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Introductory Chapter: Traveling Salesman Problem - An Overview

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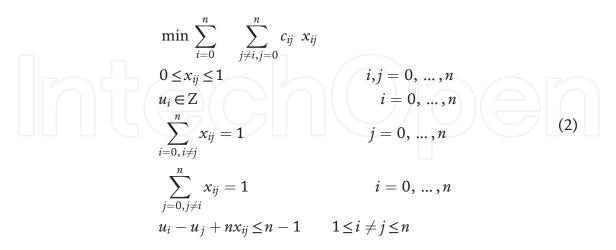
1. Introduction

The traveling salesman problem (TSP) is considered one of the seminal problems in computational mathematics. Considered as part of the Clay Mathematics Institute Millennium Problem with its assertion of $\mathscr{P} = \mathscr{N}\mathscr{P}$ [1], the TSP problem has been well researched during the past five decades.

The TSP problem can be described as the following: consider a number of cities which must be visited by a traveling salesman, only once, arriving once and departing once and starting and ending at the same city. Given the pairwise distances between cities, what is the best order in which to visit them, so as to minimize the overall distance traveled?

Mathematically, the equation for the TSP can be given as in Eq. (1):

$$x_{ij} = \begin{cases} 1 & \text{the path goes from city } i \text{ to city } j \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where $x_{ij} = 1$ if city i is connected with city j, and $x_{ij} = 0$ otherwise. For $i = 0, \ldots, n$, let u_i be an artificial variable and finally take c_{ij} to be the distance from city i to city j. The objective function can be then formulated as Eq. (2):

$$\min \sum_{i=0}^{n} \sum_{j \neq i, j=0}^{n} c_{ij}\ x_{ij}$$

$$0 \leq x_{ij} \leq 1 \qquad\qquad i, j = 0, \ldots, n$$
$$u_i \in \mathbb{Z} \qquad\qquad\qquad i = 0, \ldots, n$$
$$\sum_{i=0, i \neq j}^{n} x_{ij} = 1 \qquad\qquad j = 0, \ldots, n \tag{2}$$
$$\sum_{j=0, j \neq i}^{n} x_{ij} = 1 \qquad\qquad i = 0, \ldots, n$$
$$u_i - u_j + n x_{ij} \leq n - 1 \qquad 1 \leq i \neq j \leq n$$

2. Complexity

The complexity of the TSP is still unknown. Using a brute force approach to test each and every tour, for a tour of n cities, it will be (n−1)! possibilities and it will have a time complexity of $O(n!)$. However, using the dynamic programming approach, the complexity can be derived of a tour of n cities, which can be divided into n−2 subsets each of size n−1, with the constraint that all such subsets don't have

the nth city in them. Therefore, there are a maximum of $O(n2^n)$ such subproblems, which can be solved in *linear* time. The time complexity is therefore $O(n^2 2^n)$. Both space and time complexity of the TSP problem can be considered as *exponential*.

3. History

The genesis of the TSP problem is difficult to pinpoint. Some literature point to widespread usage since the 1950's [2], after the *48 state problem* posed by Hassler Whitney in the 1930's induced a lot of interest. The subsequent second world war really ingrained the use of operations research into this domain. An excellent detailed history is given in [3], where TSP is considered as a part of the history of Combinatorial Optimization.

The TSP problem over time has evolved into many different branches, each with different constraints:

Symmetric TSP (STSP) - the basic TSP problem, where the distance between city *i* and city *j* is the same as from city *j* and city *i*.

Asymmetric TSP (ATSP) - modified TSP, where the distance between city *i* and city *j* is *not* the same as from city *j* and city *i*.

Hamiltonian Cycle Problem (HCP) - is a problem where finding if a path in an *undirected* or *directed* graph *G* that visits each vertex *V* exactly once exists.

Sequential Ordering Problem (SOP) - Given a set of *n* cities and distances for each pair of cities, find a *Hamiltonian path* from city *1* to city *n* of minimal length, which takes given precedence constraints (such as requiring some nodes to be visited prior) into account.

Capacitated Vehicle Routing Problem (CVRP) - Given *n-1* nodes, 1 depot (with resources) and distances between the nodes, the objective is to satisfy demand at each node using vehicles with identical capacity with the shortest tour.

Case Enough TSP (CETSP) - a problem developed for radio frequency identification (RFID), where close proximity is enough to a node. The objective is to trace the shortest path interlinking the different nodes.

TSP with Neighborhoods (TSPN) - where a collection of $\mathscr{L}$ regions in the plane, called *neighborhoods* is given and the objective is to seek the shortest tour to visit all these neighborhoods.

4. Current literature

Linear programming and deterministic methods have been seen as the early solvers, however, intractability of this problem has led to a general decline in these mathematical formulations. Within the past few decades with the rise of computational power, a new branch of algorithms called *meta-heuristics* generally based on evolutionary dynamics have become more synonymous with solving combinatorial optimization problems. Based around naturally occurring phenomena, these algorithms treat each problem as a black box with the aim of finding feasibly good solutions within acceptable space and time constraints. A vast repository of literature exists for the TSP problem, and the TSP Library is an excellent starting off resource point [4].

4.1 Deterministic approaches

Some of the latest literature on the TSP problem is divided into three components. The first is the exact and approximation algorithms, which try and produced

efficient and reasonably good quality solutions. Some of the latest approaches are given below.

1. 2-Opt Algorithm [5]

2. Branch and Cut Algorithm [6]

3. Approximate and Exact Algorithms [7]

4. Branch and Bound [8]

4.2 Evolutionary approaches

The second aspect is evolutionary algorithms. A vast number of these algorithms are now in existence and have been applied to the TSP problem from the seminal work on the Ant Colony Optimization by Dorigo and Gambardella [9] to the following current research.

1. Artificial Bee Colony [10]

2. Differential Evolution [11]

3. Genetic Algorithm [12]

4. Tree Seed Algorithm [13]

5. Spider Monkey [14]

6. Ant Colony Optimization [15]

7. Harmony Search Algorithm [16]

8. Pigeon Inspired Optimization [17]

4.3 High performance computing

The third aspect is application based, specifically high-performance computing. With the wider dissemination of parallel computing, especially multi-core and graphic processor unit based approaches, many algorithms have been parrallized. Some of the latest approaches from literature is given as:

1. Multi-Core approach [18]

2. OpenMP [19]

3. CUDA [20]

5. Future direction

Even though a number of problems, especially in the combinatorial and scheduling domain have increased over the past decade, the TSP problem have remained a vital area of research. This is primarily for it being generally equated to the

intractably quandary of $\mathscr{P} = \mathscr{N}\mathscr{P}$, with its far reaching consequences in other fields such as encryption etc. It is the belief that a combination of smart heuristics employed on super-computers with parallel programming paradigms will be the future direction of tacking large-scale TSP problems.

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