Chapter

Impact of Hybrid-Enabling Technology on Bertrand-Nash Equilibrium Subject to Energy Sources

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Abstract

In this chapter, we quantify an optimal level of subsidy for the sharing of hybrid-enabling technology innovation in an energy market while examining its Bertrand-Nash equilibrium. We formulate this as a Stochastic Differential Game (SDG) and analyze the stability of the Stuckenberg, Nash and cooperative equilibria via a feedback control strategy. We then adopt limit expectation and variance of the improvement degree to identify the influence of the external environment on the decision maker. We show that the game depends on its parameters and the equilibria chosen. Ultimately, our use of short-run price competition characterized by strategic supplies for renewable and fossil resources provides a more robust model than that presented by Bertrand-Edgworth with endogenous capacity. As a result, we highlight that R&D investments in hybrid-enabling technology can ensure immediate reliability and affordability within energy production and implementation of policy instruments.

Keywords: Bertrand duopoly game, cooperative game, hybrid-enabling technology, Nash non-cooperative game, Stackelberg game, stochastic differential game

1. Introduction

In recent years, many researchers have developed models to discuss the importance of lowering carbon emissions and its potential impact on society by examining economic growth, international trade, and health benefits. Khan et al. [1] examined the relationship between green logistics indices, economic, environmental, and social factors through the perspective of Asian emerging economies. By adopting a Fully Modified OLS (FMOLS) Model and Dynamic OLS (DOLS) they claimed that logistics operations, particularly the efficiency of customs clearance processes, quality of logistics services and trade and transport-related infrastructure are positively and significantly correlated with per capita income, manufacturing value added and trade openness, whereas greater logistics operations are negatively associated with social and environmental problems including, climate change, global warming, carbon emissions, and poisoning atmosphere. Khan et al. [2] examined the potential relationship between public health expenditures, logistics performance indices, renewable energy, and ecological sustainability in members of the Association of Southeast Asian Nations by applying the structural equation modeling approach. They showed that the use of renewable energy in logistics operations will improve environmental and economic performance to reduce emissions, whereas environmental performance is negatively correlated with public health expenditures, indicating that greater environmental sustainability can improve human health and economic growth. In [3], economic growth and environmental sustainability in the South Asian Association for Regional Cooperation using the data from the South Asian Association for Regional Cooperation (SAARC) member countries from 2005 to 2017 was examined. Adopting the panel autoregressive distributed lag technique to examine the hypotheses, they find that environmental sustainability is strongly and positively associated with national scale-level green practices, including renewable energy, regulatory pressure, eco-friendly policies, and the sustainable use of natural resources. In [4], the consumption of renewable energy with international trade and environmental quality in Nordic countries from 2001 to 2018 is investigated. Their findings concluded that renewable energy is strongly and positively associated with international trade in Nordic countries. Furthermore, [5] adopted multi-criteria-decision-making techniques to examine barriers in the sustainable supply chain management (SSCM) when firms are facing heavy pressure to adopt green practices in their supply chain (SC) operations to achieve better socio-environmental sustainability.

Around the world governments, businesses and individuals have committed to reducing carbon emission. As a result, the energy economy is highly exposed to these processes. As industries push for renewable energies, technology will need to step in to ensure reliability of the power supply. Therefore, there remains a need for exploiting the role of hybrid technology, its dynamics, limitations on the reduction of pollution levels and policy implementation within the wider carbon emissions debate. This is due to the vital role hybrid technology plays in energy production processes and the ability for the energy system to offer a better energy security. Development of such lower carbon emission policies has potential benefits to the environment and ecological sustainability to those economies. However, within many of these environmental policy models, technology is incorporated as an exogenous variable and limited attention is given to endogenous technology, other technological breakthroughs, potential government subsidies or collaborative innovations to integrate low carbon technology in environmental economics. Such interventions will promote the renewable energy sector to use natural resources and undertake public-private partnership investments to minimize dependence on fossil fuel derived energy.

To investigate the effects of hybrid-enabling technology when producing energy to meet consumption demand, we assume energy producing firms follow the Bertrand game paradigm. In the presence of government subsidy for the development and sustainability of renewable energy, tax on pollution created by energy producing firms will motivate them to undertake Research & Development (R&D) measures to improve hybrid enabling technologies to further reduce the level of carbon pollution. As a result, from an economic point of view it is an interesting question to examine the Bertrand-Nash equilibrium under such a dynamic environment. This chapter examines this concept via a Stochastic Differential game paradigm.

Many researchers have applied game theory to study carbon reduction behavior in electricity markets. In [6], the Cournot equilibria in an oligopolistic electricity market subject to a linear demand function is examined. In [7], the power suppliers bidding behavior is evaluated under the supply function equilibrium (SFE) paradigm, where the market power of an independent system operator (ISO) is modeled as a bi-level multi-objective problem. In [8], the equilibrium strategies in random-demand procurement auctions in the electricity market is obtained and presented a method for explicit calculation of the bid strategies is presented. [9] proposed a Nash bargaining game model to examine how governments can determine the taxes and subsidies in a competitive electricity market whilst achieving their environmental objectives. In [10–14], the role of government as a leading player who intervenes in competitive electricity markets to promote environmental protection is evaluated. In [15], the role of government, when managing environmental sustainability in a complete electricity market in a Stackelberg game paradigm is examined. In [16], a more robust trans-boundary industrial pollution reduction strategy for global emission collaborations is presented. The dynamics of each country's quantity of pollution is modeled as a Brownian motion with Jumps to capture the systematic jumps caused by surprise effects arising from policy uncertainties within the economy. However, a crucial limitation within many of these environmental policy models, is that technological change is incorporated as an exogenous variable and does not consider the role of endogenous hybrid-enabling technology or other technological breakthroughs, hence limiting the dynamics of these models.

We quantify an optimal level of subsidy for the sharing of hybrid-enabling technology in an energy market under a Bertrand-duopoly game. We formulate a Stochastic Differential Game (SDG) to analyze the stability of the Stackelberg, Nash and cooperative equilibria via a feedback control strategy. We then adopt limit expectation and variance of the improvement degree to identify the influence of external environment limitations on the decision maker. We show that the game depends on its parameters and the equilibria chosen. We consider an electricity market composed of power plants I and II, with each one having the choice between fossil fuels (F) (e.g., natural gas, petroleum or coal) and renewable sources (R) (e.g., biomass, solar, wind, wave, geothermal or hydroelectric). Such hybrid power plants play a crucial ameliorating role in managing the long-standing problem of climate change and ensure immediate reliability and affordability of energy production, whilst reducing Greenhouse Gas (GHG) emissions.

In this model, we consider a Bertrand duopoly game for two power plants under endogenous hybrid-enabling technology. In the first stage the matrix of prices $\left(p_{ij}\right)_{ij\in\{F,R\}}$, (where $\left(p_{ij}\right)$ is the price of energy i, given that the opponent player's energy j) is determined as a Nash equilibrium of the game where each player wants to optimize his/her demand. We then search for the Nash equilibria, and the optimal proportions that maximizes the $(\Pi)_{ij}, ij \in \{F,R\}$ subject to the source type of energy that has been used. Once all these parameters have been fixed, the game becomes dynamic due to the evolution of a hybrid-enabling technology level $K(t)$, prompted by Research and Developments (R&D) measures undertaken by each power plant. Hence, each player must fix a time-dependent effect level associated with this hybrid-enabling technology. In doing so, this study makes the following contributions to existing game theory/energy economics literature:

i. stochastic endogenous hybrid-enabling technology innovation is introduced into a two-player stochastic differential game with random interference factors, which capture uncertain external environment factors and the internal limitations within the shared hybrid-enabling technology decision process. In doing so, we provide a framework to quantify the impacts of market power on prices.

ii. we show that both power plants invest in R&D measures and that the limit of expectation and variance of the improvement degree can be applied to identify the influence of random factors.

iii. mathematically, we show that the issue of the game depends on the parameters of the game and the type of equilibrium one considers.

iv. by applying the HJB equation we obtain the optimal effort level and the optimal level of subsidy for sharing hybrid-enabling technology via feedback equilibrium strategies whilst examining the Stackelberg equilibria, Nash equilibria and cooperative equilibria under Bertrand duopoly.

v. we reveal that for a given level of payoff distribution the Stackelberg equilibria under endogenous hybrid-technology innovation and the sharing paradigm dominate the Nash equilibria.

vi. we show that in Stackelberg and Nash games, optimal hybrid-enabling technology innovation is proportional to the government subsidy, but the variance improvement degree of the Stackelberg game is different to the results of the Nash game.

vii. our characterization of the short run price competition by strategic supplies for renewable and fossil resources, provides a more robust model than that presented by Bertrand-Edgworth, in which price competition with fixed (endogenous) capacities was used.

viii. our model shows that robust cost-reducing R&D investments with effective hybrid-enabling technology innovation strengthens an innovator's competitive position and the Stackelberg structure emerges as an equilibrium outcome, allowing each power plant to optimally use energy sources to produce electricity while maximizing their payoffs.

Therefore, under a Stochastic Differential Game (SDG) paradigm with uncertainty, each power plant can optimally use energy sources to produce electricity while maximizing their payoffs. Each power plant is capable of using fossil fuels (F) and renewable sources (R) to produce electricity at any time. To maintain the generality of the proposed model, this model is not limited to a specific energy source. Hence, the terms ″F″ and ″R″, are used throughout the paper. On the other hand the government encourages power plants to conform to a maximum accepted level of carbon emissions through strategies such as the imposition of tariffs on polluters as well as incentives for those who choose to undertake R&D measures to reduce their emission levels in order to maintain environmental sustainability. R&D spending is costly, and the presumption is that R&D spending is somehow connected to increased innovation, revenue growth and profits.

In recent years, researchers have incorporated the theory of SDGs, originated from [14, 17–20] to analyzed environmental issues. Especially [21] analyzed (two player) zero-sum stochastic differential games in a rigorous way, and proved that the upper and lower value functions of such games satisfy the dynamic programming principle whilst being the unique viscosity solutions of their associated Hamilton-Jacobi-Bellman-Isaacs equations.

In Section 2, the proposed model and elements of evolutionary game theory are presented. In Section 3 by implementing the Stackelberg game we examine feedback Stackelberg equilibria, optimal level of subsidy for the shared hybrid-enabling technology from its counterpart and the limit of expectation and variance. In Section 4 by implementing a Nash game we examine feedback Nash equilibria and the limit of expectation and variance under hybrid-enabling technology. In Section 5 by implementing a cooperative game we examine feedback equilibria and the limit of expectation and variance under hybrid-enabling technology. In Section 6, comparative analysis of equilibrium results are described. Section 7 concludes the study. Appendix at the end of the chapter contains proofs.

2. Model setup

We propose that the production process of electricity leads to emissions and is proportional to the power industry's use of energy source. We assume that there are two power plants (Player I) and (Player II) in the energy market and each power plant is capable of using *fossil fuels* (F) and *renewable sources* (R) to generate power at any given time t. To reduce the level of Green House Gas (GHG)-emissions into the atmosphere (accordance with [22] Protocol), the government will set a maximum emission quantitative level, that is directly linked to the power industry's use of energy source F, when producing electricity. Government encourages the power industry to undertake necessary hybrid-enabling technology to reduce their GHG-emission levels to the maximum accepted quantitative level, $\overline{\eta}^F$, and improve efficiency in renewables. We assume that the power plants change their strategies over time based on payoff comparisons based on hybrid-enabling technological advances. This contradicts with classical non cooperative game theory that analyzes how rational players will behave through static solution concepts such as the Nash equilibrium (NE) (i.e., a strategy choice for each player whereby no individual has a unilateral incentive to change his or her behavior).

Under the theory of evolutionary games, the production strategies in the *absence of any superior hybrid-enabling technological advances*, allows the power plants to play a symmetric two-person 2×2 bi-matrix game. Thus, for each power plant, we define the set Σ as its pure strategy given by the set of non-negative prices $[0, \infty)$. According to the Bertrand game all firms setting the lowest price will split market demand equally (Hotelling type) and the profit can be calculated subject to the electricity prices and the associated cost functions.

Then each iteration of an evolutionary game, where two matched power plants in accordance with Bertrand paradigm compete with each market and play a one-shot non-zero-sum game, represents the benchmark game of the population. If $\left(p_{ij}, p_{ji}\right)$ is the matrix of prices of power plants, respectively, then via Proposition 1 (given below), it will allow us to derive Nash equilibria of prices for these two matched power plants. On the demand side we assume that the preferences are quadratic as in [23].

We define the continuous demand function $\left(D_{ij}\right)$, for each power plant as

$$D_{ij} = a_{ij} - \beta_{ij}\left(p_{ij} + \tau_i\right) + \gamma_{ij}\left(p_{ji} + \tau_j\right), \quad i,j \in \{F, R\} \tag{1}$$

where D_{ij} is the demand function for the power plants employing the energy source $i \in \{F, R\}$ against the power plant which use the energy source $j \in \{F, R\}$. τ_i is the tariff imposed by government subject to the power source i. For example government impose a tariff-rate quota (TRQs) (τ_F), for *fossil fuels* (F) and a feed-in-tariff (FITs) (τ_R), for *renewable sources* (R). p_{ij} is the electricity price of the power plant that uses the energy source i, versus the power plant that employs the energy source j. $a_{ij} > 0$, is the constant market base for the power plant that employs the energy source i versus the one which use the energy source j. The parameters $\beta_{ij} > 0$ and $\gamma_{ij} > 0$, are independent constants that captures the demand sensitivity of a power plant subject to its own price β_{ij} and its rival's price γ_{ij}. Eq. (1), concludes that the goods in the market are gross substitutes and that the demand function D_{ij}, is increasing in the price of the rival firm p_{ji}.

The government's tariff policy for the power plants with respect to their source of energy for long-time periods are transparent, and this information is available to the public. Therefore, it is assumed that the competitive power plants follow the government's financial legislation, having the capability and technological skills to produce electricity from specific sources at any given time to meet energy demand. Then for each time-period the power plant will consider the tariff-rate quota or feed-in-tariff and adopt a pricing strategy for the selected energy source. Hence, we conclude that the production rate of the power plants is equal to the corresponding demand rates with a negligible internal consumption and waste rate.

To apply the Backward induction technique to investigate the equilibrium prices, demand, and profits, we define the profit function for each power plant as

$$\begin{aligned}\Pi_{ij} &= \left(p_{ij} - C_i - v_i\right)D_{ij} - F_i \\ &= \left(p_{ij} - C_i - v_i\right)\left(a_{ij} - \beta_{ij}\left(p_{ij} + \tau_i\right) + \gamma_{ij}\left(p_{ji} + \tau_j\right)\right) - F_i,\end{aligned} \tag{2}$$

where $i, j \in \{F, R\}$ and $C_i > 0$ is the unit production cost of the power plant when using energy source i. $v_i > 0$, for any additional R&D unit cost for undertaking hybrid-enabling technology, for a power plant that rely on an energy source i, ($F_i > 0$ is the initial setup cost of the power plants when using the energy source i). We also assume that $\left(p_{ij} - C_i - v_i\right) > 0$. The firms' technologies are represented by their reduced cost functions. This assumes that all factor markets are perfectly competitive and – both here and in the models of imperfect competition in the output market – are not influenced by any strategic behavior of the firms in other markets. We will make alternative assumptions about those technologies. In the first assumption, pollution is proportional to output and firms do not have any further abatement technologies.

Proposition 1. *The equilibrium price for the power plants under* (τ_F, τ_R), *is given as* $p_{ij} = \Lambda_{ij} + C_i + v_i$.

Proof. Via the first order conditions of the profit function (Eq. (2)), obtain

$$\begin{aligned}\frac{\partial \Pi_{ij}}{\partial p_{ij}} = a_{ij} - 2\beta_{ij}\left(p_{ij} - C_i - v_i\right) + \gamma_{ij}\left(p_{ji} - C_j - v_j\right) + \gamma_{ij}(\tau_j + C_j + v_j) \\ -\beta_{ij}(\tau_i + C_i + v_i) = 0.\end{aligned} \tag{3}$$

Defining $\Lambda_{ij} = p_{ij} - C_i - v_i$ and using Λ_{ij} and Λ_{ji}, rewrite the first order conditions as:

$$\begin{cases} 2\beta_{ij}\Lambda_{ij} - \gamma_{ij}\Lambda_{ji} & = a_{ij} + \gamma_{ij}(\tau_j + C_j + v_j) - \beta_{ij}(\tau_i + C_i + v_i), \\ 2\beta_{ji}\Lambda_{ji} - \gamma_{ji}\Lambda_{ij} & = a_{ji} + \gamma_{ji}(\tau_i + C_i + v_i) - \beta_{ji}(\tau_j + C_j + v_j). \end{cases} \tag{4}$$

Simultaneously solving Eq. (4), obtain

$$\Lambda_{ij} = \frac{2\beta_{ji}a_{ij} + \gamma_{ij}a_{ji} + \beta_{ji}\gamma_{ij}\left(\tau_j + \left(C_j + v_j\right)\right) + \left(\gamma_{ij}\gamma_{ji} - 2\beta_{ji}\beta_{ij}\right)\left(\tau_i + \left(C_i + v_i\right)\right)}{\left(4\beta_{ji}\beta_{ij} - \gamma_{ij}\gamma_{ji}\right)} \tag{5}$$

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such that $\beta_{ji}\beta_{ij} \neq \frac{\gamma_{ij}\gamma_{ji}}{4}$. p_{ij}^* and p_{ji}^* are obtained via $p_{ij} = \Lambda_{ij} + C_i + v_i$ and are the optimum prices if the profit functions are concave on p_{ij} and on p_{ji}. Then via the second order conditions, obtain the maximum point in the set as: $\frac{\partial^2 \Pi_{ij}}{\partial p_{ij}^2} = -2\beta_{ij} < 0$. Since $\beta_{ij} > 0$, implies that the second derivative of the profit function in equilibrium is negative confirming that the profit function is concave at this point.

Proposition 2. *At equilibrium prices the power plant's demand and profit under* (τ_F, τ_R), *can be obtained as*

$$\begin{aligned} D_{ij}^* &= \beta_{ij}\Lambda_{ij}^* = \beta_{ij}\left(\theta_{ij} + \omega_{ij}\tau_i^* + \chi_{ij}\tau_j^*\right), \\ \Pi_{ij}^* &= \beta_{ij}\left(\Lambda_{ij}^*\right)^2 - F_i = \beta_{ij}\left(\theta_{ij} + \omega_{ij}\tau_i^* + \chi_{ij}\tau_j^*\right)^2 - F_i, \end{aligned} \tag{6}$$

where

$$\theta_{ij} = \frac{\left(2\beta_{ji}a_{ij} + \gamma_{ij}a_{ji} + \beta_{ji}\gamma_{ij}(C_j + v_j) + \left(\gamma_{ij}\gamma_{ji} - 2\beta_{ji}\beta_{ij}\right)(C_i + v_i)\right)}{\left(4\beta_{ji}\beta_{ij} - \gamma_{ij}\gamma_{ji}\right)}, \tag{7}$$

$$\omega_{ij} = \frac{\left(\gamma_{ij}\gamma_{ji} - 2\beta_{ji}\beta_{ij}\right)}{\left(4\beta_{ji}\beta_{ij} - \gamma_{ij}\gamma_{ji}\right)} \text{ and } \chi_{ij} = \frac{\beta_{ji}\gamma_{ij}}{\left(4\beta_{ji}\beta_{ij} - \gamma_{ij}\gamma_{ji}\right)} \tag{8}$$

Proof. Obtain the results by substituting Λ_{ij}^* from Proposition 1 into Eqs. (1) and (2) and simplifying.

Remark 1. The only outcome where neither power plant has an incentive to deviate is when $p_{ij} = p_{ji} = c_i$, which will be the Nash or Bertrand equilibrium for the game. The intuition behind this result is that power plants will keep *undercutting* the price of its rival until price equals marginal cost. In the long run price changes with marginal cost and industry production increases with demand and falls with marginal cost. One way for a power plant to avoid the Bertrand paradox and earn economic profit in a Bertrand setting is to have a competitive cost advantage over its rival.

2.1 Production decisions of power plants with homogenous hybrid/enabling technology

Restricting ourselves to a two matched symmetric two-person bi- matrix game in random contest in a one-population evolutionary game, we define the payoff (utility) in **Table 1**.

Note 1. Power and the payoff are measured on a utility scale consistent with the power plant's preference ranking. Furthermore, [24–26] have applied symmetric two-person bi-matrix game in random contest to study evolutionary stable games.

		Power Plant II	
	Production Strategy	Fossil Fuel	Renewable Sources
Power Plant I	Fossil Fuel	(Π_{FF}, Π_{FF})	(Π_{FR}, Π_{RF})
	Renewable Sources	(Π_{RF}, Π_{FR})	(Π_{RR}, Π_{RR})

Table 1.
Bi matrix for two power plants by different energy sources.

Then via equation $\Pi_{ij}^* = \beta_{ij}\left(\Lambda_{ij}^*\right)^2 - F_i = \beta_{ij}\left(\theta_{ij} + \omega_{ij}\tau_i + \chi_{ij}\tau_j\right)^2 - F_i$, in Proposition 1, and the payoff matrix of the power plant I is given by:

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} \Pi_{F,F} & \Pi_{F,R} \\ \Pi_{R,F} & \Pi_{R,R} \end{bmatrix} = \begin{bmatrix} \beta_{F,F}\Lambda_{F,F}^2 - F_F & \beta_{F,R}\Lambda_{F,R}^2 - F_F \\ \beta_{R,F}\Lambda_{R,F}^2 - F_R & \beta_{R,R}\Lambda_{R,R}^2 - F_R \end{bmatrix}. \tag{9}$$

Obviously the bimatrix of the power plant II, is given by:

$$A = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{bmatrix} = \begin{bmatrix} \Pi_{F,F} & \Pi_{R,F} \\ \Pi_{F,R} & \Pi_{R,R} \end{bmatrix} = \begin{bmatrix} \beta_{F,F}\Lambda_{F,F}^2 - F_F & \beta_{R,F}\Lambda_{R,F}^2 - F_R \\ \beta_{F,R}\Lambda_{F,R}^2 - F_F & \beta_{R,R}\Lambda_{R,R}^2 - F_R \end{bmatrix}. \tag{10}$$

Proposition 3. *The Nash equilibrium for the Bi-matrix game G, is given as*

$$\left(\frac{(\Pi_{R,R} - \Pi_{F,R})}{(\Pi_{F,F} - \Pi_{R,F} - \Pi_{FR} + \Pi_{R,R})}, \frac{(\Pi_{R,R} - \Pi_{F,R})}{(\Pi_{F,F} - \Pi_{F,R} - \Pi_{R,F} + \Pi_{R,R})}\right). \tag{11}$$

Proof. Suppose players I and II use mixed strategies (x,1-x) and (y,1-y), respectively, where

i. The probability that player I choosing row 1 is x and the probability that player I choosing row 2 is 1-x.

ii. The probability that player II choosing row 1 is y and the probability that player II choosing row 2 is 1-y.

Then the value of the game for Player I is

$$\begin{aligned} v_1(x,y) &= xy(\Pi_{F,F}) + x(1-y)(\Pi_{F,R}) + (1-x)y(\Pi_{R,F}) + (1-x)(1-y)(\Pi_{R,R}) \\ &= ((\Pi_{F,F} - \Pi_{F,R} - \Pi_{R,F} + \Pi_{R,R})y + (\Pi_{F,R} - \Pi_{R,R}))x + ((\Pi_{R,F} - \Pi_{R,R})y + \Pi_{R,R}), \end{aligned} \tag{12}$$

and the value of the game for Player II is

$$\begin{aligned} v_2(x,y) &= xy(\Pi_{F,F}) + x(1-y)(\Pi_{R,F}) + (1-x)y(\Pi_{F,R}) + (1-x)(1-y)(\Pi_{R,R}) \\ &= ((\Pi_{F,F} - \Pi_{R,F} - \Pi_{F,R} + \Pi_{R,R})x + (\Pi_{F,R} - \Pi_{R,R}))y + ((\Pi_{R,FR} - \Pi_{R,R})x + \Pi_{R,R}). \end{aligned} \tag{13}$$

Suppose (X, Y) yields a Nash equilibrium. Then for the given payoffs having $0 < x < 1$ implies that

$$v_1 = (\Pi_{F,F} - \Pi_{F,R} - \Pi_{R,F} + \Pi_{R,R})y + (\Pi_{F,R} - \Pi_{R,R}) = 0. \tag{14}$$

Otherwise Player I can change x slightly and do better.
Similarly, for $0 < y < 1$,

$$v_2 = (\Pi_{F,F} - \Pi_{R,F} - \Pi_{FR} + \Pi_{R,R})x + (\Pi_{F,R} - \Pi_{R,R}) = 0. \tag{15}$$

Otherwise Player II can change y slightly and do better. It follows that the unique Nash equilibrium (x,y), has

$$\left(\frac{(\Pi_{R,R} - \Pi_{F,R})}{(\Pi_{F,F} - \Pi_{R,F} - \Pi_{FR} + \Pi_{R,R})}, \frac{(\Pi_{R,R} - \Pi_{R,F})}{(\Pi_{F,F} - \Pi_{F,R} - \Pi_{R,F} + \Pi_{R,R})}\right). \tag{16}$$

Remark 1. Since the power plants plays a symmetric two person bimatrix game G, having two pure strategies $\Pi_{F,F} \neq \Pi_{R,F}$, $\quad \Pi_{R,R} \neq \Pi_{FR}$, imply that G, has an evolutionary stable strategy. Then the Nash equilibrium is an outcome in which the strategy chosen by each player is the best reply to the strategy chosen by the other. This best reply strategy yields the highest payoff to the player choosing it, given the strategy chosen by the co-player, [27, 28].

2.2 Production decisions of power plants under endogenous hybrid/enabling technological advances

Both players will undertake R&D measures on hybrid-enabling technology to ensure immediate reliability and affordability in energy production whilst reducing GHG-emissions. We assume that the strategic effects implemented by power plant I (Player I), has improved hybrid-enabling technology to generate energy and utilize energy sources in a much efficient way. This gives a superior advantage to power plant I overpower plant II (Player II) and both power plants are rational to maximize their profits. Although Power plant II has heterogeneous resources to hybrid-enabling technology, from a practical point of view it is logical for power plant I to share this technology with power plant II, because the price competition is typically characterized by a second-mover advantage. Many researchers have investigated the effects of these commitments in Cournot, Bertrand and Stackelberg setups. See [29–31]. Due to the government incentives, tariff-rate quota, feed-in-tariff and R&D incentive measures, the power companies will be competitive to improve their efficiency. Let $L^R(t)$ denotes the *R&D effort level* of technological improvements on renewable sources at time t, and $L^F(t)$ denotes the *R&D effort level* of technological improvements on fossil fuel at time t, of Player I. $\tilde{L}^R(t)$ denotes the *R&D effort level* of technological improvements on renewable sources at time t, and $\tilde{L}^F(t)$ denotes the *R&D effort level* of technological improvements on fossil fuel at time t, of Player II. For, further consideration, the sharing cost of advanced hybrid-enabling technology (Player I) and inferior hybrid-enabling technology (Player II) is denoted as $C_{\mathrm{I}}(t)$ and $C_{\mathrm{II}}(t)$, which are the quadratic functions of the effect level of Player I and Player II at time t, respectively. Consider

$$C_{\mathrm{I}}\left(L^R(t), L^F(t), t\right) = \frac{1}{2}\left(\beta^R(t)\left(L^R(t)\right)^2 + \beta^F(t)\left(L^F(t)\right)^2\right), \tag{17}$$

and

$$C_{\mathrm{II}}\left(\tilde{L}^R(t), \tilde{L}^F(t), t\right) = \frac{1}{2}\left(\tilde{\beta}^R(t)\left(\tilde{L}^R(t)\right)^2 + \tilde{\beta}^F(t)\left(\tilde{L}^F(t)\right)^2\right), \tag{18}$$

where $0 < \left(\beta^R(t), \beta^F(t), \tilde{\beta}^R(t), \tilde{\beta}^F(t)\right) \leq 1$ and lower the $\left(\beta^R(t), \beta^F(t), \tilde{\beta}^R(t), \tilde{\beta}^F(t)\right)$, more effective is the technological development.

Let $K(t)$ denote the evolution of the hybrid-enabling technology at time t, due to R&D collaborative innovation system of Player I and Player II at time t. The dynamics of hybrid-technology is governed by the stochastic differential equation (SDE):

$$\begin{cases} dK(t) = \left[\vartheta_1(t)\left(L^R(t), L^F(t)\right) + \vartheta_2(t)\left(\tilde{L}^R(t), \tilde{L}^F(t)\right) - \xi K(t)\right]dt + \varphi\sqrt{K}dW(t) \\ K(0) = K_0 > 0. \end{cases} \tag{19}$$

$\xi \in (0, 1]$, is the attenuation coefficient of hybrid-enabling technology. Let $\vartheta_1(t) = \left(\vartheta_1^R(t) + \vartheta_1^F(t)\right)$ and $\vartheta_2(t) = \left(\vartheta_2^R(t) + \vartheta_2^F(t)\right)$ denote the influence of the effort level of hybrid-enabling technology sharing on collaboration innovation between Player I and Player II, at time t. $W(t)$ is a standard Brownian motion and $\varphi\left(\sqrt{K}(t)\right)$ random interference factor on hybrid-enabling technology.

Let $\Pi(t)$ denotes the total profit under the hybrid-enabling technology system at time t. Let $(\alpha_1(t), \alpha_2(t))$ and $(\beta_1(t), \beta_2(t))$ denote the influence of the effort level hybrid-enabling technology on the total profit of Player I and player II, respectively, at time t, namely, the marginal return coefficient of hybrid-enabling technology. Total profit function can be expressed as:

$$\Pi(t) = \left(\alpha_1(t)L^R(t) + \alpha_2(t)L^F(t)\right) + \left(\beta_1(t)\tilde{L}^R(t) + \beta_2(t)\tilde{L}^F(t)\right) + (\Gamma + \delta)K(t), \quad (20)$$

where

$$\alpha_1(t) = \frac{\Pi_{R,R}(t)}{\Pi_{F,F}(t) - \Pi_{R,F}(t) - \Pi_{F,R}(t) + \Pi_{R,R}(t)}, \quad (21)$$

$$\alpha_2(t) = \frac{-\Pi_{F,R}(t)}{\Pi_{F,F}(t) - \Pi_{R,F}(t) - \Pi_{F,R}(t) + \Pi_{R,R}(t)}, \quad (22)$$
$$\Gamma = \Gamma_{(\mathrm{I})} + \Gamma_{(\mathrm{II})}, \delta = \delta_{(\mathrm{I})} + \delta_{(\mathrm{II})}, \text{and}$$

$$\beta_1(t) = \frac{\Pi_{R,R}(t)}{\Pi_{F,F}(t) - \Pi_{R,F}(t) - \Pi_{F,R}(t) + \Pi_{R,R}(t)}, \quad (23)$$

$$\beta_2(t) = \frac{-\Pi_{R,F}(t)}{\Pi_{F,F}(t) - \Pi_{R,F}(t) - \Pi_{F,R}(t) + \Pi_{R,R}(t)}. \quad (24)$$

Γ is the influence of the hybrid-enabling technology innovation on total revenue $\delta \in (0, 1]$; δ is the total government subsidy coefficient of hybrid-enabling technology based on increments of advances in hybrid-enabling technology.

Proposition 4. *At least one of the Power Plants has a second mover advantage.*

Proof. Demand function $D_{ij}\left(p_{ij}, p_{ji}\right) > 0$, given by Eq. (4), is twice continuously differentiable and

$$\frac{\partial D_{ij}\left(p_{ij}, p_{ji}\right)}{\partial p_{ij}} = -\beta_{ij} < 0, \textit{and } \frac{\partial D_{ij}\left(p_{ij}, p_{ji}\right)}{\partial p_{ji}} = \gamma_{ij} > 0 \forall \left(p_{ij}, p_{ji}\right) \in P_{\mathrm{I}} \times P_{\mathrm{II}}. \quad (25)$$

The first inequality says that each demand is downward sloping in own price, and the second that goods are substitutes (each demand increases with the price of the other good). [32] shows that in case of symmetric firms, there is a second-mover (first-mover) advantage for both players when each profit function is strictly concave in own action and strictly increasing (decreasing) in rival's action, and reaction curves are upward (downward) sloping.

Then a sufficient condition on the super-modularity of the profit function is obtained via the profit function Π_{ij}, given by Eq. (4):

$$\left[\frac{\partial D_{ij}\left(p_{ij}, p_{ji}\right)}{\partial p_{ji}} + \left(p_{ij} - C_i - v_i\right)\frac{\partial^2 D_{ij}\left(p_{ij}, p_{ji}\right)}{\partial p_{ji} p_{ij}}\right] E(K(t)) > 0, \quad (26)$$

where E is the expectations. The main implication of this is that it leads to reaction correspondences that are non-decreasing (in the sense that each selection is non-decreasing) but need not be single-valued or continuous. This has a very appealing and precise interpretation: The price elasticity of Power Plant i's demand increases in the rival's price, [33]. This is a very intuitive and general condition, though clearly not a universal one. It is satisfied in particular if $\frac{\partial^2 D_{ij}(p_{ij}, p_{ji})}{\partial p_{ji} p_{ij}} > 0$, if a higher price by a Power Plant's rival does not lower the responsiveness of the Power plant's demand to a change in own price.

We further assume that the total revenue is allocated between two players and $\theta(t)$ is the payoff distribution coefficient of player I at time t and $\theta(t) \in [0, 1]$. Although Player II has heterogeneous resources of hybrid-enabling technology, Player I can produce electricity more efficiently with lower GHG-emission, ensure immediate reliability and affordability in energy production. Then Player II, can acquire practical outcomes of this hybrid-enabling technological advances. To promote the hybrid-enabling technology, Player II (leader) determine an optimal sharing effort level and an optimal subsidy. Then Player I (follower) choose their optimal sharing effort level according to the optimal sharing effort level and subsidy. This leads to a Stackelberg equilibrium. Let $\omega(t) = (\omega_1(t), \omega_2(t))$, denote the subsidy for hybrid-enabling technology, with Player II willing to pay to Payer I under collaboration. The objective functions of power plant I and power plant II satisfy the following partial differential equations

$$\begin{aligned} &J_{(\mathrm{I})}(K_0) \\ &= \max_{\{L_S^R, L_S^F\} \geq 0} E\left\{ \int_0^\infty e^{-\rho_1 t} \left[\theta(t) \left(\alpha_1(t) L^R(t) + \alpha_2(t) L^F(t) + \beta_1(t) \tilde{L}^R(t) + \beta_2(t) \tilde{L}^F(t) \right.\right.\right. \\ &\left.\left.\left. + (\Gamma + \delta) K(t) \right) \right) - \frac{1}{2} \beta^R(t)(1 - \omega_1) \left(L^R(t)\right)^2 - \frac{1}{2} \beta^F(t)(1 - \omega_2)\left(L^F(t)\right)^2 \right] dt \right\}, \end{aligned} \tag{27}$$

and

$$\begin{aligned} J_{(\mathrm{II})}(K_0) = &\max_{\{\tilde{L}_S^R, \tilde{L}_S^F, \omega(t)\} \geq 0} E\left\{ \int_0^\infty e^{-\rho_2 t} \left[(1 - \theta(t)) \left(\alpha_1(t) L^R(t) + \alpha_2(t) L^F(t) + \beta_1(t) \tilde{L}^R(t) \right.\right.\right. \\ &\left. + \beta_2(t) \tilde{L}^F(t) + (\Gamma + \delta) K(t) \right) - \frac{1}{2} \tilde{\beta}^R(t) \left(L^R(t)\right)^2 - \frac{1}{2} \tilde{\beta}^F(t) \left(\tilde{L}^F(t)\right)^2 \\ &\left.\left. - \frac{1}{2} \omega_1 \beta^R(t) \left(L^R(t)\right)^2 - \frac{1}{2} \omega_2 \beta^F(t) \left(L^F(t)\right)^2 \right] dt \right\}, \end{aligned} \tag{28}$$

where ρ_1 and ρ_2 are the discount rates of Player I and Player II, respectively. In this feedback control strategy $L_S^R(t) \geq 0$, $L_S^F(t) \geq 0$, L S R (t) ≥ 0, $\tilde{L}_S^R(t) \geq 0$ and $\tilde{L}_S^F(t) \geq 0$, are the control variables and $\omega(t) = (\omega_1(t), \omega_2(t)) \in (0, 1)$. $K(t) > 0$ is the state variable. In feedback control process, it is assumed that players at every point in time have access to the current system and can make decisions accordingly to that state. Consequently, the players can respond to any disturbance in an optimal way. Hence, feedback strategies are robust for deviations and players can react to disturbances during the evolution of the game and adapt their actions accordingly, [34].

3. A Stackelberg game under heterogeneous technology

Theory of strong Stackelberg reasoning is an improved version of an earlier theory [35], which provides an explanation of coordination in all dyadic (two-player) common interest games. It provides an explanation of why players tend to choose strategies associated with a payoff-dominant Nash equilibrium. Its distinctive assumption is that players behave as though their co-players will anticipate any strategy choice and invariably choose a best reply to it. Stackelberg strategies resulting from this form of reasoning do not form Nash equilibria. The theory makes no predictions, because a non-equilibrium outcome is inherently unstable, leaving at least one player with a reason to choose differently and thereby achieve a better payoff. Strong Stackelberg reasoning is a simple theory, according to which players in dyadic games choose strategies that would maximize their own payoffs if their co-players could invariably anticipate their strategy choices and play counter-strategies that yield the maximum payoffs for themselves. The key assumption is relatively innocuous, first because game theory imposes no constraints on players' beliefs, apart from consistency requirements, and second because the theory does not assume that players necessarily believe that their strategies will be anticipated, merely that they behave as though that is the case, as a heuristic aid to choosing the best strategy. Strong Stackelberg reasoning is, in fact, merely a generalization of the minorant and majorant models introduced by [36] and used to rationalize their solution of strictly competitive games.

To promote the sharing of hybrid-enabling technology, the Player II (the leader) determine an optimal sharing effort sharing level and an optimal subsidy scheme. Then the Player I (the follower) choose his/her optimal sharing level according to the optimal sharing effort level and subsidy. This leads to a Stackelberg equilibrium.

Proposition 5. *If above conditions are satisfied, the feedback Stackelberg leader (Player II)-follower (Player I) and equilibria is given as:*

$$L_S^R = \frac{\alpha_1(2-\theta)(\rho_2+\xi)(\rho_1+\xi) + \vartheta_1^R(\Gamma+\delta)((2-2\theta)(\rho_1+\xi)+\theta(\rho_2+\xi))}{2\beta^R(\rho_2+\xi)(\rho_1+\xi)}, \tag{29}$$

$$L_S^F = \frac{\alpha_2(2-\theta)(\rho_2+\xi)(\rho_1+\xi) + \vartheta_1^F(\Gamma+\delta)((2-2\theta)(\rho_1+\xi)+\theta(\rho_2+\xi))}{2\beta^F(\rho_2+\xi)(\rho_1+\xi)}, \tag{30}$$

$$\tilde{L}_S^R = \frac{(1-\theta)(\beta_1(\rho_2+\xi)+(\Gamma+\delta))\vartheta_2^R}{\tilde{\beta}^R(\rho_2+\xi)}, \tag{31}$$

$$\tilde{L}_S^F = \frac{(1-\theta)(\beta_2(\rho_2+\xi)+(\Gamma+\delta))\vartheta_2^F}{\tilde{\beta}^F(\rho_2+\xi)}. \tag{32}$$

where L_S^R, L_S^F are the optimal effort level of hybrid-enabling technological improvements shared on renewable sources and fossil fuel at time t by Player I, respectively. $\tilde{L}_S^R, \tilde{L}_S^F$ are the optimal effort level of technological improvements shared on renewable sources and fossil fuel at time t by Player II, respectively.

The optimal level of subsidy for sharing hybrid-enabling on renewable sources is given by

$$\omega_1 = \begin{cases} \dfrac{\alpha_1(2-3\theta)+\vartheta_1^R[2a_2-a_1]}{\alpha_1(2-\theta)+\vartheta_1^R[2a_2+a_1]}, & 0 \le \theta \le \dfrac{2}{3} \\ 0. & \text{otherwise} \end{cases} \tag{33}$$

Similarly, the optimal level of subsidy for sharing hybrid-enabling technology on fossil fuel is given by:

$$\omega_2 = \begin{cases} \dfrac{\alpha_2(2-3\theta) + \vartheta_1^F[2a_2 - a_1]}{\alpha_2(2-\theta) + \vartheta_1^F[2a_2 + a_1]}, & 0 \le \theta \le \dfrac{2}{3} \\ 0. & \text{otherwise} \end{cases} \tag{34}$$

The optimal sharing payoff functions under hybrid-enabling technology on renewable sources and on fossil fuel for Player I and Player II are given below

$$V_S^{(\mathrm{I})}(K) = \frac{\theta(\Gamma+\delta)}{(\rho_1+\xi)}K + b_1, \quad V_S^{(\mathrm{II})}(K) = \frac{(1-\theta)(\Gamma+\delta)}{(\rho_2+\xi)}K + b_2, \tag{35}$$

where a_1, a_2, b_1 and b_2 are given in the proof.

Proof. We define the optimal revenue functions for Player I and Player II under hybrid-enabling technology as $V_S^{(\mathrm{I})}(K)$ and $V_S^{(\mathrm{II})}(K)$, respectively, which are continuously differentiable. Applying HJB equation to $V_S^{(\mathrm{I})}(K)$, for Player I, we obtain

$$\begin{aligned}\rho_1 V_S^{(\mathrm{I})}(K) = \max_{\{L_S^R, L_S^F\}\ge 0} \Bigg\{ & \Big[\theta\Big(\alpha_1 L_S^R(t) + \alpha_2 L_S^F + \beta_1 \tilde{L}_S^R + \beta_2 \tilde{L}_S^F + (\Gamma+\delta)K\Big)\Big] \\ & -\frac{1}{2}\beta^R(1-\omega_1)\left(L_S^R\right)^2 - \frac{1}{2}\beta^F(1-\omega_2)\left(L_S^F\right)^2 \\ & + \frac{\partial V_S^{(\mathrm{I})}(K)}{\partial K}\left[\vartheta_1\left(L_S^R, L_S^F\right) + \vartheta_2\left(\tilde{L}_S^R, \tilde{L}_S^F\right) - \xi K\right] + \frac{1}{2}\frac{\partial^2 V_S^{(\mathrm{I})}(K)}{\partial K^2}\varphi^2(K)\Bigg\}.\end{aligned} \tag{36}$$

Via the first order conditions, we obtain the optimal values $\left(L_S^R, L_S^F\right)$ for Player I as:

$$L_S^R = \frac{\theta\alpha_1 + V'^{(\mathrm{I})}_S(K)\vartheta_1^R}{\beta^R(1-\omega_1)}, \tag{37}$$

$$L_S^F = \frac{\theta\alpha_2 + V'^{(\mathrm{I})}_S(K)\vartheta_1^F}{\beta^F(1-\omega_2)}, \tag{38}$$

where $\frac{\partial V_S^{(\mathrm{I})}(K)}{\partial K} \equiv V'^{(\mathrm{I})}_S(K)$. The optimal sharing revenue function, $V'^{(\mathrm{II})}_S(K)$, for Player II and the associated HJB equation is

$$\begin{aligned}\rho_2 V_S^{(\mathrm{II})}(K) = \max_{\{\tilde{L}_S^R, \tilde{L}_S^F\}\ge 0} \Bigg\{ & \Big[(1-\theta)\Big(\alpha_1 L_S^R + \alpha_2 L_S^F + \beta_1 \tilde{L}_S^R + \beta_2 \tilde{L}_S^F + (\Gamma+\delta)K\Big)\Big] \\ & -\frac{1}{2}\tilde{\beta}^R\left(L_S^R(t)\right)^2 - \frac{1}{2}\tilde{\beta}^F\left(\tilde{L}_S^F\right)^2 - \frac{1}{2}\omega_1\beta^R\left(L_S^R\right)^2 - \frac{1}{2}\omega_2\beta^F\left(L_S^F\right)^2 \\ & + \frac{\partial V_S^{(\mathrm{II})}(K)}{\partial K}\left[\vartheta_1\left(L_S^R, L_S^F\right) + \vartheta_2\left(\tilde{L}_S^R, \tilde{L}_S^F\right) - \xi K\right] + \frac{1}{2}\frac{\partial^2 V_S^{(\mathrm{II})}(K)}{\partial K^2}\varphi^2(K)\Bigg\}.\end{aligned} \tag{39}$$

Substituting the results of Eqs. (37) and (38) into Eq. (39), obtain

$$\begin{aligned}\rho_2 V_S^{(\mathrm{II})}(K) = \max_{\left\{\tilde{L}_S^R, \tilde{L}_S^F\right\}\geq 0} \Bigg\{ \Bigg[(1-\theta)\Bigg(\frac{\alpha_1\left(\theta\alpha_1 + V'^{(\mathrm{I})}_S(K)\vartheta_1^R\right)}{\beta^R(1-\omega_1)} + \frac{\alpha_2\left(\theta\alpha_2 + V'^{(\mathrm{I})}_S(K)\vartheta_1^F\right)}{\beta^F(1-\omega_2)} \\ + \beta_1\tilde{L}_S^R + \beta_2\tilde{L}_S^F + (\Gamma+\delta)K\Bigg) - \frac{1}{2}\tilde{\beta}^R\left(\tilde{L}_S^R(t)\right)^2 - \frac{1}{2}\tilde{\beta}^F\left(\tilde{L}_S^F\right)^2 \\ - \frac{1}{2}\omega_1\beta^R\left(\frac{\theta\alpha_1 + V'^{(\mathrm{I})}_S(K)\vartheta_1^R}{\beta^R(1-\omega_1)}\right)^2 - \frac{1}{2}\omega_2\beta^F\left(\frac{\theta\alpha_2 + V'^{(\mathrm{I})}_S(K)\vartheta_1^F}{\beta^F(1-\omega_2)}\right)^2\Bigg] \\ + \frac{\partial V_S^{(\mathrm{II})}(K)}{\partial K}\left[\frac{\vartheta_1^R\left[\theta\alpha_1 + V'^{(\mathrm{I})}_S(K)\vartheta_1^R\right]}{\beta^R(1-\omega_1)} + \frac{\vartheta_1^F\left[\theta\alpha_2 + V'^{(\mathrm{I})}_S(K)\vartheta_1^F\right]}{\beta^F(1-\omega_2)} + \vartheta_2\left(\tilde{L}_S^R, \tilde{L}_S^F\right) - \xi K\right] \\ + \frac{1}{2}\frac{\partial^2 V_S^{(\mathrm{II})}(K)}{\partial K^2}\varphi^2(K)\Bigg\}. \end{aligned} \tag{40}$$

Via the first order conditions of (Eq. (40)), we obtain the optimal values $\left(\tilde{L}^R, \tilde{L}^F\right)$ for Player II as:

$$\tilde{L}_S^R = \frac{(1-\theta)\beta_1 + V'^{(\mathrm{II})}_S(K)\vartheta_2^R}{\tilde{\beta}^R}, \tag{41}$$

$$\tilde{L}_S^F = \frac{(1-\theta)\beta_2 + V'^{(\mathrm{II})}_S(K)\vartheta_2^F}{\tilde{\beta}^F}. \tag{42}$$

And the optimal value for (ω_1, ω_2)

$$\omega_1 = \frac{\alpha_1(2-3\theta) + \vartheta_1^R\left[2V'_S{}^{(\mathrm{II})}(K) - V_S{}'^{(\mathrm{I})}(K)\right]}{\alpha_1(2-\theta) + \vartheta_1^R\left[2V'_S{}^{(\mathrm{II})}(K) + V_S{}'^{(\mathrm{I})}(K)\right]}, \tag{43}$$

and

$$\omega_2 = \frac{\alpha_2(2-3\theta) + \vartheta_1^F\left[2V'_S{}^{(\mathrm{II})}(K) - V_S{}'^{(\mathrm{I})}(K)\right]}{\alpha_2(2-\theta) + \vartheta_1^F\left[2V'_S{}^{(\mathrm{II})}(K) + V_S{}'^{(\mathrm{I})}(K)\right]}. \tag{44}$$

Hence, the solution of the HJB equation is an unary function with K (K as the independent variable), we define $V_S^{(\mathrm{I})} = a_1K + b_1$ and $V_S^{(\mathrm{II})} = a_2K + b_2$, where a_1, b_1, a_2, and b_2 are constants that need to be solved. Simplifying (Eq. (39)), obtain:

$$\rho_1 V_S^{(\mathrm{I})}(K) = \theta\left(\alpha_1\left(\frac{\theta\alpha_1 + a_1\vartheta_1^R}{\beta^R(1-\omega_1)}\right) + \alpha_2\left(\frac{\theta\alpha_2 + a_1\vartheta_1^F}{\beta^F(1-\omega_2)}\right) + \beta_1\left(\frac{(1-\theta)\beta_1 + a_2\vartheta_2^R}{\tilde{\beta}^R}\right)\right. \tag{45}$$

$$
\begin{aligned}
&+\beta_2\left(\frac{(1-\theta)\beta_2+a_2\vartheta_2^F}{\tilde{\beta}^F}\right)+(\Gamma+\delta)K\Bigg)-\frac{1}{2}\beta^R(1-\omega_1)\left(\frac{\theta\alpha_1+a_1\vartheta_1^R}{\beta^R(1-\omega_1)}\right)^2 \\
&-\frac{1}{2}\beta^F(1-\omega_2)\left(\frac{\theta\alpha_2+a_1\vartheta_1^F}{\beta^F(1-\omega_2)}\right)^2-\xi K a_1 \\
&+\left[\frac{\vartheta_1^R\left[\theta\alpha_1+a_1\vartheta_1^R\right]}{\beta^R(1-\omega_1)}+\frac{\vartheta_1^F\left[\theta\alpha_2+a_1\vartheta_1^F\right]}{\beta^F(1-\omega_2)}\right]a_1 \\
&+\left[\frac{\vartheta_2^R\left[(1-\theta)\beta_1+a_2\vartheta_2^R\right]}{\tilde{\beta}^R}+\frac{\vartheta_2^F\left[(1-\theta)\beta_2+a_2\vartheta_2^F\right]}{\tilde{\beta}^F}\right]a_1,
\end{aligned}
\tag{46}
$$

and simplifying (Eq. (40)), obtain:

$$
\begin{aligned}
\rho_2 V_S^{(\mathrm{II})}(K)=(1-\theta)&\Bigg(\frac{\alpha_1\left(\theta\alpha_1+a_1\vartheta_1^R\right)}{\beta^R(1-\omega_1)}+\frac{\alpha_2\left(\theta\alpha_2+a_1\vartheta_1^F\right)}{\beta^F(1-\omega_2)}+\beta_1\left(\frac{(1-\theta)\beta_1+a_2\vartheta_2^R}{\tilde{\beta}^R}\right) \\
&+\beta_2\left(\frac{(1-\theta)\beta_2+a_2\vartheta_2^F}{\tilde{\beta}^F}\right)+(\Gamma+\delta)K\Bigg)-\frac{1}{2}\tilde{\beta}^R\left(\frac{(1-\theta)\beta_1+a_2\vartheta_2^R}{\tilde{\beta}^R}\right)^2 \\
&-\frac{1}{2}\tilde{\beta}^F\left(\frac{(1-\theta)\beta_2+a_2\vartheta_2^F}{\tilde{\beta}^F}\right)^2-\frac{1}{2}\omega_1\beta^R\left(\frac{\theta\alpha_1+a_1\vartheta_1^R}{\beta^R(1-\omega_1)}\right)^2-\frac{1}{2}\omega_2\beta^F\left(\frac{\theta\alpha_2+a_1\vartheta_1^F}{\beta^F(1-\omega_2)}\right)^2 \\
&+\left[\frac{\vartheta_1^R\left[\theta\alpha_1+a_1\vartheta_1^R\right]}{\beta^R(1-\omega_1)}+\frac{\vartheta_1^F\left[\theta\alpha_2+a_1\vartheta_1^F\right]}{\beta^F(1-\omega_2)}\right]a_2-\xi K a_2 \\
&+\left[\frac{\vartheta_2^R\left[(1-\theta)\beta_1+a_2\vartheta_2^R\right]}{\tilde{\beta}^R}+\frac{\vartheta_2^F\left[(1-\theta)\beta_2+a_2\vartheta_2^F\right]}{\tilde{\beta}^F}\right]a_2.
\end{aligned}
\tag{47}
$$

This implies that,

$$
a_1=\frac{\theta(\Gamma+\delta)}{(\rho_1+\xi)},\quad b_1=\frac{\Phi_1}{\rho_1},\quad a_2=\frac{(1-\theta)(\Gamma+\delta)}{(\rho_2+\xi)},\quad b_2=\frac{\Phi_2}{\rho_2},
\tag{48}
$$

where

$$
\begin{aligned}
\Phi_1=&\left(\alpha_1\theta-\frac{\left(\theta\alpha_1+a_1\vartheta_1^R\right)}{2}+\vartheta_1^R a_1\right)\left(\frac{\theta\alpha_1+a_1\vartheta_1^R}{\beta^R(1-\omega_1)}\right) \\
&+\left(\alpha_2\theta-\frac{\left(\theta\alpha_2+a_1\vartheta_1^F\right)}{2}+\vartheta_1^F a_1\right)\left(\frac{\theta\alpha_2+a_1\vartheta_1^F}{\beta^F(1-\omega_2)}\right) \\
&+\left(\beta_1\theta+\vartheta_2^R a_1\right)\left(\frac{(1-\theta)\beta_1+a_2\vartheta_2^R}{\tilde{\beta}^R}\right) \\
&+\left(\beta_2\theta+\vartheta_2^F a_2\right)\left(\frac{(1-\theta)\beta_2+a_2\vartheta_2^F}{\tilde{\beta}^F}\right)>0,
\end{aligned}
\tag{49}
$$

and

$$\Phi_2 = \left((1-\theta)\alpha_1 - \frac{\omega_1\left(\theta\alpha_1 + a_1\vartheta_1^R\right)}{2(1-\omega_1)} + \vartheta_1^R a_2\right)\frac{\left(\theta\alpha_1 + a_1\vartheta_1^R\right)}{\beta^R(1-\omega_1)}$$
$$+\left((1-\theta)\alpha_2 - \frac{\omega_2\left(\theta\alpha_2 + a_1\vartheta_1^F\right)}{2(1-\omega_2)} + \vartheta_1^F a_2\right)\frac{\left(\theta\alpha_2 + a_1\vartheta_1^F\right)}{\beta^F(1-\omega_2)}$$
$$+\left((1-\theta)\beta_1 - \frac{\left((1-\theta)\beta_1 + a_2\vartheta_2^R\right)}{2} + \vartheta_2^R a_2\right)\left(\frac{(1-\theta)\beta_1 + a_2\vartheta_2^R}{\tilde{\beta}^R}\right)$$
$$+\left((1-\theta)\beta_2 - \frac{\left((1-\theta)\beta_2 + a_2\vartheta_2^F\right)}{2} + \vartheta_2^F a_2\right)\left(\frac{(1-\theta)\beta_2 + a_2\vartheta_2^F}{\tilde{\beta}^F}\right) > 0. \tag{50}$$

Substituting the results of a_1 and a_2 into Eqs. (37), (38), (41) and (42), and simplifying, we obtain the optimal effort level of hybrid-enabling technological improvements. By substituting optimal values given in Eqs. (48)–(50) into Eqs. (46) and (47) obtain the optimal sharing payoff functions under hybrid-enabling technology on renewable sources and fossil fuel for Player I and Player II.

3.1 The limit of expectation and variance

The payoff of Player I and Player II, under the Stackelberg game paradigm is related to the improvement degree of hybrid-enabling technology via Proposition 4. To analyze the limit of expectations and variance under Stackelberg game equilibrium rewrite (Eq. (19)) as follows.

$$\begin{cases} dK(t) = [\mu_1 + \mu_2 - \xi K(t)]dt + \varphi\sqrt{K}dW(t) \\ K(0) = K_0 > 0, \end{cases} \tag{51}$$

where

$$\mu_1 = \vartheta_1\left[\frac{\alpha_1(2-\theta)(\rho_2+\xi)(\rho_1+\xi) + \vartheta_1^R(\Gamma+\delta)((2-2\theta)(\rho_1+\xi)+\theta(\rho_2+\xi))}{2\beta^R(\rho_2+\xi)(\rho_1+\xi)}\right.$$
$$\left.+\frac{\alpha_2(2-\theta)(\rho_2+\xi)(\rho_1+\xi) + \vartheta_1^F(\Gamma+\delta)((2-2\theta)(\rho_1+\xi)+\theta(\rho_2+\xi))}{2\beta^F(\rho_2+\xi)(\rho_1+\xi)}\right], \tag{52}$$

and

$$\mu_2 = \vartheta_2\left[\frac{(1-\theta)(\beta_1(\rho_2+\xi)+(\Gamma+\delta))\vartheta_2^R}{\tilde{\beta}^R(\rho_2+\xi)} + \frac{(1-\theta)(\beta_2(\rho_2+\xi)+(\Gamma+\delta))\vartheta_2^F}{\tilde{\beta}^F(\rho_2+\xi)}\right]. \tag{53}$$

Proposition 6. *The limit of expectation $E(K(t))$, and variance $D(K(t))$ in the Stackelberg game feedback equilibrium must satisfy*

$$E(K(t)) = \frac{\mu_1+\mu_2}{\xi} + e^{-\xi t}\left(\tilde{K}_0 - \frac{\mu_1+\mu_2}{\xi}\right), \quad \lim_{t\to\infty} E(K(t)) = \frac{\mu_1+\mu_2}{\xi}. \tag{54}$$

$$D(K(t)) = \frac{\varphi^2\left[(\mu_1+\mu_2) - 2\left(\mu_1+\mu_2-\xi\tilde{K}_0\right)e^{-\xi t} + \left(\mu_1+\mu_2-2\xi\tilde{K}_0\right)e^{-2\xi t}\right]}{2\xi^2} \tag{55}$$

$$\lim_{t\to\infty} D(E(K(t))) = \frac{\varphi^2(\mu_1+\mu_2)}{2\xi^2}, \tag{56}$$

Proof. Applying Itô's lemma to (Eq. (51)), obtain:

$$\begin{cases} d(K(t))^2 = \left[2(\mu_1+\mu_2+\varphi^2)K - 2\xi K^2\right]dt + 2\varphi K\sqrt{K}dW(t) \\ (K(0))^2 = \tilde{K}_0^2 > 0. \end{cases} \tag{57}$$

Then $E(K(t))$ and $E(K(t))^2$ can be defined as:

$$\begin{cases} dE(K(t)) = [\mu_1+\mu_2 - \xi K(t)]dt \\ K(0) = K_0 > 0. \end{cases} \tag{58}$$

$$\begin{cases} dE(K(t))^2 = \left[[2(\mu_1+\mu_2+\varphi^2)K]E(K) - 2\xi E\left(K^2\right)\right]dt \\ (K(0))^2 = K_0^2 > 0, \end{cases} \tag{59}$$

Solving the above non-homogeneous linear differential equation, will obtain the results.

4. Nash non cooperative game

Under Nash-non-cooperative game setting, Player I and Player II simultaneously and independently choose their optimal efforts levels of heterogeneous hybrid-enabling technology sharing concept to maximize their profits.

Proposition 7. *If above conditions are satisfied, the feedback non-cooperative game Nash equilibria will be:*

$$L_N^R = \frac{\theta[\alpha_1(\rho_1+\xi) + (\Gamma+\delta)]}{\beta^R(\rho_1+\xi)}, L_N^F = \frac{\theta[\alpha_2(\rho_1+\xi) + (\Gamma+\delta)]}{\beta^F(\rho_1+\xi)}. \tag{60}$$

$$\tilde{L}_N^R = \frac{(1-\theta)[\beta_1(\rho_2+\xi) + (\Gamma+\delta)]}{\tilde{\beta}^R(\rho_2+\xi)}, \tilde{L}_N^F = \frac{(1-\theta)[\beta_2(\rho_2+\xi) + (\Gamma+\delta)]}{\tilde{\beta}^F(\rho_2+\xi)}, \tag{61}$$

where L_N^R, L_N^F are the optimal level of hybrid-enabling technological advantage on renewable sources and on fossil fuel at time t for Player I, respectively. $\tilde{L}_N^R, \tilde{L}_N^F$ are the optimal level of hybrid-enabling technological advantage on fossil fuel and on renewable sources at time t for Player II, respectively.

The optimal sharing payoff functions under hybrid-enabling technology on renewable sources and on fossil fuel for Player I and Player II are given below

$$V_N^{(\mathrm{I})}(K) = \frac{\theta\left(\Gamma+\delta_{)}\right)}{(\rho_1+\xi)}K + \hat{b}_1, \quad V_N^{(\mathrm{II})}(K) = \frac{(1-\theta)(\Gamma+\delta)}{(\rho_2+\xi)}K + \hat{b}_2, \tag{62}$$

where $\hat{b}_1$ and $\hat{b}_2$ are given in the proof.
Proof. See Appendix A.

4.1 The limit of expectation and variance

Proposition 8. *The limit of expectation $E(K(t))$ and variance $D(K(t))$ in the Nash non-cooperative game feedback equilibrium must satisfy*

$$E(K(t)) = \frac{\hat{\mu}_1 + \hat{\mu}_2}{\xi} + e^{-\xi t}\left(\hat{K}_0 - \frac{\hat{\mu}_1 + \hat{\mu}_2}{\xi}\right), \quad \lim_{t\to\infty} E(K(t)) = \frac{\hat{\mu}_1 + \hat{\mu}_2}{\xi}. \tag{63}$$

$$D(K(t)) = \frac{\varphi^2\left[(\tilde{\mu}_1 + \tilde{\mu}_2) - 2(\tilde{\mu}_1 + \tilde{\mu}_2 - \xi\tilde{K}_0)e^{-\xi t} + (\tilde{\mu}_1 + \tilde{\mu}_2 - 2\xi\tilde{K}_0)e^{-2\xi t}\right]}{2\xi^2} \tag{64}$$

$$\lim_{t\to\infty} D(E(K(t))) = \frac{\varphi^2(\tilde{\mu}_1 + \tilde{\mu}_2)}{2\xi^2}. \tag{65}$$

where $\tilde{\mu}_1 = \frac{\theta\beta^F[\alpha_1(\rho_1+\xi)+(\Gamma+\delta)]+\theta\beta^R[\alpha_2(\rho_1+\xi)+(\Gamma+\delta)]}{\beta^R\beta^F(\rho_1+\xi)}$ *and* $\tilde{\mu}_2 = \frac{(1-\theta)\beta^F[\beta_1(\rho_2+\xi)+(\Gamma+\delta)]+(1-\theta)\beta^R[\beta_2(\rho_2+\xi)+(\Gamma+\delta)]}{\beta^R\beta^F(\rho_2+\xi)}$.

Poof of Proposition 8 is like the derivation of Proposition 6.

5. Cooperative game

Under cooperative game paradigm, Player I and Player II will choose to collaborate/share their hybrid-enabling technology development knowledge while sharing the payoff function in order to maximize their total payoffs. As a result, hybrid-enabling technology can be improved through this effort as well.

Proposition 9. *If above conditions are satisfied, then the feedback cooperative equilibria are defined as*

$$L_c^R = \frac{(\alpha_1 + \beta_1)(\rho + \xi) + (\Gamma + \delta)(\vartheta_1 + \vartheta_2)}{(\rho + \xi)\beta^R}, \quad L_c^F = \frac{(\alpha_2 + \beta_2)(\rho + \xi) + (\Gamma + \delta)(\vartheta_1 + \vartheta_2)}{(\rho + \xi)\beta^F}, \tag{66}$$

and the optimal cooperative payoff function under hybrid-enabling technology on renewable sources and on fossil fuel, respectively. $V_c(K) = \frac{(\Gamma+\delta)}{(\rho+\xi)}K + \overline{b}$.

where $\overline{b}$, is given in the proof.

Proof. The objective function (optimal sharing payoff function) satisfies the following equation.

$$J(K_0) = \max_{\{L_c^R, L_c^R\}0} E\left\{\int_0^\infty e^{-\rho t}\left[\left(\alpha_1 L_c^R(t) + \alpha_2 L_c^F + \beta_1 \tilde{L}_c^R + \beta_2 \tilde{L}_c^F + (\Gamma + \delta)K\right)\right.\right. \tag{67}$$

Then the optimal revenue sharing function satisfies the following HJB equation

$$\begin{aligned} \rho V_c(K) = \max_{\{L_c^R, L_c^F\}\geq 0} &\left\{\left[\alpha_1 L_c^R(t) + \alpha_2 L_c^F + \beta_1 \tilde{L}_c^R + \beta_2 \tilde{L}_c^F + (\Gamma + \delta)K)\right]\right. \\ &-\frac{1}{2}\beta^R\left(L_c^R\right)^2 - \frac{1}{2}\beta^F\left(L_c^F\right)^2 \\ &\left.+\frac{\partial V_c(K)}{\partial K}\left[\vartheta_1\left(L_c^R, L_c^F\right) + \vartheta_2\left(\tilde{L}_c^R, \tilde{L}_c^F\right) - \xi K\right] + \frac{1}{2}\frac{\partial^2 V_c(K)}{\partial K^2}\varphi^2(K)\right\} \end{aligned} \tag{68}$$

Via the first order conditions, now obtain the optimal values $\left(L_c^R, L_c^F\right)$ as:

$$L_c^R = \frac{(\alpha_1 + \beta_1) + V'_c(K)(\vartheta_1 + \vartheta_2)}{\beta^R}, \tag{69}$$

$$L_c^F = \frac{(\alpha_2 + \beta_2) + V'_c(K)(\vartheta_1 + \vartheta_2)}{\beta^F}. \tag{70}$$

Substituting the results of Eqs. (69) and (70), obtain

$$\begin{aligned}\rho V_c(K) = &\max_{\left\{L_c^R, L_c^F\right\} \geq 0} \left\{ (\alpha_1 + \beta_1)\left(\frac{(\alpha_1 + \beta_1) + V'_c(K)(\vartheta_1 + \vartheta_2)}{\beta^R}\right)\right. \\ &+ (\alpha_2 + \beta_2)\left(\frac{(\alpha_2 + \beta_2) + V'_c(K)(\vartheta_1 + \vartheta_2)}{\beta^F}\right) - \frac{1}{2}\beta^R\left(\frac{(\alpha_1 + \beta_1) + V'_c(K)(\vartheta_1 + \vartheta_2)}{\beta^R}\right)^2 \\ &- \frac{1}{2}\beta^F\left(\frac{(\alpha_2 + \beta_2) + V'_c(K)(\vartheta_1 + \vartheta_2)}{\beta^F}\right)^2 - \frac{\partial V_c(K)}{\partial K}\xi K \\ &+ \frac{\partial V_c(K)}{\partial K}\left[(\vartheta_1 + \vartheta_2)\left(\frac{(\alpha_1 + \beta_1) + V'_c(K)(\vartheta_1 + \vartheta_2)}{\beta^R}\right)\right.\end{aligned} \tag{71}$$

Hence, the solution of the HJB equation is an unary function with K, $V_c = \overline{a}K + \overline{b}$, where $\overline{a}$ and $\overline{b}$ are constant that need to be solved. This implies that

$$\overline{a} = \frac{(\Gamma + \delta)}{(\rho + \xi)}. \tag{72}$$

$$\begin{aligned}\overline{b} = &\left((\alpha_1 + \beta_1) - \frac{((\alpha_1 + \beta_1) + \overline{a}(\vartheta_1 + \vartheta_2))}{2} + (\vartheta_1 + \vartheta_2)\overline{a}\right)\left(\frac{(\alpha_1 + \beta_1) + \overline{a}(\vartheta_1 + \vartheta_2)}{\beta^R}\right) \\ &+\left((\alpha_2 + \beta_2) - \frac{((\alpha_2 + \beta_2) + \overline{a}(\vartheta_1 + \vartheta_2))}{2} + (\vartheta_1 + \vartheta_2)\overline{a}\right)\left(\frac{(\alpha_2 + \beta_2) + \overline{a}(\vartheta_1 + \vartheta_2)}{\beta^F}\right) > 0.\end{aligned} \tag{73}$$

Substituting the results of Eqs. (72) and (73), into $V_c = \overline{a}K + \overline{b}$, will obtain the results.

5.1 The limit of expectation and variance

Proposition 10. The limit of expectation and variance in cooperative game feedback equilibrium satisfy

$$E(K(t)) = \frac{\overline{\mu}_1 + \overline{\mu}_2}{\xi} + e^{-\xi t}\left(\tilde{K}_0 - \frac{\overline{\mu}_1 + \overline{\mu}_2}{\xi}\right), \qquad \lim_{t \to \infty} E(K(t)) = \frac{\overline{\mu}_1 + \overline{\mu}_2}{\xi}. \tag{74}$$

$$D(K(t)) = \frac{\varphi^2\left[(\overline{\mu}_1 + \overline{\mu}_2) - 2(\overline{\mu}_1 + \overline{\mu}_2 - \xi K_0)e^{-\xi t} + (\overline{\mu}_1 + \overline{\mu}_2 - 2\xi K_0)e^{-2\xi t}\right]}{2\xi^2} \tag{75}$$

$$\lim_{t \to \infty} D(E(K(t))) = \frac{\varphi^2(\overline{\mu}_1 + \overline{\mu}_2)}{2\xi^2}, \tag{76}$$

where $\overline{\mu}_1 = \frac{(\alpha_1+\beta_1)(\rho+\xi)+(\Gamma+\delta)(\vartheta_1+\vartheta_2)}{(\rho+\xi)}$ and $\overline{\mu}_2 = \frac{(\alpha_2+\beta_2)(\rho+\xi)+(\Gamma+\delta)(\vartheta_1+\vartheta_2)}{(\rho+\xi)}$.
Proof. Poof of Proposition 10 is like the derivation of Propositions 6 and 8.

6. Comparative analysis of equilibrium results

Proposition 11. *The outcome of the game depends on the parameters of the game and the type of the equilibrium one considers.*

Proof. (i) Player I, will participate in a Stackelberg game to share more hybrid-enabling technology under the condition that Player II pay much more extra cost for hybrid-enabling technology

$$\frac{\alpha_1(2-\theta)(\rho_2+\xi)(\rho_1+\xi)+\vartheta_1^R(\Gamma+\delta)((2-2\theta)(\rho_1+\xi)+\theta(\rho_2+\xi))}{2\beta^R(\rho_2+\xi)(\rho_1+\xi)} - \frac{(1-\theta)(\beta_1(\rho_2+\xi)+(\Gamma+\delta))\vartheta_2^R}{\tilde{\beta}^R(\rho_2+\xi)} > 0, \tag{77}$$

and

$$\frac{\alpha_2(2-\theta)(\rho_2+\xi)(\rho_1+\xi)+\vartheta_1^F(\Gamma+\delta)((2-2\theta)(\rho_1+\xi)+\theta(\rho_2+\xi))}{2\beta^F(\rho_2+\xi)(\rho_1+\xi)} - \frac{(1-\theta)(\beta_2(\rho_2+\xi)+(\Gamma+\delta))\vartheta_2^F}{\tilde{\beta}^F(\rho_2+\xi)} > 0. \tag{78}$$

(ii) Player I will prefer to participate in a cooperative game over a non-cooperative game with Player II under the condition such that

$$\frac{(\alpha_1+\beta_1)(\rho+\xi)+(\Gamma+\delta)(\vartheta_1+\vartheta_2)}{(\rho+\xi)\beta^R} - \frac{\theta[\alpha_1(\rho_1+\xi)+(\Gamma+\delta)]}{\beta^R(\rho_1+\xi)} > 0, \tag{79}$$

and

$$\frac{(\alpha_2+\beta_2)(\rho+\xi)+(\Gamma+\delta)(\vartheta_1+\vartheta_2)}{(\rho+\xi)\beta^F} - \frac{\theta[\alpha_2(\rho_1+\xi)+(\Gamma+\delta)]}{\beta^F(\rho_1+\xi)} > 0. \tag{80}$$

(iii) The total payoff for Player I under a Stackelberg game exceeds the total payoff of Nash non-cooperative game with Player II under the condition such that

$$\frac{\alpha_1(2-\theta)(\rho_2+\xi)(\rho_1+\xi)+\vartheta_1^R(\Gamma+\delta)((2-2\theta)(\rho_1+\xi)+\theta(\rho_2+\xi))}{2\beta^R(\rho_2+\xi)(\rho_1+\xi)} - \frac{(\alpha_1+\beta_1)(\rho+\xi)+(\Gamma+\delta)(\vartheta_1+\vartheta_2)}{(\rho+\xi)\beta^R} > 0, \tag{81}$$

and

$$\frac{\alpha_2(2-\theta)(\rho_2+\xi)(\rho_1+\xi)+\vartheta_1^F(\Gamma+\delta)((2-2\theta)(\rho_1+\xi)+\theta(\rho_2+\xi))}{2\beta^F(\rho_2+\xi)(\rho_1+\xi)} - \frac{(\alpha_2+\beta_2)(\rho+\xi)+(\Gamma+\delta)(\vartheta_1+\vartheta_2)}{(\rho+\xi)\beta^F} > 0. \tag{82}$$

(iv). Player II will prefer to participate in a cooperative game over a non-cooperative game with Player I under the condition such that

$$\frac{(\alpha_1+\beta_1)(\rho+\xi)+(\Gamma+\delta)(\vartheta_1+\vartheta_2)}{(\rho+\xi)\beta^R}-\frac{(1-\theta)[\beta_1(\rho_2+\xi)+(\Gamma+\delta)]}{\tilde{\beta}^R(\rho_2+\xi)}>0, \tag{83}$$

and

$$\frac{(\alpha_2+\beta_2)(\rho+\xi)+(\Gamma+\delta)(\vartheta_1+\vartheta_2)}{(\rho+\xi)\beta^F}-\frac{(1-\theta)[\beta_2(\rho_2+\xi)+(\Gamma+\delta)]}{\tilde{\beta}^F(\rho_2+\xi)}>0. \tag{84}$$

Proposition 12. *For any $K \geq 0$, under the condition that Player II pay an extra cost for sharing hybrid-enabling technology. Then the optimal sharing payoff of hybrid-enabling technology of Player I reaches higher than the optimal sharing payoff under the condition that player II does not provide extra cost. This implies that $V_S^{(\mathrm{I})}(K) \geq V_N^{(\mathrm{I})}(K)$. Similarly, the optimal sharing payoff of hybrid-enabling technology of Player II reaches higher than the optimal sharing payoff under the condition that Player II do not provide extra cost, such that $V_S^{(\mathrm{II})}(K) \geq V_N^{(\mathrm{II})}(K)$.*

Proof. When $0 \leq \theta \leq \frac{2}{3}$, establish that

$$\begin{aligned}\Delta V^{(\mathrm{I})}(K) = V_S^{(\mathrm{I})}(K)-V_N^{(\mathrm{I})}(K) &= \frac{\theta(\Gamma+\delta)}{(\rho_1+\xi)}K+b_1-\frac{\theta(\Gamma+\delta)}{(\rho_1+\xi)}K+\hat{b}_1 \\ &= b_1-\hat{b}_1>0,\end{aligned} \tag{85}$$

and

$$\begin{aligned}\Delta V^{(\mathrm{II})}(K) = V_S^{(\mathrm{II})}(K)-V_N^{(\mathrm{II})}(K) &= \frac{(1-\theta)\left(\Gamma+\delta_)\right)}{(\rho_1+\xi)}K+b_2-\frac{(1-\theta)(\Gamma+\delta)}{(\rho_1+\xi)}K+\hat{b}_2 \\ &= b_2-\hat{b}_2>0.\end{aligned} \tag{86}$$

7. Concluding remarks

In this chapter a complete study of an energy market by considering a Bertrand duopoly game with two power plants using endogenous hybrid-enabling technology was presented. Numerous game paradigms were articulated and defined including Stackelberg, Nash non-cooperative and cooperative games as well as their relevant equilibria via a feedback control strategy. Mathematically, the necessary conditions under which a power plant will move from taking part in a non-cooperative Nash game to participate as a leader in a Stackelberg game was derived. In doing so, this model allowed us to quantify the optimal level of subsidy for sharing the hybrid-enabling technology. We then adopted the concept of limit expectation and variance of the improvement degree to identify the influence of random factors of external environment and limitations of the decision maker. It is found that for a given level of payoff distribution the Stackelberg equilibria with technological enhancements, the knowledge sharing paradigm dominates the Nash equilibria. In both Stackelberg and Nash games, optimal technological enhancements for power plants were found to be proportional to the government subsidy, but the variance improvement degree of the Stackelberg game differed to the results of the Nash non-cooperative game due to the influence of random factors.

Furthermore, we have shown that due to optimal price reaction functions being upward sloping, the subsidy level plays a decisive role on the payoff function of power plant II as the leader in a Stackelberg game. This model shows that cost reducing R&D investments with efficient hybrid-enabling technology innovation/s strengthens one's competitive bargaining position via the level of subsidy for Power Plant I to become a follower in the Stackelberg game. By analyzing this stochastic differential game model, we capture the government subsidy incentive as well as the subsidy that the leader (Power Plant II) pays the follower (Power Plant I) to share hybrid-enabling technology.

The proposed quantitative framework could assist policymakers when determining the appropriate R&D incentives for the development of hybrid-enabling technology within the energy market to achieve desired short and long-term environmental objectives with respect to budget limitations and environmental considerations.

A. Appendix

Proof. The optimal profit function for power plant I satisfies the following HJB equation such that $V_N^{(\mathrm{I})}(K)$:

$$\begin{aligned}\rho_1 V_N^{(\mathrm{I})}(K) = \max_{\{L_N^R, L_N^F\} \geq 0} \Bigg\{ & \left[\theta\left(\alpha_1 L_N^R(t) + \alpha_2 L_N^F + \beta_1 \tilde{L}_N^R + \beta_2 \tilde{L}_N^F + (\Gamma + \delta)K\right)\right] \\ & - \frac{1}{2}\beta^R \left(L_N^R\right)^2 - \frac{1}{2}\beta^F \left(L_N^F\right)^2 \\ & + \frac{\partial V_N^{(\mathrm{I})}(K)}{\partial K}\left[\vartheta_1\left(L_N^R, L_N^F\right) + \vartheta_2\left(\tilde{L}_N^R, \tilde{L}_N^F\right) - \xi K\right] + \frac{1}{2}\frac{\partial^2 V_N^{(\mathrm{I})}(K)}{\partial K^2}\varphi^2(K)\Bigg\}\end{aligned} \tag{A.1}$$

Via the first order conditions, first obtain the optimal values $\left(L_N^R, L_{(\mathrm{I})}^R\right)$ for power plant I as:

$$L_N^R = \frac{\theta\alpha_1 + V'^{(\mathrm{I})}_N(K)\vartheta_1^R}{\beta^R} = \frac{\theta\alpha_1 + a_1\vartheta_1^R}{\beta^R}, \tag{A.2}$$

$$L_N^F = \frac{\theta\alpha_2 + V'^{(\mathrm{I})}_N(K)\vartheta_1^F}{\beta^F} = \frac{\theta\alpha_2 + a_1\vartheta_1^F}{\beta^F} \tag{A.3}$$

where $\frac{\partial V_N^{(\mathrm{I})}(K)}{\partial K} \equiv V'^{(\mathrm{I})}_N(K)$. The HJB for power plant (II), using $\frac{\partial V_N^{(\mathrm{II})}(K)}{\partial K} = V'^{(\mathrm{II})}_N(K)$, then obtain

$$\begin{aligned}\rho_2 V_N^{(\mathrm{II})}(K) = \max_{\{\tilde{L}_N^R, \tilde{L}_N^F\}} \Bigg\{ & \left[(1-\theta)\left(\alpha_1 L_N^R(t) + \alpha_2 L_N^F + \beta_1 \tilde{L}_N^R + \beta_2 \tilde{L}_N^F + (\Gamma + \delta)K\right)\right] \\ & - \frac{1}{2}\tilde{\beta}^R \left(\tilde{L}_N^R\right)^2 - \frac{1}{2}\tilde{\beta}^F \left(\tilde{L}_N^F\right)^2 \\ & + \frac{\partial V_N^{(\mathrm{I})}(K)}{\partial K}\left[\vartheta_1\left(L_N^R, L_N^F\right) + \vartheta_2\left(\tilde{L}_N^R, \tilde{L}_N^F\right) - \xi K\right] + \frac{1}{2}\frac{\partial^2 V_N^{(\mathrm{I})}(K)}{\partial K^2}\varphi^2(K)\Bigg\}.\end{aligned} \tag{A.4}$$

Substituting Eqs. (A.2) and (A.3) results to Eq. (A.4) and via the first order conditions, we obtain the optimal values $\left(\tilde{L}_N^R, \tilde{L}_N^F\right)$ for power plant II as:

$$\tilde{L}_N^R = \frac{(1-\theta)\beta_1 + V'^{(\mathrm{II})}_N(K)\vartheta_2^R}{\tilde{\beta}^R} = \frac{(1-\theta)\beta_1 + a_2\vartheta_2^R}{\tilde{\beta}^R}, \tag{A.5}$$

$$\tilde{L}_N^F = \frac{(1-\theta)\beta_2 + V'^{(\mathrm{II})}_N(K)\vartheta_2^F}{\tilde{\beta}^F} = \frac{(1-\theta)\beta_2 + a_2\vartheta_2^F}{\tilde{\beta}^F}. \tag{A.6}$$

Hence, the solution of the HJB equation is an unary function with K, such that $V_N^{(\mathrm{I})} = \hat{a}_1K + \hat{b}_1$, $V_N^{(\mathrm{II})} = \hat{a}_2K + \hat{b}_2$. Hence, finally $\hat{a}_1, \hat{b}_1, \hat{a}_2,$ and $\hat{b}_2$ as:

$$\hat{a}_1 = \frac{\theta(\Gamma+\delta)}{(\rho_1+\xi)}, \quad \hat{b}_1 = \frac{\hat{\Phi}_1}{\rho_1}, \quad \hat{a}_2 = \frac{(1-\theta)(\Gamma+\delta)}{(\rho_2+\xi)}, \quad \hat{b}_2 = \frac{\hat{\Phi}_2}{\rho_2}, \tag{A.7}$$

where

$$\begin{aligned}
\hat{\Phi}_1 = &\left(\alpha_1\theta - \frac{(\theta\alpha_1 + a_1\vartheta_1^R)}{2} + \vartheta_1^R a_1\right)\left(\frac{\theta\alpha_1 + a_1\vartheta_1^R}{\beta^R}\right)\\
&+\left(\alpha_2\theta - \frac{(\theta\alpha_2 + a_1\vartheta_1^F)}{2} + \vartheta_1^F a_1\right)\left(\frac{\theta\alpha_2 + a_1\vartheta_1^F}{\beta^F(1-\omega_2)}\right)\\
&+\left(\beta_1\theta + \vartheta_2^R a_1\right)\left(\frac{(1-\theta)\beta_1 + a_2\vartheta_2^R}{\tilde{\beta}^R}\right) + \left(\beta_2\theta + \vartheta_2^F a_1\right)\left(\frac{(1-\theta)\beta_2 + a_2\vartheta_2^F}{\tilde{\beta}^F}\right) > 0.
\end{aligned} \tag{A.8}$$

and

$$\begin{aligned}
\hat{\Phi}_2 = &\left((1-\theta)\alpha_1 + \vartheta_1^R a_2\right)\frac{(\theta\alpha_1 + a_1\vartheta_1^R)}{\beta^R(1-\omega_1)}\\
&+\left((1-\theta)\alpha_2 + \vartheta_1^F a_2\right)\frac{(\theta\alpha_2 + a_1\vartheta_1^F)}{\beta^F(1-\omega_2)}\\
&+\left((1-\theta)\beta_1 - \frac{((1-\theta)\beta_1 + a_2\vartheta_2^R)}{2} + \vartheta_2^R a_2\right)\left(\frac{(1-\theta)\beta_1 + a_2\vartheta_2^R}{\tilde{\beta}^R}\right)\\
&+\left((1-\theta)\beta_2 - \frac{((1-\theta)\beta_2 + a_2\vartheta_2^F)}{2} + \vartheta_2^F a_2\right)\left(\frac{(1-\theta)\beta_2 + a_2\vartheta_2^F}{\tilde{\beta}^F}\right) > 0.
\end{aligned} \tag{A.9}$$

Substituting a_1&a_2 into Eqs. (A.2), (A.3), (A.5) and (A.6) and simplifying, obtain the optima effort level of technological improvements. By substituting the above results into Eqs. (A.1) and (A.4) we obtain the optimal sharing payoff functions, for power plant (I) and power plant (II).

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