Chapter

Analysis and Two-Dimensional Modeling of Directional Coupler Based on Two Coplanar Lines

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Abstract

This chapter is dedicated to physical modeling and numerical characterization of directional coupler based on two coplanar lines using the general theory of coupled lines. The modeling in this chapter is two-dimensional due to the chosen numerical method (MOMs), for that purpose the analysis is divided into steps, we started by analyzing and modeling a micro-coplanar line in the quasi-TEM approximation using Green's functions and the integral equation method then we conclude by using the telegraphist equations and the results of the first step to modeling a couple of micro-coplanar lines.

Keywords: micro-coplanar lines CPW, directional coupler, Green's functions, integral equation method, MOMs, quasi-TEM approximation, general theory of coupled lines

1. Introduction

It is well known that MICs are based on the use of the technology of planar circuits printed partially or entirely, on a flat surface of a dielectric, by an etching operation. The entire circuit can be produced in large numbers at low cost by photolithography.

The technical characteristics of MICs are their small size, their low weight, and their high reliability.

At the end of the 1970s, the advancement of planar circuit technology coupled with the rapid development of microwave semiconductor components, particularly MESFET on gallium arsenide and advances in manufacturing materials technologies, were at the origin of the emergence of microwave monolithic integrated circuit (MMIC) technology. In this technology, the passive and active circuits and their interconnections are produced in large numbers on the same substrate.

In MIC technology, the structure of the planar waveguide consists of block elements according to the development of various functional components or subsystems. The study of planar waveguide structures was an important research subject in the field of MIC circuits. in recent years, the explosive development of commercial microwave applications for the general public, has considerably increased research activities in this field on the one hand, to explore the various new configurations of planar circuits, on the other hand to precisely characterize their electrical performance.

Planar transmission lines are the most essential point in PCM circuits. in the late sixties, with the availability of dielectrics with high dielectric constants of low loss dielectric materials and with the increasing demand for miniaturized microwave circuits for space applications. The intensity of interest in micro-ribbon circuits was renewed. This resulted in the rapid development of the use of micro-stipe lines.

At that time, two other types of planar transmission lines were also invented: they are slotted lines, and coplanar (CPW) respectively proposed by S.B. Cohn and C.P. Wen.

With the growth in operating frequencies, particularly in the millimeter band, the use of the traditional microstrip line becomes problematic because of the increase in losses, the presence of higher order modes and parasitic couplings. In this regard, the interest in uniplanar waveguide structures, using only a one substrate face, was renewed.

Uniplanar transmission structures include the coplanar lines which is modeled in this chapter, the slotted lines, and the two-ribbons coupled lines. These structures have many advantages over microstrip lines, such as easy production of successive parallel connections of passive or active components without the need to resort to metallized holes to a ground plane, low dispersion [1, 2].

The physical modeling and then the numerical characterization of these planar transmission lines have been an important axis in the field of scientific research in recent years. Many techniques and numerical methods have been developed and used for the numerical characterization of uniplanar structures. In general, the numerical methods for the study of MMICs can be classify into two groups: the first includes the integral methods like MOMs, NEWTON-COTES FORMULAS, … and the other group is derivative like the FDTD, TLM … DF method.

The analysis in this chapter is devoted to modeling a directional coupler in CPW technology by modeling one and two micro-coplanar lines on a dielectric substrate, the problem starts with solving the poison's equation which is transformed to the integral equation in which the unknown is the charge density ρ on the metal strip, by using the method of the Green's function [3] and the conformal mapping [4] technique. Subsequently we apply the numerical method (MOMs) [5] to solve the integral equation for obtaining the unknown function which is used to determine the variation of capacitance C, the value of the characteristic impedance Zc, and the effective permittivity εeff.

The second part of this analysis is to search the expression of the characteristic impedance for the odd Z_{co} and even Z_{ce} modes as a function of the geometric dimensions of the directional coupler in CPW technology and the coupling coefficient K as a function of the gap ? between the two strip lines by using the general theory of coupled lines (telegraphist equations) and the results of the first part.

2. Statement of the problem in quasi-TEM mode

In this part we focused to the formulation of the problem studied by determining the different characteristics of one and two micro-coplanar lines in the quasi-TEM approximation, using the integral equation method using the Green's function technique and the conformal mapping. The integral equation method is suitable for planar structures and it's most used to solve the electromagnetic problems.

The problem starts with solving the poison's equation Eq. (1) to obtaining the linear charge density ρ on the central metal strip shown in **Figure 1**.

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\psi(x, y) = -\frac{1}{\varepsilon}\rho(x, y) \tag{1}$$

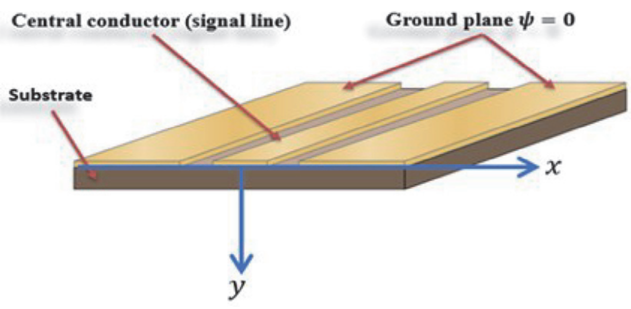

Figure 1.
Micro-coplanar line.

With $\rho(x, y) = \rho(x)$ for $y = 0$ and $x \in$ strip and $\rho(x, y) = 0$ elsewhere.

The determination of $\rho(x, y = 0)$ makes it possible to evaluate the capacity C per unit length of the coplanar line, and that will allow us to determine the other characteristic parameters.

To solve the poison's equation, we inverting the Laplacian operator using the Green's function to the integral operator Eq. (2) to form the integral equation:

$$\psi(x, y) = \frac{1}{\varepsilon}\int G(x, y|x_0, y_0)\rho(x)dl \tag{2}$$

The determination of the Green's function G corresponding to the studied problem constitute the most delicate step. Once this function is obtained, the second step is to solve numerically the integral equation by the method of MOMs.

3. Determination of the Green's function

This function used to form the integral equation Eq.(2), thereby it represents the inverse of Laplacian operator Eq.(3), where the point (x, y) is said field point created by a unit charge $(1C)$ at the point (x_0, y_0) said source point.

$$G_0(x, y|x_0, y_0) = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)^{-1} \tag{3}$$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)G_0(x, y|x_0, y_0) = -\delta(x - x_0)(y - y_0) \tag{4}$$

With δ is the Dirac function, so the Green's function G_0 is written in the following form:

$$G_0(x, y|x_0, y_0) = -\frac{1}{2\pi}\, ln\sqrt{(x - x_0)^2 + (y - y_0)^2} + cte \tag{5}$$

To calculate the Green's function corresponds to the coplanar line without the central strip shown in **Figure 2a**, first we calculate the Green's function of the electrical potential created by a distribution of charges between two infinite ground planes shown in **Figure 2b** using the multiple image method [2], given by [3] as follow:

$$G_0(x, y|x_0, y_0) = -\frac{1}{4\pi}\sum_{n=-\infty}^{n=+\infty} ln\left(\frac{(y - y_0)^2 + (x - x_0 - 2na)^2}{(y - y_0)^2 + (x + x_0 - 2na)^2}\right) \tag{6}$$

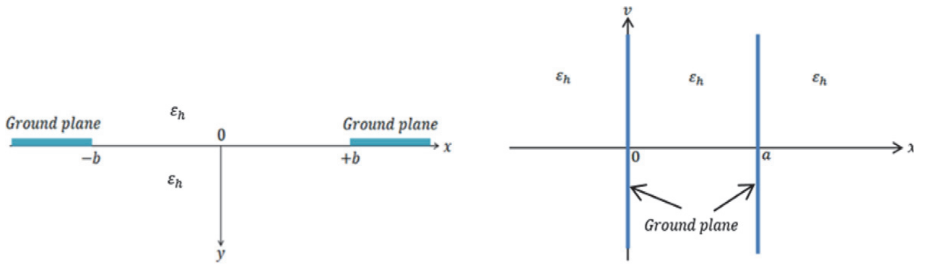

Figure 2.
(a) Micro-coplanar line without central conductor; (b) structure with two infinite ground planes.

With a is the gap between the two infinite metal plate **Figure 2a**, its sum given by [6] as:

$$G_0(x, y|x_0, y_0) = -\frac{1}{4\pi}\ ln\left(\frac{sin^2\frac{\pi}{2a}(x - x_0) + sh^2\frac{\pi}{2a}(y - y_0)}{sin^2\frac{\pi}{2a}(x + x_0) + sh^2\frac{\pi}{2a}(y - y_0)}\right) \tag{7}$$

The next step consists of applying a suitable conformal mapping which makes it possible to transform the Green's function into the given structure in **Figure 2a**, it's given as:

$$G(x, y|x_0, y_0) = -\frac{1}{4\pi}\ ln\left(\frac{sin^2\left(\frac{1}{2}\left(cos^{-1}\left(\frac{x}{b.z}\right) - cos^{-1}\left(\frac{x_0}{b.z_0}\right)\right)\right) + sh^2\left(\frac{1}{2}\left(ch^{-1}(z) - ch^{-1}(z_0)\right)\right)}{sin^2\left(\frac{1}{2}\left(cos^{-1}\left(\frac{x}{b.z}\right) + cos^{-1}\left(\frac{x_0}{b.z_0}\right)\right)\right) + sh^2\left(\frac{1}{2}\left(ch^{-1}(z) - ch^{-1}(z_0)\right)\right)}\right) \tag{8}$$

With: $z = \left(\frac{\alpha+\beta}{2}\right)^{1/2}$; $z_0 = \left(\frac{\alpha_0+\beta_0}{2}\right)^{1/2}$; and $\alpha = \left(\frac{x}{b}\right)^2 + \left(\frac{y}{b}\right)^2$; $\beta = \left(\alpha^2 - 4\left(\frac{x}{b}\right)^2\right)$.

The Green's function Eq. (8) is also a reciprocal function [7], and it's can be expressed as the superposition of two functions for non-homogeneous [8] middle, and for isotropic or anisotropic [9–10] middles.

So, for a non-homogeneous middle shown in the **Figure 3**, the Green's function is written as [3]:

$$G(x, y|x_0, y_0) = -\frac{1}{4\pi}\left(ln\left(\frac{sin^2(R^-) + sh^2(T^-)}{sin^2(R^+) + sh^2(T^-)}\right) + R.\,ln\left(\frac{sin^2(R^-) + sh^2(T^+)}{sin^2(R^+) + sh^2(T^+)}\right)\right) \tag{9}$$

With: $R^{\pm} = \frac{1}{2}\left(cos^{-1}\left(\frac{x}{b.z}\right) \pm cos^{-1}\left(\frac{x_0}{b.z_0}\right)\right)$; $T^{\pm} = \frac{1}{2}\left(ch^{-1}(z) \pm ch^{-1}(z_{0.})\right)$

And: $R = \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_1 + \varepsilon_2}$.

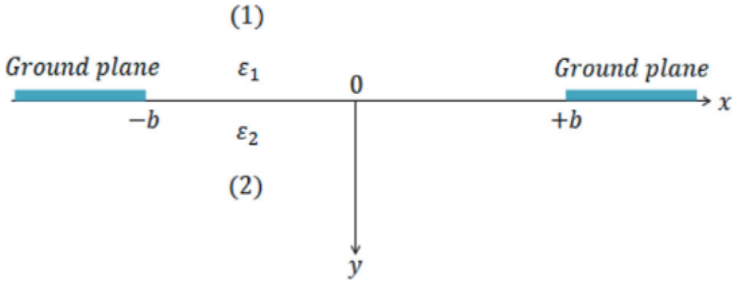

Figure 3.
Micro-coplanar (non-homogeneous middle).

4. Numerical characterization of the integral equation

The linear charge density ρ on the metal strip shown in **Figure 4** is related to the distribution potential ψ in the cross-section of the coplanar line, and it's zero elsewhere. For that reason, the integral equation Eq. (2) can be written as:

$$\psi(x,y) = \frac{1}{\varepsilon}\int_{-w/2}^{w/2} G(x,y|x_0,y_0).\rho(x_0,y_0)dl_0 \tag{10}$$

The central strip conductor is considered infinitely thin, that allows us to write:

$$\psi(x) = \frac{1}{\varepsilon}\int_{-w/2}^{w/2} G(x|x_0).\rho(x_0)dx_0 \tag{11}$$

Assuming that the central strip is submitted to a unit potential, so the equation Eq. (11) is convenient to solve numerically using the method of moments. This method is divided into two stages, firstly by developing $\rho(x_0,y_0)$ in series of N basic functions Eq. (12) in the form of rectangular pulses Eq. (13), secondly by using the Galerkin procedure which allows to write equation Eq. (11) as a linear equations system Eq. (14):

$$\rho_j(x_0) = \sum_{j=1}^{N} \alpha_j f_j(x_0) \tag{12}$$

$$f_j(x_0) = \begin{cases} 1 & if\ x \in \left[x_j - \frac{h}{2}; x_j + \frac{h}{2}\right] \\ 0 & alsewhere \end{cases} \tag{13}$$

$$\sum_{i=1}^{N} \psi_i(x_0) = \sum_{i=1}^{N}\sum_{j=1}^{N} \frac{\alpha_j}{\varepsilon_e}\int_{x_j-\frac{h}{2}}^{x_j+\frac{h}{2}} G(x_i|x_j)dx_0 \tag{14}$$

So, the linear equations system (14) can be written in the following matrix form:

$$[\alpha_j] = [M_{ij}]^{-1}.[\psi_j] \tag{15}$$

With $[M_{ij}]$ is the matrix of the Green's function obtained by the inversion of the linear equations system Eq. (14), defined on the central strip of the structure

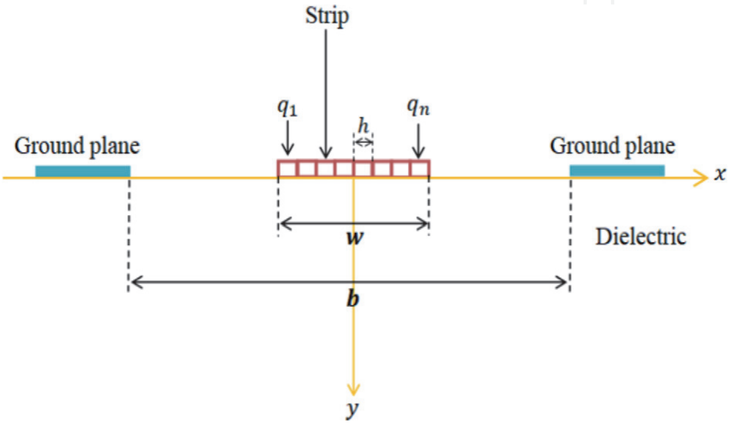

Figure 4.
Discretization of the central strip.

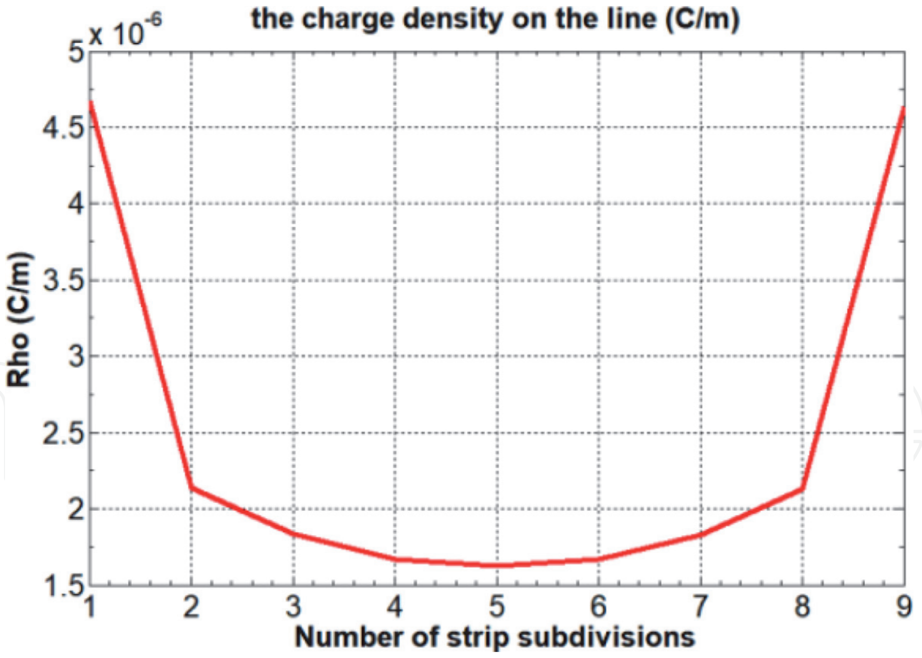

Figure 5.
Variation of the charge density on the metal strip.

studied show in **Figure 4**. And **Figure 5** shows the variation of the charge density $\rho_j(x_0)$ (solution of the integral equation) as a function of subdivisions N of the central strip of with w.

5. Characteristics of the coplanar line

In this part we present the numerical results of the variation of the characteristic impedance Z_c of the transmission line shown in **Figure 6** and of the variation of the effective dielectric permittivity ε_{eff} as a function of the ratio w/b shown in **Figure 7**. The line is assumed to be lossless $(R = G = 0)$, then its characteristic impedance is given by:

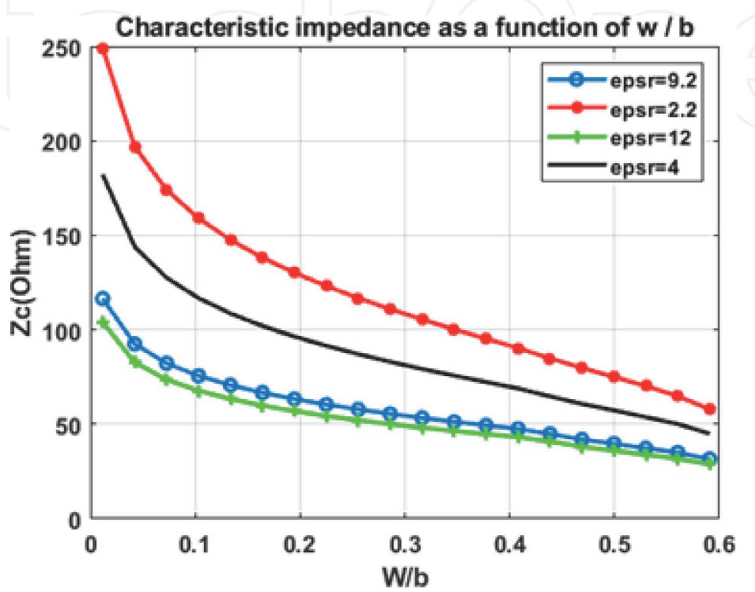

Figure 6.
Variations of the characteristic impedance for different ε_{eff}.

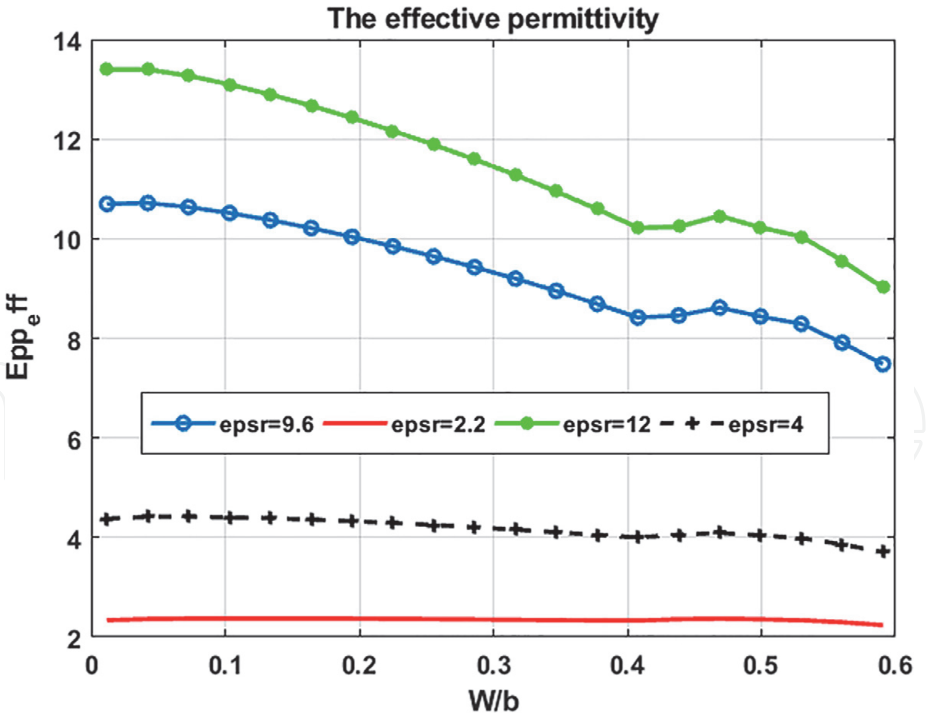

Figure 7.
Variations of the effective permittivity.

$$Z_c = \sqrt{\frac{L}{C}} = v_p L = \frac{1}{v_p C} \tag{16}$$

$$C = \frac{\sum_{j=1}^{N} \rho_j}{\psi} \tag{17}$$

$$L = \frac{1}{v_{pa}^2 C} = \frac{1}{\varepsilon_0 \mu_0 C} \tag{18}$$

$$\varepsilon_{eff} = \left(\frac{v_p}{v_{pa}}\right)^2 \tag{19}$$

With v_p is the propagation velocity in the coplanar line and v_{pa} is the propagation velocity in the air, while L and C is the is the inductance and capacity per unit length of the micro-coplanar line. And ψ is the unit potential.

6. Analysis of a directional coupler in CPW technology

A coupled coplanar line configuration consists of two transmission lines placed parallel to each other and in proximity as shown in **Figure 8**. In such a configuration there is a continuous coupling between the electromagnetic fields of the two lines. Coupled lines are utilized extensively as basic elements for coplanar directional coupler which is the subject of this study, filters, amplifiers, and a variety of other useful circuits.

Because of the coupling of electromagnetic fields, a pair of coupled lines can support two different modes of propagation. These modes have different characteristic impedances Z_{co} for odd mode, and Z_{ce} for even mode. The general theory of coupled lines (telegraphist equations) is used as method of analysis to determine those impedances for each mode of propagation, and to calculate the coupling coefficient K.

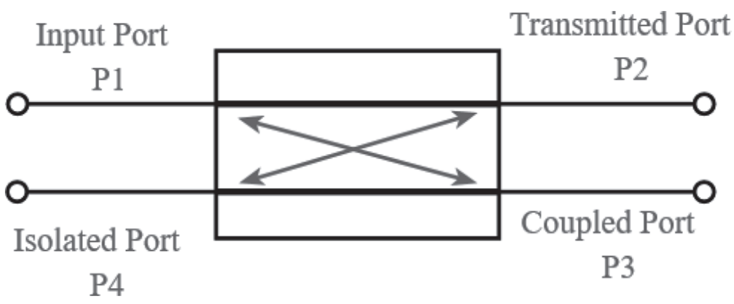

Figure 8.
Directional coupler.

We suppose that the propagation is along the axis OZ, the telegraphist equation is written [10]:

$$\begin{cases} -\frac{d}{dz}[V] = [Z].[I] \\ -\frac{d}{dz}[I] = [Y].[I] \end{cases} \tag{20}$$

With $[Z]$ is the impedance matrix and $[Y]$ represents the admittance matrix of the directional coupler. This system can be written as:

$$\begin{cases} -\frac{dv_1}{dz} = Z_1 i_1 + Z_m i_2 \\ -\frac{dv_2}{dz} = Z_m i_1 + Z_2 i_2 \\ -\frac{di_1}{dz} = Y_1 v_1 + Y_m v_2 \\ -\frac{di_2}{dz} = Y_m v_1 + Y_m v_2 \end{cases} \tag{21}$$

With Z_1, Z_2 are the impedances of the coupled lines per unit length, and Y_1, Y_2 their admittance, where Z_m, Y_m are the mutual impedance per unit length and the mutual admittance per unit length respectively.

The system Eq. (21) is a system of homogeneous first-order differential equations with constant coefficients. The resolution of this system gives the voltage and the current for even and odd propagated modes is as follow:

$$v_1 = \left(A_1 e^{-\gamma_e Z} + A_2 e^{-\gamma_e Z}\right) + \left(A_3 e^{-\gamma_o Z} + A_4 e^{-\gamma_o Z}\right) \tag{22}$$

$$v_2 = R_e\left(A_1 e^{-\gamma_e Z} + A_2 e^{-\gamma_e Z}\right) + R_o\left(A_3 e^{-\gamma_o Z} + A_4 e^{-\gamma_o Z}\right) \tag{23}$$

$$i_1 = Y_{e1}\left(A_1 e^{-\gamma_e Z} - A_2 e^{-\gamma_e Z}\right) + Y_{o1}\left(A_3 e^{-\gamma_o Z} - A_4 e^{-\gamma_o Z}\right) \tag{24}$$

$$i_2 = Y_{e2} R_e\left(A_1 e^{-\gamma_e Z} - A_2 e^{-\gamma_e Z}\right) + Y_{o2} R_o\left(A_3 e^{-\gamma_o Z} - A_4 e^{-\gamma_o Z}\right) \tag{25}$$

Where $R_{e,o}$ and $Y_{e1,2;o1,2}$ are functions depending on the impedance and admittance of the coupled line. As a result, the propagation constants of the two modes are expressed as a function of linear capacitances and inductances:

$$\begin{aligned} \gamma_{e,o} &= j\beta_{e,o} \\ &= j\frac{\omega}{\sqrt{2}}\sqrt{(L_1C_1 + L_2C_2 - 2L_mC_m) \pm \sqrt{(L_2C_2 - L_1C_1)^2 + 4(L_mC_1 - L_2C_m)(L_mC_2 - L_1C_m)}} \end{aligned} \tag{26}$$

$$R_{e,o} = \frac{(L_2C_2 - L_1C_1) \pm \sqrt{(L_2C_2 - L_1C_1)^2 + 4(L_mC_2 - L_1C_m)(L_mC_1 - L_2C_m)}}{2(L_mC_2 - L_1C_m)} \quad (27)$$

Therefore, the characteristic impedances of the two coupled lines for even and odd mode are:

$$Z_{ce} = \frac{\omega}{\beta_e}\left(L_1 - \frac{L_m}{R_o}\right) = \frac{\beta_e}{\omega}\left(\frac{1}{C_1 - R_eC_m}\right) \quad (28)$$

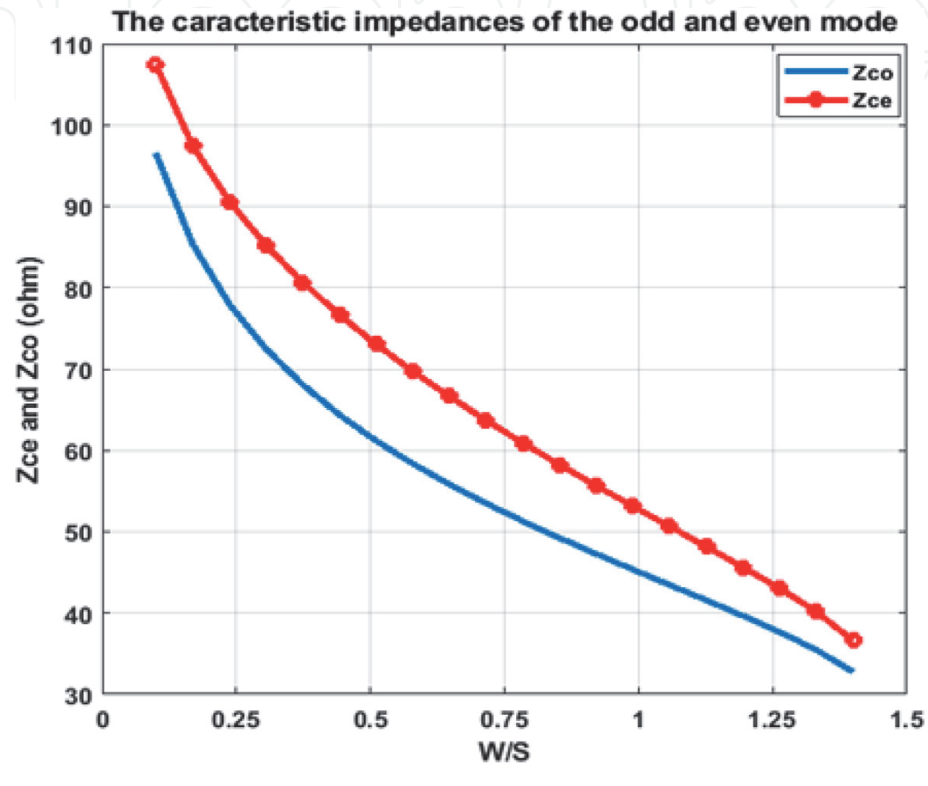

Figure 9.
Variations of the characteristic impedance for even and odd mode.

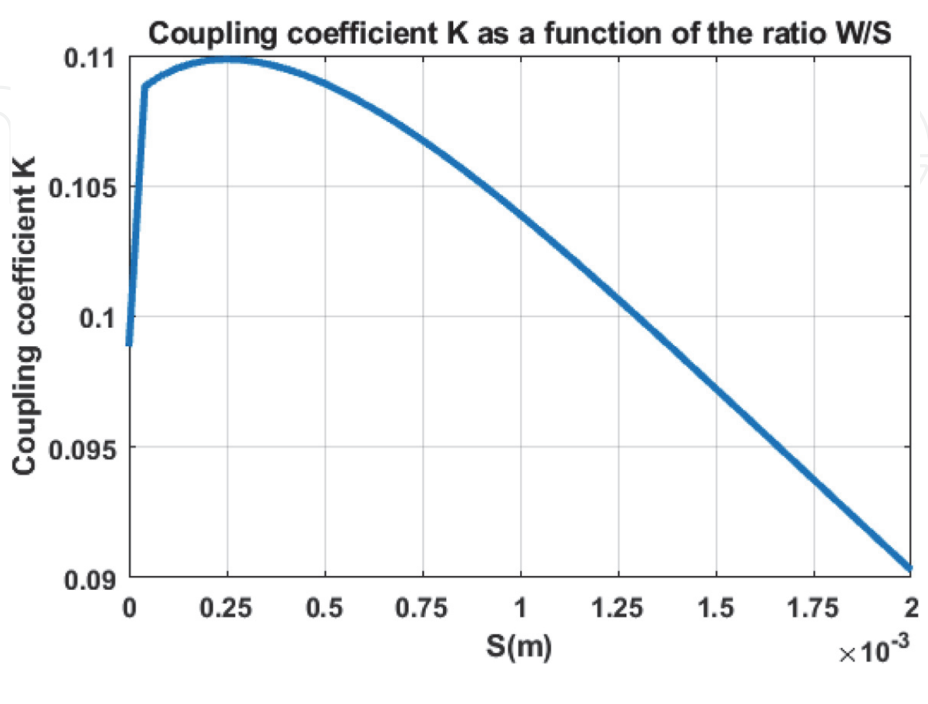

Figure 10.
Variations of the coupling coefficient.

$$Z_{co} = \frac{\omega}{\beta_o}\left(L_1 - \frac{L_m}{R_e}\right) = \frac{\beta_o}{\omega}\left(\frac{1}{C_1 - R_o C_m}\right) \tag{29}$$

For the coupling coefficient K, it is given by the following formula:

$$K = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}} \tag{30}$$

Figures 9 and **10** show the variations of Z_{co}, and Z_{ce} as a function of the ratio W / S, and the variation of K the coupling coefficient as a function of the gap S between the coupled lines respectively.

7. Conclusion

The analysis presented in this chapter has made it possible to modelized the most important characteristic parameters of a directional coupler in CPW (coplanar waveguide) technology, Such as the characteristic impedances for different modes of propagation: odd and even in quasi TEM mode by using the general theory of coupled lines. The directional coupler studied is based on two micro-coplanar lines in which we utilize the integral equation method (solved numerically by using the MOMs) associated with the Green's function, and conformal mapping method to describe the different characteristics of one and two micro-coplanar line.

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