Chapter

Using Electromagnetic Properties to Identify and Design Superconducting Materials

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Abstract

Superconductors have a wide array of applications, such as medical imaging, supercomputing, and electric power transmission, but superconducting materials only operate at very cold temperatures. Thus, the quest to engineer room temperature superconductors is currently a hot topic of research. To accomplish this mission, it is important to have a complete understanding of the material properties that are being used to create these superconductors. Understanding the atomic and electromagnetic properties of the prospective materials will provide tremendous insight into the best choice for the materials. Therefore, a theoretical model that incorporates electromagnetic field theory and quantum mechanics principles is utilized to explain the electrical and magnetic characteristics of superconductors. This model can be used to describe the electrical resistance response and why it vanishes at the material's critical temperature. The model can also explain the behavior of magnetic fields and why some superconducting materials completely exclude magnetic fields while other superconductors partially exclude these fields. Thus, this theoretical analysis produces a model that describes the behavior of both type I and type II superconductors. Since there are subtle differences between superconductors and perfect conductors, this model also accounts for this distinction and explains why superconductors behave differently than perfect conductors. Therefore, this theory addresses the major properties associated with superconducting materials and thus will aid researchers in the pursuit of designing room temperature superconductors.

Keywords: conductivity, permittivity, permeability, resistivity, resonance, Schrödinger wave equation

1. Introduction

Superconductors are materials in which electricity can flow indefinitely because electrons can move through the material without losing energy. Superconductivity is a state in which a material's electrical properties or characteristics are altered when the temperature reaches a sufficiently low value. This temperature is known as the material's critical temperature and when the material falls below this temperature, two phenomena will result. One event that occurs is the electrical resistance drops to zero (or electric fields inside the material must vanish). The other outcome which takes place is that magnetic fields diminish inside the material. For some materials the magnetic field becomes zero (if a field exists in the material prior to the temperature change it becomes zero, and if a field is applied after the temperature change, it will not enter the material) [1–5].

Superconductors are known to exhibit zero electrical resistance or infinite electrical conductivity when the temperature of the superconductor reaches its critical temperature. Electrical conductivity is well described on the macroscale and can be modeled by Ohm's Law. Although macroscale theories usually are not appropriate on a microscale, Ohm's Law remains valid at the atomic level [6]. Bloch's theorem states that for perfect periodic lattices, conduction electrons will lose no energy and thus experience zero resistance [7]. However, since it is impractical to obtain a perfect lattice and since some materials exhibit superconductivity and others do not demonstrate this behavior (e.g., gold, silver, copper), additional theories and/or explanations are needed to explain the phenomenon of superconductivity.

Research is currently underway to develop superconducting materials at room temperature. Electrical engineers, physicists, and material scientists are engaged in this research to understand, identify, and create materials that exhibit superconducting properties. Thus, if superconductivity research can advance this science and develop room temperature materials, many industries will be revolutionized [8–13].

One goal of superconductivity research is to identify and/or create materials to transmit energy or information efficiently and dependably. Because electrical resistance in wires will lead to wasted energy and information loss during transmission, superconductivity research is important in unlocking these mysteries to explain the nature of these materials. Successfully accomplishing the goal in these research areas will revolutionize electrical power transmission and information technology [8–10].

Another major goal in superconductivity research is to develop room temperature superconductors for use in medical imaging. High intensity magnetic fields are needed for MRI and NMR imaging and in order to produce these large fields, superconducting wires are used. The image quality as well as the cost to operate these imaging systems depends upon the use of superconducting magnets. Creating and maintaining the necessary large magnetic fields requires a substantial amount of energy. If this energy is consumed by the resistance in the wires, then the quality and/or operating cost will suffer. Achieving the goal of creating room temperature superconductors will further reduce the cost to operate and transform the medical industry [11–13].

Regarding electrical field properties, electrical resistivity is a fundamental material property that leads to electrical resistance in a material. Electrical resistance is a function of the dimensions of the material that is being utilized or analyzed, whereas electrical resistivity is based on atomic interactions with conduction electrons (thus resistivity is independent of the material dimensions). Electrical resistivity is a function of temperature and a material's resistivity will generally decrease as its temperature decreases [14]. When a material's temperature decreases, conduction electrons will interact less with lattice atoms and therefore they will lose less energy. When a superconductor enters the superconducting state and its electrical resistance vanishes, electrons will flow through that material unimpeded and current flows indefinitely [15].

Regarding magnetic field properties, the Meissner effect is a condition that results in magnetic fields vanishing inside the interior of superconductors (provided that the magnetic field is small). For these small magnetic fields, whether the field is present before or after the material is cooled below its critical temperature and whether the material is a type I or type II superconductor, that magnetic field will be expelled from the superconductor's interior. If the magnetic field increases and has a value between the upper and lower critical fields, type II superconductors will enter a mixed state in which a portion of the magnetic field will penetrate the superconductor's interior [1–5].

To truly understand why some materials are superconductors and why others are not, and to advance research in superconductivity, theoretical models are needed. Various microscopic or atomic theories have been developed to explain superconductivity [16]. Thermodynamic theories have been developed to explain certain aspects of superconductivity behavior, however, there have been inconsistencies in these theories or explanations [17]. Researchers understand that even though theories may be incorrect or partially correct, those theories can push the science forward and provide some insight into a material's behavior. For example, the BCS theory and the London equations are two of the most successful theories on superconductivity, but they have limitations in certain aspects of their explanations. In particular, the BCS theory was developed several decades before high temperature superconductors were discovered. As a result, it does well in describing the behavior of type I materials, but it is inadequate in explaining how high temperature superconductors operate [18–20]. Furthermore, the BCS theory is imperfect and insufficient in explaining several fundamental properties of superconducting materials such as the Meisner effect [18]. Again, a theory does not have to explain every aspect of a material's behavior, but if it cannot explain the fundamental properties, then it is inadequate. Therefore, a comprehensive theory that explains the electromagnetic properties of all superconducting materials is needed.

There is not a complete theory on superconductivity that explains the electromagnetic properties of type I and type II superconducting materials. Because these previously developed models are incomplete, one model may explain one aspect, but it cannot address another. A given theory can handle certain characteristics of superconductors, but it fails in other areas. So, if scientific research is going to make significant advances, a comprehensive theory is needed.

A general theoretical model has been developed to explain the relationship between conductivity and temperature [21, 22]. This model used atomic analysis and solid state physics principles to develop the theory and explain why electrical conductivity is dependent on temperature. The relationship between conductivity and temperature is first derived and then its accuracy is demonstrated through comparisons to known linear responses from platinum and nickel. Again, a model does not have to be entirely correct in order to be useful and move the science forward. This aforementioned model does not and did not intend to account for superconducting effects. Therefore, the model presented herein will provide missing pieces of the puzzle and demonstrate that it is sufficient to characterize properties of superconductors.

The theory presented here accounts for the electromagnetic properties of both type I and type II superconductors. The theoretical model has been obtained by using quantum mechanics and analyzing conduction electron interactions with atoms in a lattice. This analysis is then used to specify how the electrical resistance of a material will respond. Analysis reveals that certain conditions will allow electrons to move through a material without interference. Under these conditions, the material will act as a superconductor. This theoretical model will then be used to explain the Meisner effect or the response of superconducting materials to external magnetic fields. Since type I and type II superconducting materials display different characteristics, the model will be used to explain this difference. The theory can be used to explain the difference between perfect conductors and superconductors. Finally, the frequency spectrum of a generic material's dielectric response is analyzed to demonstrate the feasibility of this model in explaining superconductivity.

2. Why a new resistance model is needed

To determine the electrical resistivity of a material (or the electrical resistance since resistivity and resistance are proportional), Matthiessen's rule states that the total resistivity is comprised of the sum of the individual resistivities associated with electron interactions with lattice phonons as well as lattice imperfections [1]. In equation form the resistivity is $\rho = \rho_p + \rho_i$ and thus the equivalent electrical resistance equation is

$$R = R_p + R_i \tag{1}$$

where R_p is the resistance that arises due to phonons and is a function of temperature, and R_i is the resistance that arises due to lattice imperfections (e.g., defects, impurities, grain boundaries, etc.) and is independent of temperature. In essence, Matthiessen's rule is a 'series' approach to electrical resistance, but this approach alone cannot account for the electrical resistance of superconductors since all resistance terms would have to become zero in order for the total electrical resistance to become zero. **Figure 1** shows the individual resistances and the total or sum of these resistance for a typical material. This figure demonstrates that a series approach alone cannot explain electrical resistance of superconductors. As a result, modifications must be made to Matthiessen's rule or a different approach has to be taken to account for superconducting effects in materials. Rather than using a series concept to analyze and explain electrical resistance, a general approach using parallel concepts will be used.

The two-fluid model and the resistively shunted junction have been used to describe superconductivity and it incorporates a parallel approach to phenomenologically describe electrical resistance [23, 24]. These approaches use a 'regular' channel to represent normal or non-superconducting electrons and a 'superconductor' channel to represent superconducting or Cooper-paired electrons. It is presupposed that the Matthiessen series model can be incorporated into the regular channel and therefore the regular channel can model the non-superconducting response of materials. However, since Cooper-paired electrons cannot explain the behavior of high-temperature superconducting materials, these models need to be modified to include or reflect the underlying physics as well as the behavior of all types of superconductors.

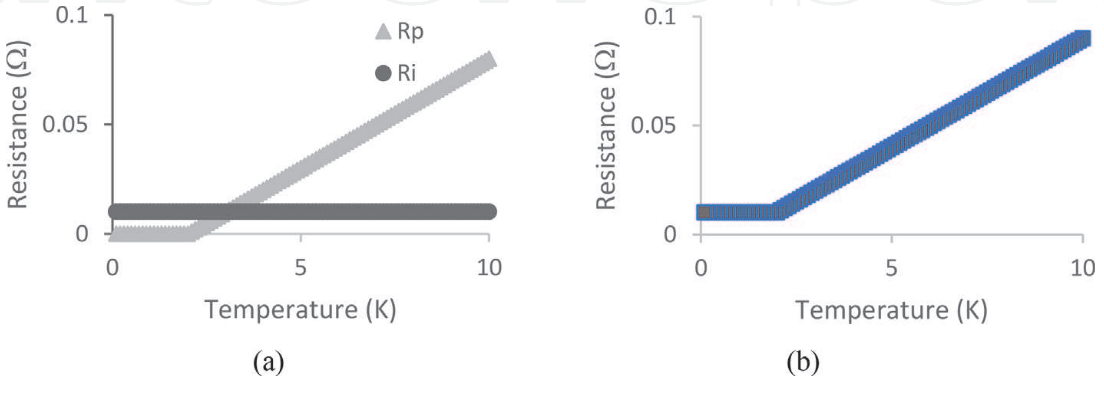


Figure 1.
(a) Example of electrical resistances for a material associated with conduction electron interaction with lattice atoms (phonons) R_p, as well as conduction electron interaction with lattice imperfections R_i. (b) Example of the total resistance using Matthiessen's rule in which the resistance is the sum of the individual resistances. Using Matthiessen's rule makes it impossible to explain why electrical resistance vanishes in superconductors.

3. Electrical resistance analysis

Information about this model has been developed and published [21, 22, 25, 26]. Since the conduction electrons shown in **Figure 2** travel down two different paths, they will encounter two different electrical resistances. Resistance R_1 will represent the resistance that conduction electrons experience if they are not in the space or pathway of atoms. Resistance R_2 will represent the resistance that conduction electrons experience if they are in the path of atoms. Since these resistors are in parallel, they have an equivalent resistance represented by R as given in Eq. (2).

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \tag{2}$$

To develop and understand the theoretical model, consider a generic atomic lattice shown in **Figure 2**. The model is a two-dimensional lattice structure with periodic atoms and conduction electrons that travel through the lattice. An electron will either travel through a path that contains atoms, or it will not. Atoms will vibrate at a rate that is a function of temperature. These vibrations will enlarge or expand the space that contains atoms or conversely, as temperature decreases, the space between atoms will increase. A higher proportion of conduction electrons in the space between atoms will lead to higher electrical conductivity. By analyzing a unit cell within the lattice, an equation can be obtained that represents the response for the entire lattice.

This model will yield an equation that describes the relationship between conductivity (or resistivity) and temperature. This model utilizes the concept that when electrons are in the space or path containing atoms, these atoms will always impede the flow of the electrons. In the case of superconducting materials at temperatures below their critical temperature, this will not be the case. So, in order to address superconductivity with this model, additional elements will be factored into the analysis.

When conduction electrons are in the region where they will only directly encounter other electrons, the electrical resistivity in this region will be temperature dependent and will have a response that linearly decreases as temperature decreases. A typical example of this response is shown in **Figure 3** where the

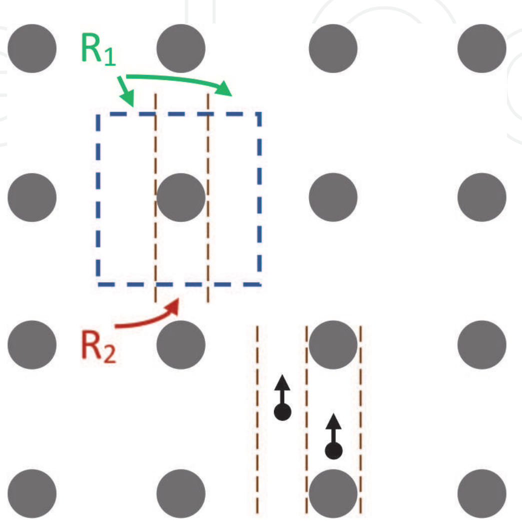


Figure 2.
Illustration of the atomic lattice showing electrons (small circles) and atoms (large circles). In a non-superconducting state, an electron will experience very little resistance (leftmost electron) if it travels between atoms (resistance R_1) and much resistance (rightmost electron) if it travels in the path of atoms (resistance R_2). These are the two cases that can occur within a unit cell (dotted box).

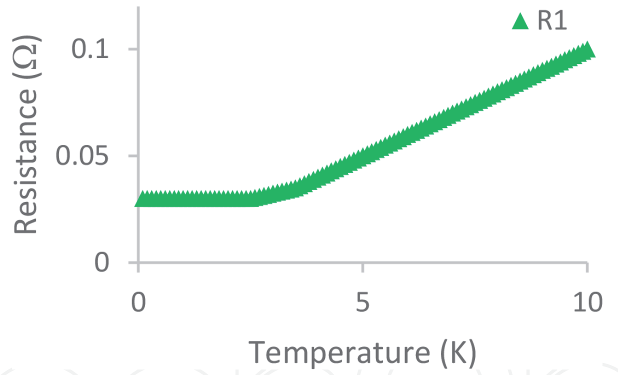


Figure 3.
Example of electrical resistance as a function of temperature for conduction electrons that travel in pathways between atoms and will not directly contact atoms. This graph represents the resistance of resistor R_1.

resistivity ρ and resistance R_1 generally decreases with decreasing temperature. Because of lattice defects, grain boundaries, and impurities, resistance R_1 will not reach zero when the temperature is zero, but it will have a residual value. This represents the resistance in the region where atoms do not exist regardless of whether the material is superconducting or non-superconducting.

Resistance R_2 represents electrons that will travel in regions where they will encounter atoms. This resistance will have two different responses depending upon whether the material is a superconductor or not. If the material is a superconductor, then at some critical temperature, the atoms will offer no resistance or they will be invisible to conduction electrons, but above this critical temperature, there will be a non-zero temperature dependent response. An example of this response is shown in **Figure 4a**. For non-superconducting materials, the electrical resistance R_2 will not become zero like it does for superconductors at the critical temperature. The resistance will have some temperature dependent response and have some non-zero value when the temperature reaches zero. An example of this response is shown in **Figure 4b**.

This evaluation provides an analysis of the resistances that can (and will) be used in the theoretical model. To fully incorporate superconducting effects into the theoretical model, the interaction between the atoms and conduction electrons must be examined. This will provide details that explain when and why certain materials become superconductors. The analysis of the conduction electron and atom interaction will be accomplished by utilizing the Schrödinger wave equation and quantum mechanics.

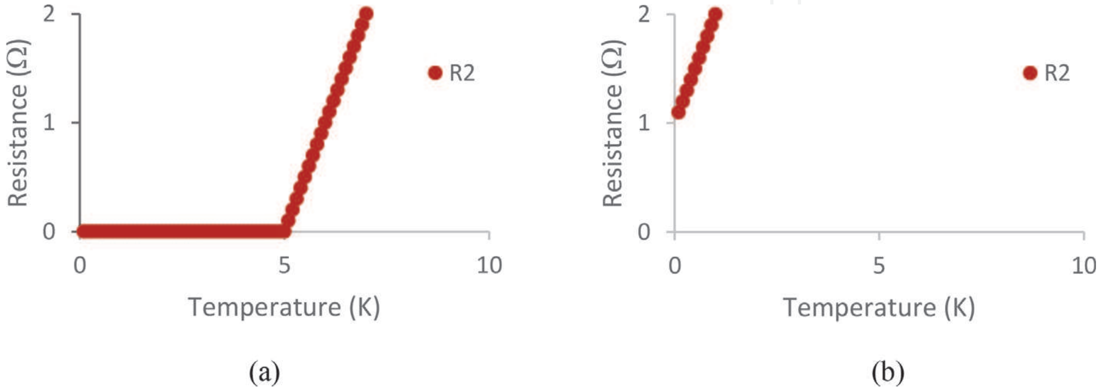


Figure 4.
Example of electrical resistance as a function of temperature for conduction electrons traveling in the pathway of atoms (a) in a superconducting material [type I] and (b) in a non-superconducting material. These graphs represent the resistance of resistor R_2.

4. Atomic theory analysis

Through the wave particle duality theorem, it is understood that objects behave like waves and like particles. Thus, atomic particles will display both particle-like properties as well as wave-like properties. To highlight this theorem, electrons are indeed particles with electrical charge and mass, but they are also waves that travel with a wavelength and frequency [1]. Traveling electrons can be characterized by the Schrödinger wave equation as shown in Eq. (3).

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x,t)}{\partial x^2} + V\psi(x,t) = i\hbar\frac{\partial\psi(x,t)}{\partial t} \tag{3}$$

In this equation, V represents the potential energy that a traveling electron will encounter at a particular time and location. Because many electrical charges exist in this model, the overall potential energy can be represented by the sum of the potentials associated with the charges that a traveling electron will encounter. So

$$V = \sum \frac{qQ}{4\pi\varepsilon x} \tag{4}$$

where q is the charge of a traveling electron, Q represents the charge of a nearby object (e.g., nucleus or another electron), ε is the permittivity or dielectric constant, and x is the distance between the traveling electron and nearby object.

Because there are many charges in the system, the exact solution to the Schrödinger wave equation will be extremely complex. However, the general solution to the wave equation, or Eq. (3) is given by

$$\psi(x,t) = Ae^{\,j(kx-\omega t)}u_k(x) \tag{5}$$

where k is the wavenumber, ω is the frequency of the traveling electron wave, and $u_k(x)$ is the Bloch function and has the periodicity of the lattice [1]. If the potential energy, V, is zero, then the electron will be a free particle and thus the wavenumber, k, will be a real number. This means that the wave can be modeled with a sinusoidal response and thus the electron will travel unimpeded.

Thus, if and when the permittivity becomes large enough, the potential energy will become sufficiently small and thus as conduction electrons travel through the lattice, they will not be attenuated. Therefore, if or when the permittivity becomes infinitely large, atoms will offer no resistance to moving electrons.

5. Magnetic field analysis

The electrical resistance response of a material is a major characteristic to determine if it is a superconductor. However, the response of a material to magnetic fields is also a major factor in determining if the material is a superconductor as well as what type of superconductor the material is. Therefore, magnetic field analysis is needed and must be incorporated into the theoretical model. To integrate a material's magnetic field response into the model, electromagnetic field theory will be considered.

Maxwell's equations can be used to understand how electromagnetic fields interact with matter [27]. Information such as how much of an electromagnetic wave will be transmitted or reflected at an interface between two different materials, and a wave's velocity, frequency, and wavelength in a material can be determined by analyzing Maxwell's equations. To gain a clear understanding of why the Meissner effect occurs in superconductors, Ampere's law (one of Maxwell's equations), will be used. In differential equation form

$$\nabla x\overline{B} = \mu\left(\overline{J} + \varepsilon\frac{d\overline{E}}{dt}\right) \tag{6}$$

where B is the magnetic flux, μ is the permeability of the material, ε is the permittivity of the material, J is the current density (and $\overline{J} = \sigma\overline{E}$), and E is the electric field. Electric fields and magnetic fields are related, and Eq. (6) shows that a magnetic field can be produced from an electric current and from a time changing electric field.

In addition to using Ampere's law as given in Eq. (6), the theoretical model will incorporate other electromagnetic equations and concepts because of the interdependence of the electric and magnetic fields. Specifically, Maxwell-Faraday's equation will be used or equivalently in differential equation form, $\nabla x\overline{E} = -\frac{d\overline{B}}{dt}$, and the force on charged particles due to external fields will be used or equivalently $\overline{F} = q\left(\overline{E} + \overline{v}\,x\,\overline{B}\right)$.

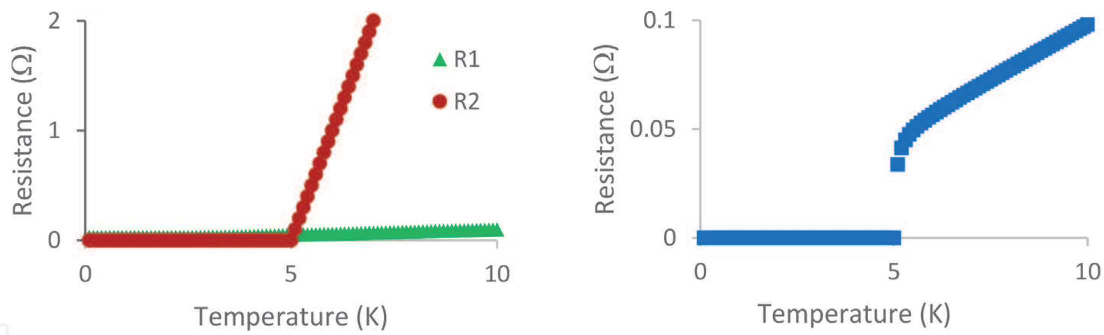
Eq. (6) reveals that the magnetic field is related to the electric field (or the time derivative of the electric field), but this equation also illustrates that this relationship is dependent upon the material's permittivity. Since a material's permittivity will behave as outlined and described previously, the main properties that characterize superconductors, zero electrical resistance and zero interior magnetic field are interrelated and therefore will be coupled.

6. Electrical resistance results

To demonstrate that the theoretical model functions properly and characterizes the behavior of superconducting materials, the resistance models, the parallel resistor concept, and the Schrödinger wave equation (and its solutions), will be used. Consequently, the model will be validated by demonstrating that it accurately describes the electrical resistance of both superconducting and non-superconducting materials. Furthermore, the distinction between the resistance of type I and type II superconductors will be made.

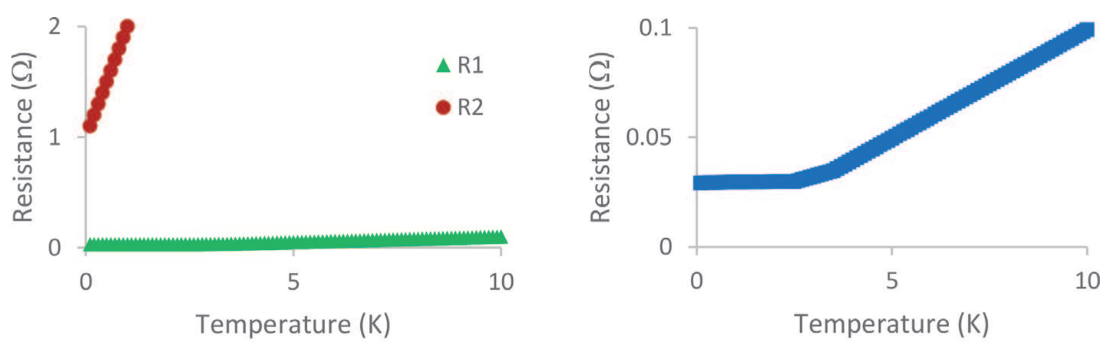
First the case for superconducting materials is considered. Resistances R_1 and R_2 are displayed on the same graph as a function of temperature as shown in **Figure 5a**. These curves are representative of the two resistances that would occur for superconductors. The theoretical model is represented by parallel resistors, so when these values are combined using the parallel resistor equation, the result is shown in **Figure 5b**. It is seen that the equivalent resistance has a linearly decreasing slope until the temperature reaches the transition temperature T_C. When it reaches this temperature, the resistance then abruptly goes zero. This is typical for the electrical resistance response of a superconducting material [1–5].

Next, the case for non-superconducting materials is considered. Resistances R_1 and R_2 are displayed on the same graph as a function of temperature as shown in **Figure 6a**. These curves are symbolic of the two resistance types that would occur for non-superconductors. Again, the theoretical model is represented by parallel resistors, so when these values are combined using the parallel resistor equation, the result is shown in **Figure 6b**. It is seen that the equivalent resistance resembles resistance R_1. This result is accurate and expected because resistance R_2 is always much larger than resistance R_1 and since parallel resistors will resemble the smaller



(a) (b)

Figure 5.
Example of electrical resistances for a type I superconducting material. (a) Graph showing the resistance of electrons traveling in the 'gaps' (represented by resistor R_1) and the resistance of electrons traveling in the pathway of atoms (represented by resistor R_2). (b) Graph showing the total resistance of a superconducting material when the two resistors R_1 and R_2 are combined in parallel. There is no resistance from the atoms when the material reaches its transition temperature.



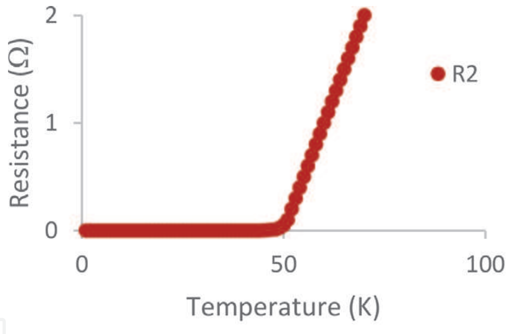
(a) (b)

Figure 6.
Example of electrical resistances for a non-superconducting material. (a) Graph showing the resistance of electrons traveling in the 'gaps' (represented by resistor R_1) and the resistance of electrons in the pathway of atoms (represented by resistor R_2). (b) Graph showing the total resistance of a non-superconducting material when the two resistors R_1 and R_2 are combined in parallel. The resistance of a non-superconductor will not reach zero regardless of the material's temperature.

resistor value, the total resistance will approximately equal R_1. As a result, when the resistance of non-superconductors is analyzed over a wide temperature range, there is no abrupt change in resistance because R_2 does not have a transition temperature. Therefore, conduction electrons will never experience zero resistance when they encounter atoms in this material. This is typical for the electrical resistance response of a non-superconducting material [1–5].

The previous analysis focused on revealing the electrical properties of type I superconductor materials. But the same theoretical model can be used to explain the behavior of type II superconductors materials. It is noted that there is a subtle difference between the electrical resistance response of type I materials and type II materials. At the transition temperature, the resistance of type I materials has a sharper transition to zero (i.e., the transition occurs over a small temperature range) whereas type II materials exhibit a slower transition (i.e., the transition occurs over a large temperature range) [28]. This difference in the transition temperature range can be explained using the following analysis.

Resistance R_1 (which is the resistance seen by conduction electrons that will not directly encounter atoms) will be the same for type I and type II materials. However, resistance R_2 (which is the resistance seen by the conduction electrons that directly interact with lattice atoms) for type II materials will be different than the



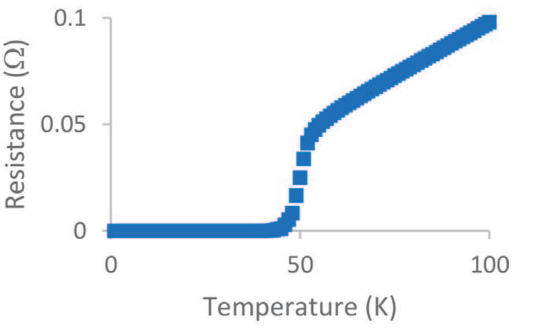


Figure 7.
*Example of electrical resistances for a type II superconducting material. (a) Graph of electrical resistance R_2 for electrons traveling in the pathway of atoms and (b) graph of the total electrical resistance for a type II superconducting material. The total electrical resistance is obtained by combining resistances R_2 (as shown in this figure) and R_1 (as shown in **Figure 2**) in a parallel manner.*

resistance of type I materials. The difference is that R_2 (or in essence the permittivity of the atoms), will have a response that is not linear near the transition temperature. Whereas type I materials will have a resistance with a sharp response as the temperature approaches the critical temperature, type II materials will have a resistance that gradually approaches zero near the critical temperature. The type II material response is shown in **Figure 7a**. When the resistance shown in **Figure 7a** is used to represent resistance R_2, the equivalent resistance will have the response shown in **Figure 7b**. This equivalent resistance response is obtained experimentally for typical type II materials [28].

7. Magnetic field results

The theoretical model will now be validated by demonstrating that it accurately describes the response of both superconducting and non-superconducting materials to applied magnetic fields. When superconducting materials are above their critical temperature they will behave as other materials and thus external magnetic fields can exist on the interior of the superconductor. Upon cooling the superconductor to a temperature below its critical temperature, regardless of whether a magnetic field is applied before or after cooling, type I superconductors will have no interior magnetic field [1–5]. However, the manner in which the magnetic field is excluded from the interior of the superconductor in these two cases is different. These two cases of applying the magnetic field before and after cooling will be analyzed. To simplify this analysis, it is assumed that there is no initial current flowing through the superconductor, and thus current density J is zero (it is noted that this assumption will not affect the magnetic field analysis or results).

First, the case of applying the magnetic field after cooling a type I superconductor below its critical temperature is examined. In the area where atoms are located, the permittivity ε will be infinite and initially since there are no fields, the electric field is zero and dE/dt is zero. Now, when a magnetic field is applied, this field changes in space and time. The change goes from zero to a value that is non-zero and finite. So, ∇xB will be non-zero and thus dE/dt will become non-zero and finite and as a result, $(\varepsilon)(dE/dt)$ will be infinite. Since Ampere's law as shown in Eq. (6) must be true, $(\mu)(\varepsilon)(dE/dt)$ must be finite and this will only be true if the permeability $\mu = 0$. Since μ is zero, there will be no magnetic flux in the material. In essence, conduction electrons on the material's surface will circulate according to

$\overline{F} = q\overline{v}\ x\ \overline{B}$ and produce magnetic fields that will cancel the external field. Thus, there will be no net magnetic flux inside the material (it is noted that the magnetic field will decay exponentially according to the London penetration depth [1, 2, 29]). When the external magnetic field is removed, the conduction electrons stop circulating and return to their normal motion. The top half of **Figure 8** illustrates the magnetic field's behavior when a type I superconductor is cooled before the magnetic field is applied. Note that if the external magnetic field becomes too large, it will stretch and alter the atoms in the material, the surface current will be affected, and the permittivity ε of the atoms will no longer be infinite. As a result, the permeability μ will no longer be zero and therefore, the magnetic field will be able to penetrate the material.

Next the case of applying the magnetic field before cooling a type I superconductor below its critical temperature is examined. Initially the permittivity ε, the permeability μ, and the magnetic flux B inside the material will all be non-zero and finite. Then when the material is cooled and reaches the transition or critical temperature, the permittivity will change from finite to infinite. As a result, conduction electrons will circulate to counteract the applied field according to $\overline{F} = q\overline{v}\ x\ \overline{B}$. These circulating charges will reduce the external magnetic field. As a result of the changing magnetic flux, a changing electric field E will be produced, and therefore dE/dt will become non-zero. Since $\nabla x B$ will now be non-zero and finite, then the term $(\mu)(\varepsilon)(dE/dt)$ must be finite. This will be true if the permeability becomes zero or $\mu = 0$. If μ is zero, there will be no magnetic flux in the material, so the field will get expelled from the interior of the superconductor. Again, when the external magnetic field is removed, the conduction electrons stop circulating and return to their normal motion. The bottom half of **Figure 8** illustrates the magnetic field's behavior when a type I superconductor is cooled after the magnetic field is applied. Again, note that if the external magnetic field becomes too large, it will stretch and alter the atoms and subsequently surface currents and superconducting properties will be destroyed (thus the electrical resistance will no longer be zero and a magnetic field can penetrate the material).

The previous analysis focused on revealing the magnetic properties or magnetic field response of type I superconductor materials. The model (along with the same analysis) is not only capable of explaining why type II superconductors exhibit their response, but it is able to explain the behavior of type II materials. Type II superconductors have two critical magnetic fields: a lower field T_{CL} and a higher field T_{CH}. If the applied magnetic field is less than T_{CL}, the type II material behaves as a type I material and excludes all of the magnetic flux from its interior. If the applied

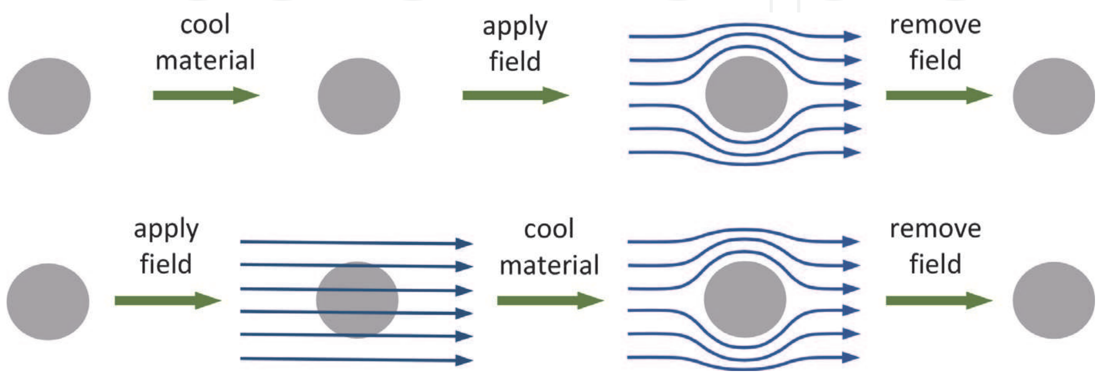


Figure 8.
Illustration of the response of a superconductor to an external magnetic field. The top half of this figure shows the response of the superconductor if the material is cooled before the field is applied and the bottom half of the figure shows the response if the field is applied before the material is cooled. Once the material is cooled, there will be no magnetic field inside the superconductor.

magnetic field is greater than T_{CH}, the type II material also behaves like type I materials and will allow all of the magnetic field to penetrate its interior. However, when the applied field is between T_{CL} and T_{CH} only a portion of the external field will penetrate type II materials. This is the mixed state in which superconducting and non-superconducting regions exist in the material [1–5]. Based on the state of the surface current and the state of the permittivity of the atoms in the material, the three different states of type II superconductors (i.e., superconducting, mixed, and normal) can be explained.

In the presence of a magnetic field, the atoms in the material will be stretched and altered. As long as the external magnetic field is smaller than T_{CL}, the atoms are not altered significantly. So, surface currents will exist, and the permittivity of the atoms will be infinite, and the material will exist in the superconducting state. This response is similar to type I materials, so no additional analysis is necessary.

The permittivity of the atoms in the material will vary as a function of the magnetic field (as well as temperature). So, when the external magnetic field exceeds T_{CL}, the permittivity of the atoms will be affected. Thus, the permittivity ε will not be infinite and based on Ampere's law, the permeability μ will no longer be zero. So, a magnetic field will be able to exist in the material. The surface currents will remain until the magnetic field reaches the upper critical field T_{CH}. The surface currents will block most of the magnetic field but some of the external field will penetrate through the surface. Since the permeability is not zero, the magnetic field will be able to exist in the interior of the material. This creates a pattern of superconducting and non-superconducting regions and forms an Abrikosov lattice [30]. This is the mixed state.

Finally, if the magnetic field is continually increased and reaches T_{CH}, the atoms in the material become stretched to the point where they interfere with all of the surface currents. Since the surface currents will not be able to flow unimpeded and since the permittivity of the atoms will not be infinite, the material moves from the mixed state to a non-superconducting state.

8. Superconductors vs. perfect conductors

If a model that explains superconductivity is going to be completely correct, it must also be able to explain why superconductors are different from perfect conductors. This theoretical model can explain this difference. The main difference between a perfect conductor and a superconductor is in the response to a magnetic field that is applied before the material is cooled to the transition temperature. A superconductor will exclude the magnetic field when it reaches the transition temperature whereas a perfect conductor will allow the magnetic field to remain [28, 31]. The magnetic field analysis for superconductors has been performed using the theoretical model, and the model explains the behavior. A similar analysis will be utilized to explain the behavior of perfect conductors.

A perfect conductor is defined as a material that has no resistivity or equivalently zero conductivity when the material is below the transition temperature. So, to compare the perfect conductor to a superconductor, $\sigma = \infty$ for the perfect conductor, but $\varepsilon = \infty$ for superconductors. Although the perfect conductor will have infinite conductivity, its other electromagnetic parameters, ε and μ, remain normal. Since $\sigma = \infty$ and $J = \sigma E$, the electric field E must vanish in the material to keep the current density J finite. As a result, dE/dt must be zero. Furthermore, the term $(\mu)(\varepsilon)(dE/dt)$ must be zero since the first two terms are finite and dE/dt will be zero. Ampere's law requires that $\nabla x B$ and $(\mu)(\varepsilon)(dE/dt)$ must be equal, so $\nabla x B$ must be zero and therefore B cannot change spatially. Thus, the perfect conductor will

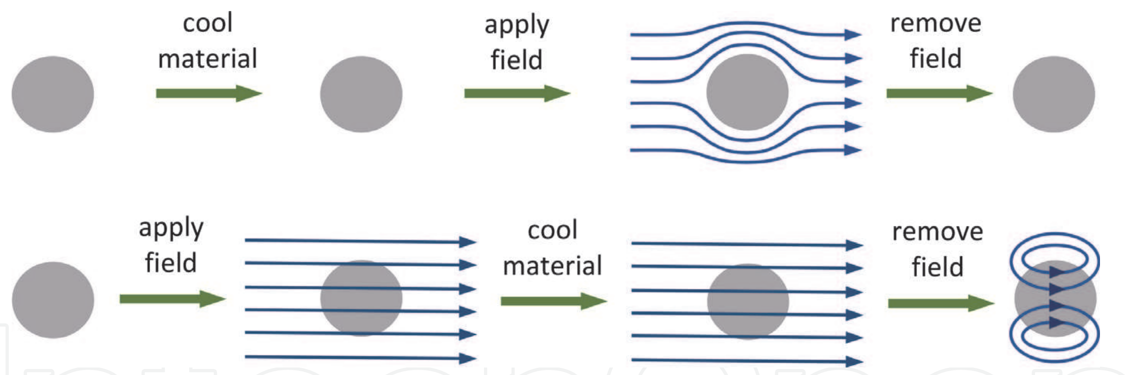


Figure 9.
Illustration of the response of a perfect conductor to an external magnetic field. The top half of this figure shows the response of the perfect conductor if the material is cooled before the field is applied and the bottom half of the figure shows the response if the field is applied before the material is cooled. Regardless of when cooling occurs, the material maintains the magnetic field that it had when the external field is removed.

contain the same amount of magnetic flux before and after the material cools to its transition temperature.

Thus, if a magnetic field is applied after the perfect conductor is cooled, that field will not be able to penetrate that material (because the material's interior had no magnetic field prior to cooling). The top half of **Figure 9** illustrates the magnetic field's behavior when a perfect conductor is cooled before the magnetic field is applied. Additionally, if a magnetic field is present in a perfect conductor, it will remain in that material after it is cooled to its transition temperature (since the magnetic flux cannot change spatially, it will remain in the material). The bottom half of **Figure 9** illustrates the magnetic field's behavior when a perfect conductor is cooled after the magnetic field is applied. Therefore, this theoretical model can explain the response of perfect conductors to applied magnetic fields.

9. Permittivity examination

Superconductors exhibit the properties of zero electrical resistance as well as magnetic field exclusion because the material's permittivity becomes infinite. It can be shown that the permittivity of atoms can become infinite under specified conditions [27]. Atoms possess a resonant frequency and if they are excited at this frequency, their permittivity becomes infinitely large. So, if or when the frequency of the conduction electrons aligns with the resonant frequency of the atoms, sufficient conditions will be met (and the permittivity of the material will become infinite).

In general, a graph of a material's permittivity as a function of frequency displays three notable regions. In the microwave region or below 10^9 Hz the permittivity is constant, in the infrared region or at approximately 10^{12} Hz the permittivity displays a spike or discontinuity, and in the ultraviolet region or at approximately 10^{15} Hz the permittivity displays a second spike or discontinuity. A typical graph of this response is shown in **Figure 10** [32].

These spikes or discontinuities reveal that the permittivity goes to infinity at these frequencies. Therefore, these types of graphs alone indicate that the permittivity can be infinitely large and thus according to the theoretical model, the potential energy vanishes in the Schrödinger wave equation. When the potential energy term becomes zero, conduction electrons experience no impedance and thus superconductivity results. So, if conduction electrons in a material exist with a frequency that matches the frequency at one of these permittivity spikes, then the necessary conditions will exist that lead to superconductivity.

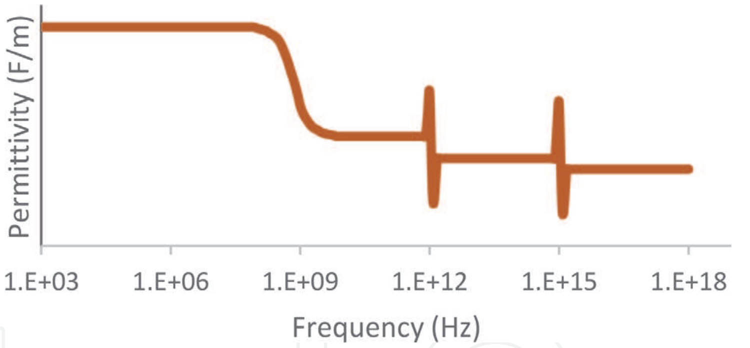


Figure 10.
Illustration of the permittivity as a function of frequency for a generic material. These first discontinuity, which occurs around 10^{12} Hz represents the resonance that occurs for atom-atom interactions. The second discontinuity, which occurs around 10^{15} Hz represents the resonance that occurs for electron-atom interactions.

The frequency of conduction electrons can be determined by using the Planck-Einstein relationship between energy and frequency. In equation form, $E = hf$, where E is the energy of the electron, h is Planck's constant, and f is the frequency of the electron. It is understood that conduction electrons have larger energies compared to other electrons in a material. These energies are approximately equal to the Fermi energy and will depend upon the material. Fermi energies have been determined for many materials and can range from approximately 1 to 10 eV. Using an average energy value of 5 eV, this corresponds to a frequency of 10^{15} Hz [1].

Comparing conduction electron frequencies to the discontinuities in the permittivity spectrum, it is seen that these two frequencies can align at 10^{15} Hz. Depending upon a material's characteristics, these two frequencies will align for some materials but will not align for others. This general analysis confirms that the mechanisms and analysis associated with the theoretical model are plausible and thus will lead to the phenomena of superconductivity.

10. Summary and discussion

A theoretical model has been developed that explains why some materials behave like superconductors (and thus display the corresponding electrical and magnetic properties), and why other materials do not. The theoretical model produces results in which electrical resistance is a function of temperature as well as results that explain why magnetic fields can or cannot exist in these materials. These theoretical results are validated by experimental results regardless of whether a material is classified as a type I superconductor, a type II superconductor, or a non-superconductor.

It is reasonable to state that atoms have a temperature dependent frequency response since energy, temperature, and an atom's motion are directly related [33]. The theoretical model and material properties can be used to help design superconducting materials that achieve zero resistance at specified temperatures. Having knowledge of how atoms function in materials will enhance the ability of scientists and engineers to create materials with specific properties. Therefore, by understanding the temperature and frequency relationship of atoms, engineers can manufacture or create materials that operate with zero resistance under desired conditions [34]. Moreover, since type II superconductors are better than type I materials at withstanding higher magnetic fields (before they exhibit non-superconducting properties), engineers can also use this knowledge of atomic behavior in materials to create superconductors with desired properties.

The electrical resistance for superconducting materials is linear for temperatures above the critical temperature, zero for temperatures below the critical temperature, and has a transition region at the transition temperature. For type I materials, the transition occurs within a narrow temperature range (approximately 10^{-3} K), whereas for type II materials the transition occurs over a large temperature range (approximately 1 K) [28]. The value and slope of the transition to zero resistance can be affected by many factors such as purity and the presence of isotopes [28, 35, 36]. Because the electrical resistance response of type II materials is prone to these lattice irregularities, they can be considered as type I materials that are either impure materials or composite materials. Therefore, it is reasonable to alter the theoretical model for type I materials and modify the characteristics of the atoms to obtain the response for type II materials.

It has been determined that permittivity is the electromagnetic parameter that is of importance in determining whether a material is a superconductor or not. Because permittivity is the governing parameter and not conductivity, ultimately it would probably be better to model the superconducting channel as a capacitor rather than a resistance. These two electrical circuit elements are similar in that they will limit electrical current. However, a capacitor would offer impedance rather than resistance and thus frequency effects could be modeled [37, 38]. Nevertheless, since impedance and resistance are similar concepts, if frequency effects are not important to an application (and in many applications, frequency effects are not important), the main characteristics associated with superconductors can still be modeled quite well using the resistor concept.

Sometimes a classical physics approach to modeling a system can provide accurate results and insight into the behavior of that system [39–41]. When a classical approach is used to analyze atoms in a material, it can be shown that the atoms have a resonant frequency. Furthermore, it can be shown that the atoms have a permittivity that is dependent upon this resonant frequency [27]. If the frequency of conduction electrons matches the resonant frequency of the atoms, the permittivity will become infinitely large and the material will behave as a superconductor.

Based on this resonant frequency analysis, at sufficiently low temperatures, superconducting atoms will have a resonant frequency that aligns with the frequency of conduction electrons. This allows conduction electrons to travel through the material without encountering electrical potentials. However, the conduction electron and atom frequency alignment will not occur at elevated temperatures due to effects of thermal energy on atoms. Conversely, reducing the temperature of non-superconducting materials will not lead to superconductivity because the atoms (or molecules) in non-superconducting materials have a resonant frequency that will not match the frequency of the conduction electrons.

In applying atomic theory to the resistance analysis, note that the resonant frequency of the atoms is normally 'hidden' at elevated temperatures. An additional frequency will be superimposed onto the resonant frequency of the atoms due to thermal energy. As a result, the resonant frequency will be masked if the temperature is above the critical temperature [42–44]. When the temperature of the superconductor is lowered to the threshold or critical value, the atoms in the material will display their resonance and therefore the material will behave as a superconductor.

In addition to verifying the electrical properties, this theoretical model also accounts for the magnetic field properties associated with superconductors. In other words, the model explains why the Meissner effect occurs. A material's permittivity and permeability are linked through Ampere's law. So, for a material that behaves as a superconductor, when its permittivity becomes infinitely large at sufficiently low temperatures, its permeability must go to zero. Therefore, the material must exclude magnetic fields from its interior. Surface currents produce magnetic fields and will account for this cancelation.

This theoretical model can also explain why the mixed state occurs in type II superconductors (i.e., the model explains why the magnetic field can exist in specified regions of the material). A magnetic field can penetrate the material because when a strong field is applied, the atoms are stretched and distorted and as a result the permittivity will no longer be infinite [45]. Perpetual currents will not be able to exist in the interior of the material to shield the external magnetic field. However, surface currents that exist above the atoms will be able to shield some of the external magnetic field. A fraction of the magnetic field will penetrate this surface shield and will exist inside the material.

To further explain the mixed state or why the magnetic field phenomena occurs in type II materials, consider the following. Superconductors have well known relationships between temperature and resistivity as well as between temperature and magnetic field strength. Because of these relationships, when a magnetic field is applied to a superconductor, that field will have an effect on the electrical resistance [46–48]. Since the magnetic field will alter the relationship between the atoms and electrons, if the field becomes large enough, it will alter the resonant frequency of the atoms. As a result, the permittivity ε will no longer be infinite, but it will become finite. Thus, the atoms in the bulk of the material will exhibit electrical resistance (and electrical currents will not flow indefinitely) and the atoms will not have the ability to block external magnetic fields.

11. Conclusion

A theoretical model has been developed based on atomic level analysis and electromagnetic field theory to explain why some materials exhibit superconductor properties and other materials do not. Specifically, the theoretical model addresses electrical resistance and the material's response to magnetic fields. The model explains why the electrical resistance of superconductors becomes zero or why conduction electrons are unimpeded by atoms. The model also explains why superconductors exhibit the Meissner effect and exclude magnetic fields from their interior. Additionally, the theoretical model is general enough such that it can describe the behavior of all materials (whether they are superconductors or not), but the model is specific enough such that it can explain the behavior of both type I or type II superconductors. Furthermore, the theoretical model also describes or distinguishes the characteristics of superconductors and perfect conductors and thus it is able to differentiate the behavior of these two materials.

Because many theories on superconductivity address specific aspects of superconductors and do not (or cannot) address other aspects, those theories have constraints and thus there are restrictions on the information they provide. Based on the approach and analysis that was used to construct this theoretical model (i.e., atomic physics and electromagnetic field theory), it provides insight into the physical mechanisms that cause materials to become superconducting. Therefore, this theoretical model should aid science and engineering researchers in the quest to develop room temperature superconductors that can be used to produce intense magnetic fields needed for medical imaging, as well as zero electrical resistance needed for supercomputing, and electric power transmission.

Conflict of interest

The author has no conflict of interest.

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