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Estimation of Mean Response Time of Multi-Agent Systems Using Petri Nets

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1. Introduction

Performance analysis of multi-agent system can be done by experiments with real system, simulation or analytic methods. Now, multi-agent technologies, e.g., (Deloach et al., 2001; JADE), are often based on Unified Modeling Language (UML) (Booch et al., 1999; UML, 2007) or its modifications. The following analytical approaches: queuing network models (Kahkipuro, 1999), stochastic automata networks (Steward et al., 1995), stochastic Petri nets (King & Pooley, 1999), stochastic process algebra (Pooley, 1999), Markov chains can be used in performance evaluation of multi-agent systems.

In this chapter, an analytical approach, which is based on Petri nets, is developed. This approach is applied to performance evaluation of layered multi-agent system. These layers are associated with the following types of agents: manager, bidder, and searcher ones. Time-out mechanisms are used in communication between agents. Our method is based on approximation using Erlang distribution. Erlang distributions create the family of distributions with different number of stages. In the paper (Babczyński & Magott, 2006a), an approximation method which is based on Erlang distribution has been applied for the above layered multi-agent system. In that paper, there was no bounds for time of waiting for messages from the agents. In present chapter, time-out mechanisms are used in communication between the agents. The chapter is an extension of the paper (Babczyński & Magott, 2006b) where PERT based approach was presented. Accuracy of our approximation method is verified using simulator. This simulator has been previously used in simulation experiments with the following multi-agent systems: personalized information system (Babczyński et al., 2004a), industrial system (Babczyński et al., 2004b), system with static agents and system with mobile agent (Babczyński et al., 2005). These systems have been expressed in standard FIPA (FIPA) which the JADE technology (JADE) is complied with.

The chapter is organized as follows. In section 2, the multi-agent system is described. Then our approximation method is presented. In section 4, accuracy of our approximation method is verified by comparison with simulation results. Finally, there are conclusions.

2. Layered multi-agent system

We consider the layered multi-agent information retrieval (MAS) system given at Fig. 1. The MAS includes: one manager type agent (MTA) as Fat Agent, two bidder type agents (BTAs) as Thin Agents, and searcher type agents (STAs) as Thin Agents. One BTA co-operates with a number of STAs.

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Fig. 1. Layered multi-agent information retrieval system

After receiving a request from an user, the *MTA* sends messages to the *BTAs* in order to inform them about the user’s request. Then the timer of the *MTA* is started, and the *MTA* is waiting for two responses from the *BTAs*. The waiting time is limited by the termination time *tm*. Having two responses from the *BTAs*, the *MTA* prepares the response for the user. If the maximal waiting time *tm* has elapsed then the *MTA* prepares the response for the user having information received from the *BTAs* until the *tm* has elapsed. In this case, the *MTA* has the response from one *BTA* or it has no response.

After receiving a request from the *MTA*, the *BTA* sends messages to all *STAs* co-operating with this *BTA*. Then the timer of the *BTA* is started, and the *BTA* is waiting for responses from all its *STAs* but no longer than the termination time *tb*. Having responses from all its *STAs*, the *BTA* prepares the response for the *MTA*. If the maximal waiting time *tb* has elapsed then the *BTA* prepares the response for the *MTA* having information received from the *STAs* until the *tb* has elapsed. In this case, the *BTA* has less responses from the *STAs* than the number of its *STAs*.

Fig. 2. Petri net model of layered multi-agent information retrieval system

The *STA* prepares the response by the Data Base (*DB*) searching. Each *STA* is associated with one *DB*. The probability of finding the response in the *DB* is denoted by *f_rate*. Time

unit is second, and it will be omitted. If there is required information in the DB, then searching time is expressed by uniform distribution over the time interval $[0, b)$. Hence, the expected searching time, provided there is the required information in the DB, is equal to $b/2$. Searching time is equal to b with the probability $1-f_rate$.

Message transmission times between the MTA and the BTA, and between the BTA and the STA are given by n stage Erlang distributions with parameter λ for each stage. Each stage is described by exponential distribution with this parameter. Random variable expressed by Erlang distribution is denoted by $E_{n,\lambda}$. Exponential distribution is a special case of Erlang distribution, i.e., $E_{1,\lambda}$. In examination of the accuracy of our approximation method, it will be assumed that $n=2, \lambda=1$.

In Fig. 2, rectangles represent timed transitions with non-zero firing time, while dashes represent immediate transitions with zero firing time. Immediate transitions have higher priority than timed transitions. Firing of transition t1 illustrates that the MTA sends messages to the BTAs. Presence of tokens in places p4 and p5 denotes that the timer for termination time t_m has been started. If there are tokens in all input places of transition t2 during time interval of length t_m , then this transition can be fired in last time instant of this interval. Firing time of transition t3 expresses transmission time of message from the MTA to a BTA. Firing of transition t4 indicates that the BTA has send m messages to m STAs. Firing time of t6 is equal to transmission time of the message from the BTA to a STA. From the other side, firing of transition t4 causes that there are tokens in all input places of transition t5. The time t_b is termination time associated with the BTA. If there are tokens in all input places of transition t5 during time interval of length t_b , then this transition can be fired in last time instant of this interval. If there is a token in place p10, then transitions t7, t8, respectively, are fired with probabilities $f_rate, 1-f_rate$, respectively. Transitions t9, t10, respectively, illustrates STA searching process in DB, provided required information is found, is not found, respectively. Transition t11 expresses transmission of message from the STA to the BTA. m tokens in place p14 shows that m messages from the STAs have been received by the BTA. There are races of transitions t5, t13 to be fired. If a token is added to place p15 earlier than the firing time t_b has elapsed, then transition t13 is fired. Transition t15 is connected with transmission of message from the BTA to the MTA. Firing of transition t16 denotes that the MTA has received all messages from all BTAs. The token in place p5 is engaged in races of transitions t2, t17 to be fired. A token in place p21 indicates that the user has received a response.

3. Erlang distribution based approximation method

Now we will explain how the expected value of time of the response to the users request is approximated. First, we show what kind of operations will be considered. Then, we recall the functions used in further part of the section. Next, the method of the approximation will be shown. Finally, the approximation of the mean response time of the MAS will be given.

3.1 Operations on random variables

In the chapter, the approximation of the following operations will be described. In all cases we assume the independence of random variables.

Sum of m random variables. For RVs X_1 to X_m we introduce the *Sum* operation.

$$Sum(X_1, \dots, X_m) = X_1 + \dots + X_m \quad (1)$$

Because the RVs X_1 to X_m are independent, the following equations, for expected values (E) and variations (Var) of the RVs, are true.

$$\begin{aligned}
 E(\text{Sum}(X_1, \dots, X_m)) &= \sum_{k=1}^m E(X_k) \\
 \text{Var}(\text{Sum}(X_1, \dots, X_m)) &= \sum_{k=1}^m \text{Var}(X_k)
 \end{aligned}
 \tag{2}$$

Maximum of m identically distributed RVs. For m independent RVs X_1 to X_m we define the *Max* operation.

$$\text{Max}(X_1, \dots, X_m) \tag{3}$$

The cumulative distribution function (CDF) is given by the formula:

$$F_{\text{Max}(X_1, \dots, X_m)}(t) = (F_{X_k}(t))^m \tag{4}$$

where $F_{X_k}(t)$ is the common CDF of RVs $X_1, \dots, X_m$.

Cut-off by time-out event. For the RV X and time-out T we have the *Tout* operation.

$$\text{Tout}(X, T) \tag{5}$$

The CDF is given by the formula:

$$F_{\text{Tout}(X, T)}(t) = \begin{cases} F_X(t) & \text{for } t < T \\ 1 & \text{for } t \geq T \end{cases} \tag{6}$$

where $F_X(t)$ is the CDF of the RV X .

Approximation operation. Additionally, we define an *Apx* operation, which stands for the approximation of a RV by the Erlang distributed one.

$$\text{Apx}(X) = E_{n, \lambda} \tag{7}$$

where $E_{n, \lambda}$ is the Erlang distributed RV with n stages and parameter λ for each stage.

This approximation will be used for RVs resulted from the *Sum* and the *Max* operations. The details of the approximation will be shown in section 3.4.

3.2 Gamma functions

In the below described approximation, the Γ function will be used. Now we recall some equations and facts associated with this function (MathWorld). The (complete) Γ function for real value of p is defined by the following integral.

$$\Gamma(p) \equiv \int_0^\infty x^{p-1} e^{-x} dx \tag{8}$$

Two incomplete functions are also defined, the upper incomplete Γ function and the lower incomplete γ function.

$$\Gamma(p, \lambda t) \equiv \int_{\lambda t}^\infty x^{p-1} e^{-x} dx, \quad \gamma(p, \lambda t) \equiv \int_0^{\lambda t} x^{p-1} e^{-x} dx \tag{9}$$

The following obvious equation is true for the all values of p and λ, t .

$$\Gamma(p) = \Gamma(p, \lambda t) + \gamma(p, \lambda t) \quad (10)$$

When the parameter $p=n \in \mathbb{N}$, the Γ function reduces to the factorial.

$$\Gamma(n) = (n-1)! \quad (11)$$

As for the Γ function from the equation 5, the incomplete functions can also be reduced to elementary functions for the parameter $p=n \in \mathbb{N}$.

$$\begin{aligned} \Gamma(n, \lambda t) &= (n-1)! e^{-\lambda t} \sum_{k=0}^{n-1} \frac{(\lambda t)^k}{k!} \\ \gamma(n, \lambda t) &= (n-1)! \left(1 - e^{-\lambda t} \sum_{k=0}^{n-1} \frac{(\lambda t)^k}{k!} \right) \end{aligned} \quad (12)$$

3.3 Erlang distribution

Some probability distributions of time characteristics are approximated by Erlang distribution.

The probability density function and the CDF of Erlang distribution with n stages and with parameter λ are given (MathWorld) by expressions:

$$f_{E_{n,k}}(t) = \frac{\lambda^n t^{n-1} e^{-\lambda t}}{(n-1)!}, \quad F_{E_{n,k}}(t) = \frac{\gamma(n, \lambda t)}{\Gamma(n)} = 1 - \sum_{k=0}^{n-1} \frac{\lambda^k t^k e^{-\lambda t}}{k!} \quad (13)$$

The random variable (RV) with this distribution will be denoted by $E_{n,\lambda}$. This RV can be interpreted as sum of n RVs with exponential distribution and each with parameter λ . The expected value and the variance for this RV are equal to $E(E_{n,\lambda}) = n/\lambda$ and $Var(E_{n,\lambda}) = n/\lambda^2$, respectively.

For the RV X (with any distribution), the squared coefficient of variation (SCV) of the X is defined by the formula:

$$SCV(X) \equiv \frac{Var(X)}{E(X)^2} \quad (14)$$

where: $E(X)$ is the expected value of X , $Var(X)$ is the variance of X . The SCV for the $E_{n,\lambda}$ is equal to $SCV(E_{n,\lambda}) = 1/n$.

3.4 Two moments approximation

In the Section 3.1, the $Apx(X)$ operation was introduced. The result of the operation is the Erlang distributed random variable $E_{n,\lambda}$. In order to determine the parameters of demanded Erlang distribution, the following procedure can be used.

1. For the RV X under approximation, calculate the moments

$$E(X), Var(X) \text{ and the coefficient } SCV(X)$$

2. Determine the number n of stages of the Erlang distribution

$$n = \text{round}(1/SCV(X))$$

3. Obtain the λ parameter from the equation:

$$\lambda=n/E(X)$$

While the last two steps of the procedure are common for all distributions of approximated RVs, the first one is different for RVs resulted from operations *Sum* and *Max*, introduced in the Section 3.1.

Sum($X_1,\cdots,X_m$) The moments are calculated from the formulae 2.

Max($X_1,\cdots,X_m$) In this case, the moments are calculated by numerical integrating the following formulae (Abdelkader, 2003)

$$\begin{aligned} E(M) &= \int_0^\infty \left(1 - (F_{X_k}(t))^m\right) dt \\ Var(M) &= \left(2 \int_0^\infty t \left(1 - (F_{X_k}(t))^m\right) dt\right) - E(M)^2 \end{aligned} \tag{15}$$

where: $F_{X_k}(t)$ is the common CDF of RVs $X_1, \dots, X_m$ as in the formula 2, M is the shortcut for $Max(X_1,\cdots,X_m)$.

This operation will be executed on the Erlang distributed RVs obtained as a result of the approximation.

3.5 Example

Now, we show how to apply the approximation method to the layered multi-agent system described in the section 2.

The timed model of the system can be written using the operations defined in the section 3.1. First, we suppose that time-out mechanism is not used.

Fig. 3. Petri net of the first analysis step

Let us consider the subnet contained in dashed part of Fig. 3. It represents the RV of the STA searching time in the DB denoted by U_{b,f_rate} . This RV has the probability density function:

$$f_{U_{b,f_rate}}(t) = \begin{cases} f_rate \cdot 1/b & \text{for } t \in [0, b) \\ (1 - f_rate) \cdot \delta(t - b) & \text{for } t = b \\ 0 & \text{otherwise} \end{cases} \tag{16}$$

where $\delta(t-b)$ is the Dirac delta distribution in point b .
Expected value, variance, and SCV for this RV are given by the following expressions:

$$\begin{aligned} E(U_{b,f_rate}) &= \frac{b(2 - f_rate)}{2} \\ Var(U_{b,f_rate}) &= \frac{b^2 f_rate(4 - 3f_rate)}{12} \\ SCV(U_{b,f_rate}) &= \frac{f_rate(4 - 3f_rate)}{3 \cdot (2 - f_rate)^2} \end{aligned} \tag{17}$$

Fig. 4. Petri net of the second analysis step
Let us consider the subnet contained in dashed part of Fig. 4. It illustrates the probability distribution of the RV X of the length of the time interval between the time instant when the BTA sends the request to given STA and the time instant when the BTA receives the response from this STA. This RV is given by the expression:

$$X = Sum(E_{n,\lambda}, U_{b,f_rate}, E_{n,\lambda}) \tag{18}$$

We suppose that RVs of the transmission times between agents and RVs of the searching processes in the DBs are independent. Hence, according to the expressions 2, the expected value, the variance, and the SCV for the RV X are expressed by the following formulae:

$$\begin{aligned} E(X) &= 2 \frac{n}{\lambda} + b - \frac{1}{2} b \cdot f_rate \\ Var(X) &= 2 \frac{n}{\lambda^2} + \frac{1}{3} b^2 \cdot f_rate - \frac{1}{4} b^2 \cdot f_rate^2 \\ SCV(X) &= \frac{24n + (4f_rate - 3f_rate^2)\lambda^2 b^2}{3(4n + (2 - f_rate)\lambda b)^2} \end{aligned} \tag{19}$$

For analysed multi-agent system, the RVs of the transmission times between the agents are two stage Erlang distributions with the parameter $\lambda=1$ for each stage, and will be denoted by $E_{2,1}$.
The RV X is approximated by the RV:

$$E_{n_X,\lambda_X} = Apx(X)$$

(20)

using the procedure described in the section 3.4.

Fig. 5. Petri net of the third analysis step

Let us consider the subnet contained in dashed part of Fig. 5. It models m STAs associated to one BTA. Let $(E_{n_X,\lambda_X})_k$ be such a RV E_{n_X,λ_X} that approximates the length of the time interval between the time instant when the BTA sends the request to k^{th} STA and the time instant when the BTA receives the response from this STA. Equal values of mean completion times of each sequence: transmission from the BTA to the STA, searching in the DB, transmission from the STA to the BTA has been selected because this strategy usually gives the greatest error. In this case, the RV Y of the BTA waiting time for all responses from the STAs is

$$Y = Max((E_{n_X,\lambda_X})_1, \dots, (E_{n_X,\lambda_X})_m).$$

(21)

The CDF and the probability density function of the RV Y are given by the expressions:

$$F_Y(t) = \left(\frac{\gamma(n,\lambda t)}{(n-1)!} \right)^m$$
$$f_Y(t) = \frac{m t^{n-1} e^{-\lambda t} \lambda^n (\gamma(n,\lambda t))^{m-1}}{((n-1)!)^m}$$

(22)

The RV Y is now approximated by the RV

$$E_{n_Y,\lambda_Y} = Apx(Y)$$

(23)

using the procedure described in the section 3.4. The moments needed in the procedure are calculated using the formulae 15.

Let us consider the subnet contained in dashed part of Fig. 6. It represents such a situation that the *BTA* waits for the responses from the *m* *STAs* but not longer than for the termination time *tb*, i.e., time-out mechanism is applied. Therefore, we analyse the RV E_{n_Y, λ_Y} truncated in the *tb*. This RV will be denoted by:

Fig. 6. Petri net of the fourth analysis step

The CDF and the k^{th} moment of the RV W are given by the expressions:

$$F_W(t) = \begin{cases} 0 & \text{for } t < 0 \\ F'_W(t) & \text{for } 0 \leq t \leq tb \\ 1 & \text{otherwise} \end{cases}$$

where $F'_W(t) = \frac{\gamma(n_Y + 1, \lambda_Y t) + \lambda_Y^{n_Y} t^{n_Y} e^{-\lambda_Y t}}{n_Y!}$.

$$\mu_W^{(k)} = \frac{\gamma(n_Y + k, \lambda_Y \cdot tb)}{(n_Y - 1)! \lambda_Y^k} + \frac{tb^k \Gamma(n_Y, \lambda_Y \cdot tb)}{(n_Y - 1)!}$$
$$E(W) = \mu_W^{(1)}; \quad Var(W) = \mu_W^{(2)} - (\mu_W^{(1)})^2$$

$$\tag{25}$$

The RV W is not approximated.

Let us consider the subnet contained in dashed part of Fig. 7. It models the RV of the length of the time interval between the time instant when the *MTA* sends the request to given *BTA* and the time instant when the *MTA* receives the response from this *BTA* is denoted by the RV:

$$Z = Sum(E_{2,1}, W, E_{2,1}) \tag{26}$$

Fig. 7. Petri net of the fifth analysis step

The expected value of the time of receiving of a response by the *MTA* (or user), i.e. response time, is approximated in the similar way as the expected value of the *RVs* *X* and *Y* have been approximated giving the *RV*:

$$E_{n_Z, \lambda_Z} = \text{Apx}(Z) .$$

(27)

Fig. 8. Petri net of the sixth analysis step

Let us consider the subnet contained in dashed part of Fig. 8. It expresses two *BTAs* that are associated with the *MTA*. In this case, the *RV* *Q* of the *MTA* waiting time for all responses

from *BTAs* is:

$$Q = \text{Max}((E_{n_Z,\lambda_Z})_1,(E_{n_Z,\lambda_Z})_2) \; .$$

(28)

The expected value and the variance of the *RV* Q are calculated from the formulae 15.
The *RV* Q is now approximated by the *RV*:

$$E_{n_Q,\lambda_Q} = \text{Apx}(Q)$$

(29)

using the procedure described in the section 3.4.

Fig. 9. Petri net of the seventh analysis step

Let us consider the subnet contained in dashed part of Fig. 9. It models that the *MTA* waits for the responses from two *BTAs* not longer than for the termination time tm . Therefore, we truncate the *RV* E_{n_Q,λ_Q} in the tm . This *RV* will be denoted by:

$$R = \text{Tout}(E_{n_Q,\lambda_Q},tm) \; .$$

(30)

This *RV* is not approximated, we calculate the expected value $E(R)$ in similar way as $E(W)$ has been computed from equations 25.

Fig. 10. The final Petri net

Summing it up, we can write our example model as:

$$\begin{aligned} X &= Sum(E_{2,1}, U_{b,f_rate}, E_{2,1}) \\ Y &= Max(Apx(X_1), \dots, Apx(X_m)) \\ W &= Tout(Apx(Y), tb) \\ Z &= Sum(E_{2,1}, W, E_{2,1}) \\ Q &= Max(Apx(Z_1), Apx(Z_2)) \\ R &= Tout(Apx(Q), tm) \end{aligned}$$

(31)

4. Accuracy of the approximation method

In order to evaluate the accuracy of the approximation method, the simulation for: the MAS containing m STAs for each BTA, where $m=3,10$, have been performed. For each MAS, the following values of $f_rate=0.1, 0.3, 0.6$, and 0.9 have been considered. The transmission time between agents is given by the RV $E_{2,1}$. Hence, the mean transmission time between the agents is equal to $E(E_{2,1}) = 2$, and the mean transmission time in both directions is equal to $E(tr) = 4$.

In Table 1, the percentage errors of the mean response time for: the maximal searching time in the DB equal to $b=16$, the termination times $tb=20, tm=27$ are given. First, let us suppose that time-out mechanism is not considered. The RV of the length of time interval between the time instant when the BTA sends the message to given STA and time interval when the BTA receives the response from this STA is $X_{16,f_rate} = Sum(E_{2,1}, U_{16,f_rate}, E_{2,1})$. The expected value of the RV U_{b,f_rate} is equal:

$$E(U_{b,f_rate}) = f_rate \cdot b / 2 + (1 - f_rate) \cdot b$$

(32)

Hence, $E(U_{16,0.9}) = 8.8$, $E(U_{16,0.1}) = 15.2$, and as a result $E(X_{16,0.9}) = 12.8$, $E(X_{16,0.1}) = 19.2$. Now let us consider the MAS with the time-out mechanisms. Therefore, $tb \approx E(X_{16,0.1})$.

In spite of $b / E(E_{2,1}) = 8$ and $2E(tr) < E(U_{16,0.9})$, i.e., the uniform distribution of the RV of the searching time is dominating the Erlang distribution of the RV of the transmission times, the Erlang distribution based approximation is very good (maximal error in the Table 1 is equal to 2.6%).

$m \backslash f_rate$	0.1	0.3	0.6	0.9
3	0.7%	-0.4%	-2.5%	-2.6%
10	1.4%	1.3%	0.7%	-0.1%

Table 1. Percentage errors of the mean response time for: the maximal searching time $b=16$, the termination times $tb=20, tm=27$

In Table 2, the percentage errors of the mean response time for: the maximal searching time $b=16$, the termination times $tb=15, tm=20$ are given. In this case: $1.3 \cdot tb \approx E(X_{16,0.1})$. The

approximation error is smaller than that for the $tb=20$ (Table 1).

$m \backslash f_rate$	0.1	0.3	0.6	0.9
3	0.14%	0.17%	-0.19%	-0.80%
10	0.22%	0.08%	-0.28%	0.04%

Table 2. Percentage errors of the mean response time for: the maximal searching time $b=16$, the termination times $tb=15$, $tm=20$

In Table 3, the percentage errors of the mean response time for: the maximal searching time $b=16$, the termination times $tb=30$, $tm=40$ are given. In this case: $1.5 \cdot E(X_{16,0.1}) \approx tb$. Now approximation error is clearly greater than in previous two tables.

$m \backslash f_rate$	0.1	0.3	0.6	0.9
3	7.3%	6.4%	4.3%	2.5%
10	11.7%	10.9%	12.1%	7.7%

Table 3. Percentage errors of the mean response time for: the maximal searching time $b=16$, the termination times $tb=30$, $tm=40$

In table 4, the percentage errors of the mean response time for: the maximal searching time $b=32$, the termination times $tb=38$, $tm=45$ are given. The expected values of analysed RVs are the following: $E(U_{32,0.9})=17.6$, $E(U_{32,0.1})=30.4$, and as a result $E(X_{32,0.9})=21.6$, $E(X_{32,0.1})=34.4$. Hence, $1.1 \cdot E(X_{32,0.1}) \approx tb$. Additionally, $b/E(E_{2,1})=16$. In this case, the uniform distribution of the RV of the searching time dominates the Erlang distribution of the RV of the transmission times stronger than for the Tables 1 and 2. The approximation is still very good.

$m \backslash f_rate$	0.1	0.3	0.6	0.9
3	0.5%	-0.7%	-3.1%	-2.5%
10	0.9%	0.7%	0.6%	1.0%

Table 4. Percentage errors of the mean response time for: the maximal searching time $b=32$, the termination times $tb=38$, $tm=45$

In Table 5, the percentage errors of the mean response time for: the maximal searching time $b=32$, the termination times $tb=380$, $tm=450$ are given. In this case, $b/E(E_{2,1})=16$. The approximation errors are much greater than previously. However, it is not realistic choice of parameters, because the termination time tb is more than 10 times greater than the mean

time of the RV $X_{32,0.1}$.

$m \backslash f_rate$	0.1	0.3	0.6	0.9
3	17.2%	11.9%	9.8%	7.6%
10	28.2%	22.3%	24.6%	21.0%

Table 5. Percentage errors of the mean response time for: the maximal searching time $b=32$, the termination times $tb=380$, $tm=450$

5. Conclusions

The approximation method of the mean response time for layered multi-agent system has been presented. This system has three layers of agents, namely, manager, bidder, and searcher type ones denoted by abbreviations *MTA*, *BTA*, *STA*. After receiving a request from an user, the *MTA* sends the messages to the *BTAs* in order to inform them about the user request. After receiving a request from the *MTA*, the *BTA* sends the messages to all *STAs* co-operating with this *BTA*. In the communication, the time-out mechanisms are used. The *STA* prepares the response for the *BTA* by the Data Base (*DB*) searching. The probability of finding the response in the *DB* is denoted by *f_rate*. Searching time is expressed by uniform distribution over the time interval $[0,b)$. Message transmission times between the *MTA* and the *BTA* and between the *BTA* and the *STA* are given by the random variables (*RVs*) of n stage Erlang distribution. In verification of accuracy of the method by simulation experiments, the transmission times have been expressed by the *RV* of two-stage one. In the approximation method, the *RV* with n stage Erlang distribution is used. It has been obtained from the simulation, that the sum of the *RV* of the Erlang distribution (representing the transmission time) and the *RV* of searching time with uniform distribution can be approximated by the other *RV* of Erlang distribution with suitable number of stages. This is true even if the expected value of the *RV* of the uniform distribution is clearly greater than the expected value of the *RVs* of the transmission times. Let us analyse the *RV* of the length of the time interval between the time instant when the *BTA* sends the message to given *STA* and the time instant when the *BTA* receives the response from this *STA*. Let this *RV* be denoted by X . If the maximal time when the *BTA* is waiting for the responses from the *STAs* (termination time) tb is equal or smaller than the expected value $E(X)$ of the *RV* X then the approximation is very good. For the analysed cases, the maximal error is 3.1%. For the performed simulation experiments, if the tb is approximately equal to $1.5 \cdot E(X)$ then the maximal approximation error is about 12%. If the tb is about 10 times greater than the $E(X)$ then, for analysed cases, the approximation error is about 28%. However, this is not realistic choice of the tb . Many multi-agent systems have layered structure with the following agents: client assistant, brokers, execution agents. The presented performance approximation method can be used for finding the mean time of response on client request for this class of systems. In the future, we will try to get a better approximation using the general phase type distribution

(Bobbio et al., 2004) and hypoexponential distribution (Magott & Skudlarski, 1993) instead of the Erlang one.

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