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Symplectic Geometry and Its Applications on Time Series Analysis

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Abstract

This chapter serves to introduce the symplectic geometry theory in time series analysis and its applications in various fields. The basic concepts and basic elements of mathematics relevant to the symplectic geometry are introduced in the second section. It includes the symplectic space, symplectic transformation, Hamiltonian matrix, symplectic principal component analysis (SPCA), symplectic geometry spectrum analysis (SGSA), symplectic geometry mode decomposition (SGMD), and symplectic entropy (SymEn), etc. In addition, it also briefly reviews the applications of symplectic geometry on time series analysis, such as the embedding dimension estimation, nonlinear testing, noise reduction, as well as fault diagnosis. Readers who are familiar with the mathematical preliminaries may omit the second section, i.e. the theory part, and go directly to the third section, i.e. the application part.

Keywords: symplectic geometry, symplectic principal component analysis (SPCA), symplectic geometry spectrum analysis (SGSA), symplectic geometry mode decomposition (SGMD), symplectic entropy (SymEn), chaotic time series, embedding dimension, feature extraction

1. Introduction

From the viewpoint of mathematical systems, the time series observed in physics are usually regarded as coming from the Lagrangian systems, also called the conventional systems. The systems can be analyzed by the conventional Euclidean geometry [1]. However, the systems in practice are usually nonlinear and complex. Thus, a lot of interesting time series in nature are complex due to nonlinear phenomena derived from nonlinear dynamical systems [2]. The nonlinear dynamical systems have been described by Hamiltonian systems and dealt with by using symplectic geometry [3]. Symplectic geometry is an even dimensional geometry living on even dimensional spaces. Different from the conventional Euclidean geometry that measures 1-dimensional lengths and angles, the symplectic geometry studies the metric properties (such as area) and can preserve the system structure in the phase space [4]. Apart from applications on the classical dynamical systems to solve the equation problems, symplectic geometry has been also used on the studies of nonlinear time series [5–8].

According to Takens' embedding theorem, a time series can be reconstructed into an attractor in phase space [9]. The reconstructed attractor is a geometrical object that can reflect the underlying dynamical system. In order to better understand the nature of the underlying system, the attractor and its properties are characterized in the phase space by various mathematical methods, such as dimension, fractal geometry, Lyapunov exponent, entropy and symplectic geometry [1, 5, 10, 11]. For dimension, fractal geometry, Lyapunov exponent, entropy, there are a more extensive discussion with mathematical details in some research literatures [12–15]. Here, we only introduce how to apply symplectic geometry theory to extract the information from the reconstructed attractor and its application on physics, engineering and biomedical engineering.

2. Mathematical fundamental

2.1 Reconstruction of the system dynamics in phase space from a time series

The reconstruction from a time series of observation is the first and most crucial step in nonlinear time series analysis. It is also the basis of applications of symplectic geometry on time series analysis. Takens' embedding theorem allows us to reconstruct an equivalent attractor of the underlying dynamical system by embedding one time series. The theorem proves that the reconstructed attractor has the same dynamical characteristics as the attractor of the original system if the embedding dimension m is sufficiently large. Let a time series of observation x_1, x_2, \dots, x_n . n is the number of samples. The reconstructed attractor can be given in N -dimensional space R^N by the time-delay embedding [5]:

$$\mathbf{X} = (\mathbf{X}_1, \mathbf{X}_2, \dots, \mathbf{X}_m) = \begin{pmatrix} x_1 & x_2 & \dots & x_m \\ x_2 & x_3 & \dots & x_{m+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_N & x_{N+1} & \dots & x_n \end{pmatrix}, \quad (1)$$

where the number of dots in the attractor is $m = n - N + 1$, the embedding dimension is N . X is also called as the trajectory matrix of the original system in phase space. The corresponding program is given by matlab software as follows:

```
function matrixSignal = signalMatrix(x, N)
% -----Construct data matrix-----
%
% Synopsis:
% matrixSignal = signalMatrix(x, N)
%
% Description:
% It constructs a data matrix from a time series as a column vector, i.e., a
% reconstruction attractor.
%
% Input:
% x a time series with the length n.
% N [1x1] Output dimension; N > 1 (default N = dim);
```

```

%
% Outputs:
% matrixSignal [N x M] a data matrix (M = n-N + 1).
%
if nargin < 2, N = 2; end
n = length(x);
M = n - N + 1;

matrixSignal = zeros(N, M);
for i = 1:N
matrixSignal(i, :) = x(i:M + i - 1);
end
    
```

2.2 Hamilton matrix from the reconstructed attractor

In the symplectic spaces, Hamiltonian system is the analysis fundamental for the real physical processes [4, 5]. A real system should be first described by a suitable Hamiltonian system, i.e. an even dimensional matrix. For a time series, its Hamiltonian matrix H can be defined by using its reconstructed attractor X .

Definition 2.1 Let X be a d -dimensional matrix in a real number field R^d . The matrix \bar{X} can be given by removing the mean values of the columns of the X . We define the covariance matrix A of the matrix X :

$$A = \bar{X} \cdot \bar{X}^T. \quad (2)$$

Here, A is a $d \times d$ real number matrix.

Definition 2.2 For a $d \times d$ matrix A , the Hamiltonian matrix H can be defined:

$$H = \begin{pmatrix} A & \mathbf{0} \\ \mathbf{0} & -A^T \end{pmatrix}. \quad (3)$$

Here, H is a $2d \times 2d$ matrix.

2.3 Mathematical preliminaries in symplectic geometry

Symplectic geometry focuses on the study of area measure in symplectic space R^{2n} . Its basic concepts and basic properties are related but different from those of a Euclidean geometry (see **Table 1**).

In Euclidean space, the inner product is denoted as the measure of the length. The unit matrix is I , i.e. the main diagonal elements are 1, and the other elements are 0. Corresponding to the unit matrix I in Euclidean space, the unit matrix in symplectic space is defined as the unit symplectic matrix J , an even dimensional matrix:

$$J = J_{2n} = \begin{bmatrix} 0 & +I_n \\ -I_n & 0 \end{bmatrix}, \quad (4)$$

The properties of the matrix J have:

$$|J| = 1, \quad (5)$$

$$J^2 = -I, \quad (6)$$

$$J^T = J^{-1} = -J, \quad (7)$$

Geometry space	Symplectic space	Euclidean space
Space dimension	$2n$ -dimension	n -dimension
Unit matrix	unit symplect matrix: $J_{2n} = \begin{pmatrix} \mathbf{0} & +I_n \\ -I_n & \mathbf{0} \end{pmatrix}.$	unit matrix: $I_n = \underbrace{\begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & \dots & 0 & 1 \end{pmatrix}}_n.$
Determinant of unit matrix	$ J_{2n} = 1$	$ I_n = 1$
Product calculation	symplectic inner product $\langle \mathbf{x}, \mathbf{y} \rangle$ $\langle \mathbf{x}, \mathbf{y} \rangle = (\mathbf{x}, J_{2n}\mathbf{y})$ $= \mathbf{x}^T J_{2n} \mathbf{y}$	Inner product (\mathbf{x}, \mathbf{y}) $(\mathbf{x}, \mathbf{y}) = (\mathbf{x}, I_n \mathbf{y})$ $= \mathbf{x}^T I_n \mathbf{y}$ $= \mathbf{x}^T \mathbf{y}$
Calculation measure	area	length
Orthogonality	$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T J_{2n} \mathbf{y} = 0.$	$(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T I_n \mathbf{y}$ $= \mathbf{x}^T \mathbf{y}$ $= 0$
Space basis	Adjoint symplectic orthonormal basis $\mathbf{Q} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m, \mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_m\}, m \leq n$; when determinant $ \mathbf{Q} = 1$, the basis \mathbf{Q} is normal.	Orthogonal basis $\mathbf{W} = \{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m\}, m \leq n$; when $ \mathbf{W} = 1$, the basis \mathbf{W} is normal.
Orthogonal matrix	Symplectic matrix \mathbf{S} $\mathbf{S}^T \mathbf{J} \mathbf{S} = \mathbf{J}.$	Orthogonal matrix \mathbf{W} $\mathbf{W}^T \mathbf{I} \mathbf{W} = \mathbf{W}^T \mathbf{W} = \mathbf{I}.$
Analysis matrix	Hamiltonian matrix \mathbf{H} $\mathbf{H}^T = \mathbf{J} \mathbf{H} \mathbf{J}.$	Symmetry matrix \mathbf{A} $\mathbf{A}^T = \mathbf{A} = \mathbf{I} \mathbf{A} \mathbf{I}.$
Matrix transformation	Hamiltonian transformation $\langle \mathbf{x}, \mathbf{H} \mathbf{y} \rangle = \langle \mathbf{y}, \mathbf{H} \mathbf{x} \rangle.$	Symmetry transformation $(\mathbf{x}, \mathbf{A} \mathbf{y}) = (\mathbf{y}, \mathbf{A} \mathbf{x}).$
Eigenvalues of the matrix	The eigenvalues of \mathbf{H} are $\pm \mu$.	The eigenvalues μ of \mathbf{A} are real.
Eigenvectors of the matrix	The eigenvectors of \mathbf{H} are symplectic orthogonal.	The eigenvectors of \mathbf{A} are orthogonal.

Table 1.
 The comparison between symplectic geometry and Euclidean geometry.

$$\mathbf{J} \mathbf{J}^{-1} = \mathbf{J}^{-1} \mathbf{J} = \mathbf{I}. \quad (8)$$

Definition 2.3 For any two n -dimensional vectors $\mathbf{x}_{2n \times 1}$ and $\mathbf{y}_{2n \times 1}$, the normal symplectic inner product is defined by using the inner product of Euclidean space:

$$\langle \mathbf{x}, \mathbf{y} \rangle = (\mathbf{x}, J_{2n} \mathbf{y}) = \sum_{i=1}^n (x_i y_{n+i} - x_{n+i} y_i) = \mathbf{x}^T J_{2n} \mathbf{y}. \quad (9)$$

The normal symplectic inner product is also denoted briefly as the symplectic inner product in a real vector space R^{2n} . When $n = 1$, there is:

$$J_2 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad (10)$$

$$\langle \mathbf{x}, \mathbf{y} \rangle = (\mathbf{x}, \mathbf{J}_2 \mathbf{y}) = \mathbf{x}^T \mathbf{J}_2 \mathbf{y} = \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix} = x_1 y_2 - x_2 y_1. \quad (11)$$

The symplectic inner product is a bilinear antisymmetric nonsingular cross product. In symplectic space, the length of any vectors is equal to 0. But there exists the concept of symplectic orthogonal cross-course.

Definition 2.4 Let \mathbf{x} and \mathbf{y} be a $2n$ -dimensional real vector. If their symplectic inner product is equal to zero, i.e.:

$$\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^T \mathbf{J} \mathbf{y} = 0, \quad (12)$$

then \mathbf{x} and \mathbf{y} are symplectic orthogonal. Otherwise, they are called as symplectic adjoint.

Definition 2.5 If a vector set $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m, \mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_m\}$ in the real symplectic space \mathbf{R}^{2n} ($m \leq n$) is an adjoint symplectic orthonormal vector set, then the vectors \mathbf{x}_i and \mathbf{y}_i ($i = 1, \dots, m, \mathbf{x}_i \in \mathbf{R}^{2n}, \mathbf{y}_i \in \mathbf{R}^{2n}$) satisfy

$$\langle \mathbf{x}_i, \mathbf{y}_j \rangle = \mathbf{x}_i^T \mathbf{J}_{2n} \mathbf{y}_j = \begin{cases} a_{ii} \neq 0, & i = j \\ 0, & i \neq j \end{cases}, \quad (13)$$

$$\langle \mathbf{x}_i, \mathbf{x}_j \rangle = 0, \quad (14)$$

$$\langle \mathbf{y}_i, \mathbf{y}_j \rangle = 0, \quad (15)$$

where $i, j = 1, 2, \dots, m$. It is called as an adjoint symplectic orthonormal basis in the $2n$ -dimensional symplectic space. If $a_{ii} = 1$, the vector set $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m, \mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_m\}$ is a normal adjoint symplectic orthonormal vector set (a normal adjoint symplectic orthonormal basis in the space \mathbf{R}^{2n}).

The orthogonal of the Euclidean space is different from the symplectic orthogonal. If vectors \mathbf{x} and \mathbf{y} in the space \mathbf{R}^n are orthonormal, then they satisfy:

$$(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \mathbf{y} = 0, \quad (16)$$

where $\mathbf{x} \neq \mathbf{y}$.

If a vector set $\{\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_m\} \in \mathbf{R}^n$ is an orthonormal vector set, then any two vectors in the set satisfy:

$$(\mathbf{x}_i, \mathbf{x}_j) = 0, \quad (17)$$

where $i, j = 1, 2, \dots, m, i \neq j$. Eq. (17) is similar to Eqs. (14) and (15). In the n -dimensional Euclidean space, the set $\{\mathbf{x}_i\}$ is denoted as an orthonormal basis. If $\|\mathbf{x}_i\| = 1$, the orthonormal basis is a normal orthonormal basis.

Theorem 2.1 Let $\{\alpha_i\}$ be a normal adjoint symplectic orthonormal basis in a $2n$ -dimensional symplectic space Φ . Let the coordinates of any vectors β and γ in Φ be $\{x_1, x_2, \dots, x_n, x_{n+1}, \dots, x_{2n}\}^T$ and $\{y_1, y_2, \dots, y_n, y_{n+1}, \dots, y_{2n}\}^T$, respectively. Referring to the basis $\{\alpha_i\}$, the coordinates can be described as:

$$x_i = \langle \beta, \alpha_{n+i} \rangle, x_{n+i} = -\langle \beta, \alpha_i \rangle, y_i = \langle \gamma, \alpha_{n+i} \rangle, y_{n+i} = -\langle \gamma, \alpha_i \rangle, \quad (18)$$

where $i = 1, 2, \dots, n$. Then the symplectic inner product of β and γ is as follows:

$$\langle \beta, \gamma \rangle = \sum_{i=1}^n (x_i y_{n+i} - x_{n+i} y_i) = \mathbf{x}^T \mathbf{J}_{2n} \mathbf{y}. \quad (19)$$

Thus, the symplectic inner product operation is transformed to the matrix operation of ordinary vectors or matrices by applying a normal adjoint symplectic orthonormal basis.

Definition 2.6 Let \mathbf{S} is a $2n \times 2n$ matrix, if \mathbf{S} satisfies:

$$\mathbf{JSJ}^{-1} = \mathbf{S}^{-T}, \text{ or } \mathbf{S}^T \mathbf{JS} = \mathbf{J}, \quad (20)$$

then \mathbf{S} is a symplectic matrix and the determinant $|\mathbf{S}| = 1$ or -1 . Meanwhile, the inverse matrix and the transpose matrix of a symplectic matrix are a symplectic matrix, respectively. The symplectic matrix \mathbf{S} is similar to an orthogonal matrix \mathbf{W} in Euclidean space, like Eq. (20):

$$\mathbf{W}^T \mathbf{IW} = \mathbf{W}^T \mathbf{W} = \mathbf{I}. \quad (21)$$

Theorem 2.2 The product of symplectic matrixes is also a symplectic matrix.

Proof:

Let \mathbf{S}_i ($i = 1, 2, \dots, n$) be a symplectic matrix. The product matrix \mathbf{M} :

$$\mathbf{M} = \prod_{i=1}^n \mathbf{S}_i. \quad (22)$$

According to the above definition of symplectic matrix, there are:

$$\mathbf{JS}_i \mathbf{J}^{-1} = \mathbf{S}_i^{-T}, \quad i = 1, 2, \dots, n \quad (23)$$

$$\mathbf{J}^{-1} \mathbf{J} = \mathbf{I}, \quad (24)$$

$$\begin{aligned} \mathbf{JMJ}^{-1} &= \mathbf{J} \left(\prod_{i=1}^n \mathbf{S}_i \right) \mathbf{J}^{-1} \\ &= \mathbf{J} (\mathbf{S}_1 \mathbf{S}_2 \dots \mathbf{S}_n) \mathbf{J}^{-1} \\ &= \mathbf{JS}_1 \mathbf{J}^{-1} \mathbf{JS}_2 \mathbf{J}^{-1} \dots \mathbf{J}^{-1} \mathbf{JS}_n \mathbf{J}^{-1} \\ &= (\mathbf{JS}_1 \mathbf{J}^{-1}) (\mathbf{JS}_2 \mathbf{J}^{-1}) (\mathbf{J} \dots \mathbf{J}^{-1}) (\mathbf{JS}_n \mathbf{J}^{-1}), \\ &= \mathbf{S}_1^{-T} \mathbf{S}_2^{-T} \dots \mathbf{S}_n^{-T} \\ &= (\mathbf{S}_1 \mathbf{S}_2 \dots \mathbf{S}_n)^{-T} \\ &= \mathbf{M}^{-T} \end{aligned} \quad (25)$$

Thus, the product of symplectic matrixes is also a symplectic matrix.

Definition 2.7 If a $2n \times 2n$ matrix \mathbf{H} is a Hamiltonian matrix, then the matrix \mathbf{H} satisfies the following properties:

$$\mathbf{JHJ}^{-1} = -\mathbf{H}^T, \mathbf{JHJ} = \mathbf{H}^T, \text{ or } (\mathbf{JH})^T = \mathbf{JH}, \quad (26)$$

$$\langle \mathbf{x}, \mathbf{Hy} \rangle = \langle \mathbf{y}, \mathbf{Hx} \rangle, \quad (27)$$

where \mathbf{x} and \mathbf{y} are $2n$ -dimensional vectors. In other words, if an even-dimensional matrix \mathbf{H} satisfies these properties above, the matrix \mathbf{H} is a Hamiltonian matrix. In Euclidean space, a symmetric matrix \mathbf{A} is similar to a Hamiltonian matrix \mathbf{H} , like Eqs. (26) and (27):

$$\mathbf{IAI} = \mathbf{A} = \mathbf{A}^T, \quad (28)$$

$$(\mathbf{x}, \mathbf{A}\mathbf{y}) = (\mathbf{y}, \mathbf{A}\mathbf{x}). \quad (29)$$

Theorem 2.3 Let a matrix \mathbf{A} be a $n \times n$ real number matrix, if it can be built into a $2n \times 2n$ matrix \mathbf{H} in symplectic space in the following pattern:

$$\begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & -\mathbf{A}^T \end{pmatrix}. \quad (30)$$

Then the matrix \mathbf{H} is a Hamilton matrix.

Proof:

Let $\mathbf{H} = \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & -\mathbf{A}^T \end{pmatrix}$, then

$$\begin{aligned} \mathbf{J}\mathbf{H}\mathbf{J}^{-1} &= \mathbf{J} \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & -\mathbf{A}^T \end{pmatrix} \mathbf{J}^{-1} \\ &= \begin{pmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{I}_n & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & -\mathbf{A}^T \end{pmatrix} \begin{pmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{I}_n & \mathbf{0} \end{pmatrix}^{-1} \\ &= \begin{pmatrix} -\mathbf{A}^T & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{pmatrix}, \quad (31) \\ &= - \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & -\mathbf{A}^T \end{pmatrix}^T \\ &= -\mathbf{H}^T \end{aligned}$$

where \mathbf{J} is the $2n \times 2n$ unit symplectic matrix. In terms of Definition 2.7, the matrix \mathbf{H} is a $2n \times 2n$ Hamiltonian matrix.

Theorem 2.4 Let a $2n \times 2n$ matrix \mathbf{H} be a Hamiltonian matrix. Then its properties keep unchanged at symplectic similar transform. That is, a Hamiltonian matrix \mathbf{H} through a series of symplectic similar transforms is still a Hamiltonian matrix.

Proof:

According to Definition 2.6, let the matrix \mathbf{S} be a symplectic transform matrix. Then, the inverse matrix \mathbf{S}^{-1} is also a symplectic matrix. For a Hamiltonian matrix \mathbf{H} , let $\mathbf{S}\mathbf{H}\mathbf{S}^{-1}$ be the matrix \mathbf{M} under the symplectic similar transformation of the matrices \mathbf{S} and \mathbf{S}^{-1} . Thus,

$$\begin{aligned} \mathbf{J}(\mathbf{M})\mathbf{J}^{-1} &= \mathbf{J}(\mathbf{S}\mathbf{H}\mathbf{S}^{-1})\mathbf{J}^{-1} \\ &= (\mathbf{J}\mathbf{S}\mathbf{J}^{-1})(\mathbf{J}\mathbf{H}\mathbf{J}^{-1})(\mathbf{J}\mathbf{S}^{-1}\mathbf{J}^{-1}) \\ &= \mathbf{S}^{-T}(-\mathbf{H}^T)\mathbf{S}^T \\ &= -(\mathbf{S}\mathbf{H}\mathbf{S}^{-1})^T \\ &= -\mathbf{M}^T \end{aligned} \quad (32)$$

Therefore, \mathbf{M} is also a Hamiltonian matrix. Moreover, the matrix \mathbf{M} is similar to the matrix \mathbf{H} . Therefore, the Hamiltonian matrix \mathbf{H} can keep unchanged at symplectic similar transform in symplectic space.

The eigenvalues of a Hamiltonian matrix have the specific characteristics of the Hamiltonian matrix. However, the eigenvalues may be complex or repeated eigenvalues. In order to obtain the real eigenvalues of a Hamiltonian matrix H , symplectic QR decomposition method is applied to deal with the Hamiltonian H :

1. Let a $2n \times 2n$ matrix H be $(A^T \ G; \ F \ -A)$, then

$$\begin{aligned} \mathbf{N} &= \mathbf{H}^2 \\ &= \begin{pmatrix} \mathbf{A}^T & \mathbf{G} \\ \mathbf{F} & -\mathbf{A} \end{pmatrix}^2, \end{aligned} \quad (33)$$

2. Build a $2n \times 2n$ symplectic matrix Q and satisfy:

$$\mathbf{Q}^T \mathbf{N} \mathbf{Q} = \begin{pmatrix} \mathbf{B} & \mathbf{R} \\ \mathbf{0} & \mathbf{B}^T \end{pmatrix}, \quad (34)$$

$$\mathbf{B} = \begin{pmatrix} b_{11} & b_{12} & \cdots & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & \cdots & b_{2n} \\ 0 & b_{32} & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & b_{nn-1} & b_{nn} \end{pmatrix}. \quad (35)$$

Here B is an upper Hessenberg matrix. Besides, the matrix Q can be a $2n \times 2n$ Householder matrix.

3. Use the symplectic QR decomposition method to obtain eigenvalues:

$$\mu(B) = \{\mu_1, \mu_2, \dots, \mu_n\}. \quad (36)$$

4. The eigenvalues of the Hamiltonian matrix H with multiplicity n are $\lambda_i = \sqrt{\mu_i}$, $i = 1, 2, \dots, n$; $\lambda_{n+i} = -\lambda_i$ is also an eigenvalue with multiplicity n .

In symplectic space, the symplectic QR decomposition method allows the primary $2n$ -dimensional space transform into n dimensional space to resolve the eigenvalues of the Hamiltonian H , where the matrix Q is a symplectic unitary matrix. Thus, the consuming time of the calculation is only one fourth the number of floating-point operations. In general, one makes use of a Householder matrix instead of the matrix Q .

Theorem 2.5 If a $2n \times 2n$ matrix Q is a Householder matrix, then the matrix Q is a symplectic unitary matrix.

Proof:

Let a Householder matrix Q

$$\mathbf{Q} = \mathbf{Q}(k, \omega) = \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{pmatrix}, \quad (37)$$

$$\mathbf{P} = \mathbf{I}_n - \frac{2\omega\omega^*}{\omega^*\omega}, \quad (38)$$

$$\omega = (0, \dots, 0, \omega_k, \dots, \omega_n)^T \neq 0, \quad (39)$$

where, “*” means the conjugate transposition. Then, there is

$$\mathbf{P}^* = \mathbf{P}, \quad (40)$$

$$\begin{aligned} \mathbf{P}^* \mathbf{P} &= \mathbf{P}^2 \\ &= \left(\mathbf{I}_n - \frac{2\boldsymbol{\omega}\boldsymbol{\omega}^*}{\boldsymbol{\omega}^* \boldsymbol{\omega}} \right) \left(\mathbf{I}_n - \frac{2\boldsymbol{\omega}\boldsymbol{\omega}^*}{\boldsymbol{\omega}^* \boldsymbol{\omega}} \right), \\ &= \mathbf{I}_n \end{aligned} \quad (41)$$

$$\begin{aligned} \mathbf{Q}^* \mathbf{J} \mathbf{Q} &= \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{pmatrix}^* \begin{pmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{I}_n & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{0} & \mathbf{P}^* \mathbf{P} \\ -\mathbf{P}^* \mathbf{P} & \mathbf{0} \end{pmatrix}, \\ &= \begin{pmatrix} \mathbf{0} & \mathbf{I}_n \\ -\mathbf{I}_n & \mathbf{0} \end{pmatrix} \\ &= \mathbf{J} \end{aligned} \quad (42)$$

$$\begin{aligned} \mathbf{Q}^* \mathbf{Q} &= \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{pmatrix}^* \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{P}^* \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P}^* \mathbf{P} \end{pmatrix}, \\ &= \begin{pmatrix} \mathbf{I}_n & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_n \end{pmatrix} \\ &= \mathbf{I}_{2n} \end{aligned} \quad (43)$$

Therefore, the Householder matrix \mathbf{Q} is a symplectic unitary matrix.

2.4 Mathematical fundamental on applications

2.4.1 Symplectic geometry spectrums of the reconstructed attractor from a time series

In symplectic space, the reconstructed attractor can keep its properties unchanged [5, 6]. Its symplectic geometry spectrums can be given by the symplectic geometry theory above. On the basis of Section 2.1 and 2.2, one can build a Hamiltonian matrix \mathbf{M} from a time series of the observation. Due to the structure characteristics of the matrix \mathbf{M} , its eigenvalues can be calculated by the $2n$ -dimensional symplectic space reducing into n -dimensional space. In terms of Theorem 1.5, a $2n \times 2n$ symplectic Householder matrix \mathbf{Q} can be constructed. The matrix \mathbf{P} in the matrix \mathbf{Q} can be calculated by the matrix \mathbf{A} in the matrix \mathbf{M} . The specific steps are as follows:

1. Let \mathbf{A} be

$$\begin{aligned} \mathbf{A} &= \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix} \\ &= \begin{pmatrix} a_{11} & \mathbf{A}_{12}^{(1)} \\ \mathbf{a}_{21}^{(1)} & \mathbf{A}_{22}^{(1)} \end{pmatrix} \end{aligned} \quad (44)$$

If the vector $\alpha_{21}^{(1)} \neq 0$, set $\mathbf{S}^{(1)}$ be the first column vector of \mathbf{A} :

$$\mathbf{S}^{(1)} = \begin{pmatrix} a_{11}^{(1)} \\ a_{21}^{(1)} \\ \vdots \\ a_{n1}^{(1)} \end{pmatrix} = \begin{pmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{n1} \end{pmatrix}, \quad (45)$$

then, there is:

$$\alpha^{(1)} = \|\mathbf{S}^{(1)}\|_2, \quad (46)$$

$$\rho^{(1)} = \|\mathbf{S}^{(1)} - \alpha^{(1)}\mathbf{E}^{(1)}\|_2, \quad (47)$$

$$\boldsymbol{\omega}^{(1)} = \frac{\mathbf{S}^{(1)} - \alpha^{(1)}\mathbf{E}^{(1)}}{\rho^{(1)}}, \quad (48)$$

where $\mathbf{E}^{(1)} = (1, 0, \dots, 0)^T$ is a $n \times 1$ unit column vector.

Then, the elementary reflective matrix $\mathbf{P}^{(1)}$ can be calculated:

$$\mathbf{P}^{(1)} = \mathbf{I} - 2\boldsymbol{\omega}^{(1)}(\boldsymbol{\omega}^{(1)})^T. \quad (49)$$

So, there is

$$\begin{aligned} \mathbf{A}^{(2)} &= \mathbf{P}^{(1)}\mathbf{A} \\ &= \begin{pmatrix} \sigma_1 & a_{12}^{(2)} & \cdots & a_{1n}^{(2)} \\ 0 & a_{22}^{(2)} & \cdots & a_{2n}^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & a_{n2}^{(2)} & \cdots & a_{nn}^{(2)} \end{pmatrix}. \end{aligned} \quad (50)$$

Continue to deal with $\mathbf{A}^{(2)}$ by repeating the above steps, let $\mathbf{S}^{(2)}$ be

$$\mathbf{S}^{(2)} = \begin{pmatrix} 0 \\ a_{22}^{(2)} \\ \vdots \\ a_{n2}^{(2)} \end{pmatrix}. \quad (51)$$

Then,

$$\alpha^{(2)} = \|\mathbf{S}^{(2)}\|_2, \quad (52)$$

$$\rho^{(2)} = \|\mathbf{S}^{(2)} - \alpha^{(2)}\mathbf{E}^{(2)}\|_2, \quad (53)$$

$$\boldsymbol{\omega}^{(2)} = \frac{\mathbf{S}^{(2)} - \alpha^{(2)}\mathbf{E}^{(2)}}{\rho^{(2)}}, \quad (54)$$

where $\mathbf{E}^{(2)} = (0, 1, 0, \dots, 0)^T$ is a $n \times 1$ unit column vector.

Then, the elementary reflective matrix $\mathbf{P}^{(2)}$ can be calculated:

$$\mathbf{P}^{(2)} = \mathbf{I} - 2\boldsymbol{\omega}^{(2)}\left(\boldsymbol{\omega}^{(2)}\right)^T. \quad (55)$$

Thus, we can get $\mathbf{A}^{(3)}$ with all zeros elements except the first and second non-zero elements:

$$\mathbf{A}^{(3)} = \mathbf{P}^{(2)}\mathbf{A}^{(2)} = \begin{pmatrix} \sigma_1 & a_{12}^{(3)} & a_{13}^{(3)} & \dots & a_{1n}^{(3)} \\ 0 & \sigma_2 & a_{23}^{(3)} & \dots & a_{2n}^{(3)} \\ 0 & 0 & a_{33}^{(3)} & \vdots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & a_{n3}^{(3)} & \dots & a_{nn}^{(3)} \end{pmatrix}. \quad (56)$$

Repeat the same steps until $\mathbf{A}^{(n)}$ becomes an upper triangle matrix, one can construct a Householder matrix \mathbf{P} as follows:

$$\mathbf{P} = \mathbf{P}^{(n)}\mathbf{P}^{(n-1)}\dots\mathbf{P}^{(1)}. \quad (57)$$

Thus, a symplectic Householder matrix \mathbf{Q} can be built to make the Hamiltonian matrix \mathbf{M} transform as an upper Hessenberg matrix, namely:

$$\begin{aligned} \mathbf{QMQ}^T &= \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{pmatrix} \begin{pmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & -\mathbf{A}^T \end{pmatrix} \begin{pmatrix} \mathbf{P} & \mathbf{0} \\ \mathbf{0} & \mathbf{P} \end{pmatrix}^T \\ &= \begin{pmatrix} \mathbf{PAP}^T & \mathbf{0} \\ \mathbf{0} & -\mathbf{PA}^T\mathbf{P}^T \end{pmatrix} \\ &= \begin{pmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & -\mathbf{B}^T \end{pmatrix} \end{aligned} \quad (58)$$

$$\mu(\mathbf{A}) = \mu(\mathbf{B}), \quad (59)$$

where μ means the eigenvalue. The matlab program is as follow:

```
function [P, R] = householder (A)
% -----Solve Householder Transform Matrix-----
%
% Synopsis:
% [P, R] = householder (A)
%
% Description:
% It solves a Householder matrix from a data matrix, i.e., a
% reconstruction attractor.
%
% Input:
% A [mRow x mCol] a data matrix.
%
```

```

% Ouputs:
% P [mRow x mRow] a Householder matrix
% R [mRow x mCol] an upper triangle matrix

[mRow, mCol] = size(A);
if mRow>mCol
    A = A';
    [mRow, mCol] = size(A);
end
I_matrix = eye(mRow);
m = min([mRow, mCol]);
p = I_matrix;
for i = 1:m.
    S = A(:,i);
    if i > 1.
        S(1:i-1) = 0;
    end
    alpha = sqrt(S'*S);
    delta1 = S-alpha*I_matrix(:,i);
    delta = sqrt(delta1'*delta1);
    if delta==0
        delta = eps;
    end
    omega = delta1/delta;
    p = I_matrix-2*omega*omega';
    A = p*A;
    P = p*P;
end
R = A;
return

```

For the attractor matrix \mathbf{X} of a time series, its symplectic geometry spectrums \mathbf{SGS} are calculated by the eigenvalues of the \mathbf{A} in descending order, that is:

$$SGS_i = \log \left(\frac{\sigma_i}{tr(\sigma_i)} \right), \quad (60)$$

$$\sigma = \mu^2(\mathbf{X}) = \mu(\mathbf{A}), \sigma_1 = \mu_{\max}^2, \dots, \sigma_n = \mu_{\min}^2, \quad (61)$$

where $i = 1, \dots, n$. n is the dimension of the attractor \mathbf{X} .

2.4.2 Embedding dimension estimate of the reconstructed attractor from a time series

To estimate the embedding dimension is usually the first step of nonlinear analysis [5]. For a time series, it is important to resolve a suitable embedding dimension of the observed system. Due to the measure-preserving characteristic of symplectic geometry, symplectic geometry spectrums can be used to estimate the embedding dimension of the system from a time series. With the increase of the dimension n in Eq. (61), the change of the symplectic geometry spectrums \mathbf{SGS} in Eq. (60) tends to be flat at $i = d$ ($i \in (1, n)$) and enters the noise floor area, $SGS_1 > SGS_2 > \dots > SGS_d > SGS_{d+1} \geq \dots \geq SGS_n$. That is, the eigenvalues exist $\sigma_1 > \sigma_2 > \dots > \sigma_d > \sigma_{d+1} \geq \dots \geq \sigma_n$, then d is defined as the embedding dimension of the time series for the reconstruction system.

2.4.3 Symplectic entropy (SymEn) of a time series

Symplectic entropy (SymEn) is a kind of entropy measure for a dynamic system in symplectic space [16]. Based on the symplectic geometry spectrums, the SymEn measures the energy distribution in symplectic space of a dynamic system from a time series. The distribution of the energy of the system is described by the eigenvalues σ in the relevant symplectic orthonormal bases of the symplectic space. In each base direction, the probability of the energy distribution can be given as follows:

$$p_i = \frac{\sigma_i}{\sum_{i=1}^n \sigma_i}, \quad (62)$$

where i denotes the i th base direction in the symplectic space, $\sum_{i=1}^n p_i = 1$, $0 \leq p_i \leq 1$.

Then,

$$\text{SymEn} = - \sum_{i=1}^n p_i \log(p_i). \quad (63)$$

The matlab program is as follows:

```
function SymEn = SymplecticEntropy(A)
    [Q, R] = householder(A);
    delta = diag(R);
    sum_delta = sum(delta);
    p = delta./sum_delta;
    SymEn = -sum(p.*log(p));
    Return
```

The SymEn value represents the uncertainty of the entropy about the underlying probability distribution of a dynamic system in symplectic space, called Symplectic Entropy.

2.4.4 Symplectic principal component analysis (SPCA) of a time series

Symplectic principal component analysis (SPCA) is a kind of principal component analysis (PCA) to map the dynamic system from a time series into the symplectic space [17]. Due to the preserving-measure nature of symplectic geometry, symplectic principal components elucidate the dominant features of a time series for an underlying system. The principal components corresponding to larger eigenvalues capture the key relationship between the variables in symplectic space. The components corresponding to smaller eigenvalues are regarded to relate primarily to the less important components or noise in the time series. The analysis of eigenvalues are also called as the symplectic geometry spectrums analysis (SGSA) [6, 18, 19]. The corresponding components are also regarded as symplectic geometry mode decomposition (SGMD) [7, 8, 20, 21]. According to the symplectic geometry spectrums above, if the number of the chosen symplectic principal components is k , the corresponding principal eigenvector matrix \mathbf{p} can be constructed by using the first k eigenvectors of the matrix \mathbf{P} in the matrix \mathbf{Q} . The corresponding principal

eigenvalues are the first k eigenvalues in the symplectic geometry spectrum. If $k = n$, $\mathbf{p} = \mathbf{P}$. Otherwise, $\mathbf{p} \subset \mathbf{P}$. Then the reestimated attractor matrix $\hat{\mathbf{X}} = \mathbf{p}(\mathbf{p}^T \mathbf{X})$, where $\mathbf{p}^T \mathbf{X}$ is defined the transformation coefficient matrix \mathbf{S} . If \mathbf{p}_i is the i th eigenvector in \mathbf{P} corresponding to the i th eigenvalue σ_i in the symplectic geometry spectrum, \mathbf{S}_i will be the i th principal component coefficients, or called the projection of the \mathbf{p}_i th direction in the symplectic space:

$$\mathbf{S}_i = \mathbf{p}_i^T \mathbf{X} = \mathbf{X}^T \mathbf{p}_i. \quad (64)$$

The corresponding \mathbf{p}_i th principal component matrix $\hat{\mathbf{X}}_i$ is given as follows:

$$\hat{\mathbf{X}}_i = \mathbf{p}_i \mathbf{S}_i. \quad (65)$$

Then, the reestimated attractor matrix is equal to the sum of $\hat{\mathbf{X}}_i$, $i = 1, \dots, n$.

$$\hat{\mathbf{X}} = \sum_{i=1}^n \hat{\mathbf{X}}_i. \quad (66)$$

The reestimated time series \mathbf{x}^r is equal to the sum of each principal component, i.e. the sum of projections in different directions. If $i = 1$, the reestimated time series is a reduced noise data based on the first principal component.

3. Applications

Symplectic geometry theory has been applied to deal with a time series in fields of physics, engineering, biomedical engineering [6–8, 11, 16–24], since Lei et al. (2002) first proposed a symplectic geometry method to estimate the appropriate embedding dimension from a time series [5]. Here, we will introduce four research cases in terms of the above theorem and properties of symplectic geometry for the time series analysis.

Case 1: Embedding dimension estimation for Lorenz chaotic time series [5].

Lorenz chaotic system was accidentally discovered by Edward Norton Lorenz [25], an American meteorologist, in 1963 when he was studying weather forecast, and was known as the first chaotic attractor. Since then, people began to study chaos, a random-like phenomenon. Lorenz chaotic time series x comes from Lorenz chaotic system, which is a three-dimensional dynamical system as follows [5]:

$$\begin{aligned} \dot{x} &= \sigma(y - x) \\ \dot{y} &= \gamma x - y - xz \\ \dot{z} &= -bz + xy \end{aligned}, \quad (67)$$

where $\sigma = 10$, $b = 8/3$, $\gamma = 28$. The state variable x is chosen as the analyzed data. The sampling interval is 0.005. The length n is 1000 points.

The attractor reconstructed from Lorenz chaotic time series x can reflect the Lorenz system. Here, the dimension of the reconstructed attractor is estimated by the above symplectic geometry method. Let the embedding dimension d be 3: 5: 23, where $i = 1: d$. The matlab program is as follows:

```
% Compute a Lorenz chaotic time series
% Example:
```

```
% state = [5 5 5];
% Ts = 0.005;
% N = 10000;
% y = calculate_lorenz(state, Ts, N);
% x = y(:,1);

function y = calculate_lorenz(state, Ts, N).

if nargin < 1
state = [5 5 5];
Ts = 0.005;
N = 10000;
end
if nargin == 1
Ts = 0.005;
N = 10000;
end
if nargin == 2
N = 10000;
end
% set time span with specific times for the solution
T = 0:Ts:N*Ts;
% set a scalar relative error tolerance 'RelTol' (1e-3 by default).
% and a vector of absolute error tolerances 'AbsTol' (all components 1e-6% by
default).
options = odeset('RelTol',1e-4,'AbsTol',[1e-4 1e-4 1e-5]);
% solve Lorenz chaotic system
[t,y] = ode45('lorenzeq1',T,state,options);
return
function ydot = lorenzeq(t,y)
% Lorenz equation
b = 8/3;
r = 28;
delta = 10;
A = [-delta delta 0;r - 1 -y(1);y(2) 0 -b];
ydot = A*y;
return



---


% Calculate the embedding dimension.
state = [5 5 5];
Ts = 0.005;
N = 10000;
y = calculate_lorenz(state, Ts, N);
x = y(:,1);
figure.
for N = 3:5:23
X = signalMatrix(x,N);
A = X*X';
[Q, R] = householder(A);
delta = diag(R);
sum_delta = sum(delta);
d = log10(delta./sum_delta);
n = length(d);
plot(1:n, d, 'b*-' )
```

```

hold on
end
ylabel('log10(\delta_{\it{i}}/tr(\delta_{\it{i}}))')
xlabel('{\it{d}} = 3:5:23')
axis([0 25 -15 0])

```

Figure 1a shows the symplectic geometry spectrums SGS of x without noise according to the above equations based on symplectic geometry theory. We can see that the symplectic geometry spectrums turn abruptly into a flat area from $i = 6$, i.e. $\sigma_1 > \sigma_2 > \dots > \sigma_5 \gg \sigma_{5+1} \geq \dots \geq \sigma_d$. So, the embedding dimension of the time series x can be estimated at 6. But from the **Figure 1b**, we can see that it is difficult for the SVD method to determinate the embedding dimension from the time series x . The results indicate that the symplectic geometry method could better determinate the embedding dimension from a time series due to its preserving-measure properties.

Case 2: Embedding dimension estimation for the surface EMG signal [5].

In the practical engineering research, a lot of time series data due to their complexity are considered to be nonlinear, such as the surface EMG signal in biomedical engineering. As a kind of non-invasive measure for the contracting skeletal muscles, the surface EMG signal reflects some information about the muscle, limb movements and loading of the bones and joints. It has been widely applied to assess biomechanical and motor control deficits and other functional disorders, as well as to diagnose neuromuscular problems. However, due to noise interference, the study of surface EMG signal is still a great challenge in biomedical engineering. Many researches indicate that the surface EMG signal is complex and nonlinear.

The embedding dimension estimation of the surface EMG signal is usually critical to analyze its nonlinear features. As an example, we use the above symplectic geometry method to estimate the embedding dimension of the surface EMG signal during forearm supination. The length of the surface EMG signal is 1000 points. The data sampling frequency is 1 kHz. **Figure 2a** shows the raw surface EMG signal.

Figure 2b gives the symplectic geometry spectrums SGS of the data in **Figure 2a**. From **Figure 2b**, the symplectic geometry spectrums SGS change slowly at $d = 6$ and turn into noise floor with the increase of the index i . Then, the embedding dimension can be estimated at 6 for the surface EMG signal during forearm supination.

Case 3: SymEn analysis of vibration signals on rolling bearings [11].

In the rotating machinery systems, it is extremely important for rolling bearings to detect faults from vibration signals. The Case Western Reserve University (CWRU) Bearing Data Center provides a website database for the vibration signals

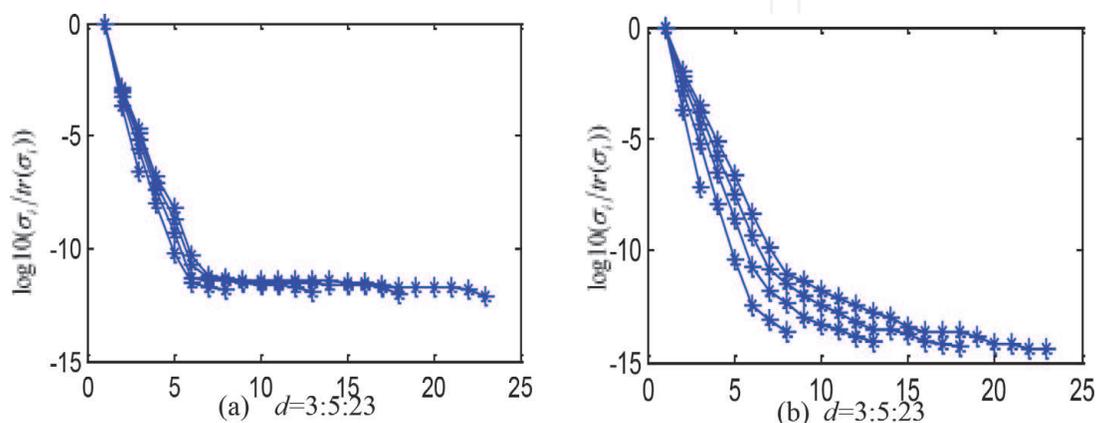


Figure 1.

The embedding dimension estimation of Lorenz chaos series with no noise based on: (a) the symplectic geometry method; (b) the SVD method.

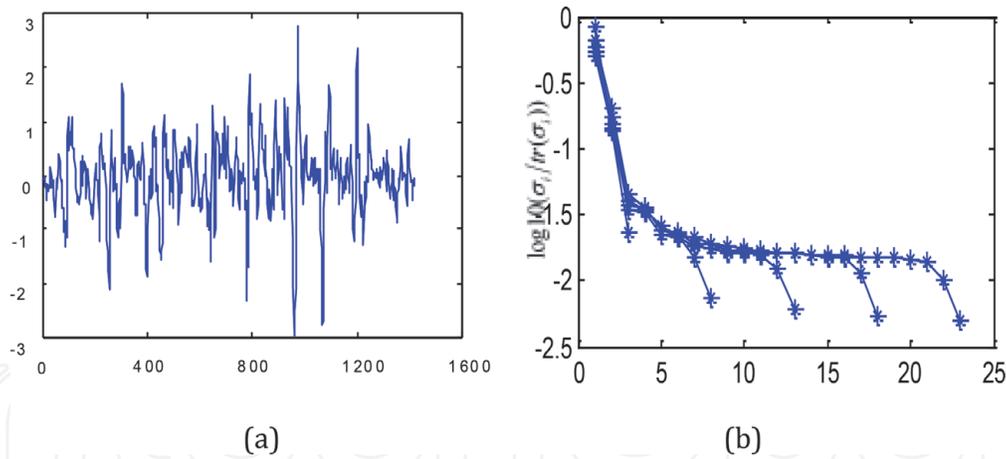


Figure 2. The embedding dimension analysis of the surface EMG signal based on the symplectic geometry spectrums: (a) Typical surface EMG signal during forearm supination; (b) The symplectic geometry spectrums of the surface EMG data in (a), where abscissa is the analysis dimension $d = 3, 8, 13, 18, 23$, ordinate is $SGS_i = \log(\sigma_i / \text{tr}(\sigma_i))$, where the index $i = 1: d$.

of bearings (<http://csegroups.case.edu/bearingdatacenter/pages/welcome-case-western-reserve-university-bearing-data-center-website>). From the website, the acceleration vibration data sets for 6205-2RS JEM of SKF deep-groove ball bearings are obtained to detect their fault categories. The corresponding sampling frequency is 12 kHz, the shaft speed 1730 r/min. The analyzed data sets include No.100 for normal condition (NC), No.212 for inner race fault (IRF), No.225 for rolling element fault (REF), and No.261 for outer race fault (ORF) at 12 o'clock position. The data of each set consist of the vibration signals at the housing of the drive end (DE) bearing and that of the fan end (FE) bearing, which the faults are at the drive end. The corresponding fault depth and diameter are 0.21 inches and 0.53 mm, respectively.

Symplectic geometry preserves the nature of a dynamic system under symplectic similar transformations. As an entropy measure in symplectic geometry, the SymEn value of a time series measures the lack of information in a dynamic system to reflect its properties. For the complexity of a rolling bearing, the SymEn estimate is applied to test its nonlinear characteristics from the vibration signals. **Figure 2** shows the SymEn values of the vibration signals at the drive end and their surrogate data sets based on the null hypothesis of a Gaussian linear stochastic process. Here, the length of each data is 6000 points. The embedding dimension $d = 7$.

Meanwhile, the 39 sets of surrogate data are generated by the iterated amplitude adjusted Fourier transform (IAAFT) algorithm in the 95% confidence level [26]. From **Figure 3**, we can see that there are the significant differences between these SymEn values of the vibration signals of a rolling bearing and their surrogate data sets. The results indicate that the vibration data could contain nonlinear characteristics. The original vibration signals are not from a Gaussian linear stochastic process in the 95% confidence level but from a nonlinear dynamical system. It conforms that the rolling bearing system is a complex nonlinear dynamical system.

Due to the complexity of rolling bearings, it is often thought that the high dimensional features can better identify the faults of rolling bearings [27–29]. However, the SymEn method can availablely extract the low-dimensional features to identify the faults of rolling bearings from vibration signals quite precisely. **Figure 4** shows the four working states of rolling bearings, i.e., NC, ORF, REF, and IRF, based on 2-dimensional features. The abscissa is the SymEn estimates of vibration signals at the drive end. The ordinate is those estimates of vibration signals at the fan end. We can see that the four states are obviously different.

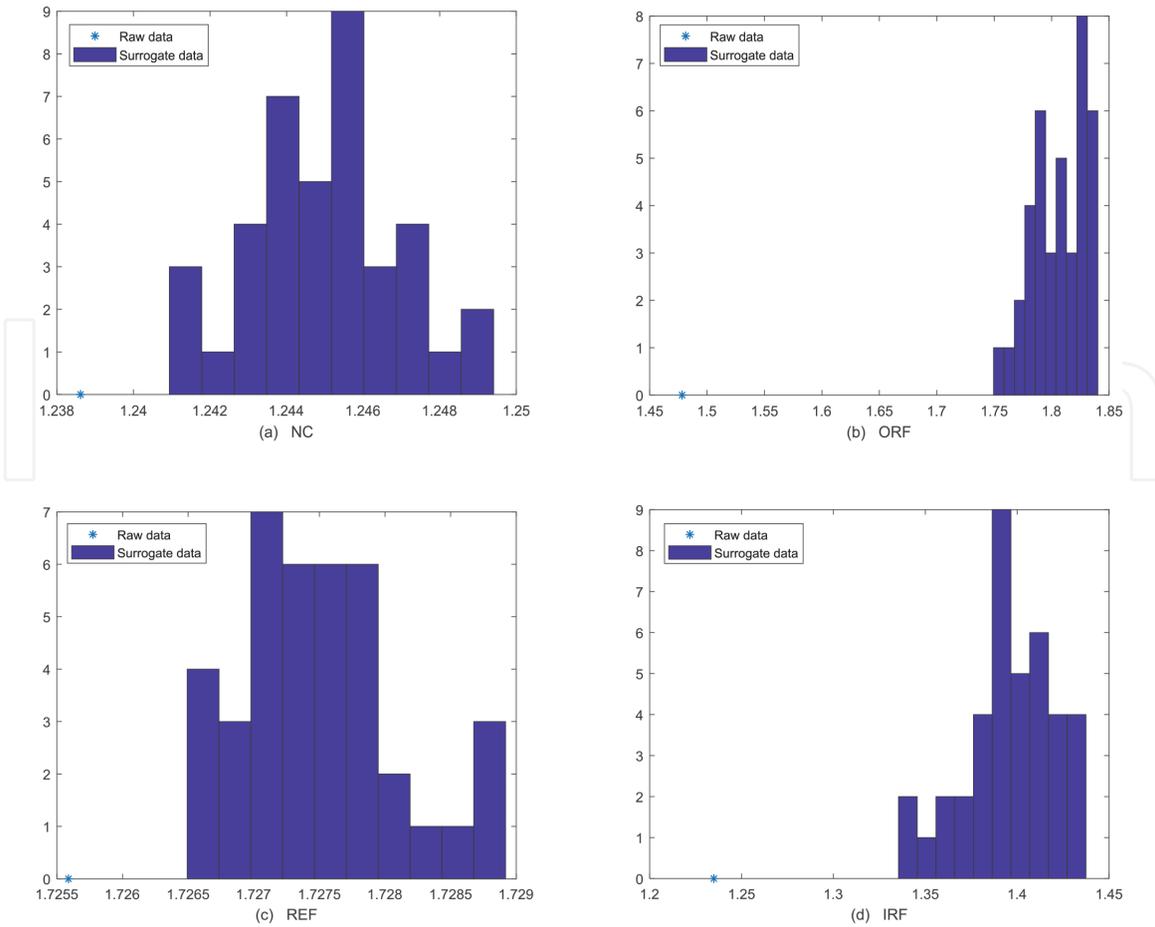


Figure 3. The nonlinear analysis of vibration signals based on the SymEn method: (a) for the normal condition (NC); (b) for the outer race fault (ORF); (c) for the rolling element fault (REF); (d) for the inner race fault (IRF). The abscissa is the SymEn values of vibration signals and their surrogate data.

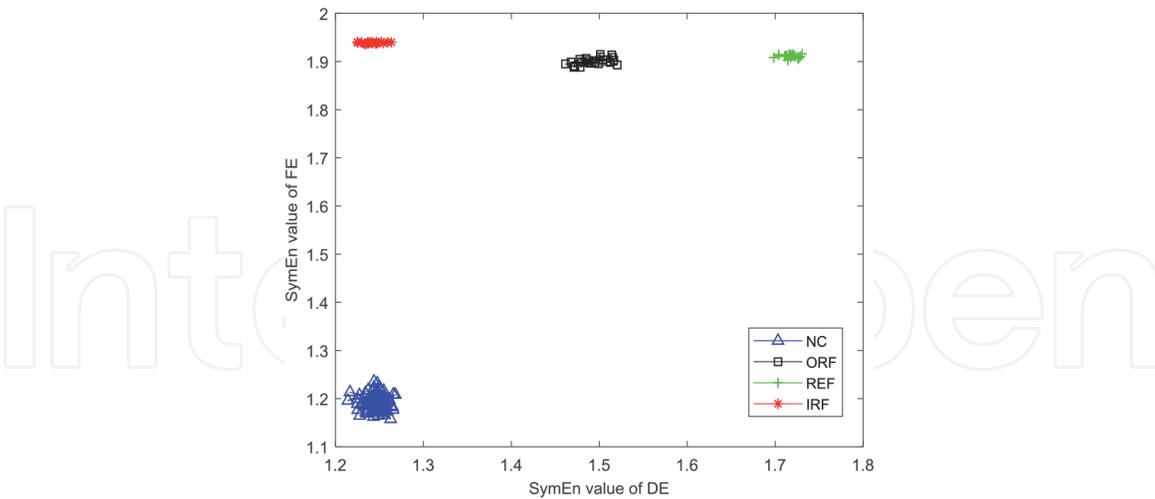


Figure 4. The states analysis of rolling for bearings with the SymEn estimates.

There are 100% accuracies by RBF classifier for the four states of the rolling bearings. **Figure 5** plots the histogram of error values between output classes and target classes for the SymEn estimates as features of vibration signals.

Case 4: Noise reduction analysis of vibration signals based on SPCA [17, 30].

In the practical engineering measurement, the vibration data of rolling bearings have often become contaminated with noise. The noise reduction is also beneficial to analyze the measured data. The SPCA method preserves the intrinsic nonlinear nature of the raw data. The symplectic principal components can better retrieve

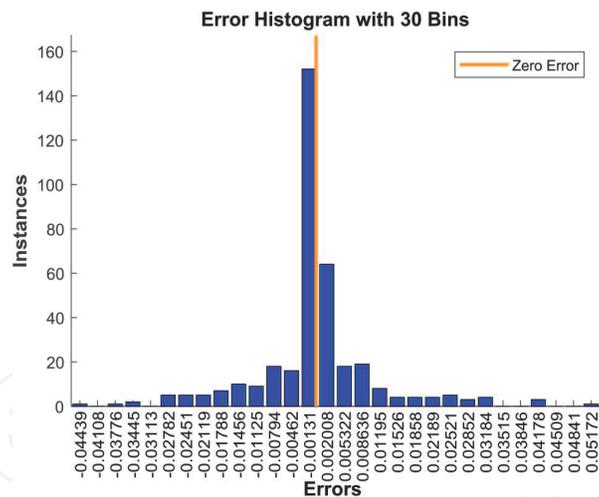


Figure 5.
 The analysis of error values identification accuracies of four states.

dominant patterns from the noisy data. For the vibration signals of rolling bearings, the first symplectic principal component is used two times continuously to reduce the noise in the data.

The specific analysis procedures are as follows:

1. Build a Hamiltonian matrix from the measured data in terms of Eq. (1), Definition 2.1, 2.2 and Theorem 2.3;
2. Use the Eq. (44)–(59) to compute a symplectic Householder transform matrix Q for the symplectic QR decomposition in the SPCA method;
3. Construct the first symplectic principal component eigenvector matrix p_1 ;
4. Calculate the first symplectic principal component coefficients S_1 , i.e.:

$$S_1 = p_1^T X = X^T p_1;$$

5. Get the first denoised data x_1 from the reestimated matrix in the following:

$$\hat{X}_1 = p_1 S_1;$$

6. Let the first denoised data x_1 into the first step, and repeat the above steps, then obtain the second denoised data x_3 .

Figure 6 shows the effect of denoising for the vibration signals of rolling element fault (REF), No.225 data in the CWRU database [11]. For the rolling element fault at the drive end, the fault state can be seen clearly by the second reducing noise (see **Figure 6a**). For the vibration signals at the fan end without faults, the periodical characteristics in the normal state can be shown after the two reducing noise (see **Figure 6b**).

Moreover, the noise reduction method based on the symplectic geometry has been used to denoise several time series data of Lorenz chaotic system, duffing chaotic system, Chua's chaotic system with noise, as well as the sunspot number [30]. The details can be found in literatures [17, 30].

Besides, the symplectic geometry method also further integrate other approaches to better investigate the fault extraction and identification for rotating systems, such as symplectic geometry mode decomposition [19] with power

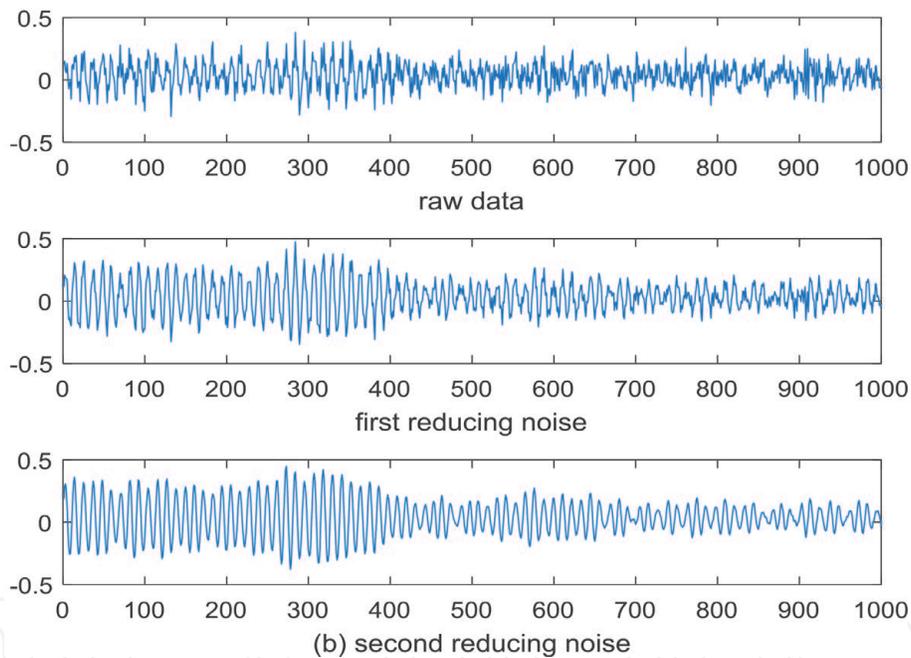
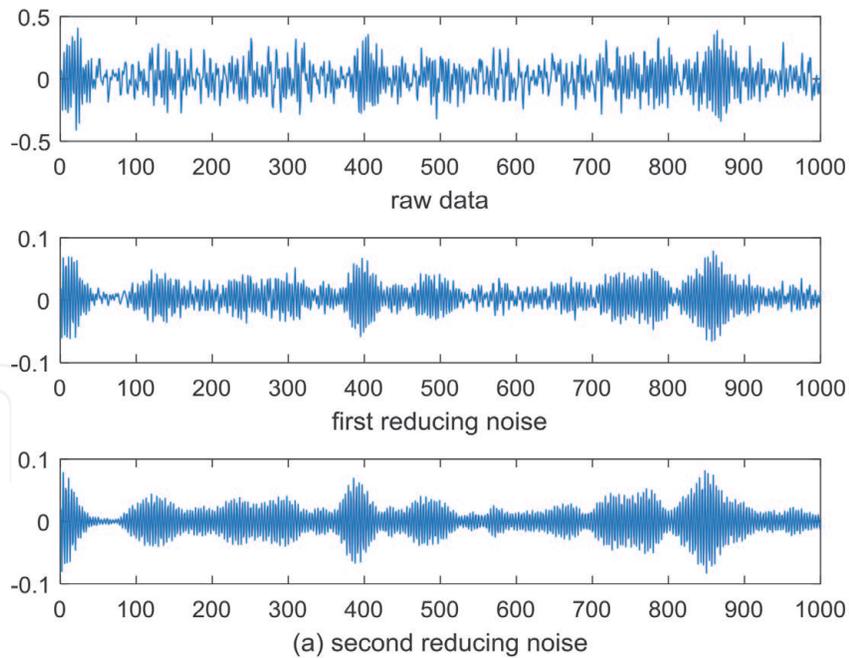


Figure 6. The two times denoising analysis for the vibration signals of rolling element fault (REF) in No.225 data from the CWRU database. (a) The abscissa is the number of data points; (b) the ordinate is the amplitude (v) of the data.

spectral entropy [7] as well as Lagrange multiplier [20], symplectic transformation based variational Bayesian learning [21].

4. Conclusions and future research

This chapter introduces the symplectic geometry theory in the research field of the time series analysis in view of the complexity of a time series. Corresponding to Euclidean geometry, the basic concepts and basic elements of mathematics of the symplectic geometry are given, such as the symplectic space, symplectic transformation, Hamiltonian matrix, symplectic entropy (SymEn), symplectic principal

component analysis (SPCA), and so on. Based on the symplectic geometry theory, the symplectic geometry spectrum analysis (SGSA), the symplectic entropy (SymEn) method and the symplectic geometry mode decomposition (SGMD) method are demonstrated to investigate the principal characteristics of a time series in the symplectic space. Meanwhile, the corresponding matlab programs are given. At last, in order to facilitate readers to learn, use and develop the symplectic geometry method, some applications of symplectic geometry on time series analysis are presented, such as the embedding dimension estimation, nonlinear testing, fault diagnosis, as well as noise reduction.

The embedding dimension estimation is often the first step in nonlinear time series analysis. Case 1 and 2 show the embedding dimension estimation of Lorenz chaotic time series and the surface EMG signal based on symplectic geometry spectrum. Moreover, the symplectic entropy method is applied to detect the nonlinearity of vibration signals on rolling bearings and identify the faults of vibration signals on rolling bearings (see Case 3). Considering the noise pollution in the practical engineering measurement, to dispose of the noise problem is very necessary for the measured time series analysis. Case 4 uses the SPCA method based on symplectic geometry to investigate the denoise of the vibration signals for rolling element fault (REF) from the CWRU database.

Symplectic geometry provides a new research idea for data analysis in practice. Although the symplectic geometry theory has been developed and applied on the nonlinear time series analysis, the related research based on symplectic geometry still needs to be further developed. Many future challenges in the research of symplectic geometry theory and various applications on a number of diverse aspects need to be developed furtherly. This chapter is only to provide a snapshot of some current trends and future challenges in the research of symplectic geometry theory on the time series analysis.

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Conflict of interest

The authors declare no conflict of interest.

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References

- [1] Tao T. An Introduction to Measure Theory. Graduate Studies in Mathematics, 2011, vol.126, published by American Mathematical Society. ISBNs: 978-0-8218-6919-2 (print); 978-1-4704-1187-9 (online). DOI: <http://dx.doi.org/10.1090/gsm/126>.
- [2] Kantz H, Schreiber T. Nonlinear time series analysis. Cambridge University Press, 1997.
- [3] Wimberger S. Nonlinear Hamiltonian systems. Chaptered in Nonlinear Dynamics and Quantum Chaos, 2014; 21–102.
- [4] Yao W, Zhong WX, Lim CW. Symplectic elasticity. World Scientific Press, 2009. DOI: 10.1142/6656
- [5] Lei M, Wang ZZ, Feng ZJ. A method of embedding dimension estimation based on symplectic geometry. Physics Letters A. 2002; 303(2–3): 179–189.
- [6] Xie HB, Dokos S, Sivakumar B, Mengersen K. Symplectic geometry spectrum regression for prediction of noisy time series. Physical Review E. 2016; 93: 052217.
- [7] Zheng Z, Xin G. Fault feature extraction of hydraulic pumps based on symplectic geometry mode decomposition and power spectral entropy. Entropy. 2019; 21: 476–1-24.
- [8] Jiang Y, Chen WZ, Li MY. Symplectic geometry decomposition-based features for automatic epileptic seizure detection. Computers in Biology and Medicine. 2020; 116: 103549.
- [9] Takens F. Detecting Strange Attractors in Turbulence. In: Dynamical Systems and Turbulence, eds. D. A. Rand and L. –S. Young, Lecture Notes in Mathematics. Warwick, 1980, vol.898. (Springer-Verlag, New York, 1980), 366–381.
- [10] Pincus SM. Approximate entropy as a measure of system complexity. Proceedings of the National Academy of Sciences USA. 1991; 88(6): 2297–2301.
- [11] Lei M, Meng G, Dong GM. Fault detection for vibration signals on rolling bearings based on the symplectic entropy method. Entropy. 2017; 19: 607–1-17.
- [12] Richman JS, Moorman JR. Physiological time-series analysis using approximate entropy and sample entropy. American Journal of Physiology Heart & Circulatory Physiology. 2000; 278: H2039-H2049.
- [13] Skokos Ch. The Lyapunov characteristic exponents and their computation. Chaptered in Dynamics of Small Solar System Bodies and Exoplanets. 2009; 63–135.
- [14] Vibe K, Vesin JM. On chaos detection methods. International Journal of Bifurcation and Chaos. 1996; 6(3): 529–543.
- [15] Fraser AM. Information and entropy in strange attractors. IEEE Trans. Inf. Theory IT. 1989; 35: 245.
- [16] Lei M, Meng G, Zhang WM, Wade J, Sarkar N. Symplectic entropy as a novel measure for complex systems. Entropy. 2016; 18: 412–1-18.
- [17] Lei M, Meng G. Symplectic principal component analysis: A new method for time series analysis. Mathematical Problems in Engineering. 2011; 2011: 793429.
- [18] Xie HB, Guo TR, Sivakumar B, Liew AW, Dokos S. Symplectic geometry spectrum analysis of nonlinear time series. Proceedings of The Royal Society A. 2014; 470: 20140409–1-17.

- [19] Cheng J, Yang Y, Li X, Pan HY, Cheng JS. An early fault diagnosis method of gear based on improved symplectic geometry mode decomposition. *Measurement* 2020; 151: 107140.
- [20] Pan HY, Yang Y, Zheng JD, Cheng JS. A noise reduction method of symplectic singular mode decomposition based on Lagrange multiplier. *Mechanical Systems and Signal Processing*. 2019; 133: 106283.
- [21] Pan HY, Yang Y, Zheng JD, Cheng JS. Symplectic transformation based variational Bayesian learning and its applications to gear fault diagnosis. *Measurement* 2019; 147: 106827.
- [22] Lei M, Wang ZH, Feng ZJ. The application of symplectic geometry on nonlinear dynamics analysis of the experimental data. In: 14th International Conference on Digital Signal Processing Proceeding; 1–3 July 2002; Santorini, Greece. 2002. v1–2, p.1137–1140
- [23] Niu X, Qu F, Wang N. Evaluating Sprinters' Surface EMG Signals Based on EMD and Symplectic Geometry. *Journal of Ocean University of Qingdao*. 2005; 35(1): pp125–129. (in Chinese)
- [24] Xie H, Wang Z, Huang H. Identification determinism in time series based on symplectic geometry spectra. *Physics Letters A*, 2005,342 (1–2):156–161.
- [25] Lorenz EN. Deterministic nonperiodic flow. *Journal of Atmospheric Sciences*. 1963; 20: 130–141.
- [26] Schreiber T, Schmitz A. Improved surrogate data for nonlinearity tests. *Physical Review Letters*. 1996; 77: 635–638. DOI: 10.1103/PhysRevLett.77.635.
- [27] Yang J, Zhang Y, Zhu Y. Intelligent fault diagnosis of rolling element bearing based on SVMs and fractal dimension. *Mechanical Systems and Signal Processing*. 2007; 21:2012–2014. DOI:0.1016/j.ymssp.2006.10.005.
- [28] Prieto MD, Cirrincione G, Espionsa AG, Ortega JA, Henao H. Bearing fault detection by a novel condition-monitoring scheme based on statistical-time features and neural networks. *IEEE Transactions on Industrial Electronics* 2013; 30: 3398–3407. DOI:10.1109/TIE.2012.2219838.
- [29] Wu S, Wu C, Wu T, Wang C. Multi-scale analysis based ball bearing defect diagnostics using mahalanobis distance and support vector machine. *Entropy* 2013; 15: 416–433. DOI:10.3390/e15020416.
- [30] Lei M, Meng G. A noise reduction method for continuous chaotic systems based on symplectic geometry. *Journal of Vibration Engineering & Technologies*. 2015; 3(1): 13–24.