0 \left(\frac{c_0}{\sqrt{2}} d_{a8b} + c_8 d_{a8c} d_{b8c} + c_{15} d_{a8c} d_{b15c}\right) + \\ &+ <u_{15}>_0 \left(\frac{c_0}{\sqrt{2}} \ d_{a15b} + c_8 \ d_{a15c} \ d_{b8c} + c_{15} \ d_{a15c} \ d_{b15c}\right) \end{aligned} \tag{43}$$

where

$$\rho^{ab} = (2\pi)^3 \sum_{n \neq a} \delta^4 (p_n - q) < 0|\overline{D}^a|n> <n|\overline{D}^b|0> \tag{44}$$

f_a - decay constants, $<u^i>_0$ - vacuum expectation value of the operator u^i. Because the vacuum expectation values of operators u^8, u^{15} and the spectral density δ^{ab} are proportional to the squared parameters of symmetry breaking, they are further neglected [12]. Approximately from Eq. (43) we obtain

$$m_a^2 \ f_a^2 \ \delta^{ab} = \frac{1}{\sqrt{2}} \left(\frac{c_0}{\sqrt{2}} + c_8 \ d_{a8b} + c_{15} \ d_{a15b}\right) <u^0>_0 \tag{45}$$

The masses of the mesons are given as follows

$$\begin{aligned} m_\pi^2 \ f_\pi^2 &= \frac{1}{2\sqrt{3}} \left(\sqrt{3} \ c_0 + \sqrt{2} \ c_8 + c_{15}\right) <u^0>_0 \\ m_K^2 \ f_K^2 &= \frac{1}{2\sqrt{3}} \left(\sqrt{3} \ c_0 - \frac{1}{\sqrt{2}} \ c_8 + c_{15}\right) <u^0>_0 \end{aligned} \tag{46}$$

$$m_D^2 \; f_D^2 = \frac{1}{2\sqrt{3}}\left(\sqrt{3}\; c_0 + \frac{1}{\sqrt{2}}\; c_8 - c_{15}\right) < u^0 >_0$$

In the limit of the exact chiral $SU_2 * SU_2$ sub-symmetry there is the following constraint

$$\sqrt{3}\; c_0 + \sqrt{2}\; c_8 + c_{15} = 0 \tag{47}$$

so the pion is massless.

Let us make some remarks. The task of the Cabibbo angle calculation in SU_4 symmetry using the procedure of symmetry breaking has been done in [6]. In Ebrahim's earlier paper [5] the parameters of the $SU_4 * SU_4$ symmetry breaking have been found.

$$\frac{c_8}{c_0} = -\frac{2\sqrt{2}}{\sqrt{3}}\frac{m_K^2 f_K^2 - m_\pi^2 f_\pi^2}{m_K^2 f_K^2 + m_D^2 f_D^2} \qquad \frac{c_{15}}{c_0} = -\frac{1}{\sqrt{3}}\frac{3m_D^2 f_D^2 - m_K^2 f_K^2 - 2m_\pi^2 f_\pi^2}{m_K^2 f_K^2 + m_D^2 f_D^2} \tag{48}$$

In [6] the numerical values of parameters (48) have been used to calculate the rotation angle (interpreted as the Cabibbo angle). The $SU_2 * SU_2$ invariant Hamiltonian density breaking $SU_4 * SU_4$ symmetry has been rotated through an angle $2\,\Theta$ about the seventh axis and the coefficients of the operators u^a (a = 0, 8, 15) have been identified with the parameters of symmetry breaking

$$\begin{aligned} H_{SB}(\Delta S = 0) = c_0 u^0 + \frac{\sqrt{3}}{2} c_8 \sin^2\Theta\;\; u^3 + c_8\left(1 - \frac{3}{2}\sin^2\Theta\right)\; u^8 \\ - \left(\sqrt{3} c_0 + \sqrt{2} c_8\right)\; u^{15} \end{aligned} \tag{49}$$

It seems to us that there are some errors in the numerical calculations of the author. The use of the numerical values of the parameters (48) has not been necessary. On the grounds of theoretical formulas only, indeed from the Eq. (7) in Ref. [5] and the Eq. (10) in A3-Ebrahim, it follows that

$$\sin^2\Theta = \frac{2m_\pi^2 f_\pi^2}{2m_K^2 f_K^2 + m_\pi^2 f_\pi^2} \tag{50}$$

Then the value of Θ is given by

$$\sin^2\Theta = (0.215)^2 \tag{51}$$

instead of

$$\sin^2\Theta = -0.04 \tag{52}$$

from Eqs. (10) in [6]. Formula (50) has the same form as in SU_3 symmetry. In agreement with our expectation the angle Θ is described by parameters of the pion and the strange meson.

In Ebrahim's method the $SU_4 * SU_4$ symmetry breaking the Hamiltonian density is parametrized by the factors c_0, c_8, c_{15}. The parameters of symmetry breaking are expressed by the masses and decay constants of the mesons and they are fixed (Eq. (7) in [5]). In the limit of the exact $SU_2 * SU_2$ sub-symmetry the factors c_0, c_8, c_{15} should satisfy the constraint (47) but it is possible only if $m_\pi = 0$ namely the parameters of symmetry breaking are not expressed by real (measured in experiment) masses of mesons. In [6] Ebrahim breaks the $SU_4 * SU_4$ symmetry in the limit of the exact $SU_2 * SU_2$ sub-symmetry by the rotation of the $SU_2 * SU_2$ invariant Hamiltonian density through an angle 2 Θ about the seventh axis. The factors of the rotated Hamiltonian density are identified with the parameters of symmetry breaking (Eq. (7) in [5]). Solving a set of equations the author gets the factors c_0, c_8, c_{15} dependent on the rotation angle and on the real mesons masses already. The masses of mesons standing in the formula which describes the parameters of symmetry breaking are determined by the method of symmetry breaking and they have a real value for the real realization of the symmetry breaking only. In this case the rotation angle does not matter a parameter of the symmetry violation. It seems to us that such an interpretation is not satisfactory. The expression of meson masses as a function of the rotation angle (as a measure of symmetry violation) seems to be more natural. In the present paper the other interpretation of the symmetry breaking and the method of calculating the rotation angle is proposed. We describe our method as follows.

Before the $SU_4 * SU_4$ symmetry in the limit of the exact $SU_2 * SU_2$ sub-symmetry is broken the masses of mesons have been expressed by the factors c_0, c_8, c_{15} which satisfy the constraint (47). After symmetry breaking a new set of factors c_0', c_g', c_{15}' dependent on the old factors c_0, c_8, c_{15} and on the rotation angle is introduced. The new factors are identified with the coefficients by the operators u^i of the rotated Hamiltonian density (49).

$$c_0' = c_0 \qquad c_g' = c_8\left(1 - \frac{3}{2} sin^2\Theta\right) \qquad c_{15}' = -\sqrt{3}c_0 - \sqrt{2}c_8 \tag{53}$$

Meson masses are expressed by new factors and they are the function of the rotation angle as a measure of symmetry violation.

$$m_\pi^2 f_\pi^2 = \frac{1}{2\sqrt{3}}\left(\sqrt{3}c_0' + \sqrt{2}c_g' + c_{15}'\right) < u^0 >_0 = -\frac{\sqrt{3}}{2\sqrt{2}} c_8 \sin^2\Theta < u^0 >_0 \tag{54}$$

$$m_K^2 f_K^2 = \frac{1}{2\sqrt{3}}\left(\sqrt{3}c_0' - \frac{c_8'}{\sqrt{2}} + c_{15}'\right) < u^0 >_0 = -\frac{\sqrt{3}}{2\sqrt{2}} c_8 \left(1 - \frac{1}{2}\sin^2\Theta\right) < u^0 >_0$$

$$m_D^2 f_D^2 = \frac{1}{2\sqrt{3}}\left(\sqrt{3}c_0' + \frac{c_8'}{\sqrt{2}} - c_{15}'\right) < u^0 >_0 = \left(c_0 + \frac{\sqrt{3}}{2\sqrt{2}}\ c_8\left(1 - \frac{1}{2}\sin^2\Theta\right)\right) < u^0 >_0$$

It seems to be more natural that the meson masses are functions of the parameters of symmetry breaking (54) than inversely the parameters of symmetry breaking are functions of meson masses which are not consistent with the experimental data and are dependent on the method of symmetry breaking. This interpretation is consistent with the fact that the mass generation of the mesons is a consequence of symmetry breaking. From Eq. (54) we obtain the formula for the angle Θ as in Eq. (50). Let us consider the other variant of symmetry breaking described in [6]. Ebrahim, using his method, broke the $SU_4 * SU_4$ symmetry in the limit of the exact $SU_3 * SU_3$ symmetry by the rotation of $SU_3 * SU_3$ invariant Hamiltonian density about the fourteenth axis in the SU_4 space. The rotation angle Θ' is identified with the Cabibbo angle. The formula describing the angle Θ' should be given as follows

$$\sin^2\Theta' = \frac{3m_K^2 f_K^2}{2\left(m_K^2 f_K^2 + m_D^2 f_D^2\right)} \tag{55}$$

(in Eqs. (4a) in [8] there is the factor 3/2). The rotation of the Hamiltonian density about the fourteenth axis is considered in [14] too. The D meson is interpreted as a Goldstone boson. Putting aside the agreement of the numerical value of the angle Θ' with the experimental data it seems to us that the angle connected with the rotation about the fourteenth axis cannot be interpreted as the Cabibbo angle, because the rotation is performed inside the doublet (s, c). Then the states with the different electric charges are mixed. The interpretation that the D meson is a Goldstone boson is also unsatisfactory. If the $SU_4 * SU_4$ symmetry is broken in such a way that the $SU_2 * SU_2$ sub-symmetry is still exact, so the K meson becomes massive but the pion is still massless. Such a symmetry breaking cannot be accepted, results contradict the experimental data. The next breaking of the exact $SU_3 * SU_3$ symmetry is connected with the mixing of s and c quarks. The rotation angle cannot be interpreted as the Cabibbo angle for the reasons given above. It seems that the hierarchy of symmetry breaking is extended and the breaking of the $SU_4 * SU_4$ symmetry taken as a whole cannot be connected with the Cabibbo angle. This is possible, however, for $SU_4 * SU_4$ symmetry breaking in the limit of exact $SU_2 * SU_2$ sub-symmetry. Then results are in agreement with our expectation.

Our method described above is used· to calculate an angle ϕ which is connected with the rotation about the tenth axis in SU_4 space. Then the $SU_4 * SU_4$ symmetry is broken by the rotation of the $SU_2 * SU_2$ invariant Hamiltonian density through an angle 2ϕ about the tenth axis.

$$H_{SB}(\Delta C = 0) = c_0 u^0 + \left(\sqrt{2}c_0 + \frac{\sqrt{3}}{2}c_8\right)\sin^2\phi \; u^3 + \left(c_8 \; \left(\frac{1}{2\sqrt{2}}\left(\frac{4}{\sqrt{3}}c_0 + \sqrt{2}c_8\right)\sin^2\phi\right) \; u^8 + \right.$$
$$\left. + \left(-\sqrt{3}\; c_0 + \sqrt{2}\; c_8 + \left(\frac{4}{\sqrt{3}}c_0 + \sqrt{2}c_8\right)\sin^2\phi\right) \; u^{15} \right. \tag{56}$$

Using the factors from the Hamiltonian density (56) the masses of mesons are given as follows

$$m_\pi^2 f_\pi^2 = \left(c_0 + \frac{\sqrt{6}}{4}\right)\sin^2\phi \;\; < u^0 >_0 \tag{57}$$

$$m_K^2 f_K^2 = -\left(\frac{\sqrt{6}}{4} - \frac{1}{2}\left(c_0 + \frac{\sqrt{6}}{4}c_8\right)\sin^2\phi\right) < u^0 >_0$$

$$m_D^2 f_D^2 = \left(1 - \frac{1}{2}\sin^2\phi\right)\left(c_0' + \frac{\sqrt{6}}{4}c_8\right) < u^0 >_0$$

so

$$\sin^2\phi = \frac{2m_\pi^2 f_\pi^2}{2m_D^2 f_D^2 + m_\pi^2 f_\pi^2} \tag{58}$$

In agreement with our expectation the angle ϕ is a function of the mass of the pion (as a measure of the $SU_2 * SU_2$ violation) and is connected with the parameters of the charmed meson (mixing in (u, c) doublet). For the mass m_D = 1862 MeV and f_D/f_π = 0.974 [13] one gets

$$\sin^2\phi = 0.076 \tag{59}$$

The small value of the angle ϕ is the effect of the large mass of the charmed quark. From (59) results that only mixing in the (u, c) system is excluded, the value of the angle ϕ contradicts the experimental data. The simultaneous mixing in both doublets are, however, still possible. Fritzsch [15] considers also the mixing in (u, c) system. The mixing angle is calculated on the grounds of quark masses and does not contradict the results obtained above. Although the value of the angle ϕ is relatively small, it is significant: the sum of the angles $\Theta + \phi$ is larger than the value of the angle measured experimentally, called Cabibbo angle. This fact cannot be explained by the limits of experimental errors. Let us note that the angles (50) and (58) are calculated for the case where quarks are mixed separately. The angles from formula (32) cannot be identified with those from Eqs. (50) and (58). In the case of simultaneous mixing in both doublets the relation between the angles is more complicated. To find the relation, the $SU_2 * SU_2$ invariant Hamiltonian density is rotated through an angle 2ϕ about the tenth axis and afterwards by an angle 2Θ about the seventh axis. The sequence of the rotations is insignificant, because

$$\left[F^7, F^{10}\right] = 0 \tag{60}$$

The rotated Hamiltonian density is given by

$$\begin{aligned} H_{SB}(\Delta S = \Delta C = 0) &= c_0 u^0 + \left(\frac{\sqrt{3}}{2}\ c_8\ \ \sin^2\Theta + \left(\sqrt{2}\ c_0 + \frac{\sqrt{3}}{2} c_8\right) \sin^2\phi\right) u^3 + \qquad (61) \\ &+ \left(\frac{1}{\sqrt{3}}\left(\sqrt{2} c_0 + \frac{\sqrt{3}}{2} c_8\right) \sin^2\phi + c_8\left(1 - \frac{3}{2}\sin^2\Theta\right)\right)\ u^8 + \\ &+ \left(-\sqrt{3}\ c_0 - \sqrt{2}\ c_8 + \frac{2\sqrt{2}}{\sqrt{3}}\left(\sqrt{2} c_0 + \frac{\sqrt{3}}{2} c_8\right) \sin^2\phi\right)\ u^{15} \end{aligned}$$

The meson masses are given as follows

$$\begin{aligned} m_\pi^2 f_\pi^2 &= \frac{1}{2\sqrt{3}}\left(\sqrt{6}\left(\sqrt{2}\ c_0 + \frac{\sqrt{3}}{2}\ c_8\right) \sin^2\phi - \frac{3}{\sqrt{2}}\ c_8\ \ \sin^2\Theta\right)\ <u^0>_0 \qquad (62) \\ m_K^2 f_K^2 &= \frac{1}{2\sqrt{3}}\left(\frac{3}{\sqrt{6}}\left(\sqrt{2}\ c_0 + \frac{\sqrt{3}}{2}\ c_8\right) \sin^2\phi - \frac{3}{\sqrt{2}}\ c_8\ \left(1 - \frac{1}{2}\sin^2\Theta\right)\right)\ <u^0>_0 \\ m_D^2 f_D^2 &= \frac{1}{2\sqrt{3}}\left(2\sqrt{3}\ c_0 - \frac{3}{\sqrt{6}}\ \left(\sqrt{2} c_0 + \frac{\sqrt{3}}{2} c_8\right) \sin^2\phi + \frac{3}{\sqrt{2}}\ c_8\ \left(1 - \frac{1}{2}\sin^2\Theta\right)\right)\ <u^0>_0 \end{aligned}$$

Now the angles Θ and ϕ cannot be described independently. The following relation is obeyed.

$$\begin{aligned} &2\ m_\pi^2 f_\pi^2 + 2\ \left(m_K^2 f_K^2 + m_D^2 f_D^2\right)\ \sin^2\Theta\ \ \sin^2\phi = \qquad (63) \\ &= \left(2\ m_K^2 f_K^2 + m_\pi^2 f_\pi^2\right)\ \sin^2\Theta + \left(2\ m_D^2 f_D^2 + m_\pi^2 f_\pi^2\right)\ \sin^2\phi \end{aligned}$$

or equivalently

$$1 + \left(\frac{1}{\sin^2\Theta} + \frac{1}{\sin^2\phi} - 1\right) \sin^2\Theta\ \ \sin^2\phi = \frac{\sin^2\Theta}{\sin^2\Theta_0} + \frac{\sin^2\phi}{\sin^2\phi_0} \tag{64}$$

where

$$\sin^2\Theta_0 = \frac{2m_\pi^2 f_\pi^2}{2m_K^2 f_K^2 + m_\pi^2 f_\pi^2} \qquad \sin^2\phi_0 = \frac{2m_\pi^2 f_\pi^2}{2m_D^2 f_D^2 + m_\pi^2 f_\pi^2} \tag{65}$$

The angles Θ and ϕ from Eq. (64) concern a simultaneous mixing in doublets (d, s) and (u, c) respectively and they can be identified with those from Eq. (32). The condition (64) limits the values of the angles Θ and ϕ. The maximal values of the angles Θ_0 and ϕ_0 are given by Eq. (65). The value of the function

$$f(\Theta, \phi) = \sin(\Theta + \phi) \tag{66}$$

is also limited. A numerical calculation shows that there is an extremum (a maximum) of function (66) on the condition (64) for

$$\Theta_m = 0.20452 \qquad \phi_m = 0.02575 \qquad \sin(\Theta_m + \phi_m) = 0.2282 \tag{67}$$

It is worth noticing that the extremum of function (66) on condition (64) can be identified with the measured Cabibbo angle. It is not excluded that symmetry breaking is realized in the maximal allowed case, so the effective angle of mixing would correspond to the maximum of the function (66).

3. Bonds for the Kobayashi-Maskawa mixing parameters in a model with hierarchical symmetry breaking

A simultaneous mixing in (d, s) and (u, c) sectors has also been taken into account [15, 16], but due to the large mass of the c quark, the influence of the mixing in the (u, c) sector can be treated as a perturbation. At the six-quark level the quark mixing is described by three Cabibbo-like flavor mixing angles and the phase parameter responsible for CP-non-conservation [17]. The charged weak current in the $SU_6 * SU_6$ chiral symmetry

$$J_\mu = (\overline{u}, \overline{c}, \overline{t}) \ \gamma_\mu \ (1 - \gamma_5) \ U \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{68}$$

is described by a unitary matrix U, which can be put in 21 different forms [18], however only the standard Kobayashi-Maskawa matrix [19] will be used further.

$$U = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & -c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & -c_1 s_2 s_3 + c_2 s_3 e^{i\delta} \end{pmatrix} \tag{69}$$

where $s_i = \sin\Theta_i$, $c_i = \cos\Theta_i$.

The matrix (69) can be expressed as follows

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} \tag{70}$$

$$U = U_2 \ U_\delta \ U_1 \ U_3 \tag{71}$$

and it can mix quarks either in the negative or in the positive electric charge subspace.

A simultaneous mixing in both spaces was also considered [10]. From the form of the matrix (70) the following variants of the quark mixing are allowed:

$$A \;:\; U = U_2(s-b)\; U_\delta\; U_1(d-s)\; U_3(s-b) \tag{72}$$

$$B \;:\; U = U_2(c-t)\; U_\delta\; U_1(d-s)\; U_3(s-b) \tag{73}$$

$$C \;:\; U = U_2(c-t)\; U_\delta\; U_1(u-c)\; U_3(s-b) \tag{74}$$

$$D \;:\; U = U_2(c-t)\; U_\delta\; U_1(u-c)\; U_3(c-t) \tag{75}$$

where $U_k(x-y)$ denotes the mixing of x and y quarks by the matrix U_k. It is known that the Cabibbo angle cannot be explained by the mixing in the (u-c) sector only [15, 16], so the variants C and D must be rejected. Let us examine the variant B.

The charged weak current (68) with the matrix (69) for the variant B can be expressed as follows

$$J_\mu = R\; J_\mu(0)\; R^{-1} \tag{76}$$

where

$$J_\mu(0) = (\overline{u}, \overline{c}, \overline{t})\;\gamma_\mu\;(1-\gamma_5)\; I \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{77}$$

$$R = e^{-2i\,\Theta_3\,Q^{21}} e^{-2i\,\Theta_1\,Q^{7}} e^{-i\,\delta X} e^{2i\,\Theta_2\,Q^{32}} \tag{78}$$

$$X = \frac{4}{\sqrt{10}}\,Q^{24} - \frac{1}{\sqrt{15}}\,Q^{35} \tag{79}$$

where Q^k is the $6*6$ matrix representation of the k-th generator of SU_6 group. To get the values of the angles Θ_i the Gell-Mann-Oakes-Renner model will be used [9]. If the electromagnetic mass splitting of u-d quarks is neglected the Hamiltonian density breaking the chiral $SU_6 * SU_6$ symmetry is given as follows

$$H_0 = c_0 u^0 + c_8 u^8 + c_{15} u^{15} + c_{24} u^{24} + c_{35} u^{35} \tag{80}$$

where $c_0, \ldots, c_{35}$ are the symmetry breaking parameters, u^i (i = 0,1, ..., 35) are the scalar components Of the $(\overline{6}, 6) + (6, \overline{6})$ representation of the chiral $SU_6 * SU_6$ group. From the GMOR model, neglecting the vacuum expectation values of operators u^k (k = 8, 15, 24, 35) and the spectral density ρ^{ab} as proportional to the squared parameters of the symmetry breaking [12, 16], we get the approximate relation for masses of the pseudo-scalar mesons

$$m_a^2\; f_a^2\; \delta^{ab} = \frac{1}{\sqrt{3}} \left(\frac{c_0}{\sqrt{3}} + c_8\; d_{a8b} + c_{15}\; d_{a15b} + c_{24}\; d_{a24b} + c_{35}\; d_{a35b} \right) < u^0 >_0 \tag{81}$$

where f_a are the decay constants, d_{aib} - symmetric constants of the SU_6 group, $< u^0 >_0$ - the vacuum expectation value of the operator u^0. From (81) we obtain

$$\pi = m_\pi^2 \ f_\pi^2 \ = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} + \frac{c_8}{\sqrt{3}} + \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right) \ < u^0 >_0 \tag{82}$$

$$K = m_K^2 \ f_K^2 \ = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} - \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right) \ < u^0 >_0 \tag{83}$$

$$D = m_D^2 \ f_D^2 \ = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} + \frac{c_8}{2\sqrt{3}} - \frac{c_{15}}{\sqrt{6}} + \frac{c_{24}}{\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right) \ < u^0 >_0 \tag{84}$$

$$B = m_B^2 \ f_B^2 \ = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} + \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{2\sqrt{6}} - \frac{3c_{24}}{2\sqrt{10}} + \frac{c_{35}}{\sqrt{15}}\right) \ < u^0 >_0 \tag{85}$$

$$T = m_T^2 \ f_T^2 \ = \frac{1}{\sqrt{3}}\left(\frac{c_0}{\sqrt{3}} + \frac{c_8}{2\sqrt{3}} + \frac{c_{15}}{2\sqrt{6}} + \frac{c_{24}}{2\sqrt{10}} - \frac{2c_{35}}{\sqrt{15}}\right) \ < u^0 >_0 \tag{86}$$

By the symmetry breaking, the massless quark x can become massive if it is mixed with the other massive y. The rotation angle is then described by the masses of pseudo-scalar mesons. If the $SU_n * SU_n$ symmetry with the exact $SU_k * SU_k$ subsymmetry is broken to the exact $SU_{k-1} * SU_{k-1}$ symmetry, the rotation angle is a function of masses of a pseudo-scalar meson belonging to n-multiplet of the $SU_n * SU_n$ group and the meson which has become massive [19]. We demand the quarks to become massive due to the hierarchical symmetry breaking, so the highest exact symmetry of the Hamiltonian density, which Can be assumed, is $SU_4 * SU_4$ (at least one quark in the each sector must be massive). Oakes and the others [4, 20, 21] in order to get the Cabibbo angle value in the $SU_3 * SU_3$ or $SU_4 * SU_4$ symmetry, have rotated the Hamiltonian density breaking the chiral symmetry in the same way as the weak charged current. In a model with hierarchical symmetry breaking such a procedure cannot be used. Let us notice that from the form (5) of the rotation operator R it follows that the quarks are mixed in the following sequence: (c-t), a phase rotation, (d-s), (s-b), so for the exact $SU_4 * SU_4$ symmetry the massless quarks d and s would be mixed as the first (in the negative electric charge subspace) and then the generation of their masses would not be possible. The quark s would become massive in the next stage of the symmetry breaking after the mixing with the massive quark b. So, in order to get the massive both d and s quarks, they should be mixed in the inverse sequence. In the first stage of the symmetry breaking the exact $SU_4 * SU_4$ symmetry is broken to the exact $SU_2 * SU_2$ symmetry, in the 2^{nd} stage even the $SU_2 * SU_2$ symmetry is no longer exact. The next mixing stages are connected either with the mass generation of the c quark (variant B) or with the repeated mixing of massive s and b quarks (variant A). In our procedure the Hamiltonian density breaking the chiral $SU_6 * SU_6$ symmetry will be rotated in the inverse sequence in comparison with the rotation of the weak charged current.

$$H_{SB} = R_1 \ H_0 \ R_1^{-1} \tag{87}$$

where

$$R_1 = e^{2i \ \Theta_2 \ Q^{32}} e^{-iX \ \delta} e^{-2i \ \Theta_1 \ Q^7} e^{-2i \ \Theta_3 \ Q^{21}} \tag{88}$$

The exact $SU_4 * SU_4$ symmetry implies the following relations

$$c_8 = c_{15} = 0 \qquad \sqrt{5} \ c_0 + c_{35} = 0 \tag{89}$$

So, the $SU_4 * SU_4$ invariant Hamiltonian density is given as

$$H_E = c_0 \left(u^0 - \sqrt{5}\ u^{35}\right) + c_{24} \left(u^{24} - \sqrt{\frac{3}{2}}\ u^{35}\right) \tag{90}$$

or equivalently

$$H_E = P\ \overline{q}_6\ q_6 - V\ \overline{q}_5\ q_5 \tag{91}$$

where

$$P = \sqrt{c_0} + V \qquad V = \frac{5}{\sqrt{10}}\ c_{24} \tag{92}$$

The symmetry-breaking Hamiltonian density

$$H_{SB} = R_1\ H_E\ R_1^{-1} \tag{93}$$

retaining the flavor-conservation part only is given as follows

$$H_{SB} = \overline{q}_6 q_6 P c_2^2 - \overline{q}_5 q_5 V c_3^2 + \overline{q}_4 q_4 P s_2^2 - \overline{q}_3 q_3 V c_1^2 s_3^2 - \overline{q}_2 q_2 V s_1^2 s_3^2 \tag{94}$$

Let us notice that the phase transformation does not produce terms $\overline{q}_i q_i$ since the operator (79) commutes with the scalar components u^k. The flavor-conservation on each stage of the symmetry breaking has been assumed. The Hamiltonian density (94) can be written as a function of the operators if, so the coefficients of $u^{k\prime}$s are given as

$$c_0' = c_0 \tag{95}$$

$$c_{34}' = \frac{V}{2}\ s_1^2\ s_3^2 \tag{96}$$

$$c_8' = \frac{V}{2\sqrt{3}}\ \left(2\ c_1^2\ s_3^2 - s_1^2\ s_3^2\right) \tag{97}$$

$$c_{15}' = -\frac{1}{2\sqrt{6}}\ \left(3\ P\ s_2^2 + V\ s_3^2\right) \tag{98}$$

$$c_{24}' = \frac{1}{2\sqrt{10}}\ \left(4\ V - 5\ V\ s_3^2 + P\ s_2^2\right) \tag{99}$$

$$c_{35}' = -\frac{1}{2\sqrt{15}}\ \left(5\ P + V - 6\ P\ s_2^2\right) \tag{100}$$

Now, after the symmetry breaking, the pseudo-scalar masses (82–86) will be described as functions of the coefficients c_i' (i = 0, 3, 8, 15, 24, 35) [7, 16].

$$\pi = Z\ V\ s_1^2\ s_3^2 \tag{101}$$

$$K = Z\ V\ s_3^2\ \left(1 - \frac{1}{2}\ s_1^2\right) \tag{102}$$

$$D = -Z\ \left(P\ s_2^2 - \frac{1}{2}\ V\ s_1^2\ s_3^2\right) \tag{103}$$

$$B = Z\ V\ \left(1 - s_3^2\ \left(1 - \frac{1}{2}\ s_1^2\right)\right) \tag{104}$$

$$T = -Z \left(P\ c_2^2 - \frac{1}{2}\ V\ s_1^2\ s_3^2 \right) \tag{105}$$

where

$$Z = -\frac{1}{2\sqrt{3}}\ \ < u^0 >_0 \tag{106}$$

The Cabibbo angle Θ_1 is expressed in the same form as at four-quark level in the $SU_4 * SU_4$ symmetry [7, 16],

$$s_1^2 = \frac{\pi}{2\ K + \pi} \tag{107}$$

Because

$$s_1^2\ s_3^2 = \frac{\pi}{K + B} \quad \Rightarrow \quad s_3^2 = \frac{K + \frac{\pi}{2}}{K + B} \tag{108}$$

In an agreement with our prediction the angle Θ_3 connected with the mixing of s and b quarks is expressed by the parameters of the strange and beautiful mesons. The angle Θ_1 however, connected with mixing d and s quarks and breaking of the $SU_2 * SU_2$ symmetry is expressed by the masses of the pion and the kaon. The angle Θ_2 connected with the mixing in the (c-t) sector is given as

$$s_2^2 = \frac{D - \frac{\pi}{2}}{D + T - \pi} \tag{109}$$

Let us notice that if we do not demand the flavor-conservation on each stage of the symmetry breaking, after the rotation around the 21^{st} axis the terms $\overline{q}_5 q_3$, $\overline{q}_3 q_5$ in the broken Hamiltonian density arise. In the second stage (the rotation around the 7^{th} axis) there will be in H_{SB} the following terms: $\overline{q}_5 q_3$, $\overline{q}_3 q_5$, $\overline{q}_2 q_3$, $\overline{q}_3 q_2$, $\overline{q}_5 q_2$, $\overline{q}_2 q_5$. Because

$$\left[X, \overline{q}_3\ q_5\right] = i\ \delta\ \overline{q}_3\ q_5 \qquad \left[X, \overline{q}_5\ q_3\right] = -i\ \delta\ \overline{q}_5\ q_3 \tag{110}$$

after the phase rotation there will arise in the H_{SB} the following terms: $\overline{q}_3\ q_5\ e^{i\delta}$, $\overline{q}_5\ q_3\ e^{-i\delta}$, ... In the variant B the matrix U_2 has mixed c and t quarks so that in the flavor-conservation part of the broken Hamiltonian density the phase factor $e^{i\delta}$ cannot appear. But if the matrix U_2 mixes s and b quarks again, due to the following relations

$$e^{-2i\Theta_2 Q^{21}} \overline{q}_3 q_5 e^{2i\Theta_2 Q^{21}} = \overline{q}_3 q_5 c_2^2 - \overline{q}_5 q_3 s_2^2 + \frac{1}{2}\left(\overline{q}_5 q_5 - \overline{q}_3 q_3\right)\ \sin 2\Theta_2 \tag{111}$$

$$e^{-2i\Theta_2,\ Q^{21}} \overline{q}_5 q_3 e^{2i\Theta_2 Q^{21}} = \overline{q}_5 q_3 c_2^2 - \overline{q}_3 q_5 s_2^2 + \frac{1}{2}\left(\overline{q}_5 q_5 - \overline{q}_3 q_3\right) \sin 2\Theta_2 \tag{112}$$

the following terms: $\overline{q}_5\ q_5\ e^{i\delta}$, $\overline{q}_5\ q_5\ e^{-i\delta}$, $\overline{q}_3\ q_3\ e^{i\delta}$, $\overline{q}_3\ q_3\ e^{-i\delta}$ appear in the broken Hamiltonian density.

We assume that the symmetry is broken by the quarks mixing in the following sequence: (s-b), (d-s), a phase rotation, (s-b) and the flavor will not be conserved in the intermediate stages of the symmetry breaking, but it will be conserved in the broken symmetry taken as a whole. The assumptions given above are consistent with the variant A. Let us take it into account.

We assume the exact $SU_4 * SU_4$ symmetry. The Hamiltonian density is given by Eq. (91). After symmetry breaking, the flavor conserving part of the broken Hamiltonian density H_{SB} is given as

$$H_{(\Delta F=0)} = \overline{q}_6 q_6 P - \overline{q}_5 q_5 V(\alpha - A) - \overline{q}_3 q_3 V(\beta + A) - \overline{q}_2 q_2 V\gamma \tag{113}$$

where

$$\alpha = c_1^2 s_2^2 s_3^2 + c_2^2 c_3^2 \tag{114}$$

$$\beta = c_1^2 c_2^2 s_3^2 + s_2^2 c_3^2 \tag{115}$$

$$\gamma = s_1^2 s_3^2 \tag{116}$$

$$A = \frac{1}{2} \cos\Theta_1 \sin 2\Theta_2 \sin 2\Theta_3 \cos\delta \tag{117}$$

Since

$$\alpha + \beta + \gamma = 1 \tag{118}$$

α can be eliminated from (113).

The coefficients by the operators u^k are as follows

$$c_0' = c_0 \tag{119}$$

$$c_3' = \frac{V}{2}\gamma \tag{120}$$

$$c_8' = \frac{V}{2\sqrt{3}}(2\beta + 2A - \gamma) \tag{121}$$

$$c_{15}' = -\frac{V}{2\sqrt{6}}(\beta + A + \gamma) \tag{122}$$

$$c_{24}' = \frac{V}{2\sqrt{10}}(4 - 5(\beta + A + \gamma)) \tag{123}$$

$$c_{35}' = -\sqrt{5}c_0 - \sqrt{\frac{3}{2}}c_{24} \tag{124}$$

Let us notice that the functions β and A occur in Eqs. (121–123) as a sum $\beta + A$ only. So, only two functions can be expressed independently. Because there is no mixing in the positive electric charge subspace we shall not use relations describing mesons D and T. The following relations are obeyed

$$\pi = ZV\gamma \tag{125}$$

$$K = ZV\left(\beta + A + \frac{\gamma}{2}\right) \tag{126}$$

$$B = ZV\left(1 - \beta - A - \frac{\gamma}{2}\right) \tag{127}$$

so we immediately obtain

$$s_1^2 s_3^2 = \frac{\pi}{K+B} \tag{128}$$

as in the variant B, but at the moment the angles Θ_1 and Θ_3 cannot be calculated separately. Putting the experimental value

$$\cos\Theta_1 = 0.9737 \qquad (\sin\Theta_1 = 0.2278) \tag{129}$$

as an input [22], we get

$$\sin\Theta_3 = 0.136 \qquad (\Theta_3 = 7.8^\circ) \tag{130}$$

for

$$m_\pi = 0.139 \ \ GeV \tag{131}$$

$$m_K = 0.495 \ \ GeV \qquad f_K = 1.28 \quad [21] \tag{132}$$

$$m_B = 5.2 \ \ GeV \qquad f_B = 0.86 \quad [22] \tag{133}$$

The angle Θ_3 was calculated by Fritzsch [15] also for the following quark masses ratios:

$$m_u \ : \ m_d \ : \ m_s \ : \ m_c = 1 \ : 1.78 \ : \ 35.7 \ : 285 \tag{134}$$

and the limit for the angle Θ_2

$$\Theta_2 < \sqrt{\frac{m_c}{m_t}} = 0.33 \tag{135}$$

For the assumptions given above Fritzsch obtained the following boundary

$$\sin\Theta_3 < 0.09 \qquad (\Theta_3 < 5^\circ) \tag{136}$$

However there is no agreement between descriptions of the quark masses ratios. The other authors [23] give smaller difference between quark masses

$$m_u \ : \ m_d \ : \ m_s \ : \ m_c = 1 \ : 1.1 \ : \ 6.4 \ : 23.6 \tag{137}$$

Thus for the ratio (137) we get the following limit for the angle Θ_3

$$\sin\Theta_3 < 0.163 \tag{138}$$

The value of the angle Θ_3 (130) is consistent with the boundary (138). The value (130) is close to the value given by Białas [24] and consistent with results in [25, 26] as well as the experimental boundary:

$$\sin\Theta_3 < 0.42 \qquad [24] \tag{139}$$

$$|\sin\Theta_3| = 0.28 \begin{cases} +0.21 \\ -0.28 \end{cases} \qquad [19] \tag{140}$$

Let us consider the relation between the angle Θ_2 and the phase parameter δ. From (125)–(127) we obtain

$$\beta + A = \frac{K - \frac{\pi}{2}}{K + B} \tag{141}$$

or equivalently

$$\frac{K + \frac{\pi}{2}}{K + B} - \frac{\gamma}{s_1^2} = s_2^2 \left(1 + \gamma \left(1 - \frac{2}{s_1^2}\right)\right) + A \tag{142}$$

denoting

$$\xi = \frac{\left(K + \frac{\pi}{2}\right) - \frac{\pi}{s_1^2}}{K + B} \tag{143}$$

$$\eta = 1 + \frac{\pi}{K + B} \tag{144}$$

$$\rho = \frac{1}{2} \cos \Theta_1 \sin 2\Theta_3 \tag{145}$$

we get

$$\cos \delta = \frac{\xi - s_2^2 \eta}{\rho \sin 2\Theta_2} \tag{146}$$

It is worth noting that if we take the constraint on the Cabibbo angle Θ_1 from the four- quark level [7, 16], which is the same as given by Eq. (107), the parameter ξ (143) will be exactly equal to zero, hence we get

$$\cos \delta = -\frac{\eta}{2\rho} \tan \Theta_2 \tag{147}$$

Because $|cos\ \delta| \leq 1$, so from (147)

$$|\Theta_2| < \left| \arctan \frac{2\rho}{\eta} \right| \tag{148}$$

and we get also a boundary on the angle Θ_2

$$\sin \Theta_2 < 0.265 \qquad (\Theta_2 < 15.4^\circ) \tag{149}$$

The value (149) is in a good agreement with the results given by Fritzsch [15], Białas [24], Shrock, Treiman, Wang [22], Barger, Long, Pakvasa [25] and experimental limits [27], respectively:

$$9^\circ < \Theta_2 < 19^\circ \tag{150}$$

$$\sin \Theta_2 = 0.23 \tag{151}$$

$$|\sin \Theta_2| < 0.25 \qquad (m_t = 15 \ \text{GeV}) \tag{152}$$

$$\sin \Theta_2 < 0.5 \qquad (m_t = 30 \ \text{GeV}) \tag{153}$$

The Eq. (146) can be written as follows

$$x^2 \left(\eta^2 + 4 \rho^2 \cos^2 \delta\right) - 2x \left(\xi\eta + 2\rho^2 \cos^2 \delta\right) + \xi^2 = 0 \tag{154}$$

where

$$x = \sin^2\Theta_2 \tag{155}$$

To get a real value of the angle Θ_2 the determinant of the square Eq. (154) cannot be negative, so

$$16\rho^2\cos^2\delta\left(\xi\eta + \rho^2\cos^2\delta - \xi^2\right) \geq 0 \tag{156}$$

hence

$$1 \geq \cos^2\delta \geq \frac{\xi(\xi - \eta)}{\rho^2} \tag{157}$$

For (129, 130) we get

$$\eta = 0.9647 \qquad \rho = 0.1309 \tag{158}$$

If the parameter ξ. which can be identified with a change of the Cabibbo angle description by a transition to the higher symmetries, is slightly less than zero, the phase parameter δ will be bounded ($|\delta|$ should be nearly zero, as the Cabibbo angle description should not change strongly by a transition to higher symmetries, on the other hand the Eq. (157) gives a boundary on the parameter

$$\xi > -0.0175 \tag{159}$$

From (147)

$$\text{sign}(\cos\delta) = -\text{sign}(\tan\Theta_2) \tag{160}$$

so, for the angle Θ_2 lying in the first quadrant, it follows $\frac{\pi}{2} < \delta < \pi$ and from (159) there is a lower limit for the phase δ. For an input given by the Eqs. (129, 130, 132) we get

$$\xi = 0.002 \tag{161}$$

so there is no boundary on δ, since sign $\xi = +1$. Let us notice, that a small change of the f_K can change the sign of the parameter ξ. Following Fuchs [28], in a chiral perturbation theory at the $SU_3 * SU_3$ level

$$\frac{f_K}{f_\pi} = 1 + \frac{3\left(m_K^2 - m_\pi^2\right)}{64\pi^2 f_\pi^2}\ \ln\frac{\Lambda}{4\mu^2} + O(\varepsilon) \tag{162}$$

where μ^2 is the average meson squared mass and Λ is a cut-off parameter, which is estimated to be near $4m_N^2$, it implies

$$\frac{f_K}{f_\pi} = 1.15 \tag{163}$$

$$\xi = -0.00179 \tag{164}$$

$$\cos^2\delta > 0.1 \tag{165}$$

Taking into account (160) we obtain

$$109^\circ < \delta < 180^\circ \tag{166}$$

Since f_K is treated as a variable and can depend on the energy scale via Λ parameter and the symmetry breaking parameters ε, the boundary of the phase due to the Eqs. (143, 157) can be expected. For sign (cos δ) = −1 there is a lower limit of the angle Θ_2 also. A variant $\cos\delta > 0$ is allowed but the angle Θ_2 corresponding to this variant is too severely limited and it is not consistent with the experimental data [27].

We have shown that the weak mixing angles at the six-quark level can be estimated in terms of the masses of pseudo-scalar mesons. The calculation of mixing angles is possible by using the hierarchical symmetry breaking leading to a quark masses generation. A number of independent mixing angles that can be calculated on the ground of the given above model is equal to a number of degrees of freedom connected with the symmetry breaking and the quarks mixing in the fixed electric charge subspace (let us notice that in the variant B after the rotation around the 21^{st} axis and next around the 7^{th} one, even the exact $SU_2 * SU_2$ symmetry did not remain; however the angle Θ_2 connected with the mixing in the positive electric charge subspace could be calculated). An assumption that in the hierarchical symmetry breaking the flavor does not have to be conserved on each stage of the symmetry breaking, while it is conserved in the broken symmetry taken as a whole, has allowed the author to introduce to the broken Hamiltonian density a phase angle responsible for CP-non-conservation. The experimental value of the Cabibbo angle treated as an input has allowed the author to calculate the angle Θ_3 and to find the relation connecting the angle Θ_2 and the phase parameter δ. Limits of trigonometric functions values imply boundaries on the angle Θ_2 and the phase δ. The kaon decay constant is a sensitive parameter, which can introduce CP-non-conservation to the chiral perturbation theory. Boundaries for the angle Θ_2 and the phase δ vs. f_K can be also found.

4. The standard six-quark model with a hierarchical symmetry breaking

The simultaneous mixing of quarks in both negative and positive electric charge sub-spaces is considered. Quark mixing in each space is described by the Kobayashi-Maskawa matrix. In order to get a right number of independent mixing parameters only one angle θ_7 common for both sub-spaces has been adjusted. Since the electromagnetic mass splitting of u and d quarks has been taken into account the real K-M mixing angles can be calculated explicitly. As an input only meson masses and f_x factors (treated as factors in matrix elements between one meson state and vacuum according to PCAC) are needed. Physical quark mixing is realized for maximal allowed symmetry breaking and it corresponds to vanishing of θ_7, which implies that only quark mixings with mass generation are permitted. Bounds of the phase δ have been also found.

The Kobayashi-Maskawa mixing matrix (69) is usually considered to mix quarks in the negative electric charge subspace. It can be written also as (70) and it can mix quarks either in the negative or in the positive electric charge subspace. A simultaneous mixing in both spaces [13] was also taken onto account.

Let us assume that quarks are mixed in both sub-spaces simultaneously, then the charged weak current is expressed as follows

$$J_\mu = (\bar{u}, \bar{c}, \bar{t})\ \gamma_\mu\ (1-\gamma_5)\ U_+\ U_- \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{167}$$

where

$$U_+ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_5 & s_5 & 0 \\ -s_5 & c_5 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_2} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_6 & s_6 \\ 0 & -s_6 & c_6 \end{pmatrix} \tag{168}$$

$$U_- = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_7 & s_7 \\ 0 & -s_7 & c_7 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_1} \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix} \tag{169}$$

The matrices U_+ and U_- mix quarks in spaces with charges +2/3 and $-$ 1/3 respectively. The Kobayashi-Maskawa mixing matrix is parametrized by four independent parameters only, while in the product of the matrices (168) and (169) in the current (167) there are eight mixing angles. In order to get the effective mixing matrix in the K-M form with the right number of independent mixing parameters, we must adjust the angles in such a way as to get the effective matrix with only four independent angles. We shall demand the following elements U_{11}, U_{12}, U_{13}, U_{21}, U_{31} of the effective matrix to be real and the complex phase to exist in the elements U_{22}, U_{23}, U_{32}, U_{33} only, as in the original K-M matrix. The only solution is

$$\theta_6 = -\theta_7 \tag{170}$$

Hence in the matrix $U_+ U_-$ there will be effectively only four parameters: θ_2, $\theta_C = \theta_1 + \theta_5$, θ_3 and $\delta = \delta_1 + \delta_2$. The current (167) can be expressed as follows

$$J_\mu = R_2 \;\; R_1 \;\; J_\mu(0) \;\; R_1^{-1} \;\; R_2^{-1} \tag{171}$$

where

$$J_\mu(0) = (\overline{u}, \overline{c}, \overline{t}) \;\; \gamma_\mu \;\; (1 - \gamma_5) \;\; I \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{172}$$

$$R_1 = e^{-2i\,\theta_3\,Q^{21}} e^{-2i\,\theta_1\,Q^{7}} \;\; e^{-iX\delta_1} \;\; e^{-2i\,\theta_7\,Q^{21}} \tag{173}$$

$$R_2 = e^{-2i\,\theta_2\,Q^{32}} e^{-iY\delta_2} \;\; e^{-2i\,\theta_5\,Q^{10}} \;\; e^{-2i\,\theta_7\,Q^{32}} \tag{174}$$

where X = (79) and $Y = \frac{\sqrt{15}}{3} Q^{35}$. Q^k is the $6 * 6$ matrix representation of the k-th generator of SU_6 group. In variants A (72) and B (73), because of quark mixing in (d, s, b) sector only, the electromagnetic mass splitting of u and d quarks was neglected. For the simultaneous mixing in both (d, s, b) and (u, c, t) sectors the calculation of the angles θ_i explicitly is not possible (see below formulas (187) and (188). The Hamiltonian density breaking the chiral $SU_6 * SU_6$ symmetry is given as follows

$$H_0 = \sum_{j=1}^{6} \;\; c_{j^2-1} \;\; u^{j^2-1} \tag{175}$$

where c_i are the symmetry breaking parameters, u^i- the scalar components of the $(\overline{6}, 6) + (6, \overline{6})$ of the chiral $SU_6 * SU_6$ group. From the GMOR model we obtain the following relations for masses of pseudo-scalar mesons for $SU_6 * SU_6$ symmetry:

$$\pi = m_\pi^2\ f_\pi^2 = Z\ \left(\frac{c_0}{\sqrt{3}}\ + \frac{c_8}{\sqrt{3}}\ + \frac{c_{15}}{\sqrt{6}}\ + \frac{c_{24}}{\sqrt{10}}\ + \frac{c_{35}}{\sqrt{15}}\right) \tag{176}$$

$$K^+ = m_{K^+}^2\ f_{K^+}^2 = Z\ \left(\frac{c_0}{\sqrt{3}}\ + \frac{c_3}{2}\ - \frac{c_8}{2\sqrt{3}}\ + \frac{c_{15}}{\sqrt{6}}\ + \frac{c_{24}}{\sqrt{10}}\ + \frac{c_{35}}{\sqrt{15}}\right)$$

$$K^0 = m_{K^0}^2\ f_{K^0}^2 = Z\ \left(\frac{c_0}{\sqrt{3}}\ - \frac{c_3}{2}\ - \frac{c_8}{2\sqrt{3}}\ + \frac{c_{15}}{\sqrt{6}}\ + \frac{c_{24}}{\sqrt{10}}\ + \frac{c_{35}}{\sqrt{15}}\right)$$

$$D^+ = m_{D^+}^2\ f_{D^+}^2 = Z\ \left(\frac{c_0}{\sqrt{3}}\ - \frac{c_3}{2}\ + \frac{c_8}{2\sqrt{3}}\ - \frac{c_{15}}{\sqrt{6}}\ + \frac{c_{24}}{\sqrt{10}}\ + \frac{c_{35}}{\sqrt{15}}\right)$$

$$D^0 = m_{D^0}^2\ f_{D^0}^2 = Z\ \left(\frac{c_0}{\sqrt{3}}\ - \frac{c_3}{2}\ + \frac{c_8}{2\sqrt{3}}\ - \frac{c_{15}}{\sqrt{6}}\ + \frac{c_{24}}{\sqrt{10}}\ + \frac{c_{35}}{\sqrt{15}}\right)$$

$$B^+ = m_{B^+}^2\ f_{B^+}^2 = Z\ \left(\frac{c_0}{\sqrt{3}}\ + \frac{c_3}{2}\ + \frac{c_8}{2\sqrt{3}}\ + \frac{c_{15}}{2\sqrt{6}}\ - \frac{3c_{24}}{2\sqrt{10}}\ + \frac{c_{35}}{\sqrt{15}}\right)$$

$$T^+ = m_{T^+}^2\ f_{T^+}^2 = Z\ \left(\frac{c_0}{\sqrt{3}}\ - \frac{c_3}{2}\ + \frac{c_8}{2\sqrt{3}}\ + \frac{c_{15}}{2\sqrt{6}}\ + \frac{c_{24}}{2\sqrt{10}}\ - \frac{2c_{35}}{\sqrt{15}}\right)$$

In a model with hierarchical symmetry breaking the highest exact symmetry, which can be assumed, is the $SU_4 * SU_4$ one. At least one quark in each sector must be massive. Following the procedure described in [29] the Hamiltonian density breaking the chiral $SU_6 * SU_6$ symmetry will be rotated in the opposite direction by comparison with the rotation of the weak charged current.

$$H_{SB} = R_{21}\ R_{11}\ H_E\ R_{11}^{-1}\ R_{21}^{-1} \tag{177}$$

where

$$\begin{aligned} R_{11} &= e^{-2i\,\theta_7\,Q^{21}}\,e^{-iX\delta_1}\,e^{-2i\,\theta_1\,Q^7}\ \ e^{-2i\,\theta_3\,Q^{21}} \\ R_{21} &= e^{-2i\,\theta_7\,Q^{32}}\,e^{-iY\delta_2}\ \ e^{2i\,\theta_5\,Q^{10}}\ \ e^{2i\,\theta_2\,Q^{32}} \end{aligned} \tag{178}$$

The exact $SU_4 * SU_4$ symmetry implies that

$$c_3 = c_8 = c_{15} = \sqrt{5}c_0 + c_{35} = 0 \tag{179}$$

The $SU_4 * SU_4$ invariant Hamiltonian density is given as

$$H_E = P\overline{q}_6 q_6 - V\overline{q}_5 q_5 \tag{180}$$

where

$$P = \sqrt{12}c_0 + V \qquad\qquad V = \frac{5}{\sqrt{10}}c_{24} \tag{181}$$

We shall assume that in the model with hierarchical symmetry breaking the flavor will not be conserved in the intermediate stages of the symmetry breaking, but it will be conserved in the broken symmetry taken as a whole. The symmetry breaking Hamiltonian density retaining onlyÂ· the flavor-conserving part is given as follows

$$\begin{aligned} H_{(\Delta F=0)} = {} & \overline{q}_6 q_6 P(\lambda - M) - \overline{q}_5 q_5 V(\alpha - A) + \overline{q}_4 q_4 P(\rho + M) - \overline{q}_3 q_3 V(\beta + A) - \overline{q}_2 q_2 V\gamma \\ & + \overline{q}_1 q_1 P\tau \end{aligned} \tag{182}$$

where

$$\alpha = c_1^2 s_3^2 s_7^2 + c_3^2 c_7^2 \tag{183}$$
$$\beta = c_1^2 s_3^2 c_7^2 + c_3^2 s_7^2$$
$$\gamma = s_1^2 s_3^2$$
$$A = \frac{1}{2} \sin 2\theta_7 \sin 2\theta_3 c_1 \cos \delta_1$$

$$\lambda = s_2^2 c_5^2 s_7^2 + c_2^2 c_7^2 \tag{184}$$
$$\rho = s_2^2 c_5^2 c_7^2 + c_2^2 s_7^2$$
$$\tau = s_2^2 s_5^2$$
$$M = -\frac{1}{2} \sin 2\theta_7 \sin 2\theta_2 c_5 \cos \delta_2$$

The broken Hamiltonian density (182) can be expressed as a function of operators u^k (k = 0, 3, 8, 15, 24, 35). The coefficients of the operators u^k are as follows

$$c_0' = c_0 \tag{185}$$
$$c_3' = \frac{1}{2}(P\tau + V\gamma)$$
$$c_8' = \frac{1}{2\sqrt{3}}(P\tau + V(2\beta' - \gamma))$$
$$c_{15}' = \frac{1}{2\sqrt{6}}(P(\tau - 3\rho') - V(\beta' + \gamma))$$
$$c_{24}' = \frac{1}{2\sqrt{10}}(P(\tau + \rho') - V(5\beta' + 5\gamma - 4))$$
$$c_{35}' = \frac{1}{2\sqrt{15}}(6P(\tau + \rho') - 5P - V)$$

where

$$\beta' = \beta + A \qquad \rho' = \rho + M \tag{186}$$

After symmetry breaking the pseudo-scalar masses (176) will be described as functions of the coefficients c_i' [16].

$$\begin{aligned}
&\pi = \frac{Z}{2}(P\tau - V\gamma) && \\
&K^+ = \frac{Z}{2}(P\tau - V\beta') && K^0 = -\frac{Z}{2}V(\beta' + \gamma) \\
&D^0 = \frac{Z}{2}P(\rho' + \tau) && D^+ = \frac{Z}{2}(P\rho' - V\gamma) \\
&B^+ = \frac{Z}{2}(P\tau + V(\beta' + \gamma - 1)) && T^+ = \frac{Z}{2}(P(1 - \tau - \rho') - V\gamma)
\end{aligned} \tag{187}$$

From (183), (184), (186) and (187) we get

$$\beta' = \frac{K^0 + K^+ - \pi}{2B^+ + 3K^0 - K^+ - \pi} \qquad \gamma = \frac{K^0 - K^+ + \pi}{2B^+ + 3K^0 - K^+ - \pi}$$
$$\rho' = \frac{D^0 + D^+ - \pi}{2T^+ + 3D^0 - D^+ - \pi} \qquad \tau = \frac{D^0 - D^+ + \pi}{2T^+ + 3D^0 - D^+ - \pi} \tag{188}$$

(contrary to the case of mixing in (d, s, b) sector only (variant A in [29] the electromagnetic mass splitting of u and d quarks cannot be neglected; if we put arbitrarily $c_3 = 0$ in Eq. (175) as in variants A and B in [29], the parameters $\gamma, \beta'\tau, \rho'$ could not be calculated separately. We would obtain only three nonlinear relations connecting these parameters with meson masses). Since

$$\alpha + \beta + \gamma = \lambda + \rho + \tau = 1 \tag{189}$$

putting (183, 184, 186) to (188) and eliminating θ_2 and θ_3 from the obtained set of four equations we get

$$f_1(\theta_1, \delta_1) = \tan\theta_7 = -f_5(\theta_5, \delta_2) \tag{190}$$

where

$$f_1(\theta_1, \delta_1) = \frac{B_1 \mp \sqrt{B_1^2 - A_1 C_1}}{A_1} \tag{191}$$

$$A_1 = s_1^2(1 - \beta') - \gamma \qquad B_1 = \sqrt{\gamma}\sqrt{s_1^2 - \gamma}c_1 \, cos \, \delta_1 \qquad C_1 = \gamma - s_1^2(\beta' + \gamma) \tag{192}$$

$$f_5(\theta_5, \delta_2) = \frac{B_5 \mp \sqrt{B_5^2 - A_5 C_5}}{A_5} \tag{193}$$

$$A_5 = s_5^2(1 - \rho') - \tau \qquad B_5 = \sqrt{\tau}\sqrt{s_5^2 - \tau}c_5 \, cos \, \delta_2 \qquad C_5 = \tau - s_5^2(\rho' + \tau) \tag{194}$$

We considered in [16] the simultaneous mixing in (d, s) and (u, c) sectors in the $SU_4 * SU_4$ symmetry. The mixing angles Θ and ϕ could not be calculated separately, however the nonlinear formula connecting both angles and pseudo-scalar masses was found

$$2\pi + 2(K + D)\sin^2\Theta \sin^2\phi = (2K + \pi)\sin^2\Theta + (2D + \pi)\sin^2\phi \tag{195}$$

A numerical calculation showed that there is an extremum (a maximum) of the function (66) with condition (195) for the angles $\Theta_m + \phi_m$ very close to the experimentally measured Cabibbo angle. This fact suggests that the symmetry breaking is realized in the maximal allowed case, so the effective angle of mixing would correspond to the maximum of function (66). As in [16] we shall look for the extremum of the function

$$f(\theta_1, \theta_5) = \sin(\theta_1 + \theta_5) \tag{196}$$

with condition (190). The following set of equations must be obeyed

$$f_1(\theta_1, \delta_1) + f_5(\theta_5, \delta_2) = 0 \qquad \frac{\partial f_1(\theta_1, \delta_1)}{\partial \delta_1} = 0 \tag{197}$$

$$\frac{\partial f_1(\theta_1, \delta_1)}{\partial \theta_1} - \frac{\partial f_5(\theta_5, \delta_2)}{\partial \theta_5} = 0 \qquad \frac{\partial f_5(\theta_5, \delta_2)}{\partial \delta_2} = 0$$

From (197) we get

$$C_1 = 0 \qquad C_5 = 0 \tag{198}$$

respectively, which implies that the separation constant

$$\tan \theta_7 = 0 \tag{199}$$

This means that the maximal allowed symmetry breaking occurs only for independent mixing of quarks in both sectors.

Let us consider the action of the operators R_{11} and R_{21} on quarks. The operator R_{11} mixes quarks in the negative electric charge subspace in the following sequence: (s-b)(θ_3), (d-s)(θ_1), a phase rotation (δ_1), (s-b)(θ_7), however the operator R_{21} mixes quarks as follows: (c-t) (θ_2), (u-c)(θ_5). a phase rotation (δ_2), (c-t)(θ_7). By the exact $SU_4 * SU_4$ symmetry only b and t quarks are massive. After the symmetry breaking a massless quark can become massive if it mixes with the other massive one. By the mixing in the sector with the charge - 1/3 the quark s has become massive in the first stage of the hierarchical symmetry breaking, after mixing with the quark b (the rotation on the angle θ_3 generated by the operator Q^{21}), the quark d has become massive in the second stage after mixing with the already massive quarks (the rotation on the angle θ_1). The next rotation by the angle θ_7, and mixing of s and b quarks are not connected with the symmetry breaking, because the mixing quarks have been already massive. There is analogical situation in the sector with the charge +2/3. The c and u quarks have become massive due to the hierarchical symmetry breaking (rotations on angles θ_2 and θ_5, respectively), however the rotation by the angle θ_7 and mixing of c and t quarks are also not connected with the symmetry breaking. Thus, from (199) it results that the physical quark mixing is realized only in the symmetry breaking with the quark masses generation. Putting (199) to (183) and (184) and comparing with (188) we get

$$\sin^2\theta_1 = \frac{K^0 - K^+ + \pi}{2K^0} \qquad \sin^2\theta_5 = \frac{D^0 - D^+ + \pi}{2D^0} \tag{200}$$
$$\sin^2\theta_3 = \frac{2K^0}{2B^+ + 3K^0 - K^+ - \pi} \qquad \sin^2\theta_2 = \frac{2D^0}{2T^+ + 3D^0 - D^+ - \pi}$$

so $\theta_C = \theta_1 + \theta_5$ depends on the parameters of mesons belonging only to the SU_4 multiplet. Let us notice that in comparison to the variant A in [29], taking into account the quark mixing in the (u, c, t) sector allowed the author to calculate the Cabibbo angle from the model and the angles θ_2 and θ_3. Let us compare the value of the calculated angle $\theta_C = \theta_1 + \theta_5$ realized for the maximal symmetry breaking with the experimentally measured Cabibbo angle value [22].

$$\cos\theta = 0.9737 \pm 0.0025 \tag{201}$$

The well known values of meson masses were taken from [30]. f_π, f_{K^+}, ... were assumed as the factors in the matrix elements between one meson state and the vacuum according to PCAC, so for meson multiplets with the isospin 1/2 the factors for charged and neutral mesons are the same [31]. There exist many conjectures concerning the values of f_x. They widely differ in magnitude, depending on the particular approach to the estimation of the matrix element $< 0|v_x|x >$ and so far they have no reliable experimental support. Only in the case of f_K there is a fair consensus that the value is around 1.28 [12, 13, 31, 32]. For a calculation we took as f_D for comparison's sake values significantly different

$$f_D = 0.974 \quad [10] \qquad\qquad f_D = 0.65 \quad [29] \tag{202}$$

which gives

$$\cos\left(\theta_1 + \theta_5\right) = 0.9799 \qquad\qquad \cos\left(\theta_1 + \theta_5\right) = 0.9709 \tag{203}$$

respectively, very close to the experimental value (201), as in the case of the $SU_4 * SU_4$ broken symmetry [16]. It seems to us that such a well agreement in both $SU_4 * SU_4$ and $SU_6 * SU_6$ symmetries is not accidental and the symmetry breaking is indeed realized for the maximal allowed case.

Putting (198) to (197) we find the relation connecting both phase parameters

$$\cos\ \delta_2 = \xi\ \cos\ \delta_1 \tag{204}$$

where

$$\xi = \sqrt{\frac{\gamma\rho'\left(1-\gamma-\beta\right)\left(\rho'+\tau\right)}{\tau\beta'\left(1-\tau-\rho'\right)\left(\beta'+\gamma\right)}} \tag{205}$$

The effective phase parameter $\delta = \delta_1 + \delta_2$ is bounded for $\xi \neq 1$. Indeed, the Eq. (204) has solution only for

$$|\delta| \begin{cases} > \arccos \dfrac{1}{\xi} & \text{if } \left(\xi > 1\right) \\ > \arccos\left(\xi\right) & \text{if } \left(\xi < 1\right) \end{cases} \tag{206}$$

It is worth noticing that even for $\xi \to \infty$ or $\xi \to 0$ the second and third quadrant for δ is still allowed.

Appendix

From the Gell-Mann Oakes Renner model for $SU_6 * SU_6$ symmetry we obtain the following relation for masses of pseudo-scalar mesons

$$m_a^2 f_a^2 \delta^{ab} + \int \frac{dq^2}{q^2}\rho^{ab} = i < 0|\left[\overline{Q}^a, \overline{D}^b|0> \right. = \sum_{i=1}^{6}\left(\sum_{j=1}^{6} c_{j^2-1} d_{i^2-1,a,c} d_{j^1-1,b,c}\right) < u^{i^2-1} >_0 \tag{207}$$

where

$$\rho^{ab} = (2\pi)^3 \sum_{n\neq a} \delta^4\left(p_n - q\right) < 0|\overline{D}^a|n> < n|\overline{D}^b|0> \tag{208}$$

d_{abc} symmetric constants of the SU_6 group, $< u^i >_0$ - vacuum expectation value of the operator u^i, $Q^i \pm \overline{Q}^i = \int d3^x V_0^\alpha(x) \pm \int d^3x A_0^\alpha(x)$ - the generators of the $SU_6 * SU_6$ group

$$D^a = \partial^\mu V^a_\mu(x) \qquad \overline{D}^a = \partial^\mu A^a_\mu(x) \qquad [12] \tag{209}$$

Because the vacuum expectation values of operators u^i: i = 3, 8, 15, 24, 35 and the spectral density ρ^{ab} are proportion to the squared parameters Of symmetry breaking, they were neglected. Approximately we obtain

$$m_a^2 f_a^2 = \frac{1}{\sqrt{3}} \left(\sum_{j=1}^{6} c_{j^2-1} d_{j^2-1,a,a} \right) < u_0 >_0 \tag{210}$$

Because the symmetric constants of SU_6 group: $d_{113} = d_{223} = d_{333} = 0$ the masses of neutral and charged pions are not differentiated, however there is the electromagnetic mass splitting of the other meson multiplets (see Eq. (176)).

The experimental data [30] gives

$$\Delta m_K = m_{K^0} - m_{K^+} = 4.003 \ MeV \qquad \Delta m_D = m_{D^0} - m_{D^+} = -5.3 \ MeV \tag{211}$$

so

$$sign \ \ \Delta m_K = -sign \ \ \Delta m_D \tag{212}$$

Let us notice that from (17613) we get

$$sign \ \ (K^0 - K^+) = -sign \ \ (D^0 - D^+) \tag{213}$$

so the direction of the electromagnetic mass splitting by the factor c_3 responsible for this effect is consistent with the experimental data. On the other hand

$$sign \ \ \Delta m_\pi = m_{\pi^0} - m_{\pi^+} = -4.603 MeV \tag{214}$$

so the electromagnetic mass splitting of pions is of the same order as kaons or D mesons. It suggests that the neglected terms in approximate formula (207) are of the order of the factor c_3. This means that such an approximation does not generate error greater than the electromagnetic ma s splitting of pion in meson masses description.

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