Chapter

Adjustment of the PID Gains Vector Due to Parametric Variations in the Plant Model in Terms of Internal Product

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Abstract

The tuning of the gains of a controller with proportional-integral-derivative (PID) actions has been prevalent in the industry. The adjustment of these gains in PID controllers is often determined by classical methods, such as Ziegler-Nichols, and trial and error. However, these methods fail to deliver satisfactory performance and often do not meet specific project demands because of the inherent complexity of industrial processes, such as plant parameter variations. To solve the tuning problem in highly complex industrial processes, a controller adjustment method based on the internal product of PID terms is proposed, and a propagation matrix (PM) is generated by the numerator coefficients of the plant transfer function (TF). In the proposed method, each term of the PID controller is influenced by each of the numerator and the denominator coefficients. Mathematical models of practical plants, such as unloading and resumption of bulk solids by car dumpers and bucket wheel resumption, were employed to evaluate the proposed method. The obtained results demonstrated an assertive improvement in the adjustment gains from PID actions, thereby validating it as a promising alternative to conventional methods.

Keywords: parameter variations, industrial processes, internal product, propagation matrix, PID actions

1. Introduction

The tuning of the parameters of PID controllers is challenging and requires expertise to achieve superior performance [1]. PID controllers are extensively used in the industries. However, the controllers are often implemented without a derivative action because of the highly sensitive tuning of parameters, which affects the efficiency of the controller [2]. This study presents a methodology for tuning three terms of the PID controller simultaneously to ensure overall efficiency of the controller [3].

The advantage of the PID controller tuning methodology, which is based on the internal product of the PID terms that generates the propagation matrix (PM), is that a vector of the specified parameters of a characteristic polynomial can be projected, and an error vector is obtained on comparison with the parameters of the characteristic plant polynomial [4, 5]. The method minimizes the error from the specified parameters, thereby facilitating the design of a high-performance PID controller. The method also enables the allocation of the poles by direct replacement using specified parameters, thereby ensuring the desired operating point of the control system.

This chapter presents a formulation proposal to resolve the PID controller tuning problem. The proposal is based on the dot product of the gain vector parameters of the controller and the rows of the propagation matrix. The dot product represents the changes in the behavior of the plant that are determined by the parametric variations in the coefficients of the TF polynomial characteristic.

The following topics and proposal development are presented in the remainder of this chapter. In Section 2, a preliminary on the transfer functions of the plant and PID controller are presented. In terms of internal product, the main properties of PID controllers and the development of proposed method are presented in Section 3. Taking into account three industrial plants of the mining sector, computational evaluation experiments of the PID tuning proposal are presented in Section 4. Finally, the conclusion of the work is presented in Section 5.

2. Preliminaries

Adjusting the PID controller gain parameters is not a trivial task and requires in-depth knowledge from the experts. In this work, the problem of tuning PID controllers is based on original studies regarding polynomial compensators and problems strongly related to the specification of parameters to meet operational constraints in plant dynamics, presented as a particular form of compensators in the s domain [6, 7].

2.1 Mathematical model of the plant in terms of transfer function

The plant's dynamic system is represented by ordinary differential equations (ODE), described by TFs in the s domain (Laplace transform). The ODE concept in terms of TFs established in this chapter is in accordance with the block diagram shown in **Figure 1**, where the closed loop system relates the input and output signals: $R(s)$ is reference input, $W(s)$ is disturbance signal and $V(s)$ is noise signal, $Y(s)$ is plant output and $Y_m(s)$ is plant output measured by the sensor, U(s) and E(s) are the control effort and closed loop error, respectively, that are internal control system performance variables.

Applying the Laplace transforms to the control elements of **Figure 1**, the generalized TF with a polynomial structure in the s domain is obtained that is given by

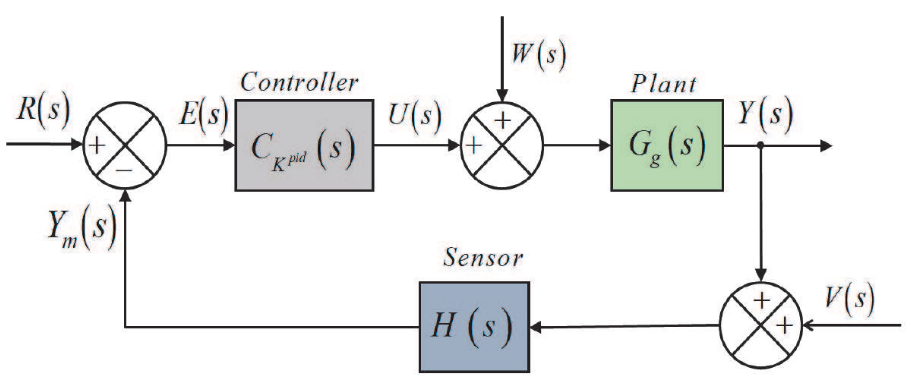

Figure 1.
Canonical block diagram of the closed-loop control system.

$$G_g(s) = \frac{b_o s^m + b_1 s^{m-1} + \ldots + b_{m-1} s + b_m}{a_o s^n + a_1 s^{n-1} + \ldots + a_{n-1} + a_n}, \tag{1}$$

where $G_g(s)$ is the general TF of the control system blocks diagram, related to **Figure 1**, n is the order of the plant model and the number of poles that are entered into the system and m is the number of zeros, which is associated with the PID actions of the controller. As an imposition of the controller gains values, the coefficients a_i and b_k are the adjustable parameters to compensate for the parametric variations of the plant. In the proposed formulation, the b_k coefficients are kept constant and adjustment are made only to the a_i parameters.

2.2 PID controller model

The controller model associated with the TF given in Eq. (1) is customized to perform the actions of the controller's PID terms, where $n = 1$, $m = 2$, $a_0 = 1$, the PID actions are represented by the transfer function that is given by

$$C_{K^{pid}}(s) = \frac{K_D s^2 + K_P s + K_I}{s}, \tag{2}$$

where $C_{K^{pid}}(s)$ is the controller model associated with the TF given in Eq. (1).

Adjustments of parameters that meet the project specifications, can be found in a large number of scientific and technical publications in controle specialized books, conferences and high quality journals [8]. The importance of developing methods for adjusting parameters of PID controllers and systematizing applications in industrial processes of real-world plants, has the objective of meeting the project specifications contained in technological advances, in order to guarantee the optimal adjustment of the parameters of the PID term of the controller [9, 10]. The challenge of tuning with optimal performance of the parameters of a PID controller, started around 1920 and continues to the present days [11–13].

The parameters of the PID controllers are adjusted to adapt to the tuning needs in a combination of proportionality associated with the proportional action, lead associated with the derivative action, and delay associated with the integral action of the error signal. However, there are still many problems that can be solved with computational intelligence-based algorithms. The purpose of this work is to contribute with a method of tuning PID controllers, which can support the development of electronic devices that contribute to technological advancement and the evolution of industry 4.0 with logical planning units, for optimal, robust decision-making and adaptability [14, 15]. Such units must be based on digital control technologies and embedded systems [16] in real time [17], to be reliably deployed in real-world systems [18].

To meet the demands of design specitifications, the proposed solution contributes to the evolution in approaches of optimal and adaptive control, providing the optimization of the figures of merit [19], ensuring a solution with satisfactory performance, meeting the requirements specified in projects, in a way that minimizes efforts of computational cost and control.

TF is specified in the factored form, that is, by the roots of the numerator and denominator polynomials associated with Eq. (1). TF in the factored form is represented in terms of product, where the designer inserts the specified or desired parameters. TF in the form of a product is given by

$$G_g(s) = K \frac{\prod_{k=1}^{m} (s - s_{zk})}{\prod_{i=1}^{n} (s - s_{pi})}, \tag{3}$$

where $s_{pi} = \omega_{di}$ are the poles and $s_{zk} = \sigma_k \omega_{dk}$ are the zeros of the dynamic system. The poles and zeros of the system are represented by the pair (ζ, ω_n), the first component is the damping factor and the second is the undamped natural frequency, $\sigma_i = \zeta_i \omega_{ni}$, and $\omega_{di} = \omega_{ni}\sqrt{1-\zeta_i^2}$.

3. PID adjustment problem

The PID controller adjustment problem is formulated based on the parametric difference between the specified coefficients and the original coefficients of the TF denominator polynomial. This formulation is based on the references [20, 21]. Where the authors present the development of models for optimized online optimization that is based on computational intelligence approaches.

The formulation of the proposed PID controller adjustment problem is presented in this section and is illustrated by the block diagram of **Figure 1**. Thus, in the context of the proposal, the performance matrix of the control system provides the means to determine the values of the parameters of the gain vector of the PID controller K^{pid}.

The models are represented in the form of internal produc <,>, which is a notation widely used in this text as the product of two vectors (internal product) and the models are called internal product models of the plant. The internal product is the appropriate form for analysis, allowing the designer to observe the impact of the earnings vector parameters K^{pid} (K_D, K_P and K_I), in the output of the dynamic system associated with the polynomial coefficients of the TF denominator.

The PID controller model, in terms of the ODE equations and the Laplace transform, obtains the TF of the PID controller in terms of the dot product, which is given by

$$C_{K^{pid}}(s) = \frac{\langle K^{pid}, s^{pid} \rangle}{s}, \tag{4}$$

$$K^{pid} = [K_D \; K_P \; K_I]^T, \tag{5}$$

and s^{pid} is vector-powered in s of the PID gains associated with transfer function numerator that is given by

$$s^{pid} = \left[s^2 \; s^1 \; s^0\right]^T. \tag{6}$$

3.1 PID model in the form of internal product

Inserting the characteristics of the plant, through the values of the coefficients of the polynomials of the numerator and the denominator (poles and zeros), associated with the mathematical models in terms of TFs given in Eq. (1) in internal product form $\langle \cdot \rangle$ and in Eq. (3) in generic form, which is given by

$$G_p(s) = K\frac{\langle b_k, s^m \rangle}{s^n + \langle a_i, s^{n-1} \rangle}, \tag{7}$$

where $G_p(s)$ is the TF in the form of internal product and the coefficients a_i with $i = 1, \ldots, n$ and b_k with $k = 0, \ldots, m$ are a combination of the s_{pi} poles and s_{zk} zeros.

3.2 Open-loop transfer function

The open-loop FT or direct branch of the control system is given by

$$G_p^{OL}(s) = K\frac{\langle K^{pid}, s^{pid}\rangle\langle b_k, s^m\rangle}{s^{n+1} + s\langle a_i, s^{n-1}\rangle}, \tag{8}$$

where $G_p^{OL}(s)$ is the TF of the plant in the open loop and K the gain of the plant and $n > m$.

The structure of TF is determined by the relationship $n + 1 \geq 2 + m$. For $n + 1 = 2 + m$, the system is proper and for $n + 1 < 2 + m$ the system is strictly proper, thereby establishing a general relationship between the order of the PID controller and the order of plant dynamics. This relationship ensures that the system structure is adequate, not allowing the system to present a nonpractical structure. In this way, it establishes that the relationship of the closed loop system is given by

$$n_{cl} = m^{PID} + n, \tag{9}$$

where m^{PID} can only assume 0 (zero) or 1 (one) values. The PID is observed to impose a proper TF, if the closed-loop system is of order $n + 1$ and the numerator is of order $\left(m^{PID} + m\right)$.

According to the block diagram of **Figure 1**, the TFs $Y(s)/R(s)$, $W(s) = 0$, and $V(s) = 0$ are given by

$$G_p^{CL}(s) = \frac{C_{K^{pid}}(s)G(s)}{1 + C_{K^{pid}}(s)G(s)H(s)}. \tag{10}$$

3.3 Propagation of PID terms x b_k coefficients

The development of the polynomials of the numerator (zeros) and the denominator (poles) consists of the propagation of the gain vector K^{pid} of the controller by the numerator coefficients (b_k) associated with the coefficients of the denominated (a_i) TF of the plant. The equationing of the problem is given in the form of an internal product that weights the coefficients of the polynomial of zeros in the closed-loop and additive to the dynamics of the closed-loop transfer function.

3.3.1 Polynomial of zeros

When replacing Eqs. (1) and (2) in Eq. (10), the numerator polynomial of the closed-loop TF is obtained, which is given by

$$N^{CL}(s) = C_{K^{pid}}(s)G(s). \tag{11}$$

Expanding and ordering Eq. (11), one obtains

$$\begin{aligned} N^{CL}(s) = \; & (b_0K_D + b_{-1}K_P + b_{-2}K_I)s^{m+m^{pid}} \\ & +(b_1K_D + b_0K_P + b_{-1}K_I)s^{m+m^{pid}-1} \\ & +(b_2K_D + b_1K_P + b_0K_I)s^m \\ & +(b_3K_D + b_2K_P + b_1K_I)s^{m-1} \end{aligned}. \tag{12}$$

$$\begin{aligned} &\dots\dots\dots\dots\dots\dots\dots \\ &+(b_m K_D + b_{m-1} K_P + b_{m-2} K_I) s^2 \\ &+(b_{m+1} K_D + b_m K_P + b_{m-1} K_I) s^1 \; . \\ &+(b_{m+2} K_D + b_{m+1} K_P + b_m K_I) s^0 \end{aligned}$$

In terms of inner product, the general polynomial form of the closed-loop numerator polynomial is given by

$$N^{CL}(s) = \sum_{i=0}^{m_{cl}} \langle K^{pid}, \overline{b}_{k-1} \rangle s^{m_{cl}-i}, \tag{13}$$

where $m_{cl} = m + m^{pid}$ and vector $\overline{b}_k$ of the polynomial of zeros of the closed-loop system is given by

$$\overline{b}_k = [b_k \quad b_{k-1} \quad b_{k-2}]. \tag{14}$$

In similar way Eq. (13), one obtains the closed-loop denominator polynomial is given by

$$\begin{aligned} D^{CL}(s) &= s^{n+n_{D-pid}} \\ &+ \sum_{i=0}^{n+n_{D-pid}-1} (b_k K_D + b_{k-1} K_P + b_{k-2} K_I + a_{i+1}), \end{aligned} \tag{15}$$

where $n_{D-pid} = 0$ or 1. When $n_{D-pid} = 0$, the PID controller structure have the terms derivative and proportional. When $n_{D-pid} = 1$, the structure of the PID controller has an integrator term that increases the order of the system by 1, starting with the three terms: proportional, derivative and integrative [21]. In this work, when $n_{D-pid} = 0$, the proposal is to specify an additional a_i^s coefficient., to ensure that the PID controller has the three terms.

3.3.2 Characteristic polynomial

The general form of the closed-loop denominator polynomial is given by

$$P^{CL}(s) = s^{n_{cl}} + \sum_{i=1}^{n_{cl}-1} \left(a_i + \langle K^{pid}, \overline{b}_{k-1} \rangle\right) s^{n_{cl}-i}, \tag{16}$$

where $P^{CL}(s)$ is the general form of the characteristic polynomial of the closed-loop plant and n_{cl} is the order of referred polynomial.

the characteristic closed-loop polynomial for unit feedback ($H(s) = 1$) is obtained in a similar way and given by

$$P_p^{CL}(s) = 1 + C_{K^{pid}}(s) G(s). \tag{17}$$

$P_p^{CL}(s)$ is the characteristic polynomial of the closed loop. The representation of the problem in the form of an internal product that relates the coefficients of the zero polynomial with the coefficients of the closed plant dynamics is the basis for obtaining the characteristic closed-loop polynomial.

3.4 Proposed method

The problem is formulated based on the propagation matrix generated from the dot product between the terms of the earnings vector K^{pid} with the coefficients of the numerator b_k associated with the coefficients of the denominator a_i of the TF of plant. The propagation matrix product associated with the TF numerator coefficients of the plant, give rise to a new characteristic polynomial based on new specified operating points, which are imposed by new zeros and new poles.

3.4.1 Propagation matrix of PID design

The design is based on the propagation matrix, allowing the designer to specify new points of operation that improve the performance of the controller acting on the plant dynamics, where changes in the order and coefficients of the characteristic polynomial can be observed through the internal product of the zero and gain coefficients of PID controllers.

The problem is formulated based on the propagation matrix $\overline{B}$, which is a consequence of the interaction between the parameters of the gain vector K^{pid} (K_D, K_P, K_I) with the coefficients of the plant TF numerator, this matrix is represented by

$$\overline{B} = \begin{bmatrix} b_0 & 0 & 0 \\ b_1 & b_0 & 0 \\ b_2 & b_1 & b_0 \\ b_3 & b_2 & b_1 \\ \cdots & \cdots & \cdots \\ b_m & b_{m-1} & b_{m-2} \\ 0 & b_m & b_{m-1} \\ 0 & 0 & b_m \end{bmatrix}. \tag{18}$$

One case notice in [20] that the diagonals are not repeated, and they vary according to the order n of the system's characteristic polynomial. The propagation of the gains is weighted by the coefficients of the numerator polynomial. The closed-loop TF of the plant is given in terms of the product of the gains K_D, K_P, and K_I with the coefficients b_k, b_{k-1}, and b_{k-2} of the closed-loop TF.

The law of formation of the propagation Matrix (18) is ruled by $m+2$ rows and 3 columns. The rows represent the order of the system, starting with the propagation in the dynamics of order $s^{n_{cl}}$ and ending in the dynamics of order s^0 zero. The columns represent the gains of the controller in the poles and zeros of the plant dynamics.

3.4.2 Proposed characteristic polynomial

The proposed characteristic polynomial based on propagation matrix of PID controller gains idea is presented. From the system of equations that represents the actions of the PID controller in the plant dynamics, the formulation of the adjustment problem is established from the perspective of the inner product of the gains and the coefficients of the polynomials of the zeros of the closed-loop TF. In the case of the characteristic polynomial, the inner product is added to its coefficients. This way, the mechanism of gain adjustment is represented for allocations of zeros or poles.

From Eq. (17), the equation system that has an unknown vector K^{pid} and design specifications a_i^s, $i = 1,\ 2, \ldots, n+1$ is assembled. In scalar form, this system of equations is represented by

$$a_i + \left\langle K^{pid}, \overline{b}_k \right\rangle = a_i^s \Rightarrow \left\langle K^{pid}, \overline{b}_k \right\rangle = a_i^s - a_i \Rightarrow \left\langle K^{pid}, \overline{b}_k \right\rangle = a_i^e. \tag{19}$$

where $\overline{b}_k$ vector is assembled with the rows of the $\overline{B}$ matrix.

Expanding the scalar representation of Eq. (19), the system of equations to be solved is given by

$$\left\langle K^{pid}, \overline{b}_k \right\rangle = a_i^e \Rightarrow \begin{cases} K_D b_0 & + & K_P 0 & + & K_I 0 & = & a_1^e \\ K_D b_1 & + & K_P b_0 & + & K_I 0 & = & a_2^e \\ K_D b_2 & + & K_P b_1 & + & K_I b_0 & = & a_3^e \\ K_D b_m & + & K_P b_{m-1} & + & K_I b_{m-2} & = & a_4^e \\ K_D b_0 & + & K_P b_m & + & K_I b_{m-1} & = & a_5^e \\ \vdots & + & \vdots & + & \vdots & = & \vdots \\ K_D 0 & + & K_P 0 & + & K_I b_0 & = & a_n^e \end{cases} \tag{20}$$

The formulation of the problem presented in Eq. (19) and expanded in Eq. (20) is the starting point for the development of forms of parametric variation problems of TFs, as well as, for the establishment of operational points.

To determine the numerical values of the parameters K^{pid}, the following rules are presented: rule-1) the $\overline{B}$ matrix is assembled via Eq. (18), where b_k is the coefficients of TF numerator polynomial; rule-2) the new a_i^s parameters of the characteristic polynomial are specified; rule-3) the dot product of the parameters of the gain vector K^{pid} is made with the rows of the matrix $\overline{B}$, associated with the original a_i parameters of the characteristic polynomial and with the specified a_i^s parameters; rule-4) the system of equations given in Eq. (20) and rule-5 the system of equations given in rule-4 (Eq. (20)) is solved to determine the numerical values of the parameters of the gains vector K^{pid}.

4. Experiments

The experimental results are evaluated in three plants with mathematical models in terms of TF obtained with real data, being: Plant I of second order, with a zero; Third order plan II, with two zeros and fourth order plant III, with three zeros.

4.1 Plant I

Plant I, is a car dumper, which is used to unload solids in bulk, this equipment has the capacity to move up to 4,000 tons per hour (t/h). The general mathematical model of Plant I in TF is given by

$$G_{P_I}^G(s) = \frac{b_0}{s^2 + a_1 s + a_2}, \tag{21}$$

where, $G_{P_I}^G(s)$ is the TF of Plant I, it is a second order plant with zero at infinity.

The product of the TF numerator of Plant I given in Eq. (21) associated with the TF numerator of the controller given in Eq. (2) is given by

$$\left[C_{P_I}^{K^{pid}}(s)G_{P_I}^{G}(s)\right]_N = \left(K_D s^2 + K_P s + K_I\right)b_0 = K_D b_0 s^2 + K_P b_0 s + K_I b_0. \tag{22}$$

The product of the TF denominator of Plant I given in Eq. (21) associated with the TF denominator of the controller given in Eq. (2) is given by

$$\left[C_{P_I}^{K^{pid}}(s)G_{P_I}(s)\right]_D = s\left(s^2 + a_1 s^2 + a_2 s\right) = s^3 + a_1 s^2 + a_2 s. \tag{23}$$

The characteristic polynomial of Plant I is given by

$$P_I(s) = s^3 + (a_1 + K_D b_0)s^2 + (a_2 + K_P b_0)s + K_I b_0. \tag{24}$$

System equations of Plant I related to Eq. (19) in the form $Ax = b$ is given by

$$\left\langle K^{pid}, \overline{b}_i \right\rangle = a_i^e \Rightarrow \begin{cases} a_1 + K_D b_0 = a_1^s; \\ a_2 + K_D b_0 = a_2^s; \\ a_3 + K_D b_0 = a_3^s; \end{cases}$$
$$\Rightarrow \begin{cases} K_D b_0 = a_1^s - a_1; \\ K_P b_0 = a_2^s - a_2; \\ K_I b_0 = a_3^s - a_3; \end{cases} \tag{25}$$
$$\Rightarrow \begin{cases} 1)\ K_D b_0 = a_1^e; \\ 2)\ K_P b_0 = a_2^e; \\ 3)\ K_I b_0 = a_3^e. \end{cases}$$

Placing the systems of equations given in (25) in matrix form, we have

$$\left\langle K^{pid}, \overline{B} \right\rangle = a_i^e \Rightarrow \begin{bmatrix} b_0 & 0 & 0 \\ 0 & b_0 & 0 \\ 0 & 0 & b_0 \end{bmatrix} \times \begin{bmatrix} K_D \\ K_P \\ K_I \end{bmatrix} = \begin{bmatrix} a_1^e \\ a_2^e \\ a_3^e \end{bmatrix}. \tag{26}$$

The transfer function of Plant I related to Eq. (21) is given by

$$G_{P_I}(s) = \frac{0.438}{1s^2 + 0.0861s + 0.0421s}. \tag{27}$$

The a_i coefficients of the transfer function of the Plant - I related to Eq. (27) are given by

$$a_i \Rightarrow \begin{cases} a_1 = 0.0861; \\ a_2 = 0.0421; \\ a_3 = 0. \end{cases} \tag{28}$$

The specified coefficients a_i^s of Plant I are given by

$$a_i^s \Rightarrow \begin{cases} a_1^s = 0.8604; \\ a_2^s = 0.421; \\ a_3^s = 0.0641. \end{cases} \tag{29}$$

The error calculation a_i^e for Plant I associated with the coefficients a_i^s given in (29) and with the coefficients a_i given in (28) is given by

$$a_i^e \Rightarrow \begin{cases} a_1^s - a_1 = a_1^e = 0.8604 - 0.0861 = 0.7743; \\ a_2^s - a_2 = a_2^e = 0.421 - 0.0421 = 0.3789; \\ a_3^s - a_3 = a_3^e = 0.0641 - 0 = 0.0641. \end{cases} \tag{30}$$

The calculation of the K^{pid} gains vector of Plant I, is done by replacing the numerical values of Eq. (27) in the system of equations given in (25) and (26), is given by.

$$\begin{bmatrix} 0.438 & 0 & 0 \\ 0 & 0.438 & 0 \\ 0 & 0 & 0.438 \end{bmatrix} \times \begin{bmatrix} K_D \\ K_P \\ K_I \end{bmatrix} = \begin{bmatrix} 0.7743 \\ 0.3789 \\ 0.0641 \end{bmatrix}. \tag{31}$$

The Plant I given in Eq. (21) related to Eq. (21) has only the coefficient b_0, with that, the system of equations generated, related to the system of equations given in (25), has three equations and three unknowns.

$$K^{pid} \Rightarrow \begin{cases} i)\ 0.438K_D = 0.7749 \Rightarrow K_D = 0.7749/0.431 \Rightarrow K_D = 1.78; \\ ii)\ 0.438K_P = 0.3789 \Rightarrow K_P = 0.3789/0.431 \Rightarrow K_P = 0.87; \\ iii)\ 0.438K_I = 0.0641 \Rightarrow K_I = 0.0641/0.438 \Rightarrow K_I = 0.15. \end{cases} \tag{32}$$

Solving the system of equations given in (32), you can start with any of the equations to find the numerical values of K_D, K_P and K_I, since they are independent. With the numerical values of the earnings K_D, K_P and K_I, it replaces in the simulator developed in the MATLAB/SIMULINK software to monitor the performance of the proposed method.

Figure 2 shows the performance of the PID-Specified controller, which has the transfer function parameters specified by the designer and the $K_{Specified}^{pid}$ gain vector determined by the internal product of the vector of gains with the propagation matrix in purchase with the controller with the gains determined by the second method of ZN.

4.2 Plant II

Plant II, is a solid bulk reclaimer, which is used to recover bulk for ship loading, this equipment has the capacity to move up to 8,000 tons per hour (t/h).

$$G_{P_{II}}(s) = \frac{b_1 s + b_0}{s^3 + a_1 s^2 + a_2 s + a_3}, \tag{33}$$

where $G_{P_{II}}^G(s)$ is the TF of Plant II, it is a third order plant with zero at infinity.

The product of the TF numerator of Plant II given in Eq. (33) associated with the TF numerator of the controller given in Eq. (2) is given by

$$\begin{aligned} \left[C_{P_{II}}^{pid}(s) G_{P_{II}}(s)\right]_N &= \left(K_D s^2 + K_P s + K_I\right)\left(b_1 + b_0\right) \\ &= K_D b_1 s^3 + \left(K_D b_0 + K_I b_1\right) s^2 + K_I b_0. \end{aligned} \tag{34}$$

The product of the TF denominator of Plant II given in Eq. (33) associated with the TF denominator of the controller given in Eq. (2) is given by

$$\left[C_{P_{II}}^{pid}(s)G_{P_{II}}(s)\right]_D = s\left(s^3 + a_1s^2 + a_2s + a_3\right) = s^4 + a_1s^3 + a_2s^2 + a_3s. \tag{35}$$

The characteristic polynomial of Plant II is given by

$$\begin{aligned} P_{P_{II}}(s) = s^4 + (a_1 + K_Db_1)s^3 \\ + (a_2 + K_Db_0 + K_Pb_1)s^2 \\ + (a_3 + (K_Pb_0 + K_Ib_1)s \\ + K_Ib_0. \end{aligned} \tag{36}$$

System equations of Plant II related to Eq. (19) in the form $Ax = b$ is given by

$$\begin{aligned} \langle K^{pid}, \overline{b}_i \rangle = a_i^e \Rightarrow & \begin{cases} a_1 + K_Db_1 = a_1^s; \\ a_2 + K_Db_0 + K_Pb_1 = a_2^s; \\ a_3 + K_Pb_0 + K_Ib_1 = a_3^s; \\ a_4 + K_Ib_0 = a_4^s; \end{cases} \\ \Rightarrow & \begin{cases} K_Db_1 = a_1^s - a_1; \\ K_Db_0 + K_Pb_1 = a_2^s - a_2; \\ K_Pb_0 + K_Ib_1 = a_3^s - a_3; \\ K_Ib_0 = a_4^s - a_4; \end{cases} \\ \Rightarrow & \begin{cases} 1)\ K_Db_1 = a_1^e; \\ 2)\ K_Db_0 + K_Pb_1 = a_2^e; \\ 3)\ K_Pb_0 + K_Ib_1 = a_3^e; \\ 4)\ K_Ib_0 = a_4^e. \end{cases} \end{aligned} \tag{37}$$

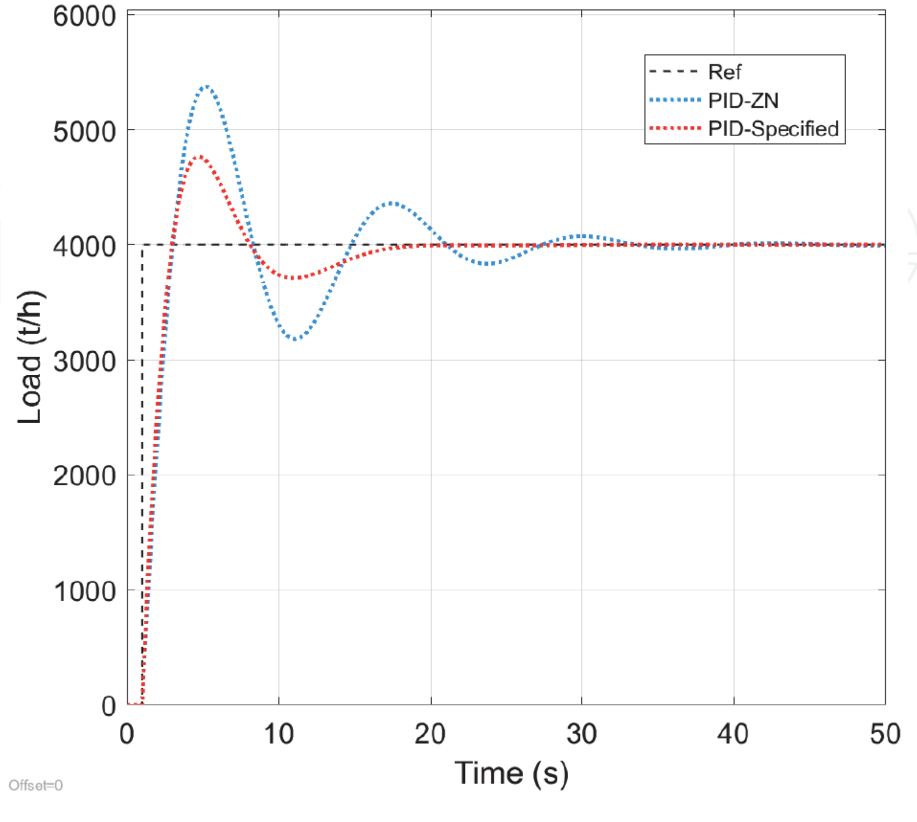

Figure 2.
Plant I - PID-ZN x PID-specified.

Placing the systems of equations given in (37) in the matrix form, we have

$$\langle K^{pid}, \overline{B} \rangle = a_i^e \Rightarrow \begin{bmatrix} b_0 & 0 & 0 \\ b_1 & b_0 & 0 \\ 0 & b_1 & b_0 \\ 0 & 0 & b_1 \end{bmatrix} \times \begin{bmatrix} K_D \\ K_P \\ K_I \end{bmatrix} = \begin{bmatrix} a_1^e \\ a_2^e \\ a_3^e \\ a_4^e \end{bmatrix}. \tag{38}$$

The transfer function of Plant II related to Eq. (33) is given by

$$G_{P_{II}}(s) = \frac{0.1812s + 0.087}{s^3 + 0.3853s^2 + 0.117s + 0.01567}. \tag{39}$$

The a_i coefficients of the Plant TF - II related to Eq. (39) are given by

$$a_i \Rightarrow \begin{cases} a_1 = 0.3853; \\ a_2 = 0.117; \\ a_3 = 0.01567; \\ a_4 = 0. \end{cases} \tag{40}$$

The specified coefficients a_i^s of Plant II are given by

$$a_{4i}^{e\,s} \Rightarrow \begin{cases} a_1^s = 1.7133; \\ a_2^s = 0.8542; \\ a_3^s = 0.2670; \\ a_4^s = 0.0478. \end{cases} \tag{41}$$

The error calculation a_i^e for Plant II associated with the coefficients a_i^s given in (41) and with the coefficients a_i given in (40) is given by

$$a_{4i}^{e\,e} \Rightarrow \begin{cases} a_1^e = a_1^s - a_1 = 1.7133 - 0.3853 = 1.328; \\ \mathrm{a}_2^e = a_2^s - a_2 = 0.7372 - 0.117 = 0.6202; \\ a_3^e = a_3^s - a_3 = 0.2514 - 0.01567 = 0.2358; \\ a_4^e = a_4^s - a_4 = 0.0478 - 0 = 0.0478. \end{cases} \tag{42}$$

The calculation of the K^{pid} gains vector is performed by replacing the numerical values of Eq. (39) in the system of equations given in (37) and (38).

$$\begin{bmatrix} 0.087 & 0 & 0 \\ 0.1812 & 0.087 & 0 \\ 0 & 0.1812 & 0.087 \\ 0 & 0 & 0.1812 \end{bmatrix} \times \begin{bmatrix} K_D \\ K_P \\ K_I \end{bmatrix} = \begin{bmatrix} 1.3280 \\ 0.9538 \\ 0.0996 \\ 0.0478 \end{bmatrix}. \tag{43}$$

The plant II given in Eq. (39) related to Eq. (33) has the b_0 coefficients and b_1, with this, the system of equations generated, for the system of equations related to the system of equations given in (37), has four equations and three unknowns.

$$K^{pid} \Rightarrow \begin{cases} i)\, K_D b_1 = a_1^e \Rightarrow K_D = 1.328/0.1812 \Rightarrow K_D = 7.329; \\ ii)\, K_D b_0 + K_P b_1 = a_2^e \Rightarrow K_P = (0.9538 - 0.6376)/0.1812 \Rightarrow K_P = 1.745; \\ iii)\, \mathrm{K}_P b_0 + K_I b_1 = a_3^e \Rightarrow K_I = (2514 - 0.1518)/0.1812 \Rightarrow K_I = 0.549; \\ iv)\, K_I b_0 = a_4^e \Rightarrow K_I = 0.0478/0.087 \Rightarrow K_I = 0.549. \end{cases} \tag{44}$$

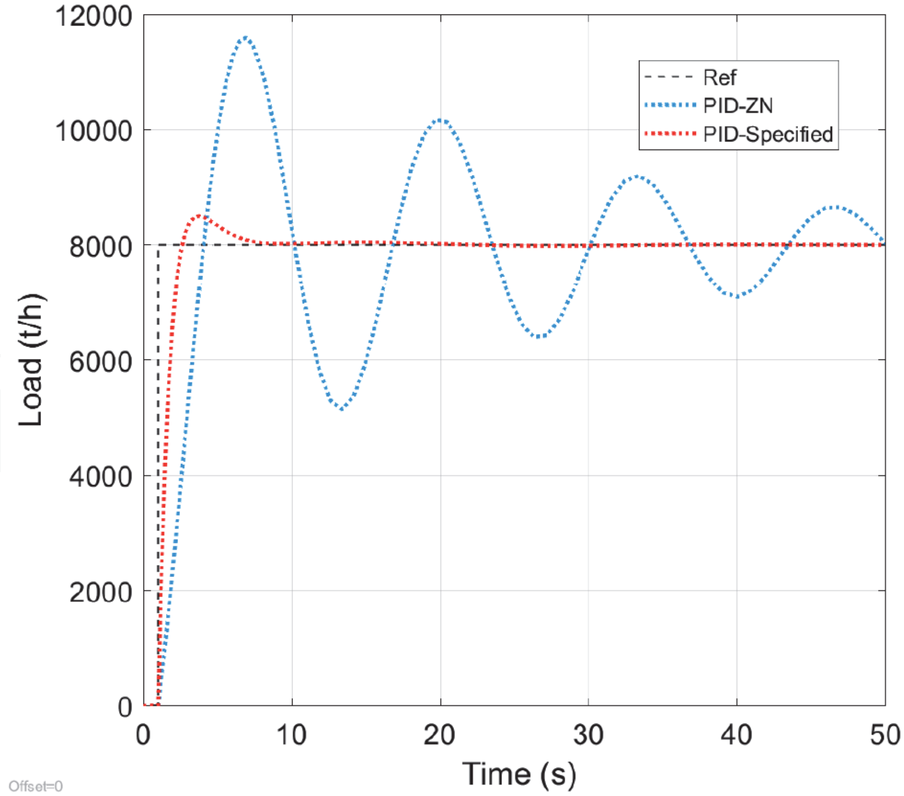

Figure 3.
Plant II - PID-ZN x PID-specified.

Solving the system of equations given in (44), first, solve Equation i) to find the numerical value of K_D. Then, replace the numerical value of K_D in Equation ii) and find the numerical value of K_P. To find the numerical value of K_I, solve equation iv) or replace the values of K_P in equation iii). With the numerical values of the gains K_D, K_P and K_I, it replaces in the simulator developed in the MATLAB/SIMULINK software to monitor the performance of the proposed method.

Figure 3 shows the performance of the PID-Specified controller, which has the transfer function parameters specified by the designer and the $K^{pid}_{Specified}$ gain vector determined by the internal product of the vector of gains with the propagation matrix in purchase with the controller with the gains determined by the second method of ZN.

4.3 Plant III

Plant III, is a car dumper with two feeders, which is used to unload solids in bulk, this equipment has the capacity to move up to 8,000 tons per hour (t/h). The general mathematical model of Plant III in TF is given by

$$G^G_{P_{III}}(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4}. \tag{45}$$

where, $G^G_{P_{III}}(s)$ is the TF of Plant III, it is a fourth order plant with zero at infinity.

The product of the TF numerator of Plant III given in Eq. (45) associated with the TF numerator of the controller given in Eq. (2) is given by

$$\begin{aligned}\left[C^{pid}_{P_{III}}(s)G_{P_{III}}(s)\right]_N &= \left(K_Ds^2+K_Ps+K_I\right)\left(b_2s^2+b_1s+b_0\right)\\ &= K_Db_2s^4+(K_Db_1+K_Pb_2)s^3\\ &\quad +(K_Db_0+K_Pb_1+K_I)s^2\\ &\quad +(K_Pb_0+K_Ib_1)s\\ &\quad +K_Ib_0.\end{aligned} \tag{46}$$

The product of the TF denominator of Plant III given in Eq. (45) associated with the TF numerator of the controller given in Eq. (2) is given by

$$\begin{aligned}\left[C^{pid}_{P_{III}}(s)G_{P_{III}}(s)\right]_D &= s\left(s^4+a_1s^3+a_2s^2+a_3s+a_4\right)\\ &= s^5+a_1s^4+a_2s^3+a_3s^2+a_4s.\end{aligned} \tag{47}$$

The characteristic polynomial of Plant III is given by

$$\begin{aligned}P_{P_{III}}(s) = s^5 &+ a_1+K_Db_2s^4\\ &+a_2+(K_Db_1+K_Pb_2)s^3\\ &+a_3+(K_Db_0+K_Pb_1+K_Ib_2)s^2\\ &+a_4+(K_Pb_0+K_Ib_1)s\\ &+K_Ib_0.\end{aligned} \tag{48}$$

System equations da Planta III related to Eq. (19) in the form $Ax = b$ is given by

$$\begin{aligned}\left\langle K^{pid},\overline{b}_i\right\rangle = a_i^e &\Rightarrow \begin{cases} a_1+K_Db_2=a_1^s;\\ a_2+K_Db_1+K_Pb_2=a_2^s;\\ a_3+K_Db_0+K_Pb_1+K_Ib_2=a_3^s;\\ a_4+K_Pb_0+K_Ib_1=a_4^s;\\ a_5+K_Ib_0=a_5^s.\end{cases}\\ &\Rightarrow \begin{cases} K_Db_2=a_1^s-a_1;\\ K_Db_1+K_Pb_2=a_2^s-a_2;\\ K_Db_0+K_Pb_1+K_Ib_2=a_3^s-a_3;\\ K_Pb_0+K_Ib_1=a_4^s-a_4;\\ K_Ib_0=a_5^s-a_5.\end{cases}\\ &\Rightarrow \begin{cases} 1)K_Db_2=a_1^e;\\ 2)K_Db_1+K_Pb_2=a_2^e;\\ 3)K_Db_0+K_Pb_1+K_Ib_2=a_3^e;\\ 4)K_Pb_0+K_Ib_1=a_4^e;\\ 5)K_Ib_0=a_5^e.\end{cases}\end{aligned} \tag{49}$$

Placing the systems of equations given in (49) in the matrix form, we have

$$\left\langle K^{pid},\overline{B}\right\rangle = a_i^e \Rightarrow \begin{bmatrix} b_0 & 0 & 0\\ b_1 & b_0 & 0\\ b_2 & b_1 & b_0\\ 0 & b_2 & b_1\\ 0 & 0 & b_2\end{bmatrix} \times \begin{bmatrix} K_D\\ K_P\\ K_I\end{bmatrix} = \begin{bmatrix} a_1^e\\ a_2^e\\ a_3^e\\ a_4^e\\ a_5^e\end{bmatrix}. \tag{50}$$

The transfer function of Plant III related to Eq. (45) is given by

$$G_{P_{III}}(s) = \frac{0.959s^2 + 0.1698s + 0.1593}{s^4 + 0.1767s^3 + 0.3463s^2 + 0.029s + 0.02331}. \tag{51}$$

The a_i coefficients of the Plant TF - III related to Eq. (51) are given by

$$a_i \Rightarrow \begin{cases} a_1 = 0.1767; \\ a_2 = 0.3463; \\ a_3 = 0.029; \\ a_4 = 0.2331; \\ a_5 = 0. \end{cases} \tag{52}$$

The specified coefficients a_i^s of Plant III are given by

$$a_i^s \Rightarrow \begin{cases} a_1^s = 1.8358; \\ a_2^s = 8.619; \\ a_3^s = 8.9947; \\ a_4^s = 1.4277; \\ a_5^s = 1,3254. \end{cases} \tag{53}$$

The error calculation a_i^e for Plant III associated with the coefficients a_i^s given in (53) and with the coefficients a_i given in (52) is given by

$$a_i^e \Rightarrow \begin{cases} a_1^e = a_1^s - a_1 = 1.8358 - 0.1767 = 1.659; \\ a_2^e = a_2^s - a_2 = 8.619 - 0.3463 = 4.9066; \\ a_3^e = a_3^s - a_3 = 8.9947 - 0.029 = 8.9657; \\ a_4^e = a_4^s - a_4 = 1.4277 - 0.0233 = 1.4044; \\ a_5^e = a_5^s - a_5 = 1.3254 - 0 = 1.3254. \end{cases} \tag{54}$$

The calculation of the gain vector K^{pid} is done by replacing the numerical values of Eq. (51) in the system of equations given in (49) and (50).

$$\begin{bmatrix} 0.1593 & 0 & 0 \\ 0.1698 & 0.1593 & 0 \\ 0.959 & 0.1698 & 0.1593 \\ 0 & 0.959 & 0.1698 \\ 0 & 0 & 0.959 \end{bmatrix} \times \begin{bmatrix} K_D \\ K_P \\ K_I \end{bmatrix} = \begin{bmatrix} 1.6591 \\ 4.9066 \\ 9.0712 \\ 2.7381 \\ 1.3254 \end{bmatrix}. \tag{55}$$

The Plant III given in Eq. (51) related to Eq. (45), has the coefficients b_0, b_1 and b_2, with that, the generated system of equations, for the system of equations related to the system of equations given in (49), has five equations and three unknowns.

$$K^{pid} \Rightarrow \begin{cases} i)\, K_D b_2 = a_1^e \Rightarrow K_D = 1.6591/959 \Rightarrow K_D = 1.73; \\ ii)\, K_D b_1 + K_P b_2 = a_2^e \Rightarrow K_P = 4.6128/0.959 \Rightarrow K_P = 4.81; \\ iii)\, K_D b_0 + K_P b_1 + K_I b_2 = a_3^e \Rightarrow K_I = 7.978/0.959 \Rightarrow K_I = 8.32; \\ iv)\, K_P b_0 + K_I b_1 = a_4^e \Rightarrow K_I = 1.4127/0.1698 \Rightarrow K_I = 8.32; \\ v)\, K_I b_0 = a_5^e \Rightarrow K_I = 1.3254/0.1593 \Rightarrow K_I = 8.32. \end{cases} \tag{56}$$

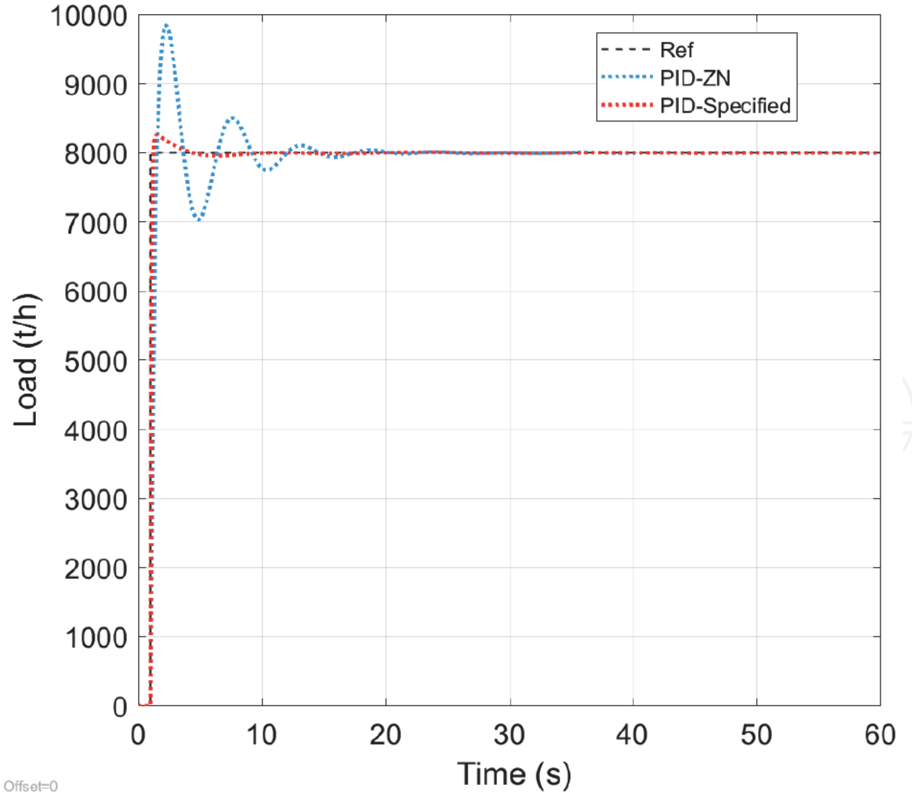

Figure 4.
Plant III - PID-ZN x PID-specified.

Solving the system of equations given in (56), first, solve Equation i) to find the numerical value of K_D. Then, replace the numerical value of K_D in Equation ii) and find the numerical value of K_P. To find the numerical value of K_I, solve Equation v) or replace the values of K_D and K_P in Equation iii) or you can substitute the value of K_P in Equation iv). With the numerical values of the gains K_D, K_P and K_I, it replaces in the simulator developed in the MATLAB/SIMULINK software to monitor the performance of the proposed method.

Figure 4 shows the performance of the PID-Specified controller, which has the transfer function parameters specified by the designer and the $K^{pid}_{Specified}$ gain vector determined by the internal product of the vector of gains with the propagation matrix in purchase with the controller with the gains determined by the second method of ZN.

5. Conclusions

The study presented a methodology for adjusting the gains of a PID controller in terms of the internal product of the gains and the propagation matrix. In addition, the relevance of the matrix was shown, which enabled impact assessment of the PID actions associated with the plant parameters. The proposed methodology complied with the project specifications and ensured high controller efficiency without suppressing the PID terms caused by the adjustments. The three PID controller terms were adjusted. Therefore, this methodology can be considered as an alternative to conventional methods for the computation of K_D, K_P, and K_I gains parameters in practical applications and highly complex control plants.

Acknowledgements

The authors would like to thank PPGEE of the UFMA for the resources to develop this work. We are especially grateful to FAPEMA for research incentive and infrastructure. We acknowledge the Department of Computer Engineering of the UEMA for making this research possible. Finally, we also acknowledge CAPES and CNPq for promoting and supporting the advanced studies that contributed to this work.

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