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Topology and Geometry of Serial and Parallel Manipulators

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1. Introduction

The evolution of requirements for mechanical products toward higher performances, coupled with never ending demands for shorter product design cycle, has intensified the need for exploring new architectures and better design methodologies in order to search the optimal solutions in a larger design space including those with greater complexity which are usually not addressed by available design methods. In the mechanism design of serial and parallel manipulators, this is reflected by the need for integrating topological and geometric synthesis to evaluate as many potential designs as possible in an effective way.

In the context of kinematics, a mechanism is a kinematic chain with one of its links identified as the base and another as the end-effector (EE). A manipulator is a mechanism with all or some of its joints actuated. Driven by the actuated joints, the EE and all links undergo constrained motions with respect to the base (Tsai, 2001). A serial manipulator (SM) is a mechanism of open kinematic chain while a parallel manipulator (PM) is a mechanism whose EE is connected to its base by at least two independent kinematic chains (Merlet, 1997). The early works in the manipulator research mostly dealt with a particular design; each design was described in a particular way. With the number of designs increasing, the consistency, preciseness and conciseness of manipulator kinematic description become more and more problematic. To describe how a manipulator is kinematically constructed, no normalized term and definition have been proposed. The words architecture (Hunt, 1982a), structure (Hunt, 1982b), topology (Powell, 1982), and type (Freudenstein & Maki, 1965; Yang & Lee, 1984) all found their way into the literature, describing kinematic chains without reference to dimensions. However, some kinematic properties of spatial manipulators are sensitive to certain kinematic details. The problem is that with the conventional description, e.g. the topology (the term topology is preferred here to other terms), manipulators of the same topology might be too different to even be classified in the same category. The implementation of the kinematic synthesis shows that the traditional way of defining a manipulator's kinematics greatly limits both the qualitative and quantitative designs of spatial mechanisms and new method should be proposed to solve the problem. From one hand, the dimension-independent aspect of topology does not pose a considerable problem to planar manipulators, but makes it no longer appropriate to describe spatial manipulators especially spatial PMs, because such properties as the degree

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of freedom (DOF) of a manipulator and the degree of mobility (DOM) of its EE as well as the mobility nature are highly dependent on some geometric elements. On the other hand, when performing geometric synthesis, some dimensional and geometric constraints should be imposed in order for the design space to have a good correspondence with the set of manipulators which can satisfy the basic design requirements (the DOF, DOM and the mobility nature), otherwise, a large proportion of the design space may have nothing to do with the design problem in hand. As for the kinematic representation of PMs, one can hardly find a method which is adequate for a wide range of manipulators and commonly accepted and used in the literature. However, in the classification (Balkan et al., 2001; Su et al., 2002), comparison studies (Gosselin et al., 1995; Tsai & Joshi, 2001) (equivalence, isomorphism, similarity, difference, etc.) and manipulator kinematic synthesis, an effective kinematic representation is essential. The first part of this work will be focused on the topology issue.

Manipulators of the same topology are then distinguished by their kinematic details. Parameter (Denavit & Hartenberg, 1954), dimension (Chen & Roth, 1969; Chedmail, 1998), and geometry (Park & Bobrow, 1995) are among the terms used to this end and the ways of defining a particular manipulator are even more diversified. When performing kinematic synthesis, which parameters should be put under what constraints are usually dictated by the convenience of the mathematic formulation and the synthesis algorithm implementation instead of by a good delimitation of the searching space. Another problematic is the numeric representation of the topology and the geometry which is suitable for the implementation of global optimization methods, e.g. genetic algorithms and the simulated annealing. This will be the focus of the second part of this work.

2. Preliminary

Some basic concepts and definitions about kinematic chains are necessary to review as a starting point of our discussion on topology and geometry. A kinematic chain is a set of rigid bodies, also called links, coupled by kinematic pairs. A kinematic pair is, then, the coupling of two rigid bodies so as to constrain their relative motion. We distinguish upper pairs and lower pairs. An upper kinematic pair constrains two rigid bodies such that they keep a line or point contact; a lower kinematic pair constrains two rigid bodies such that a surface contact is maintained (Angeles, 2003). A joint is a particular mechanical implementation of a kinematic pair (IFToMM, 2003). As shown in Fig. 1, there are six types of joints corresponding to the lower kinematic pairs - spherical (S), cylindrical (C), planar (E), helical (H), revolute (R) and prismatic (P) (Angeles, 1982). Since all these joints can be obtained by combining the revolute and prismatic ones, it is possible to deal only with revolute and prismatic joints in kinematic modelling. Moreover, all these joints can be represented by elementary geometric elements, i.e., point and line. To characterize links, the notions of simple link, binary link, ternary link, quaternary link and n-link were introduced to indicate how many other links a link is connected to. Similarly, *binary joint*, *ternary joint* and *n-joint* indicate how many links are connected to a joint. A similar notion is the connectivity of a link or a joint (Baron, 1997). These basic concepts constitute a basis for kinematic analysis and kinematic synthesis.

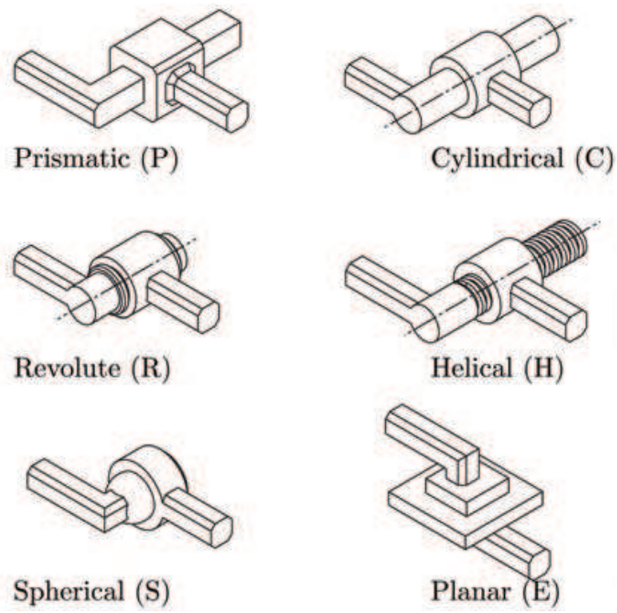


Figure 1: Lower Kinematic Pairs

3. Topology

For kinematic studies, the kinematic description of a mechanism consists of two parts, one is qualitative and the other quantitative. The qualitative part indicates which link is connected to which other links by what types of joints. This basic information is referred to as structure, architecture, topology, or type, respectively, by different authors. When dealing with complex spatial mechanisms, the qualitative description alone is of little interest, because the kinematic properties of the corresponding mechanisms can vary too much to characterize a mechanism. This can be demonstrated by the single-loop 4-bar mechanisms shown in Fig. 2. Without reference to dimensions, all mechanisms shown in Fig. 2 are of the same kinematic structure but have very distinctive kinematic properties and therefore are used for different applications— mechanism a) generates planar motion, mechanism b) generates spherical motion, mechanism c) is a Bennett mechanism (Bennett, 1903), while mechanism d) permits no relative motion at any joints. Fig. 3 shows an example of parallel mechanisms having the same kinematic structure—mechanism a) has 3 DOFs whose EE has no mobility, mechanism b) has 3 DOFs whose EE has 3 DOMs in translation, mechanism c) permits no relative motion at any joints.

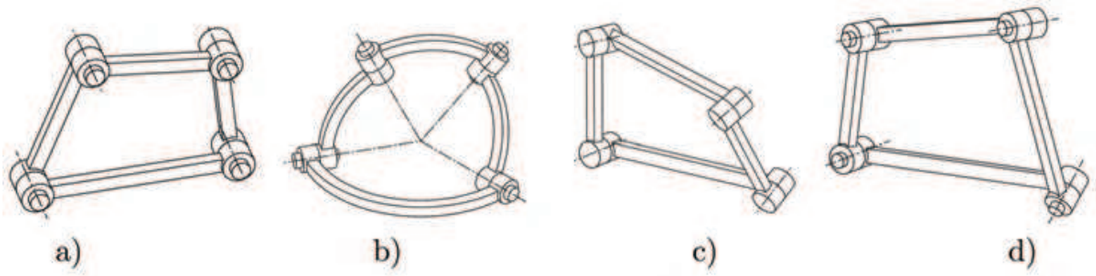
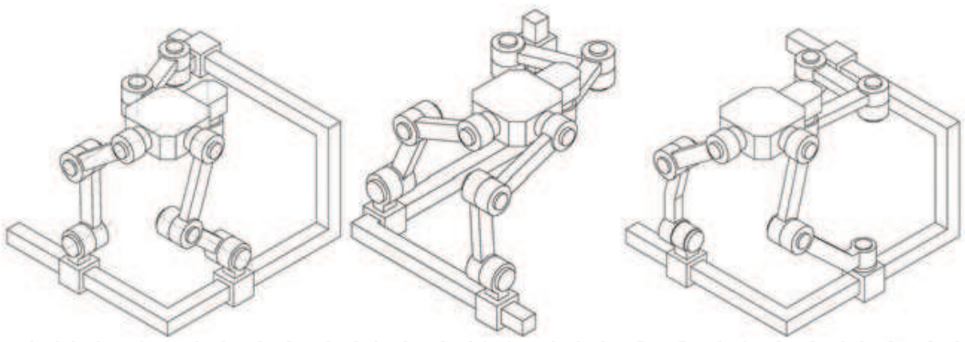


Figure 2: 4-bar mechanisms of different geometries



a) b) c)

Figure 3: 3-PRRR parallel mechanisms

A particular mechanism is thus described, in addition to the basic information, by a set of parameters which define the relative position and orientation of each joint with respect to its neighbors. For complex closed-loop mechanisms, an often ignored problem is that certain parameters must take particular values or be under certain constraints in order for the mechanism to be functional and have the intended kinematic properties. In absence of these special conditions, the mechanisms may not even be assembled. More attention should be payed to these particular conditions which play a qualitative role in determining some important kinematic properties of the mechanism. For kinematic synthesis, not only do the eligible mechanisms have particular kinematic structures, but also they feature some particular relative positions and orientations between certain joints. If this particularity is not taken into account when formulating the synthesis model, a great number of mechanisms generated with the model will not have the required kinematic properties and have to be discarded. This is why the topology and geometry issue should be revisited, the special joint dispositions be investigated and an adapted definition be proposed.

Since the 1960s, a very large number of manipulator designs have been proposed in the literature or disclosed in patent files. The kinematic properties of these designs were studied mostly on a case by case basis; characteristics of their kinematic structure were often not investigated explicitly; the constraints on the relative joint locations which are essential for a manipulator to meet the kinematic requirements were rarely treated in a topology perspective.

Constraints are introduced mainly to meet the functional requirements, to simplify the kinematic model, to optimize the kinematic performances, or from manufacturing considerations. These constraints can be revealed by investigating the underlying design ideas.

For a serial manipulator to generate planar motion, all its revolute joints need to be parallel and all its prismatic joints should be perpendicular to the revolute joints. For a serial manipulator to generate spherical motion, the axes of all its revolute joints must be concurrent (McCarthy, 1990). For a parallel manipulator with three identical legs to produce only translational motion, the revolute joints of the same leg must be arranged in one or two directions (Wang, 2003).

A typical example of simplifying the kinematic model is the decoupling of the position and orientation of the EE of a 6-joint serial manipulator. This is realized by having three consecutive revolute joint axes concurrent. A comprehensive study was presented in (Ozgoren, 2002) on the inverse kinematic solutions of 6-joint serial manipulators. The study

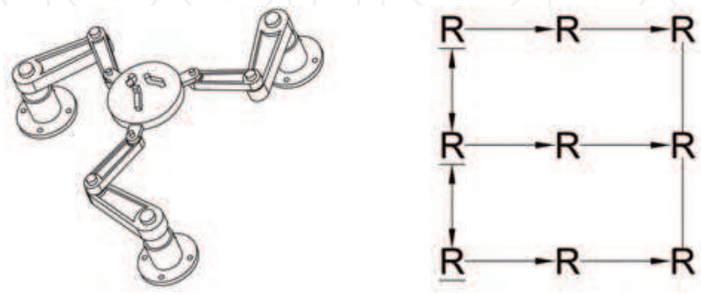
reveals how the inverse kinematic problem is simplified by making joint axes parallel, perpendicular or intersect.

Based on the analysis of the existing kinematic design, the definition of the manipulator topology and geometry is proposed as the following:

- the *kinematic composition* of a manipulator is the essential information about the number of its links, which link is connected to which other links by what types of joints and which joints are actuated;
- the *characteristic constraints* are the minimum conditions for a manipulator of given kinematic composition to have the required kinematic properties, e.g. the DOF, the DOM;
- the *topology* of a manipulator is its kinematic composition plus the characteristic constraints;
- The *geometry* of a manipulator is a set of constraints on the relative locations of its joints which are unique to each of the manipulators of the same topology.

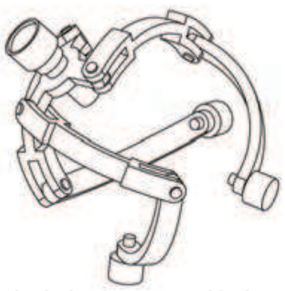
Hence, topology also has a geometric aspect such as parallelism, perpendicularity, coplanar, and even numeric values and functions on the relative joint locations which used to be considered as geometry. By definition, geometry no longer includes relative joint locations which are common to all manipulators of the same topology because the later are the characteristic constraints and belong to the topology category. A manipulator can thus be much better characterized by its topology.

Taking the basic ideas of graph representation (Crossley, 1962; Crossley, 1965) and layout graph representation (Pierrot, 1991), we propose that the kinematic composition be represented by a diagram having the graph structure so as to be eventually adapted for automatic synthesis. The joint type is designated as an upper case letter, *i.e.*, **R** for revolute, **P** for prismatic, **H** for helical, **C** for cylindrical, **S** for spherical and **E** for planar. Actuated joints are identified by a line under the corresponding joint. The letters denoting joint types are placed at the vertices of the diagram, while the links are represented by edges. Fig. 4 and Fig. 5 are two examples of representation of kinematic composition. Each joint has two joint elements, to which element a link is connected is indicated by the presence or absence of the arrow. Any link connected to the same joint element is actually rigidly attached and no relative motion is possible. The most left column represents the base carrying three actuated revolute joints while the most right column the EE. The EE is connected to the base by three identical kinematic chains composed of three revolute joints respectively. It is noteworthy that the two different manipulators have exactly the same kinematic composition. The diagram must bear additional information in order to appropriately represent the topology.

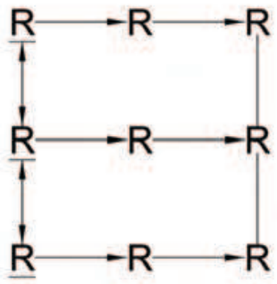


a) Physical manipulator b) Diagram

Figure 4: Kinematic Composition of a Planar 3-RRR parallel manipulator



a) Physical manipulator



b) Diagram

Figure 5: Kinematic Composition of a Spherical 3-RRR parallel manipulator

When dealing with manipulators composed of only lower kinematic pairs, the characteristic constraints are the relative locations between lines. Constraints on relative joint axis locations can be summarized as the following six and only six possible situations shown in Fig.6. Superimposing the characteristic constraint symbols on the kinematic composition diagrams shown in Fig. 4 and 5, we get the diagrams shown in Fig. 7 and 8.

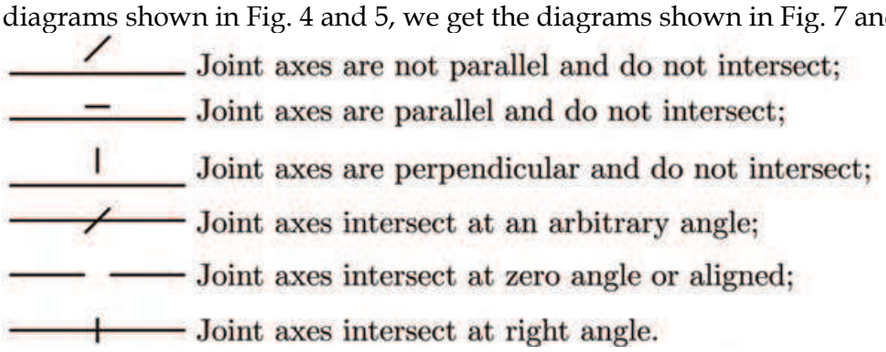
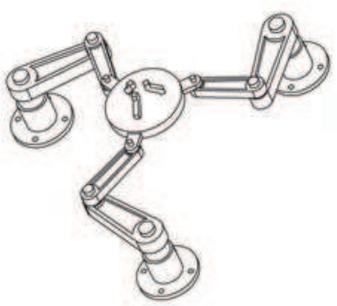
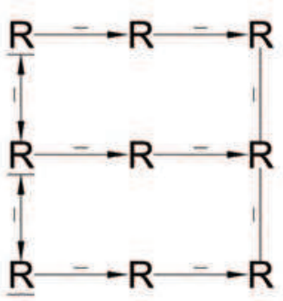


Figure 6: Graphic symbols for characteristic constraints



a) Physical manipulator



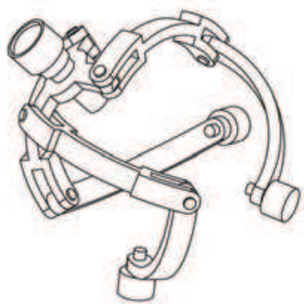
b) Topological diagram

Figure 7: Diagram of a planar parallel manipulator with characteristic constraints

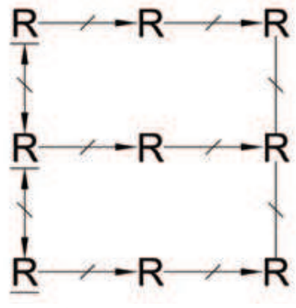
When implementing the automatic topology generation of a SM composed of only revolute and prismatic joints, the topology is represented by 6 integers, *i.e.*

- *n*: number of joints.
- x_0: kinematic composition. Its bits 0 to n − 1 represent respectively the joint type of joints 1 to n with 1 for revolute and 0 for prismatic.

- x_1: bits 0 to n − 2 indicate respectively whether the axes of joints 2 to n − 1 intersect the immediate preceding joint axis.
- x_2: each two consecutive bits characterize the orientation of the corresponding joint relative to the immediate preceding joint with 00 for parallel, 01 for perpendicular, and 10 for the general case.
- x_3: supplementary constraint identifying joints whose axes are concurrent. All joint axes whose corresponding bits are set to 1 are concurrent.
- x_4: supplementary constraint identifying joints whose axes are parallel. All joint axes whose corresponding bits are set to 1 are parallel.



a) Physical manipulator



b) Diagram

Figure 8: Diagram of a spherical parallel manipulator with characteristic constraints

With this numerical representation, topological constraint can be imposed on a general kinematic model to carry out geometric synthesis to ensure that the search is performed in designs with the intended kinematic properties. The binary form makes the representation very compact. No serial kinematic chain should have more than 3 prismatic joints, so all values for x_0 of 6 joint kinematic chains take only 42B (byte) storage. Those for x_1 take 31B while those for x_2 243B. Without supplementary constraints which are applied between non adjacent joints, the maximum number of topologies is 316386 (some topologies, those with two consecutive parallel prismatic joints for example, will not be considered for topological synthesis purpose). All topologies without supplementary constraint can be stored in a list, making the walk through quite straightforward. Applying supplementary constraints while walking through the list provides a systematic way for automatic topology generation.

4. Geometry

In the kinematic synthesis of SMs, the most successively employed geometric representation is the Denavit-Hartenberg notation (Denavit & Hartenberg, 1954). For PMs, the Denavit-Hartenberg notation is more or less adapted to suit the particularity of the manipulator being studied, especially for reducing the number of parameters and simplifying the formulation and solution of the kinematic model (Baron et al., 2002). One major problem of the later in implementing computer aided geometric synthesis is the computation of the initial configuration. Once a new set of parameters are generated, the assembly of each design take too much computation and sometimes the computation don't converge at all. This may be du to the complexity of the kinematic model or that the set of parameters correspond to no manipulator in the real domain. It also arrives that only within a subspace of the entire workspace, a particular design possesses the desired kinematic properties,

making the computation useless outside the subspace. A PM (Fig. 9) presented in (Zlatanov et al., 2002) is a good example of this kind. Depending on the initial configuration, the manipulation can be a translational one or spherical one. Another problem encountered when performing computer aided synthesis is that the entire set of equations is underdetermined, while a subset of the set is overdetermined. It seems that the set of parameters correspond to no functional manipulator. But manipulators having such mathematic equations do exist. The PM shown in Fig. 10 has 8 DOF for the system on the whole and its EE has 3 DOM. The two *PRRR* legs form an overdetermined system, but the system on the whole is underdetermined.

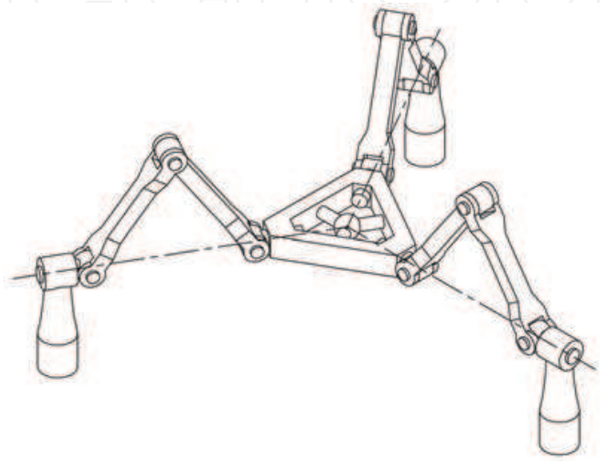


Figure 9: 3-RRRRR [28]

To improve the efficiency of the computation algorithms, an initial configuration seems to be an effective solution. So, for PMs, we proposed that the geometry definition be always accompanied by an initial configuration to start with and the evaluation computation is carried out mainly in certain neighborhood of the initial configuration.

The most challenging part of the kinematic synthesis is the integration of the topological synthesis and geometric synthesis. From the best of knowledge of the authors, the most systematic study in this regard is that presented in (Ramstein, 1999). In (Ramstein, 1999), the synthesis problem is formulated as an global optimization problem with genetic algorithms as solution tools. The joint type is represented by boolean numbers with 1 for prismatic and 0 for revolute. The synthesis results are far from what were expected. The problem is that the population does not migrate as much as expected from one topology region to another, making the synthesis concentrate on a very few topologies.

Since the joint type is represented by discrete numbers, a joint can only be either prismatic or revolute, nothing in between, which greatly limites the diversity and the migration of the solution population. With the simulated annealing techniques, similar situations have been observed by the authors.

Inspired by this observation, the basic concept of fuzzy logic and the fact that a prismatic joint is actually a revolute joint at infinity, we introduce the concept: **joint nature** which is a non negative real number to characterize the level of the "revoluteness" of a joint. This allows us to deal with the prismatic joints and the revolute ones in the same way and permit a joint to evolve between revolute and prismatic. Although a joint in between is meaningless in real application, this increases the migration channels for the solution populations and

probability of finding the global optima. Before proposing the joint nature definition, it should be inspected how a revolute joint mathematically evolves toward prismatic joint.

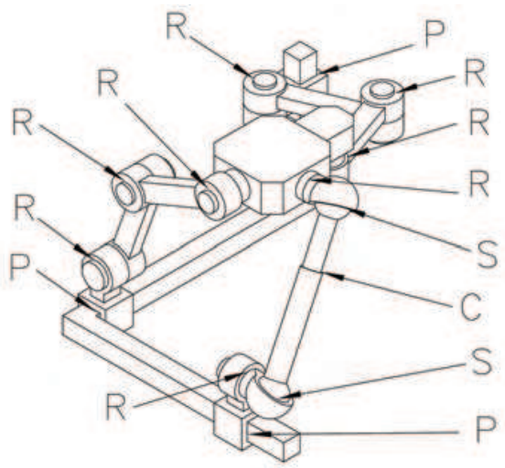


Figure 10: An overconstrained mechanism with redundant joints

Nomenclature

- b : subscript to identify the base;
- e : subscript to identify the end-effector;
- F_i : reference frame attached to *link i*;
- G_i : 3 × 3 orientation matrix of F_i with respect to F_{i-1} at the initial configuration;
- G_{hi} : 4 × 4 homogeneous orientation matrix of F_i with respect to F_{i-1} at the
- initial configuration;
- ${}^d\rho_c$: 3 × 1 position vector of the origin of F_c in F_d;
- ρ_i : 3 × 1 position vector of the origin of F_i in F_{i-1};
- p_i : 3 × 1 position vector of the origin of F_i in F_b
- A_i : 3 × 3 orientation matrix of F_i with respect to F_{i-1};
- ${}^dQ_c$: 3 × 3 orientation matrix of F_c with respect to F_d;
- Q_c : 3 × 3 orientation matrix of F_c with respect to F_b;
- $R_z(\theta)$: 3 × 3 rotation matrix about z axis with θ being the rotation angle:

$$R_z(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix};$$

- $\mathbf{R}_{hz}(\theta)$: 4 × 4 homogeneous rotation matrix about z axis with θ being the rotation angle;
- $\mathbf{B}_x(r)$: 4 × 4 homogeneous translation matrix along x axis with r being the translation distance;
- $\mathbf{C}_i$: 4 × 4 homogeneous transformation matrix of F_i in F_{i-1};
- $\mathbf{H}_i$: 4 × 4 homogeneous transformation matrix of F_i in F_b;
- ${}^d\mathbf{H}_c$: 4 × 4 homogeneous transformation matrix of F_c in F_d;
- e_i : the k_{th} canonical vector which is defined as

$$e_k = \left[\underbrace{0 \ldots 0}_{k-1} 1 \underbrace{0 \ldots 0}_{n-k} \right]^T$$

whose dimension is implicit and depends on the context;

- ${}^{d}\mathbf{T}_c$: tangent operator of F_c in F_d expressed in F_b;
- ${}^{f,d}\mathbf{T}_c$: tangent operator of F_c in F_d expressed in F_f;
- ${}^{d}t_c$: tangent vector of F_c in F_d expressed in F_b;
- ${}^{f,d}t_c$: tangent vector of F_c in F_d expressed in F_f;
- t_c : tangent vector of F_c in F_b expressed in F_b.

Suppose two links coupled by a revolute joint and a reference frame is attached to each of them; at an initial configuration, the origins of the two reference frames $\boldsymbol{F}_{i-1}$ and $\boldsymbol{F}_i$ coincide; the joint axis is parallel to the z-axis of $\boldsymbol{F}_{i-1}$ and intersects the negative side of the x-axis of $\boldsymbol{F}_{i-1}$ at right angle (Fig. 11).

The relative orientation and position are given as

$$\boldsymbol{A}_i = \mathbf{R}_z(\theta_i)\boldsymbol{G}_i \tag{1}$$

$$\rho_i = -r_i e_1 + r_i \mathbf{R}_z(\theta_i) e_1 \tag{2}$$

$$\rho_i = \begin{bmatrix} r_i \cos(\theta_i) - r_i \\ r \sin(\theta_i) \\ 0 \end{bmatrix} = \begin{bmatrix} -2r_i \sin^2(\theta_i / 2) \\ r_i \sin(\theta_i) \\ 0 \end{bmatrix} \tag{3}$$

Instead of taking θ_i as joint variable, we define

$$\mathrm{q}_i = \mathrm{r}_i \theta_i \tag{4}$$

to measure the relative pose of the two links and q_i is referred to as normalized joint variable. In addition, we define

$$w_i = \frac{1}{r_i} \tag{5}$$

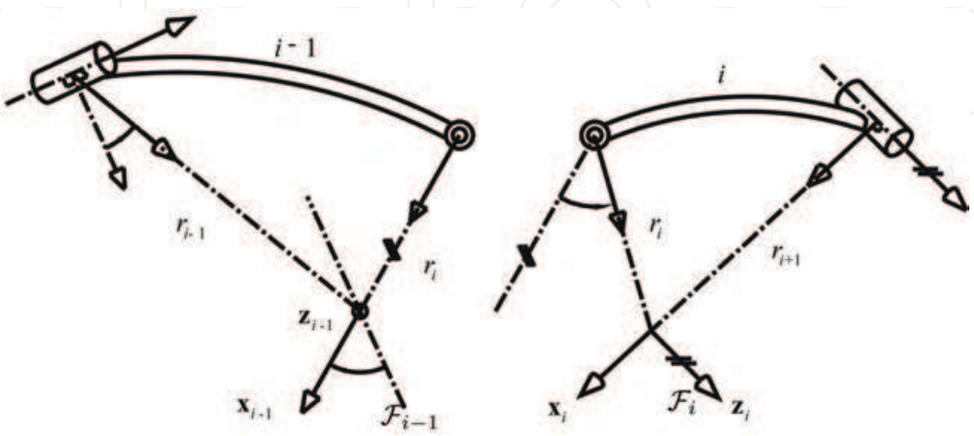


Figure 11: Two links coupled by a revolute joint

Then from equations (3), (4), and (5), we have

$$\rho_i = \begin{bmatrix} -2\sin^2(w_i q_i / 2)/ w_i \\ \sin(w_i q_i)/ w_i \\ 0 \end{bmatrix} \tag{6}$$

It is evident that

$$\lim_{w_i \to 0} \rho_i = \begin{bmatrix} 0 \\ q_i \\ 0 \end{bmatrix} \tag{7}$$

$$\lim_{w_i \to 0} A_i = \lim_{w_i \to 0} [R_z(w_i q_i) G_i] = G_i$$

Equation (7) is just the relative pose of the two links when they are coupled by a prismatic joint. With the above formulation, revolute joints and prismatic ones can be treated in a unified way and the normalization of the joint variable is the key to achieve this.

Definition*: the nature of a joint in a kinematic chain is represented by a pair (k,w) where k is a natural number identifying its orientation from other joints, while w is a non negative number characterizing its membership to revolute joint.*

In fact, *w* characterizes the distance of a revolute joint with respect to the origin of the global reference and represent a prismatic joint when it is equal to 0.

The topology of a fully parallel mechanism of *n*-DOF is represented by *n* matrices with each matrix representing a subchain from the base to the end-effector:

$$\begin{bmatrix} k_{j,1} & k_{j,2} & \cdots & k_{j,m_j-1} & k_{j,m_j-1} \\ w_{j,1} & w_{j,2} & \cdots & w_{j,m_j-1} & w_{j,m_j-1} \end{bmatrix}, \text{ j = 1, 2, ... , } n \tag{8}$$

where m_j is the total number of joints of j^{th} subchain.

This numerical representation is aimed at simultaneous synthesis of both topology and geometry.

For geometric representation, instead of describing separately the geometry of each link, we describe an initial configuration. This is done by giving the coordinates of all joint axes with respect to the global reference frame.

Definition: *the location of a joint axis at an initial configuration is represented by a triple ($\hat{n}$, $\hat{m}$,w) where $\hat{n}$ is a unit vector defining the orientation of the joint axis, $\hat{m}$ is a unit vector indicating the direction of the moment of $\hat{n}$ with respect to the origin of the global reference frame, w is the nature of the joint.*

It is here that the topology information is integrated into the geometric definition.

The *Plücker* coordinates of the joint axis is simply

$$l = \begin{bmatrix} w\hat{n} \\ \hat{m} \end{bmatrix} \tag{9}$$

With this representation, it should be avoided to position the joint such that its axis is too close to the origin of the global reference frame, because this will lead to parameter

singularity, that is w will approach infinity. This does not limit the representation method, because it is the relative location of the joints that defines the geometry, changing the reference frame does not change the geometry.
The topology and geometry of a fully parallel mechanism of n-DOF is represented by n matrices with each matrix representing a subchain from the base to the EE:

$$\begin{bmatrix} \hat{n}_{j,1} & \hat{n}_{j,2} & \dots & \hat{n}_{j,m_j-1} & \hat{n}_{j,m_j} \\ \hat{m}_{j,1} & \hat{m}_{j,2} & \dots & \hat{m}_{j,m_j-1} & \hat{m}_{j,m_j} \\ w_{j,1} & w_{j,2} & \dots & w_{j,m_j-1} & w_{j,m_j} \end{bmatrix}, \text{j} = 1, 2, \dots, n \quad (10)$$

where m_j is the total number of joints of j^{th} subchain.
Those are the design parameters, they are continuous and suffer from no parameter singularity problem.

5. Kinematic modelling of general PMs

The reference frames for all links are defined at the initial configuration and this is done by following the rules given below:

1. Locate the reference frame for the EE such that no joint axis passes through its origin (Fig. 12);

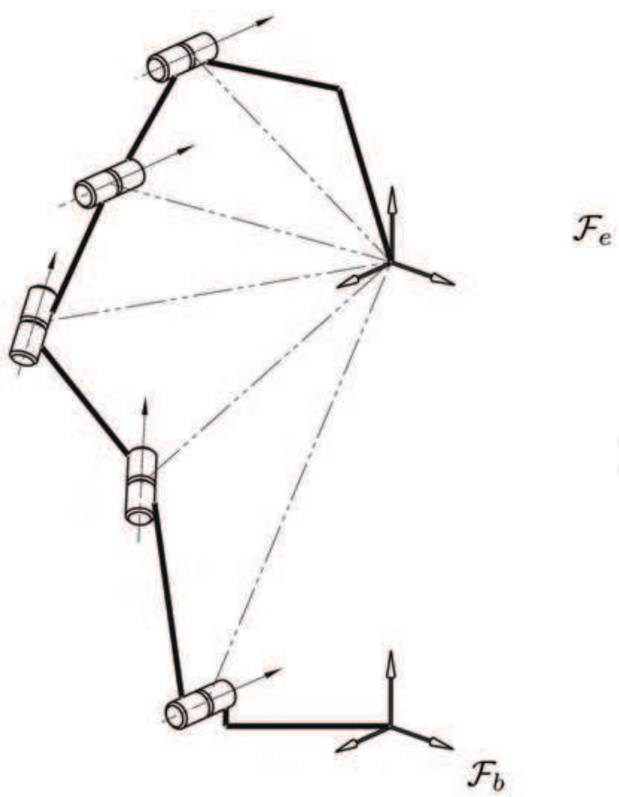


Figure 12: Frame assignment for the EE

2. Change the reference frame of the topological and geometric parameters to the EE frame: recall that ${}^{b}\rho_{e}$ and ${}^{b}\mathbf{Q}_{e}$ denote respectively the position and the orientation of the EE frame in the base frame. For every joint (the subscript is dropped off for simplicity), if ${}^{b}w = 0$ then

$$\begin{aligned} {}^{e}\hat{n} &= {}^{e}\mathbf{Q}_{b}\,{}^{b}\hat{n} \\ {}^{e}\hat{\boldsymbol{m}} &= {}^{e}\mathbf{Q}_{b}\,{}^{b}\hat{\boldsymbol{m}} \\ {}^{e}w &= 0 \end{aligned} \tag{11}$$

otherwise, let P be a point on the axis, ${}^{b}r$ and ${}^{e}r$ denote its positions in the base frame and in the EE frame respectively, we then have

$$\begin{aligned} {}^{e}\hat{\boldsymbol{n}} &= {}^{e}\mathbf{Q}_{b}\,{}^{b}\hat{\boldsymbol{n}} \\ {}^{e}\boldsymbol{r} &= {}^{e}\mathbf{Q}_{b}({}^{b}\boldsymbol{r} - {}^{b}\boldsymbol{\rho}_{e}) \end{aligned}$$

$${}^{e}\boldsymbol{m} = {}^{e}\boldsymbol{r} \times {}^{e}\hat{\boldsymbol{n}} = {}^{e}\mathbf{Q}_{b}({}^{b}\boldsymbol{r} \times {}^{b}\hat{\boldsymbol{n}} - {}^{b}\boldsymbol{\rho}_{e} \times {}^{b}\hat{\boldsymbol{n}}) \tag{12}$$

Let $[{}^{b}\rho_{e}\ \mathrm{x}]$ denote the cross product matrix associated with ${}^{b}\rho_{e}$, since

$${}^{b}\boldsymbol{r} \times {}^{b}\hat{\boldsymbol{n}} = {}^{b}\boldsymbol{m} = {}^{b}\hat{\boldsymbol{m}}/{}^{b}w \tag{13}$$

by substituting equation (13) into (12), we have

$${}^{e}\boldsymbol{m} = -{}^{e}\mathbf{Q}_{b}\left[{}^{b}\boldsymbol{\rho}_{e}\times\right]{}^{b}\hat{\boldsymbol{n}} + {}^{e}\mathbf{Q}_{b}\,{}^{b}\hat{\boldsymbol{m}}/{}^{b}w \tag{14}$$

then, the *Plücker* coordinates of the axis in the EE frame can be computed as

$$\begin{bmatrix} {}^{e}\hat{\boldsymbol{n}} \\ {}^{e}\boldsymbol{m} \end{bmatrix} = \begin{bmatrix} {}^{e}\mathbf{Q}_{b} & \mathbf{O} \\ -{}^{e}\mathbf{Q}_{b}\left[{}^{b}\boldsymbol{\rho}_{e}\times\right] & {}^{e}\mathbf{Q}_{b} \end{bmatrix} \begin{bmatrix} {}^{b}\hat{\boldsymbol{n}} \\ {}^{b}\hat{\boldsymbol{m}}/{}^{b}w \end{bmatrix} \tag{15}$$

Finally, ${}^{e}w = 1/\left\|{}^{e}m\right\|_{2}$ and ${}^{e}\hat{m} = {}^{e}m/{}^{e}w$.

3. Links of subchain j from the base to the EE are identified by *link*(j, 0) to *link*(j,m_j), the base being *link*(j, 0) and the EE being *link*(j,m_j); joint coupling *link*(j, i–1) and *link*(j, i) is identified by *joint*(j, i); frame $F_{j,i}$ is attached to *link*(j, i)(*Fig.* 13); the base and the EE have multiple rigidly attached frames with each of them corresponding to an individual subchain;
4. The reference frame for *link*(j, i) is defined such that

$${}^{e}\mathbf{Q}_{j,i} = \left[\ {}^{e}\hat{\boldsymbol{m}}_{j,i+1} \times {}^{e}\hat{\boldsymbol{n}}_{j,i+1},\ \ {}^{e}\hat{\boldsymbol{m}}_{j,i+1},\ \ {}^{e}\hat{\boldsymbol{n}}_{j,i+1}\ \right] \tag{16}$$

$${}^{e}\rho_{j,i} = 0 \tag{17}$$

the z-axis of $F_{j,i}$ being parallel to the axis of *joint*(j, i + 1) and the x-axis intersecting the the axis of *joint*(j, i + 1) and pointing from the intersecting point to the origin of the EE frame (Fig. 14). The y-axis is determined as usual by the right-hand rule.

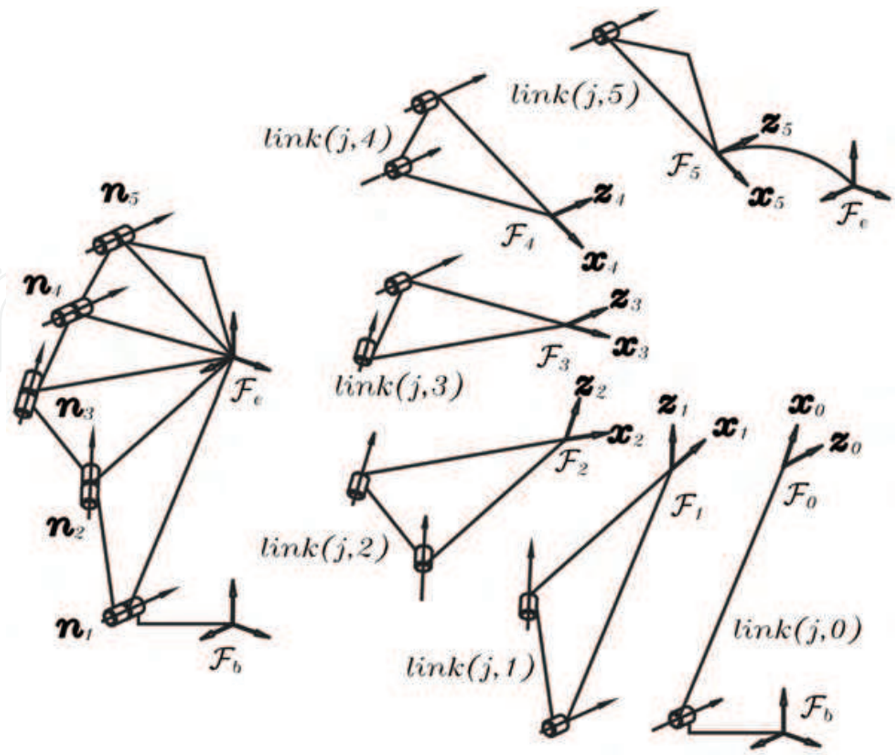


Figure 13: Link reference frames

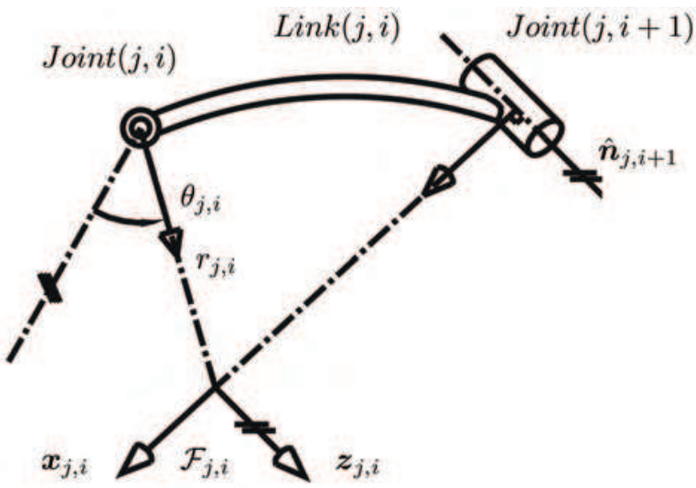


Figure 14: Reference frame definition for *link*(*i, j*)

5. The normalized joint variable of *joint*(*j, i*) is denoted by $q_{j,i}$, the rotation angle with respect to the initial configuration is denoted by $\theta_{j,i}$ and

$$\theta_{j,i} = w_{j,i}\, q_{j,i} \tag{18}$$

6. Compute the link geometry matrices from ${}^{b}Q_{e}$, ${}^{e}Q_{j,0}$, $\cdots$, and ${}^{e}Q_{j,mj}$: for $G_{j,1}$ to $G_{j,mj-1}$

$$G_{i,j} = {}^{j,i-1}Q_e\, {}^{e}Q_{j,i} \tag{19}$$

$G_{j,0}$, $G_{j,mj}$, and $G_{j,e}$ are treated differently, *i.e.*

$$G_{j,0} = {}^{b}Q_e\, {}^{e}Q_{j,0} \tag{20}$$

$$G_{j,mj} = 1 \tag{21}$$

$$G_{j,e} = {}^{j,mj}Q_e \tag{22}$$

The sequence of links in each subchain has a corresponding sequence of homogeneous transformations that defines the pose of each link relative to its neighbor in the chain. The pose of the EE is therefore constrained by the product of these transformations through every subchain. With the above frame assignment, the pose of *link*(*j, i*) with respect to *link*(*j, i* − 1) is given as

$$\mathbf{C}_{j,i} = \mathbf{B_x}(-\frac{1}{w_{j,i}})\mathbf{R_{hz}}(w_{j,i}q_{j,i})\mathbf{B_x}(\frac{1}{w_{j,i}})\mathbf{G_h}_{j,i} \tag{23}$$

The corresponding 3 × 3 orientation matrix is given as

$$\mathbf{A}_{j,i} = \mathbf{R_z}(w_{j,i}q_{j,i})\mathbf{G}_{j,i} \tag{24}$$

The corresponding position is given as

$$\boldsymbol{\rho}_{j,i} = -\frac{\boldsymbol{e}_1}{w_{j,i}} + \mathbf{R_z}(w_{j,i}q_{j,i})\frac{\boldsymbol{e}_1}{w_{j,i}} \tag{25}$$

This leads to

$$\boldsymbol{\rho}_{j,i} = \begin{bmatrix} \frac{1}{w_{j,i}}\cos(w_{j,i}q_{j,i}) - \frac{1}{w_{j,i}} \\ \frac{1}{w_{j,i}}\sin(w_{j,i}q_{j,i}) \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{2}{w_{j,i}}\sin^2\left(\frac{w_{j,i}q_{j,i}}{2}\right) \\ \frac{1}{w_{j,i}}\sin(w_{j,i}q_{j,i}) \\ 0 \end{bmatrix} \tag{26}$$

When $w_{j,i}$ approaches 0, we have

$$\lim_{w_{j,i}\to 0}\mathbf{A}_{j,i} = \mathbf{G}_{j,i} \tag{27}$$

$$\lim_{w_{j,i}\to 0}\boldsymbol{\rho}_{j,i} = \begin{bmatrix} 0 \\ q_{j,i} \\ 0 \end{bmatrix} \tag{28}$$

This corresponds to the situation of a prismatic joint.
The pose of the EE under the structure constraint of subchain *j* is

$$\mathbf{H}_e = \mathbf{H}_{j,0}\left(\prod_{i=1}^{m_j}\mathbf{C}_{j,i}\right)\mathbf{C}_{j,e},\ i = 1, 2, \cdots, m_j \tag{29}$$

In terms of orientation and position, equation (29) can be written as

$$\mathbf{Q}_e = \mathbf{Q}_{j,0}\left(\prod_{i=1}^{m_j}\mathbf{A}_{j,i}\right)\mathbf{A}_{j,e},\ i = 1, 2, \cdots, m_j \tag{30}$$

$$\mathbf{Q}_{j,i} = \mathbf{Q}_{j,0}\prod_{k=1}^{i}\mathbf{A}_{j,k},\ k = 1, 2, \cdots, i \tag{31}$$

$$\boldsymbol{p}_{j,i} = \boldsymbol{p}_{j,0} + \sum_{k=1}^{i}\left(\mathbf{Q}_{j,k-1}\boldsymbol{\rho}_{j,k}\right),\ k = 1, 2, \cdots, i \tag{32}$$

$$\boldsymbol{p}_{j,e} = \boldsymbol{p}_{j,0} + \sum_{i=1}^{m_j}\left(\mathbf{Q}_{j,i-1}\boldsymbol{\rho}_{j,i}\right) + \mathbf{Q}_{j,m_j}\boldsymbol{\rho}_{j,e},\ i = 1, 2, \cdots, m_j \tag{33}$$

Equations (31) and (32) are used to compute the orientation and position of links other than the base and the EE.

For a PM of n degree of freedom, the n subchains are closed by rigidly attaching together their fist link frames and last link frames respectively. The structure equations are obtained by equating the transformation products defined by equation (29) of all subchains, *i.e.*, $\forall\, j, k = 1, 2, \cdots, n$ and $j \neq k$

$$\mathbf{H}_{j,0}\left(\prod_{i=1}^{m_j}\mathbf{C}_{j,i}\right)\mathbf{C}_{j,e} = \mathbf{H}_{k,0}\left(\prod_{i=1}^{m_k}\mathbf{C}_{k,i}\right)\mathbf{C}_{k,e} \tag{34}$$

It is obvious that this kinematic formulation is not aimed at simplifying the forward or inverse kinematic solutions, but for the simultaneous topological and geometric synthesis with numeric method, genetic algorithms in particular. The initial population will be generated using the numeric topological representation proposed in Section 3 and the reproduction performed while respecting the characteristic constraints. The implementation of the synthesis for translational PMs is being carried out in our laboratory.

6. Conclusion

By introducing characteristic constraints, kinematic chains of serial and parallel manipulators can be better characterized. This is essential for both topology synthesis and geometry synthesis. On the one hand, topology synthesis of spatial manipulator is no longer dimension-independent; most of the topology syntheses are actually the search for some special geometric constraints which play a key role in determining the fundamental kinematic properties. On the other hand, it is necessary to identify the characteristic constraints when performing geometry synthesis in order for the design space to correspond appropriately to the manipulators having the intended kinematic properties. The graph structure of the proposed topological representation makes it possible to implement computer algorithms in order to perform systematic enumeration, comparison and classification of serial and parallel manipulators. The geometric representation is well adapted for computer aided simultaneous topological and geometric synthesis by introducing the concepts of initial configuration and the joint nature, making it possible to

represent revolute joints and prismatic joints in a unified way. Then a singularity-free parametrization of both topology and geometry was proposed. After that, joint variables were normalized, which enables the joint type to be seamlessly incorporated into kinematic model, it is no longer necessary to reformulate the kinematic model when a revolute joint is replaced by a prismatic one or vice versa. The effectiveness of the propose kinematic modelling remains to be evaluated.

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Parallel Manipulators, New Developments

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Parallel manipulators are characterized as having closed-loop kinematic chains. Compared to serial manipulators, which have open-ended structure, parallel manipulators have many advantages in terms of accuracy, rigidity and ability to manipulate heavy loads. Therefore, they have been getting many attentions in astronomy to flight simulators and especially in machine-tool industries.The aim of this book is to provide an overview of the state-of-art, to present new ideas, original results and practical experiences in parallel manipulators. This book mainly introduces advanced kinematic and dynamic analysis methods and cutting edge control technologies for parallel manipulators. Even though this book only contains several samples of research activities on parallel manipulators, I believe this book can give an idea to the reader about what has been done in the field recently, and what kind of open problems are in this area.

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