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Probe on Rupture Theory of Soft-Matter Quasicrystals

Hui Cheng and Tian-You Fan

Abstract

In this chapter, a probe on an important aspect, rupture theory of soft matter, is discussed, in which the soft matter and soft-matter quasicrystals are introduced at first. Then, we discuss the behaviour of the matter. For the soft-matter quasicrystals, there are basic equations describing their dynamics; due to the work of the second author of this chapter, this provides a fundamental for studying the rupture feature quantitatively. For general soft matter, there are no such equations so far, whose rupture theory quantitatively is not available at moment. The discussion on the soft-matter quasicrystals may provide a reference for other branches of soft matter.

Keywords: soft matter, quasicrystals, generalized dynamics, equation of state, crack, stress intensity factor, generalized Griffith criterion of rupture

1. Introduction

Soft-matter quasicrystals belong to a category of soft matter. It is well known that the soft matter is an intermediate phase between solid and liquid, which presents the behaviour of both solid and liquid, the first nature of which is fluidity, and behaves as a complex structure. Hence, the soft matter is named as a complex liquid or a structured liquid. Soft-matter quasicrystals are soft-matter with quasi-periodic symmetry. In this sense, they are a category of soft matter with highest symmetry so far. The high symmetry of the matter presents symmetry breaking and leads to importance of elementary excitations. This helps us to set up their dynamic equations and provide the fundamental for the rupture study.

2. Soft matter and the rupture problem

The liquid crystals, polymers, colloids, surfactants, and so on are in common named as soft matter. The 12-fold symmetry quasicrystals were observed most frequently in the soft matter [1–7]. At the same time, cracks in soft matter [8, 9] cannot be ignored and should be prevented [10] from the material safety. Other crack and rupture problems in soft matter can be referred from [11–18]. This shows that the study on crack and rupture for soft matter including soft-matter quasicrystals is very important.

It is well known that the failure of brittle structural materials has been well studied. According to the work of Griffith [19], the existence and propagation of crack is the reason of the failure of these materials. Griffith used the exact solution of a crack in an infinite plate and calculated the crack strain energy. The energy is

the function of crack size. He further calculated crack energy release rate and suggested that when the release rate equals to the surface tensile of the material, the solid will be a failure. His hypothesis was proved by experiments. This is the famous Griffith criterion. Afterward, the classical Griffith theory was developed by Irwin et al. for studying quasi-brittle failure, where the strain energy release rate was replaced by so-called stress intensity factor and the surface tensile was replaced by the fracture toughness of the material. This is the engineering approach of the Griffith theory, and played an important role in engineering application. The failure of ductile materials is also related with the existence and propagation of crack, but the problem has not been well studied due to the plastic deformation around the crack tip. The plastic deformation is a nonlinear irreducible process. The problem is extremely complex physically and mathematically. The failure of soft matter will be more complex than the ductile structural materials because of the existence of the fluidity. Especially, the experimental results are few of reported. As a most preliminary probe for studying the problem of soft matter, we try to draw from the idea of the classical Griffith work, i.e., to study the crack stability/instability, we can use the so-called following Griffith-Irwin criterion

$$K_1(a, \sigma) = K_{IC}(T, f(s)) \quad (1)$$

in which $K_1(a, \sigma)$ represents elastic stress intensity factor, which a function of crack size a and applied external stress σ , can be determined by stress analysis of cracked materials, and the $K_{IC}(T, f(s))$ fracture toughness, a material constant but influenced by temperature and the structure factor $f(s)$, and the suffice I expresses mode I, i.e., the opening mode fracture (and the mode II is shearing mode, or slip mode fracture, and mode III is longitudinal shear mode, or tearing mode fracture, we here discuss only the opening mode). If the value of $K_1(a, \sigma)$ is greater than that of $K_{IC}(T, f(s))$, then the crack will propagate and the material will fracture.

Of course, the criterion (1) is only a reference for the soft matter, and a further analysis will be given in the subsequent sections.

3. Soft-matter quasicrystals and their generalized dynamics

There are a quite lot of references concerning the crack and rupture problems in soft matter; however, the quantitative analyses are not so much, because most branches of soft matter science are in qualitative stage so far. Either theoretical research or engineering application, the rupture problem of the soft matter needs a quantitative analysis.

Recently, generalized dynamics of soft-matter quasicrystals has been developed [20–24], which may become another quantitative branch in soft matter apart from the liquid crystals science. The generalized dynamics of soft-matter quasicrystals provides a tool for analyzing quantitatively rupture problem of the matter, whose result may be references of other categories of soft matter.

Soft-matter quasicrystals look like other categories of soft matter which belong to complex fluid; at mean time they present highly symmetry. For the high ordered phase, the symmetry breaking and elementary excitation principle are important. By using the Landau-Anderson [25, 26] symmetry breaking and elementary excitation principle, there are phonon and phason elementary excitations. As a class of soft matter, the fluidity is the substantive nature of the soft-matter quasicrystals; so Fan [20–24] introduced another elementary excitation—fluid phonon apart from phonons and phasons; of course, the concept of the fluid phonon is originated from the Landau School [27]. The introducing of fluid phonon requires a supplemented equation and

the equation of state as well, which is also completed by Fan and co-worker [28]. With these bases, the generalized dynamics of soft-matter quasicrystals is set up.

For the need of the present chapter, we list the two-dimensional equations of the dynamics of soft-matter quasicrystals of 5- and 10-fold symmetry as follows:

$$\left. \begin{aligned}
 & \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \\
 & \frac{\partial(\rho V_x)}{\partial t} + \frac{\partial(V_x \rho V_x)}{\partial x} + \frac{\partial(V_y \rho V_x)}{\partial y} = -\frac{\partial p}{\partial x} + \eta \nabla^2 V_x + \frac{1}{3} \eta \frac{\partial}{\partial x} \nabla \cdot \mathbf{V} \\
 & + M \nabla^2 u_x + (L + M - B) \frac{\partial}{\partial x} \nabla \cdot \mathbf{u} \\
 & + R_1 \left(\frac{\partial^2 w_x}{\partial x^2} + 2 \frac{\partial^2 w_y}{\partial x \partial y} - \frac{\partial^2 w_x}{\partial y^2} \right) - R_2 \left(\frac{\partial^2 w_y}{\partial x^2} - 2 \frac{\partial^2 w_x}{\partial x \partial y} - \frac{\partial^2 w_y}{\partial y^2} \right) \\
 & - (A - B) \frac{1}{\rho_0} \frac{\partial \delta \rho}{\partial x} \\
 & \frac{\partial(\rho V_y)}{\partial t} + \frac{\partial(V_x \rho V_y)}{\partial x} + \frac{\partial(V_y \rho V_y)}{\partial y} = -\frac{\partial p}{\partial y} + \eta \nabla^2 V_y + \frac{1}{3} \eta \frac{\partial}{\partial y} \nabla \cdot \mathbf{V} \\
 & + M \nabla^2 u_y + (L + M - B) \frac{\partial}{\partial y} \nabla \cdot \mathbf{u} \\
 & + R_1 \left(\frac{\partial^2 w_y}{\partial x^2} - 2 \frac{\partial^2 w_x}{\partial x \partial y} - \frac{\partial^2 w_y}{\partial y^2} \right) + R_2 \left(\frac{\partial^2 w_x}{\partial x^2} + 2 \frac{\partial^2 w_y}{\partial x \partial y} - \frac{\partial^2 w_x}{\partial y^2} \right) \\
 & - (A - B) \frac{1}{\rho_0} \frac{\partial \delta \rho}{\partial y} \\
 & \frac{\partial u_x}{\partial t} + V_x \frac{\partial u_x}{\partial x} + V_y \frac{\partial u_x}{\partial y} = V_x + \Gamma_{\mathbf{u}} [M \nabla^2 u_x + (L + M) \frac{\partial}{\partial x} \nabla \cdot \mathbf{u} + \\
 & R_1 \left(\frac{\partial^2 w_x}{\partial x^2} + 2 \frac{\partial^2 w_y}{\partial x \partial y} - \frac{\partial^2 w_x}{\partial y^2} \right) - R_2 \left(\frac{\partial^2 w_y}{\partial x^2} - 2 \frac{\partial^2 w_x}{\partial x \partial y} - \frac{\partial^2 w_y}{\partial y^2} \right)] \\
 & \frac{\partial u_y}{\partial t} + V_x \frac{\partial u_y}{\partial x} + V_y \frac{\partial u_y}{\partial y} = V_y + \Gamma_{\mathbf{u}} [M \nabla^2 u_y + (L + M) \frac{\partial}{\partial y} \nabla \cdot \mathbf{u} + \\
 & R_1 \left(\frac{\partial^2 w_y}{\partial x^2} - 2 \frac{\partial^2 w_x}{\partial x \partial y} - \frac{\partial^2 w_y}{\partial y^2} \right) + R_2 \left(\frac{\partial^2 w_x}{\partial x^2} + 2 \frac{\partial^2 w_y}{\partial x \partial y} - \frac{\partial^2 w_x}{\partial y^2} \right)] \\
 & \frac{\partial w_x}{\partial t} + V_x \frac{\partial w_x}{\partial x} + V_y \frac{\partial w_x}{\partial y} = \Gamma_{\mathbf{w}} [K_1 \nabla^2 w_x + \\
 & R_1 \left(\frac{\partial^2 u_x}{\partial x^2} - 2 \frac{\partial^2 u_y}{\partial x \partial y} - \frac{\partial^2 u_x}{\partial y^2} \right) + R_2 \left(\frac{\partial^2 u_y}{\partial x^2} + 2 \frac{\partial^2 u_x}{\partial x \partial y} - \frac{\partial^2 u_y}{\partial y^2} \right)] \\
 & \frac{\partial w_y}{\partial t} + V_x \frac{\partial w_y}{\partial x} + V_y \frac{\partial w_y}{\partial y} = \Gamma_{\mathbf{w}} [K_1 \nabla^2 w_y + \\
 & R_1 \left(\frac{\partial^2 u_y}{\partial x^2} + 2 \frac{\partial^2 u_x}{\partial x \partial y} - \frac{\partial^2 u_y}{\partial y^2} \right) - R_2 \left(\frac{\partial^2 u_x}{\partial x^2} - 2 \frac{\partial^2 u_y}{\partial x \partial y} - \frac{\partial^2 u_x}{\partial y^2} \right)] \\
 & p = 3 \frac{k_B T}{l^3} \left(\frac{\rho}{\rho_0} + \frac{\rho^2}{\rho_0^2} + \frac{\rho^3}{\rho_0^3} \right)
 \end{aligned} \right\} \quad (2)$$

in which u_i denotes the phonon field, w_i the phason field, V_i the fluid velocity field, C_{ijkl} is the phonon elastic constant tensor, K_{ijkl} phason elastic constant tensor, and R_{ijkl} and R_{klij} are the phonon-phason coupling elastic constant tensors.

A and B are the constants describing density variation, $g = \rho \mathbf{V}$, and η is the fluid viscosity, k_B is the Boltzmann constant, T is the absolute temperature, and $l \sim 10 - 100$ nm is the characteristic size of the soft matter, respectively.

4. An example of stress analysis of soft-matter quasicrystals

With Eq. (2), we can carry out a stress analysis of some fundamental specimens with crack of soft-matter quasicrystals; we here give only a computational example as shown in **Figure 1**.

If we want to obtain further information on deformation and motion of the material, we must solve the equations under appropriate initial and boundary conditions. To solve the problem, a specimen made by the matter should be optioned, which is subjected to some initial and boundary conditions. Here, the corresponding conditions of the specimen shown in **Figure 1** are as follows:

$$\begin{aligned}
 t = 0 : V_x = V_y = 0, u_x = u_y = 0, w_x = w_y = 0, p = p_0; \\
 y = \pm H, |x| < W : V_x = V_y = 0, \sigma_{yy} = \sigma_0 f(t), \sigma_{yx} = 0, H_{yy} = H_{yx} = 0, p = p_0; \\
 x = \pm W, |y| < H : V_x = V_y = 0, \sigma_{xx} = \sigma_{xy} = 0, H_{xx} = H_{xy} = 0, p = p_0; \\
 y = 0, |x| < a : V_x = V_y = 0, \sigma_{yy} = \sigma_{yx} = 0, H_{yy} = H_{yx} = 0, p = p_0
 \end{aligned} \tag{3}$$

In the present computation we take, $2H = 0.01$ m, $2W = 0.01$ m, $2a = 0.0024$ m, $\sigma_0 = 0.01$ MPa, $\rho_0 = 1.5 \times 10^3$ kg/m³, $\eta = 0.1$ Pa · s (or 0.2 Pa · s), $\zeta = 0$, $L = 10$ MPa, $M = 4$ MPa, $K_1 = 0.5L$, $R = R_1 = 0.04M$ (or 0.06M), $R_2 = 0$, $\Gamma_u = 4.8 \times 10^{-17}$ m³ · s/kg, $\Gamma_w = 4.8 \times 10^{-19}$ m³ · s/kg, $A \sim 0.2$ MPa, $B \sim 0.2$ MPa, and p_0 denotes 1 atm.

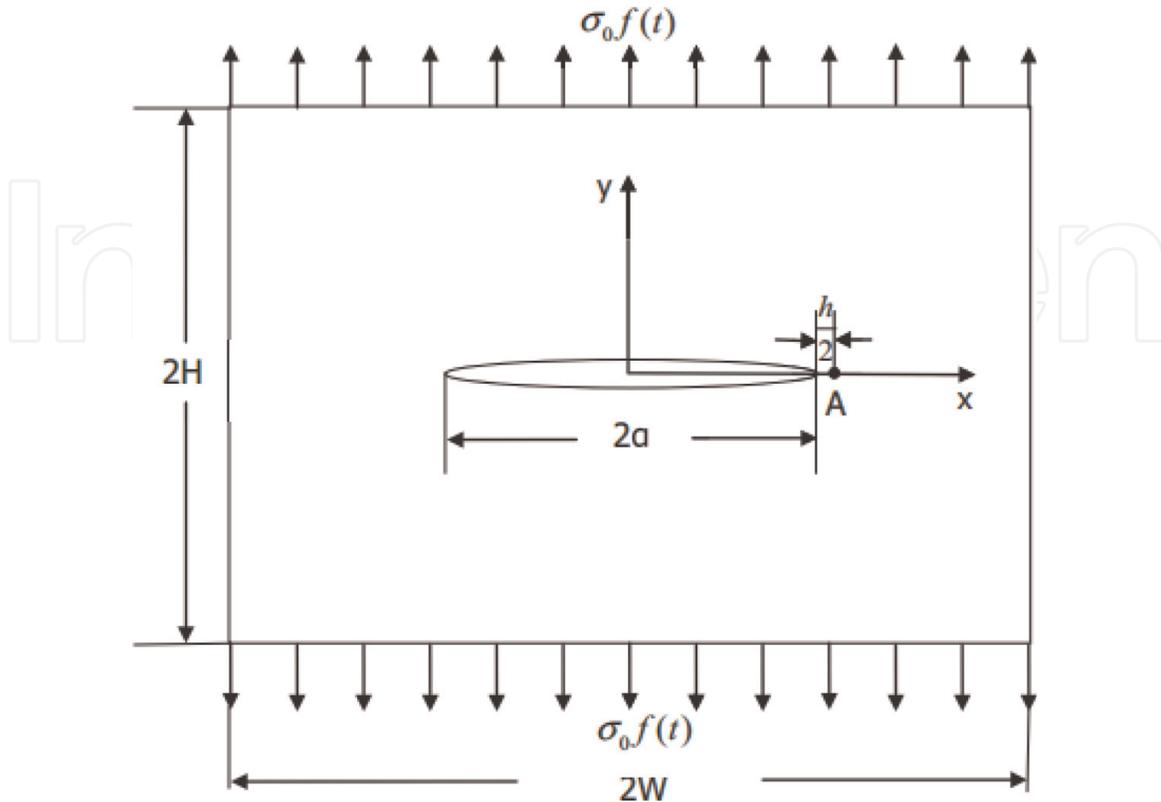


Figure 1. Specimen of soft-matter quasicrystals of 5- and 10-fold symmetries with a Griffith crack under tension.

The initial and boundary value problem (3) of Eq. (2) can be solved by the finite difference method to solve the boundary value problem (**Figure 2**), e.g.,

$$\begin{aligned}\frac{\partial^2 u_x}{\partial x^2} &= \frac{u_x(i+1, j) - 2u_x(i, j) + u_x(i-1, j)}{h^2}, \\ \frac{\partial^2 u_x}{\partial y^2} &= \frac{u_x(i, j+1) - 2u_x(i, j) + u_x(i, j-1)}{h^2}, \\ \frac{\partial u_x}{\partial t} &= \frac{u_x(k+1) - u_x(k)}{\tau}.\end{aligned}$$

so determine the phonon and phason displacement fields and fluid phonon velocity field, then the phonon and phason strain tensors.

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), w_{ij} = \frac{\partial w_i}{\partial x_j}, \quad (4)$$

and the fluid phonon deformation rate tensor

$$\dot{\xi}_{ij} = \frac{1}{2} \left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right) \quad (5)$$

and according to constitutive laws

$$\left. \begin{aligned}\sigma_{ij} &= C_{ijkl} \varepsilon_{ik} + R_{ijkl} w_{kl}, \\ H_{ij} &= K_{ijkl} w_{ij} + R_{klij} \varepsilon_{kl}, \\ p_{ij} &= -p \delta_{ij} + \sigma'_{ij} = -p \delta_{ij} + 2\eta \dot{\xi}_{kl},\end{aligned} \right\} \quad (6)$$

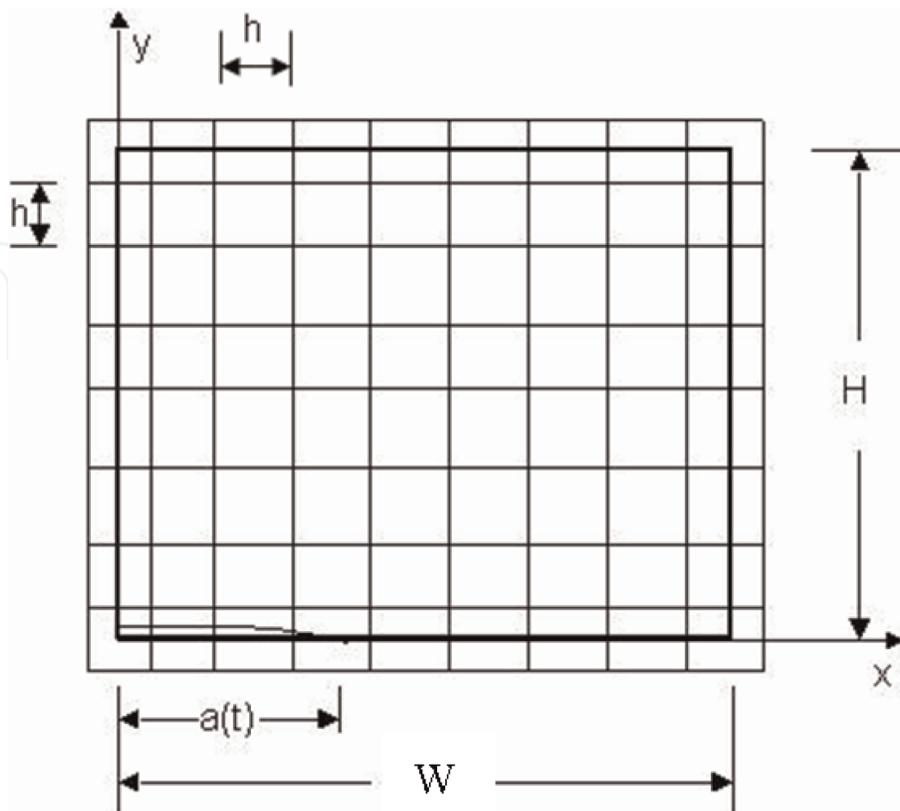


Figure 2.
 The scheme of grid of the difference for a part of the specimen.

we obtain the phonon stresses σ_{ij} and fluid phonon stresses p_{ij} , respectively, in which recall C_{ijkl} the phonon elastic constant tensor, K_{ijkl} phason elastic constant tensor, R_{ijkl} and R_{klij} the phonon-phason coupling elastic constant tensors, etc., refer to [24].

Due to the complexity of the equations, the computation is complex too. For the dynamic problems, i.e., there are manmade damping terms $\theta\rho$, $\theta\rho V_x$, and so on, the iterative computation is easily stable, and for the static problems, we still take the iterative computation; however, the choosing of the manmade damping coefficient θ depends upon experience.

The computational results are mostly dependent on the ratio value of σ_0/p_0 , i.e., the ratio of amplitudes of the phonon stress and fluid phonon stress apart from other factors.

5. Significance of fluid stress to crack initiation of growth and crack propagation

In the crack and fracture of brittle or quasibrittle failure of structural materials, the stress analysis is a basic task, from which one can determine the stress intensity factor, and then use the Griffith-Irwin criterion to analyze the crack stability/instability. For the Mode I crack, the tensile stress $\sigma_{yy}(x, y, t)$ is the most important, which leads to the crack surface opening and so the crack propagation.

After our computation, in soft matter including soft-matter quasicrystals, apart from the phonon stress $\sigma_{yy}(x, y, t)$, there is the fluid phonon stress $p_{yy}(x, y, t)$ which is pressure and leads to crack closing (**Figures 3 and 4**).

In principle, the Griffith theory holds for describing brittle and quasi-brittle rupture of structural materials (or engineering materials). However, for soft matter including soft-matter quasicrystals, there is fundamental difference with the structural materials. The key reason about this is the existence of fluid effect; in terminology of soft-matter quasicrystal study, it is also called the existence of fluid phonon. According to our analysis, the effect of fluid stress intensity factor is opposite to the elastic stress intensity factor (or by using the terminology of soft-matter quasicrystals study, the elastic stress intensity factor is also called the phonon stress

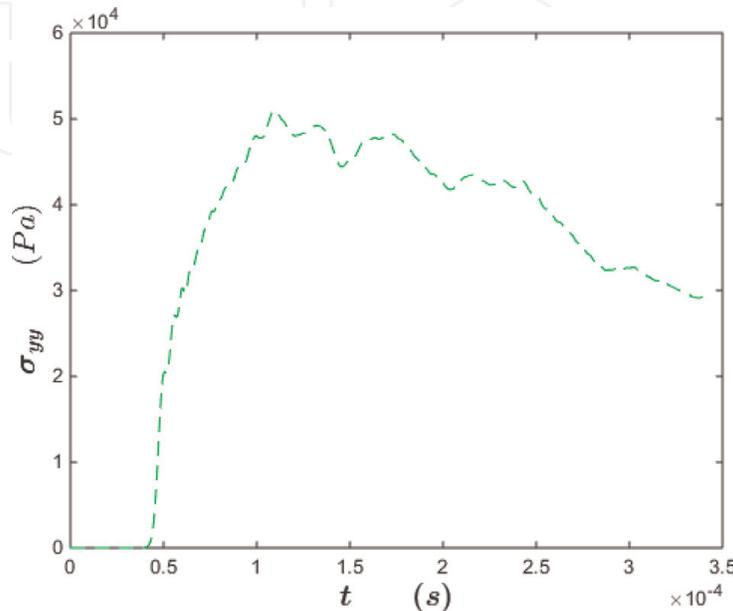


Figure 3. Normal stress $\sigma_{yy}(x, 0, t)$ of phonon field at the point A of specimen versus time.

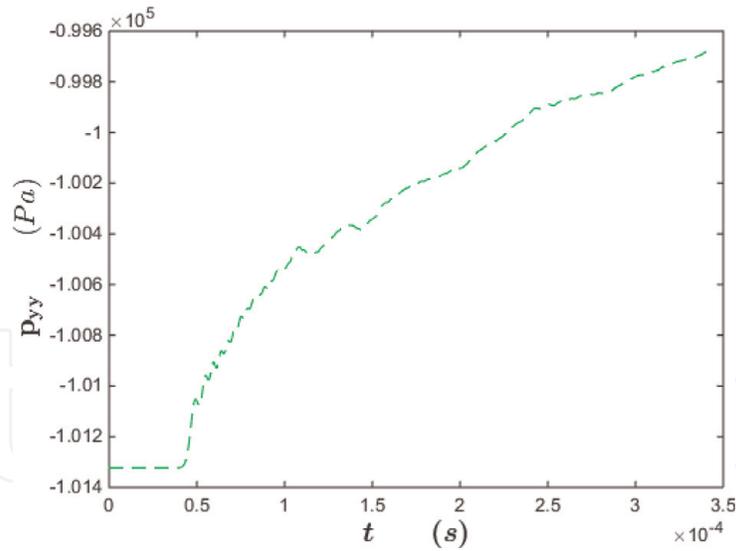


Figure 4.
 Normal stress $p_{yy}(x, 0, t)$ of fluid field at the point A of specimen versus time.

intensity factor). K_1^{phonon} denotes the phonon stress intensity factor and K_1^{fluid} denotes the fluid phonon stress intensity factor which can be defined as follows:

$$K_1^{\text{phonon}} = \lim_{x \rightarrow a_0^+} \sqrt{\pi(x - a_0)} \sigma_{yy}(\mathbf{x}, 0, t) \quad (7)$$

$$K_1^{\text{fluid}} = \lim_{x \rightarrow a_0^+} \sqrt{\pi(x - a_0)} p_{yy}(\mathbf{x}, 0, t) \quad (8)$$

The computational results are dependent upon the ratio σ_0/p_0 , while the fluid stress intensity factor is expressed by K_1^{fluid} , which is also a function of time in general, because $K_1^{\text{phonon}} > 0$ leads to crack surface opening, and according to our computation, $K_1^{\text{fluid}} < 0$ leads to crack surface closing. The total stress intensity factor is denoted by

$$K_1^{\text{total}} = K_1^{\text{phonon}} + K_1^{\text{fluid}}. \quad (9)$$

Therefore, the value of K_1^{total} is smaller than K_1^{phonon} . K_1 is the most often used in engineering design parameter in fracture mechanics and, hence, must be understood if we are to design fracture tolerant materials used in bridges, buildings, aircraft, or even bells. Polishing cannot detect a crack. Typically, if a crack can be seen, it is very close to the critical stress state predicted by the stress intensity factor. The magnitude of \tilde{K}_1 depends on sample geometry, the size and location of the crack, and the magnitude and the modal distribution of loads on the material.

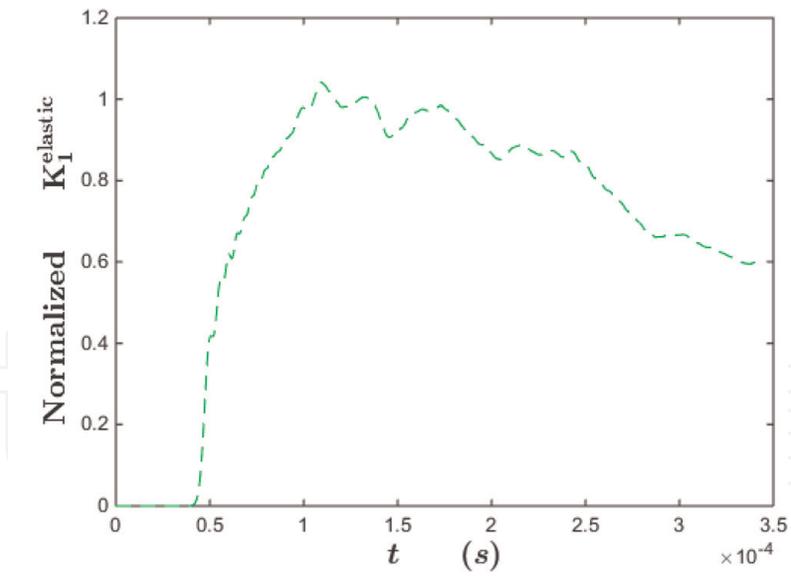
The normalized elastic dynamic stress intensity factor is defined as

$$\tilde{K}_1(t) = K_1(t) / \sqrt{\pi a_0} \sigma_0 \quad (10)$$

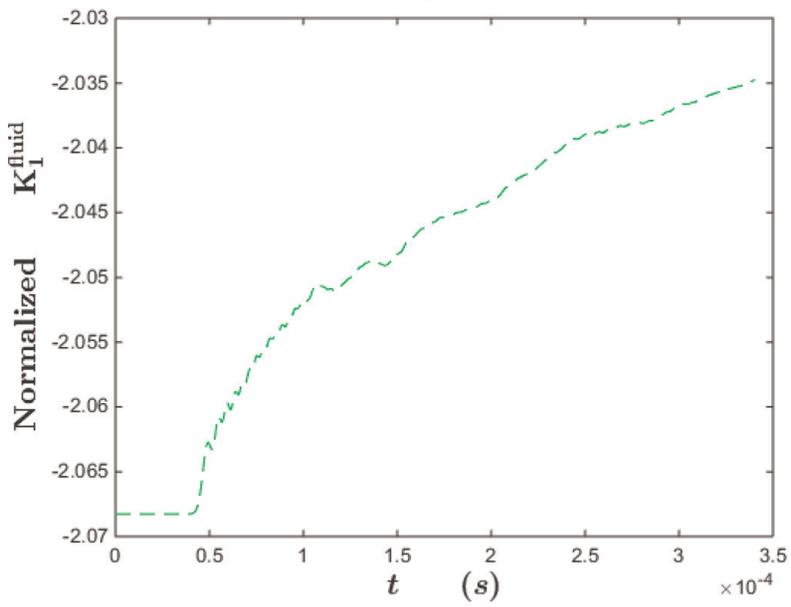
6. Possible criterion of rupture

For soft matter, there is fracture criterion:

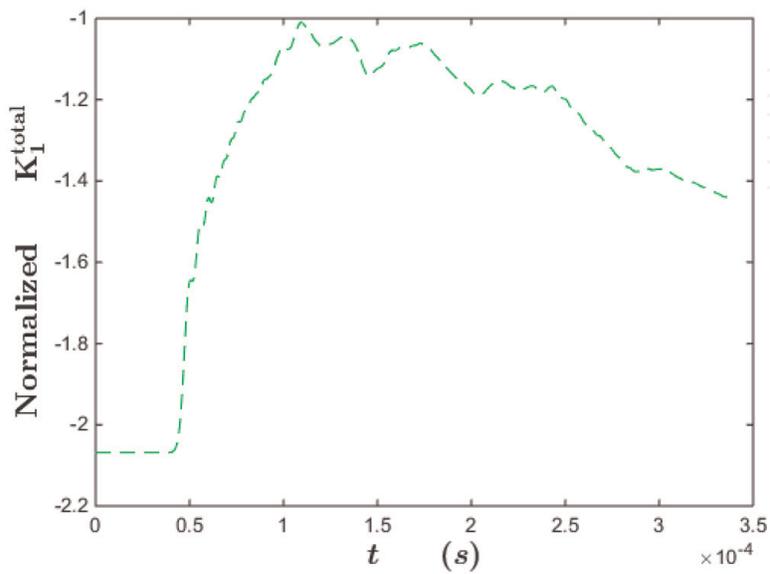
$$K_1^{\text{total}} = K_{IC}(T). \quad (11)$$



(a)



(b)



(c)

Figure 5. For soft-matter quasicrystals with 5- and 10-fold symmetries: (a) normalized phonon (elastic) DSIF versus time; (b) normalized fluid DSIF versus time; and (c) normalized total DSIF versus time.

The Griffith theory can be extended to soft matter. If $K_1^{\text{total}}(t) > K_{IC}(T)$ then crack surface to be opening, and crack propagate leads to fracture; otherwise $K_1^{\text{total}}(t) < K_{IC}(T)$ then crack surface to be closing, and crack cannot propagate, the material is in safe. The conclusion is that the physical state around crack tip in soft matter quasicrystals is dependent on the competition of phonon stress and fluid stress.

7. The references for other soft matter

The example given in previous section is only the most preliminary discussion on the probe of rupture of soft-matter quasicrystals. It does not mean any in-depth and exact theory of the problem. The significance of the discussion lies in providing a possible drawing for other categories of soft matter in the quantitative study of rupture problems. The formulation of the present study may be referenced by other branches of soft matter, in which the constitutive laws for themselves are needed, and phason field can be excluded for non-quasicrystalline materials, and so on. With these considerations, the rupture theory for the other categories of soft matter out of the soft-matter quasicrystals may be set up.

8. Conclusion and discussion

From **Figure 5**, we also can see that the real value of fluid DSIF is negative; thus, we can say it causes the crack closed. When comparing (a) with (b), the absolute value of fluid DSIF is larger than the elastic ones; thus, the crack cannot propagate. Then, we can conclude that in the quasicrystal of soft matter, the fluid effect is important, and we should do the study under the hydrodynamics theory.

The equation set of generalized dynamics of soft-matter quasicrystals is complete; it is possible to provide the hydrodynamics study on the mass distribution, deformation, and motion, including rupture analysis of the new phase. In particular, the dynamics stress intensity factor (DSIF) was evaluated, and the computation presents highly stability. In the results, there are two regimes: one is elastic DSIF and the other is fluid DSIF. The former leads to crack surface opening and the latter leads to crack surface closing. We can conclude that whether or not the crack open or close depends on the competition between the elastic DSIF and fluid DSIF. Of course, the theoretical analysis is anxious for the experimental verification currently, which is our attempt work in future.

The above introducing is rough and preliminary, in fact, around the crack tip, due to the stress concentration and high stress grad, nonelastic stress appears, the Griffith theory will not be effective, i.e., the theory based on the stress intensity factor will not be valid, one must carry out nonlinear analysis, e.g. the theory based on the crack tip opening displacement, in which the size of the plastic zone near the crack tip is also considered, refer to Fan [24], which may help us to overcome the difficulty mentioned above. However this is a very difficult question which has not been solved so far.

Acknowledgements

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Conflict of interest

The authors declare no conflict of interest.

A. Appendix

A.1 The final governing equations of generalized dynamics of soft-matter quasicrystals with 10-fold symmetry in three dimension

In the text, Eq. (2) is the planar field form only. In book [24], it provides an explicit form of the equations and lists below soft-matter quasicrystals with 10-fold symmetry:

$$\begin{aligned}
 & \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{V}) = 0 \\
 & \frac{\partial(\rho V_x)}{\partial t} + \frac{\partial(V_x \rho V_x)}{\partial x} + \frac{\partial(V_y \rho V_x)}{\partial y} + \frac{\partial(V_z \rho V_x)}{\partial z} = -\frac{\partial p}{\partial x} + \eta \nabla^2 V_x + \frac{1}{3} \eta \frac{\partial}{\partial x} \nabla \cdot \mathbf{V} \\
 & + \left(C_{66} \frac{\partial^2}{\partial y^2} + C_{44} \frac{\partial^2}{\partial z^2} \right) u_x + (C_{12} + C_{66}) \frac{\partial^2 u_y}{\partial x \partial y} + (C_{13} + C_{44} - C_{11}) \frac{\partial^2 u_z}{\partial x \partial z} + (C_{11} - B) \frac{\partial}{\partial x} \nabla \cdot \mathbf{u} \\
 & + R \frac{\partial}{\partial x} \nabla_1 \cdot \mathbf{w} - R \frac{\partial}{\partial y} \left(\frac{\partial w_x}{\partial y} - \frac{\partial w_y}{\partial x} \right) - (A - B) \frac{1}{\rho_0} \frac{\partial \delta \rho}{\partial x} \\
 & \frac{\partial(\rho V_y)}{\partial t} + \frac{\partial(V_x \rho V_y)}{\partial x} + \frac{\partial(V_y \rho V_y)}{\partial y} + \frac{\partial(V_z \rho V_y)}{\partial z} = -\frac{\partial p}{\partial y} + \eta \nabla^2 V_y + \frac{1}{3} \eta \frac{\partial}{\partial y} \nabla \cdot \mathbf{V} \\
 & + (C_{12} + C_{66}) \frac{\partial^2 u_x}{\partial x \partial y} + \left(C_{66} \frac{\partial^2}{\partial x^2} + C_{11} \frac{\partial^2}{\partial y^2} + C_{44} \frac{\partial^2}{\partial z^2} \right) u_y + (C_{13} + C_{44}) \frac{\partial^2 u_z}{\partial y \partial z} + (C_{11} - B) \frac{\partial}{\partial y} \nabla \cdot \mathbf{u} \\
 & - R \frac{\partial}{\partial x} \left(\frac{\partial w_x}{\partial y} - \frac{\partial w_y}{\partial x} \right) - R \frac{\partial}{\partial y} \nabla_1 \cdot \mathbf{w} - (A - B) \frac{1}{\rho_0} \frac{\partial \delta \rho}{\partial y} \\
 & \frac{\partial(\rho V_z)}{\partial t} + \frac{\partial(V_x \rho V_z)}{\partial x} + \frac{\partial(V_y \rho V_z)}{\partial y} + \frac{\partial(V_z \rho V_z)}{\partial z} = -\frac{\partial p}{\partial z} + \eta \nabla^2 V_z + \frac{1}{3} \eta \frac{\partial}{\partial z} \nabla \cdot \mathbf{V} \\
 & + \left(C_{44} \frac{\partial^2}{\partial x^2} + C_{44} \frac{\partial^2}{\partial y^2} + (C_{33} - C_{13} - C_{44}) \frac{\partial^2}{\partial z^2} \right) u_z + (C_{13} + C_{44} - B) \frac{\partial}{\partial z} \nabla \cdot \mathbf{u} - (A - B) \frac{1}{\rho_0} \frac{\partial \delta \rho}{\partial z} \\
 & \frac{\partial u_x}{\partial t} + V_x \frac{\partial u_x}{\partial x} + V_y \frac{\partial u_x}{\partial y} + V_z \frac{\partial u_x}{\partial z} \\
 & = V_x + \Gamma_{\mathbf{u}} \left[\left(C_{11} \frac{\partial^2}{\partial x^2} + C_{66} \frac{\partial^2}{\partial y^2} + C_{44} \frac{\partial^2}{\partial z^2} \right) u_x + (C_{12} + C_{66}) \frac{\partial^2 u_y}{\partial x \partial y} \right. \\
 & \left. + (C_{13} + C_{44}) \frac{\partial^2 u_z}{\partial x \partial z} + R \frac{\partial}{\partial x} \nabla_1 \cdot \mathbf{w} - R \frac{\partial}{\partial y} \left(\frac{\partial w_x}{\partial y} - \frac{\partial w_y}{\partial x} \right) \right] \\
 & \frac{\partial u_y}{\partial t} + V_x \frac{\partial u_y}{\partial x} + V_y \frac{\partial u_y}{\partial y} + V_z \frac{\partial u_y}{\partial z} \\
 & = V_y + \Gamma_{\mathbf{u}} \left[(C_{12} + C_{66}) \frac{\partial^2 u_x}{\partial x \partial y} + \left(C_{66} \frac{\partial^2}{\partial x^2} + C_{11} \frac{\partial^2}{\partial y^2} + C_{44} \frac{\partial^2}{\partial z^2} \right) u_y \right. \\
 & \left. + (C_{13} + C_{44}) \frac{\partial^2 u_z}{\partial y \partial z} - R \frac{\partial}{\partial x} \left(\frac{\partial w_x}{\partial y} - \frac{\partial w_y}{\partial x} \right) - R \frac{\partial}{\partial y} \nabla_1 \cdot \mathbf{w} \right] \\
 & \frac{\partial u_z}{\partial t} + V_x \frac{\partial u_z}{\partial x} + V_y \frac{\partial u_z}{\partial y} + V_z \frac{\partial u_z}{\partial z} = V_z + \Gamma_{\mathbf{u}} \left[(C_{13} + C_{44}) \left(\frac{\partial^2 u_x}{\partial x \partial z} + \frac{\partial^2 u_y}{\partial y \partial z} \right) \right. \\
 & \left. + \left(C_{44} \frac{\partial^2}{\partial x^2} + C_{44} \frac{\partial^2}{\partial y^2} + C_{33} \frac{\partial^2}{\partial z^2} \right) u_z \right] \\
 & \frac{\partial w_x}{\partial t} + V_x \frac{\partial w_x}{\partial x} + V_y \frac{\partial w_x}{\partial y} + V_z \frac{\partial w_x}{\partial z} \\
 & = \Gamma_{\mathbf{w}} \left[K_1 \nabla_1^2 w_x + K_4 \frac{\partial^2 w_x}{\partial z^2} + K_2 \frac{\partial}{\partial y} \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y} \right) w_y + R \frac{\partial}{\partial x} \left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y} \right) - R \frac{\partial}{\partial y} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) \right] \\
 & \frac{\partial w_y}{\partial t} + V_x \frac{\partial w_y}{\partial x} + V_y \frac{\partial w_y}{\partial y} + V_z \frac{\partial w_y}{\partial z} \\
 & = \Gamma_{\mathbf{w}} \left[K_1 \nabla_1^2 w_y + K_4 \frac{\partial^2 w_y}{\partial z^2} + R \frac{\partial}{\partial x} \left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x} \right) + R \frac{\partial}{\partial y} \left(\frac{\partial u_x}{\partial x} - \frac{\partial u_y}{\partial y} \right) \right] \\
 & p = f(\rho) = 3 \frac{k_B T}{l^3 \rho_0^3} (\rho_0^2 \rho + \rho_0 \rho^2 + \rho^3)
 \end{aligned} \tag{A1}$$

in which $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$, $\nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z}$, $\mathbf{V} = \mathbf{i}V_x + \mathbf{j}V_y + \mathbf{k}V_z$, $\mathbf{u} = \mathbf{i}u_x + \mathbf{j}u_y + \mathbf{k}u_z$, and $C_{11}, C_{12}, C_{13}, C_{33}, C_{44}, C_{66} = (C_{11} - C_{12})/2$ are the phonon elastic constants, K_1, K_2, K_3, K_4 are the phason elastic constants, R is the phonon-phason coupling constant, η is the fluid dynamic viscosity, Γ_u and Γ_w the phonon and phason dissipation coefficients, and A and B the material constants due to variation of mass density, respectively.

Equation (A1) is the final governing equations of dynamics of soft-matter quasicrystals of 10-fold symmetry in three-dimensional case with field variables $u_x, u_y, u_z, w_x, w_y, V_x, V_y, V_z, \rho$ and p ; the amount of the field variables is 10, and the amount of field equations is 10 too; among them are: (A1a) (the first of (A1)) is the mass conservation equation, (A1b)–(A1d) (the second to fourth of (A1)) are the momentum conservation equations or generalized Navier-Stokes equations, (A1e)–(A1g) (the fifth to seventh of (A1)) are the equations of motion of phonons due to the symmetry breaking, (A1h) and (A1i) (the eighth to ninth of (A1)) are the phason dissipation equations, and (A1j) (the tenth of (A1)) is the equation of state, respectively. The equations are consistent to be mathematical solvability, if there is lack of the equation of state, the equation system is not closed, and has no meaning mathematically and physically. This shows that the equation of state is necessary.

The equation set (A1) is the three-dimensional form of the equations, and in our solution, we computed only the two-dimensional form, i.e., the plane field form; in this special case, the 5- and 10-fold symmetry quasicrystals have the same governing equations, but in the three-dimensional case, the equation set (A1) is valid only for the 10-fold symmetry quasicrystals.

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