Chapter

Cement-Based Piezoelectricity Application: A Theoretical Approach

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Abstract

The linear theory of piezoelectricity has widely been used to evaluate the material constants of single crystals and ceramics, but what happens with amorphous structures that exhibit piezoelectric properties such as cement-based? In this chapter, we correlate the theoretical and experimental piezoelectric parameters for small deformations after compressive stress–strain, open circuit potential, and impedance spectroscopy on cement-based. Here, in detail, we introduce the theory of piezoelectricity for large deformations without including a functional for the energy; also, we show two generating equations in terms of a free energy's function for later it will be reduced to constitutional equations of piezoelectricity for infinitesimal deformations. Finally, here is shown piezoelectric and electrical parameters of gold nanoparticles mixed to cement paste: the axial elasticity parameter $Y = 323.5 \pm 75.3$ $[kN/m^2]$, the electroelastic parameter $\gamma = -20.5 \pm 6.9$ $[mV/kN]$, and dielectric constant $\varepsilon = (939.6 \pm 82.9)\varepsilon_0$ $[F/m]$, which have an interpretation as linear theory parameters s^D_{ijkl}, g_{kij} and ε^T_{ik} discussed in the chapter.

Keywords: piezoelectricity, cement-based, nano-composites, constitutional equations, impedance spectroscopy

1. Introduction

The direct piezoelectric effect creates an electric polarization on a continuum medium due to applied stress. The polarization can be macroscopic (effect over continuum medium) and nanoscopic and microscopy scales (effect over atoms, molecules, and electrical domains). Once the Curie brothers discovered the piezoelectric effect in 1880 [1], piezoelectricity investigations led to more data and constructed models based on crystallography to explain the electricity generation since electro-optics and thermodynamic. Voigt in 1894 proposed a piezoelectric parameter related to the strain of material; since the thermodynamic theory, he constructed a non-linear model and expressed the free energy of a piezoelectric crystal in terms of the electric field, strain, electric and elastic deformation potentials, temperature, pyroelectric and piezoelectric parameters [2]. Currently, we can see these constants in the constitutive equations of piezoelectricity. During 1956 and 1963, Toupin and Eringen used a variational formulation to construct a functional in terms of internal energy and derive the constitutive equations [3, 4]. Then, in 1971 Tiersten proposed to use the conservation equations of mass, electrical charge, linear momentum, angular momentum, and energy, adding a Legendre transformation to include a thermodynamic functional in terms of the free energy, achieving a reduction of the number of constitutive equations from 7 to 4 to facilitate theoretical calculations [5]. These constitutional equations and their linear approach gave support to the theoretical calculus of piezoelectric parameters of crystalline structures, e.g., zinc-blende [6, 7], zinc oxide [8, 9], and other crystals with similar symmetric of quartz [10, 11]. Finally, between 1991 and 2017, Yang has proposed modifications for the Legendre transformation of Tiersten, and he has included two models to describe the polarization in a deformable continuum medium [12, 13].

According to electrostatic theory, the macroscopic polarization $\vec{P}$ can be written in terms of electric charge distribution likewise with the electric field $\vec{E}$ induced into the continuum medium. Besides, the polarization starts with continuum medium deformation $\vec{S}$ for applied stress. Considering: (i) Uniqueness for the parameters that relate the polarization and the electric field, (ii) Stress produces equal deformations in each cell of crystal, (iii) The deformations lead dipole and quadrupole moments affecting the piezoelectric parameters directly. Based on the above considerations, the linear constitutive equations of piezoelectricity can be written, as Martin said [6].

This chapter book is thought to be a working example that connects the piezoelectricity theory and experimental data of electromechanical and electrical properties. These data were obtained on cement paste mixed with gold nanoparticles.

2. Constitutional equations in detail and correlation with the piezoelectricity of cement-based composites

In this section, we have selected Yang's differential approach to obtain the constitutional equations of piezoelectricity. The differential derivation shows the physics involved in the conservation laws differently. For example, it shows that the electric body couple and the Cauchy stress tensor are asymmetric. Also, it relates the local electric field with the electric interaction between the differential elements of the lattice continuum and the electronic continuum.

2.1 Conservation laws applied to a polarized continuum from differential approximation

Regarding the study of the piezoelectric properties of cement paste, it is necessary to describe the material separately as two continua medium, as from the piezoelectric phenomenon, the crystal and their symmetry would have a lattice (positive charge) and an electronic component (negative charge) Those continua can be separated by mechanical stress. Their physical properties could change according to the coordinate systems or states. Therefore, the body will study in two states (reference state and current state), as is shown in **Figure 1**.

2.1.1 Electric charge conservation

The conservation electric charge in the body takes importance with an infinitesimal displacement on the medium's current state to get polarization. For this reason, the phenomenon described in the above state is known as the two continuum medium model. The electronic continuum comes under an infinitesimal

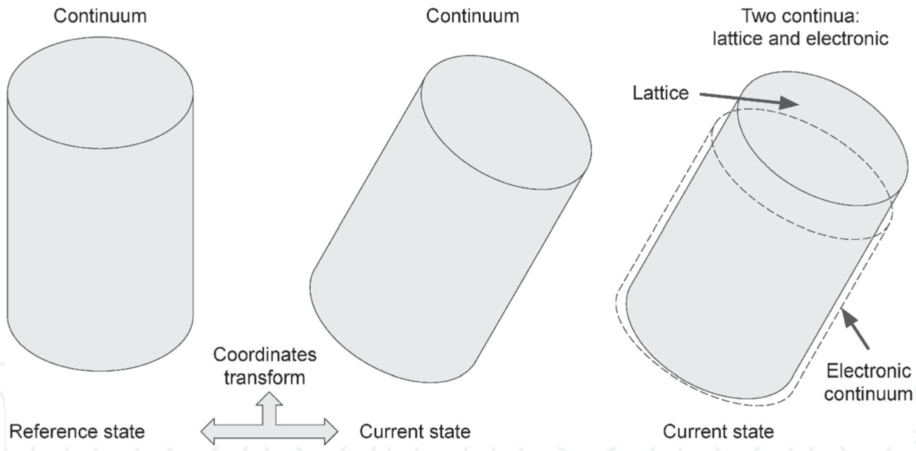

Figure 1.
States of a deformable and polarizable continuum.

displacement η respect to the lattice. It is produced by body deformation, as described in **Figure 2**. From the assumption of the lattice and electronic continuum, the medium have equal volumes, some variation of infinitesimal displacement respect coordinates in the current state should be zero and can be written as

$$\eta_{k,k} = \frac{\partial \eta_k(y)}{\partial y_k} = 0 \tag{1}$$

Furthermore, if it is taken the two continuous mediums, the electric charge density must be neutral to consider only the piezoelectric effect.

$$\mu^l\left(\vec{y}\right) + \mu^e\left(\vec{y} + \vec{\eta}\right) = 0 \tag{2}$$

We can show that the gradient of infinitesimal displacement Eq. (1) and the neutrality condition of electric charge density Eq. (2) are sufficient to explain the polarization in a deformable continuum

$$\vec{P} = \mu^l\left(\vec{y}\right)\left[-\vec{\eta}\left(\vec{y}\right)\right] = \mu^e\left(\vec{y} + \vec{\eta}\right)\vec{\eta} \approx \mu^e\left(\vec{y}\right)\vec{\eta} \tag{3}$$

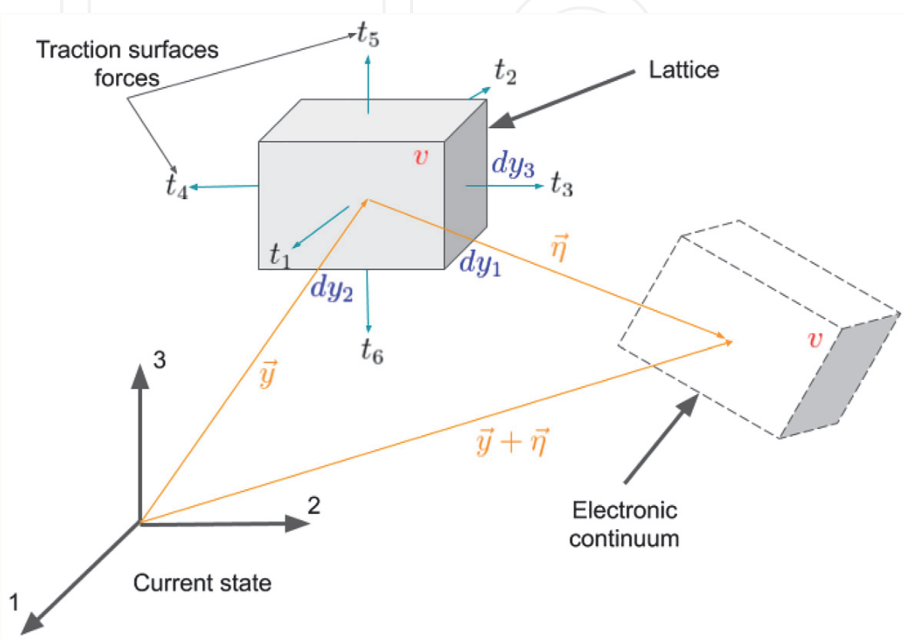

Figure 2.
Volume elements of electronic and lattice continuum medium.

2.1.2 Energy conservation

Once the body is deformed, electronic and lattice continua electric charges apply a quasi-static electric field. Tiersten et al. called it Maxwelliam electric field E_k. It is interacting on two continuum mediums producing an electrical force on each one. The other forces acting on the body are traction and body forces. The traction force t_k per unit, the area is working on the surfaces of volume elements of the lattice (see **Figure 2**). Also, it can be written in terms of Cauchy stress tensor τ_{jk} as $t_k = n_j \tau_{jk}$, where n_j is the normal vector. Moreover, body force refers to an external force acting on the body, for example, gravity. The previous three forces are necessary to get the conservation laws, including energy.

From the three forces above, it is possible to construct the Eqs. (4) and (5), linear and angular momentum conservation, respectively.

$$\rho \dot{u}_k = f_k^E + \tau_{mk,m} + \rho f_k \tag{4}$$

$$c_i^E + \varepsilon_{ijk} \tau_{jk} \left(\vec{y} \right) = 0 \tag{5}$$

where f_k^E is the electric force on the body and start from the dot product of polarization P_j with the electric field gradient $E_{k,i}$; Besides, c_i^E is called electric coupling, and it is the cross product of P_j with E_k; finally, f_k is the external force on the body per unit mass.

Then, replacing the electric coupling c_i^E by the cross product between P_j and E_k, we can rewrite Eq. (5) as follow

$$\varepsilon_{ijk} P_j E_k \left(\vec{y} \right) + \varepsilon_{ijk} \tau_{jk} \left(\vec{y} \right) = 0 \tag{6}$$

Factoring the permutation tensor ε_{ijk}

$$\varepsilon_{ijk} \left[P_j E_k + \tau_{jk} \right] = 0 \tag{7}$$

The term in square brackets from Eq. (7) is a symmetry tensor that can be written as

$$\tau^S{}_{jk} = P_j E_k + \tau_{jk} \tag{8}$$

The term $P_j E_k$ is called the Maxwell stress tensor $T^E{}_{jk}$. On the other hand, multiplying Eq. (5) by ε_{iqr} is obtain

$$\begin{aligned} \varepsilon_{iqr} c_i^E + \varepsilon_{iqr} \varepsilon_{ijk} \tau_{jk} \left(\vec{y} \right) &= 0 \\ \delta_{jq} \delta_{kr} \tau_{jk} \left(\vec{y} \right) - \delta_{jr} \delta_{kq} \tau_{jk} \left(\vec{y} \right) &= -\varepsilon_{iqr} c_i^E \\ \tau_{qr} \left(\vec{y} \right) - \tau_{rq} \left(\vec{y} \right) &= -\varepsilon_{iqr} c_i^E \end{aligned} \tag{9}$$

From Eq. (9) we can conclude that the Cauchy stress tensor ($t_k = n_j \tau_{jk}$) is asymmetry. Now, we get the total energy inside the continuum medium as a combination of kinetic and internal energy ε^{in} [13], both per mass unit. Here is performed the powers added due to the three forces above.

$$\begin{aligned}\frac{d}{dt}\left(\frac{1}{2}u_k u_k \rho dv + \varepsilon^{in}\rho dv\right) = {} & \mu^l\left(\vec{y}\right)E_k\left(\vec{y}\right)u_k\left(\vec{y}\right)dv \\ & + \mu^e\left(\vec{y}+\vec{\eta}\right)E_k\left(\vec{y}+\vec{\eta}\right)\left[u_k\left(\vec{y}\right)+\dot{\eta}_k\right]dv + t_k u_k ds \\ & + \rho f_k u_k dv\end{aligned} \tag{10}$$

The following steps from Eq. (10) conduct to develop conservation energy law. **In the first step**, we work on the **total energy term**. Then, the product rule to the total energy term is applied:

$$\begin{aligned}\frac{d}{dt}\left(\frac{1}{2}u_k u_k \rho dv + \varepsilon^{in}\rho dv\right) &= \rho dv\frac{d}{dt}\left(\frac{1}{2}u_k u_k + \varepsilon^{in}\right) + \left(\frac{1}{2}u_k u_k + \varepsilon^{in}\right)\frac{d}{dt}(\rho dv) \\ &= \rho dv\frac{d}{dt}\left(\frac{1}{2}u_k u_k + \varepsilon^{in}\right) + \left(\frac{1}{2}u_k u_k + \varepsilon^{in}\right)\frac{d}{dt}(\rho dv)\end{aligned} \tag{11}$$

The term $\frac{d}{dt}(\rho dv)$ represents the mass conservation. Therefore, this term is null in Eq. (11).

$$\frac{d}{dt}\left(\frac{1}{2}u_k u_k \rho dv + \varepsilon^{in}\rho dv\right) = \rho dv\frac{d}{dt}\left(\frac{1}{2}u_k u_k + \varepsilon^{in}\right) \tag{12}$$

Then, here is writes the add of derivatives from kinetic and internal energies

$$\frac{d}{dt}\left(\frac{1}{2}u_k u_k \rho dv + \varepsilon^{in}\rho dv\right) = \rho dv\; u_k\frac{d}{dt}(u_k) + \rho dv\frac{d}{dt}\left(\varepsilon^{in}\right) \tag{13}$$

In the second step, the **electric power term** will be developed by Taylor's expansion.

$$\begin{aligned}&\mu^l\left(\vec{y}\right)E_k\left(\vec{y}\right)u_k\left(\vec{y}\right)dv + \mu^e\left(\vec{y}+\vec{\eta}\right)E_k\left(\vec{y}+\vec{\eta}\right)\left[u_k\left(\vec{y}\right)+\dot{\eta}_k\right]dv \\ &\approx \mu^l\left(\vec{y}\right)E_k\left(\vec{y}\right)u_k\left(\vec{y}\right)dv + \mu^e\left(\vec{y}+\vec{\eta}\right)\left[E_k\left(\vec{y}\right)+E_{k,i}\left(\vec{y}\right)\eta_i\right]\left[u_k\left(\vec{y}\right)+\dot{\eta}_k\right]dv\end{aligned} \tag{14}$$

From Eq. (14), it will solve the dot product between the electric field and velocity,

$$\begin{aligned}&= \mu^l\left(\vec{y}\right)E_k\left(\vec{y}\right)u_k\left(\vec{y}\right)dv \\ &+\mu^e\left(\vec{y}+\vec{\eta}\right)\left[E_k\left(\vec{y}\right)u_k\left(\vec{y}\right)+E_k\left(\vec{y}\right)\dot{\eta}_k+E_{k,i}\left(\vec{y}\right)\eta_i u_k\left(\vec{y}\right)+E_{k,i}\left(\vec{y}\right)\eta_i\dot{\eta}_k\right]dv\end{aligned} \tag{15}$$

The second-order term $\eta_i\dot{\eta}_k = \eta_i\dot{\eta}_i = \frac{1}{2}\frac{d}{dt}\eta_i^{\,2} \approx 0$ is zero, taking into account the infinitesimal displacement. Then, factorize $E_k\left(\vec{y}\right)u_k\left(\vec{y}\right)dv$ in Eq. (15), it takes a form:

$$= \left[\mu^l\left(\vec{y}\right)+\mu^e\left(\vec{y}+\vec{\eta}\right)\right]E_k\left(\vec{y}\right)u_k\left(\vec{y}\right)dv + \left[E_k\left(\vec{y}\right)\dot{\eta}_k + E_{k,i}\left(\vec{y}\right)\eta_i u_k\left(\vec{y}\right)\right]dv \tag{16}$$

Replacing Eq. (2) in Eq. (16), we obtain:

$$= \mu^e\left(\vec{y} + \vec{\eta}\right)E_k\left(\vec{y}\right)\dot{\eta}_k dv + \mu^e\left(\vec{y} + \vec{\eta}\right)E_{k,i}\left(\vec{y}\right)\eta_i u_k\left(\vec{y}\right)dv \tag{17}$$

With Eq. (3), Eq. (17) takes the form:

$$= \mu^e\left(\vec{y} + \vec{\eta}\right)E_k\left(\vec{y}\right)\dot{\eta}_k dv + P_i E_{k,i}\left(\vec{y}\right)u_k\left(\vec{y}\right)dv \tag{18}$$

In Eq. (18), the term $\mu^e\left(\vec{y} + \vec{\eta}\right)E_k\left(\vec{y}\right)\dot{\eta}_k$ is called the electric power of the body w^E And the term $P_i E_{k,i}$ is the electric force between the lattice and electronic volume elements. It was defined in Eq. (4) above.

$$\mu^l\left(\vec{y}\right)E_k\left(\vec{y}\right)u_k\left(\vec{y}\right)dv + \mu^e\left(\vec{y} + \vec{\eta}\right)E_k\left(\vec{y} + \vec{\eta}\right)\left[u_k\left(\vec{y}\right) + \dot{\eta}_k\right]dv = w^E dv + f_k^{\ E} u_k\left(\vec{y}\right)dv \tag{19}$$

In the third step, we will solve the **traction force** in terms of Cauchy stress tensor. Then, the power due to the traction force takes the form:

$$\begin{aligned} t_k u_k ds = \; & \tau_{mk} n_m u_k ds \\ = \; & \tau_{1k}\left(\vec{y} + \frac{1}{2}dy_1\hat{i}_1\right)u_k\left(\vec{y} + \frac{1}{2}dy_1\hat{i}_1\right)dy_2 dy_3 \\ & - \tau_{1k}\left(\vec{y} - \frac{1}{2}dy_1\hat{i}_1\right)u_k\left(\vec{y} - \frac{1}{2}dy_1\hat{i}_1\right)dy_2 dy_3 \\ & + \tau_{2k}\left(\vec{y} + \frac{1}{2}dy_2\hat{i}_2\right)u_k\left(\vec{y} + \frac{1}{2}dy_2\hat{i}_2\right)dy_3 dy_1 \\ & - \tau_{2k}\left(\vec{y} - \frac{1}{2}dy_2\hat{i}_2\right)u_k\left(\vec{y} - \frac{1}{2}dy_2\hat{i}_2\right)dy_3 dy_1 \\ & + \tau_{3k}\left(\vec{y} + \frac{1}{2}dy_3\hat{i}_3\right)u_k\left(\vec{y} + \frac{1}{2}dy_3\hat{i}_3\right)dy_1 dy_2 \\ & - \tau_{3k}\left(\vec{y} - \frac{1}{2}dy_3\hat{i}_3\right)u_k\left(\vec{y} - \frac{1}{2}dy_3\hat{i}_3\right)dy_1 dy_2 \end{aligned} \tag{20}$$

Now, we apply Taylor's expansion to Cauchy stress tensor τ_{1k} and velocity u_k. Next, the last result will be implemented in the components: two τ_{2k} and three τ_{3k}

$$\begin{aligned} & \tau_{1k}\left(\vec{y} + \frac{1}{2}dy_1\hat{i}_1\right)u_k\left(\vec{y} + \frac{1}{2}dy_1\hat{i}_1\right)dy_2 dy_3 - \tau_{1k}\left(\vec{y} - \frac{1}{2}dy_1\hat{i}_1\right)\mathrm{u}_k\left(\vec{y} - \frac{1}{2}dy_1\hat{i}_1\right)dy_2 dy_3 \\ & \approx \left[\tau_{1k}\left(\vec{y}\right) + \frac{1}{2}dy_1\tau_{1k,1}\left(\vec{y}\right)\right]\left[u_k\left(\vec{y}\right) + \frac{1}{2}dy_1 u_{k,1}\left(\vec{y}\right)\right]dy_2 dy_3 \\ & -\left[\tau_{1k}\left(\vec{y}\right) - \frac{1}{2}dy_1\tau_{1k,1}\left(\vec{y}\right)\right]\left[u_k\left(\vec{y}\right) - \frac{1}{2}dy_1 u_{k,1}\left(\vec{y}\right)\right]dy_2 dy_3 \end{aligned} \tag{21}$$

The products give

$$\begin{aligned}&\approx\left[\tau_{1k}u_k+\tau_{1k}\frac{1}{2}dy_1u_{k,1}+\frac{1}{2}dy_1\tau_{1k,1}u_k+\frac{1}{4}{dy_1}^2\tau_{1k,1}u_{k,1}\right]dy_2dy_3\\&-\left[\tau_{1k}u_k-\tau_{1k}\frac{1}{2}dy_1u_{k,1}-\frac{1}{2}dy_1\tau_{1k,1}u_k+\frac{1}{4}{dy_1}^2\tau_{1k,1}u_{k,1}\right]dy_2dy_3\end{aligned}\tag{22}$$

Adding similar terms:

$$\approx\left[\tau_{1k}\left(\vec{y}\right)dy_1u_{k,1}\left(\vec{y}\right)+dy_1\tau_{1k,1}\left(\vec{y}\right)u_k\left(\vec{y}\right)\right]dy_2dy_3\tag{23}$$

Factorizing dy_1 that multiply to dy_2 and dy_3 it will transform into the volume element dv.

$$\approx\tau_{1k}\left(\vec{y}\right)u_{k,1}\left(\vec{y}\right)dv+\tau_{1k,1}\left(\vec{y}\right)u_k\left(\vec{y}\right)dv\tag{24}$$

Reply the proceeding since Eq. (21) to Eq. (24) for the components τ_{2k} and τ_{3k} yield

$$\begin{aligned}t_ku_kds=&\tau_{1k}\left(\vec{y}\right)u_{k,1}\left(\vec{y}\right)dv+\tau_{1k,1}\left(\vec{y}\right)u_k\left(\vec{y}\right)dv+\tau_{2k}\left(\vec{y}\right)u_{k,2}\left(\vec{y}\right)dv\\&+\tau_{2k,2}\left(\vec{y}\right)u_k\left(\vec{y}\right)dv+\tau_{3k}\left(\vec{y}\right)u_{k,3}\left(\vec{y}\right)dv+\tau_{3k,3}\left(\vec{y}\right)u_k\left(\vec{y}\right)dv\end{aligned}\tag{25}$$

Each one of the components has information about two opposite faces of the volume element. Then, adding index m it reduces the Eq. (25) to:

$$t_ku_kds=\tau_{mk}\left(\vec{y}\right)u_{k,m}\left(\vec{y}\right)dv+\tau_{mk,m}\left(\vec{y}\right)u_k\left(\vec{y}\right)dv\tag{26}$$

From the results of Eq. (13), Eq. (19), and Eq. (26) into Eq. (10), we obtain

$$\begin{aligned}\rho dv\,u_k\frac{d}{dt}(u_k)+\rho dv\frac{d}{dt}\left(\varepsilon^{in}\right)=&\,w^Edv+{f_k}^Eu_k\left(\vec{y}\right)dv+\tau_{mk}\left(\vec{y}\right)u_{k,m}\left(\vec{y}\right)dv\\&+\tau_{mk,m}\left(\vec{y}\right)u_k\left(\vec{y}\right)dv+\rho f_ku_kdv\end{aligned}\tag{27}$$

Here is factoring the terms that contain $u_k\left(\vec{y}\right)dv$ to the left side and the terms with the volume differential dv to the right side.

$$\left[\rho\frac{d}{dt}(u_k)+{f_k}^E+\tau_{mk,m}\left(\vec{y}\right)+\rho f_k\right]u_k\left(\vec{y}\right)dv=\left\{w^E-\rho\frac{d}{dt}\left[\varepsilon^{in}\right]+\tau_{mk}\left(\vec{y}\right)u_{k,m}\left(\vec{y}\right)\right\}dv\tag{28}$$

From Eq. (28), the term in the square bracket is null by Eq. (4). Then, we obtain Eq. (29) for energy conservation that depends on internal energy.

$$\rho\dot{\varepsilon}^{in}=w^E+\tau_{mk}\left(\vec{y}\right)u_{k,m}\left(\vec{y}\right)\tag{29}$$

Remember from Eq. (19) that electric power can be written as:

$$w^E=\mu^e\left(\vec{y}+\vec{\eta}\right)E_m\left(\vec{y}\right)\frac{d\eta_m}{dt}=E_m\left(\vec{y}\right)\left\{\frac{d}{dt}\left[\mu^e\left(\vec{y}+\vec{\eta}\right)\eta_m\right]-\frac{d\mu^e\left(\vec{y}+\vec{\eta}\right)}{dt}\eta_m\right\}$$

$$w^E = E_m\left(\vec{y}\right)\left\{\dot{P}_m - \dot{\mu}^e\left(\vec{y}+\vec{\eta}\right)\eta_m\right\} \tag{30}$$

Another form of electric charge conservation is $\dot{\mu}^e\left(\vec{y}+\vec{\eta}\right) + \mu^e\left(\vec{y}+\vec{\eta}\right)u_{i,i} = 0$, it will simplify the Eq. (30) to:

$$w^E = E_m\left(\vec{y}\right)\left\{\dot{P}_m + \mu^e\left(\vec{y}+\vec{\eta}\right)u_{i,i}\eta_m\right\} \tag{31}$$

The mass conservation $\dot{\rho} + \rho u_{i,i} = 0$ has a similar mathematical structure as charge conservation. Therefore, the gradient of the speed $u_{i,i}$ in Eq. (31) was replaced

$$w^E = E_m\left(\vec{y}\right)\dot{P}_m + E_m\left(\vec{y}\right)\mu^e\left(\vec{y}+\vec{\eta}\right)\frac{-\dot{\rho}}{\rho}\eta_m = E_m\dot{P}_m - \frac{\dot{\rho}}{\rho}E_mP_m$$

$$w^E = \frac{E_m}{\rho}\left[\rho\dot{P}_m - \dot{\rho}P_m\right] \tag{32}$$

Eq. (32) has been used on Eq. (29)

$$\rho\dot{\varepsilon}^{in} = \frac{E_m}{\rho}\left[\rho\dot{P}_m - \dot{\rho}P_m\right] + \tau_{mk}\left(\vec{y}\right)u_{k,m}\left(\vec{y}\right) \tag{33}$$

With Legendre transformation showing in Eq. (34), Tiersten replaced the internal energy ε^{in} by free energy χ [5]. This transformation diminishes the number of constitutional equations. Besides, it offers a quantitative interpretation that can not get from the internal energy resulting in more useful for those who perform piezoelectricity experiments. After Section 2.3, we could see the χ will depend on the gradient of potential in the reference state and deformation tensor.

$$\chi = \varepsilon^{in} - E_m\frac{P_m}{\rho} \tag{34}$$

Upon differentiating respect to time the Eq. (34).

$$\dot{\chi} = \dot{\varepsilon}^{in} - \dot{E}_m\frac{P_m}{\rho} - E_m\frac{\dot{P}_m}{\rho} + E_m\frac{P_m}{\rho^2}\dot{\rho} \tag{35}$$

Clear the term $\rho\dot{\varepsilon}^{in}$

$$\rho\dot{\varepsilon}^{in} = \rho\dot{\chi} + \dot{E}_mP_m + E_m\dot{P}_m - E_m\frac{P_m}{\rho}\dot{\rho} \tag{36}$$

Using Eq. (36) on Eq. (29), we obtain:

$$\begin{aligned}
\rho\dot{\chi} + \dot{E}_mP_m + E_m\dot{P}_m - E_m\frac{P_m}{\rho}\dot{\rho} &= \frac{E_m}{\rho}\left[\rho\dot{P}_m - \dot{\rho}P_m\right] + \tau_{mk}\left(\vec{y}\right)u_{k,m}\left(\vec{y}\right)\\
\rho\dot{\chi} + \dot{E}_mP_m + E_m\dot{P}_m - E_m\dot{\rho}\frac{P_m}{\rho} &= E_m\dot{P}_m - E_m\dot{\rho}\frac{P_m}{\rho} + \tau_{mk}\left(\vec{y}\right)u_{k,m}\left(\vec{y}\right)
\end{aligned} \tag{37}$$

The similar terms $E_m\dot{P}_m$ and $-E_m\dot{\rho}\frac{P_m}{\rho}$ in Eq. (37), are clear. Finally, we have rewritten **energy conservation** in terms of free energy, electrical (electric field and polarization vector), and mechanical (Cauchy stress tensor) components, as is shown in Eq. (38).

$$\rho\dot{\chi} = \tau_{mk}\left(\vec{y}\right)u_{k,m}\left(\vec{y}\right) - \dot{E}_m P_m \tag{38}$$

2.2 Transformation of fundamental physical quantities in piezoelectricity to the reference state

There are several reasons to consider two coordinate systems (reference and current state) for continuum. Firstly, it is not mathematically simple to describe the movement of each particle that compounds a continuum as seen on the gradient of velocity $u_{k,m}$ in Eq. (38); it is more appropriate to propose a coordinate system that describes the continuum in the reference system. The material behavior could be affected by the characteristics of the current state, too. For example, fluids and solids can change their mechanical behavior while changing the shape [14]. Hence, we refer to our study material (cement-based composites) whom we know the physical properties in the reference state X_L. To explain the behavior of a material, we must include physical quantities respect the reference state X_L: potential gradient W_K, polarization P_L, electric displacement $\mathcal{D}_L$, volume free charge density ρ^E, mass density ρ^0 and the second Piola-Kirchhoff stress $T^S{}_{KL}$ [15]. It raises by the transformation of symmetric tensor $\tau^S{}_{mk}$ in the current state to reference state, and relate the traction force with areas, both in the reference state. While the first Piola-Kirchhoff stress is connecting the traction force and electric force in the current state with regions in the reference state.

This section will describe the **transformation of energy conservation from the current state to the reference state**, using Eq. (8), the symmetric tensor modifies Eq. (38).

$$\rho\dot{\chi} = \tau^S{}_{mk}u_{k,m} - P_m E_k u_{k,m} - \dot{E}_m P_m \tag{39}$$

2.2.1 Electric field and gradient of potential

To transform the electric field to a reference state, here will use follow:

$$W_K = E_m y_{m,K} \tag{40}$$

The gradient of the potential W_K is multiplying both sides by $X_{K,m}$

$$W_K X_{K,m} = E_m y_{m,K} X_{K,m} = E_{\mathrm{m}} \frac{\partial y_m}{\partial X_K}\frac{\partial X_K}{\partial y_m} \tag{41}$$

Therefore,

$$E_m = W_K X_{K,m} \tag{42}$$

The derivative respect to time of E_m becomes

$$\dot{E}_m = \frac{d}{dt}[W_K X_{K,m}] = \frac{d}{dt}(W_K)X_{K,m} + W_K\frac{d}{dt}(X_{K,m}) \tag{43}$$

The term $X_{K,m}$ from Eq. (43) is developed as follow

$$X_{K,m} = \delta_{KL} X_{L,m} = \frac{\partial y_k}{\partial X_L} \frac{\partial X_K}{\partial y_k} X_{L,m} \tag{44}$$

Derivative $X_{K,m}$ respect to time

$$\begin{gathered} \frac{d}{dt}(X_{K,m}) = \frac{d}{dt}\left(\frac{\partial y_k}{\partial X_L}\frac{\partial X_K}{\partial y_k}X_{L,m}\right) = \frac{d}{dt}\left(y_{k,L}X_{K,k}X_{L,m}\right) \\ \frac{d}{dt}(X_{K,m}) = \frac{d}{dt}\left(y_{k,L}\right)X_{K,k}X_{L,m} + \frac{d}{dt}(X_{K,k})y_{k,L}X_{L,m} + \frac{d}{dt}(X_{L,m})y_{k,L}X_{K,k} \end{gathered} \tag{45}$$

Partial derivate of y and X are written in Leibniz notation.

$$\frac{d}{dt}(X_{K,m}) = u_{k,L}X_{K,k}X_{L,m} + \frac{d}{dt}(X_{K,k})\frac{\partial y_k}{\partial X_L}\frac{\partial X_L}{\partial y_m} + \frac{d}{dt}(X_{L,m})\frac{\partial y_k}{\partial X_L}\frac{\partial X_K}{\partial y_k} \tag{46}$$

The products of partial derivate are reduced to Kronecker delta.

$$\frac{d}{dt}(X_{K,m}) = u_{k,L}X_{K,k}X_{L,m} + \frac{d}{dt}(X_{K,k})\delta_{km} + \frac{d}{dt}(X_{L,m})\delta_{KL} \tag{47}$$

The index into $X_{K,k}$ and $X_{L,m}$ were exchanging due to commutation Kronecker deltas.

$$\frac{d}{dt}(X_{K,m}) = u_{k,L}X_{K,k}X_{L,m} + \frac{d}{dt}(X_{K,m}) + \frac{d}{dt}(X_{K,m}) \tag{48}$$

In Eq. (48) was delete the term $\frac{d}{dt}(X_{K,m})$ in both sides

$$0 = u_{k,L}X_{K,k}X_{L,m} + \frac{d}{dt}(X_{K,m}) \tag{49}$$

Clearing $\frac{d}{dt}(X_{K,m})$ we obtain

$$\frac{d}{dt}(X_{K,m}) = -u_{k,L}X_{K,k}X_{L,m} \tag{50}$$

Substituting Eq. (50) into Eq. (43) becomes

$$\dot{E}_m = \frac{d}{dt}[W_K X_{K,m}] = \dot{W}_K X_{K,m} - u_{k,L}X_{K,k}X_{L,m}W_K \tag{51}$$

Then, we replace the Eq. (51) into Eq. (39).

$$\rho\dot{\chi} = \tau^S{}_{mk}u_{k,m} - P_m E_k u_{k,m} - P_m\left(\dot{W}_K X_{K,m} - u_{k,L}X_{K,k}X_{L,m}W_K\right) \tag{52}$$

The index L was changed by k, into $u_{k,L}$, due to $X_{L,m}$

$$\rho\dot{\chi} = \tau^S{}_{mk}u_{k,m} - P_m E_k u_{k,m} - P_m\left(\dot{W}_K X_{K,m} - u_{k,m}X_{K,k}W_K\right) \tag{53}$$

In Eq. (53), the term $X_{K,k}W_K$ is the electric field concerning the current state.

$$\rho\dot{\chi} = \tau^S{}_{mk}u_{k,m} - P_m E_k u_{k,m} - P_m\dot{W}_K X_{K,m} + P_m u_{k,m}E_k \tag{54}$$

From Eq. (54) the term $P_m E_k u_{k,m}$ was removes to get

$$\rho\dot{\chi} = \tau^S{}_{mk} u_{k,m} - P_m \dot{W}_K X_{K,m} \tag{55}$$

2.2.2 Polarization vector

In this subsection, we will perform the transformation of the polarization vector to the reference state.

$$P_L = J X_{L,i} P_i \tag{56}$$

Where J is the Jacobian, multiplying Eq. (56) by $J^{-1} y_{m,L}$ we obtain

$$J^{-1} y_{m,L} P_L = J^{-1} y_{m,L} J X_{L,i} P_i = \delta_{mi} P_i \tag{57}$$

To get

$$P_m = J^{-1} y_{m,L} P_L \tag{58}$$

From Eq. (58) into Eq. (55) results in

$$\begin{gathered}
\rho\dot{\chi} = \tau^S{}_{mk} u_{k,m} - J^{-1} y_{m,L} P_L \dot{W}_K X_{K,m} \\
\rho\dot{\chi} = \tau^S{}_{mk} u_{k,m} - J^{-1} P_L \dot{W}_K \frac{\partial y_m}{\partial X_L} \frac{\partial X_K}{\partial y_m} \\
\rho\dot{\chi} = \tau^S{}_{mk} u_{k,m} - J^{-1} P_L \dot{W}_K \delta_{KL} \\
\rho\dot{\chi} = \tau^S{}_{mk} u_{k,m} - J^{-1} P_K \dot{W}_K
\end{gathered} \tag{59}$$

Until now, in Eq. (59), we have obtained a partial transformation, and still missing transform the symmetric Cauchy stress tensor $\tau^S{}_{mk}$.

2.2.3 Second Piola-Kirchhoff stress

The symmetric tensor $\tau^S{}_{mk}$ is related with second Piola-Kirchhoff stress $T^S{}_{KL}$ through a reverse transformation as follow:

$$\tau^S{}_{mk} = J^{-1} y_{m,K} y_{k,L} T^S{}_{KL} \tag{60}$$

Eq. (60) into Eq. (59) results in

$$\rho\dot{\chi} = J^{-1} y_{m,K} y_{k,L} T^S{}_{KL} u_{k,m} - J^{-1} P_K \dot{W}_K \tag{61}$$

The gradient of velocity $u_{k,m}$ can be separated on antisymmetric tensor $\omega_{mk} = \frac{1}{2}(u_{k,m} - u_{m,k})$ plus a symmetric tensor $d_{mk} = \frac{1}{2}(u_{k,m} + u_{m,k})$.

$$\begin{gathered}
\rho\dot{\chi} = J^{-1} y_{m,K} y_{k,L} T^S{}_{KL} (\omega_{mk} + d_{mk}) - J^{-1} P_K \dot{W}_K \\
\rho\dot{\chi} = J^{-1} y_{m,K} y_{k,L} T^S{}_{KL} \omega_{mk} + J^{-1} y_{m,K} y_{k,L} T^S{}_{KL} d_{mk} - J^{-1} P_K \dot{W}_K
\end{gathered} \tag{62}$$

From Eq. (61), the product between symmetric tensor $T^S{}_{KL}$ and antisymmetric tensor ω_{mk} result be null

$$\rho\dot{\chi} = J^{-1} y_{m,K} y_{k,L} T^S{}_{KL} d_{mk} - J^{-1} P_K \dot{W}_K \tag{63}$$

The term $y_{m,K} y_{k,L} d_{mk}$ will be solved as following

$$\begin{gathered}
y_{m,K} y_{k,L} d_{mk} = y_{m,K} y_{k,L} \frac{1}{2}(u_{k,m} + u_{m,k}) = \frac{1}{2}\left(u_{k,m} y_{m,K} y_{k,L} + u_{m,k} y_{m,K} y_{k,L}\right) \\
y_{m,K} y_{k,L} d_{mk} = \frac{1}{2}\left(\frac{\partial u_k}{\partial y_m}\frac{\partial y_m}{\partial X_K}\frac{\partial y_k}{\partial X_L} + \frac{\partial u_m}{\partial y_k}\frac{\partial y_m}{\partial X_K}\frac{\partial y_k}{\partial X_L}\right) = \frac{1}{2}\left(\frac{\partial u_k}{\partial X_K}\frac{\partial y_k}{\partial X_L} + \frac{\partial u_m}{\partial X_L}\frac{\partial y_m}{\partial X_K}\right) \\
y_{m,K} y_{k,L} d_{mk} = \frac{1}{2}\left(u_{k,K} y_{k,L} + u_{m,L} y_{m,K}\right)
\end{gathered} \tag{64}$$

We interchange the index k to m in $u_{k,K}$.

$$\begin{gathered}
y_{m,K} y_{k,L} d_{mk} = \frac{1}{2}\left(u_{m,K} y_{k,L} + u_{m,L} y_{m,K}\right) = \frac{1}{2}\left(\dot{y}_{m,K} y_{k,L} + \dot{y}_{m,L} y_{\mathrm{m},K}\right) \\
y_{m,K} y_{k,L} d_{mk} = \frac{1}{2}\frac{d}{dt}\left(y_{m,K} y_{k,L}\right) = \frac{d}{dt}\left[\frac{1}{2}\left(y_{m,K} y_{k,L} - \delta_{KL}\right)\right]
\end{gathered} \tag{65}$$

With $m = k$ the term $\frac{1}{2}\left(y_{m,K} y_{k,L} - \delta_{KL}\right)$ is known as the finite strain tensor E_{KL} in the reference state, and $\underset{\sim}{E}$ whit an uppercase index will represent the electric field vector in the reference state. Then, we reduce $y_{m,K} y_{k,L} d_{mk}$ to:

$$y_{m,K} y_{k,L} d_{mk} = \frac{d}{dt}(E_{KL}) = \dot{E}_{KL} \tag{66}$$

Substituting Eq. (66) into Eq. (63), we obtain

$$\rho\dot{\chi} = J^{-1} T^S{}_{KL} \dot{E}_{KL} - J^{-1} P_K \dot{W}_K \tag{67}$$

Factoring the inverse of Jacobian, we get

$$\rho\dot{\chi} = J^{-1}\left(T^S{}_{KL} \dot{E}_{KL} - P_K \dot{W}_K\right) \tag{68}$$

Multiplying both sides into Eq. (68) by the Jacobian gives

$$J\rho\dot{\chi} = JJ^{-1}\left(T^S{}_{KL} \dot{E}_{KL} - P_K \dot{W}_K\right) \tag{69}$$

Using mass transformation to the reference state $\rho^0 = \rho J$ into Eq. (69), we get a new equation for energy conservation in terms of physical quantities in the reference state. Symmetric tensor $T^S{}_{KL}$, strain tensor E_{KL}, polarization P_K and gradient of potential W_K.

$$\rho^0 \dot{\chi} = T^S{}_{KL} \dot{E}_{KL} - P_K \dot{W}_K \tag{70}$$

2.3 Constitutional equations from free energy

The conservation laws are valid for any piezoelectric material, including cement-based composites. However, a specific material's piezoelectric properties are determined by a set of functions that describes free energy, symmetric tensor, and polarization. Once we replace these functions into Eq. (70), we will get the piezoelectricity's constitutional equations. Take into account Eq. (70), we can propose the next dependence to the functions

$$\begin{aligned} \chi &= \chi(E_{KL}, W_K) \\ T^S{}_{KL} &= T^S{}_{KL}(E_{KL}, W_K) \\ P_K &= P_K(E_{KL}, W_K) \end{aligned} \tag{71}$$

Derivation respect to time the free energy into Eq. (71) as follow

$$\dot{\chi} = \frac{\partial \chi}{\partial E_{KL}} \dot{E}_{KL} + \frac{\partial \chi}{\partial W_K} \dot{W}_K \tag{72}$$

Substituting Eq. (72) into Eq. (70), we obtain

$$\rho^0 \frac{\partial \chi}{\partial E_{KL}} \dot{E}_{KL} + \rho^0 \frac{\partial \chi}{\partial W_K} \dot{W}_K = T^S{}_{KL} \dot{E}_{KL} - P_K \dot{W}_K \tag{73}$$

Both sides of Eq. (73) were compared to deduce two transformations, which resulting symmetric tensor $T^S{}_{KL}$ and polarization P_K. The transformations use free energy as a generating function, as shown in Eq. (75).

$$T^S{}_{KL} = \rho^0 \frac{\partial \chi}{\partial E_{KL}} \tag{74}$$

$$P_K = -\rho^0 \frac{\partial \chi}{\partial W_K} \tag{75}$$

The mathematical structure of the free energy function will define the order of constitutional equations. There are functions for the free energy of piezoelectric materials from order 1 to order 3 [15]. It means that piezoelectric material behavior depends on the free energy function and its parameters. Here is an example of free energy function with order three

$$\begin{aligned} \rho^0 \chi = \ & \frac{1}{2} c_{ABCD} E_{AB} E_{CD} - e_{ABC} W_A E_{BC} - \frac{1}{2} \chi^E{}_{AB} W_A W_B + \frac{1}{6} c_{ABCDEF} E_{AB} E_{CD} E_{EF} \\ & + \frac{1}{2} d_{ABCDE} W_A E_{BC} E_{DE} - \frac{1}{2} b_{ABCD} W_A W_B E_{CD} - \frac{1}{6} \chi^E{}_{ABC} W_A W_B W_C \\ & + \frac{1}{24} c_{ABCDEFGH} E_{AB} E_{CD} E_{EF} E_{GH} + \frac{1}{6} d_{ABCDEFG} W_A E_{BC} E_{DE} E_{FG} \\ & + \frac{1}{4} a_{ABCDEF} W_A W_B E_{CD} E_{EF} + \frac{1}{6} d_{ABCDE} W_A W_B W_C E_{DE} \\ & - \frac{1}{24} \chi^{\mathrm{E}}{}_{ABCD} W_A W_B W_C W_D + \ \dots, \end{aligned} \tag{76}$$

The parameters are called elasticity c, piezoelectric e, electric permeability χ^E, odd electrolytic d, electrostrictive b, and electroelastic force even a.

2.3.1 The linear approach of piezoelectricity

We take on order one approach from Eq. (76) to free energy χ. Then, replacing it in Eq. (74) and Eq. (75) to obtain

$$T^S{}_{AB} = \frac{\partial}{\partial E_{AB}}\left(\frac{1}{2}c_{ABCD}E_{AB}E_{CD} - e_{ABC}W_A E_{BC} - \frac{1}{2}\chi^E{}_{AB}W_A W_B\right) \tag{77}$$

$$P_A = -\frac{\partial}{\partial W_A}\left(\frac{1}{2}c_{ABCD}E_{AB}E_{CD} - e_{ABC}W_A E_{BC} - \frac{1}{2}\chi^E{}_{AB}W_A W_B\right) \tag{78}$$

The approximation is possible if we consider an infinitesimal deformation, weak electric field, and low amplitude displacements around the reference state. Hence, it approaches require a nomenclature exchange for physical quantities. Thus, second Piola-Kirchhoff stress will be replaced by infinitesimal Cauchy stress tensor $T^S{}_{KL} \rightarrow T_{ij}$; finite strain tensor will be exchanged by infinitesimal strain tensor $E_{KL} \rightarrow S_{kl}$; potential gradient, polarization, and displacement electric vector are similar either reference or current state: $W_K \rightarrow E_k$, $P_L \rightarrow P_i$, and $\mathcal{D}_L \rightarrow D_i$. Then, Eqs. (77) and (78) follow:

$$T_{ij} = \frac{\partial}{\partial S_{ij}}\left(\frac{1}{2}c_{ijkl}S_{ij}S_{kl} - e_{ijk}E_i S_{jk} - \frac{1}{2}\chi^E{}_{ij}E_i E_j\right) \tag{79}$$

$$P_i = -\frac{\partial}{\partial E_i}\left(\frac{1}{2}c_{ijkl}S_{ij}S_{k1} - e_{ijk}E_i S_{jk} - \frac{1}{2}\chi^E{}_{ij}E_i E_j\right) \tag{80}$$

Here is considering symmetry to parameters elastic c_{ijkl}, piezoelectric e_{kij}, and electric χ_{ik} when they have odd permutations. Differentiating the Eq. (79) and Eq. (80), we obtain

$$T_{ij} = c_{ijkl}S_{kl} - e_{kij}E_k \tag{81}$$

$$P_i = e_{ikl}S_{kl} + \chi^E{}_{ik}E_k \tag{82}$$

The polarization can be written in terms of electric displacement vector too.

$$P_i = D_i - \varepsilon_0 E_i \tag{83}$$

From Eq. (83) into Eq. (82) gives

$$D_i - \varepsilon_0 E_i = e_{ikl}S_{kl} + \chi^E{}_{ik}E_k \tag{84}$$

Solving D_i,

$$D_i = e_{ikl}S_{kl} + \varepsilon_0 E_i + \chi^E{}_{ik}E_k = e_{ikl}S_{kl} + \varepsilon_0\delta_{ik}E_i + \chi^E{}_{ik}E_k \tag{85}$$

Factoring E_k,

$$D_i = e_{ikl}S_{kl} + \left(\varepsilon_0\delta_{ik} + \chi^E{}_{ik}\right)E_k \tag{86}$$

where the term $\varepsilon_0\delta_{ik} + \chi^E{}_{ik}$ is defined as dielectric constant ε_{ik}. Finally, we have the linear constitutional equation for the electric displacement vector.

$$D_i = e_{ikl}S_{kl} + \varepsilon_{ik}E_k \tag{87}$$

We have seen several forms to present the linear constitutional equations in piezoelectricity. Next, we include another form of constitutional equations shown in the IEEE standard for piezoelectricity. It can be obtained inverting the matrix formed by Eq. (81) and Eq. (82).

$$D_i = d_{ikl} S_{kl} + \varepsilon_{ik}^{T} E_k \tag{88}$$

$$S_{ij} = s_{ijkl}^{D} T_{kl} + g_{kij} D_k \tag{89}$$

The electromechanical properties are defined by piezoelectric charge d_{ikl} and voltage g_{kij} constants. Unlike parameters c_{ijkl} and e_{ikl} These new piezoelectric constants are taken out directly from experiments, as shown in the next section.

2.4 Electromechanical and electrical properties of cement-based composites

Incorporating piezoelectric nanocomposites into cement paste improves its piezoelectric and mechanical properties [16] due to increased deformable crystal structures. Zeolites, oxides, and carbon nanotubes are the most used cement-based composites to improve these properties [17]. Chen et al. also report some piezoelectric parameters of cement-based composites such as piezoelectric charge d_{33}, voltage g_{33}. And the coupling factor K_t. As was mentioned in the previous section, these piezoelectric parameters come from linear piezoelectricity theory. However, the crystalline structure of Calcium Silicate Hydrate (C-S-H) that compose the cement is a complex system described by linear theory. It could also be combined with statistical physics and mean-field homogenization theory tools to get the macroscale properties [18]. Here are show piezoelectric and electrical parameters of gold nanoparticles mixed to cement paste, which we hope to lead to our system's constitutional equations.

Next, we introduce a brief description of the gold nanoparticles' physical synthesis [19, 20]. They are produced by laser ablation at 532 nm. A gold plate at 99.9999% purity is put inside a beaker filled with 50 mL of ultrapure water. Then, the pulse laser spot with an energy of 30 mJ beats the gold plate by 10 minutes, as shown in **Figure 3**.

At the time, the gold nanoparticles were brought to be characterized by dynamical light scattering (DLS). If not done quickly, the gold nanoparticles were agglomerated. These measures are required because the gold nanoparticles directly affect the piezoelectric properties of cement cylinders. Some results of gold nanoparticle sizes are shown in **Figure 4**.

Also, the gold nanoparticles in water must be mixed quickly with the cement. The ratio of water/cement used was 0.47 mL/g. Then, the admixture was poured into cylindrical molds that contained copper wires as follows in **Figure 5**.

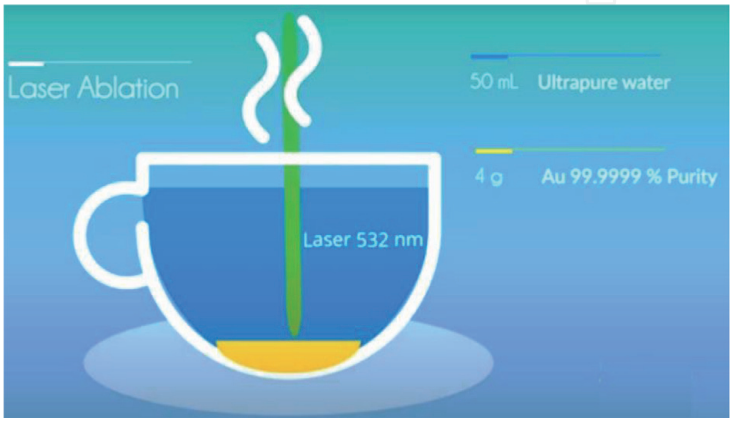

Figure 3.
Scheme of nanoparticle physical synthesis by laser ablation.

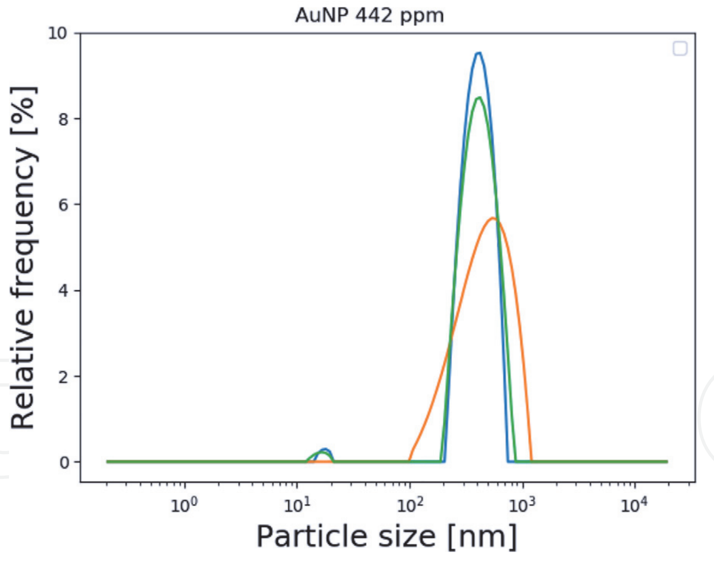

Figure 4.
The particle size distribution of gold nanoparticles suspended in water to concentration 442 ppm.

Figure 5.
Molds and dimensions of cement cylinders.

The cement cylinders were dried one day. Then it leaves curing for 28 days and finally to thermal treatment one day more. After 14 days, electromechanical measurements were performed, as shown in **Figure 6**.

Electromechanical measurements consist of two measurements performed in parallel: the cement cylinders under compressive strength test in the axial direction, open circuit potential (OCP) measurements in the electrodes of cement cylinders. From mechanical and electrical data, we calculated an electroelastic parameter with units $[mV/kN]$, it has the same interpretation of piezoelectric parameter e in linear theory. From **Figure 7**, an example of voltage-force curves for identically cement samples with gold nanoparticles is shown. We did get from the above measurements the axial elasticity parameter:

$$Y = 323.5 \pm 75.3 \left[kN/m^2\right] \tag{90}$$

The axial piezoelectric parameter:

$$\gamma = -20.5 \pm 6.9 \, [mV/kN]. \tag{91}$$

For a total deformation $S = 0.57 \pm 0.09 \, [mm]$ in the axial direction.

The electrical properties of cement cylinders were obtained from the imaginary part of impedance; an example of these curves in **Figure 8**. From impedance data

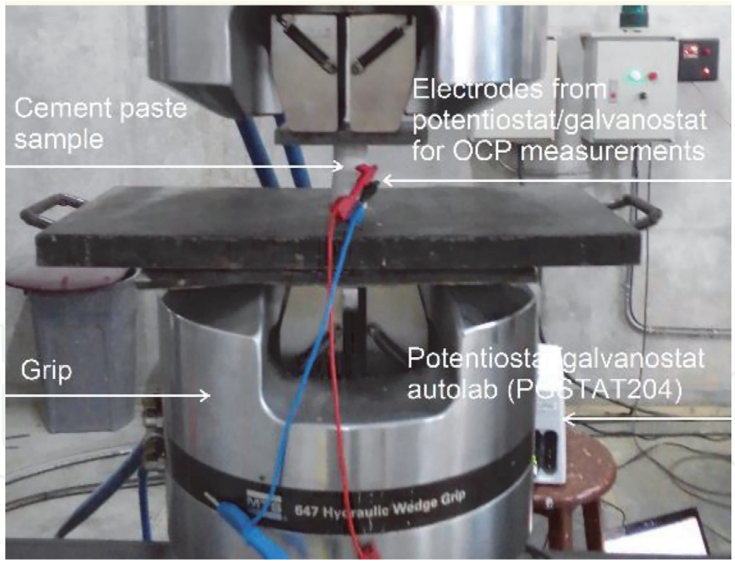

Figure 6.
Experimental setup of electromechanical measurements.

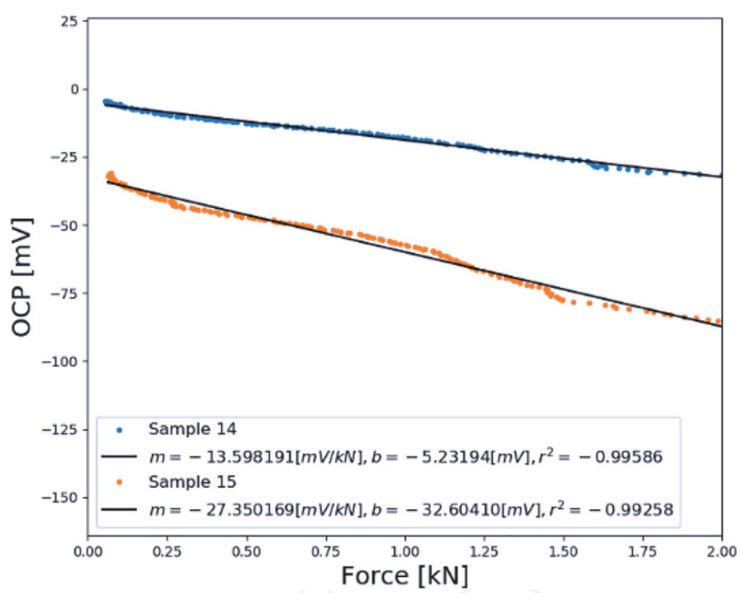

Figure 7.
OCP-force curves from cement cylinders with gold nanoparticles concentrated to 658 ppm.

can perform a transformation to get a real part of the capacitance C'. It has frequency dependence as follow

$$C'(\omega) = \frac{1}{\omega Z''} \tag{92}$$

The geometry of copper electrodes (an approximation to parallel plates) is related to capacitance. Therefore, we can calculate the dielectric parameter ε since 1 MHz; this parameter is a real number that depends on the frequency and is given by

$$\varepsilon(\omega)\varepsilon_0 = \frac{d * C'}{A} \tag{93}$$

where ε_0 is the electric permittivity of free space, A is the transversal section, and d is the thickness between electrodes.

From the data in **Figure 8** and Eq. (92) and Eq. (93), we obtain the dielectric constant:

$$\varepsilon = (939.6 \pm 82.9)\varepsilon_0 \tag{94}$$

Where ε_0 has unit $[F/m]$. The piezoelectric and electrical properties of cement paste mixed with gold nanoparticles exhibit reproducibility and linearity of the piezoelectric parameter.

2.5 Future studies and remarks

The Piezoelectric parameters are an initial point to beginning a new connection with piezoelectricity theory by inverse modeling and constructing new free energy functions and constitutional equations. To catch out with researchers in this scope, we suggest thinking about the next research questions; how is the piezoelectric parameter presented related to the piezoelectric parameter formulated by linear theory for piezoelectricity? Is the free energy function of order one sufficient to describe cement paste's piezoelectric with gold nanoparticles? How to develop a new function for free energy that models cement paste's piezoelectric behavior of cement paste with gold nanoparticles?

In this chapter, we have intended to contribute to the theory of piezoelectricity for large deformations without including an energy function. **Figure 9** shows a possible use around IoT as intelligent sensing of devices based on cement-based composites' piezoresistivity. Without reaching into depth in the technical and engineering aspect that smart construction, active sensing system entails; we highlight how the Eqs. (88) and (89) that relate the electromechanical properties and that are defined by piezoelectric charge d_{ikl} and voltage g_{kij} constants are present as indicators to improve the detection resolution in large structures with large deformations.

The sensors analyze the deformations, temperature, relative humidity, and other critical parameters of the concrete in real-time. This data is captured via wireless communication (WAN/BLE) and deployed on a secure and scalable platform (Cloud) capable of collecting data to facilitate remote decision making with

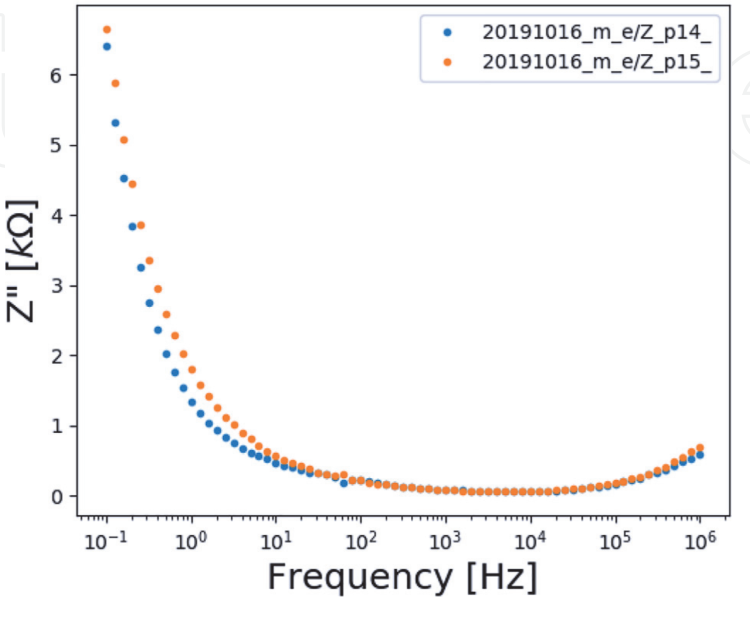

Figure 8.
The imaginary part of electrical impedance represented in a Bode plot was performed on two cement cylinders with gold nanoparticles concentrated to 658 ppm.

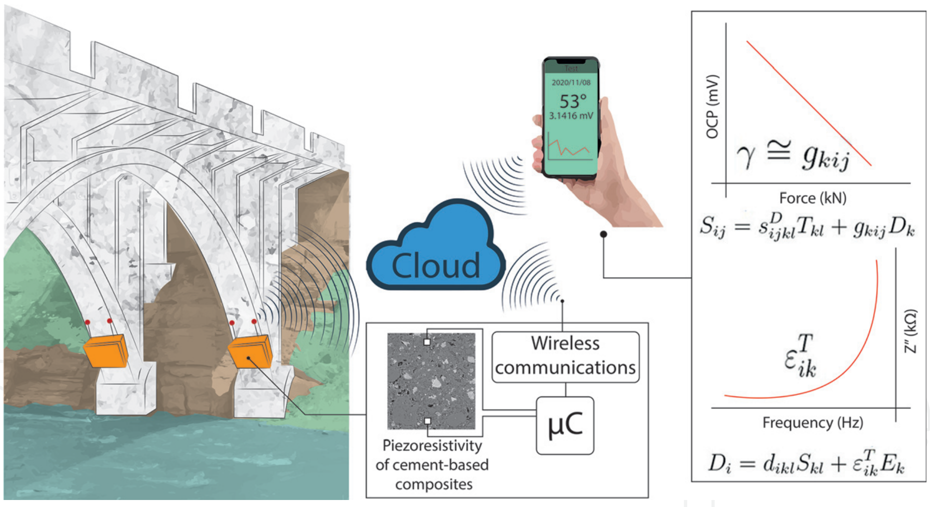

Figure 9.
The image shows a network of IoT sensors based on cement-based composites piezoresistivity as an active part of smart construction.

information from deep within the concrete. The experimental control of the NPs embedded within the cement paste's dispersions and piezoresistive responses is essential to have a good signal-to-noise ratio within the sensing. Knowing the coupling between the electromechanical equations from a theoretical approach is another crucial factor in making viable these technological solutions.

3. Conclusions

This chapter proposed a mathematical physicist construction of the linear theory of piezoelectricity since classical movement laws and the conservation of their physical quantities (mass, charge, linear momentum, angular momentum, and energy) over time. This construction takes parts of Eringen, Tiersten, and Yang's research without including the variational formulation or energy functional to deduce the constitutional equations. We have also presented some results of piezo-electric and dielectric constants obtained for cement mixed to gold nanoparticles. We got the axial elasticity parameter $Y = 323.5 \pm 75.3\ [kN/m^2]$, the electroelastic parameter $\gamma = -20.5 \pm 6.9\ \ [\mathrm{mV/kN}]$, and dielectric constant $\varepsilon = (939.6 \pm 82.9)\varepsilon_0[F/m]$, which can be compared with parameters s^D_{ijkl}, g_{kij} and ε^T_{ik} respectively presents into constitutional equations discussed in the chapter.

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Conflict of interest

The authors declared no potential conflicts of interest concerning the research, authorship, and/or publication of this book chapter.

Appendices and nomenclature

In the reference state, the continuum has a volume V, and mass density ρ^0.

In the current state, the continuum has a volume v, mass density ρ, electronic charge density μ^e and lattice charge density μ^l. Besides, In the current state with infinitesimal displacement η, the electronic charge does not change its volume.

The capital letter in the index is for the reference state X_K And the lowercase letters to the current state y_i. Also, the index in the physics quantities can denote a vector. For example X_K, y_i, u_i; or a tensor, for example E_{KL}, τ_{jk}. Another form to present a vector quantity is the right-pointing arrow $\vec{y}$.

The velocity of the continuum is denoted by lower case letter u, and just makes sense in the current state.

The partial derivate is denoted by comma separation in the indexes.
For example $y_{i,i}$.

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