Chapter

SIR Model with Homotopy to Predict Corona Cases

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Abstract

In this chapter, we will discuss SIR model to study the spread of COVID-2019 pandemic of India. We will give the prediction of corona cases using homotopy method. The HM is a method for solving the ordinary differential equations. The SIR model consists of three ordinary differential equations. In this study, we have used the data of COVID-2019 Outbreak of India on 20 Jan 2021. In this data, Recovered is 102656163, Active cases are 189245 Susceptible persons are 189347782 for the experimental purpose. Data about a wide variety of infectious diseases has been analyzed with the help of SIR model. Therefore, this model has been already well tested for infectious diseases by various scientists and researchers.

Keywords: SIR model, homotopy, differential equation, corona, graph, table

1. Introduction

Novel Coronavirus, assigned as 2019-nCoV, emerged in Wuhan, China, toward the end of 2019. As of January 24, 2020, as many as 830 cases had been analyzed in nine nations: China, Thailand, Japan, South Korea, Singapore, Vietnam, Taiwan, Nepal, and the United States [1–3]. Twenty-six fatalities happened, chiefly in patients who had genuine basic sickness. Albeit numerous subtleties of the rise of this infection.

In 2019, the Centers for Disease Control and Prevention (CDC) started monitoring the outbreak of a new coronavirus, SARS-CoV-2, which causes the respiratory illness now known as COVID-19. Authorities first identified the virus in Wuhan, China. More than 74,000 people have contracted the virus in China. Health authorities have identified many other people with COVID-19 around the world, including many in the United States. On January 31, 2020, the virus passed from one person to another in the U.S. The World Health Organization (WHO) have declared a public health emergency relating to COVID-19. Since then, this strain has been diagnosed in several U.S. residents. The CDC have advised that it is likely to spread to more people. COVID-19 has started causing disruption in at least 25 other countries.

All the adjoining nations of India have revealed positive COVID-19 cases. To secure against the lethal infection, the Indian government have taken fundamental and severe measures, including setting up wellbeing check posts between the public lines to test whether individuals entering the nation have the infection. Various nations have presented salvage endeavors and reconnaissance measures for residents wishing to get back from China. The exercise gained from the SARS episode was first that the absence of lucidity and data about SARS debilitated China's worldwide standing and hampered its financial development. The episode of SARS in China was disastrous and has prompted changes in medical care and clinical frameworks. Contrasted and China, the capacity of India to counter a pandemic is by all accounts a lot of lower. A new report announced that influenced relatives had not visit the Wuhan market in China, proposing that SARS-CoV-2 may spread without showing side effects. Analysts accept that this wonder is typical for some infections. India, with a populace of more than 1.34 billion—the second biggest populace on the planet—will experience issues treating serious COVID-19 cases on the grounds that the nation has just 49,000 ventilators, which is a negligible sum. On the off chance that the quantity of COVID-19 cases increments in the country, it would be a fiasco for India.

As the characteristics of a potential vaccine become better known, mathematical models can be used to explore alternative scenarios about effectively distributing a vaccine in order to limit transmission and protect the most vulnerable population groups.

Coronaviruses can spread in the following ways:

Coughing and sneezing without covering the mouth can disperse droplets into the air. Touching or shaking hands with a person who has the virus can pass the virus between individuals. Making contact with a surface or object that has the virus and then touching the nose, eyes, or mouth.

The National Institutes of Health (NIH) suggest that several groups of people have the highest risk of developing complications due to COVID-19. These groups include:

1. Young children
2. People aged 65 years or older
3. Pregnant women.

Coronaviruses will contaminate most individuals at a few time amid their lifetime. Coronaviruses can change viably, which makes them so infectious. To anticipate transmission, individuals ought to remain at domestic and rest whereas side effects are dynamic. They ought to moreover maintain a strategic distance from near contact with other individuals. They should also avoid close contact with other people. Covering the mouth and nose with a tissue or handkerchief while coughing or sneezing.

2. Analysis of SIR model

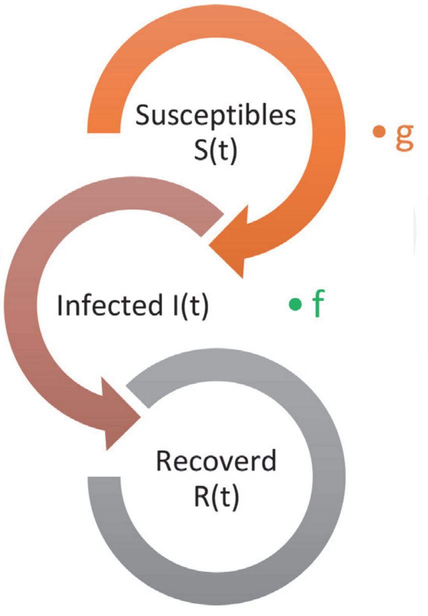
SIR model is first introduced by W.O. Kermach and A.G Mckendrick in 1927. SIR model is a best model of an infectious disease. This model divided the population into the three groups. The groups name is.

S (t) is the Susceptible people at the time.

I (t) is the infected people at the time.

R(t) is the recovered people at the time.

This model is constructing the ordinary differential equations in this model time t is the independent variable and S, I, R is the dependent variables. These groups have taken the number of people on every day. Yet, the data is transitions with time, as human being act from one group to another group. Illustration, human being in group **S** will act to the group **I**, that is the infected. Furthermore, infected person, **I** will act to the recovered **R** group that is they are recover or die from the disease. This method has been used successfully many times before in spreading disease like yellow fever, plague, fever, influenza, avian influenza, etc. Therefore, we have made the differential equations of COVID 19 using this method. This method is very helpful in giving a mathematical model to COVID.



$$\frac{ds}{dt} = -gsi. \tag{1}$$

$$\frac{di}{dt} = gsi - fi \tag{2}$$

$$\frac{dr}{dt} = fi \tag{3}$$

where t is the independent variable s, i, r is dependent variables i.e.
s is denote the susceptible person at the time t
i is denote the infected person at the time t
r is denote the recovered person at the time t
g is transmission coefficient
f is recovery
If $s > 0$, $i > 0$ then

$$\frac{ds}{dt} = -gsi < 0, \forall s > 0, i > 0 \tag{4}$$

$$\frac{\mathrm{di}}{\mathrm{dt}} = \mathrm{i}(\mathrm{gs} - \mathrm{f}) \tag{5}$$

$$gs - f = 0$$

$$s = \frac{f}{g}$$

$$s - \frac{f}{g} = 0 \tag{6}$$

$$\text{So } \frac{di}{dt} < 0 \text{ if } s < \frac{f}{g} \tag{7}$$

$\frac{di}{dt} > 0$ if $s > \frac{f}{g}$ is defined the direction diagram of trajectories
We find the trajectories

$$\frac{di}{ds} = \frac{\frac{di}{dt}}{\frac{ds}{dt}} = \frac{gsi - fi}{-gsi}$$

$$\frac{di}{ds} = -1 + \frac{f}{gs}$$

$$di = -1ds + \frac{f}{gs}\, ds$$

$$i = -s + \frac{f}{g} \log s + c$$

Initial conditions

$$s(0) = s_0$$

$$i(0) = i_0$$

(If)

$$s \to 0, i \to -\infty$$

$$s \to \infty, i \to -\infty$$

1. It is impossible for the disease to infect all the susceptible person.

2. $s_0 > \frac{f}{g}$ for an epidemic to occur.

3. $s_0 < \frac{f}{g}$ disease dig out.

4. $\frac{gs_0}{f} > 1$ then number of infected is increase.

5. $s + i + r$ is the total population.

3. Mathematical modeling of COVID 19

3.1 Case study of India

In this research work we have discussed about the COVID 19 disease. also know about how many people got sick in India due to pandemic disease COVID19. In this article, we have given a mathematical model to COVID 19 with the help of SIR model [4–6]. We have taken data of how many people had become ill in India by COVID 19 on 20 January 2021 and using this data , we have created a mathematical model of COVID 19 with the help of SIR model. We have created three differential equations by taking the original data and solving those equations by the Homotopy Perturbation method (HPM).We got out the numerical solutions and made a table, and with the help of that table, we tried to tell what is the position of COVID 19 in India by making the graphs. There are a lot of methods which solves the differential equations [7–9], but we have used the HPM method. This method solves the biggest and most difficult equations very easily and with less of calculations.We will solve the Differential Equations of COVID 19, which is made with the help of SIR model. In these equations, we took the data of 20 January 2021 COVID 19 people of India who were caught by COVID 19 epidemics.

Total confirmed cases on 20 January 2021 in India is 10611728.

Death 152907

Recovered is 102656163

Active cases are 189245

Susceptible persons are 189347782

So, we take the

$$s(0) = 18.9347782$$

$$i(0) = 1.0611728$$

$$r(0) = 10265163 + 152907 = 1.0418070$$

$$g = \frac{active\ \ cases\ of\ \ india\ on\ 20\ january\ 2021\ for\ \ COVID\ 19}{susceptible\ people\ of\ \ india\ on\ 20\ january\ 2021\ for\ \ COVID\ 19}$$

$$g = \frac{189245}{18.9347782} = 0.00999457$$

$$f = \frac{1}{14} = 0.0714$$

$$\frac{ds}{dt} = -gsi \tag{8}$$

$$\frac{di}{dt} = gsi \quad -fi \tag{9}$$

$$\frac{dr}{dt} = fi \tag{10}$$

where t is the independent variable s, i, r is dependent variables i.e.

s is denote the susceptible person at the time t

i is denote the infected person at the time t

r is denote the recovered person at the time t

g is transmission coefficient

f is recovery

Now we will solve these equations with the help of HPM method.

By the homotopy method we get,

$(1-p)\frac{dS}{dt} + p(\frac{dS}{dt} + si)$

$$\frac{dS}{dt} = p(-0.00999457si)$$

$$(1-p)\frac{di}{dt} + p\left(\frac{di}{dt} - 0.00999457si + 0.0714i\right) \tag{11}$$

$$\frac{di}{dt} = p(\ 0.00999457si - 0.0714i) \tag{12}$$

$$\frac{dr}{dt} = p0.0714i$$

$$s = s_0 + p^1 s_1 + p^2 s_2 + \ \dots \tag{13}$$

$$i = i_0 + p^1 i_1 + p^2 i_2 + \ \dots \tag{14}$$

$$r = r_0 + p^1 r_1 + p^2 r_2 + \ \dots \tag{15}$$

Putting value s, i, r we get,

$$\frac{ds}{dt} = p(-0.00999457\{\ s_0 + p^1 s_1 + p^2 s_2 + \ \dots\}\{i_0 + p^1 i_1 + p^2 i_2 + \ \dots\}) \tag{16}$$

$$\frac{di}{dt} = p(\ 0.00999457[\{\ s_0 + p^1 s_1 + p^2 s_2 + \ \dots\}\{\ i_0 + p^1 i_1 + p^2 i_2 + \ \dots\}] - 0.0714\{i_0 + \ \ p^1 i_1 + p^2 i_2 + \ \dots\}) \tag{17}$$

$$\frac{dr}{dt} = p\ 0.0714\{i_0 + p^1 i_1 + p^2 i_2 \tag{18}$$

Both side comparing the coefficient of p we get

$$s_0 = 18.9347782$$

$$i_0 = 1.0611728$$

$$r_0 = 1.0418070$$

$$\frac{ds_1}{dt} = -0.00999457(18.9347782)(1.0611728)$$

$$s_1 = -0.2008216t$$

$$\frac{di_1}{dt} = 0.00999457(18.9347782)(1.0611728) - 0.0714(1.0611728)$$

$$i_1 = 0.1250538t$$

$$\frac{dr_1}{dt} = 0.0714\{1.0611728\}$$

$$r_1 = 0.0757677t$$

So, by the HPM we get the solution:

$$s(t) = 18.9347782 - 0.2008216t + \ \dots$$

$$i(t) = 1.0611728 + 0.1250538t + \ \dots$$

$$r(t) = \ \ 1.0418070 + 0.0757677t + \ \dots$$

We have the table

S.NO.	DATE	S	I	R
1	20/01/2021	18.9347782	1.0611728	1.0418070
2	21 /01/2021	18.7339566	1.0662266	1.1175747
3	22/01/2021	18.533135	1.06128040	1.1933424
4	23/01/2021	18.3323134	1.06633424	1.2691101
5	24/01/2021	18.1314918	1.0613880	1.3448778

S.NO.	DATE	S	I	R
6	25/01/2021	17.9306702	1.0664418	1.4206455
7	26/01/2021	17.7298486	1.0684956	1.4964132
8	27/01/2021	17.529027	1.0705494	1.5721809
9	28/01/2021	17.3282054	1.0716032	1.6479486
10	29/01/2021	17.1273838	1.0726657	1.7237163
11	30/01/2021	16.9265622	1.0747108	1.799484
12	31/01/2021	16.7257406	1.0767646	1.8752517

From the above table we can predict that infected cases of corona on 31 January which is almost same as actual cases on 31 January. The current COVID-19 pandemic is unprecedented, but the global response draws on the lessons learned from other disease outbreaks over the past several decades.

World scientists on COVID-19 then met at the World Health Organization's Geneva headquarters on 11–12 February 2020 to assess the current level of

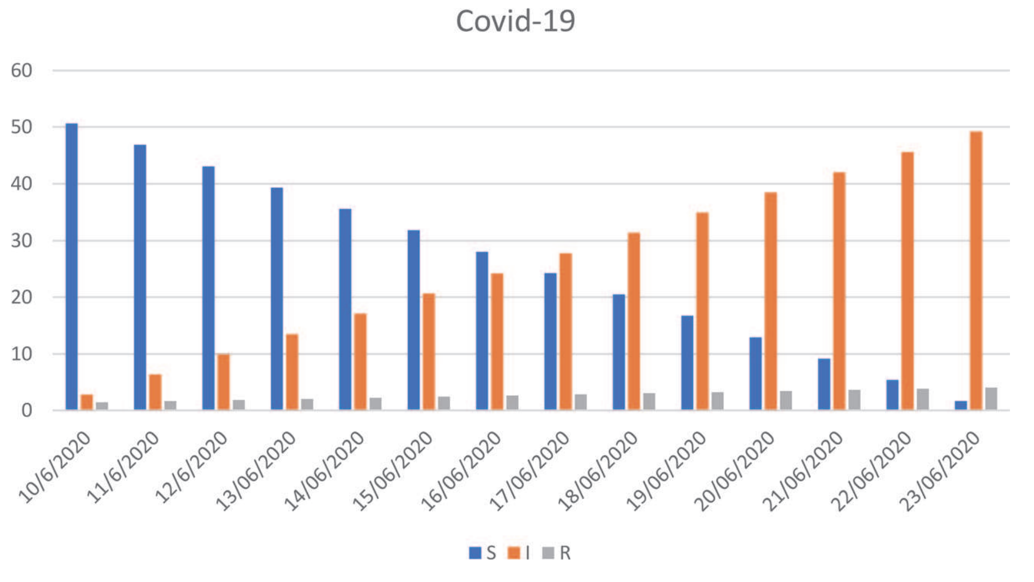


Figure 1.
SIR chart depicting no of people susceptible, infected and recovered.

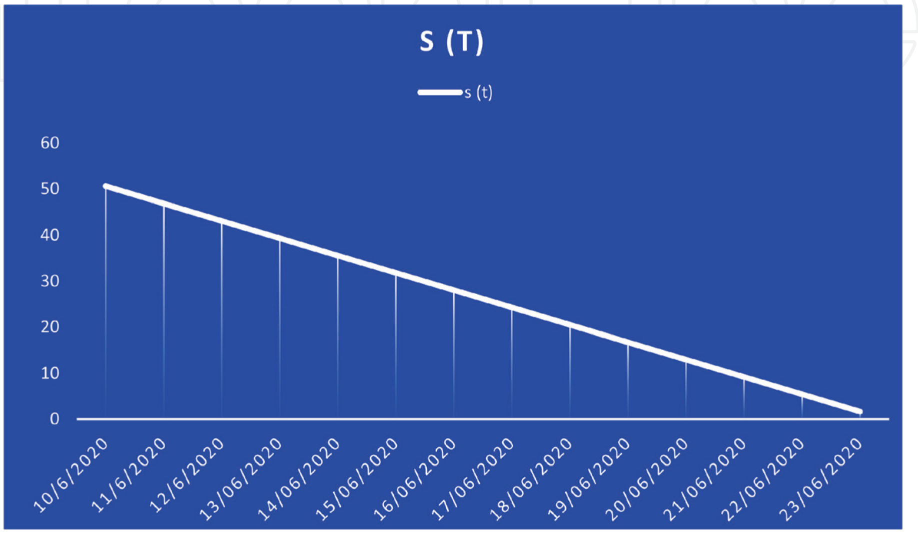


Figure 2.
SIR chart depicting no of people susceptible.

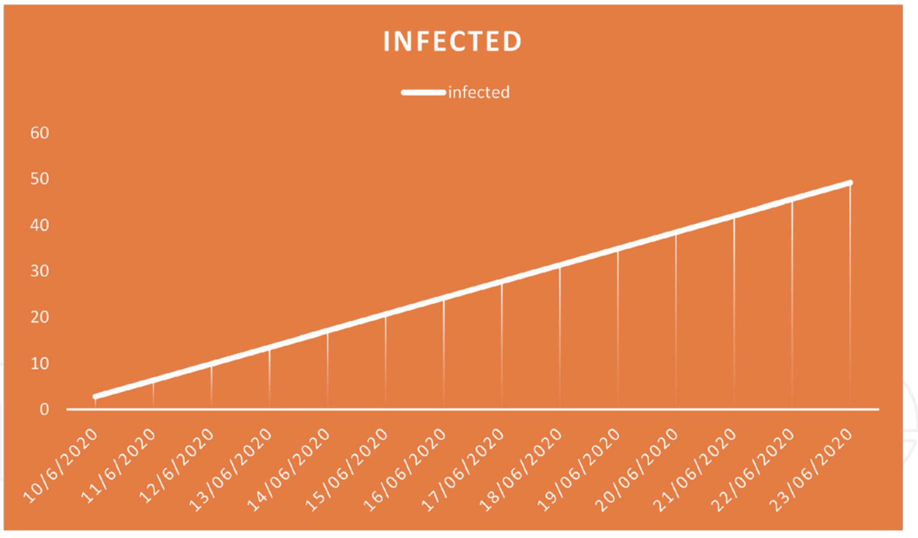


Figure 3.
SIR chart depicting no of people infected.

knowledge about the new virus, agree on critical research questions that need to be answered urgently, and to find ways to work together to accelerate and fund priority research to curtail this outbreak and prepare for those in the future see **Figures 1–3** for reference.

4. Conclusion

In this chapter, we have taken data of people affected by coronavirus in India till 20 January (1). Then we converted this data into three differential equations with the help of SIR model. We solved the equation made from SIR model with HPM. From the result of solving, we estimated the people who got infected with corona virus in the coming 5 days. We converted the result from HPM into a table and graph and from the result we saw that in the coming days, corona cases are increasing and recovering but the corona positive rate is very high, and the rate of recovery is very short. We saw that the information about Corona-positive cases being given by the Government of India was also that the rate of positive is increasing very fast, but the rate of recovery is very low. From all these, we can now say that by solving with HPM we get the result very close to the actual result. We have predicted cases of corona till Jan 31 using SIR model, risk factors for the coronavirus disease. The risk is especially high if two or three of the Cs come together.

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