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Chapter

# A New Approach for Assessing Credit Risks under Uncertainty

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## Abstract

The purpose of this chapter is to introduce a new approach for an assessment of the credit risks. The initial part of the chapter is to briefly discuss the existing models of assessment of the credit risks and justify the need for a new approach. Since a new approach is created for conditions of uncertainty, we cannot do without fuzzy mathematics. The proposed approach is based on group decision-making, where experts' opinions are expressed by trapezoidal fuzzy numbers. The theoretical basis of the offered approach is laid out in the metric space of trapezoidal fuzzy numbers. The new approach is introduced and discussed, and two realization algorithms are given. The toy example of application of the introduced approach is offered as well.

**Keywords:** group decision-making, fuzzy set, linguistic variable, parameter, credit risk, assessment, trapezoidal fuzzy number, aggregation, algorithm

## 1. Introduction

A well-founded risk assessment is a mandatory stage in the effective implementation of almost any project. This chapter discusses credit risk assessments. Credit risk assessment includes determining whether or not to give a loan to a borrower and what is the probability of bankruptcy or inability to service a loan due to financial problems. Despite the fact that during this process the lender gets a lot of information from the borrower, there is no unambiguous rule for decision-making.

There are various models for assessing these risks. Before the 2008 financial crisis, emphasis was placed on developing models for assessing the financial stability of borrowers. But when, after this crisis, many companies faced a significant risk of bankruptcy, the vector of model developers took the direction to developing effective forecast models.

Existing forecasting models can be classified into two main groups: statistical and theoretical. A description of these models can be found in the literature (see, e.g., [1]). However, in a number of cases, these models are unacceptable. Basically, they are not suitable for assessing credit risks for corporations in developing countries, as well as for assessing the risks of lending to investment projects.

Let us dwell very briefly on the reasons for the unsuitability of these models for corporations in developing countries (a more detailed analysis can be found, e.g., in [2]). For statistical models:



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- An insufficient defaulting history does not provide a relevant background for assessment of the credit risk.

- Problems arise with classification process too: a borrower may have so-called partial default status.

For theoretical models:

- A potential problem that could arise when applying this model is that majority of companies' stock is not traded on a stock exchange. In this case, a rapid evaluation of the market value of the assets is difficult.

It is very important to note that companies whose shares are not traded on the stock exchange are not so rare in developed countries.

Let us consider in more detail the credit risks when financing investment projects. It is known that an investment project involves planning over time of three main cash flows: investment, current (operating) expenses, and income.

When implementing an investment project, the investor never has a comprehensive risk assessment, since frequent changes in the dynamically developing world cannot be foreseen. Therefore, there is an unforeseen circumstance not taken into account by the project (e.g., a catastrophe), which nevertheless happened and disrupt the investment process. At the same time, the investor must be as informed as possible in order to assess the risk of his investment decisions both at the stage of project development and during the investment process itself. In addition, it is important to keep in mind that prices and volumes of products sold, as well as cash values for materials, raw materials, and other goods and services, in the future can radically differ from their expected values at the time of planning the investment project.

Thus, the incompleteness and uncertainty of the information significantly affects the effectiveness of the investment project and often poses insurmountable risks. Therefore, a project considered to be profitable, in fact, may be losing. This may occur due to the risk of deviation of the values of the design parameters from the actual values or due to the complete neglect of any factors.

Based on the foregoing and world practice, models for assessing any object of study, including risks, are suitable only if there is a sufficiently large statistical base (general population). Consequently, these methods do not result in cases under uncertainty. What to do in the absence of such statistics? The accumulated experience shows that the only way out in this case is to use expert estimates. Thus, we come to the process of group decision-making. Decision-making processes are used in quite a variety of applications. The inherent property of these processes is to represent the transformation of individual opinions of experts to the resulting one.

First you need to perform the parameterization process, i.e., identify parameters that experts should evaluate. Denote the set of selected parameters by $P = \{p_i\}, i = \overline{1,n}$. Next, you need to determine in what form and on what scale experts will evaluate the values of the selected parameters. Then generate risk criteria and aggregate expert assessments to make a decision according to the assigned criteria. A very important point is the determination of the form for expert evaluations. Here we propose to consider expert estimates in the form of fuzzy sets.

In 1965, a professor at California University (Berkeley) Lotfi A. Zadeh published a paper "Fuzzy Sets" that gave a birth to the modeling of human intellectual activity and allowed for new interpretations of some mathematical theories. According to classical mathematics, an object either belongs to some set or not, so the characteristic function of an ordinary set is defined as {0, 1} (if an element does not belong to the set, then 0, and if it does, then 1).

Lotfi A. Zadeh was the first to propose a generalization of the range of values of the membership function {0, 1} to the closed interval [0; 1]. Thus, the value of the membership function can be any real number, starting from zero and ending with unity [3]. Such sets were called "fuzzy" sets. By gradually developing the proposed approach, Zadeh introduced the concept of a fuzzy linguistic variable, which was able to model mathematically linguistic variables [4]. For example, Zadeh made it possible to express mathematically the following linguistic notions: "childhood, young, middle-aged, old." Zadeh also introduced the concept of fuzzy relations and basic operations on them.

As noted above, in our study we cannot do without expert evaluations. Since subjectivity, vagueness, and imprecision influence the assessments of experts, the use of fuzzy set theory seems to be an effective tool for our research work (see, e.g., [5, 6]).

In [2], to assess credit risks, we used precisely fuzzy relations. In the present work, we propose to use trapezoidal fuzzy numbers as a form of presentation of experts' estimates. The rationale for this choice will be given in the third section.

The chapter consists of six sections. The second section includes all the necessary information to understand the material. The theoretical basis of the offered approach is laid out, and some theoretical results are given. In the third section, the approach to assessment of credit risk is introduced and discussed. In particular, the rationale for the choice of fuzzy trapezoidal numbers as a form for the presentation of experts' estimates is given. The fourth section looks at the algorithm for realization of the proposed approach. The fifth section contains toy example of the practical application of the introduced approach. The sixth and the final section summarize the chapter.

In conclusion, we note that when reading a chapter, the reader is not required to have knowledge of higher mathematics, but only elementary knowledge of arithmetic, algebra, and geometry. Nevertheless, we will try to explain meaningfully mathematical symbols and concepts that may be unfamiliar to the reader.

## 2. Essential notions and theoretical background

In the introduction, we gave a substantive description of a fuzzy set; now we give its mathematical description.

**Definition 1.** An ordered pair $\{x, \mu(x)\}$, where $x \in X, \mu : X \to [0, 1]$ is called a fuzzy set.

Here $X$ is the universal set of real numbers (universe), $\mu(x)$ is the membership function of fuzzy set, and $\mu : X \to [0, 1]$ means that the membership function takes values from the interval [0; 1] for all $x$.

An important special case of fuzzy sets is fuzzy numbers. A *fuzzy number* is a fuzzy subset of the universal set of real numbers that has a normal and convex membership function, that is, such that: (a) there is an element of the universe in which the membership function is equal to one, and also (b) when deviating from its maximum left or right, membership function does not increase.

In this chapter we will deal with trapezoidal fuzzy numbers. Almost all the results given in this and third sections are, with minor modifications, taken from [7]; therefore we will refrain from further citation.

We denote trapezoidal fuzzy numbers in $X$ by $\tilde{R} = (a, b, c, d),\ 0 < a \le b \le c \le d$. The membership function's graph is a trapezoid with vertices $(a; 0)$, $(b; 1)$, $(c; 1)$, and $(d; 0)$.

We denote by $\Psi(X) = \left\{\tilde{R}_i = (a_i, b_i, c_i, d_i),\ i \in \mathbb{N}\right\}$ the set of all trapezoidal fuzzy numbers in the universe $X$.

The determinations of some operations on trapezoidal fuzzy numbers are given below.

$$\tilde{R}_1 = \tilde{R}_2 \Leftrightarrow a_1 = a_2, b_1 = b_2, c_1 = c_2, d_1 = d_2, \quad \tilde{R}_1, \tilde{R}_2 \in \Psi(X). \tag{1}$$

$$\tilde{R}_1 \oplus \tilde{R}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2), \quad \tilde{R}_1, \tilde{R}_2 \in \Psi(X). \tag{2}$$

$$\alpha \otimes \tilde{R} = (\alpha a, \alpha b, \alpha c, \alpha d) \ \alpha > 0, \ \tilde{R} \in \Psi(X). \tag{3}$$

**Definition 2.** Trapezoidal fuzzy number $\tilde{R}_1 = (a_i)$ is included in trapezoidal fuzzy number $\tilde{R}_2 = (b_i), \ i = \overline{1,4}$, i.e.$\tilde{R}_1 \preceq \tilde{R}_2$, if and only if

$$a_1 \le a_2, b_1 \le b_2, c_1 \le c_2, d_1 \le d_2 \tag{4}$$

It is known that fuzzy maximum and minimum of two trapezoidal fuzzy numbers is defined as follows [8]:

$$\begin{cases} \widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\} = (\max\{a_1, b_1\}, \max\{a_2, b_2\}, \max\{a_3, b_3\}, \max\{a_4, b_4\}), \\ \widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\} = (\min\{a_1, b_1\}, \min\{a_2, b_2\}, \min\{a_3, b_3\}, \min\{a_4, b_4\}). \end{cases} \tag{5}$$

Hence it follows that the above definition is equivalent to those given in the literature (see, e.g., [9, 10]):

$$\tilde{R}_1 \preceq \tilde{R}_2 \Leftrightarrow \begin{cases} \widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\} = \tilde{R}_1, \\ \widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\} = \tilde{R}_2, \end{cases} \quad \tilde{R}_1, \tilde{R}_2 \in \Psi(X). \tag{6}$$

Now we are going to introduce a metric on $\Psi(X)$ , i.e., define a distance between trapezoidal fuzzy numbers.

We say that the function $v$: $\Psi(X) \to \mathfrak{R}^+$ is *isotone valuation on* $\Psi(X)$ if

$$v\left(\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\}\right) + v\left(\widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\}\right) = v(\tilde{R}_1) + v(\tilde{R}_2) \tag{7}$$

and

$$\tilde{R}_1 \preceq \tilde{R}_2 \Rightarrow v(\tilde{R}_1) \le v(\tilde{R}_2). \tag{8}$$

The isotone valuation $v$ determines *the metric* on $\Psi(X)$:

$$\rho(\tilde{R}_1, \tilde{R}_2) = v\left(\widetilde{\max}\{\tilde{R}_1, \tilde{R}_2\}\right) - v\left(\widetilde{\min}\{\tilde{R}_1, \tilde{R}_2\}\right) \tag{9}$$

$\Psi(X)$ with isotone valuation $v$ and metric (Eq. (8)) is called a *metric space* of trapezoidal fuzzy numbers.

**Definition 3.** In the metric space, the trapezoidal fuzzy number $\tilde{R}^*$ is *the representative* of the finite collection of trapezoidal fuzzy numbers $\{\tilde{R}_j\}, \ j = \overline{1,m}, \ m = 2, 3, \ldots$ if

$$\sum_{j=1}^{m} \rho\left(\tilde{R}^*, \tilde{R}_j\right) \le \sum_{j=1}^{m} \rho(\tilde{S}, \tilde{R}_j), \quad \forall \tilde{S} \in \Psi(X) \tag{10}$$

Let us clarify the meaning of this definition. A representative of the given finite collection of trapezoidal fuzzy numbers is a trapezoidal fuzzy number such that the sum of the distances between it and all members of this collection is minimal.

For an accommodation of posterior theoretical constructions, we need to introduce a concept of *regulation* of finite collection of trapezoidal fuzzy numbers. We begin with an example.

Suppose we have the finite collection of trapezoidal fuzzy numbers:

| | $a_j$ | $b_j$ | $c_j$ | $d_j$ |
|---|---|---|---|---|
| $\tilde{R}_1$ | 7 | 7.5 | 8 | 8.8 |
| $\tilde{R}_2$ | 6 | 6.1 | 7.7 | 9 |
| $\tilde{R}_3$ | 1 | 3 | 5 | 9.5 |
| $\tilde{R}_4$ | 7.6 | 7.9 | 8.1 | 8.9 |

Compare with it the following finite collection of trapezoidal fuzzy numbers:

| | $a'_j$ | $b'_j$ | $c'_j$ | $d'_j$ |
|---|---|---|---|---|
| $\tilde{R}_1{}'$ | 1 | 3 | 5 | 8.8 |
| $\tilde{R}_2{}'$ | 6 | 6.1 | 7.7 | 8.9 |
| $\tilde{R}_3{}'$ | 7 | 7.5 | 8 | 9 |
| $\tilde{R}_4{}'$ | 7.6 | 7.9 | 8.1 | 9.5 |

We see that the matching columns in both tables consist of equal sets; at the same time the elements of the sets in the second table form nondecreasing sequences. By the regulation of the finite collection of trapezoidal fuzzy numbers $\{\tilde{R}_1, \tilde{R}_2, \tilde{R}_3, \tilde{R}_4\}$, we will mean the finite collection of trapezoidal fuzzy numbers $\left\{\tilde{R}'_1, \tilde{R}'_2, \tilde{R}'_3, \tilde{R}'_4\right\}$. The strict definition of regulation will be given below.

**Definition 4.** The finite collection of trapezoidal fuzzy numbers $\left\{\tilde{R}'_j\right\}$ is a regulation of the finite collection of trapezoidal fuzzy numbers $\{\tilde{R}_j\}$ if the finite sets $\{a_j\}$ and $\{a_j'\}$, $\{b_j\}$ and $\{b_j'\}$, $\{c_j\}$ and $\{c_j'\}$, and $\{d_j\}$ and $\{d_j'\}$ are pairwise equal and $a_1 \le a_2' \le \ldots \le a_m'$, $b_1 \le b_2' \le \ldots \le b_m'$, $c_1' \le c_2' \le \ldots \le c_m'$, $d_1' \le d_2' \le \ldots \le d_m'$, and $j = \overline{1,m}, m = 2, 3, \ldots$.

Due to this definition and Eq. (9), it is obvious that the equality

$$\sum_{j=1}^{m} \rho\left(\tilde{S}, \tilde{R}_j\right) = \sum_{j=1}^{m} \rho\left(\tilde{S}, \tilde{R}'_j\right) \tag{11}$$

holds in the metric space for any $\tilde{S} \in \Psi(X)$ and the finite collection of trapezoidal fuzzy numbers $\{\tilde{R}_j\}$, $j = \overline{1,m}, m = 2, 3, \ldots$. From Eq. (11) it follows that representatives of finite collection of trapezoidal fuzzy numbers and its regulation coincide.

It is obvious that the regulation represents a finite collection of nested trapezoidal fuzzy numbers: $\tilde{R}'_1 \preceq \tilde{R}'_2 \preceq \ldots \preceq \tilde{R}'_m$, $m = 2, 3, \ldots$.

The following theorem yields a formal definition of a representative.

**Theorem 1** [7]**.** *In the metric space of trapezoidal fuzzy numbers, the representative* $\tilde{R}^*$ *of the finite collection of trapezoidal fuzzy numbers,* $\{\tilde{R}_j\}$, $j = \overline{1,m}$, $m = 2, 3, \ldots$, *is determined as follows:*

$$\tilde{R}'_{m/2} \preceq \tilde{R}^* \preceq \tilde{R}'_{m/2+1} \text{ if } m \text{ is even}; \tag{12}$$

$$\tilde{R}^* = \tilde{R}'_{(m+1)/2} \text{ if } m \text{ is odd}. \tag{13}$$

It follows from the theorem that when the number of members in a finite collection of trapezoidal fuzzy numbers is even, a representative can take on an infinite number of values. Now we introduce the specific aggregation operator that uniquely identifies the representative (here and further on, expression $[r]$, where $r$ is a real number, denotes the integer part of this number):

$$\tilde{R}^* = \begin{cases} \left(a'_{[m/2],i} + a'_{[(m+3)/2],i}\right)/2 \ \text{if} \ \sum_{j=1}^{[(m+1)/2]} \rho\left(\tilde{R}'_j, \tilde{R}'_{[m/2]}\right) = \sum_{j=[m/2]+1}^{m} \rho\left(\tilde{R}'_j, \tilde{R}'_{[(m+3)/2]}\right), \\ a'_{[m/2],i} + \left(\frac{\sum_{j=1}^{[(m+1)/2]} \rho\left(\tilde{R}'_j, \tilde{R}'_{[m/2]}\right)}{\sum_{j=1}^{[(m+1)/2]} \rho\left(\tilde{R}'_j, \tilde{R}'_{[m/2]}\right) + \sum_{j=[m/2]+1}^{m} \rho\left(\tilde{R}'_j, \tilde{R}'_{[(m+3)/2]}\right)} \left(a'_{[(m+3)/2],i} - a'_{[m/2],i}\right)\right) \text{otherwise} \end{cases} \quad i = \overline{1,4} \tag{14}$$

**Remark 1.** It can be easily shown that the representative determined by Eq. (14) is a trapezoidal fuzzy number.

**Summary.** In this section, we presented the definitions of fuzzy sets and trapezoidal fuzzy numbers. Operations on trapezoidal fuzzy numbers are considered, and definitions of concepts that are necessary for constructing the proposed approach are given.

## 3. Credit risk assessment method

First of all, it is necessary to parameterize the risk assessment process, i.e., identify those parameters that to one degree or another affect credit risks. Such parameters may be credit history of the borrower, revenue, provision, market share, etc. The number and characteristics of risk assessment parameters are determined by an experienced lender manager.

As already mentioned in the introduction, there is always uncertainty in forecasting the values of the parameters for assessing credit risks, and, unfortunately, this fact cannot be completely avoided. An effective way out of this situation is to attract experts whose estimates are based on experience and intuition.

A very important point is the determination of a form for the submission of expert assessments. Here we present expert estimates in the form of trapezoidal fuzzy numbers. Let us justify our choice.

The expert has the opportunity to outline the following intervals (**Figure 1**):

- [a, b]—Where the parameter takes positive values, increasing from 0 to 1
- [$b$, $c$]—The confidence interval where the parameter takes values 1
- [c, d]—Where the parameter takes positive values, decreasing from 1 to 0

If, according to the expert's opinion, the parameter takes the maximum value at a single point, then $b = c$, and the estimate will take the form of a triangular fuzzy number (**Figure 2**).

If the expert is confident that the parameter reaches its maximum value in the interval [$b$, $c$], and its eastern values are nonpositive, then $a = b$, $c = d$, and the estimate takes the form of a segment of straight line $y = 1$ (**Figure 3**).

If in an extreme case the expert considers that the maximum value of the parameter is reached at a single point, and at all other points the values are zero, then $a = b = c = d$, and the graph of the membership function degenerates into the point with coordinates ($a$; 1).

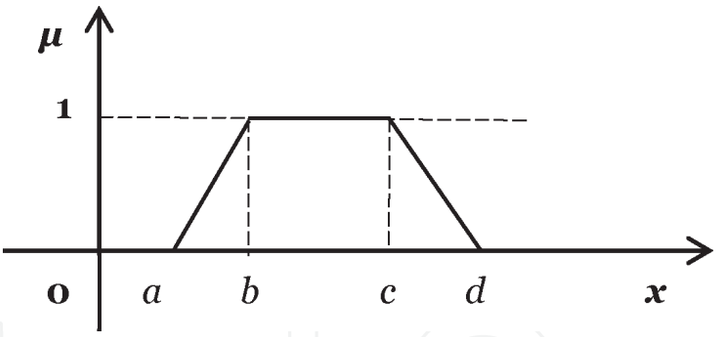



**Figure 1.**
*General form of trapezoidal fuzzy number.*

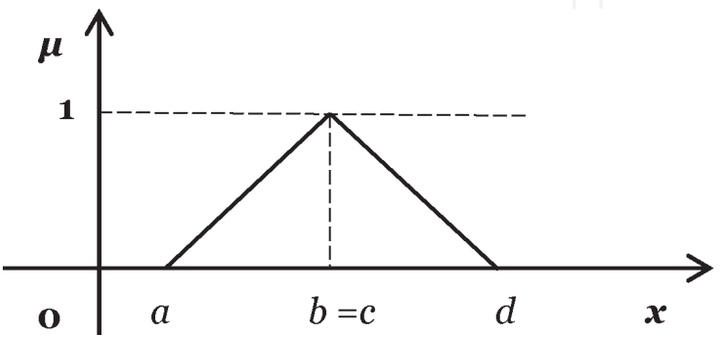



**Figure 2.**
*General form of triangular fuzzy number.*

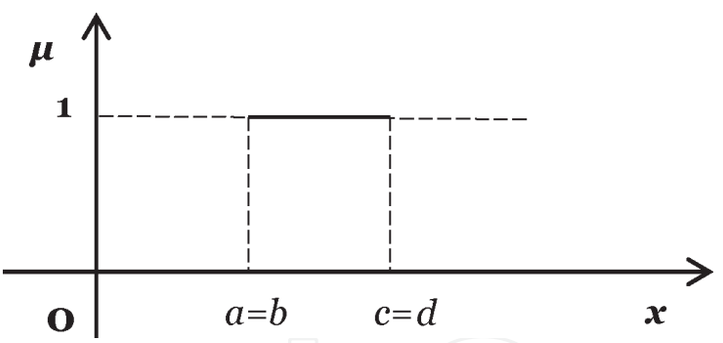



**Figure 3.**
*Segment of straight line y = 1.*

Thus, the membership function of a trapezoidal fuzzy number is common to the cases considered and can easily be reduced to particular cases.

Now we need to aggregate expert assessments for each parameter and obtain the result of group decision-making.

Suppose a group of experts estimates the rating of an alternative under some given criterion. Though the experts are professionals of the same level, their subjective estimates may be essentially different. The problem consists in processing these estimates so that a consensus could be found. In constructing any kind of aggregation method under group decision-making, the key task is to determine the well-justified weights of importance for each expert.

Let us consider the finite collection of trapezoidal fuzzy numbers formed by experts' estimates. To our mind, the representative of this collection, i.e., a trapezoidal fuzzy number such that the sum of distances between it and all other members of the given finite collection is minimal, is of particular interest.

A representative can be regarded as a kind of group consensus, but in that case the degrees of experts' importance are neglected. A representative is something like a standard for the members of the considered collection. As the weights of physical bodies are measured by comparing them with the Paris standard kilogram, it seems natural for us to determine experts' weights of importance depending on how close experts' estimates are to a representative.

Thus, the main idea of the proposed method reduces to the following. The weight of importance for each expert is determined by a function inversely proportional to the distance between his estimate and the representative of the finite collection of all experts' estimates, i.e., the smaller the distance between an expert's estimate and the representative, the larger the weight of his importance.

Let $\tilde{R}_j, j \in \{1, 2, \ldots, m\}, m = 2, 3, \ldots$ be a trapezoidal fuzzy number representing the $j$th expert's subjective estimate of the rating to an alternative under a given criterion. Estimates of all experts form the finite collection of trapezoidal fuzzy numbers $\{\tilde{R}_j\}$. By Definition 4 and formula (14), we find the regulation $\left\{\tilde{R}'_j\right\}$ and the representative $\tilde{R}^*$ of this collection. Denote the $j$th expert's aggregation weight (weight of importance) and the final result of aggregation by $\omega_j$ and $\tilde{R} = \left(\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d}\right)$, respectively.

By the above reasoning, the weights and the final result of aggregation can be defined as follows:

$$\omega_j = \frac{\left(\rho\left(\tilde{R}^*, \tilde{R}_j\right)\right)^{-1}}{\sum_{j=1}^{m}\left(\rho\left(\tilde{R}^*, \tilde{R}_j\right)\right)^{-1}}, \; m = 2, 3, \ldots \tag{15}$$

and

$$\tilde{R} = \sum_{j=1}^{m}\left(\omega_j \otimes \tilde{R}_j\right) \tag{16}$$

It is obvious that $\sum_{j=1}^{m}\omega_j = 1$. In [7] it is proved that the function in expression (15) is always continuous (the denominator does not turn into 0 in any case).

In the following proposition and its corollaries, the properties and values of the aggregation result for special cases are established.

**Proposition 1** [7]. *For any finite collection of trapezoidal fuzzy numbers* $\{\tilde{R}_j\}$, $j = \overline{1, m}, \; m = 2, 3, \ldots,$ *the following holds:*

a. $\tilde{R} = \sum_{j=1}^{m}\left(\omega_j \otimes \tilde{R}_j\right)$ *is always continuous* (*here* $\omega_j$ *is given by Eq.* (15)).

b. *If there exists at least one* $j \in \{1,2, \ldots, m\}$ *such that* $\rho\left(\tilde{R}^*, \tilde{R}_j\right) = 0$, *then* $\tilde{R} = \tilde{R}^*$.

**Corollary 1.** *If for all t,* $j \in \{1,2, \ldots, m\}$ $\tilde{R}_t = \tilde{R}_j \Rightarrow \tilde{R} = \tilde{R}^*$.

**Corollary 2.** *If the all estimates are identical then* $\omega_j = 1/m$.

**Summary**. The section introduces the approach to assessment of the credit risk. It also considers the rationale for the choice of fuzzy trapezoidal numbers as a form for the presentation of experts' estimates. The section contains important formalisms for determining the degrees of experts' importance and the result of aggregation of experts' estimates.

## 4. Realization of proposed approach

Let $m$ experts evaluate the values of $n$ parameters in the form of trapezoidal fuzzy numbers. As a result, we get a rectangular matrix of dimension $m \times n$:

$$\left(\tilde{R}_{ij}\right) = \begin{matrix} \tilde{R}_{11} & \tilde{R}_{12} & \cdots & \tilde{R}_{1n} \\ \tilde{R}_{21} & \tilde{R}_{22} & \cdots & \tilde{R}_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ \tilde{R}_{m1} & \tilde{R}_{m2} & \cdots & \tilde{R}_{mn} \end{matrix}, \quad i = \overline{1,n}, j = \overline{1,n}. \tag{17}$$

The $i$-th column of the obtained matrix represents a collection of estimates of the $j$th parameter given by the expert number $i$.

To assess the values of the parameters according to the credit risk criterion, we find a representative of each column of the matrix. As a result, we obtain the following sequence: $\left\{\tilde{R}_j^*\right\} = \tilde{R}_1^*, \tilde{R}_2^*, \ldots, \tilde{R}_n^*$.

Now we present an algorithm for finding a representative of finite collection of trapezoidal fuzzy numbers.

**Algorithm 1.**

Step 0: *Initialization: the finite collection of trapezoidal fuzzy numbers* $\left\{\tilde{R}_j\right\}$, *its regulation* $\left\{\tilde{R}_j'\right\}$, $j = \overline{1,m}$, $m = 2, 3, \ldots$. *Denote the aggregation weight of the j-th expert by* $\omega_j$ *and the final result by* $\tilde{R}$.

Step 1: *Compute the representative* $\tilde{R}^*$ *of* $\left\{\tilde{R}_j'\right\}$, $j = \overline{1,m}$, $m = 2, 3, \ldots$ *by Eq.* (14).

Step 2: *Do* Step 3 *for* $j = \overline{1,m}$.

Step 3: *Compute* $\Delta_j = \rho\left(\tilde{R}^*, \tilde{R}_j\right)$:

- If at least one $\Delta_j = 0$ then $\tilde{R} = \tilde{R}^*$;
- If $\Delta_j > 0$ for all j then compute $\omega_j$ by Eq. (15) and obtain the final result by Eq. (16).

For what follows, we need to determine a scale that can "measure" the opinions of experts regarding the risk of bankruptcy of the borrower. We use the general approach described in [11].

In almost any field, you can get a rating scale using the following principles:

a. Define a list of characteristics by which the concept (object) is evaluated.

b. Find polar characteristics in this list and form a polar scale.

c. At the poles, determine to what extent the concept possesses this characteristic (in the original, specific numerical closed intervals were used).

The collection of ratings on the scale was called the *profile* of the concept. Since the gradation values of the scale are approximate (expert opinions), with the exception of the assigned pole values, the profile represents a fuzzy set of the concept's characteristics list.

In the introduction, we mentioned the concept of a linguistic variable. This concept plays an important role in our study. Let us introduce the linguistic variable "degree of credit risk":

$$\boldsymbol{A} = \{A_1, A_2, A_3, A_4, A_5\}, \tag{18}$$

where:
$A_1$ – the degree of risk is negligible.
$A_2$ – the degree of risk is low.
$A_3$ – the degree of risk is medium.
$A_4$ – the degree of risk is high.
$A_5$ – the degree of risk is extreme.

Therefore, by constructing the linguistic variable $\boldsymbol{A}$, we have satisfied condition (a). Condition (b) is also fulfilled: the polar characteristics are "the degree of risk is negligible" and "the degree of risk is extreme." To fulfill condition (c), it is necessary to build a profile, i.e., fuzzy set describing the linguistic variable $\boldsymbol{A}$.

We construct the membership function of profile $\boldsymbol{A}$ in several stages. Here we will give a description of the stages in a general form; the reader will clarify the specifics on a practical example in the next section.

**Stage 1.** Let us evaluate the confidence of risk degrees of the linguistic variable $\boldsymbol{A}$ on the percentage scale (0–100)% as follows: $A_1 \sim [0, k_1]$, $A_2 \sim [k_1, k_2]$, $A_3 \sim [k_2, k_3]$, $A_4 \sim [k_3, k_4]$, and $A_5 \sim [k_4,100]$. Here $0 < k_1 < k_2 < k_3 < k_4 < 100$.

**Stage 2**. Since expert estimates are given in the form of trapezoidal fuzzy numbers, first of all, it is necessary to determine the boundaries of the scale of expert estimates for each characteristic of the list from $\boldsymbol{A}$. Since $m$ experts take part in the assessment process, we have $m$ trapezoidal fuzzy numbers. It seems reasonable to take the following boundaries of the scale: the left one is the minimum, and the right maximum of the abscissas of all the vertices of $m$ trapezoids, i.e.:

$$[\min\{a_i\};\ \max\{d_i\}],\ i = \overline{1,m}. \tag{19}$$

**Stage 3.** Now we will establish the conformity between the intervals of the percentage scale and the trapezoidal fuzzy numbers. Geometrically, the percentage scale that corresponds to five trapezoidal fuzzy numbers, may, for example, look like this (**Figure 4**):

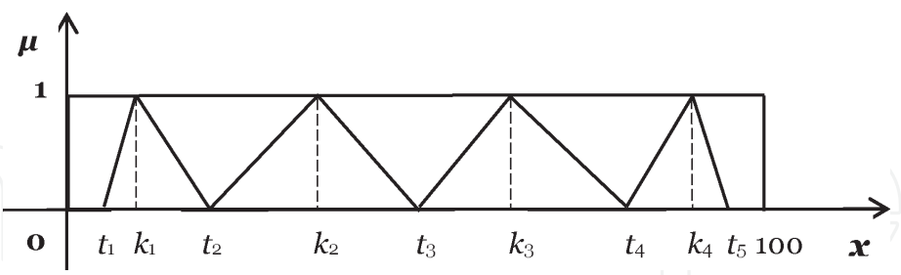



**Figure 4.**
*Conformity between the percentage scale and the trapezoidal fuzzy numbers.*

$$\tilde{A}_1 = (0, 0, k_1, t_2); \tilde{A}_{22} = (t_1, k_1, k_2, t_3); \tilde{A}_3 = (t_2, k_2, k_3, t_4); \tilde{A}_4 = (t_3, k_3, k_4, t_5);$$
$$\tilde{A}_5 = (t_4, k_4, 100, 100), \tag{20}$$

numbers $k$, $t$ are appointed by experts.

To transform the coordinate system of the percentage scale to the coordinate system for expert estimates, the following mappings should be performed $[0,100] \to [\min\{a_i\},\ \max\{d_i\}]$. Thus, we moved the origin from point (0,0) to point $(\min\{a_i\}, 0)$ and point (100,0) to point $(\max\{d_i\}, 0)$. For more simplicity let us introduce the notation:

$$\nabla = \min\{a_i\}, \Delta = \max\{d_i\},\ i = \overline{1,n}. \tag{21}$$

It is easy to see that the coefficient of proportionality between the abscissas of the primary and the new coordinate system is

$$\lambda = 0.01(\Delta - \nabla). \tag{22}$$

Thereby, the coordinates of the original trapezoidal fuzzy numbers will change as follows:

$$\begin{aligned}&\tilde{A}_1 = (\nabla, \nabla, \lambda k_1 + \nabla, \lambda t_2 + \nabla); \tilde{A}_2 = (\lambda t_1 + \nabla, \lambda k_1 + \nabla, \lambda k_2 + \nabla, \lambda t_3 + \nabla);\\ &\tilde{A}_3 = (\lambda t_2 + \nabla, \lambda k_2 + \nabla, \lambda k_3 + \nabla, \lambda t_4 + \nabla); \tilde{A}_4 = (\lambda t_3 + \nabla, \lambda k_3 + \nabla, \lambda k_4 + \nabla, \lambda t_5 + \nabla);\\ &\tilde{A}_5 = (\lambda t_4 + \nabla, \lambda k_4 + \nabla, \Delta, \Delta).\end{aligned} \tag{23}$$

So, condition (c) is also satisfied.

We continue the description of the implementation of the proposed approach. Based on Algorithm 1, we find the value of the representative of the finite collection of the trapezoidal fuzzy numbers for each parameter. Consider a representative calculated for the $i$-th parameter. The risk assessment threshold value is assigned by the manager (group of managers) of the lender. It may be that different criteria thresholds will be set for different cases, for example, for one parameter, "no more than $A_2$—the degree of risk is low," and for the other "no more than $A_3$—the degree of risk is medium."

In general, if $A_j \in \boldsymbol{A}$ (see Eq. (18)) is taken as the threshold criteria value of the parameter, then credit risk is acceptable if the following addition condition is fulfilled:

$$\tilde{R}_i \preceq \tilde{A}_j, \quad i = \overline{1,n}, \quad j = \overline{1,5}. \tag{24}$$

Here $\tilde{A}_j$ is the number corresponding to the characteristic $A_j$, while $\tilde{R}_i$ is the result of aggregation of the finite collection of expert estimates of the $i$-th parameter (see Algorithm 1).

Let us summarize the above as a generalized algorithm. So, we have the following input: $m$ expert estimates of $n$ parameters out of the set $P = \{p_i\}$, $i = \overline{1,n}$, the result of aggregation $\tilde{R}_i$ of the finite collection of expert estimates for this parameter, the threshold criteria value $A_k$, $k = \overline{1,5}$ selected from Eq. (18), and coordinates $k$, $t$ specified by the manager (group of managers) of the lender for use in Eq. (23).

**Algorithm 2.**

Step 0: *Initialization: fix* $p_i \in P, i = \overline{1,n}$, *the result of aggregation* $\tilde{R}_i$ *of the finite collection of expert estimates* $\{\tilde{R}_j\}$, $j = \overline{1,m}$, $m = 2, 3, \ldots$ *for this parameter, the threshold criteria value* $A_t, t = \overline{1,5}$, *coordinates*: $k_1, \ldots, k_4$; $t_1, \ldots, t_5$.

Step 1: *Compute* $\nabla, \Delta$ *by Eq.* (21) *and* $\lambda$ *by Eq.* (22).

Step 2: *Compute* $\tilde{A}_t, t = \overline{1,5}$ *by Eq.* (23).

Step 3: *Verification of the condition* $\tilde{R}_i \preceq \tilde{A}_j$:

- If the condition is met then the level of risk is acceptable;
- If the condition is not met then the level of risk is unacceptable.

**Summary.** The section looks at the realization of the proposed approach. The linguistic variable "degree of credit risk," polar percentage and coordinate scales are formed. The criterion for an assessment of the credit risks is generated. This section also presents two generalized algorithms for implementing the proposed approach.

## 5. Example

Here we give a toy example that will allow the reader to understand the essence of the proposed approach. To begin the practical implementation of our approach, it is necessary to determine the specific values of the isotone valuation $v$ and the metric $\rho$ (see Eq. (7)–(9)). For brevity, we denote trapezoidal fuzzy numbers by $\tilde{R} = (a_i)$. We will use the following isotone valuation $v(\tilde{R}) = \sum_{i=1}^{4} a_i$. It can be easily shown that this valuation satisfies the conditions of Eq. (7) and (8). From this it follows that distance between two trapezoidal fuzzy numbers $\tilde{R}_1, = (a_i)$ and $\tilde{R}_2 = (b_i)$ is determined as follows:

$$\rho(\tilde{R}_1, \tilde{R}_2) = \sum_{i=1}^{4} |a_i - b_i|. \tag{25}$$

Without loss of generality, we consider the process of determining the degree of risk for one parameter evaluated by three experts. For any other parameter, the procedure described below is similar. As noted above, the parameterization of the risk assessment process is carried out by the lender manager. Suppose that for evaluation the parameter "$p_1$ – revenue" has been selected.

We ask three experts to evaluate the parameter $p_1$ in the form of trapezoidal fuzzy number. As a result, we obtain

$$\{\tilde{R}_j\} = \{(1, 2, 3, 3.5), (1, 2.5, 2.8, 3), (1.5, 3, 4, 6)\} \tag{26}$$

We follow Algorithm 1, detailing it along the way.

**Algorithm 3.**

Step 0: *Initialization: the regulation of finite collection of trapezoidal fuzzy numbers* $\{\tilde{R}'_j\} = \{(1, 2, 2.8, 3), (1, 2.5, 3, 3.5), (1.5, 3, 4, 6)\}$. *Denote the aggregation weight of the j-th expert by* $\omega_j$ *and the final result by* $\tilde{R}$.

Step 1: *By Eq.* (14) *computes the representative* $\tilde{R}^* = (1.1053, 2.2105, 3.0526, 3.6315)$.

Step 2: *Do* Step 3 *for* $j = \overline{1, 3}$.

Step 3: *By Eq.* (15) *compute* $\Delta_j = \rho(\tilde{R}^*, \tilde{R}_j)$: $\Delta_1 = 0.499$, $\Delta_2 = 1.2789$, $\Delta_3 = 4.5$.

*Since all* $\Delta_j > 0$ *then by Eq.* (15) $\omega_1 = 0.666$, $\omega_2 = 0.26$, $\omega_3 = 0.074$ *and by Eq.* (16) *we obtain the finite result*:

$$\tilde{R} = (1.037, 2.204, 3.028, 3.555) \tag{27}$$

Now we form the percentage scale of the linguistic variable $\boldsymbol{A}$ (see Eq. (18)). At stage 1 of the previous section, a graduation of this scale is given in general form (see **Figure 4**). Suppose lender managers have determined the lower and upper bases of trapezoidal fuzzy numbers corresponding to the components of the linguistic variable $\boldsymbol{A}$, i.e., definition and confidence areas as a percentage:

$A_1$—the degree of risk is negligible [0, 30], [0, 20].
$A_2$—the degree of risk is low [10, 50], [20, 40].
$A_3$—the degree of risk is medium [30, 70], [40, 60].
$A_4$—the degree of risk is high [50, 90], [60, 80].
$A_5$—the degree of risk is extreme [70, 100], [80, 100].

So, $t_1 = 10$, $t_2 = 30$, $t_3 = 50$, $t_4 = 70$, $t_5 = 90$, $k_1 = 20$, $k_2 = 40$, $k_3 = 60$, and $k_4 = 80$. Thus, **Figure 4** will be converted to the form shown in **Figure 5**:

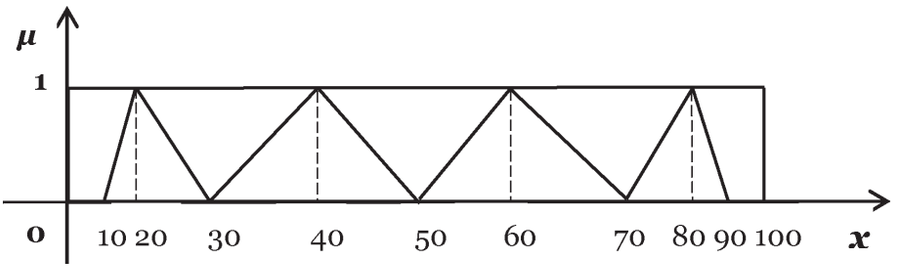



**Figure 5.**
*Graphical expression of the degree of risk in percent.*

So, we have the following input: three experts' estimates of the parameter out of the components of the linguistic variable $\boldsymbol{A}$ "revenue," the result of aggregation $\tilde{R}$ of the collection of three experts' estimates for this parameter, threshold criteria value $A_3$ selected from Eq. (18), and coordinates $k$, $t$ specified by the manager (group of managers) of the lender for use in Eq. (23).

We follow Algorithm 2, detailing it along the way.

**Algorithm 4.**

Step 0: *Initialization: the result of aggregation of the collection of expert estimates for parameter "revenue"*– $\tilde{R} = (1.037, 2.204, 3.028, 3.555)$, *threshold criteria value lender's managers "not more than medium risk", coordinates*: $10, t_2 = 30, t_4 = 70, k_2 = 40, k_3 = 60$.

Step 1: *Compute* $\nabla, \Delta$ *by Eq.* (26), *Eq.* (21) *and* $\lambda$ *by Eq.* (22), $\nabla = 1, \Delta = 6, \lambda = 0.05$.

Step 2: *Compute trapezoidal fuzzy number, corresponding to the component* $A_3$ - the degree of *risk is medium by Eq*. (23) $\tilde{A}_3 = (2.5, 3, 4, 4.5)$.

Step 3: *Verification of the condition* $\tilde{R} \preceq \tilde{A}_3$:

- the condition is satisfied and the level of risk is acceptable.

**Summary.** In this section a toy example of the practical application of the proposed approach is provided. The concrete isotone valuation and metric are considered. We calculate the risk level for one parameter based on the estimates of three experts. For other parameters and any number of experts, the process will be similar.

## 6. Conclusions

The presented work aims to propose a new approach for an assessment of the credit risks under uncertainty. The novelty of the proposed approach is the use of trapezoidal fuzzy numbers, which makes it possible to adequately form and process the experts' estimates. An important fact is that the proposed approach takes into account the degrees of experts' importance.

The main results of the work are as follows:

- A brief analysis of existing models is carried out, and the feasibility of creating a new approach is justified.
- The rationale for the presentation of experts' assessments of the credit risk in the form of trapezoidal fuzzy numbers is given.
- The linguistic variable "degree of credit risk" is formed.
- A polar percentage and coordinate scales of trapezoidal fuzzy numbers with a gradation of assigned levels are defined. The formalization of the mapping of the percentage scale to the coordinate scale is given.

- The criteria for an assessment of the credit risks are generated.
- Generalized algorithms for implementing the proposed approach are constructed.
- A toy example which illustrates the practical application of the proposed approach is provided.

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