Chapter

Introductory Chapter: Statistical and Theoretical Considerations on Magnetism in Many-Body Systems

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1. Long-range interactions

The description of systems with long-range interactions is relevant to statistical physics because we find appealing properties that deserve to be studied in detail. In this line, some variations of the Ising model that involves not only first neighbors, but also distant neighbors are employed; for instance, the Hamiltonian Mean Field (HMF), and the recently introduced dipole-type Hamiltonian Mean Field (d-HMF) model. We emphasize that the Ising model is a recurrent tool to study magnetic properties and the statistical behaviors of many-body systems in the broadest context [1–5].

In concern about long-range interactions, there are various challenges to face; these are related to the dynamics, size of the systems, and the theoretical framework to explain the behavior of systems, among others. In this type of system, we have typical consequences such as the loss of additivity and extensivity. The loss of additivity takes place for ensembles of interacting particles that cannot trivially separate into independent subsystems, which is explained by the presence of underlying interactions or correlation effects, whose characteristic lengths are comparable or more significant than the system linear size [6–8]. Additionally, the loss of extensivity frequently accompanies the loss of additivity [5, 9].

If we have a system composed of N spins that interact one to each other as a power law of inter-particle distance, the total energy of the system is E. The system divided into two subsystems, 1 and 2, both composed of $N/2$ particles with energies E_1 and E_2, respectively. If the spins in subsystem 1 are up, and those in the subsystem 2 are down, the energies satisfy $E_1 = E_2$. Since the sum of energies, $E_1 + E_2$ is not equal to the total energy E; the system is nonadditive. In general, $E < E_1 + E_2$. Besides, the extensivity ($E/N \rightarrow$constant) is another fundamental property to consider in this kind of analysis. The recent literature shows a way to recover this property through Kac's prescription that has a standard thermodynamic structure because it preserves the Euler and Gibbs-Duhem relations. Therefore, this procedure allows us to recover the extensivity, while the loss of additivity remains because of the long-range interactions [5, 9].

At equilibrium, an analytical procedure leads to solving the problem for obtaining the magnetization, the inverse temperature, the specific heat, in the canonical ensemble. Also, it is possible to get the microcanonical entropy. The caloric curve, ascertained using both, exactly coincides. At this stage, we emphasize that the solution to this system with long-range interactions becomes analytical, which solutions are not abundant in statistical physics. Nevertheless, out of equilibrium, we have data from simulations obtained from carrying out molecular dynamics.

The evolution of simulations permits us to observe several properties that are not possible to identify with any theoretical description. The interpretation comes from perceiving regularities in the dynamics of systems as HMF and d-HMF models.

2. Equilibrium

As generally noticed, at low energy, a phase of a single cluster of particles appears; at high energy, a homogeneous phase emerges.

For theoretical and numerical modeling, we consider a system of N identical, coupled, dipole-type particles with a mass equal to 1. The dynamics evolves in a periodic cell described by a one-dimensional, dipole-type HMF model (d-HMF) [9, 10] given by

$$H = \frac{1}{2}\sum_{i=1}^{N} p_i^2 + \frac{\lambda}{2N}\sum_{i\neq j}^{N}\left[\cos\left(\theta_i - \theta_j\right) - 3\cos\theta_i\cos\theta_j - \Delta_{i,j}\right], \tag{1}$$

where the variable p_i is the momentum of the particle i, and θ_i is its corresponding angle of orientation (integer $i \in [1,N]$ for the system size N). The parameter λ stands for the coupling and $\Delta_{i,j}$ suitably denotes the zero of the potential.

The equations of motion are derived by the following set:

$$p_i = \frac{-\varepsilon}{2}\left(2M_x \sin\theta_i - M_y\cos\theta_i\right), \tag{2}$$

where the components of the magnetization vector are defined as

$$\left(M_x, M_y\right) = \frac{1}{N}\sum_{i\neq j}^{N}\left(\cos\theta_i, \sin\theta_i\right). \tag{3}$$

After deriving thermodynamic properties in both, the canonical and microcanonical ensembles, the caloric curves come from the following coincident result:

$$2\beta = \frac{1}{\left(\varepsilon + \lambda\left(m^2 + \Delta/2\right)\right)}, \tag{4}$$

combined with the extremal solution

$$m = \frac{I_1\left(2\beta\lambda m\right)}{I_0\left(2\beta\lambda m\right)}, \tag{5}$$

where $I_k(\ldots)$ is the kth-order modified Bessel function, β is the inverse of temperature, ε is the energy per particle; m is the magnetization per particle. We analytically obtain the parameters that define the critical point, such as the critical inverse temperature $\beta_c = 1$, critical internal energy $\varepsilon_c = 3/2$, and critical magnetization $m_c = 0$. The specific heat diverges with the temperature $1/\beta \to 1^-$ and remains in a constant value for $1/\beta > 1$, which corresponds to an ideal gas in one dimension. Therefore, systems with magnetic behavior are recently studied in [11, 12], giving an approach to the understanding of magnetic materials.

As said before, this system denotes an analytically solvable many-body problem. This type of solutions is not common in physics and particularly in statistical mechanics, especially regarding long-range interactions [9]. Baxter gives a comprehensive overview in his book [13]. The spherical model is an additional solvable problem that includes long-range interactions.

3. Quasi-stationary states

The statistical mechanics thoroughly explains the equilibrium, which is the statistical state reached in the long-lasting evolution. Nevertheless, in intermediate intervals of the evolution time, the dynamics is abnormal, and the description is not complete. Therefore, there are at least two regimes in the evolution of systems with long-range interactions: the equilibrium and the quasi-stationary states (QSS). In the first case, we have the Boltzmann-Gibbs statistics to describe the state of the systems entirely. In the second, it is an open problem that possesses several theoretical attempts susceptible to improve them.

Generally speaking, when the modeling neglects all of the spatial structure of the system but considers long-range interactions, mean-field approximations should be good alternatives to consider. The d-HMF model is curious because it neglects the structure of systems and space interactions, and only keeps the part related to orientations, but shows axiomatic and pertinent properties.

To identify the QSS, we search for intervals where the thermodynamic values keep constant. Therefore, **Figure 1(a)** illustrates two plateaus in the behavior of magnetization per particle as a function of time, before equilibrium. A complementary perspective of the first QSS given in [10] considers the shape of the mean kinetic energy per particle, aiding us to obtain the power-law duration depicted in **Figure 1(b)** that characterizes the nature of QSS. States defined by the plateaus of mean kinetic energy per particle are lower than the canonical temperature, whose values only coincide in equilibrium [10].

Finally, this system constitutes an example of a non-symmetric HMF that shows a phase transition, the appearance of a spontaneous magnetic ordering. The symmetry is compared to the HMF model that remains invariant under a typical rotational transformation, which is not valid for the d-HMF model, but presents a second-order phase transition; therefore, a possible application found in the literature is the phase transition of non-symmetric spin glasses. Theoretical background for systems out of equilibrium is not unique and fundamental questions related

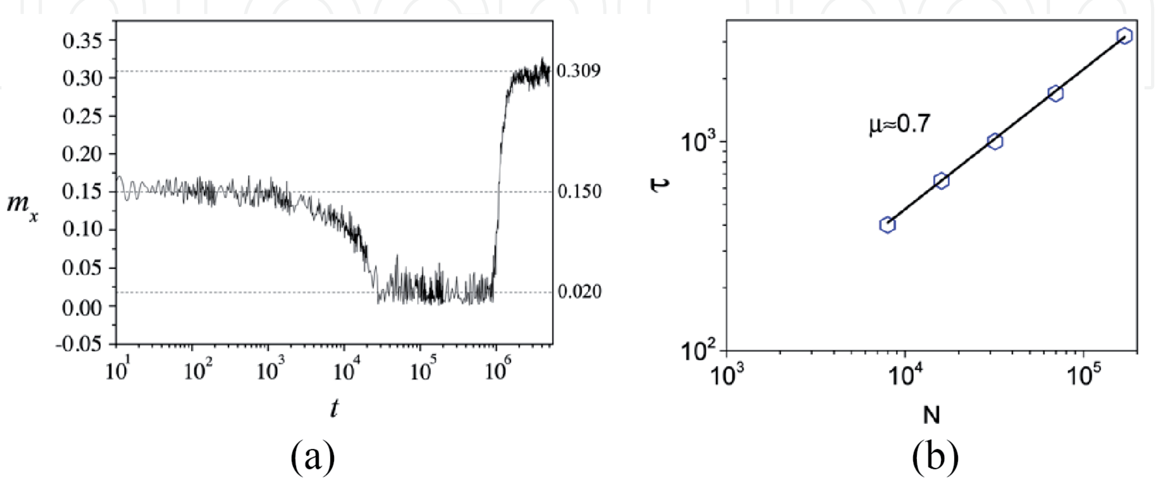

Figure 1.
The magnetization per particle is depicted as a function of the time in (a). In (b), the relaxation time τ, of the first QSS that goes to the second QSS, is depicted as a function of 1/N in log–log scale, whose power law of the duration of the first QSS in terms of the system size is obtained from the behavior of the kinetic energy.

to this issue are still open; a possible description based on a family of solutions of Vlasov equation is in progress [14].

The understanding of these thermodynamic properties can be fundamental in future demands as the manufacturing of magnetic instruments, sensors, and magnetometers.

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