Chapter

Location Selection for Smog Towers Using Zadeh's Z-Numbers Integrated with WASPAS

Janani Bharatraj

Abstract

Fuzzy sets have been extensively researched and results have been developed based on the extensions of fuzzy sets. In this chapter, fuzzy sets and its extensions are discussed. Z-numbers along with weighted sum product assessment method is used to obtain a feasible solution to the location selection problem for installation of smog towers in a densely populated locality. The degrees of freedom namely degree of membership, degree of non-membership and the degree of hesitancy have been expressed as Zadeh's Z-number with probability quotient for the degrees. Further, ranking of the alternatives based on Z-numbers and WASPAS to allocate smog towers to residential areas stricken by air pollution.

Keywords: Z-numbers, WASPAS, fuzzy set, Generalized fuzzy set, smog tower, air pollution

1. Introduction

Mathematics is a study of quantity, structure, space and change. Patterns are observed to understand the structures and reasoning is provided to real time phenomena. Mathematics can be subdivided into arithmetic, algebra, geometry and analysis. Further, there are subdivisions linking the core of mathematics to other fields like logic, set theory, empirical mathematics and more recently to the rigorous study of uncertainty, imprecision and vagueness. Set theory is a branch of mathematical logic that studies 'sets', a collection of well-defined objects. An object under consideration either 'belongs to' a set or 'does not belong' to a set. Thus classical set theory could answer membership of an element in terms of 0's and 1's. This binary logic could not translate the imprecision prevailing in the real world. The need to bridge the precise classical mathematics with the imprecise world gave birth to the concept of fuzzy sets. These sets were introduced independently by Zadeh [1] and Dieter Klaua in 1965 as an extension of classical set theory. In contrast to binary terms, fuzzy set theory allowed gradual assessment of the membership of elements in a set described by a membership function valued in the interval [0,1]. Zadeh went on to propose new operations in fuzzy logic and proved that fuzzy logic was a generalization of classical Boolean logic. He also proposed the concept of fuzzy numbers which were special case of fuzzy sets. The mathematical operations were also defined and thus fuzzy sets paved the way for many extensions, whose edifice stands strongly on the mathematics concept.

1.1 The extensions

Interval-valued fuzzy sets (IVFS) were introduced as an extension to fuzzy sets in which the membership degrees are represented by an interval value reflecting the uncertainty in assigning membership degrees. Larger the interval, more uncertainty is seen in assigning membership degrees.

Intuitionistic Fuzzy sets (IFS) is also an extension of fuzzy set introduced by Atanassov [2]. The addition of ‘degree of non-membership’ of an element improved the efficiency of modeling uncertainty.

Interval-Valued Intuitionistic Fuzzy Set (IVIFS):

Atanassov [3] combined IVFS and IFS to form IVIFS where in are all intervals in [0,1].

Neutrosophic Sets:

Having defined fuzzy sets, IVFS, IFS and IVIFS, researchers still could not handle uncertainty efficiently. The question of, “what if I had a neutral opinion?” had to be answered. Thus, Smarandache broke free the inter-dependencies of all three membership functions. Thus neutrosophic sets were defined

$$
\begin{aligned}
&A = \{(x, \mu_A(x), \upsilon_A(x))/x \in X\} \\
&\mu_A(x) : X \to [0,1], \upsilon_A(x) : X \to [0,1], \Gamma
\end{aligned}
$$

such that,

$$0 \le \mu_A(x) + \upsilon_A(x) + \Pi_A(x) \le 3.$$

Neutrosophic sets, thus, generalized all the sets with classic set theory as the foundation.

In a nutshell, the holy trinity were introduced as a trigger for astounding research all over the world.

1.2 Recent extensions

Picture Fuzzy sets (PFS):

These sets were introduced by Cuong [4] to model situations where in human opinions involved refusal towards a particular event. For instance, voting in an election could have four categories of people; people wanting to vote for a particular party, people abstaining from voting, people not wanting to vote for a party and people refusing to vote. Thus degree of refusal membership is given by $\eta_A(x)$, with

$$
\begin{gathered}
A = \{(x, \mu_A(x), \upsilon_A(x))/x \in X\} \\
\mu_A(x) : X \to [0,1], \upsilon_A(x) : X \to [0,1], \Pi_A(x) : X \to [0,1]. \\
0 \le \mu_A(x) + \upsilon_A(x) + \Pi_A(x) < 1.
\end{gathered}
$$

Pythagorean Fuzzy Sets (PyFS):

These sets were introduced as a generalization for IFS by Yager [5]. The main feature of PyFS is that it is characterized by the degrees in which the sum of the square of each of the parameters equal to 1.

Let X be a universal set. Then a Pythagorean fuzzy set A, which is a set of ordered pairs over X, is defined by the following;

$$A = \{(x, \mu_A(x), \upsilon_A(x))/x \in X\}$$

$$\mu_A(x) : X \to [0,1], \upsilon_A(x) : X \to [0,1]$$
$$0 \leq (\mu_A(x))^2 + (\upsilon_A(x))^2 \leq 1,$$
$$\Pi_A(x) = \sqrt{1 - \left((\mu_A(x))^2 + (\upsilon_A(x))^2\right)}$$

Hesitant Fuzzy Set (HFS):

HFS were introduced by Torra [6]. Hesitant fuzzy sets were defined in terms of a function that returns a set of membership values for each element in the domain.

HFS, to a large extent, were able to model uncertainty, but with in-depth research, a significant drawback appeared, namely, loss of information. To overcome this drawback, Zhu and Xu [7] proposed the concept of Probabilistic Hesitant Fuzzy Set (PHFS) which incorporates distribution information in HFS. PHFS depicts not only the hesitancy of decision-makers when they are irresolute for one thing or the other, but also hesitant distribution information.

Spherical Fuzzy Sets (SFS):

These sets were introduced as an extension to Picture fuzzy sets and Pythagorean fuzzy sets [8] (**Figure 1**).

A spherical fuzzy set $\tilde{A}_S$ of the universe of discourse U is given by

$$\mu_A(x) : X \to [0,1], \upsilon_A(x) : X \to [0,1], \Pi_A(x) : X \to [0,1]$$

With $0 \leq (\mu_A(x))^2 + (\upsilon_A(x))^2 + (\Pi_A(x))^2 \leq 1$, for any x in the universal set U.

The spherical fuzzy sets extend PFS and PyFS, but however, these sets are nothing but a particular case of Neutrosophic sets.

1.3 Decision-making techniques

Decision-making is process which involves problem-solving yielding a solution deemed to be "optimal" or satisfactory to an extent. A major part of decision-making involves analysis of finite set of alternatives with respect to a given set of criteria. The task involves ranking these alternatives based on feasibility when all

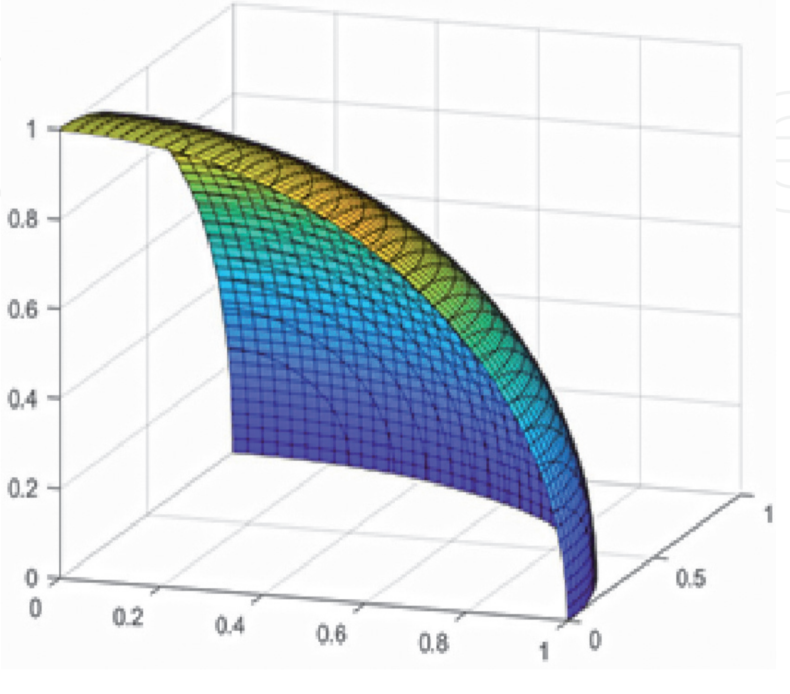

Figure 1.
Spherical fuzzy set.

the criteria are considered simultaneously. This area of decision-making has always attracted researchers and is still a highly debatable concept as there are many such methods which yield different results when applied to the same set of data. Emotion also appears to aid the decision-making process. Decision-making often occurs in uncertainty about whether one's choices will lead to benefit or harm.

Under fuzzy environment:

Decision making under uncertainty means a decision process in which the constraints or goals are fuzzy in nature, but the system need not be fuzzy. As per Bellman and Zadeh [9], fuzzy goals and constraints can be precisely defined as fuzzy sets in the space of alternatives. A fuzzy decision is then viewed as an intersection of the given goals and constraints. Decision-making under uncertainty basically translates to taking decisions in which the goals or constraints are fuzzy in nature. This implies that the constraints consists of alternatives whose boundaries are not sharply defined. An example of a fuzzy goal is "x should be in the vicinity of y", where y is a constant. Here vicinity is a source of fuzziness.

We thus divide the decision-making process into seven steps:

1. Outline the goal and outcome

2. Gather data

3. Develop alternatives

4. List pros and cons of each alternative

5. Identify the best alternative

6. Evaluate and monitor the solution

7. Examine feedback when necessary.

1.4 Zadeh's Z-numbers

A Z-number is an ordered pair of fuzzy numbers (A,B). Z-number is associated with a real-valued uncertain variable X, with the first component A, playing the role of a fuzzy restriction R(X), on the values of which X can take, written X is A. $R(X) : X\, is\, A \rightarrow \mu_A(u)$.

Here $\mu_A(u)$ is the degree to which u satisfies the constraint.

The second component B, is referred to as certainty or reliability or probability or strength of belief related to the component A.

For example, (finding an enclosed space of 900 s.m. in a densely populated area, low, not sure) [10].

1.5 Weighted aggregated sum product assessment (WASPAS)

WASPAS method was introduced by Zavadskas et al. [11]. This MCDM method is a combination of two simple decision-making techniques; Weighted Sum Model (WSM) and Weighted Product Model (WPM).

The total relative importance is given by

$$Q_i = \lambda \sum_{j=1}^{n} \overline{x}_{ij} w_j + (1-\lambda) \prod_{j=1}^{n} \left(\overline{x}_{ij}\right)^{w_j}, \lambda = 0, 0.1, 0.2, \dots, 1. \tag{1}$$

Here,

x_{ij} is the performance of ith alternative with respect to the jth criterion.

$\overline{x}_{ij}$ is the normalized value of x_{ij} evaluated as follows;

$$\overline{x}_{ij} = \frac{x_{ij}}{\max_i x_{ij}} \text{ for beneficial criteria} \tag{2}$$

$$\overline{x}_{ij} = \frac{\min_i x_{ij}}{x_{ij}} \text{ for non} - \text{beneficial criteria} \tag{3}$$

In the next section a literature review of the various aspects of decision-making using Z-numbers will be discussed.

2. Review of literature

Qiao et al. [12] proposed a simple computational method for ranking Z-numbers inspired by the concept of possibility degree of interval numbers. Outranking relations along with a weight acquisition algorithm relative to the possibility degree were developed. Finally, an extended PROMETHEE II based on the proposed ideas were developed. The same was applied to selection of travel plans.

Aliev et al. [13] suggested human-like fundamental approach for ranking Z-numbers. The approach was based on two ideas;

a. The optimality degree of Z-numbers was computed, and

b. The obtained degrees were adjusted using human opinion formalized by a degree of pessimism.

The concept was then used to solve a real time decision-making problem and results were obtained.

Peng and Wang [14] developed an innovative method for addressing MCGDM problems with Z-numbers with unknown weight information about the criteria. Cloud model was employed to analyze Z-numbers. Power aggregation operations of normal Z + -value was proposed using ratio analysis and full multiplicative form. This model was used to evaluate potential air pollution concerns.

Wang and Mao [15] developed a novel approach based on power plant location selection problem with Z-fuzzy based AHP model and successfully modeled the location selection problem.

Chatterjee and Kar [16] proposed COPRAS-Z methodology for Z-numbers. They modeled the fuzzy numbers with reliability degree to represent imprecise judgment of decision-makers in evaluating weights of criteria and selection of renewable energy alternatives.

3. Lacuna and new definition

Chakraborty et al. [17] validated the applicability of WASPAS under five real time manufacturing related problems which resulted in acceptable solutions.

Kahraman and Otay [18] used Z-numbers with AHP to select a location for solar energy PV plant using a 4-level hierarchy. Several criteria and sub-criteria were considered to understand the location selection problem with a Z-fuzzy based AHP method with a real life case study from Turkey.

Decision making techniques have been used mainly in supply chain management which covers the processes from the initial materials provision to the ultimate consumption of finished product linking all the supplier-user entities. Zarandi and Zarandi [19] proposed a modular architecture for the information agent which uses nine different modules, each of which is responsible for one or more functions for the information agent. This automated supply chain is adaptable to an ever changing business environment.

Another area which requires decision making techniques is site selection. Landfill site selection should take into account a wide range of alternative and evaluation criteria in order to reduce negative impacts on the environment. Aydi [20] presented a geographic information systems based multicriteria site selection of municipal solid waste landfill in Tunisia. The methodology involved integrating fuzzy logic and AHP to rank the best suitable landfill sites. The landfill suitability was accomplished by applying weighted linear combination that uses a comparison matrix to aggregate various scenarios associated with environmental and socio economic objectives. The study led to two candidate landfill sites best suited for the procedure.

Sadollah [21] clearly explained the role of membership functions and how to choose an appropriate membership function based on the data available. Computational time is also an important factor that decides the need of a particular type of membership function for decision making methods.

The literature review paves the way for some new definitions to be introduced. Thus, in this chapter, the degrees of freedom are combined with Z-numbers and applied to decision-making problem related to selection of location of smog towers in a densely populated area to combat air pollution problem faced by residents [22].

3.1 Definition

Let U be the universal set. Then a fuzzy subset S can be defined as

$$S = \{(x, (\mu_S(x), p), (\upsilon_S(x), q), (\Pi_S(x), r))\}$$

Where $(\mu_S(x), p), (\upsilon_S(x), q), (\Pi_S(x), r)$ are all Z-numbers with p, q and r the respective probabilities.

$$\text{Here, } \mu_S(x) \in [0, 1], \upsilon_S(x) \in [0, 1], \Pi_S(x) \in [0, 1]$$
$$\text{and } 0 \le \mu_S(x) + \upsilon_S(x) + \Pi_S(x) \le 1$$

$0 \le \mu_S^n(x) + \upsilon_S^n(x) + \Pi_S^n(x) \le 1$, n is an integer $n > 1$.

Figures 2 and **3** justify the above definition.

4. Application to location of smog towers in the capital city of Tamilnadu

The first of its kind smog towers were designed by Studio Roosegarde as a long term campaign for clean air. The seven metre tall smog-free tower uses patented positive ionization technology to let out smog free air into atmosphere. The tower is designed to clean 30,000 cubic meter of air per hour and is supposed to use small amount of green energy. A similar kind of tower was installed in New Delhi, the capital of India in a busy place called Lajpat Nagar. This tower could trap particulate matter of all sizes suspended in the air. It is capable of treating 2,50,000 to

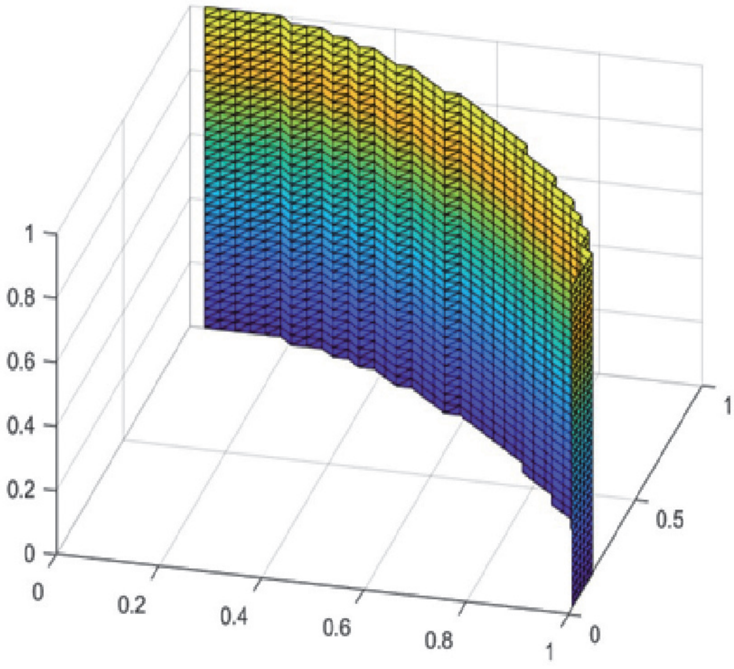

Figure 2.
Fuzzy set with n = 2.

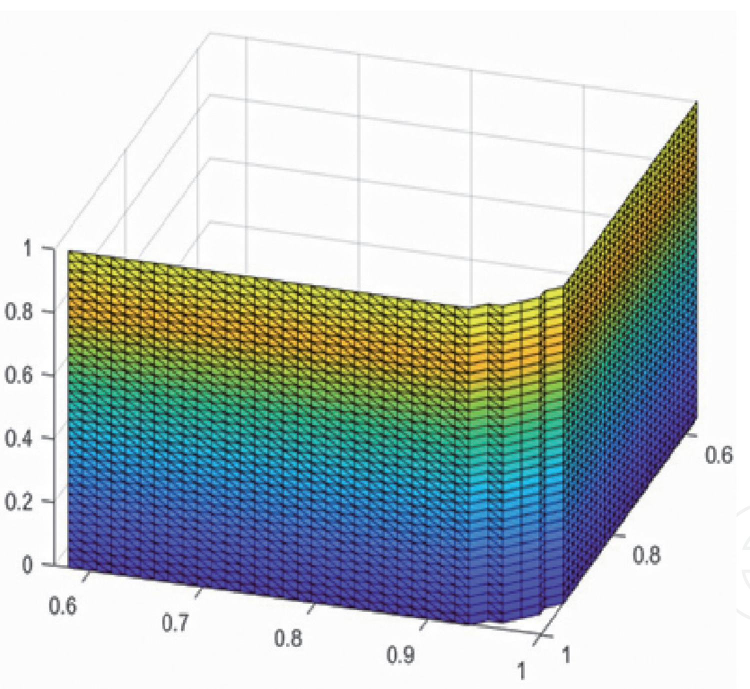

Figure 3.
Fuzzy set with n = 30.

6,00,000 cubic metre of air per day and can collect more than 75% of the particulate matter. It is a structure of concrete which has multiple layers of filters. The structure requires approximately 900 sq.m. in area for its installation. The device was designed to take in air from all angles and generate 1,30,000 cubic metres of clean pure air per hour. The 20 feet tall tower is fitted with exhaust fans to suck in polluted air and can remove upto 80% of the particulate matter ideally PM2.5 and PM10., which are the primary pollutants in Delhi's air.

The smog tower is expected to purify the air within a circumference area of almost 500 m to 750 m.

Chennai, the capital city of Southern state of Tamilnadu in India is plagued by air pollution. The sources of pollution in the city are due to transport, industries and open waste burning. The city also benefits from the land-sea breeze, limiting the contribution of sources outside the urban limit to contribute towards air pollution.

The state highway 49A popularly known as Rajiv Gandhi Salai or the IT corridor is a major road connecting Chennai with Mahabalipuram. It is a 45 km long road housing the prestigious TIDEL park, a home to a number of BPO and IT/ITES companies.

In the first 20 km stretch, 15 traffic signals are stationed, with two toll plazas. The traffic during peak hours cause a lot of pollution in spite of being close to the sea. The major junctions are the 15 signals that literally stall the vehicular movement on this road (**Figure 4**).

The first 20 km houses a small neighborhood called Perungudi. Being in the IT corridor, Perungudi is a preferred locality for booming business and software firms. It is also home to one of the two major landfills in Chennai. The dump yard is constantly in news for the burning of garbage spills despite it being banned. The area also has a sewage treatment plant. Thus, Perungudi faces the wrath of all kinds of pollution, mostly air pollution due to vehicles, dust from construction sites, stench from the sewage treatment plant along with burning of garbage. The area is also low on green cover and hence pollution has severe effect on the residents living there.

Thus, installation of smog free towers is very much required for a locality like Perungudi to fight air pollution.

Challenges and Uncertainty:

A locality like Perungudi is densely populated and lacks basic amenities even though it is part of the famous IT corridor. Ideally speaking, a smog tower should be installed in places where the vehicular movement is on a higher side. But, however such junctions with a space requirement of a minimum of 900 square metres is a big

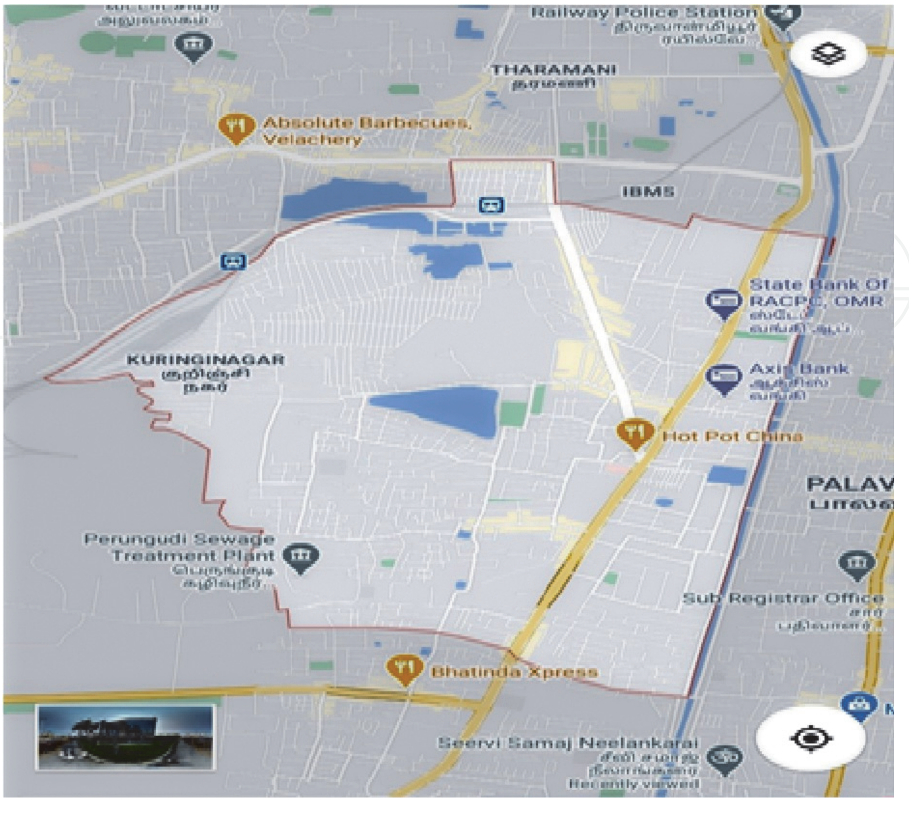

Figure 4.
Map of Perungudi, Chennai, Tamilnadu, India – Google maps.

challenge. Further, the locality has already been allocated to builders who have taken over a majority of the area for construction purpose. Thus, a lot of uncertainty is involved in location selection considering the challenges faced in terms of space requirements. Hence, fuzzy system plays a crucial role in identifying the right place to install a smog tower considering all the challenges faced in a densely crowded locality. Since, possibility of an event occurring under uncertainty is being studied here, Zadeh's Z-numbers have been combined with WASPAS method to obtain optimal solution.

In order to select a proper location for installing the smog towers, we need to look into the following criteria and understand the feasibility of allocating a place for a tower that will be beneficial on a long run.

4.1 Criteria

Some of the main criteria to set up a smog tower in a locality are listed below;
C1: Minimum area of 900 sq.m.
C2: Continuous supply of electricity.
C3: Green cover in the locality to allow solar panels as an alternative.
C4: Pollution levels in that locality.

4.2 Method

Step 1: Perungudi locality can be broadly divided into four main zones which have maximum impact due to air pollution and noise pollution. The areas; Industrial estate, Srinivasa Nagar, Telephone Nagar and Venkateswara colony are densely populated and have industries contributing to air pollution.

Thus, Four alternatives were chosen for the location of the smog tower namely A1, A2, A3 and A4.
A1: Industrial Estate
A2: Srinivasa Nagar
A3: Telephone Nagar
A4: Venkateswara Colony

Step 2: The alternatives are mapped against the criteria using Zadeh's Z numbers as follows; the degrees of freedom of A1 with respect to criterion C1 is 0.2, 0.7 and 0.1 with probability of 0.8, 0.1 and 0.1 respectively. That is, finding an enclosed space of 900 sq.m. in a densely populated area is 0.2, with the strength of belief 0.8. Likewise, the degree of non-membership is 0.7, with a probability of 0.1 and the degree of hesitancy is 0.1, with a probability of 0.1.

The decision matrix is formed from the date collected from one of the residents and is tabulated as follows (**Table 1**).

Step 3: The maximum of all membership degrees of the alternatives, the minimum of the non-membership degrees of the alternatives and the average of the hesitancy degree of the alternatives are calculated and the decision matrix is normalized using (Eq. (2) and Eq. (3)). **Table 2** shows the normalized decision matrix.

Step 4: The weighted sum and weighted product are calculated.

The total weighted sum and product assessment is tabulated as below using.

(Eq. (1)) with $\lambda = 0.5$ and weights for C1 w1 = 0.3, C2:w2 = 0.1, C3: w3 = 0.3, C4: w4 = 0.3 (**Table 3**).

Step 5: The score function is calculated using the formula

$$s(x) = \frac{\mu_S(x) + 1 - \upsilon_S(x) - \Pi_S(x)}{3}.$$

Alternatives/criteria	C1	C2	C3	C4
A1	(0.2,0.8), (0.7,0.1), (0.1,0.1)	(0.6,0.2), (0.3,0.8), (0.1,0.5)	(0.8,0.8) (0.1,0.1) (0.1,0.1)	(0.8,0.8) (0.1,0.8) (0.1,0.8)
A2	(0.1,0.9) (0.7,0.8) (0.2,0.1)	(0.4,0.2) (0.5,0.8) (0.1,0.8)	(0.3,0.8) (0.6,0.2) (0.1,0.1)	(0.8,0.8) (0.1,0.8) (0.1,0.6)
A3	(0.6,0.7) (0.3,0.5) (0.1,0.1)	(0.7,0.6) (0.2,0.8) (0.1,0.1)	(0.4,0.4) (0.4,0.6) (0.2,0.1)	(0.8,0.9) (0.1,0.8) (0.1,0.8)
A4	(0.4,0.8) (0.6,0.8) (0,0.9)	(0.7,0.7) (0.2,0.8) (0.1,0.9)	(0.3,0.5) (0.3,0.6) (0.4,0.1)	(0.7,0.9) (0.1,0.9) (0.2,0.9)

Table 1.
Decision matrix for the alternatives.

Alternatives/criteria	C1	C2	C3	C4
A1	(0.33,0.8), (0.43,0.1), (1,0.1)	(0.86,0.2), (0.67,0.8), (1,0.5)	(1,0.8) (1,0.1) (0.5,0.1)	(1,0.8) (1,0.8) (0.8,0.8)
A2	(0.17,0.9) (0.43,0.8) (0.5,0.1)	(0.57,0.2) (0.4,0.8) (1,0.8)	(0.38,0.8) (0.17,0.2) (0.5,0.1)	(1,0.8) (1,0.8) (0.8,0.6)
A3	(1,0.7) (1,0.5) (1,0.1)	(1,0.6) (1,0.8) (1,0.1)	(0.5,0.4) (0.25,0.6) (1,0.1)	(1,0.9) (1,0.8) (0.8,0.8)
A4	(0.67,0.8) (0.5,0.8) (0,0.9)	(1,0.7) (1,0.8) (1,0.9)	(0.38,0.5) (0.33,0.6) (0.5,0.1)	(0.875,0.9) (1,0.9) (0.625,0.9)

Table 2.
Normalized decision matrix.

Alternatives/ criteria	Membership Z-number	Non-membership Z-number	Hesitancy Z-number
A1	(0.75,0.1024)	(0.769,0.0064)	(0.775,0.004)
A2	(0.47, 0.11)	(0.47, 0.1024)	(0.628, 0.0048)
A3	(0.83,0.23)	(0.717,0.192)	(0.9376,0.0008)
A4	(0.65,0.25)	(0.617,0.3456)	(0.2187,0.0729)

Table 3.
Matrix obtained using WASPAS in Z-number.

Hence the ranking of alternatives is obtained as shown below.

Table 4 clearly concludes that A4 > A2 > A1 > A3.

Further, as the values of $\lambda = 0.5$ were increased and decreased, the ranking of the alternatives remained unaltered.

Thus, the uncertainty involved in allocating suitable locations for installation of smog towers could be solved using Zadeh's Z-numbers and WASPAS and a feasible solution has been obtained.

Alternatives	Score function	Ranking
A1	0.067385	3
A2	0.123704	2
A3	0.058708	4
A4	0.272859	1

Table 4.
Score and ranking of the alternatives.

4.3 Challenges and limitations

This study focusses mainly on a specific locality which is densely populated in a small part of the city. On a large scale, a city like Chennai will require at least 20 such smog towers in each locality to control air pollution. Further, the money spent on these towers in installing and maintaining would cost a lot for the local government to manage. A smog tower of this capacity would require close to Rs.30,000 just for the maintenance. Hence, such smog towers cannot be the only solution to reduce air pollution. Further research is required to prove the effectiveness of these towers in reducing air pollution and providing clean air.

4.4 Awareness and suggestions

Air pollution is one of the key factors affecting the livelihood in a city like Chennai. Government should raise awareness about using green energy and install EV charging points at key locations for people to use electric vehicles. More research and funding needs to be given to develop low cost electric vehicles which are affordable for a common man.

5. Conclusion and future directions

In this chapter, a generalized fuzzy set was discussed. Zadeh's Z-numbers were combined with WASPAS method and the problem of location selection for smog towers in the locality of Perungudi was discussed. The most feasible solution was obtained which could provide some relief to the residents suffering because of air pollution. The concept can be extended to other decision-making techniques and better results can be obtained.

Author details

Janani Bharatraj
Chennai, India

*Address all correspondence to: jananichari@gmail.com

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