Chapter

Exergy: Mechanical Nuclear Physics Measures Pressure, Viscosity and X-Ray Resonance in K-Shell in a Classical Way

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Abstract

First, the liquid drop model assumes *a priori*; to the atomic nucleus composed of protons and neutrons, as an incompressible nuclear fluid that should comply with the Navier–Stokes 3D equations (N-S3D). Conjecture, not yet proven, however, this model has successfully predicted the binding energy of the nuclei. Second, the calculation of nuclear pressure $\left(p_0 \in 1.42, 1.94\right]10^{32}Pa)$ and average viscosity $\left(\eta = 1.71 \times 10^{24}\left[fm^2/s\right]\right)$, as a function of the nuclear decay constant $k = \frac{p_0}{2\eta} = \frac{1}{T_{1/2}}$, not only complements the information from the National Nuclear Data Center, but also presents an analytical solution of (N- S3D). Third, the solution of (N-S3D) is a Fermi Dirac generalized probability function $P(x, y, z, t) = \frac{1}{1+e^{\frac{p_0}{2\eta}t-\mu\left(x^2+y^2+z^2\right)^{1/2}}}$, Fourth, the parameter μ has a correspondence with the Yukawa potential coefficient $\mu = \alpha m = 1/r$, Fifth, using low energy X-rays we visualize and measure parameters of the nuclear surface (proton radio) giving rise to the femtoscope. Finally, we obtain that the pressure of the proton is 8.14 times greater than the pressure of the neutron, and 1000 times greater than the pressure of the atomic nucleus. Analyzed data were isotopes: $9 \leq Z \leq 92$ and $9 \leq N \leq 200$.

Keywords: femtoscope, Navier Stokes 3D, nuclear viscosity, minimum entropy

1. Introduction

Neutron stars are among the densest known objects in the universe, withstanding pressures of the order of $10^{34}Pa$. However, it turns out that protons [1], the fundamental particles that make up most of the visible matter in the universe, contain pressures 10 times greater, [2, 3] $10^{35}Pa$. This has been verified from two perspectives at the Jefferson Laboratory, MIT [1–5]. High-energy physics continue to guide the study of the mechanical properties of the subatomic world.

Viscosity is a characteristic physical property of all fluids, which emerges from collisions between fluid particles moving at different speeds, causing resistance to their movement (**Figure 1**). When a fluid is forced to move by a closed surface, similar to the atomic nucleus, the particles that make up the fluid move slower in the center and faster on the walls of the sphere. Therefore, a shear stress (such as a pressure difference) is necessary to overcome the friction resistance between the

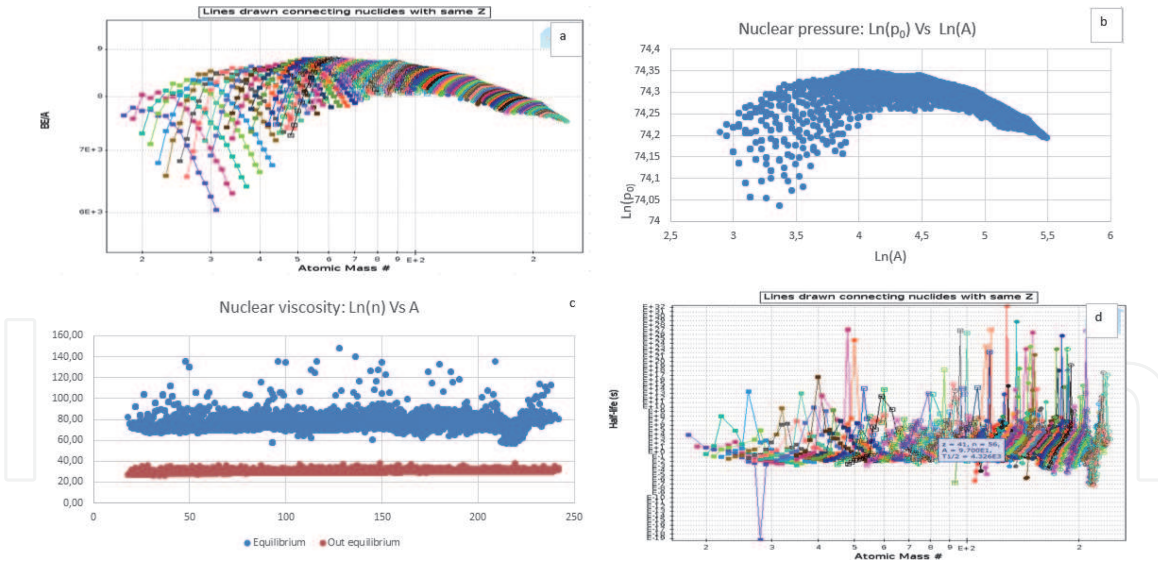

Figure 1.
Obtaining nuclear viscosity and nuclear pressure from the speed of the neutron particles in the disintegration of chemical element. ***Figure 1a****. indicates that the BE/A ratio is proportional to the nuclear pressure,* p_o *represented in* ***Figure 1b****.* ***Figure 1c*** *is the graph of the viscosity in equilibrium and out of equilibrium, the viscosity in equilibrium is greater than the viscosity at the moment of nuclear decay.* ***Figure 1d*** *is the average half-time of each isotope,* $9 \le Z \le 92$ *and* $9 \le N \le 200$.

layers of the nuclear fluid. For the same radial velocity profile, the required tension is proportional to the viscosity of the nuclear fluid or its composition given by (Z, N) [6–8].

Radioactive decay is a stochastic process, at the level of individual atoms. According to quantum theory, it is impossible to predict when a particular atom will decay, regardless of how long the atom has existed. However, for a collection of atoms of the same type [8], the expected decay rate is characterized in terms of its decay constant $k = \frac{1}{T_{1/2}}$. The half-lives of radioactive atoms do not have a known upper limit, since it covers a time range of more than 55 orders of magnitude, from almost instantaneous to much longer than the age of the universe [8, 9].

Other characteristics of the proton such as its size have been studied in many institutes such as Max Plank, where it has been measured with high precision ranges $(r_p = 0.84184(67) fm)$, providing new research methods [10, 11].

The atomic nucleus is an incompressible fluid, justified by the formula of the nuclear radius, $R = 1.2A^{\frac{1}{3}}$, where it is evident that the volume of the atomic nucleus changes linearly with $A = Z + N$, giving a density constant [11]. All incompressible fluid and especially the atomic nucleus comply with the Navier Stokes equations. We present a rigorous demonstration on the incomprehensibility of the atomic nucleus, which allows to write explicitly the form of the nuclear force $\mathbf{F}_N = -\frac{g\mu^2}{8\pi}(A-1)P(1-P)\nabla\mathbf{r}$, which facilitates the understanding of nuclear decay.

The Navier Stokes equations are a problem of the millennium [12, 13], that has not been resolved yet in a generalized manner. We present a solution that logically meets all the requirements established by the Clay Foundation [14, 15]. This solution coherently explains the incompressible nuclear fluid and allows calculations of the nuclear viscosity and nuclear pressure [1, 2].

The alpha particle is one of the most stable. Therefore it is believed that it can exist as such in the heavy core structure. The kinetic energy typical of the alpha particles resulting from the decay is in the order of 5 MeV.

For our demonstrations, we will use strictly the scheme presented by Fefferman in *http://www.claymath.org/millennium-problems* [13, 14], where six demonstrations are required to accept as valid a solution to the Navier–Stokes 3D Equation [16–18]. An understanding of the mechanics of the atomic nucleus cannot do without fluid equations.

2. Model

The velocity defined as $\mathbf{u} = -2\nu\frac{\nabla P}{P}$, with a radius noted as $r = (x^2 + y^2 + z^2)^{1/2}$ where $P(x,y,z,t)$ is the logistic probability function $P(x,y,z,t) = \frac{1}{1+e^{kt-\mu r}}$, and the expected value $E(r|r \geq 0) < C$ exist. The term P is defined in $\left((x,y,z) \in \mathbb{R}^3, t \geq 0\right)$, where constants $k > 0, \mu > 0$ and $P(x,y,x,t)$ is the general solution of the Navier–Stokes 3D equation, which has to satisfy the conditions (1) and (2), allowing us to analyze the dynamics of an incompressible fluid [12–14].

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = \nu\,\nabla^2\mathbf{u} - \frac{\nabla p}{\rho_0} \qquad \left((x,y,z) \in \mathbb{R}^3, t \geq 0\right) \tag{1}$$

With, $\mathbf{u} \in \mathbb{R}^3$ an known velocity vector, ρ_0 constant density of fluid, η dynamic viscosity, ν cinematic viscosity, and pressure $p = p_0 P$ in $\left((x,y,z) \in \mathbb{R}^3, t \geq 0\right)$.

Where velocity and pressure are depending of r and t. We will write the condition of incompressibility.

$$\nabla.\mathbf{u} = 0 \qquad \left((x,y,z) \in \mathbb{R}^3, t \geq 0\right) \tag{2}$$

The initial conditions of fluid movement $\mathbf{u}^0(x,y,z)$, are determined for $t = 0$. Where speed $\mathbf{u}^0$ must be C^∞ divergence-free vector.

$$\mathbf{u}(x,y,z,0) = \mathbf{u}^0(x,y,z) \qquad \left((x,y,z) \in \mathbb{R}^3\right) \tag{3}$$

For physically reasonable solutions, we make sure $\mathbf{u}(x,y,z,t)$ does not grow large as $r \to \infty$. We will restrict attention to initial conditions $\mathbf{u}^0$ that satisfy.

$$\left|\partial_x^\alpha \mathbf{u}^0\right| \leq C_{\alpha K}(1+r)^{-K} \qquad \text{on } \mathbb{R}^3 \ \text{ for any } \alpha \text{ and } K \tag{4}$$

The Clay Institute accepts a physically reasonable solution of (1), (2) and (3), only if it satisfies:

$$p, \mathbf{u} \in C^\infty\left(\mathbb{R}^3 \times 0, \infty)\right) \tag{5}$$

and the finite energy condition [14–16].

$$\int_{\mathbb{R}^3} |\mathbf{u}(x,y,z,t)|^2 dxdydz \leq C \qquad \text{for all } t \geq 0 \qquad (\text{bounded energy}). \tag{6}$$

The problems of Mathematical Physics are solved by the Nature, guiding the understanding, the scope, the limitations and the complementary theories. These guidelines of this research were: the probabilistic elements of Quantum Mechanics, the De Broglie equation and the Heisenberg Uncertainty principle.

2.1 Definitions

Nuclear reaction velocity coefficient.

We will use an equation analogous to concentration equation of Physical Chemistry $C = C_0 e^{kt}$, where $k = \frac{p_0}{2\eta}$, is velocity coefficient, p_0 is the initial pressure of our fluid, η the dynamic viscosity and C_0 the initial concentration of energetic fluid molecules.

It is evident that, in equilibrium state we can write $\mu r = kt$, however, the Navier–Stokes equation precisely measures the behavior of the fluids out of equilibrium, so that: $\mu r \neq kt$.

Fortunately, there is a single solution for out-of-equilibrium fluids, using the fixed-point theorem for implicit functions, $\frac{1}{1+e^{kt-\mu r}} = \frac{2}{\mu r}$, the proof is proved in *Theorem* 1.

Attenuation coefficient.

We will use the known attenuation formula of an incident flux I_0, for which $I = I_0 e^{-\mu r}$. Where, I_0 initial flux and μ attenuation coefficient of energetic molecules that enter into interaction and/or resonance with the target molecules, transmitting or capturing the maximum amount of energy [5].

Dimensional analysis and fluid elements.

We will define the respective dimensional units of each one of variables and physical constants that appear in the solution of the Navier–Stokes 3D equation [12–14, 16].

Nuclear decay $N(t) = N_0 e^{-kt}$. Where $k = \frac{1}{T_{1/2}}$ [1/s], the velocity coefficient, and $T_{1/2}$ ground state half-life.

Kinematic viscosity $\nu = \frac{\eta}{\rho_0}$, $\left[\frac{m^2}{s}\right]$.

Dynamic viscosity η, [pa.s], where pa represents pascal pressure unit.

Initial Pressure of out of equilibrium. p_0, [pa].

Fluid density ρ_0, $\left[\frac{kg}{m^3}\right]$, where kg is kilogram and m^3 cubic meters.

Logistic probability function, $P(x, y, z, t) = \frac{1}{1+e^{kt-\mu r}}$, it is a real number $0 \leq P \leq 1$.

Equilibrium condition, $r = \frac{k}{\mu} t = \frac{p_0}{2\rho_0 \nu \mu} t = |\mathbf{u}_e| t$, [m].

Fluid velocity in equilibrium, $|\mathbf{u}_e|$, $[m/s]$. Protons, neutrons and alpha particles are the elements of the fluid.

Fluid field velocity out of equilibrium, $\mathbf{u} = -2\nu\mu(1-P)\nabla r$. [m/s]. All nuclear decay is a process out of nuclear equilibrium.

Position, $r = (x^2 + y^2 + z^2)^{1/2}$, [m].

Attenuation coefficient, μ, [1/m].

Growth coefficient, $k = \frac{p_0}{2\rho_0 \nu} = \frac{p_0}{2\eta}$, [1/s].

Concentration $C = C_0 \frac{1-P}{P}$.

Theorem 1 *The velocity of the fluid is given by* $\mathbf{u} = -2\nu \frac{\nabla P}{P}$, *where* $P(x, y, z, t)$ *is the logistic probability function* $P(x, y, x, t) = \frac{1}{1+e^{kt-\mu r}}$, *and p pressure such that* $p = p_0 P$, *both defined on* $\left((x, y, z) \in \mathbb{R}^3, t \geq 0\right)$. *The function P is the general solution of the Navier Stokes equations, which satisfies conditions* (1) *and* (2).

Proof. To verify condition (2), $\nabla.\mathbf{u} = 0$, we must calculate the gradients and laplacians of the radius. $\nabla r = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$, and $\nabla^2 r = \nabla.\nabla r = \frac{\left(y^2+z^2\right)+\left(x^2+z^2\right)+\left(x^2+y^2\right)}{\left(x^2+y^2+z^2\right)^{3/2}} = \frac{2}{r}$.

$$\nabla.u = -2\nu\nabla.\frac{\nabla P}{P} = -2\nu\mu\nabla((1-P)\nabla r) \tag{7}$$

Replacing the respective values for the terms: $\nabla^2 r$ and $|\nabla r|^2$ in the Eq. (7).

$$\begin{aligned}\nabla.u &= -2\nu\mu\nabla((1-P)\nabla r)\\ &= -2\nu\mu\nabla((1-P)\nabla r)\\ &= -2\nu\mu\left[-\mu\left(P-P^2\right)|\nabla r|^2+(1-P)\nabla^2 r\right]\end{aligned} \tag{8}$$

Where the gradient modulus of $\nabla P = \mu\left(P-P^2\right)\nabla r$, has the form $|\nabla P|^2 = \mu^2\left(P-P^2\right)^2|\nabla r|^2 = \mu^2\left(P-P^2\right)^2$.

$$\nabla.u = -2\nu\mu(1-P)\left[-\mu P+\frac{2}{r}\right] = 0 \tag{9}$$

Simplifying for $(1-P)\neq 0$, we obtain the main result of this paper, which represents a fixed point of an implicit function $f(t,r)$ where $f(t,r) = P-\frac{2}{\mu r} = 0$. In Nuclear Physics, $r_0 < r < 1.2A^{1/3}$.

$$P = \frac{1}{1+e^{kt-\mu\left(x^2+y^2+z^2\right)^{1/2}}} = \frac{2}{\mu\left(x^2+y^2+z^2\right)^{1/2}} \qquad \left((x,y,z)\in\mathbb{R}^3, t\geq 0\right) \tag{10}$$

Eq. (10) has a solution according to the fixed-point theorem of an implicit function, and it is a solution to the Navier Stokes stationary equations, which are summarized in: $\nabla^2 P = \frac{2}{\mu}\nabla^2\left(\frac{1}{r}\right) = 0$. Furthermore, it is the typical solution of the Laplace equation for the pressure of the fluid $\nabla^2 p = p_0\nabla^2 P = 0$. Kerson Huang (1987).

To this point, we need to verify that Eq. (10) is also a solution of requirement (1), $\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = \nu\nabla^2\mathbf{u} - \frac{\nabla p}{\rho_0}$. We will do the equivalence $\mathbf{u} = \nabla\theta$ after we replace in Eq. (1). Taking into account that $\theta = -2\nu\ln(P)$, and that $\nabla\theta$ is irrotational, $\nabla\times\nabla\theta = 0$, we have: $(\mathbf{u}.\nabla)\mathbf{u} = (\nabla\theta.\nabla)\nabla\theta = \frac{1}{2}\nabla(\nabla\theta.\nabla\theta) - \nabla\theta\times(\nabla\times\nabla\theta) = \frac{1}{2}\nabla(\nabla\theta.\nabla\theta)$, and $\nabla^2\mathbf{u} = \nabla(\nabla.\mathbf{u}) - \nabla\times(\nabla\times\mathbf{u}) = \nabla(\nabla.\nabla\theta) - \nabla\times(\nabla\times\nabla\theta) = \nabla\left(\nabla^2\theta\right)$. Simplifying terms in order to replace these results in Eq. (1) we obtain

$$\begin{aligned}(\mathbf{u}.\nabla)\mathbf{u} &= \frac{1}{2}\nabla(\nabla\theta.\nabla\theta) = 2\nu^2\nabla\left(\frac{|\nabla P|^2}{P^2}\right)\\ \nabla^2\mathbf{u} &= \nabla(\nabla.\mathbf{u}) = \nabla\left(\nabla^2\theta\right) = 0\\ &= -2\nu\nabla\left(\frac{|\nabla P|^2}{P^2}-\frac{\nabla^2 P}{P}\right) = 0\end{aligned}$$

The explicit form of velocity is $\mathbf{u} = -2\mu\nu(1-P)\nabla r$. Next, we need the partial derivative $\frac{\partial \mathbf{u}}{\partial t}$

$$\begin{aligned}\frac{\partial \mathbf{u}}{\partial t} &= -2\mu\nu k P(1-P)\nabla r,\\ -\frac{\nabla p}{\rho_0} &= -\frac{\mu p_0}{\rho_0}P(1-P)\nabla r.\end{aligned}$$

After replacing the last four results $(\mathbf{u}.\nabla)\mathbf{u}$, $\nabla^2\mathbf{u}$, $\frac{\partial \mathbf{u}}{\partial t}$ and $-\frac{\nabla p}{\rho_0}$ in Eq. (1) we obtain (11).

$$-2\mu\nu kP(1-P)\nabla r = 2\nu^2\nabla\left(\frac{|\nabla P|^2}{P^2}\right) - \frac{\mu p_0}{\rho_0}P(1-P)\nabla r. \tag{11}$$

The Eq. (11) is equivalent to Eq. (1). After obtaining the term $\frac{|\nabla P|^2}{P^2}$ from the incompressibility equation $\nabla(\nabla^2\theta) = -2\nu\nabla\left(-\frac{|\nabla P|^2}{P^2} + \frac{\nabla^2 P}{P}\right) = 0$ and replacing in Eq. (11).

$$-2\mu\nu kP(1-P)\nabla r = 2\nu^2\nabla\left(\frac{\nabla^2 P}{P}\right) - \frac{\mu p_0}{\rho_0}P(1-P)\nabla r. \tag{12}$$

Eq. (10) simultaneously fulfills requirements (1) expressed by Eq. (12) and requirement (2) expressed by Eq. (7), for a constant $k = \frac{p_0}{2\rho_0\nu} = \frac{p_0}{2\eta}$. Moreover, according to Eq. (10), the probability $P = \frac{2}{\mu r}$ which allows the Laplace equation to be satisfied: $\nabla^2 P = \frac{2}{\mu}\nabla^2\left(\frac{1}{r}\right) = 0$. In other words, the Navier–Stokes 3D equation system is solved. ■

Implicit Function.

An implicit function defined as (10), $f(t,r) = \frac{1}{1+e^{kt-\mu r}} - \frac{2}{\mu r} = 0$ has a fixed point (t,r) of $R = \{(t,r)|0 < a \le t \le b, 0 < r < +\infty\}$, where m and M are constants, such as: $m \le M$. Knowing that the partial derivative exists: $\partial_r f(t,r) = \nu P(1-P) + \frac{2}{\mu r^2}$ we can assume that: $0 < m \le \partial_r f(t,r) \le M$. If, in addition, for each continuous function φ in $[a,b]$ the composite function $g(t) = f(t,\varphi(t))$ is continuous in $[a,b]$, then there is one and only one function: $r = \varphi(t)$ continuous in $[a,b]$, such that $f[t,\varphi(t)] = 0$ for all t in $[a,b]$.

Theorem 2 *An implicit function defined as (10)* $f(t,r) = \frac{1}{1+e^{kt-\mu r}} - \frac{2}{\mu r} = 0$ *has a fixed point* (t,r) *of* $R = \{(t,r)|0 < a \le t \le b, 0 < r < +\infty\}$. *In this way, the requirements (1) and (2) are fulfilled.*

Proof. Let C be the linear space of continuous functions in $[a,b]$, and define an operator $T : C \to C$ by the equation:

$$T\varphi(t) = \varphi(t) - \frac{1}{M}f[t,\varphi(t)].$$

Then we prove that T is a contraction operator, so it has a unique fixed point $r = \varphi(t)$ in C. Let us construct the following distance.

$$T\varphi(t) - T\psi(t) = \varphi(t) - \psi(t) - \frac{f[t,\varphi(t)] - f[t,\psi(t)]}{M}.$$

Using the mean value theorem for derivation, we have

$$f[t,\varphi(t)] - f[t,\psi(t)] = \partial_\phi f(t,z(t))[\varphi(t) - \psi(t)].$$

Where $\phi(t)$ is situated between $\varphi(t)$ and $\psi(t)$. Therefore, the distance equation can be written as:

$$T\varphi(t) - T\psi(t) = [\varphi(t) - \psi(t)]\left[1 - \frac{\partial_\phi f(t,z(t))}{M}\right]$$

Using the hypothesis $0 < m \le \partial_r f(t,r) \le M$ we arrive at the following result:

$$0 \le 1 - \frac{\partial_\phi f(t,\phi(t))}{M} \le 1 - \frac{m}{M},$$

with which we can write the following inequality:

$$|T\varphi(t) - T\psi(t)| = |\varphi(t) - \psi(t)|\left(1 - \frac{m}{M}\right) \leq \alpha\|\varphi - \psi\|. \tag{13}$$

Where $\alpha = \left(1 - \frac{m}{M}\right)$. Since $0 < m \leq M,$ we have $0 \leq \alpha < 1$. The above inequality is valid for all t of $[a, b]$. Where T is a contraction operator and the proof is complete, since for every contraction operator $T : C \to C$ there exists one and only one continuous function φ in $C,$ such that $T(\varphi) = \varphi$. Using Eq. (10), which represents the fundamental solution of the Navier–Stokes 3D equation, we verify Eq. (2), which represents the second of the six requirements of an acceptable solution. ∎

Proposition 3 *Requirement (3). The initial velocity can be obtained from:* $\mathbf{u}(x, y, z, 0) = -2\nu \frac{\nabla P}{P}$, *where each of the components* u_x, u_y *and* u_z *are infinitely derivable.*

$$\begin{aligned} \mathbf{u}(x, y, z, 0) &= \mathbf{u}^0(x, y, z) = -2\nu\mu(1 - P_0)\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right) \qquad \left((x, y, z) \in \mathbb{R}^3\right) \\ P_0 &= \frac{1}{1 + e^{-\mu r_0}} \end{aligned} \tag{14}$$

Proof. Taking the partial derivatives of $\partial_x^n\left(\frac{x}{r}\right)$, $\partial_y^n\left(\frac{x}{r}\right)$ and $\partial_z^n\left(\frac{x}{r}\right)$.

$$\begin{aligned} \partial_x^n\left(\frac{x}{r}\right) &= n\partial_x^{n-1}\left(\frac{1}{r}\right) + x\partial_x^n\left(\frac{1}{r}\right) \\ \partial_y^n\left(\frac{x}{r}\right) &= n\partial_y^{n-1}\left(\frac{1}{r}\right) + y\partial_y^n\left(\frac{1}{r}\right) \\ \partial_z^n\left(\frac{x}{r}\right) &= n\partial_z^{n-1}\left(\frac{1}{r}\right) + z\partial_z^n\left(\frac{1}{r}\right) \end{aligned} \tag{15}$$

Recalling the derivatives of special functions (Legendre), it is verified that there exists the derivative C^∞.

$$\begin{aligned} \partial_x^n\left(\frac{1}{r}\right) &= (-1)^n n!\left(x^2 + y^2 + z^2\right)^{-\frac{(n+1)}{2}} P_n\left(\frac{x}{(x^2 + y^2 + z^2)^{1/2}}\right) \\ \partial_y^n\left(\frac{1}{r}\right) &= (-1)^n n!\left(x^2 + y^2 + z^2\right)^{-\frac{(n+1)}{2}} P_n\left(\frac{y}{(x^2 + y^2 + z^2)^{1/2}}\right) \\ \partial_z^n\left(\frac{1}{r}\right) &= (-1)^n n!\left(x^2 + y^2 + z^2\right)^{-\frac{(n+1)}{2}} P_n\left(\frac{z}{(x^2 + y^2 + z^2)^{1/2}}\right) \end{aligned} \tag{16}$$

∎

Physically, this solution is valid for the initial velocity, indicated by Eq. (4), where the components of the initial velocity are infinitely differentiable, and make it possible to guarantee that the velocity of the fluid is zero when $r \to \infty$ [6–9].

Proposition 4 *Requirement (4). Using the initial velocity of a moving fluid given by* $\mathbf{u}(x, y, z, 0) = \mathbf{u}^0(x, y, z) = -2\nu\mu(1 - P_0)\left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$, *it is evident that*

$$\left|\partial_x^\alpha \mathbf{u}^0\right| \leq C_{\alpha K}(1 + r)^{-K} \qquad on\ \ \mathbb{R}^3\ \ for any \alpha\ and K$$

Proof. Using the initial velocity of a moving fluid given by $\mathbf{u}^0(x,y,z) = -2\nu\mu(1-P_0)\left(\frac{x}{r},\frac{y}{r},\frac{z}{r}\right)$, we can find each of the components: $\partial_x^\alpha u_x^0$, $\partial_y^\alpha u_y^0$ and $\partial_z^\alpha u_z^0$.

$$\left(\partial_x^\alpha \frac{x}{r}\right)^2 = \left(\alpha\partial_x^{\alpha-1}\left(\frac{1}{r}\right) + x\partial_x^\alpha\left(\frac{1}{r}\right)\right)\left(\alpha\partial_x^{\alpha-1}\left(\frac{1}{r}\right) + x\partial_x^\alpha\left(\frac{1}{r}\right)\right)$$

For the three components x, y, z the results of the partial derivatives are as follows:

$$\begin{aligned}
\left(\partial_x^\alpha \frac{x}{r}\right)^2 &= \alpha^2\left(\partial_x^{\alpha-1}\frac{1}{r}\right)^2 + 2\alpha x\partial_x^{\alpha-1}\frac{1}{r}\partial_x^\alpha\frac{1}{r} + x^2\left(\partial_x^\alpha\frac{1}{r}\right)^2 \\
\left(\partial_y^\alpha \frac{y}{r}\right)^2 &= \alpha^2\left(\partial_y^{\alpha-1}\frac{1}{r}\right)^2 + 2\alpha y\partial_y^{\alpha-1}\frac{1}{r}\partial_y^\alpha\frac{1}{r} + y^2\left(\partial_y^\alpha\frac{1}{r}\right)^2 \qquad (17) \\
\left(\partial_z^\alpha \frac{z}{r}\right)^2 &= \alpha^2\left(\partial_z^{\alpha-1}\frac{1}{r}\right)^2 + 2\alpha z\partial_z^{\alpha-1}\frac{1}{r}\partial_z^\alpha\frac{1}{r} + z^2\left(\partial_z^\alpha\frac{1}{r}\right)^2
\end{aligned}$$

Replacing Eq. (17) with the explanatory form of the Legendre polynomials, for the following terms $\partial_x^{\alpha-1}\frac{1}{r}$ and $\partial_x^\alpha\frac{1}{r}$.

$$\begin{aligned}
\partial_x^\alpha\frac{1}{r} &= (-1)^\alpha\alpha!\left(x^2+y^2+z^2\right)^{-\frac{(\alpha+1)}{2}}P_\alpha\left(\frac{x}{\left(x^2+y^2+z^2\right)^{1/2}}\right) \\
\partial_x^{\alpha-1}\frac{1}{r} &= (-1)^{\alpha-1}(\alpha-1)!\left(x^2+y^2+z^2\right)^{-\frac{\alpha}{2}}P_{\alpha-1}\left(\frac{x}{\left(x^2+y^2+z^2\right)^{1/2}}\right)
\end{aligned} \qquad (18)$$

Also, knowing that for each $\alpha \geq 0$, the maximum value of $P_\alpha(1) = 1$. We can write the following inequality

$$\begin{aligned}
&x^2\left(\partial_x^\alpha\left(\frac{1}{r}\right)\right)^2 \leq x^2(\alpha!)^2 r^{-2(\alpha+1)} \\
&2\alpha x\partial_x^{\alpha-1}\frac{1}{r}\partial_x^\alpha\frac{1}{r} \leq 2x\alpha(\alpha!)(\alpha-1)!(-1)^{2\alpha-1}r^{-2\alpha-1} \qquad (19) \\
&\alpha^2\left(\partial_x^{\alpha-1}\left(\frac{1}{r}\right)\right)^2 \leq \alpha^2((\alpha-1)!)^2 r^{-2\alpha}
\end{aligned}$$

Grouping terms for $\left(\partial_x^\alpha\frac{x}{r}\right)^2$, $\left(\partial_y^\alpha\frac{y}{r}\right)^2$ and $\left(\partial_z^\alpha\frac{z}{r}\right)^2$ we have the next expressions.

$$\begin{aligned}
\left(\partial_x^\alpha\frac{x}{r}\right)^2 &\leq r^{-2\alpha}\left[\frac{x^2(\alpha!)^2}{r^2} - \frac{2x(\alpha!)^2}{r} + \alpha^2((\alpha-1)!)^2\right] \\
\left(\partial_y^\alpha\frac{y}{r}\right)^2 &\leq r^{-2\alpha}\left[\frac{y^2(\alpha!)^2}{r^2} - \frac{2y(\alpha!)^2}{r} + \alpha^2((\alpha-1)!)^2\right] \qquad (20) \\
\left(\partial_z^\alpha\frac{z}{r}\right)^2 &\leq r^{-2\alpha}\left[\frac{z^2(\alpha!)^2}{r^2} - \frac{2z(\alpha!)^2}{r} + \alpha^2((\alpha-1)!)^2\right]
\end{aligned}$$

The module of $\left|\partial_{\mathbf{x}}^{\alpha}\mathbf{u}^0\right|$ is given by $\left|\partial_{\mathbf{x}}^{\alpha}\mathbf{u}^0\right| = \left(\left(\partial_x^{\alpha}\frac{x}{r}\right)^2 + \left(\partial_y^{\alpha}\frac{y}{r}\right)^2 + \left(\partial_z^{\alpha}\frac{z}{r}\right)^2\right)^{1/2}$. Simplifying and placing the terms of Eq. (20) we have

$$\left|\partial_{\mathbf{x}}^{\alpha}\mathbf{u}^0\right| \le r^{-2\alpha}\left[3(\alpha!)^2 + \alpha^2((\alpha-1)!)^2 - \frac{2(x+y+z)(\alpha!)^2}{r}\right]$$

Taking into consideration that $\left|\frac{x}{r}\right| \le 1,\ \left|\frac{y}{r}\right| \le 1,\ \left|\frac{z}{r}\right| \le 1$ the last term $\left|\partial_{\mathbf{x}}^{\alpha}\mathbf{u}^0\right|$ can be easily written that.

$$\left|\partial_{\mathbf{x}}^{\alpha}\mathbf{u}^0\right| \le \frac{2(\alpha!)^2}{r^{2\alpha}}\left[2 + \left|\frac{x}{r}\right| + \left|\frac{y}{r}\right| + \left|\frac{z}{r}\right|\right]$$
$$\left|\partial_{\mathbf{x}}^{\alpha}\mathbf{u}^0\right| \le \frac{10(\alpha!)^2}{r^{2\alpha}}$$

It is verified that there exists $C_\alpha = 10(\alpha!)^2$ such that if $r \to 0,$ then $\left|\partial_{\mathbf{x}}^{\alpha}\mathbf{u}^0\right| \to 0.$ Thus, we proved requirement (4). ■

According to Mathematics, and giving an integral physical structure to the study, we need to prove that there are the spatial and temporal derivatives of the velocity and pressure components, satisfying the requirement (5).

Proposition 5 *Requirement (5). The velocity can be obtained from:* $\mathbf{u}(x,y,z,t) = -2\nu\frac{\nabla P}{P}$ *and each of the components* u_x, u_y *and* u_z *are infinitely derivable.*

$$\begin{aligned} \mathbf{u}(x,y,z,t) &= 2\nu^2\left(\frac{x}{r^2}, \frac{y}{r^2}, \frac{z}{r^2}\right) \qquad \left((x,y,z) \in \mathbb{R}^3\right) \\ P(x,y,z,t) &= \frac{1}{1+e^{\frac{p_0}{2\eta}t-\mu r}} = \frac{2}{\mu r} \end{aligned} \tag{21}$$

Proof. Taking partial derivatives for $\partial_x^n\left(\frac{x}{r^2}\right),\ \partial_y^n\left(\frac{x}{r^2}\right)$ and $\partial_z^n\left(\frac{x}{r^2}\right)$.

$$\begin{aligned} \partial_x^n\left(\frac{x}{r^2}\right) &= n\partial_x^{n-1}\left(\frac{1}{r^2}\right) + x\partial_x^n\left(\frac{1}{r^2}\right) \\ \partial_y^n\left(\frac{x}{r^2}\right) &= n\partial_y^{n-1}\left(\frac{1}{r^2}\right) + y\partial_y^n\left(\frac{1}{r^2}\right) \\ \partial_z^n\left(\frac{x}{r^2}\right) &= n\partial_z^{n-1}\left(\frac{1}{r^2}\right) + z\partial_z^n\left(\frac{1}{r^2}\right) \end{aligned} \tag{22}$$

Recalling the derivatives of special functions, it is verified that the derivative C^∞ exists. These derivatives appear as a function of the Legendre polynomials $P_n(.)$.

$$\begin{aligned} \partial_x^n\left(\frac{1}{r^2}\right) &= (-1)^n n!\left(x^2+y^2+z^2\right)^{-(n+1)}P_n\left(\frac{x}{(x^2+y^2+z^2)}\right) \\ \partial_y^n\left(\frac{1}{r^2}\right) &= (-1)^n n!\left(x^2+y^2+z^2\right)^{-(n+1)}P_n\left(\frac{y}{(x^2+y^2+z^2)}\right) \\ \partial_z^n\left(\frac{1}{r^2}\right) &= (-1)^n n!\left(x^2+y^2+z^2\right)^{-(n+1)}P_n\left(\frac{z}{(x^2+y^2+z^2)}\right) \end{aligned} \tag{23}$$

There are the spatial derivatives n and the time derivative which is similar to Eq. (25). ■

Proposition 6 *Requirement (5). The pressure is totally defined by the equivalence* $p(x,y,z,t) = p_0 P(x,y,z,t)$ *and is infinitely differentiable in each of its components.*

$$p(x,y,z,t) = p_0 P(x,y,z,t) \qquad \left((x,y,z) \in \mathbb{R}^3\right) \tag{24}$$

Proof. Taking partial derivatives for $\partial_x^n\left(\frac{1}{r}\right)$, $\partial_y^n\left(\frac{1}{r}\right)$ and $\partial_z^n\left(\frac{1}{r}\right)$, recalling the derivatives of special functions of Eq. (16), it is shown that the derivative C^∞. We only have to find the time derivatives: $\partial_t^n\left(p_0 P\right) = p_0 \partial_t^n(P)$. Using Eq. (21) for P, we have.

$$\begin{aligned}
\partial_t P &= (-k)P(1-P)\\
\partial_t^2 P &= (-k)^2(1-2P)P(1-P)\\
\partial_t^3 P &= (-k)^3\left(1-6P+6P^2\right)P(1-P)\\
\partial_t^4 P &= (-k)^4\left(1-14P+36P^2-24P^3\right)P(1-P)\\
\partial_t^5 P &= (-k)^5\left(1-30P+150P^2-240P^3+120P^4\right)P(1-P)\\
\partial_t^n(P) &= \partial_t\left(\partial_t^{n-1}(P)\right)
\end{aligned} \tag{25}$$

It is always possible to find the derivative $\partial_t^n(P)$ as a function of the previous derivative, since the resulting polynomial of each derivative $n-1$ is of degree n. ∎

Proposition 7 *Requirement (6). The energy must be limited in a defined volume and fundamentally it must converge at any time, such that* $t \geq 0$.

$$\int_{\mathbb{R}^3} |\mathbf{u}(x,y,z,t)|^2 dxdydz \leq C \quad \textit{for all}\; t \geq 0 \qquad (\textit{bounded energy}).$$

Proof. We will use the explicit form of velocity given in Eq. (21) $\mathbf{u}(x,y,z,t) = 2\nu\mu(1-P)\nabla r$, to obtain the vector module: $|\mathbf{u}|^2 = 4\nu^2\mu^2(1-P)^2$. Rewriting Eq. (21), and applying a change of variable in: $dxdydx = 4\pi r^2 dr$.

$$\int_{\mathbb{R}^3} |\mathbf{u}(x,y,z,t)|^2 dxdydz = 16\pi\nu^2\mu^2 \int_{r_0}^{\infty} r^2(1-P)^2 dr \tag{26}$$

Making another change of variable $dP = \mu P(1-P)dr$. Using (10), replacing $r^2 = \left(\frac{2}{\mu P}\right)^2$ we have

$$\begin{aligned}
\int_{\mathbb{R}^3} |\mathbf{u}(x,y,z,t)|^2 dxdydz &= 16\pi\nu^2\mu^2 \int_{P_0}^{P\infty} \left(\frac{2}{\mu P}\right)^2 (1-P)^2 \frac{dP}{\mu P(1-P)}\\
&= \frac{64\pi\nu^2}{\mu} \int_{P_0}^{P\infty} \frac{1-P}{P^3} dP
\end{aligned} \tag{27}$$

Where radius $r \to \infty$, when $t \geq 0$, we have $\lim_{r\to\infty} P = \lim_{r\to\infty} \frac{1}{1+\frac{exp(kt)}{\exp(\mu r)}} = P_\infty = 1$. Moreover, physically if $r \to r_0 \approx 0$ then $t \to 0$ we have $\lim_{r\to 0} P = \lim_{r\to 0} \frac{1}{1+\frac{exp(kt)}{\exp(\mu r)}} = P_0 = \frac{1}{2}$. Here, a probability $\frac{1}{2}$ represents maximum entropy.

$$\begin{aligned}
\int_{\mathbb{R}^3} |\mathbf{u}(x,y,z,t)|^2 dxdydz &= \frac{64\pi\nu^2}{\mu} \int_{1/2}^{1} \frac{1-P}{P^3} dP = \frac{64\pi\nu^2}{\mu} \left[\frac{2P-1}{2P^2}\right]_{1/2}^{1}\\
\int_{\mathbb{R}^3} |\mathbf{u}|^2 dxdydz &\leq \frac{32\pi\nu^2}{\mu} \qquad \text{forall}\;\; t \geq 0
\end{aligned} \tag{28}$$

In this way the value of the constant C is $C = \frac{32\pi\nu^2}{\mu}$. Verifying the proposition (6) completely. In general, Eq. (10) can be written $f(t, r + r_0) = \frac{1}{1+e^{kt-\mu(r+r_0)}} - \frac{2}{\mu\mu(r+r_0)} = 0$ and in this way discontinuities are avoided when $r \to 0$, but this problem does not occur since in the atomic nucleus $r_0 < r < 1.2A^{1/3}$ is satisfied. ∎

Lemma 8 *The irrotational field represented by the logistic probability function* $P(x, y, z, t)$ *associated with the velocity* $\mathbf{u} = -2\nu\frac{\nabla P}{P}$, *can produce vortices, due to the stochastic behavior of the physical variables* p_0, η, μ. *These stochastic variations are in orders lower than the minimum experimental value.*

Proof. The implicit function representing the solution of the Navier–Stokes 3D equation, $\frac{1}{1+e^{-\mu\left(r-\frac{k}{\mu}t\right)}} = \frac{2}{\mu r}$ depends on the values of initial pressure p_0, viscosity ν and attenuation coefficient μ. Due to Heisenberg uncertainty principle, these parameters have a variation when we measure and use them, as is the case of the estimate of $\xi = r - \phi(t, k, \mu)$. The function $\phi(t, k, \mu) = \frac{k}{\mu}t$, expressly incorporates these results, when $-\infty < \xi < +\infty$. The physical and mathematical realities are mutually conditioned and allow for these surprising results. For a definite t there exist infinities (x, y, z) that hold the relationship $r = \left(x^2 + y^2 + z^2\right)^{1/2}$. Moreover, for a definite r there are infinities t that respect the fixed-point theorem and create spherical trajectories. When the physical variables k, μ vary, even at levels of 1/100 or 1/1000, they remain below the minimum variation of the experimental value. We could try to avoid the existence of trajectories on the spherical surface, for which we must assume that the fluid is at rest or it is stationary, which contradicts the Navier–Stokes 3D equation, where all fluid is in accelerated motion $\frac{\partial \mathbf{u}}{\partial t} \neq 0$. In short, if there are trajectories in the sphere as long as it is probabilistically possible, this is reduced to showing that the expected value of the radius $E[r|r \geq 0]$ exists and it is finite.

Derivation of $E(r|r \geq 0)$.

The logistic density function for ξ when $E(\xi) = 0$ and $Var(\xi) = \sigma^2$ is defined by. $h(\xi) = \frac{\mu \exp(-\xi)}{[1+\exp(-\xi)]^2}$, where $\frac{1}{\mu} = \sigma\sqrt{3}/\pi$ is a scale parameter. Given that $r = \phi(t, p_0, \eta, \mu) + \xi$ function for r is then $f(r) = \frac{\exp[-r-\phi(\bullet)/\tau]}{\tau[1+\exp(-(r-\phi(\bullet))/\tau]^2}$, to facilitate the calculations we put $\phi(\bullet) = \phi(t, k, \mu) = \frac{k}{\mu}t$. By definition, the truncated density for r when $r \geq 0$ is given by $f(r|r \geq 0) = \frac{f(r)}{P(r \geq 0)}$ for $r \geq 0$. Given that the cumulative distribution function for r is given by $F(r) = \frac{1}{1+\exp(kt-\mu r)}$, it follows that $P(r \geq 0) = 1 - F(0) = \frac{\exp(\phi(\bullet))}{1+\exp(\phi(\bullet))} = \frac{1}{1+\exp(-\phi(\bullet))}$. The derivation of $E(r|r \geq 0)$ then proceeds as follows:

$$E(r|r \geq 0) = \int_0^\infty \mu r f(r|r \geq 0) dr = \frac{1}{P(r \geq 0)} \int_0^\infty \mu r \frac{\exp[kt - \mu r]}{\{1 + \exp[kt - \mu r]\}^2} dr$$
$$E(r|r \geq 0) = \frac{1}{P(r \geq 0)} \int_{1/2}^1 \frac{2}{\mu P} (\mu P(1-P)) \frac{dP}{P(1-P)} \tag{29}$$

We replaced in Eq. (29) $dP = \mu P(1-P)dr$ and $r^2 = \left(\frac{2}{\mu P}\right)^2$ of this manner we obtain

$$E(r|r \geq 0) = \frac{1}{P(r \geq 0)} \int_{1/2}^1 \frac{2}{\mu P} (\mu P(1-P)) \frac{dP}{P(1-P)} = \frac{1}{P(r \geq 0)} \frac{2}{\mu} \log(2) \tag{30}$$

where we have used the fact that

$$P(r \geq 0) = \frac{\exp kt}{1 + \exp kt}$$
$$E(r|r \geq 0) = \frac{1}{P(r \geq 0)}\frac{2}{\mu}\log(2) \leq \frac{2}{\mu}\log(2) \quad \text{for all } t \geq 0 \tag{31}$$

where the last equality follows from an application of the L'Hopital's rule $P(r \geq 0) = \lim_{t \to \infty} \frac{\exp kt}{1+\exp kt} = 1.$ ■

3. Results

The main results of applying the Navier Stokes equation to the atomic nucleus, which behaves like an incompressible nuclear fluid, are:

- The nuclear force and the Navier Stokes force are related.
- The Cross Sections in Low energy X ray can explain the Golden Ratio $\left(\frac{\sigma_1}{\sigma_2}\right)$ that appears in the femtoscope.
- Navier Stokes Equation and Cross Section in Nuclear Physics.
- The principle of the femtoscope explains that low energy X-rays produce resonance in layer K.

3.1 The nuclear force and the Navier Stokes force are related

Firstly, we will use the concepts of Classic Mechanics and the formulation of the Yukawa potential, $\Phi(r) = \frac{g}{4\pi r}(A-1)e^{-\mu r}$ to find the nuclear force exerted on each nucleon at interior of the atomic core $\mathbf{F}_N = -\nabla\Phi(r)$. Also, replace the terms of the potential $e^{-\mu r} = \frac{1-P}{P}$ and $\frac{1}{r} = \frac{\mu}{2}P$ by the respective terms already obtained in Eq. (10).

$$\Phi(r) = \frac{g(A-1)}{4\pi r}e^{-\mu r} = \frac{g\mu(A-1)}{8\pi}(1-P) \tag{32}$$

The general form of the Eq. (32), is a function of (x, y, z, t).

$$\Phi(r,t) = \frac{g(A-1)}{4\pi r}e^{kt-\mu r} = \frac{g\mu(A-1)}{8\pi}(1-P(r,t)) \tag{33}$$

Secondly, we will obtain the Navier Stokes force equation given by:

$$\frac{d\mathbf{u}}{dt} = \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = \frac{\partial \mathbf{u}}{\partial t} + 2\nu^2\nabla\left(\frac{|\nabla P|^2}{P^2}\right) = -2\mu\nu k P(1-P)\nabla\mathbf{r} \tag{34}$$

Theorem 9 *The Nuclear Force and Navier Stokes Force are proportional inside the atomic nucleus* $\mathbf{F}_N = C\mathbf{F}_{NS}$.

Proof. Eq. (34) rigorously demonstrated by theorems and propositions 1 through 8, represent the acceleration of a particle within the atomic nucleus. According to Classical Mechanics the force of Navier Stokes applied to a particle of mass m, would have the form:

$$\mathbf{F}_{NS} = m\frac{d\mathbf{u}}{dt} = -2m\mu\nu kP(1-P)\nabla\mathbf{r}. \tag{35}$$

■

Proof. The nuclear force on its part would be calculated as follows $\mathbf{F}_N = -\nabla\Phi(r)$.

$$\mathbf{F}_N = -\nabla\Phi(r) = -\frac{g\mu}{8\pi}(A-1)\nabla P. \tag{36}$$

Replacing the term $\nabla P = \mu\left(P - P^2\right)\nabla r$ of Eq. (7), we obtain

$$\mathbf{F}_N = -\nabla\Phi(r) = -\frac{g\mu^2}{8\pi}(A-1)P(1-P)\nabla\mathbf{r}. \tag{37}$$

It is possible to write nuclear force as a function of speed.

$$\mathbf{F}_N = -\nabla\Phi(r) = -\frac{g\mu^2}{8\pi}(A-1)P(1-P)\nabla\mathbf{r}.$$

Finally, we can show that the nuclear force and force of Navier Stokes differ at most in a constant C. Equating (35) and (37), we find the value g as a function of the parameters nuclear viscosity ν, attenuation μ and growth coefficient of the nuclear reaction k, nucleon mass m and $C \neq 1$.

$$g = \frac{16m\pi\nu k}{\mu(A-1)}C \tag{38}$$

■

3.2 Cross section and Golden ratio $\left(\frac{\sigma_1}{\sigma_2}\right)$ are important elements of the femtoscope

According to NIST and GEANT4 [17], current tabulations of $\frac{\mu}{\rho}$ rely heavily on theoretical values for the total cross section per atom, σ_{tot}, which is related to $\frac{\mu}{\rho}$ by the following equation:

$$\frac{\mu}{\rho} = \frac{\sigma_{tot}}{uA} \tag{39}$$

In (Eq. 39), $u\left(= 1.6605402 \times 10^{-24} gr\right)$ is the atomic mass unit (1/12 of the mass of an atom of the nuclide 12C)[4].

The attenuation coefficient, photon interaction cross sections and related quantities are functions of the photon energy. The total cross section can be written as the sum over contributions from the principal photon interactions

$$\sigma_{tot} = \sigma_{pe} + \sigma_{coh} + \sigma_{incoh} + \sigma_{trip} + \sigma_{ph.n} \tag{40}$$

Where σ_{pe} is the atomic photo effect cross section, σ_{coh} and σ_{incoh} are the coherent (Rayleigh) and the incoherent (Compton) scattering cross sections, respectively, σ_{pair} and σ_{trip} are the cross sections for electron-positron production in the fields of the nucleus and of the atomic electrons, respectively, and $\sigma_{ph.n}$. is the photonuclear cross section 3,4.

We use data of NIST and simulations with GEANT4 for elements $Z = 11$ to $Z = 92$ and photon energies 1.0721×10^{-3} MeV to 1.16×10^{-1} MeV, and have been calculated according to:

$$\frac{\mu}{\rho} = \left(\sigma_{pe} + \sigma_{coh} + \sigma_{incoh} + \sigma_{trip} + \sigma_{ph.n}\right)/\mu A \tag{41}$$

The attenuation coefficient μ of a low energy electron beam $[10, 100]eV$ will essentially have the elastic and inelastic components. It despises Bremsstrahlung emission and Positron annihilation.

$$\sigma_{tot} = \sigma_{coh} + \sigma_{incoh} \tag{42}$$

3.3 The principle of the femtoscope explains that low energy X-rays produce resonance in K-shell

A resonance region is created in a natural way at the K-shell between the nucleus and the electrons at S-level. The condition for the photons to enter in the resonance region is given by $r_a \geq r_n + \lambda$. This resonance region gives us a new way to understand the photoelectric effect. There is experimental evidence of the existence of resonance at K-level due to photoelectric effect, represented by the resonance cross section provided by NIST and calculated with GEANT4 for each atom. In the present work we focus on the resonance effects but not on the origin of resonance region.

The resonance cross section is responsible for large and/or abnormal variations in the absorbed radiation $(I_2 - I_1)$.

$$\frac{I_2 - I_1}{\frac{I + I_1}{2}} = -\frac{\rho r}{\mu A}(\sigma_2 - \sigma_1) \tag{43}$$

Theorem 10 *Resonance region. The resonance cross section is produced by interference between the atomic nucleus and the incoming X-rays inside the resonance region, where the boundaries are the surface of the atomic nucleus and K-shell.*

The cross section of the atomic nucleus is given by:

$$\sigma_{r_n} = 4\pi r_n^2 = 4\pi A^{2/3} r_n^2 \tag{44}$$

The photon cross section at K-shell depends on the wave length and the shape of the atomic nucleus:

$$\sigma_{r_n+\lambda} = 4\pi(r_n + \lambda)^2 \tag{45}$$

Subtracting the cross sections (13) and (14) we have:

$$\sigma_\lambda = \sigma_{r_n+\lambda} - \sigma_{r_n} = 4\pi\left(2r_n\lambda + \lambda^2\right) = 4\pi\left(2r_p\lambda + 2\left(r_n - r_p\right)\lambda + \lambda^2\right) \tag{46}$$

The resonance is produced by interactions between the X-rays, the K-shell electrons and the atomic nucleus. The cross sections corresponding to the nucleus is weighted by probability pn and should have a simple dependence of an interference term. This last depends on the proton radius rp or the difference between the nucleus and proton radius $(r_n - r_p)$ according to the following relation

$$Max(\sigma_2 - \sigma_1) = \left(\frac{\sigma_1}{\sigma_2}\right)^{-b}(\sigma_2 - \sigma_1) = 4\pi\left(2r_p\lambda\right) \tag{47}$$

We note that left hand side of Eqs. (46) and (47) should have a factor larger than one due to resonance. The unique factor that holds this requirement is $\left(\frac{\sigma_1}{\sigma_2}\right)^b$ Where a, b constants.

$$\frac{8\pi\bar{r}\lambda}{(\sigma_2 - \sigma_1)} = a\left(\frac{\sigma_1}{\sigma_2}\right)^b \tag{48}$$

After performing some simulations it is shown that the thermal a represents the dimensionless Rydberg constant $a = R_\infty = 1.0973731568539 * 10^7$.

$$\frac{8000\pi\bar{r}\lambda}{(\sigma_2 - \sigma_1)} = R_\infty\left(\frac{\sigma_1}{\sigma_2}\right)^{2.5031} \tag{49}$$

We use $\sigma_1(Z), \sigma_2(Z)$ for represent cross section in resonance, R_∞ is the generalized Rydberg constant for all elements of periodic table, and Z atomic number.

Last equation with a $R^2 = 0.9935$ was demonstrated and constructed using the elements of the solution of the Navier Stokes equations.

$$\left(\frac{\sigma_1}{\sigma_2}\right) = 0.0021Z + 0.0696$$

This equation was obtained with a $R^2 = 0.9939$, and indicates that the ratio of the effective sections fully explain each element of the periodic table.

$$E^* = 2 * 10^{-5}Z^2 - 0.0003Z + 0.004$$

This equation obtained for a $R^2 = 0.9996$, complements the system of equations that allow to know the simulation values as a function of Z for $\sigma_1(Z), \sigma_2(Z)$ and $E^*(Z)$, where $\sigma_1(Z) < \sigma_2(Z)$. The Femtoscope equations further demonstrate that energy ($E^* = \min E$) is minimum and Shannon entropy ($S^* = \max S$) is maximum in resonance, because in equilibrium $\sigma_1(Z_i) = \sigma_2(Z_i)$.

The radius of the neutron can be obtained using Eq. (49) in the following way.

$$\bar{r} = \frac{R_\infty}{8000\pi a}$$

$$r_N = \frac{A}{N}\bar{r} + \frac{Z}{N}r_p$$

3.4 Navier Stokes equation and cross section in nuclear physics

The speed needs to be defined as $\mathbf{u} = -2\nu\frac{\nabla P}{P}$, where $P(x, y, z, t)$ is the logistic probability function $P(x, y, x, t) = \frac{1}{1+e^{kt-\mu r}}$, $r = (x^2 + y^2 + z^2)^{1/2}$ defined in $\left((x, y, z) \in \mathbb{R}^3, t \geq 0\right)$ This P is the general solution of the Navier Stokes 3D equations, which satisfies the conditions (50) and (51), allowing to analyze the dynamics of an incompressible fluid.

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}.\nabla)\mathbf{u} = -\frac{\nabla p}{\rho_0} \qquad \left((x, y, z) \in \mathbb{R}^3, t \geq 0\right) \tag{50}$$

Where, $\mathbf{u} \in \mathbb{R}^3$ an known velocity vector, ρ_0 constant density of fluid and pressure $p = p_0 P \in \mathbb{R}$.

With speed and pressure dependent on r and t. We will write the condition of incompressibility as follows.

$$\nabla.\mathbf{u} = 0 \qquad \left((x,y,z)\in\mathbb{R}^3, t\geq 0\right) \tag{51}$$

Theorem 11 *The velocity of the fluid given by:* $\mathbf{u} = -2\nu\frac{\nabla P}{P}$, *where* $P(x,y,z,t)$ *is6the logistic probability function* $P(x,y,x,t) = \frac{1}{1+e^{kt-\mu\left(x^2+y^2+z^2\right)^{1/2}}}$, *defined in* $\left((x,y,z)\in\mathbb{R}^3, t\geq 0\right)$ *is the general solution of the Navier Stokes equations, which satisfies conditions (50) and (51).*

Proof. Firstly, we will make the equivalence $\mathbf{u} = \nabla\theta$ and replace it in Eq. (50). Taking into account that $\nabla\theta$ is irrotational, $\nabla\times\nabla\theta = 0$, we have.

$$(\mathbf{u}.\nabla)\mathbf{u} = (\nabla\theta.\nabla)\nabla\theta = \frac{1}{2}\nabla(\nabla\theta.\nabla\theta) - \nabla\theta\times(\nabla\times\nabla\theta) = \frac{1}{2}\nabla(\nabla\theta.\nabla\theta),$$

We can write,

$$\nabla\left(\frac{\partial\theta}{\partial t} + \frac{1}{2}(\nabla\theta.\nabla\theta)\right) = \nabla(-p)$$

It is equivalent to,

$$\frac{\partial\theta}{\partial t} + \frac{1}{2}(\nabla\theta.\nabla\theta) = -\frac{\Delta p}{\rho_0}$$

where Δp is the difference between the actual pressure p and certain reference pressure p_0. Now, replacing $\theta = -2\nu\ln(P)$, Navier Stokes equation becomes.

$$\frac{\partial P}{\partial t} = \frac{\Delta p}{\rho_0}P \tag{52}$$

The external force is zero, so that there is only a constant force F due to the variation of the pressure on a cross section σ. Where σ is the total cross section of all events that occurs in the nuclear surface including: scattering, absorption, or transformation to another species.

$$\begin{aligned} F &= p\sigma_2 = p_0\sigma_1 \\ \Delta p &= p - p_0 = \left(\frac{\sigma_1}{\sigma_2} - 1\right)p_0 = -(1-P)p_0 \end{aligned} \tag{53}$$

putting (52) in (53) we have

$$\frac{\partial P}{\partial t} = -\mu k(1-P)P \tag{54}$$

In order to verify Eq. (51), $\nabla.\mathbf{u} = 0$, we need to obtain $\nabla r = \left(\frac{x}{r}, \frac{y}{r}, \frac{z}{r}\right)$, $\nabla^2 r = \nabla.\nabla r = \frac{\left(y^2+z^2\right)+\left(x^2+z^2\right)+\left(x^2+y^2\right)}{\left(x^2+y^2+z^2\right)^{3/2}} = \frac{2}{r}$.

$$\nabla.u = -2\nu\nabla.\frac{\nabla P}{P} = -2\nu\mu\nabla((1-P)\nabla r) \tag{55}$$

Replacing the respective values for the terms: $\nabla^2 P$ and $|\nabla P|^2$ of Eq. (55). The Laplacian of P can be written as follows.

$$\begin{aligned} \nabla^2 P &= \mu(1-2P)\nabla P.\nabla r + \mu\left(P-P^2\right)\nabla^2 r \\ &= \mu^2(1-2P)\left(P-P^2\right)|\nabla r|^2 + \mu\left(P-P^2\right)\nabla^2 r \\ &= \mu^2(1-2P)\left(P-P^2\right) + \mu\left(P-P^2\right)\frac{2}{r} \end{aligned} \tag{56}$$

Using gradient $\nabla P = \mu\left(P-P^2\right)\nabla r$, modulus $|\nabla P|^2 = \mu^2\left(P-P^2\right)^2|\nabla r|^2$ and $\nabla^2 P$ in (56).

$$\left[\frac{\nabla^2 P}{P} - \frac{|\nabla P|^2}{P^2}\right] = 0 \tag{57}$$

Replacing Eqs. (55) and (56) in (57) we obtain the main result of the Navier Stokes equations, the solution represents a fixed point of an implicit function $f(t,r)$ where $f(t,r) = P - \frac{2}{\mu r} = 0$.

$$P = \frac{1}{1+e^{kt-\mu\left(x^2+y^2+z^2\right)^{1/2}}} = \frac{2}{\mu\left(x^2+y^2+z^2\right)^{1/2}} \qquad \left((x,y,z)\in\mathbb{R}^3, t\geq 0\right) \tag{58}$$

■

An important result of the Navier Stokes 3D equation, applied to the nuclear fluid of an atom, allows us to advance our understanding of nuclear dynamics and nuclear force.

Corollary 12 *The nuclear decay constant k is determined by the nuclear pressure p_0, and the dynamic nuclear viscosity η, as follows: $k = \frac{p_0}{2\eta}$.*

Proof. A nuclear decay is a reaction of degree 1, which is explained by the exponential law $N(t) = N_0 e^{-kt}$. Rewriting Eq. (58) as a function of the initial nuclear pressure p_0 and the nuclear viscosity η we can prove that they are related to the nuclear decay constant, in an intrinsic way and allow to explain dynamic nuclear phenomena.

$$P(x,y,z,t) = \frac{1}{1+e^{\frac{p_0}{2\eta}t-\mu\left(x^2+y^2+z^2\right)^{1/2}}} = \frac{2}{\mu\left(x^2+y^2+z^2\right)^{1/2}} \qquad \left((x,y,z)\in\mathbb{R}^3, t\geq 0\right) \tag{59}$$

■

3.5 Model of nuclear pressure

The work in the nuclear fluid is given by the variation of the energy necessary to form that atomic nucleus, that is, by the excess mass required in the process. Using Eq. (59) we can find the explicit form of nuclear pressure $p = p_0 P = p_0 \frac{2}{\mu r}$.

$$\begin{aligned} W &= \Delta E_{mass-excess}(Z,A) = \int_0^{r_0}\int_0^{\pi}\int_0^{2\pi} p dV = \int_0^{r_0}\int_0^{\pi}\int_0^{2\pi} p_0 \frac{2}{\mu r}\left(r^2\cos\theta dr d\theta d\phi\right) \\ W &= \Delta E_{mass-excess}(Z,A) = p_0 \frac{2}{\mu}\int_0^{r_0} r dr \int_0^{\pi}\sin\theta d\theta \int_0^{2\pi} d\phi = 4\pi p_0 \frac{r_0^2}{\mu} \end{aligned} \tag{60}$$

Applying the mean value theorem of integrals, we know that there is a mean value of the nuclear pressure $\overline{p}$ and the volume of the atomic nucleus $\overline{V}$ of the integral (60)

$$W = \Delta E_{mass-excess}(Z, A) = \int_0^{r_0}\int_0^{\pi}\int_0^{2\pi} pdV = p(\zeta)V(r_0) \tag{61}$$

that according to Quantum Mechanics, $p(\zeta), V(r_0)$ are the observable values. By this restriction we can equalize Eq. (60), (61) and find the value of the initial nuclear pressure as a function of the experimentally measured nuclear pressure: $p(\zeta)\frac{4\pi}{3}r_0^3 = 4\pi p_0 \frac{r_0^2}{\mu}$ with $r_0 = 1.2A^{1/2}fm,$ from where:

$$p(\zeta) = \frac{\Delta E_{mass-excess}(Z, A)}{\frac{4}{3}\pi r_0^3} = \frac{3p_0}{\mu r_0}. \tag{62}$$

For Yukawa's potential, it is often assumed $\mu \approx \frac{1}{r_0}$.

3.6 Model of nuclear viscosity

Using the fundamental expression $k = \frac{p_0}{2\eta} = \frac{1}{T_{1/2}}$ obtained from the resolution of the Navier Stokes 3D equations, Eqs. (58) and (59), it is possible to find the average or most probable values of the variables involved pressure, nuclear viscosity and nuclear decay constant as follows: $\overline{k} = \frac{\overline{p}}{2\overline{\eta}}$.

Explicitly we find the nuclear viscosity as a function of the nuclear decay constant $\left(k = \overline{k}\right)$ and the average value of the nuclear pressure $\overline{p}$ as follows:

$$\overline{\eta} = \frac{\overline{p}}{2\overline{k}} \tag{63}$$

There is another experimental way of determining nuclear viscosity, through the fuzziness of alpha particles, protons or neutrons ejected in a nuclear decay using the fluid velocity equation $\mathbf{u} = -2\nu\mu(1-P)\nabla r,$ in which modulo $\mathbf{u} = -2\nu\mu(1-P)\nabla r,$ replacing the dynamic viscosity $\nu = \frac{\eta}{\rho_0}\left[\frac{m^2}{s}\right],$ we obtain: $|\mathbf{u}| = 2\left(\frac{\eta}{\rho_0}\right)\mu(1-P)$.

So the second way to find the nuclear viscosity has the form:

$$\eta = \frac{|\mathbf{u}|\rho_0}{2\mu(1-P)} = \frac{|\mathbf{u}|\rho_0 rP}{4(1-P)} \tag{64}$$

Eq. (64) has a more complicated form and depends on the speed of the fluid particles and the radius from which these particles leave.

3.7 Calculation of nuclear pressure and viscosity

The nuclear decay constant k is determined by the nuclear pressure p_0, and the dynamic nuclear viscosity η, as follows: $k = \frac{p_0}{2\eta}$.

4. Discussion of results

4.1 Calculation of the pressure relation of proton and neutron

The fine-structure constant, α, has several physical interpretations, we use the most known.

The fine-structure constant, $\alpha = \frac{1}{137.035999174(35)}$ is the ratio of two energies: the energy needed to overcome the electrostatic repulsion between two protons a distance of $2r$ apart, and (ii) the energy of a single photon of wavelength $\lambda = 2\pi r$ (or of angular wavelength r; according to Planck relation).

$$\alpha' = \frac{e^2}{4\pi\varepsilon_0(2r)} \Big/ \frac{hc}{\lambda} = \frac{e^2}{4\pi\varepsilon_0(2r)} \frac{2\pi r}{hc} = \frac{e^2}{4\pi\varepsilon_0(2r)} \frac{r}{\hbar c} = \frac{1}{2} \frac{e^2}{4\pi\varepsilon_0 \hbar c} = \frac{1}{2}\alpha \tag{65}$$

We can find the relationship of energies between the proton and the atomic nucleus. Knowing that the two occupy the same nuclear volume V. This relationship is identical to $\frac{1}{\alpha'}$, since the atomic nucleus interacts with the proton through the electromagnetic field and the nuclear force.

$$\frac{p_p}{p_N} = \frac{p_p V}{p_N V} = \frac{2}{\alpha} = 274.07 \tag{66}$$

Thus, we already know the pressure relation between the proton and the atomic nucleus $\frac{p_p}{p_N} = \frac{2}{\alpha}$.

Now we find the relation of pressures between the neutron and the atomic nucleus, which are under the action of the same nuclear force, F.

$$\frac{p_n}{p_N} = \frac{F/\pi(0.84184)^2}{F/\pi\left(1.2A^{1/3}\right)^2} = \frac{\left(1.2A^{1/3}\right)^2}{(0.84184)^2} \tag{67}$$

For the chemical element with maximum nuclear pressure, ${}^{62}_{28}Ni$ we have $\frac{p_n}{p_N} = \frac{\left(1.2(62)^{1/3}\right)^2}{(0.84184)^2} = 31.830$.

If we divide Eq. (67) for Eq. (66) we have.

$$\frac{p_p}{p_n} = \frac{p_p}{p_N} \Big/ \frac{p_n}{p_N} = 274.07/31.830 = 8.6104$$

4.2 An action on the nuclear surface produces a reaction in the nuclear volume and vice versa

It is created by the friction between the layers of nucleons.

Theorem 13 *An action on the nuclear surface produces a reaction in the nuclear volume and vice versa.*

Proof. The volume and the nuclear surface are connected through the Gaussian divergence theorem and the Navier Stokes equations.

For an incompressible fluid, whose velocity field $\overleftarrow{u}(x,y,z)$ is given, $\overleftarrow{\nabla}.\overleftarrow{u} = 0$ is fulfilled.

Logically, the integral of this term remains zero, that is:

$$\iiint \overleftarrow{\nabla}.\overleftarrow{u} dxdydz = 0$$

Writing the Divergence theorem. $\int\int \overleftarrow{u}.\overleftarrow{n} dS = \int\int\int \overleftarrow{\nabla}.\overleftarrow{u} dxdydz = 0,$ the first term must be equal to zero, that is:

$$\iint \overleftarrow{u}.\overleftarrow{n}dS = \iint \left\|\overleftarrow{u}\right\|\left\|\overleftarrow{n}\right\| \cos{(\alpha)}dS = 0 \rightarrow \alpha = \frac{\pi}{2}$$

The only possible trajectory is circular, because in this case the vector $\overleftarrow{n}$ is perpendicular to the surface of the sphere. In this way the equation of the outer sphere corresponding to the surface is: $x^2 + y^2 + z^2 = 1.2A^{1/3}$.

Within the nuclear fluid there are layers of nucleons that move in spherical trajectories. ■

5. Conclusions

High energy physics is the guide to low energy physics, because, in certain processes such as in the measurement of the internal pressure of protons and neutrons. However, we demonstrate that there is a trend compatibility of the two in the characterization of the atomic nucleus.

The Femtomathematics corresponds to the tools and the logic that allows to calculate the parameters of the order of $10^{-15}m$. In an indirect way, it is in full correspondence with Femtophysics.

The nuclear viscosity at equilibrium is much larger than the nuclear viscosity at the moment of nuclear decay, which is totally logical, because at equilibrium the atomic nucleus is totally compact in pressure and density, while at the moment protons, neutrons, alpha particles come out of the atomic nucleus from a nuclear decay, indicating that the nuclear viscosity decreased.

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Conflicts of interest

The author declare no conflict of interest.

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