Chapter

Evaluating the Organizational Hierarchy Using the IFSAW and TOPSIS Techniques

Mahuya Deb

Abstract

Performance evaluations in organizations are viewed as ideal instruments for evaluating and rewarding the employee's performance. While much emphasis is laid onto the administering of the evaluation techniques, not much thought has been laid out on assessing the contributions of each hierarchical level. Moreover the manifold decision making criteria can also impact the measurement of pertinent contributions because of their ambivalent characteristics. In such a scenario, intuitionistic fuzzy multi-criteria decision making can help strategists and policy makers to arrive at more or less accurate decisions. This paper restricts itself to six decision making criteria and adopts the intuitionistic fuzzy simple additive weighting (IFSAW) method and TOPSIS method to evaluate and rank the employee cadres. The results obtained were compared and both the methods revealed that the middle management displayed impeccable performance standards over their other counterparts.

Keywords: performance evaluation, organisation, intuitionistic fuzzy, IFSAW, TOPSIS

1. Introduction

Organizational fit theories have long emphasized that appropriate selection strategies can lead to superior performance compared to firms that relatively overlook the employee selection based on fit theories [1]. The extent of fit between the individual and the organization determines the labor productivity [2–4] as well as the financial performance [5–8].

The other criterions that influence the overall organizational performance are informal learning [9], workplace competencies [10], organizational citizenship behavior [11] and the like.

While many employee focused parameters are relied on while determining the organizational performance, very few researches have essayed the contributions of each of the hierarchical cadres. Performance evaluations in organizations have traditionally focused on short-term financial and technical results. But modern organizations have not just demanded a generic short-term performance assessment, but an effective means to categorize employees as vital opportunities or threats. By using measurable performance results, with a focus on the entire organization, managers will be able to determine their progress toward longterm goals and objectives [12]. Moreover, superior performance cannot be achieved by just de-layering and de-staffing. Whilst these techniques can to a certain extent eliminate the imperfections within the system, it is the overall behaviors of the employees that need a volte-face. Explicit construal of roles of the employees and managers in particular, will ensure that the managers do not slip into the comfortable and familiar role structure of grand strategists, administrative controllers, and operational implementers. Each hierarchical level or cadre needs to exemplify its cardinal responsibilities that add distinct value to an organization [13]. Identifying, weighting and evaluating the various level of managers against various criteria can be assumed as a function of multi criteria decision making process.

While focus on HR metrics has been growing off late, there is still an element of bias and ambiguity regarding the criteria that are being used rather the greatest difficulty lies in the quantification of criteria being not clearly defined. The basis for the selection of criterions is the subjective judgements by the higher authorities in organisations. These judgements/verbal descriptions do not exhibit the characteristic of being classified into a dichotomous group and are therefore treated as linguistic variables. Also the relation between the different hierarchical levels and the criterions on the basis of which they are assessed are not known precisely. This provides a framework where a different methodology is required. Thus to understand such a structure a verbal description would suffice. A formal way of dealing with them is the linguistic approach by Zadeh [14]. Its basic feature is the use of linguistic variables which are the ones whose values are words or sentences in a language in place of numerical value and a fuzzy conditional statement for expressing the relation between linguistic variables. Here the meaning of a linguistic variable is equated with a fuzzy set while the meaning of the fuzzy conditional statement with a fuzzy relation. Since its inception about a decade ago, the theory of fuzzy sets has evolved in many directions, and is finding applications in a wide variety of fields in which the phenomena under study are too complex or too ill defined to be analyzed by conventional techniques. Fuzzy set theory (FST) [15] allows for subjective evaluation by the decision maker under conditions of uncertainty and ambiguity. It helps to express irreducible observations and measurement uncertainties which are intrinsic to the empirical data. It offers far greater resources for managing complexity and controlling computational cost and allows for conversion of linguistic variables to fuzzy numbers using membership functions. Membership functions assigns to each object a grade of membership denoted by $\mu_A(x)$ which ranges between zero and one. It maps every element of the universe of discourse X to the interval [0, 1] which is written as $\mu_A : X \to [0, 1]$. Each fuzzy set is completely and uniquely defined by one particular membership function. A "direct" use of verbal descriptions of those criteria via the concepts of the fuzzy set is proposed here.

A fuzzy set is defined by

$$\overline{A} = \left\{ \left(x, \mu_{\overline{A}}(x)\right) / x \in X, \mu_{\overline{A}}(x) \in [0, 1] \right\}$$

In the pair $\left(x, \mu_{\overline{A}}(x)\right)$ the first element x belong to the classical set X, the second element $\mu_{\overline{A}}(x)$belong to the interval [0, 1] which is called the membership function or grade of membership function. This membership function is represented with the help of fuzzy number. It represents the degree of compatibility or a degree of truth of x in $\overline{A}$. The idea of fuzzy numbers was given by Dubois and Prade [16].

A fuzzy subset $\overline{A}$ of the real line R with membership function $\mu_{\overline{A}}(x) : R \to [0, 1]$ is called a fuzzy number if.

i. $\overline{A}$ is normal, (i.e.) there exist an element x_0 such that $\mu_{\overline{A}}(x_0) = 1$.

ii. $\overline{A}$ is fuzzy convex,

$$\text{i.e. } \mu_{\overline{A}}(\lambda x_1 + (1-\lambda)x_2) \geq \min\left\{\mu_{\overline{A}}(x_1), \mu_{\overline{A}}(x_2)\right\} \quad x_1, x_2 \in R, \forall \lambda \in [0,1]$$

iii. $\mu_{\overline{A}}(x)$ is upper continuous, and.

iv. supp $\overline{A}$ is bounded, where supp $\overline{A} = \left\{x \in R : \mu_{\overline{A}}(x) > 0\right\}$.

A fuzzy number $\overline{A}$ of the universe of discourse U may be characterized by a triangular distribution function parameterized by a triplet (a_1, a_2, a_3) (**Figure 1**).

Mikhailovich [17] used the fuzzy sets while solving the problem of factor causality. Dintsis [18] in his work dealt with the idea of implementing fuzzy logic for transforming descriptions of natural language to formal fuzzy and stochastic models. However, fuzzy sets lack in the idea of non -membership function. Whatever information is provided by fuzzy sets does not appear complete in context of decision making as there is no room for alternatives dissatisfying the attributes. Thus Atanassov [19] used the idea of membership value, non-membership value as well as the hesitation index to characterize an intuitionistic fuzzy set. He opined that the sum of membership value and non-membership value lies between zero and one and the hesitation index is calculated as one minus the sum of membership value and non-membership value of an element of a set. In other words some hesitation about degree of belongingness of an element of a set exists. For a fuzzy set the hesitation index is zero. The fuzzy sets along with intuitionistic fuzzy sets can depict real life application areas defined by uncertainity. Some recent applications of fuzzy systems are found in the works of [20, 21].

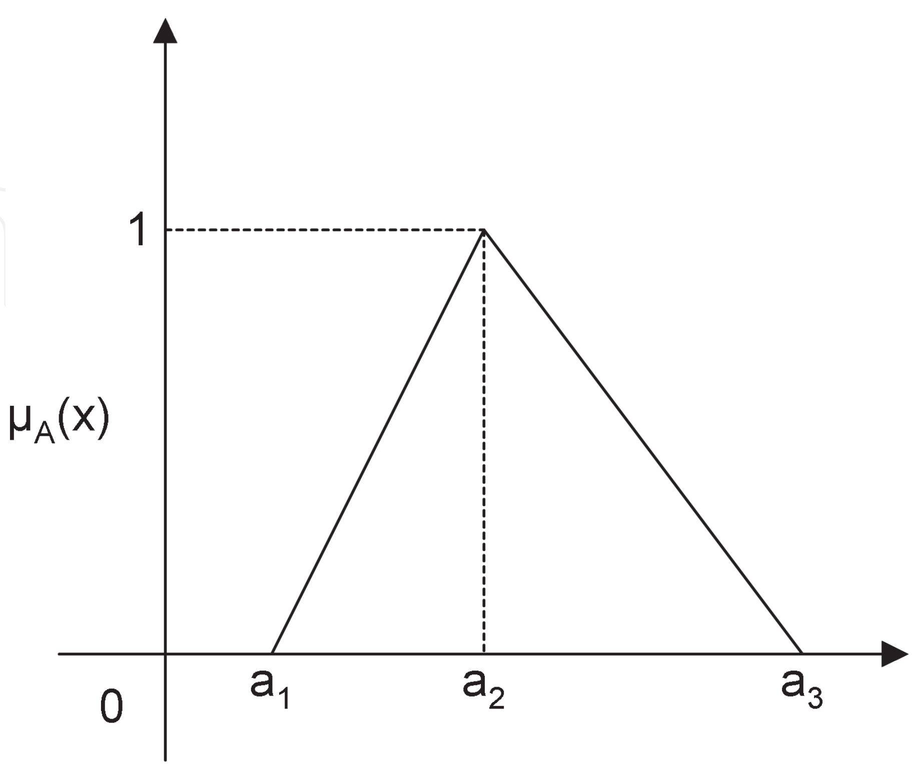

Figure 1.
Membership function of TFN.

1.1 Intuitionistic fuzzy set

Let X be a fixed set. An IFS $\tilde{A}$ in X is of the form $\tilde{A} = \{<x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x)> : x \in X\}$, where the $\mu_{\tilde{A}}(x) : X \to [0,1]$ and $\nu_{\tilde{A}}(x) : X \to [0,1]$. This represents the degree of membership and of non membership respectively of the element $x \in X$ to the set $\tilde{A}$, which is a subset of the set X, for every element of $x \in X, 0 \le \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \le 1$ [22].

The value of $\pi_A(X) = 1 - \mu_A(X) - v_A(X)$ represents the degree of hesitation (or uncertainty) associated with the membership of elements x ε X in IFS A. This is known as the intuitionistic fuzzy index of A with respect to element x.

1.2 Intuitionistic fuzzy number

An IFN $\tilde{A}$ is defined as follows [22]:

i. an intuitionistic fuzzy subset of the real line

ii. it is normal, i.e. there is any $x_0 \in R$ such that $\mu_{\tilde{A}}(x) = 1 \ \left(so \ \ v_{\tilde{A}}(x) = 0\right)$

iii. a convex set for the membership function $\mu_{\tilde{A}}(x)$i.e.

$$\mu_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \ge \min\left(\mu_{\tilde{A}}(x_1), \mu_{\tilde{A}}(x_2)\right) \forall x_1, x_2 \in R \lambda \in [0,1]$$

iv. a concave set for the non-membership function $v_{\tilde{A}}(x)$ i.e.

$$v_{\tilde{A}}(\lambda x_1 + (1-\lambda)x_2) \le \max\left(v_{\tilde{A}}(x_1), v_{\tilde{A}}(x_2)\right) \forall x_1, x_2 \in R, \lambda \in [0,1]$$

A triangular intuitionistic fuzzy number $\tilde{A} = \left(a_1, a_2, a_3; a'_1, a_2, a'_3\right)$ is a subset of intuitionistic fuzzy set on the set of real number R whose membership and non membership are defined as follows:

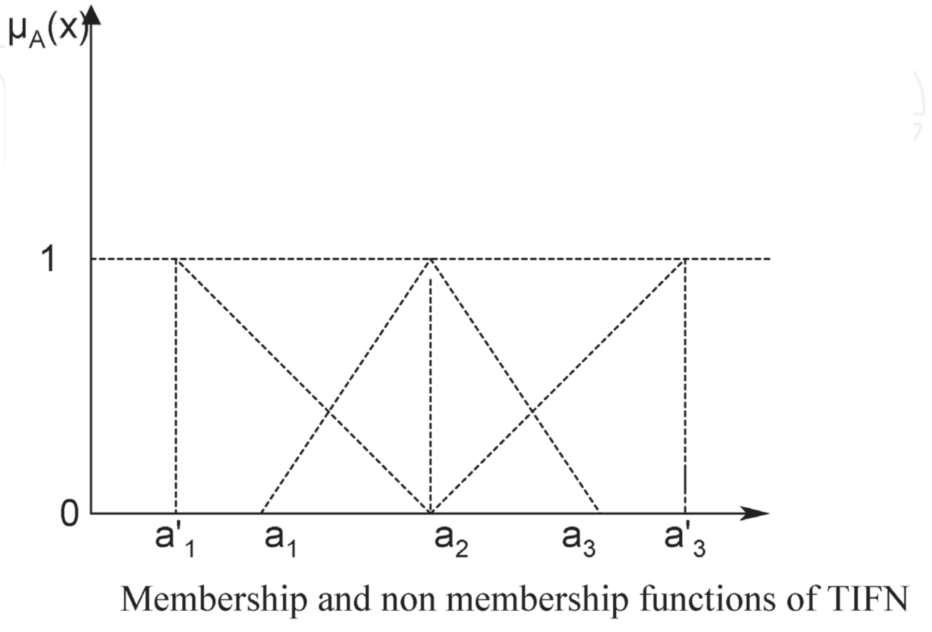

Figure 2.
A triangular intuitionistic fuzzy number.

$$\mu_{\tilde{A}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1<x\le a_2\\ \dfrac{a_3-x}{a_3-a_2}, & a_2<x\le a_3\\ 0, & otherwise\end{cases}\qquad v_{\tilde{A}}(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1'}, & a_1'<x\le a_2\\ \dfrac{x-a_2}{a_3'-a_2}, & a_2<x\le a_3'\\ 1, & otherwise\end{cases}$$

Intuitionistic fuzzy set is widely recognised and is being studied and applied in various fields be it in science, psychology and other growing fields like consumer behaviour, advertising and communications where decision making is crucial (**Figure 2**).

In this work two methods of intuitionistic fuzzy sets viz. SAW (simple additive weight method) and TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) are used for ranking the various levels of employees in an organisation. The paper is organised as follows: Section 2 begins with the basic operations of intuitionistic fuzzy sets; Section 3 and 4 explain the intuitionistic fuzzy SAW algorithm and TOPSIS methodology which are used in the paper. Section 5 illustrates the procedure for evaluating the hierarchical level using the proposed algorithms. Section 6 is the final discussion and conclusion related to the evaluation procedure.

2. Operations on intuitionistic fuzzy sets

Let A and B are IFS s of the set X, then multiplication operator is defined as follows [19]:

$$A\otimes B=\left[\mu_A(x).\mu_B(x)_{V_A}(x)+v_B(x)-v_A(x)v_B(x),1\right.\\ \left.-\{\mu_A(x).\mu_B(x)+(v_A(x)+v_B(x)-v_A(x)v_B(x)\}\right] \tag{1}$$

Let A = (μ, v) be an intuitionistic fuzzy number, a score function S of an intuitionistic fuzzy value can be represented as follows:

$$\mathrm{S(A)} = \mu\text{-v}, \mathrm{S(A)}\,\varepsilon[\text{-}1, 1] \tag{2}$$

If S (A_i) represents the largest among the values of {**S(**A_i**)**}, then the alternative A_i is the best choice.

3. Intuitionistic fuzzy simple additive weighting algorithm

This method is a simple additive weighting method developed by Hwang and Yoon [23]. According to this principle the first step ensures in obtaining a weighted sum of the performance ratings of each alternative under all attributes. Let A_1, A_2, A_3, … , A_n be n alternatives which denotes the employee cadres. Let C_1, C_2, C_3, … , C_m, be the criteria on the basis of which the evaluation is done. Further each criteria is assigned weight given by the decision makers and it is represented by a weighting vector W = {W_1, W_2, W_3, … , W_n}, where W_1, W_2, W_3, … , W_n are represented by intuitionistic fuzzy sets defined as follows:

$$\mathrm{Wj} = \left\{\mu_w(x_j), v_w(x_j), \pi_w(x_j)\right\}, \text{where } j = 1, 2, \dots, n. \tag{3}$$

The procedure for Intuitionistic fuzzy SAW is being presented as follows:

Step 1: Construct an intuitionistic fuzzy decision matrix: $\boldsymbol{R} = (\boldsymbol{r_{ij}})_{\mathbf{m\ x\ n}}$ such that $\tilde{r}_{ij}=\left(\mu_{ij},\nu_{ij},\pi_{ij}\right)$

$$\tilde{R} = \begin{bmatrix} \tilde{r}_{11} & \tilde{r}_{12} & \dots. & \tilde{r}_{1n} \\ \tilde{r}_{21} & \tilde{r}_{22} & \dots & \tilde{r}_{2n} \\ \dots & .. & .. & .. \\ \tilde{r}_{m1} & \tilde{r}_{m2} & .. & \tilde{r}_{mn} \end{bmatrix}$$

(i = 1,2, … ,m; j = 1,2, … ,n),. In $\tilde{r}_{ij}$, μ_{ij} indicates the degree that the alternative A_i satisfies C_j and ν_{ij} indicates the degree that the alternative A_i does not satisfy the attribute C_j.

Step 2: This step entails performing the transformation by using Eq. (1) and obtain the total intuitionistic fuzzy scores V (A_i) for individual vendors. This is determined by the product of intuitionistic fuzzy weight vectors (W) and intuitionistic fuzzy rating matrix (R).

$$\mathrm{V}(\mathrm{A_i}) = \mathrm{R} \odot \mathrm{W} = \sum_{i=1} \left[\left\{ \mu_{A_i}(x_j), \nu_{A_i}(x_j), \pi_{A_i}(x_j) \right\} \otimes \left\{ \mu_w(x_j), \nu_w(x_j), \pi_{w_i}(x_j) \right\} \right] \tag{4}$$

Step 3: The third step is used for ranking the alternatives. Applying Eq. (**2**) a crisp score function $S(A_1),S(A_2), \dots ,S(A_n)$ is calculated for the various alternatives. The largest value of S (A_j) among $S(A_1),S(A_2), \dots ,S(A_n)$ represents the best alternative or vendor.

Step 4: This approach is compared with Jun Ye [24] on weighted correlation coefficient under intuitionistic fuzzy environment.

4. Principle of TOPSIS for decision making with intuitionistic fuzzy set

TOPSIS methodology is proposed by [25]. The fundamental principle underlying this theory is that the alternative which is chosen entails that it has the least distance from the positive ideal- solution (i.e. alternative) and its distance is the farthest from the negative ideal- solution (i.e. alternative).

Suppose there exists n decision making alternatives given by the set $A = \{A_1, A_2, \dots, A_n\}$ from which a most preferred alternative is to be selected. These are assessed based on m attributes, both quantitative and qualitative. The set of all attributes is denoted by $X = \{x_1, x_2, \dots, x_m\}$. The ratings of different alternatives A_j on attributes x_i are expressed with intuitionistic fuzzy sets $F_{ij} = \left(\mu_{ij}, \nu_{ij}\right)$ where $\mu_{ij} \in [0,1], \nu_{ij} \in [0,1]$ and $0 \le \mu_{ij} + \nu_{ij} \le 1$. Thus, the ratings of any alternatives A_j on all m attributes x_i are expressed with intuitionistic fuzzy vector $\left(\left\langle \mu_{1j}, \nu_{1j} \right\rangle, \left\langle \left\langle \mu_{2j}, \nu_{2j} \right\rangle, \dots\dots, \left\langle \mu_{mj}, \nu_{mj} \right\rangle\right)^T$.

The intuitionistic fuzzy decision matrix is represented as $F = \left(\left(\mu_{ij}, \nu_{ij}\right)\right)_{mxn}$

$$= \begin{bmatrix} (\mu_{11}, \nu_{11}) & (\mu_{12}, \nu_{12}) & \dots. & ..(\mu_{1n}, \nu_{1n}).. \\ (\mu_{21}, \nu_{21}) & (\mu_{22}, \nu_{22}) & \dots\dots & (\mu_{2n}, \nu_{2n}) \\ \dots & \dots & \dots. & \dots. \\ (\mu_{m1}, \nu_{m1} & (\mu_{m2}, \nu_{m2}) & \dots & (\mu_{mn}, \nu_{mn}) \end{bmatrix} \tag{5}$$

It is assumed that the weights ω_i of the attributes $x_i \in X$ are real numbers known a priori i.e. the weight vector $\omega = (\omega_1, \omega_2, \omega_3, \dots \omega_m)^T$ of attributes are known.

Since the weights of the attributes are not precisely defined therefore they are treated as intuitionistic fuzzy sets i.e. the weight of each factor is expressed with the intuitionistic fuzzy set $\omega_i = \{\langle x_i, \alpha_i, \beta_i \rangle\}$ where $\alpha_i \in [0,1]$ and $\beta_i \in [0,1]$ are respectively the degree of membership and non membership respectively of the attribute $x_i \in X$. Usually $\omega_i = \{\langle x_i, \alpha_i, \beta_i \rangle\}$ is denoted by $\omega_i = \langle \alpha_i, \beta_i \rangle$ in short. The weight of all attributes is concisely expressed in the vector format as follows:

$$\begin{aligned} \omega &= (\omega_1, \omega_2, \omega_3, \ldots \omega_m)^T \\ &= (\langle \alpha_1, \beta_1 \rangle, \langle \alpha_2, \beta_2 \rangle, \ldots \langle \alpha_m, \beta_m \rangle)^T \end{aligned} \tag{6}$$

4.1 Principle and process of TOPSIS

The entire methodology can be summarized as follows:

1. Identify and determine the attributes and alternatives, denoted respectively by $A = \{A_1, A_2, \ldots A_n\}$ *and* $X = \{x_1, x_2, \ldots, x_m\}$

2. The decision maker's opinion is obtained to get ratings of the alternatives on the attributes i.e. construct the intuitionistic fuzzy decision matrix

$$F = \left(\left\langle \mu_{ij}, \nu_{ij} \right\rangle\right)_{mxn} \tag{7}$$

3. The opinion so obtained are combined to determine the weights of the attributes expressed with intuitionistic fuzzy weight vector $\omega = (\langle \alpha_i, \beta_i \rangle)_{mx1}$

4. Next the weighted intuitionistic fuzzy decision matrix $F = \left(\left\langle \mu_{ij}, \nu_{ij} \right\rangle\right)_{mxn}$ is computed using the following formula

$$\begin{aligned} \left\langle \mu_{ij}, \nu_{ij} \right\rangle &= \omega F_{ij} \\ &= \langle \alpha_i, \beta_i \rangle \left\langle \mu_{ij}, \nu_{ij} \right\rangle \\ &= \left\langle \alpha_i \mu_{ij}, \beta_i + \nu_{ij} - \beta_i \nu_{ij} \right\rangle \end{aligned} \tag{8}$$

5. For calculating the intuitionistic fuzzy positive ideal –solution and intuitionistic fuzzy negative ideal –solution the following formulas are obtained

$$\begin{aligned} A^+ &= (\langle \mu_1{}^+, \nu_1{}^+ \rangle, \langle \mu_2{}^+, \nu_2{}^+ \rangle, \ldots \langle \mu_m{}^+, \nu_m{}^+ \rangle)^T \\ A^- &= (\langle \mu_1{}^-, \nu_1{}^- \rangle, \langle \mu_2{}^-, \nu_2{}^- \rangle, \ldots \langle \mu_m{}^-, \nu_m{}^- \rangle)^T \end{aligned} \tag{9}$$

$$\begin{aligned} \textit{where } \mu_i^+ &= \max_{1 \le j \le n} \left\{ \mu_{ij} \right\} \quad \nu_i^+ = \min_{1 \le j \le n} \left\{ \nu_{ij} \right\} \\ \mu_i^- &= \min_{1 \le j \le n} \left\{ \mu_{ij} \right\} \quad \nu_i^- = \max_{1 \le j \le n} \left\{ \nu_{ij} \right\} \end{aligned} \tag{10}$$

6. The Euclidean distances of the various alternatives A_j (j = 1,2, … n) from the intuitionistic fuzzy positive ideal and intuitionistic fuzzy negative ideal solution are computed using the following equations

$$D(A_j, A^+) = \sqrt{\frac{1}{2}\left(\sum_{i=1}^{m}\left[\left(\mu_{ij} - \mu_i^+\right)^2 + \left(\nu_{ij} - \nu_i^+\right)^2 + \left(\pi_{ij} - \pi_i^+\right)^2\right]\right)} \tag{11}$$

$$D(A_j, A^-) = \sqrt{\frac{1}{2}\left(\sum_{i=1}^{m}\left[\left(\mu_{ij} - \mu_i^-\right)^2 + \left(\nu_{ij} - \nu_i^-\right)^2 + \left(\pi_{ij} - \pi_i^-\right)^2\right]\right)} \tag{12}$$

7. Thereafter the relative closeness degree λ_j of the alternatives $A_{j\ (j\ =\ 1,2\ldots,n)}$ to the intuitionistic fuzzy positive ideal solution are obtained from

$$\lambda_j = \frac{D(A_j, A^-)}{D(A_j, A^+) + D(A_j, A^-)}, j = 1, 2, \ldots, n \tag{13}$$

8. Lastly determine the ranking order of the alternatives A_j (j = 1,2 ... n) according to the non increasing order of the relative closeness degrees λ_j and the best alternative from A.

Using the two approaches the different level of workers in the organisation are assessed. For a better understanding of the situation an example is worked out below:

5. Numerical example

The example is illustrated as below:

An organization has employed six decision making criteria in order to select the most effective hierarchical level in an organization based on the following criterions.

- Instructional effectiveness (C1)
- Decision making (C2)
- Knowledge and Proficiency (C3)
- Leadership (C4)
- Organizational Citizenship Behaviour (C5)
- Flexibility and Adaptability (C6)

The hierarchical levels of an organization were broadly restricted to four and were compared based on the six decision making criteria (as indicated in **Table 1**).

Hierarchical Levels	Criterion	C1	C2	C3	C4	C5	C6
Senior Management (HL_1)		A	B	A	A	B	A
Middle Management (HL_2)		A	A	B	A	C	B
Junior Management (HL_3)		B	A	C	B	A	C
Staff (HL_4)		B	B	C	C	A	C

A-High B-Average C-Low.

Table 1.
Comparison of Hierarchical levels.

The employees are rated (A, B, C) based on the judgement provided by experts in the organisation .

The intuitionistic fuzzy decision matrix has been constructed as below (**Table 2**):

Methods Criterion	C1	C2	C3	C4	C5	C6
Senior Management (HL_1)	(.7,.1,.2)	(.5,.3,.2)	(.8,.1,.1)	(.7,.2,.1)	(.5,.3,.2)	(.8,.1,.1)
Middle Management (HL_2)	(.7,.1,.2)	(.8,.1,.1)	(.6,.3,.1)	(.8,.1,.1)	(.3,.3,.4)	(.7,.2,.1)
Junior Management (HL_3)	(.5,.1,.4)	(.7,.1,.2)	(.3,.5,.2)	(.5,.3,.2)	(.8,.1,.1)	(.3,.3,.4)
Staff (HL_4)	(.5,.4,.1)	(.6,.3,.1)	(.3,.3,.4)	(.2,.3,.5)	(.7,.2,.1)	(.2,.3,.5)

Table 2.
Intuitionistic fuzzy decision matrix.

The weights for the criteria are as below:

	C1	C2	C3	C4	C5	C6
W_i	(.2,.4,.4)	(.2,.2,.6)	(.1,.5,.4)	(.5,.3,.2)	(.3,.4,.3)	(.2,.4,.4)

Table 3.
Weights of the criteria.

The total intuitionistic fuzzy score $V(HL_i)$ for each hierarchical level is calculated as follows:

$$V(HL_1) = [(.7, .1, .2) * (.2, .4, .4)] + [(.5, .3, .2) * (.2, .2, .6)] + [(.8, .1, .1) * (.1, .5, .4)]$$
$$+[(.7, .2, .1) * (.5, .3, .2)] + [(.5, .3, .2) * (.3, .4, .3)] + [(.8, .1, .1) * (.2, .4, .4)]$$

$$V(HL_1) = [.7 * .2, .1 + .4\text{-}.1 * .4, 1\text{-}(.7 * .2 + .1 + .4\text{-}.1 * .4)] + [.5 * .2, .3 + .2\text{-}.3 * .2, 1\text{-}(.5 * .2 + .3 + .2\text{-}.3 * .2)] + [.8 * .1, .1 + .5\text{-}.1 * .5, 1\text{-}(.8 * .1 + .1 + .5\text{-}.1 * .5)]$$
$$+[.7 * .5, .2 + .3\text{-}.2 * .3, 1\text{-}(.7 * .5 + .2 + .3\text{-}.2 * .3)] + [.5 * .3, .3 + .4\text{-}.3 * .4, 1\text{-}(.5 * .3 + .3 + .4\text{-}.3 * .4)]$$
$$+[.8 * .2, .1 + .4\text{-}.1 * .4, 1\text{-}(.8 * .2 + .1 + .4\text{-}.1 * .4)]$$

$$V(HL_1) = [(.14, .46, .4) + (.1, .44, .46) + (.08, .55, .37) + (.35, .44, .21) + (.15, .58, .27) + (.16, .46, .38)]$$
$$V(HL_1) = [0.98, .013, .007]$$

Similarly, the intuitionistic fuzzy scores for other hierarchical levels are calculated as:

$$V(HL_2) = [0.99, .009, .001]$$
$$V(HL_3) = [0.82, .002, .178]$$
$$V(HL_4) = [0.6, .028, .372]$$

The score functions for each hierarchical level calculated using Eq. (2) stands as follows:

$$S(HL_1) = 0.98\text{-}.013 = 0.967$$
$$S(HL_2) = 0.99\text{-}.009 = 0.981$$
$$S(HL_3) = 0.82\text{-}0.002 = 0.818$$
$$S(HL_4) = 0.6\text{-}0.028 = 0.572$$

The hierarchical level with the largest score function value is HL_2 i.e. the middle management.

The ranking order is as below:

$$HL_2 > HL_1 > HL_3 > HL_4$$

The ranking order for the hierarchical levels is in agreement with Jun Ye [24] result on weighted correlation coefficient under intuitionistic fuzzy environment.

The TOPSIS methodology

	C1	C2	C3	C4	C5	C6
HL1	(0.7,0.1,0.2)	(0.5,0.3,0.2)	(0.8,0.1,0.1)	(0.7,0.2,0.1)	(0.5,0.3,0.2)	(0.8,0.1,0.1)
HL2	(0.7,0.1,0.2)	(0.8,0.1,0.1)	(0.6,0.3,0.1)	(0.8,0.1,0.1)	(0.3,0.3,0.4)	(0.7,0.2,0.1)
HL3	(0.5,0.1,0.4)	(0.7,0.1,0.2)	(0.3,0.5,0.2)	(0.5,0.3,0.2)	(0.8,0.1,0.1)	(0.3,0.3,0.4)
HL4	(0.5,0.4,0.1)	(0.6,0.3,0.1)	(0.3,0.3,0.4)	(0.2,0.3,0.4)	(0.7,0.2,0.1)	(0.2,0.3,0.5)

The weights for the criteria are as mentioned in **Table 3**.

The weighted IF decision matrix is obtained as:

	C1	C2	C3	C4	C5	C6
HL1	(0.14,0.04,0.08)	(0.10,0.06,0.12)	(0.08,0.05,0.04)	(0.35,0.06,0.02)	(0.15,0.12,0.06)	(0.16,0.04,0.04)
HL2	(0.14,0.04,0.08)	(0.16,0.02,0.06)	(0.06,0.15,0.04)	(0.45,0.03,0.02)	(0.09,0.12,0.12)	(0.14,0.2,0.16)
HL3	(0.10,0.04,0.16)	(0.14,0.02,0.12)	(0.03,0.25,0.08)	(0.25,0.09,0.04)	(0.24,0.04,0.03)	(0.06,0.12,0.16)
HL4	(0.10,0.16,0.04)	(0.12,0.06,0.06)	(0.03,0.15,0.16)	(0.10,0.09,0.10)	(0.21,0.08,0.03)	(0.04,0.12,0.20)

$$A^{+} = \{(0.35, 0.04), (0.45, 0.02), (0.25, 0.02), (0.21, 0.08)\}$$

$$A^{-} = \{(0.08, 0.12), (0.06, 0.80), (0.03, 0.25), (0.03, 0.16)\}$$

$$\begin{aligned}
D_1(1, A^{+}) &= \frac{1}{2}\begin{bmatrix} (0.14-0.35)^2+(0.04-0.04)^2+(0.08-0.61)^2+(0.14-0.45)^2+ \\ (0.04-0.45)^2+(0.04-0.02)^2+(0.08-0.53)^2+(0.10-0.25)^2+(0.04-0.02)^2 \\ +(0.16-0.73)^2+(0.10-0.21)^2+(0.16-0.08)^2+(0.04-0.71)^2 \end{bmatrix} \\
&= \frac{1}{2}\begin{bmatrix} 0.0441+0+0.2809+0.0961+0.0004+0.2025+0.0225+0.0004+0.3249+0.0121 \\ +0.0064+0.4489 \end{bmatrix}^{1/2} \\
&= \frac{1}{2}\sqrt{1.4392} \\
&= \frac{1}{2}\times 1.19966 \\
&= 0.59983
\end{aligned}$$

Similarly the other measures are calculated as follows:

$$D(2, A^{+}) = 0.6251$$

$$D(3, A^{+}) = 0.6462$$

$$D(4, A^{+}) = 0.5925$$

$$D(5, A^{+}) = 0.80475$$

$$D(6, A^{+}) = 0.67749$$

Also

$$D_1(1, A^-) = \frac{1}{2}\left[\begin{array}{l}(0.14-0.08)^2+(0.04-0.12)^2+(0.08-0.80)^2+(0.14-0.06)^2+\\(0.04-0.80)^2+(0.08-0.14)^2+(0.10-0.03)^2+(0.04-0.25)^2\\+(0.16-0.53)^2+(0.10-0.03)^2+(0.16-0.16)^2+(0.04-0.81)^2\end{array}\right]$$

$$= \frac{1}{2}\left[\begin{array}{l}0.0036+0+0.0064+0.5184+0.0064+0.5776+0.0036+0.0049+0+0.5929\\+0.0064+0.4489\end{array}\right]^{1/2}$$

$$= \frac{1}{2}\sqrt{1.7138}$$

$$= \frac{1}{2}\times 1.3091$$

$$= 0.6545$$

$$D(2, A^-) = 0.6900$$

$$D(3, A^-) = 0.64033$$

$$D(4, A^-) = 0.57621$$

$$D(5, A^-) = 0.619394$$

$$D(6, A^-) = 0.710$$

Now the relative closeness degree λ_j of the alternatives $A_{j\ (j\,=\,1,2\ldots,n)}$ to the intuitionistic fuzzy positive ideal solution are obtained from

$$\lambda_j = \frac{D(A_j, A^-)}{D(A_j, A^+) + D(A_j, A^-)}, j = 1, 2, \ldots, n$$

$$\lambda_1 = \frac{0.6545}{0.6545 + 0.59983} = 0.52179$$

$$\lambda_2 = \frac{0.6900}{0.6900 + 0.6251} = 0.5246$$

$$\lambda_3 = \frac{0.64033}{0.64033 + 0.6462} = 0.4977$$

$$\lambda_4 = \frac{0.57621}{0.57621 + 0.5925} = 0.4930$$

Lastly the ranking order of the alternatives A_j (j = 1,2 ... n) according to the non increasing order of the relative closeness degrees λ_j is as follows:

$$HL2 > HL1 > HL4 > HL3$$

To obtain an overall result of the two methods for finding the effectiveness of the employees the average of the two methods is sought. This is shown in the following table as below:

Hierarchical Levels	IFSAW Method	TOPSIS Method	Average	Rating
Senior Management (HL_1)	0.967(2)	0.52179(2)	0.74439	2
Middle Management (HL_2)	0.981(1)	0.5246(1)	0.7528	1
Junior Management (HL_3)	0.818(3)	0.4977(4)	0.65785	4
Staff (HL_4)	0.572(4)	0.4930(3)	0.5325	3

6. Conclusion

In this paper, the researcher worked on a first of its kind area which explored the effectiveness of the highest and the least contributions of the organizational hierarchical levels. The usage of intuitionistic fuzzy approach in the field of HR is a completely novel way of evaluating employees based on the four hierarchical levels. The approach is novel in the sense that such classification of employees using a mathematical model has hardly been used perhaps due to the fact that the parameters defining such categories can hardly be defined in concrete mathematical forms. The results indicate that the middle management is superior in terms of their performance when compared to their counterparts. The proposed method can effectively provide significant implications to policy makers, strategists and human resource professionals which help them to effectively conduct appraisals, take staffing decisions, and allocate work responsibilities and the like when the relevant information is not available or imprecise. It can also provide the decision maker the freedom to minimize the worse or maximize the better case. The method so discussed can be used for performance evaluation of individual employees as well when the attributes measuring their performance are loosely defined i.e. defined in ambiguous terms. Above all the use of intuitionistic fuzzy set in evaluating employees at various organisational levels involves computational complexity as two types of uncertainties are used. But computational complexity is no hindrance in the route to efficient results.

Author details

Mahuya Deb
Department of Commerce, Gauhati University, India

*Address all correspondence to: mahuya8@gmail.com

References

[1] Gupta AK, Govindarajan V. Resource sharing among SBUs: Strategic antecedents and administrative implications. Academy of Management journal. 1986;**29**(4):695-714

[2] Griffith, R., & Simpson, H. (2004). Characteristics of foreign-owned firms in British manufacturing. In Seeking a Premier Economy: The Economic Effects of British Economic Reforms, 1980–2000 (pp. 147–180). University of Chicago Press.

[3] Griliches Z, Regev H. Firm productivity in Israeli industry 1979–1988. Journal of econometrics. 1995; **65**(1):175-203

[4] Oulton N. Competition and the dispersion of labor productivity amongst UK companies. Oxford Economic Papers. 1998;**50**(1):23-38

[5] Brown S, Mc Hardy J, McNabb R, Taylor K. Workplace performance, worker commitment. and loyalty. Journal of Economics & Management Strategy. 2011;**20**(3):925-955

[6] Machin SJ, Stewart MB. Unions and the financial performance of British private sector establishments. Journal of Applied Econometrics. 1990;**5**(4):327-350

[7] McNabb R, Whitfield K. The impact of financial participation and employee involvement on financial performance. Scottish Journal of Political Economy. 1998;**45**(2):171-187

[8] Munday M, Peel MJ, Taylor K. The Performance of the Foreign-Owned Sector of UK Manufacturing: Some Evidence and Implications for UK Inward Investment Policy. Fiscal Studies. 2003;**24**(4):501-521

[9] Van der Klink, M., Boon, J. & Schlusmans, K. (2012). All by myself. Research into employees' informal learning experiences. International Journal of Human Resources Development and Management, 12 (1/2), 77–91.

[10] Vazirani N. Review Paper: Competencies and Competency Model–A Brief overview of its Development and Application. SIES Journal of Management. 2010;**7**(1):121-131

[11] Yaghoubi NM, Salehi M, Moloudi J. Improving Service Quality by Using Organizational Citizenship Behavior: Iranian Evidence. Iranian Journal of Management Studies. 2011;**4**(4):79-97

[12] Meybodi MZ. Alignment of Strategic Benchmarking Performance Measures: A Lean Manufacturing Perspective. Advances in Competitiveness Research. 2013;**21**(1–2):14

[13] Bartlett CA, Ghoshal S. The myth of the generic manager: new personal competencies for new management roles. California management review. 1997;**40**(1):92-116

[14] Zadeh LA. Outline of a new approach to the analysis of complex system and decision processes ,IEEE, Trans.Syst.. Lan and Cyber. C-3. 1973:28

[15] Zadeh LA. Fuzzy sets. Information and control. 1965;**8**(3):338-353

[16] Dubois D, Prade H. *Operation on fuzzy numbers*. International Journal of Systems Science. 1978;**9**(6):613-626

[17] Mikhailovich,D.M. The Fuzzy Logic Methodology for Evaluating the Causality of Factors in Organization Managemen. In: Fuzzy Logic ,DOI: 10.5772 /IntechOpen,77460,ISBN: 978–1–78984-232-6. 2019

[18] Dintsis, Daniel. Implementing Complex Fuzzy Analysis for Business

Planning Systems, Modern Fuzzy Control Systems and Its Applications. In: doi: 10.5772/67974. 2017

[19] Atanassov KT. Intuitionistic fuzzy sets. Fuzzy sets and Systems. 1986; **20**(1):87-96

[20] Volosencu C, editor. Fuzzy Logic. London, UK: Intech Open Ltd.; 2020 978-1-78984-231-9

[21] Voloşencu, C., Properties of Fuzzy Systems, WSEAS Transactions on Systems, Issue 2, Volume 8, Feruary, 2009, ISSN: 1109-2777, pg. 210–228

[22] Mahapatra GS, Roy TK. *Reliability evaluation using triangular Intuitionistic fuzzy numbers arithmetic operations*, International Journal of Computer and Information. Engineering. 2009;**3**(2): 350-357

[23] Hwang CL, Yoon K. Multiple Attribute Decision Making: Methods and Applications, A State of the Art Survey. New York, NY: Springer-Verlag; 1981

[24] Ye J. Fuzzy decision-making method based on the weighted correlation coefficient under intuitionistic fuzzy environment. European Journal of Operational Research. 2010;**205**(1):202-204

[25] Li F.D., Decision and Game Theory in Management with Intuitionistic Fuzzy Sets, Studies in Fuzziness and Soft Computing , Vol. 308, Springer Heildelberg New York Dordrecht London