Chapter

Advanced Modeling of Single Degree of Freedom System for Earthquake Ground Motion Using LabVIEW Software

R.B. Malathy, Govardhan Bhat and U.K. Dewangan

Abstract

In this paper, the structural responses at discrete time steps are evaluated to understand the linear dynamics characteristics of a structural system using LabVIEW (Laboratory Virtual Instrument Engineering Workbench) tool. Time History Analysis (THA) which is an essential procedure to design a reliable structure when the structure is subjected to dynamic loading is taken into consideration for the study. Direct integration method was used to find out the dynamic response of the structure as it is applicable for both linear as well as nonlinear range. Block diagram that perform step-by-step integration to analyze the linear single degree of freedom (SDOF) system has been prepared in LabVIEW. The processing of data is carried out till the equilibrium is satisfied at all discrete time points within the interval of solution instead of any time t. Different ground motion time histories were considered for THA and responses of the SDOF system are evaluated. The results from LabVIEW were validated and the accuracy of the algorithms generated are discussed. It is observed that the accuracy and stability of the final solution depends on the variation of displacement, velocity and acceleration that is assumed in each step. Thus, LabVIEW workbench can therefore be recognized as an effective instrument in structural engineering owing to its fast sampling features.

Keywords: LabVIEW, central difference method, wilson-θ method, SDOF system, time history analysis

1. Introduction

With the rapid development of hardware and software technology for personal computers (PCs), it is simple to effectively incorporate PCs in various precise measurement and complex control applications.VI (Virtual Instrumentation) has evolved into a thorough quest that encompasses the whole field of computer-based instrumentation leading to the large reduction of hardware. The LabVIEW (Laboratory Virtual Instrumentation Engineering Workbench) can be interfaced with several hardware, such as data acquisition cards, instrument control, and industrial automation [1]. LabVIEW is a platform and development environment for system design that focused on the framework of data flow programming. It enables the user to build programs with graphics rather than text code. It performs many applications, such as data acquisition, data interpretation, signal detection, signal processing, control and monitoring. It also simulates the vibration testing and vibration signal processing. It is an important technique that makes it easy to detect internal damage to the structure. Therefore, it is shown to be the prevailing instrument in the study of the dynamic behavior of structures which had become a major concern of mechanical, civil and aerospace engineers. To better understand the dynamic behavior, it is essential to know the modal parameters of the structure, i.e. its natural frequencies, mode shapes and damping ratios. The precise identification of these parameters can be made through the use of robust and reliable methods that belong to the field of research known as modal analysis [2].

There are different causes of vibration, such as continuous force, degradation, resonance, etc. The response of it can be understood through various control actions such as manual, automatic, sine wave generation and square wave generation on the structure. The preventive measures on the structure may be taken through analysis and monitoring of vibration signal by two processes. When the variation of force with time is known, the variation of response is formulated in time domain. This is referred to as time-domain analysis and this former signal analysis can be used to evaluate the response of any linear SDOF system to any arbitrary input. Sometimes, the force function is random and it is not possible to determine its frequency. Moreover, it may have a variable frequency over its duration and hence it is then convenient to perform the analysis in frequency domain. The frequency domain approach is also conceptually similar to the Fourier analysis procedure. However, to apply the periodic load technique to arbitrary loading, it is necessary to extend the Fourier series concept to the representation of non-periodic functions. Various researches are made in recent years to apprehend the dynamic behavior of the structure using virtual instrument engineering workbench. Sura et al., [3] analyzed the cantilever beam using the virtual instrument in which free vibrations were induced and measured in the beam. The results in the form of modal frequency were obtained for the cantilever beam which was properly fixed and he concluded that the theoretically calculated natural frequency and the experimentally calculated natural frequency are almost the same. Yao et al. [4] built a virtual earthquake simulation system instrumentation and stated that the design concept of LabVIEW is more user-friendly and efficient than others. Hu [5], describes the development of modal recognition computing tools and long-term dynamic monitoring in the LabVIEW framework. These consist mainly of two independent functional toolkits known as Structural Modal Identification (SMI) and Continuous Monitoring (CSMI) respectively. It involves checking the latest output measurements, identifying the maximum vibration amplitudes and performing statistical time series on acceleration. It generates waveform plots to represent the distribution of the frequency component and modal parameter based on automated Enhanced Frequency Domain Decomposition (EFDD) technique. An attempt is made to expand hands-on activity-based educational module through the integration of PASCO models, LabVIEW, NI hardware, sensors, and MATLAB software. Despite some existing limitations, the results successfully showed that this structure worked precisely and stably, producing good output data. It was proved as a potential tool for structural dynamics as well as Structural Health Monitoring (SHM) education and also study in which, each case of damaged structure had a distinctive property [6]. Ugo Andreaus [7] studied the experimental dynamic response of a base-isolated SDOF oscillator and formulated numerical model excited by a harmonic base acceleration using LabVIEW. The behavior of the system was well understood as the numerical simulation in LabVIEW platform efficiently agreed with the experimental investigation.

In this context of the study, an attempt is being made to propose program for time integration method in LabVIEW to predict the changes in displacement, velocity and acceleration for SDOF model for earthquake excitations. The versions of these expressions can be used for damaged structures, if the damage parameters are known. The expressions are integrated into an algorithm [8]; priory developed for Time History Analysis (THA) of structures and are analyzed here in case of central difference method and Wilson-θ method.

2. Description of the THA method

There are many numerical integration methods available to evaluate the approximate solution of equation of motions. There are two basic characteristics of these methods firstly, the differential equations of these methods are satisfied only at discrete time intervals Δt and secondly, a variation in displacement, velocity and acceleration is assumed within each time interval Δt [9]. Causevic et al., [10] discussed about non-linear dynamic time-history analysis; non-linear static method (Euro code 8); non-linear static procedure NSP (FEMA 356) and improved capacity spectrum method CSM (FEMA 440). An eight-storey reinforced concrete frame building is analyzed as the research subject. It is evident that neither of static procedures takes into consideration the damage which can be significant for long duration earthquakes. The author thus concluded that the non-linear THA was the most accurate method. Lestuzzi et al., [11] discussed about the selection of real ground motion records by considering the response of single-degree-of-freedom (SDOF) system with bilinear hysteretic model. The findings from this study are very limited, i.e., they are applicable only for building structures that can be modeled as a SDOF system. The response parameters considered are maximum displacement and ductility of the SDOF system. The study csoncludes the following points: 1.While selecting the real records of THA, the spectral acceleration records that matches with the design spectrum has to be chosen. 2. The period has to be kept as T_0 or in a range between T_0 and the period corresponding to the secant stiffness. It is observed that the mathematical computation of these methods is difficult and is time consuming and hence a requirement for alternate and efficient platform is needed.

Thus, the concept of nonlinear behavior of structures and the importance of Time history analysis (THA) is more important even though it's a century old concept. Although the linear elastic analysis and the design methods are well established, nonlinear inelastic analysis and their application to design are still evolving. The answer for the question, "Why do we need a nonlinear analysis?" lies in the fact that under extreme probable loading like earthquake; it is no longer advisable to keep the structure elastic due to the reason of yielding in structural components. Thus, a nonlinear analysis requires a clear understanding of the stress–strain curves of all the materials used in the structure, its inelastic behavior, failure criteria of the components, the capacity of its in failure modes and also the nonlinear analysis techniques. In case a single degree of freedom (SDOF) system or a multi degree of freedom (MDOF) system is subjected to a random acceleration time history, it is very difficult to solve the differential equation using the basic principle of calculus. The direct integration methods or step-by-step integration methods are used for the solutions of such problems. A very small time step Δt, is chosen and the solution is obtained from one step to the next step leading to the linear interpolation of the forces. The expression at time step (t + h) may be entirely in term of quantities at time step t or both at time step t and (t + h) which gives rise to two types of algorithm: explicit algorithm and implicit algorithm. In the former, the expressions at time step (t + h) are in terms of time step t only, whereas, in the latter, the expressions at time step (t + h) are in terms of t and (t + h). The solutions using the explicit algorithm are as easy as compared to those using the implicit algorithm.

Hence an attempt is being made to make LabVIEW programs for the widely used explicit and implicit algorithm. A brief overview of these approaches is given, followed by programming in LabVIEW platform and their validation through examples. Li [12] stated that Finite difference method optimizes the approximation for the differential operator in the central node of the considered space and provides numerical solutions to differential equations. It is noticed that the results of the central difference method approximation show a significant improvement in the accuracy along the smooth region. He also concluded that it is possible to test the function f (x) at values on the left and right of x, to obtain an optimal two-point approximation which includes abscissas that are symmetrically chosen on both sides of x. The advantage of this approach is that, its convergence speed is higher than some other finite differentiating methods, such as forward and backward differentiation. Similarly another method developed by E L Wilson for unconditionally stable linear acceleration method is Wilson θ method. This method is based on the assumption that acceleration varies linearly over an extended time step $\delta t = \theta \delta t$ [13]. Wilson-θ method is highly stable numerically as it converges rapidly to a meaningful solution. In our study, earthquake-induced ground motions of El Centro (1940) and Loma Prieta (1989) earthquake data are fed as input to the SDOF system. Seismic responses considered were in the form of acceleration, velocity, displacement and force and the application example considered was SDOF system. The accuracy which means the chosen numerical methods should converges the exact solution in terms of amplitude accuracy or amplitude decay or period accuracy or period decay was carried out in the workbench. The ground motions records were obtained from the PEER Strong Motion Database (http://peer.berke ley.edu/smcat /) [14].

3. Methodology

In the explicit method the response at time t_{n+1} is known in terms of known variables at time t_n. Thus the response values displacement, velocity and acceleration can be determined directly. Whereas, in implicit method, the response at time t_{n+1} is known in terms of the known variables at time t_n and unknown variables at time t_{n+1}. These implicit algorithms involve either an iterative scheme or solution of linear simultaneous equations because the unknown quantities appear on both sides of the equations.

3.1 Central difference method

This method is based on the finite difference approximation of the time derivative of displacement, that is, velocity and acceleration [9].

An equation of motion for an SDOF system is given as:

$$m.\ddot{u} + c.\dot{u} + k.u = F_t \tag{1}$$

m = Mass, c = Damping, k = Stiffness, $\ddot{u}$ = Acceleration, $\dot{u}$ = Velocity, u = Displacement, F_t = Force.

Initial acceleration is given as,

$$\ddot{u}_o = \frac{-m.\Delta\ddot{u}_g - c.\dot{u}_o - k.u_o}{m} \tag{2}$$

Initial displacement at i-1th time step

$$u_{i-1} = u_0 + \Delta t(\dot{u}_o) + \frac{(\Delta t)^2}{2}\ddot{u}_o \tag{3}$$

Incremental Stiffness for the ith time step,

$$k_i = \frac{m}{(\Delta t)^2} + \frac{c}{2.\Delta t} \tag{4}$$

Incremental force for ith time step,

$$F_i = -m.\ddot{u}_g - a.u_{i-1} - b.u_i \tag{5}$$

Where, a and b are constants and given as

$$a = \frac{m}{2.\beta} + \frac{\gamma.c}{\beta}, b = \frac{m}{2.\beta} + \Delta t.\left(\frac{\gamma}{2.\beta} - 1\right)c \tag{6}$$

Displacement for i + 1th time step,

$$u_{i+1} = \frac{F_i}{k_i} \tag{7}$$

Velocity for ith time step,

$$\dot{u}_i = \frac{u_{i+1} - u_{i-1}}{2\Delta t} \tag{8}$$

Acceleration for ith time step,

$$\ddot{u} = \frac{u_{i+1} - 2.u_i - u_{i-1}}{(\Delta t)^2} \tag{9}$$

3.2 Algorithm

Step 1: Initial displacement and velocity are known as initial conditions of the problem at time t = 0.
Step 2: Damping c and stiffness k are computed from the system properties.
Step 3: Acceleration at time t = 0 is computed from Eq. (2).
Step 4: Compute equivalent stiffness $k_i$from Eq. (4).
Step 5: For time step i, compute equivalent force F_i from Eq. (5).
Step 6: Compute constants a and b from Eq. (6).
Step 7: Solve for new displacement u_{i+1} from Eq. (7).
Step 8: Compute velocity and acceleration at time step i from Eq. (2) and (3).
Step 9: Repeat Steps 6 to 8 for the next time step.

3.3 Programs developed in LabVIEW for central difference method

A visual block diagram which describes the data flow within the VI is presented in the form algorithm in LabVIEW. LabVIEW accepted the input, and the algorithm was sampled and programmed through appropriate interfaces in accordance with the specification of VI, and the output data was collected. In our software, data such as damping, mass and time period were provided as an input and displacement, velocity and acceleration plot was obtained for the time history data (i.e. it can function as an analog to digital converter). Owing to the sheer quantity and simplicity of the different built-in functions, the data was thus manipulated in a wide range of forms as shown in **Figure 1**.

3.4 Wilson-θ method, linear SDOF system

The incremental equation of equilibrium known as Wilson-θ method is developed by Prof.E.L.Wilson, University of California, and Berkeley [9]. The calculations are carried out over an extended time step θΔt, where θ is an amplifier for the time step. It assumes that the variation of acceleration over the extended time step remains unchanged, that is, it is still the same as that of the original time step Δt, a linear variation.

An equation of motion for an SDOF system is given as:

$$m.\ddot{u} + c.\dot{u} + k.u = F_t \tag{10}$$

m = Mass, c = Damping, k = Stiffness, $\ddot{u}$ = Acceleration, $\dot{u}$ = Velocity, u = Displacement, F_t = Force.

Initial acceleration is given as,

$$\ddot{u}_o = \frac{-m.\Delta\ddot{u}_g - c.\dot{u}_o - k.u_o}{m} \tag{11}$$

Incremental force for i$^{\text{th}}$ time step,

$$\delta F_i = \theta\left(-m.\Delta\ddot{u}_g\right) + a.\dot{u}_i + b.\ddot{u}_i \tag{12}$$

Where, a and b are constants and given as

$$a = \frac{6.m}{q.\Delta t} + 3c, b = \frac{q.\Delta t.c}{2} + 3.m$$

Tangent Stiffness for i$^{\text{th}}$ time step,

$$k_t = k_i + \frac{3.c}{\theta.\Delta t} + \frac{6.m}{\theta(\Delta t)^2} \tag{13}$$

Solve for δu for i$^{\text{th}}$ time step,

$$\delta u_i = \frac{\delta F_i}{k_t} \tag{14}$$

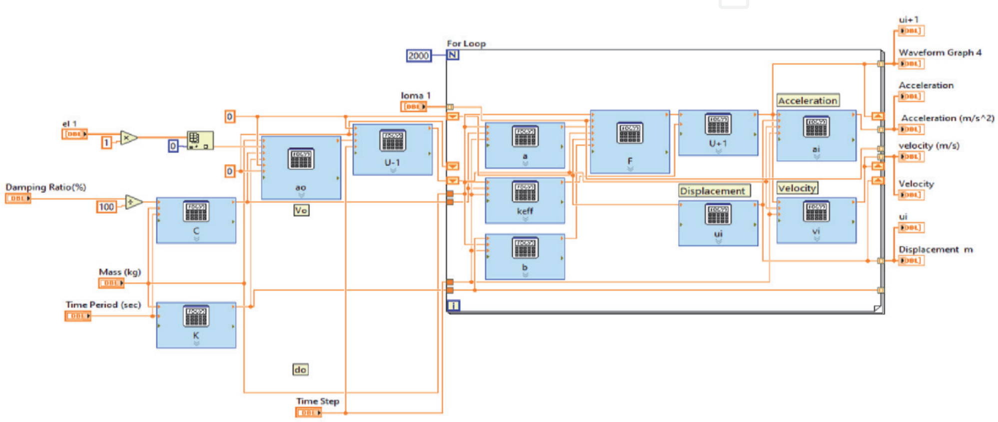

Figure 1.
Block diagram of central difference method in LabVIEW.

Solve for $\delta\ddot{u}$ for ith time step,

$$\delta\ddot{u}_i = \frac{6.\delta u_i}{(\theta.\Delta t)^2} - \frac{6.\dot{u}}{\theta.\Delta t} - 3\ddot{u}_i \tag{15}$$

Incremental Acceleration is given as

$$\ddot{u}_i = \frac{\delta\ddot{u}_i}{k_t} \tag{16}$$

Knowing incremental acceleration, incremental velocity and displacement can be calculated,

Incremental velocity,

$$\Delta\dot{u}_i = \Delta t.\ddot{u}_i + \frac{\Delta t.\Delta\ddot{u}_i}{2} \tag{17}$$

Incremental displacement,

$$\Delta u_i = \Delta t.\ddot{u}_i + \frac{(\Delta t)^2\dot{u}_i}{2} + \frac{(\Delta t)^2\Delta\ddot{u}_i}{6} \tag{18}$$

At time t_{i+1} displacement, velocity and acceleration can be calculated as

$$\begin{aligned} u_{i+1} &= u_i + \Delta u_i \\ \dot{u}_{i+1} &= \dot{u}_i + \Delta\dot{u}_i \\ \ddot{u}_{i+1} &= \ddot{u}_i + \Delta\ddot{u}_i \end{aligned} \tag{19}$$

3.5 Algorithm

Step 1: Compute $k_{t.}$

Step 2: Calculate u_{i+1},$\dot{u}_{i+1}$, $\ddot{u}_{i+1}$ using the Eqs. (15), (17) and (18).

Step3: Update c and k.

Step 4: Repeat steps 1 to 3.

It should be noted that in this method, k and c are assumed to remain constant during the extended time step and are updated at the end of the real-time increment $\mathbf{\Delta t.\theta} = \mathbf{1}$ leads to the linear acceleration method. It is recommended that $\boldsymbol{\theta}$ is taken >1.37 [15].

3.6 Programs developed in LabVIEW

The block diagram given below shows all the features that are expressed in VIs. The input signal was simulated at the first step. This was accomplished by

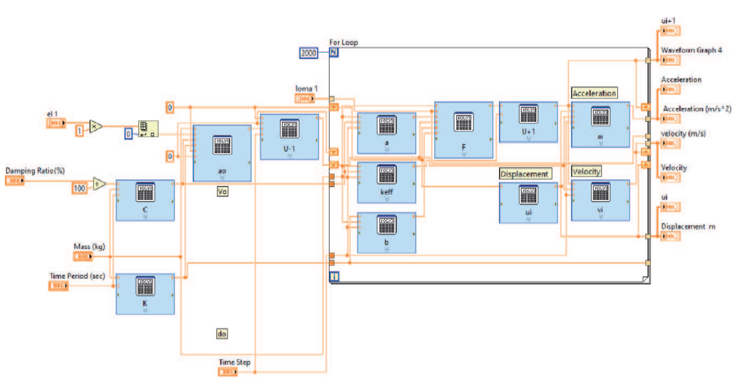

Figure 2.
Block diagram of wilson-θ method in LabVIEW.

incorporating block diagram from simulate signal express VI, which is located under the signal analysis palette. The functions, such as mathematical operations, express VIs, built analysis tools and signal simulation, were assessed from the palettes by right clicking in the block diagram or front panel, which brought up the palette menu. The frame work was thus created using the algorithm and the waveform was generated for displacement, acceleration and velocity as shown in the **Figure 2**.

4. Analytical validations

The peak ground motion recorded, magnitude and it's predominated period at real-time data storage station during the 1940 El Centro earthquake or 1940 Imperial Valley earthquake (Mw = 6.9) was considered for THA as the first analytical case study. The SDOF system that was considered has a mass of 1 kg and a damping value of 0.05. The time step that was considered for it was 0.02 s. The ground motion details (horizontal component) are given in **Table 1**. In order to further prove the efficiency of the program, the 1989 Loma Prieta earthquake was considered for analysis and the percentage variation of the LabVIEW is evaluated. The basic parameters of the SDOF system considered has a mass of 1 kg, time step 0.02 s, damping ratio 0.05 and time period of 0.513 s [16].

Earthquake	Maximum acceleration (g)	Magnitude	Predominant period
El Centro	0.296	6.9	0.588
Loma Prieta	0.276	7	0.588

Table 1.
Earthquake ground motion details (horizontal component).

5. Results and discussion

5.1 Linear SDOF system response of time integration methods and its results and discussion

El Centro and Loma Prieta earthquake ground motions were considered for the analysis. The problem was solved using the time step 0.02 to understand the displacement-time history, velocity-time history and acceleration- time history under El Centro and Loma Prieta earthquake using both the methods on LabVIEW and are shown below.

5.1.1 Response in terms of displacement of linear SDOF system

The displacement response in central difference and wilson-θ method were obtained in LabVIEW and was displayed below in **Figures 3** and **4**.

In case of El Centro earthquake maximum peak displacement of 0.0531 (m) and minimum peak displacement of 0.0456 (m) was given by Wilson-θ method and in case of Loma Prieta the maximum peak displacement of 0.033 (m) and minimum peak displacement of 0.029 (m) was again given by Wilson θ-method.

5.1.2 Response in terms of velocity of linear SDOF system

The velocity response in central difference and wilson-θ method are obtained in LabVIEW and is displayed below (**Figures 5** and **6**).

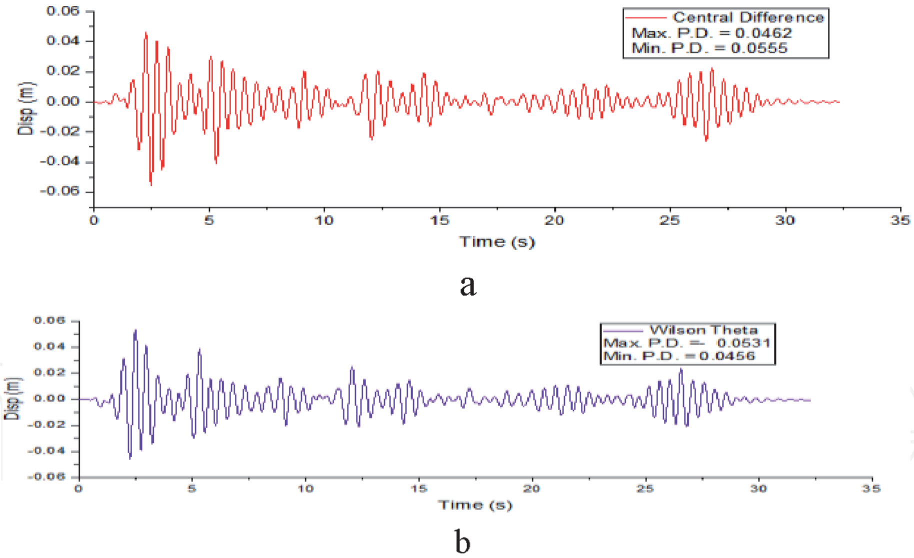

Figure 3
Displacement vs time response under El Centro earthquake (a) central difference method (b) wilson-θ method.

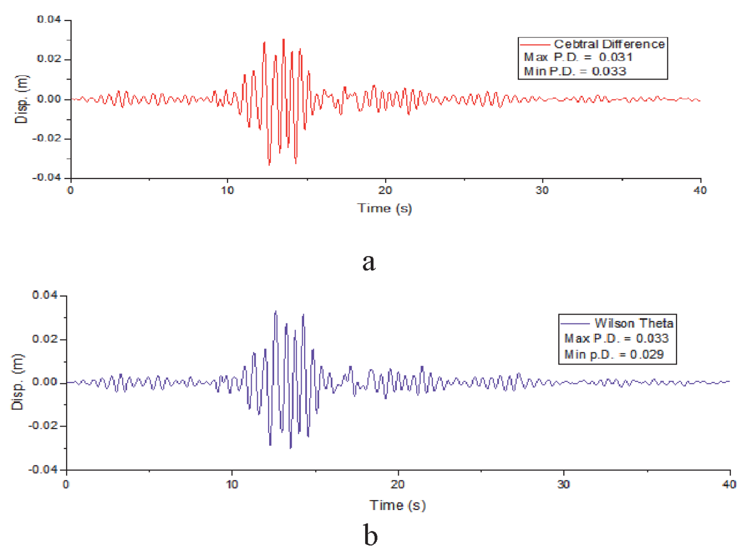

Figure 4.
Displacement vs time response under Loma Prieta earthquake (a) central difference method (b) wilson-θ method.

The complex solution determined by the central difference method in terms of velocity is contrasted with the method of wilson-θ. In the case of El Centro earthquake highest peak velocity of 0.653 (m/s.) and lowest peak velocity of 0.606 (m/s.) is responded by wilson-θ method and in case of Loma Prieta the highest peak velocity of 0.363 (m/s.) and lowest peak velocity of 0.341 (m/s.) is displayed by central difference method.

5.1.3 Response in terms of acceleration of linear SDOF system

The acceleration response in central difference and wilson-θ method were obtained in LabVIEW and are displayed below (**Figures** 7 and **8**).

Dynamic acceleration response calculated using central difference was compared with wilson-θ method. In case of El Centro earthquake motion, maximum peak acceleration of 10.32 (m/s^2) and minimum peak acceleration of 7.75 (m/s^2) was displayed by central difference method. Whereas, in case of Loma Prieta

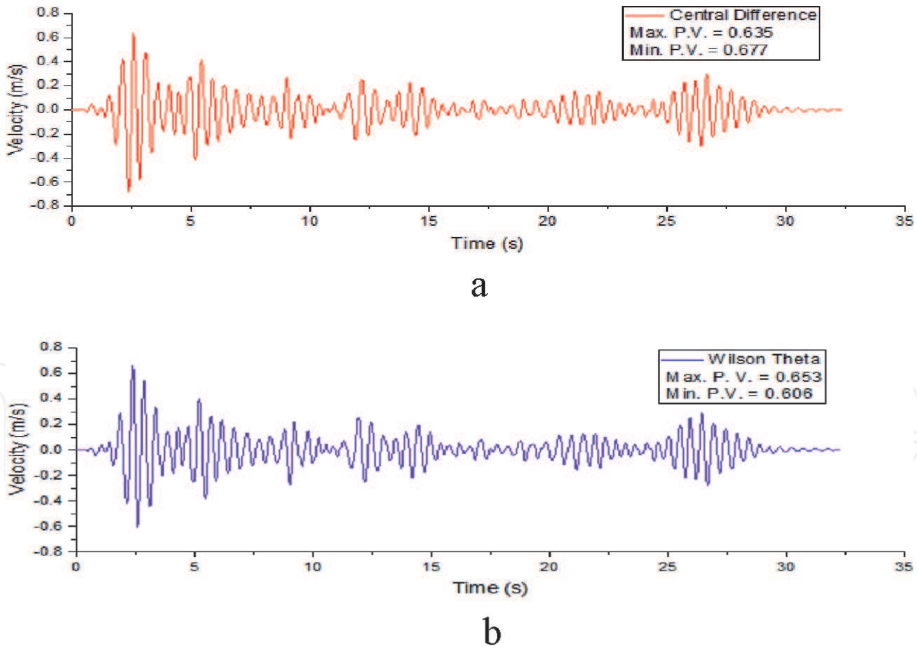

Figure 5.
Velocity vs time response under El Centro earthquake (a) central difference method (b) wilson-θ method.

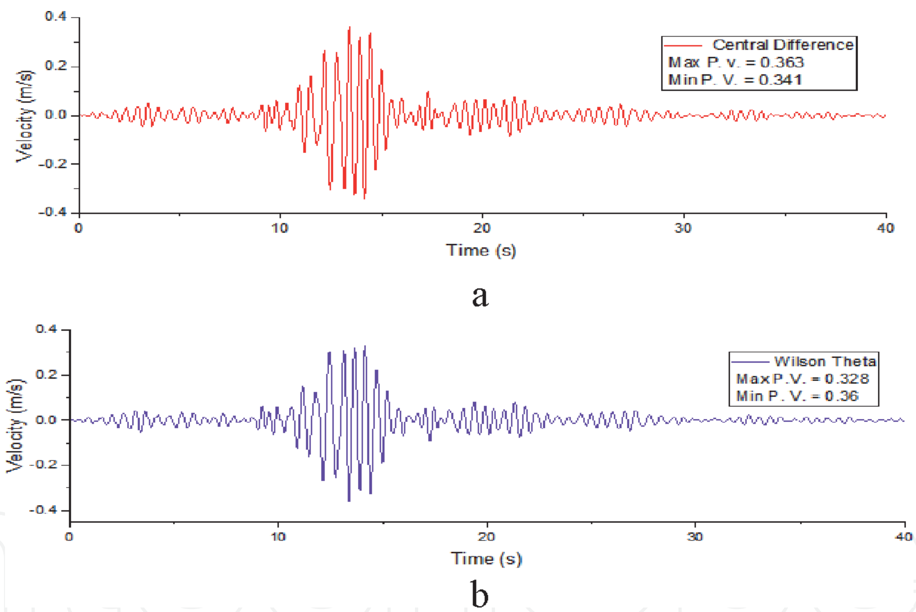

Figure 6.
Velocity vs time response under Loma Prieta earthquake (a) central difference method (b) wilson-θ method.

ground motion the maximum peak acceleration of 4.25 (m/s^2) was shown by central difference method and minimum peak acceleration of 4.24 (m/s^2) was given by wilson-θ method.

In the **Table 2** shown above, response results obtained from central difference method and wilson-θ method for El Centro earthquake was worked out. The difference in response was calculated and it was clearly seen that the variation did not exceed more than 0.08.

In the **Table 3** shown above, response results obtained from central difference method and wilson-θ method for Loma Prieta earthquake was worked out. The difference in response was calculated and it was clearly seen that the variation did not exceed more than 0.04.

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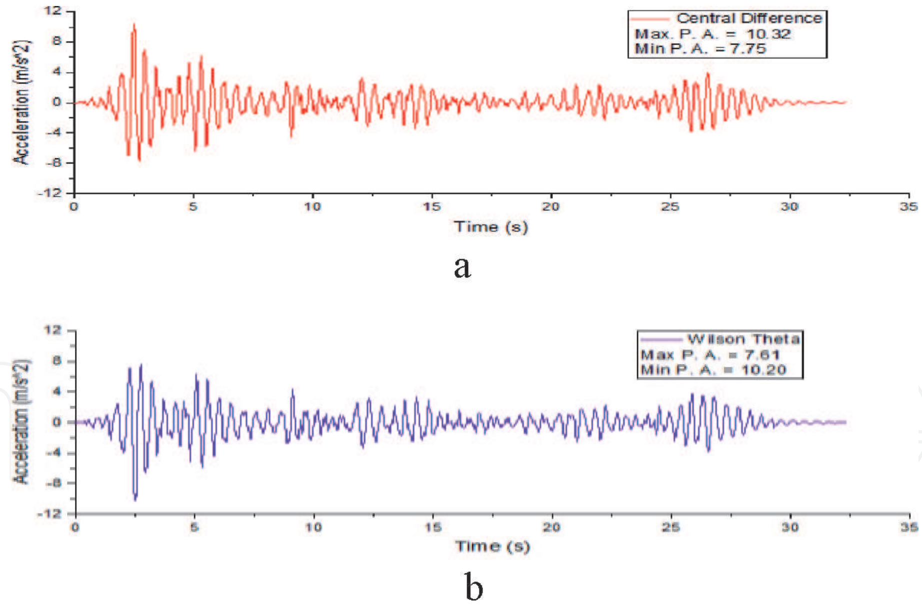

Figure7.
Acceleration vs time response under El Centro ground motion (a) central difference method (b) wilson-θ method.

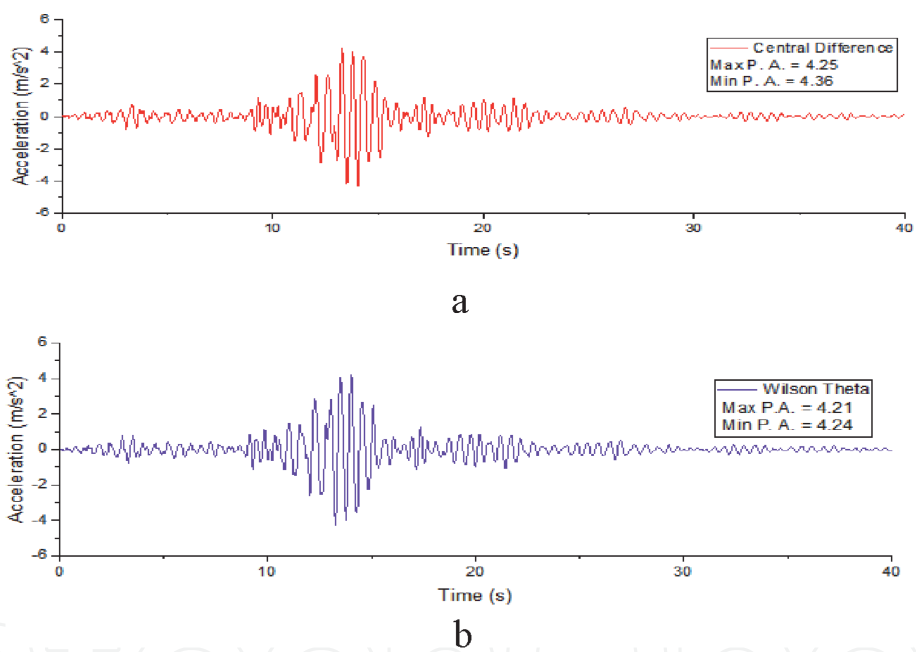

Figure 8.
Acceleration vs time response under Loma Prieta ground motion (a) central difference method (b) wilson-θ method.

El Centro	Peak Displacement(m)	Peak Velocity (m/s)	Peak acceleration (m/s^2)
CDM	0.0462	0.635	10.32
WTM	0.0531	0.653	10.20

Table 2.
Percentage variation of linear SDOF system under El Centro earthquake.

6. Conclusions

This paper summarizes, the modeling of linear SDOF system in LabVIEW software using time integration method. The comparative study with the results of an example chosen for the proposed program in LabVIEW clearly stated that the

Loma Prieta	Peak Displacement(m)	Peak Velocity (m/s)	Peak acceleration (m/s^2)
CDM	0.031	0.363	4.25
WTM	0.033	0.328	4.21

Table 3.
Percentage variation of linear SDOF under Loma Prieta earthquake.

responses obtained are accurate and hence programming in it for time integration problems will lead to trustworthy results.

These methods are thus based on two essential features:

1. Variation of displacement, velocity and acceleration are assumed within each time step. Hence, the accuracy and stability of the final solution depends on this variation.

2. The equilibrium is satisfied at all discrete time points within the interval of solution instead of any time t.

As the percentage difference between central difference method and wilson-θ method is negligible these programs can be extended to various earthquake ground motions and also for non-linear simulations. There is a need to track the system displacement, member spring force and spring stiffness to solve the problems in material nonlinearity. It should be noted that the time steps and appropriate stiffness has to be chosen very carefully to obtain accurate results. With the establishment of appropriate modeling for various other integration methods the accuracy can be further improved and time constraint problems can be easily solved.

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