Chapter

Discreteness in Time and Evaluation of the Effectiveness of Automatic Control Systems: Examples of the Effect of Discreteness on Mathematical Patterns

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Abstract

Discreteness is one of the fundamental categories in science, philosophy, mathematics, physics, and cybernetics. In the last 50 years, this concept and problem has occupied the minds of many practical engineers. There were situations in which discreteness began to play a major role, for example among the problems obstructing progress in automatic control systems and regarding the transition to digital systems. This chapter discusses the main approaches to the stability analysis of automatic control systems, proposed in fundamental works on the theory of automatic control (TAC). A proprietary approach is proposed, greatly simplifying engineering calculations, with almost no loss of analysis accuracy. It is shown, how this approach allows us to formulate new principles for the construction of seemingly well-known regulators—PID regulators and variable structure systems (VSS). In the last part of the chapter, it is proposed to analyze the famous paradoxes of science precisely from the point of view of the discreteness of the variables considered in these paradoxes. It is argued that it is discrete operations (not always correct) that are the causes of these paradoxes.

Keywords: automatic control systems, discreteness, accuracy and stability of digital systems, pulse automatic control system

1. Introduction

Digital automatic control systems (ACS) have won everywhere. Their advantages over analog are undeniable—these controllers implement control algorithms of almost any complexity, completely inaccessible to analog ACS.

They are very reliable and stable. Most often, their setup is simple and convenient, like working with mobile phones.

There are no problems with discreteness of output signals in terms of level and time for most ACS. The discreteness in time in fractions of milliseconds and in level in fractions of a percent for the overwhelming number of electromechanical ACS (the most complex of possible structures) with their working range of speed and effort changes is insignificant. Important impulse elements remain in these systems —power converters, which actively affect processes in power currents of engines.

For high-precision electromechanical systems (electric drives), the problems of discreteness of information signals and power currents remain important.

Indeed, discreteness in time and in the level of the processed signals inevitably breaks continuous ACSs and makes their behavior unpredictable. If in analog versions of the ACSs were important—the order of the differential equations describing the control object, the presence of nonlinear links and the requirements for the dynamics of the system, then in digital systems in the 70–80s of the twentieth century, time for calculating control signals became very important.

2. Statement of problems: question status

A study of the fundamental works of leading scientists of the 80s showed the following. All discrete analysis methods, pulsed digital systems, in one way or another are connected with the use of a delay link and lattice functions. These are discrete transformations of continuous channels and transfer functions—Z-transforms, D-transforms, discrete Laplace transforms, and others. What they have in common, most importantly for working with real ACS, is that ALL elements of the control system are subjected to transformations—continuous, linear, with simple and complex transfer functions. It means, that all previous developments on ACS obtained for continuous ACS, that is, stability, accuracy, quality, performance, etc. must be forgotten and remade in the language of discrete transformations and transfer functions. In this case, despite seeming a very “serious” mathematical apparatus, all these transformations, along with cumbersomeness, retain many inaccurate assumptions and reservations.

For example, in the book by Meerov et al. ([1], p. 332), it is said about inverse Z transformations:

> *“Transformation makes sense if the series converges” ...*

And on p. 350:

> *“If only the function F * (z,) is given, then ... in principle, there is no procedure for finding F (p).”*

Discrete transfer functions of the simplest links of ACS are very complex, cumbersome, and almost unacceptable for engineering calculations—in “Example 7.4” on p. 354 of the same book, discrete transfer function of an aperiodic link

$$W^*(z,\varepsilon) = \sum_{i=1}^{2} K_i z_i^{\varepsilon} \frac{z}{z - z_i} = \frac{k}{\alpha} z \left[\frac{1}{z-1} - \frac{e^{-\alpha T_p \varepsilon}}{z - e^{-\alpha T_p}} \right]$$
$$= \frac{k}{\alpha} z \frac{\left(1 - e^{-\alpha T_p \varepsilon}\right) z + \left(e^{-\alpha T_p \varepsilon} - e^{-\alpha T_p}\right)}{(z-1)\left(z - e^{-\alpha T_p}\right).} \tag{1}$$

At the end of these calculations, simplifications are made in the same book, which lead to formulas 7.138 on p. 370 with the words: “you can limit yourself to a finite number of terms in equation (7.137)” and the frequency response of the sampling link is reduced to the response of the delay:

$$W^*(j\omega) \approx \sum_{n=0}^{N} w_n e^{-j\overline{\omega}n} \tag{2}$$

It is stipulated that the clock frequency is greater than the range of frequencies under consideration.

Detailed mathematical calculations of approximately the same results are given in the later book by Tsypkina [2].

Discrete transformations with cumbersome results—paragraphs 25.3 and 25.4 and 28.2—are described in the statement:

... "with sufficiently small pulse repetition periods, the pulse system can be considered. as a continuous one containing the same continuous part and a delay element." structures shown in **Figure 1** are equivalent.

$$W^*(j\omega) \approx e^{-j\omega\frac{T}{2}} W_H(j\omega) \tag{3}$$

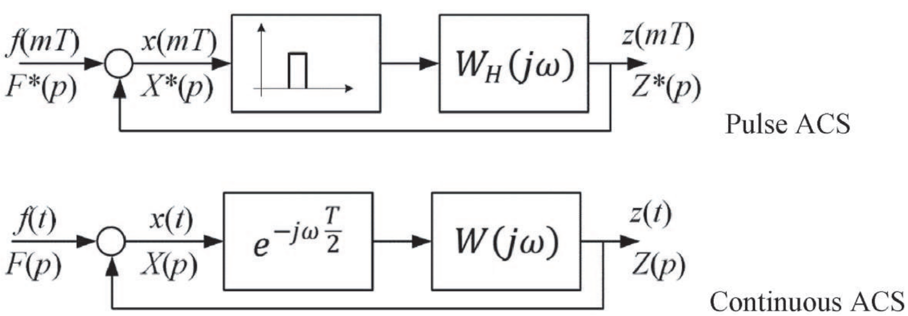

Figure 1.
Structural diagrams of ACS.

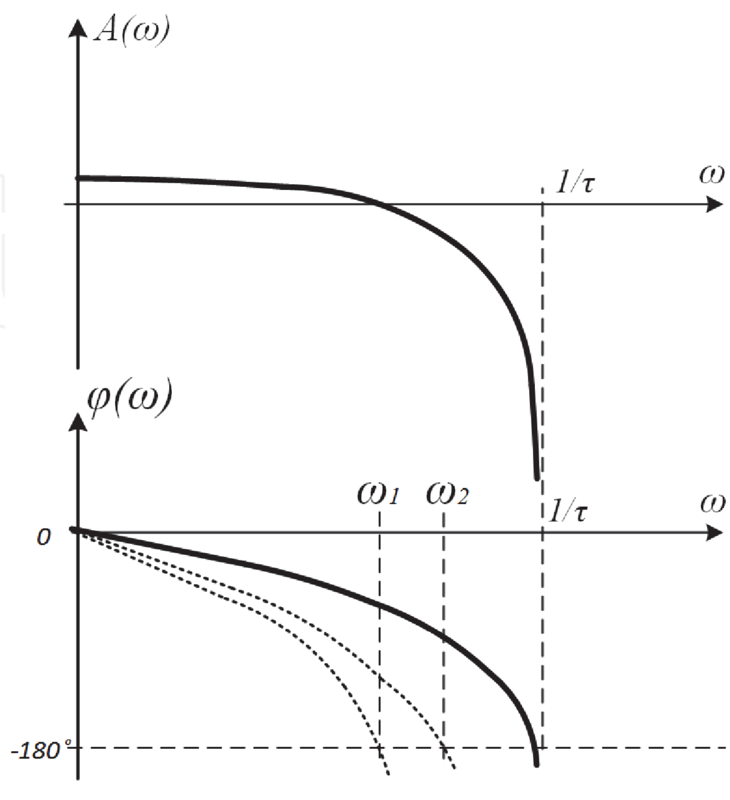

Figure 2.
Frequency characteristics of suppression link.

At the same time, it is said that the sampling time is "small," although it is not specified how small it should be and how wrong if "not small."

For engineers, this condition sounds something like this:

"The cutoff frequency of the system must be less than the quantization frequency by at least 10 times, otherwise nothing can be guaranteed." Moreover, the delay link does not change the amplitude frequency response, that is, when the condition of "smallness" of the quantization interval is satisfied, one can completely forget about it.

In this case, the phase characteristic of the delay link shown in **Figure 2** allows a formal possibility of its correction by successive links, but this is only in the case if the correction frequencies are far from the quantization frequency, and since there is a "veto" for their "rapprochement"—the prohibition of the original methodology—this possible correction is simply excluded by the method itself.

Over the past years, naturally, a lot of works on these topics have been written and published. But, practically, in almost all approaches remained the same. All methods are based on discrete Laplace transforms. The operator in these transformations is replaced by the exponential function of the delay unit, and the sampling time is included in these transformations by a parameter. Frequencies close to the clock frequency are not considered [3–10].

Thus, the "traditional" ACS theory offers two fundamental approaches:

1. Go to discrete transformations and translate ALL ACS elements into discrete formats, inevitably simplifying nonlinearities, high-order links are complex structural relationships and then operate with discrete criteria and methods.

2. "Work" only in the frequency range of 10 or more times less, than sampling rate neglecting her at all.

It does not take a lot of imagination to understand that the second approach is chosen more often in engineering calculations and studies.

One of the most commonly used devices in electromechanical systems is pulsed power amplifiers—frequency converters for asynchronous drives and voltage converters for DC drives. The switching frequency of power elements is usually in the range from 4 to 16 kHz. Mechanical processes in these systems range from 0 to 20 Hz. That is, the condition of "smallness" of the switching period of pulse elements is fulfilled. The frequency of clocking of control signals in microprocessors is most often not mentioned even in advertising materials for converters.

3. Suppression link

Many years of experience with electromechanical systems, theoretical research, and simulation showed that reducing the discretization links to the delay links according to the methods mentioned above is ineffective. This inefficiency is reflected in the inability to describe the influence of discrete links on processes in ACS, especially complex and nonlinear, and in the fact that they do not allow the formation of an effective correction of such systems. It seems appropriate to distinguish two features of the traditional representation of discretization links: firstly, the formal possibility of correcting the phase shift, and secondly, the invariability of the amplitude characteristics. To overcome these problems, it was proposed to introduce a suppression link into systems with signal sampling, the main property of which is the complete suppression of input signals with a frequency higher than or equal to the sampling frequency [11, 12]. This link breaks the connection at high frequencies without the ability to adjust this action sequentially connected link. "Included" in a closed-loop control system. it leads to instability if the cutoff frequency of the system becomes close to the sampling frequency.

4. Formula of transfer function and frequency characteristics of suppression link

The desired formula may look like this:

$$W = A(\omega)e^{j\varphi(\omega)}$$

$$\varphi(\omega) = \begin{cases} -\dfrac{K_1 \cdot (\tau\omega)}{1-\omega\tau}, & if\ \omega \le \dfrac{1}{\tau} \\ -\infty, & if\ \omega > \dfrac{1}{\tau} \end{cases} \tag{4}$$

$$A(\omega) = \begin{cases} K_2 \cdot e^{\frac{1}{\omega\tau-1}}, if\ \omega \le \dfrac{1}{\tau} \\ 0, \quad if\ \omega > \dfrac{1}{\tau} \end{cases} \tag{5}$$

$$Lg[A(\omega)] = \begin{cases} \dfrac{K_3}{\omega\tau - 1}, if\ \omega < \dfrac{1}{\tau} \\ -\infty, \quad if\ \omega > \dfrac{1}{\tau} \end{cases} \tag{6}$$

A graphical interpretation of the suppression link is shown in **Figure 2**. A of the formula (4)–(6) are phase- and amplitude-frequency characteristics. They differ from formula (3), especially in the frequency zone close to the clock frequency and show that in this frequency zone a signal is suppressed, which cannot be overcome by sequential correction, since no serial link can overcome the amplitude suppression by formula (3). The phase shift (2), at the lower frequencies, similar to the shift of the delay unit in the zone of the clock frequency, increases sharply and also cannot be seriously corrected.

Figure 3 shows the logarithmic characteristics of the suppression link—amplitude and phase characteristics.

According to these characteristics, the features of the proposed suppression link are very clearly visible.

At the clock frequency and higher, in the ACS "after" the suppression link, no sequential correction and feedback of the system will work. Disturbances at these frequencies will also not be worked out by the regulators.

Since at a frequency equal to the clock frequency, the phase takes the conditional value ($-\infty$), no sequential correction can overcome this limitation, unlike the phase characteristics of the links proposed in the sources [1, 2], which can theoretically be corrected.

As follows from the formulas and frequency characteristics of the proposed suppression link, for any sequential correction at a frequency below the quantization frequency, the phase shift will reach a critical value of $-180°$ and lead to instability of the closed loop. Depending on other parts of the system, how far from the quantization frequency this will happen?

It should be noted that the negative phase shift increases much faster than the suppression of the amplitude coefficient. So, at a frequency three times lower than

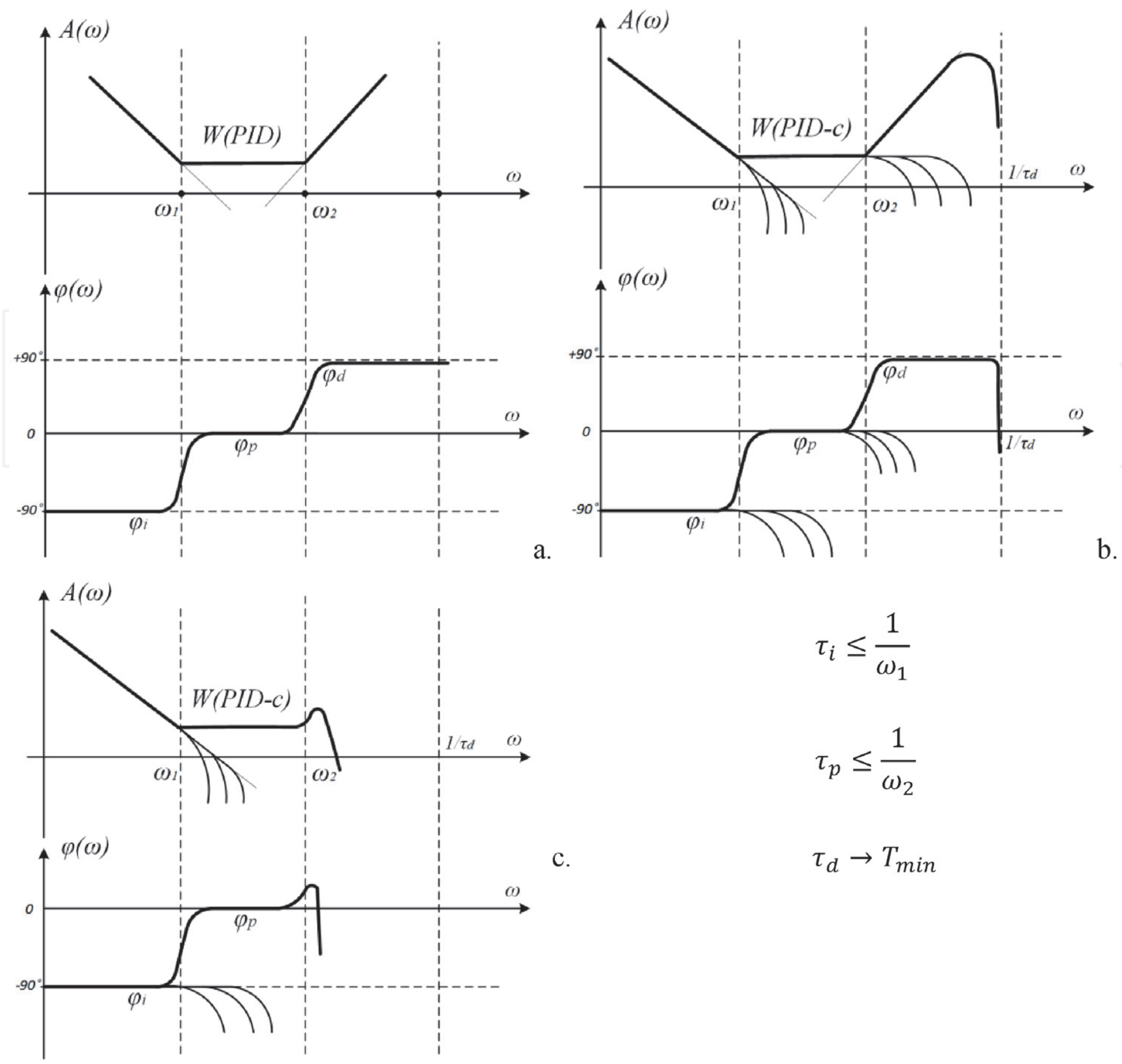

Figure 3.
Frequency characteristics of the PID controller: (a) without discrete elements, (b) with a "fast" discrete element in the D-channel, and (c) with a "slow" discrete element in the D-channel.

the clock frequency, the suppression coefficient is 0.8, and the phase shift is 90°, that is, it already significantly affects the stability of a closed system with a sampling unit.

It should be noted that according to their transfer functions, the suppression links for stability analysis of a closed loop can be converted in the same way as other dynamic links. In addition, this discreteness representation allows us to consider systems with several links, and with different sampling clocks and does not to offer cumbersome transformations. This significantly distinguishes the proposed mathematical apparatus from discrete transformations, in which each circuit of links required its own calculations of discrete transfer functions [1, 2].

Let us consider several examples of applications of these links in the structures of widely known ACS variants.

These will be proportional-integral-differential controllers (PID controllers) of control systems, variable structure systems (VSS), in which ideal sliding modes (SM) and asynchronous electric drive control systems are synthesized.

5. PID controller

It is known that the PID controller is the most widely used type of controller in industrial automation.

In this knob, the P-channel is responsible for the speed of the system and for the overall dynamics of the control loop, the D-channel provides system stability, and the I-integrator provides high static accuracy of the control system.

If we imagine the frequency characteristics of the controller as a combination of the frequency characteristics of the channels and links of suppression, it turns out, that the equivalent characteristic does not change if the discreteness of the proportional channel and the integrator is significantly slowed down (**Figure 4**). Since the links in the PID controller are connected in parallel, their resulting frequency characteristics can be determined by the "top-notch" rule. Thus you can see that the decisive role in this controller is played only by the quantization frequency of the differential channel; with its decrease (**Figure 4c**), the differentiating properties of the controller deteriorate significantly.

On a fairly simple model, these provisions are fully confirmed.

The simplicity of the model makes it easy to repeat this simulation and make sure it is correct. The control object was represented by a double integrator with an integration constant of −1 s. Here "Gain" is the channel of proportional gain with K = 10, "Deriv" is a differentiating channel with a time constant of 2.2 s, and "Trans" is an integrating channel with a time constant of 15 s.

The parameters of the PID controller, in the continuous version of the model, synthesized a process bordering on the oscillations.

In **Figures 5** and **6**, a diagram shows a 1–reference signal, 2–adjustable coordinate, 3–derivative of this coordinate, 4–signal at the output of the proportional channel of the PID controller, 5–output of the integrated channel. Continuous links simulate processes, shown in **Figure 5a**.

Then, three quantizers were introduced into the control channels. At quantization values of 0.01 s, the processes did not differ from continuous systems.

With an increase in the quantization time (0.3 s), the processes became oscillatory. The PID controller becomes equivalent to the PI controller (**Figure 3c**).

Further, in the differential channel, the discreteness is significantly reduced (0.01 s). And in other channels this discreteness still increased; so, in the proportional channel this discreteness is 0.1 s and in the integrator 0.3 s. The results are shown in **Figure 6b**.

At high speeds, the channel for differentiating the discreteness of the proportional and integral channels practically does not affect the stability of ACS. If you pay attention to the process diagrams, the following can be noted: the time of transients in

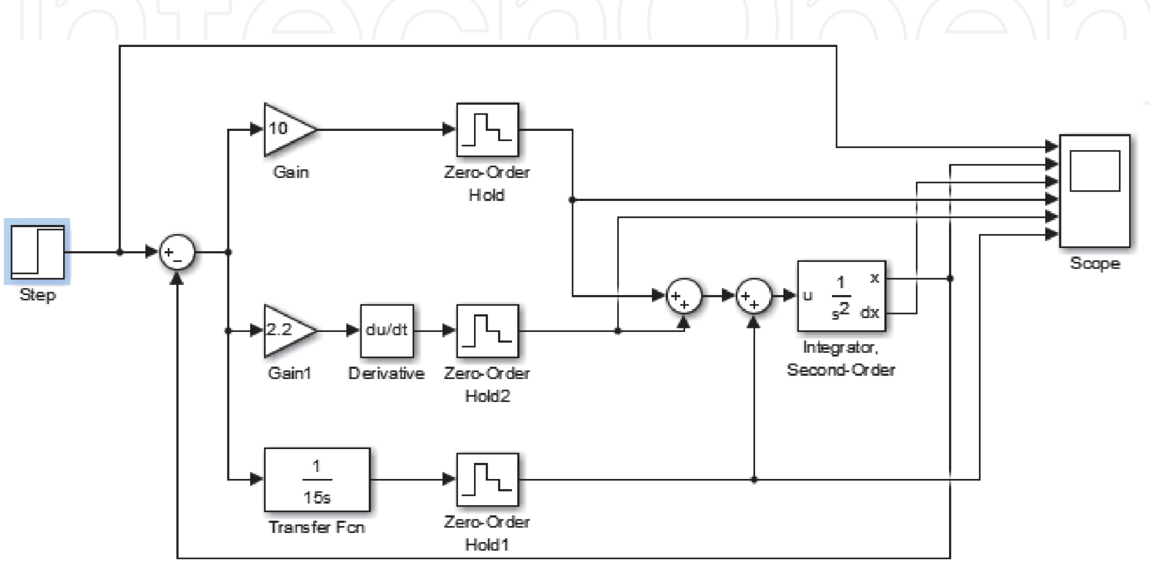

Figure 4.
Block diagram of a model of ACS with a PID controller with discrete elements.

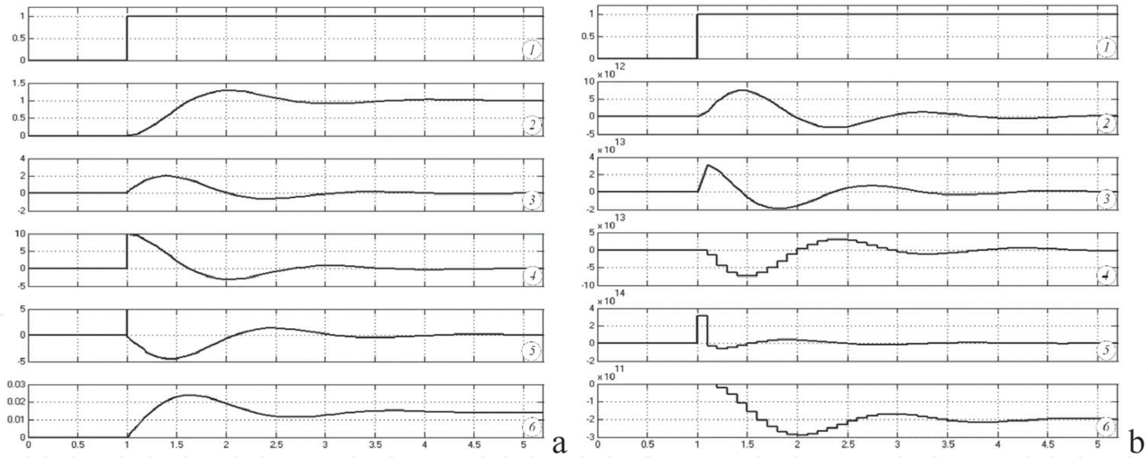

Figure 5.
Diagrams of processes: (a) in continuous ACS with a PID controller, (b) with "fast" discrete elements.

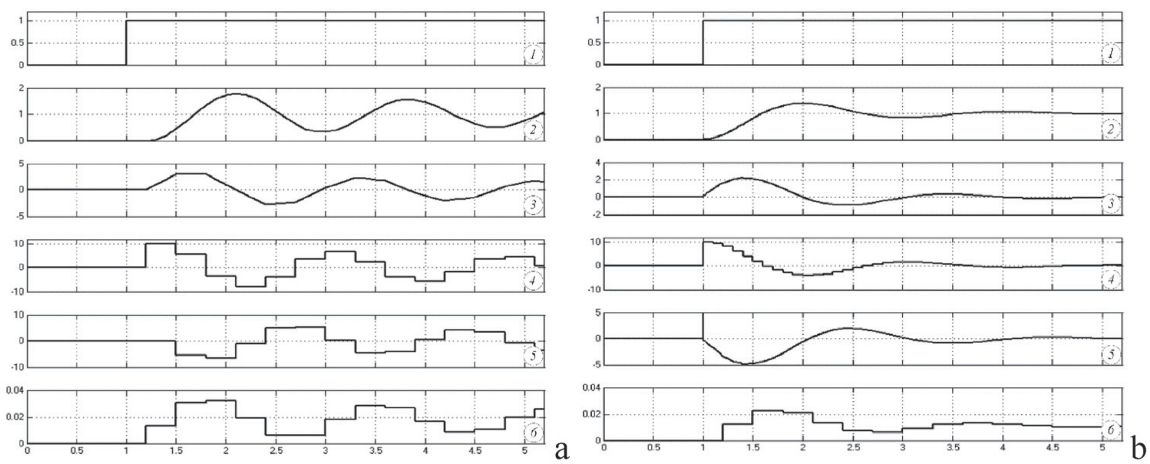

Figure 6.
Diagrams of processes: (a) with "slow" discrete elements and (b) with fast sampling in the D-channel and slow in other channels.

all diagrams is approximately the same; it does not depend on the clock frequency. The oscillation period is also unchanged. Only the degrees of vibration of the processes differ—from almost monotonic processes to unstable oscillations. This suggests that the cutoff frequency of the circuit is almost unchanged. But only the phase shifts at this point of the frequency response change. That is, changes in the quantization clocks change the cutoff frequency only slightly, since a sharp decrease in the amplitude characteristic begins near the clock frequency. And at frequencies three times smaller, the phase response shift significantly increases, which corresponds to formulas (4)–(6) of the frequency response of the link.

This shows that the sampling operation can very reasonably allocate controller resources. The integrated channel can have many discharges but a large cycle of calculations, not limited in any way by the cutoff frequency of the circuit as a whole, and the differential channel can have a fast pace of calculations, but this channel does not need accuracy, that is, in large number of discharges.

It is clear that it would hardly have been possible to find and justify such a solution using discrete transformations and related synthesis methods. According to the provisions of the theory of impulse systems set forth in classical works [1, 2, 13] and in their modern interpretations [4–7], it would be necessary to single out one impulse link and all the others "turn" to the option with a simple link. Even less likely is such a solution to be found in the neglect of the discretizer [1, 2] method, which would require a significantly higher sampling frequency compared to the transient time. Meanwhile, it is a property of discretizers to limit the frequency range of the action of links connected to it by the clock frequency, which can be very useful in correcting systems with nonlinear frequency characteristics. The most widely encountered nonlinear systems at present are asynchronous electric drives, which are discussed below.

6. VSS example

Variable-structure systems (VSSs) are an example of nonlinear control systems, the purpose of which is to obtain maximum performance in control systems. Their implementation in modern microprocessor controllers inevitably faces the problem of discreteness of control signals. Of interest is how the transfer function of the suppression link "manifests" itself in systems with a variable structure with sliding processes. **Figures 7** and **8** show the simplest VSS scheme with a sliding mode (SM). Here, CO is the control object (second-order integrator); TG, the shaper of the switching trajectory ("slip"); and C is the amplifier.

It is known that the sliding process is characterized by infinitely fast structure switching. What happens if a suppression link appears in the channel for calculating the switching path? In [14, 15, 18], the slip condition was given for an arbitrary system whose links are described by non-differential equations and frequency response. We briefly recall the main points of this conclusion.

Consider the ideal slip conditions for a second-order control object—EMS with sliding (the circuit is presented in **Figure 9**) described by the following equation:

$$\begin{cases} T^2\ddot{x} + K|x|signS = 0 \\ S = T_1\dot{x} + x \end{cases} \tag{7}$$

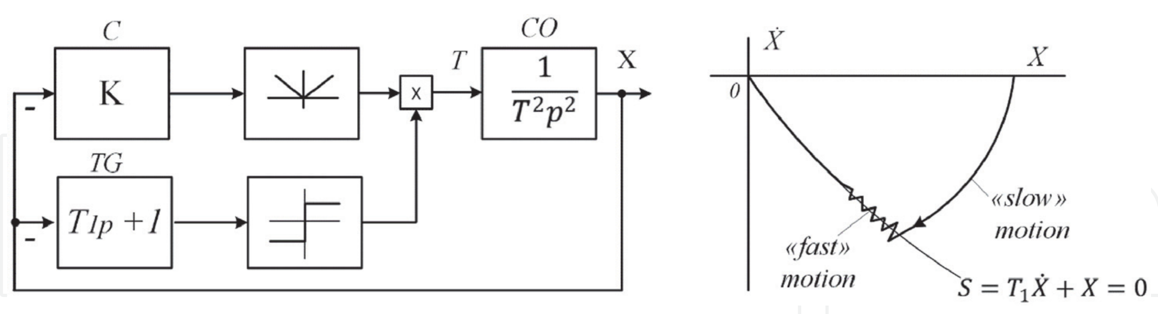

Figure 7.
Block diagram of the VSS of the second order.

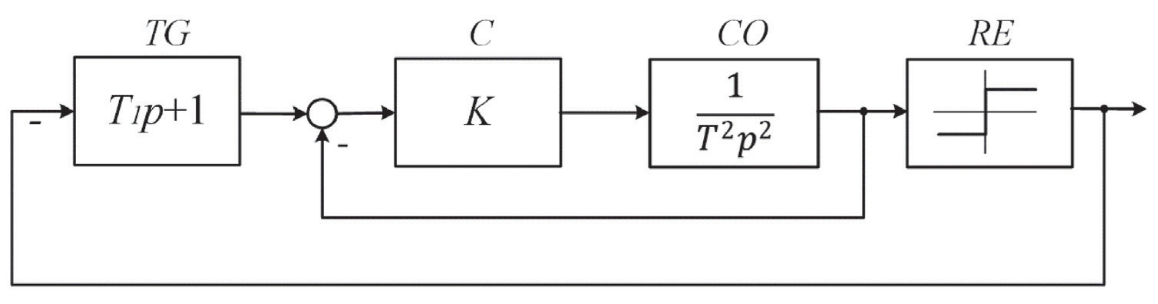

Figure 8.
Replacement block diagram of the VSS of the second order.

$$at\ x > 0\ \frac{kT_1}{T^2} - \frac{1}{T_1} \geq 0; T_1 \geq \sqrt{\frac{T^2}{k}} \tag{8}$$

The final condition links transfer functions of the controlled member ($\frac{1}{T^2p^2}$), switch trajectory generator ($T_1p + 1$), and controller (K).

Along with that, the element with cutoff frequency $\omega = \sqrt{\frac{K}{T^2}}$ is a controlled member engaged in the feedback with controller with K coefficient.

The slip condition turned out to be equivalent to the stability condition of the equivalent circuit with a relay element.

These conditions were extended to a system with arbitrary links with frequency characteristics: W_{CO} for the control object, W_{TG} for the shaper of the switching path ("slip"), and W_C for the amplifier.

The corresponding replacement block diagram is shown in **Figure 10**.

The condition (8) may be "transferred" to the frequency characteristics of EMS elements as follows: The condition of ideal sliding is met when two elements—the sliding trajectory generator and the circuit formed by the controller and controlled member—are connected in series with equivalent phase characteristic of $-90°$ minimum, and the value of $-90°$ is reached at $\omega \to \infty$.

The suggested frequency condition is met if the real part of frequency characteristics under consideration transferred to the complex space is positive:

$$\begin{gathered} Re\,[W_K \cdot W_{TG}] > 0 \\ \varphi[W_K \cdot W_{TG}] > -90^o \\ W_K = \frac{W_C \cdot W_{CO}}{1 + W_C \cdot W_{CO}} \end{gathered} \tag{9}$$

Figure 11 shows the direct correlation between the condition (8) and this assumption.

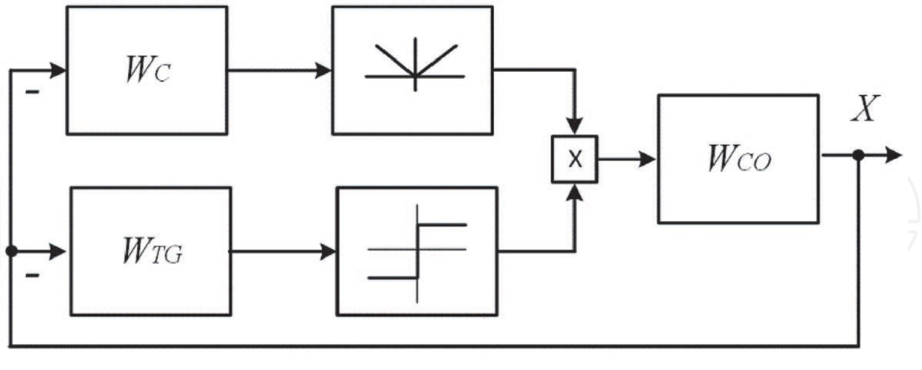

Figure 9.
Block diagram of the VSS of an arbitrary order system.

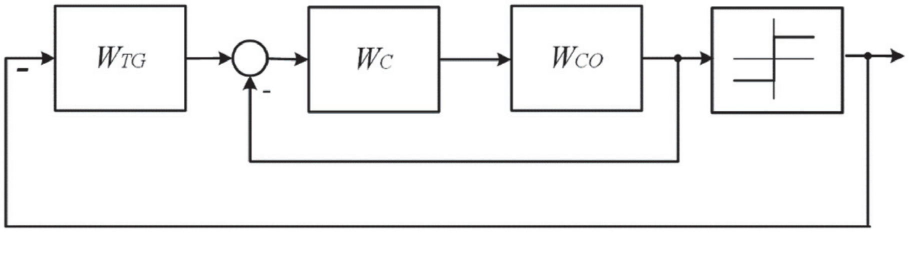

Figure 10.
Replacement block diagram of the VSS of an arbitrary order system.

1. Ideal sliding: the condition (8) is met, equivalent phase shifts of elements TG and circuit K is -90^0 minimum, frequency characteristics are presented in **Figure 12a**.

2. Unstable mode: when the condition is not satisfied in the area of the cutoff frequency, **Figure 12b**.

3. Imperfect glide: when the condition is not satisfied only in the high-frequency zone, the "slow" processes are stable. But around the sliding path, fast movements have finite amplitude and frequency. This option is most often found in real SPS and satisfies the technical requirements in systems with sliding.

Delay links primarily affect fast movements. This was dealt with in detail in all fundamental works on TAC [14, 16, 17].

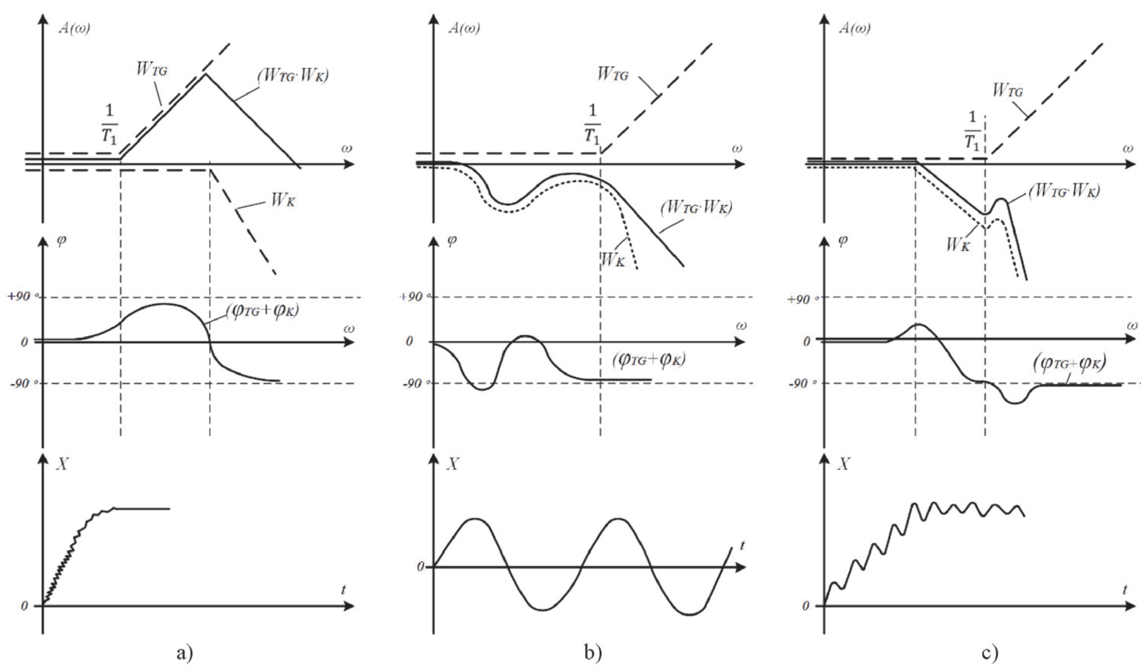

Figure 11.
Frequency characteristics of VSS: (a) with "perfect" slip, (b) if the conditions for "slow" slip are violated, (c) if the conditions for "fast" slip are not met.

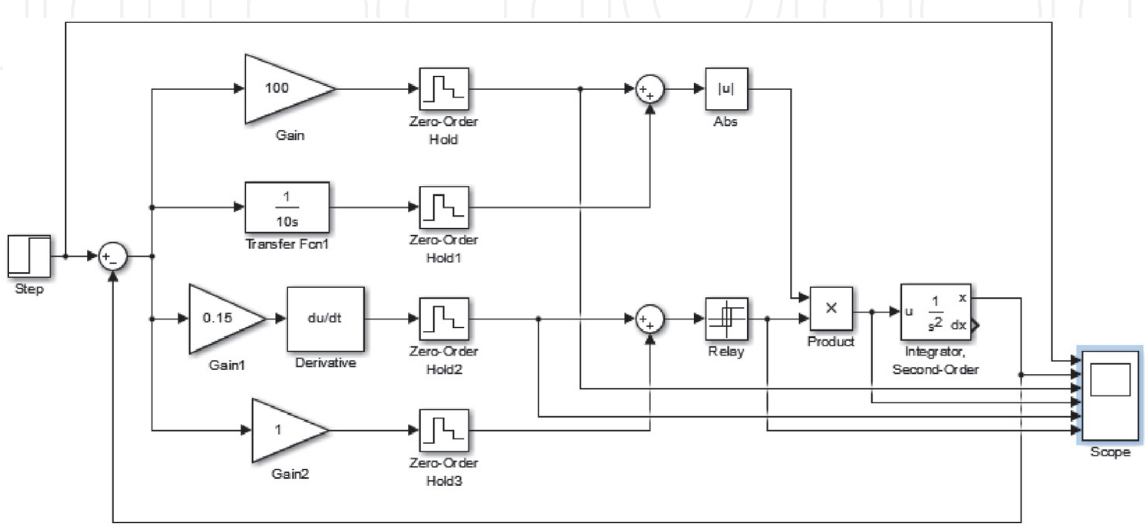

Figure 12.
Block diagram of a VSS model with discretization elements.

Let us consider how slip conditions change with the introduction of suppression units into the structures. Consider the VSS model with slip with some modifications (**Figure 12**). An integrating channel is added to the amplification channel, hysteresis is introduced into the relay element so that the slip frequency is finite. It can be assumed that the presence of suppression links in the regulator channels will violate ideal slip conditions. As it comes from the frequency characteristics of the links with the discretizer, to ensure sufficient slip parameters, fast quantization will be required in only one of the channels—differential.

Figure 13 shows the frequency characteristics of the links of the original circuit. The slip condition is satisfied in the absence of discrete elements (**Figure 13a**). If they distort the frequency characteristics of the links, as shown in **Figure 13c**, that is, in the zone of slow movements, then the process becomes unstable; in the high-frequency zone (**Figure 13b**), conditions of ideal slip are violated (infinitely high frequency and infinitesimal amplitude of "slip"), but "slow movements are stable. As can be seen from the **Figure 13b**, for the existence of "real" slip, a sufficiently high discrete frequency of only the differential channel forming the slip path

$$\tau_i \leq \frac{1}{\omega_1}; \tau_p \leq \frac{1}{\omega_2}; \tau_d \to T_{min}$$

To confirm these provisions and verify the effect of discretization and suppression links on them simulation was carried out (**Figures 14** and **15**).

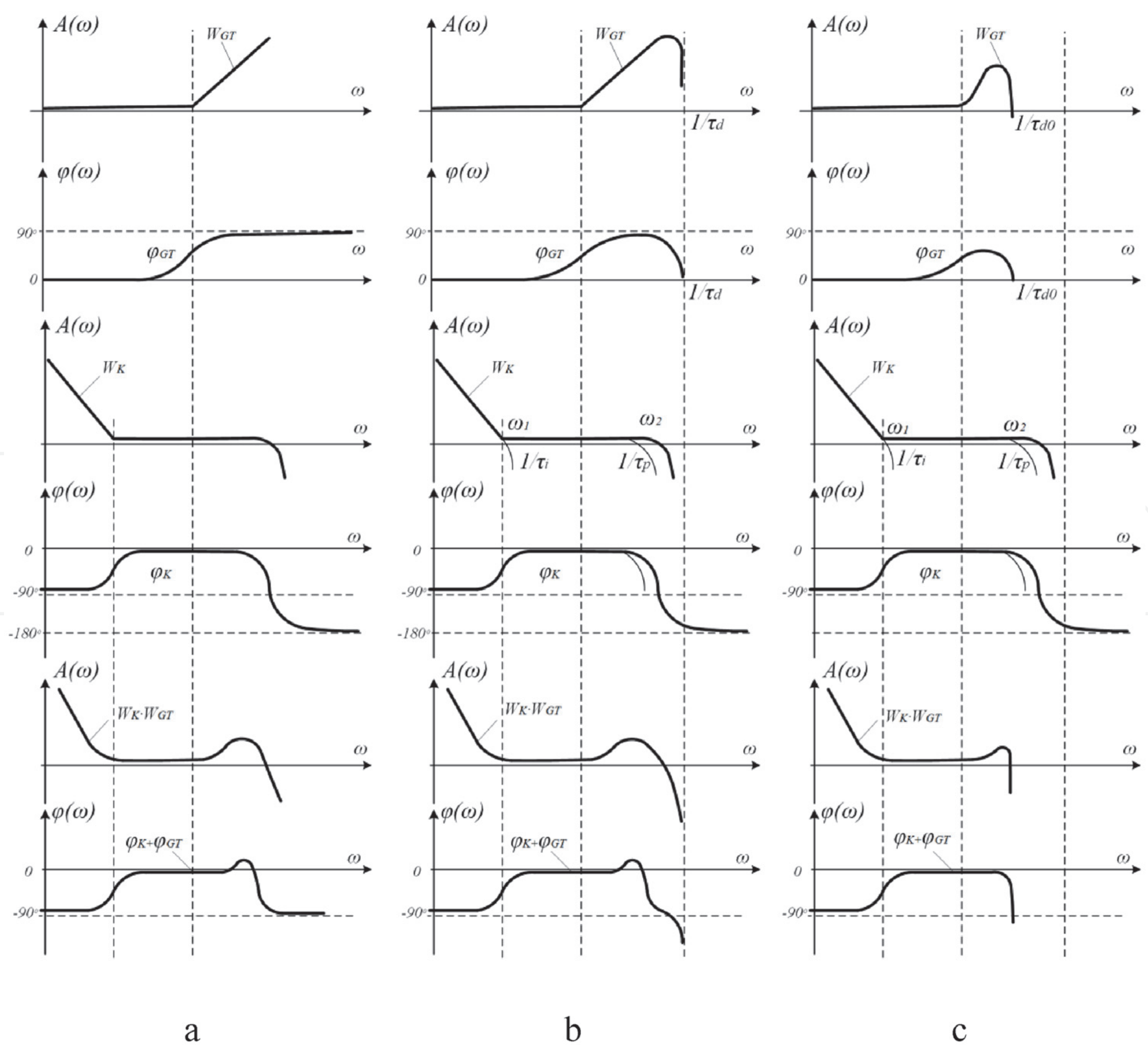

Figure 13.
Sliding conditions in VSS: (a) for continuous links, (b) for "fast" discretization elements in the D-channel, (c) for "slow" discretization in the D-channel.

Diagram shows a 1-reference signal link «step» **Figure 12**, 2-adjustable coordinate, 3-derivative of this coordinate-link «hold2», 4-signal at the output of the proportional channel controller-link «hold», 5-output of the integrated channel-link «hold1», 6-proportional channel controller-link «hold3».

When introducing discretization links into the channels of a regulator, the following results were obtained; with sampling over all channels in 0.1 s, the slip was destroyed (**Figure 15a**). At discretization of the differential channel of 0.001 s, in the remaining discrete channels the following—0.1 s and 0.3 s the process in **Figure 15b** is optimal both in accuracy and speed.

At the same time, fast movements do not correspond to perfect gliding, while slow movements completely correspond to a monotonous process. Qualitatively, the processes fully comply with the theoretical principles obtained from the analysis of the frequency characteristics of the system links for compliance with the sliding conditions.

This simulation is yet another confirmation of the effectiveness of the analysis methodology for the suppression link and controllers with different discreteness and timing of the calculations. From the time and nature of the processes, it can be seen that the sliding processes are preserved at the necessary speed of the channel for the formation of the slip function, which is determined by the fast discretization of the differential channel. Performance enhancement channels are not required. But accuracy is required. In this case, the slip condition is violated at high frequencies; it

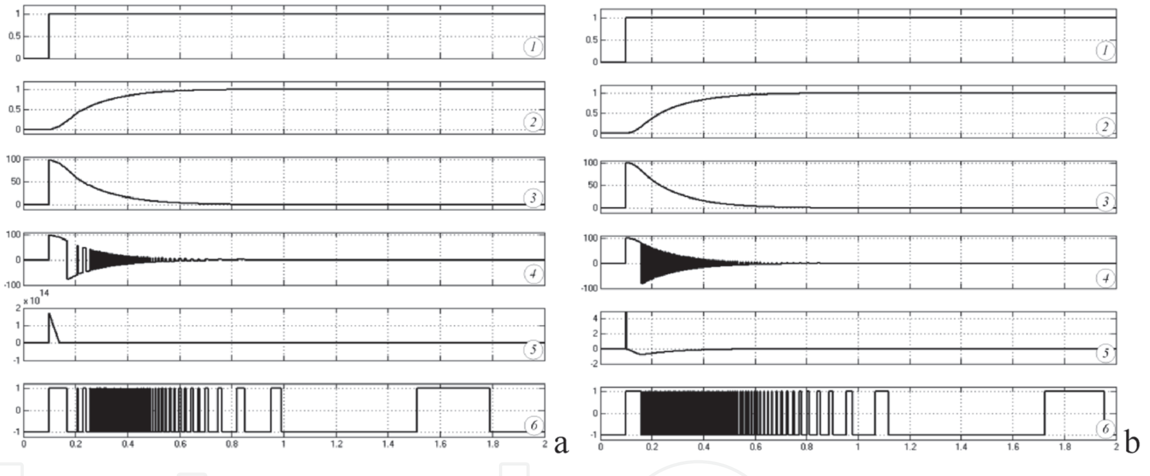

Figure 14.
Diagrams of processes in VSS with continuous elements (a) and fast discrete elements (b).

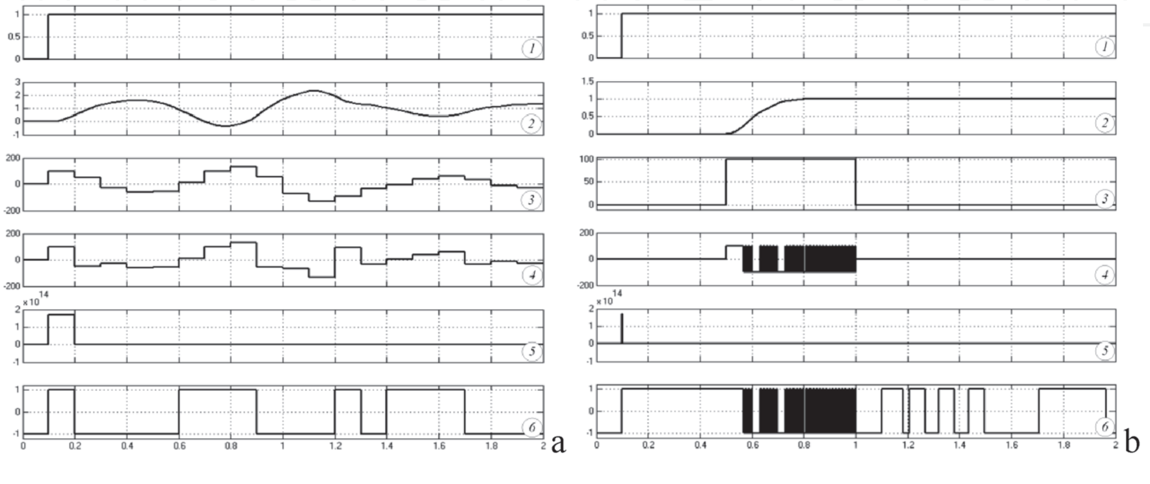

Figure 15.
Diagrams of processes: (a) in VSS with slow discrete elements, (b) in VSS with fast D-channel and slow remaining channels.

means, that fast movements are imperfect, which matches the model. This confirms the validity of the previously derived criteria for sliding along the frequency response and the effectiveness of the proposed frequency response suppression links for assessing the dynamics of even complex nonlinear control systems.

7. Correction of processes in an asynchronous electric drive

Traditionally, it is customary to be considered among electric drive engineers that the discreteness of control signals only affects the controllability of electromechanical systems, because it always "breaks" continuous connections. However, the interpretation of discretization by suppression links shows that sampling allows you to "clear" the frequency characteristics of corrective devices from "side" effects. An example is the PID controller discussed above. Under the conditions of the controller, only the differential channel "works" in the high-frequency zone. In the continuous controller, all channels are rumbled, although the integral and proportional channels are greatly weakened. The use of discrete elements at the output of each channel allows them to be completely filtered out, which cannot be done in a continuous controller and it is difficult to come to such a decision without using the concept of suppression link. A system with nonlinear dynamics is asynchronous electric drives with frequency control.

As shown in [18, 19], the traditionally applied methods and control algorithms ("transvector control") do not always provide the necessary dynamic characteristics of asynchronous electric drives.

In the same works, an alternative control algorithm is described—a dynamic positive relationship with the effective value of the stator current ("DOS+"). This connection allows you to compensate for changes in rotational speed under static and low-frequency loads [19]. In order for the communication to correct only static modes and the low-frequency region, the devices use dynamic links—low-pass filters [18]. As experiments and modeling show, these tasks are performed.

Figure 16 shows a diagram of the model of an asynchronous electric drive with corrective connections for the stator current (**Figure 16a**) and rotation speed (**Figure 16b**), and **Figures 17** and **18** show the processes of acceleration and load surges with several versions of dynamic links including a discrete element with a low sampling frequency. Static modes are well compensated. With current correction (**Figure 17**), the high-frequency oscillation in currents and speeds at different speeds is preserved by slightly changing its parameters at different speeds of rotation.

Figure 18 shows the processes in the model with additional correction for rotation speed (connection in **Figure 16b**) in which the signal passes through two parallel links—proportional with low-frequency sampling ("Zero-Order Hold1"—0.1 s) and differential with high-frequency sampling ("Zero-Order Hold2"—0.005 s).

As follows from **Figure 18**, the oscillation is completely eliminated at all speeds. In this system, there are three different discrete links: "Zero-Order Hold"—0.3 s, "Zero-Order Hold1"—0.1 s; and "Zero-Order Hold2"—0.005 s (**Figure 16b**); and the system as a whole is significantly superior to all known options for the frequency regulation of induction motors. Moreover, the system is quite easily implemented in industrial frequency converters, since it does not require high accuracy in measuring direct coordinates or in perfect processing.

Discreteness is one of the fundamental principles in science. Needless to say, the initial concepts in human thinking are discrete. We perceive the world around us as separate phenomena and objects. Only after the transition to abstract thinking, we

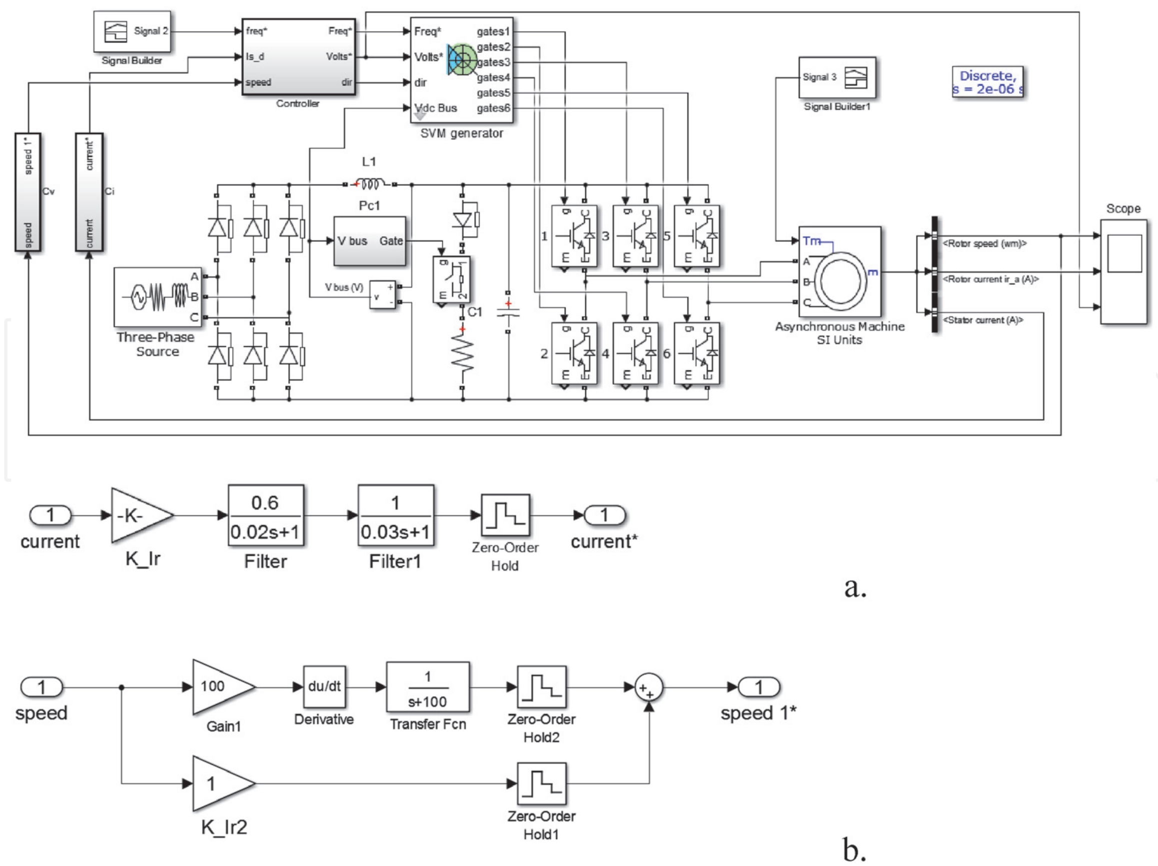

Figure 16.
Scheme of the asynchronous electric drive model and the correction of current (a) and speed (b) with discrete elements.

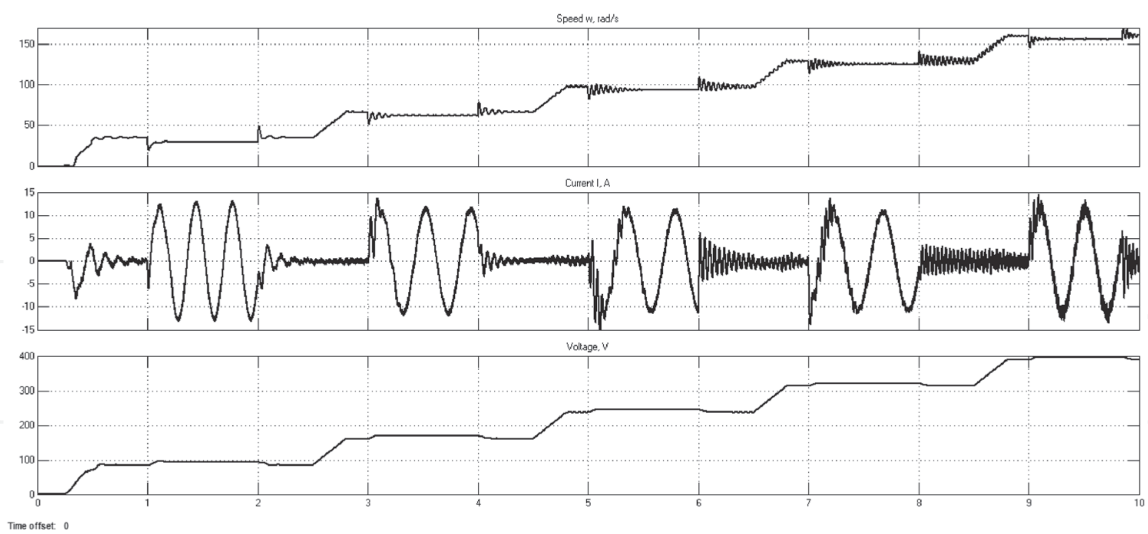

Figure 17.
Diagrams of processes in asynchronous electric drive model with stator current correction and discrete elements.

begin to link individual objects and phenomena into continuous chains. No wonder continuous mathematics, created in the seventeenth and eighteenth centuries, became one of the crowning results of almost three thousand years of our civilization.

Without this mathematics, Aristotle and Archimedes created their teachings, the whole of Ancient Rome and the millennial Byzantium created their own civilizations. But voluntarily or involuntarily, the philosophers of antiquity turned to the concepts of the continuous and discrete and received very interesting paradoxes and statements.

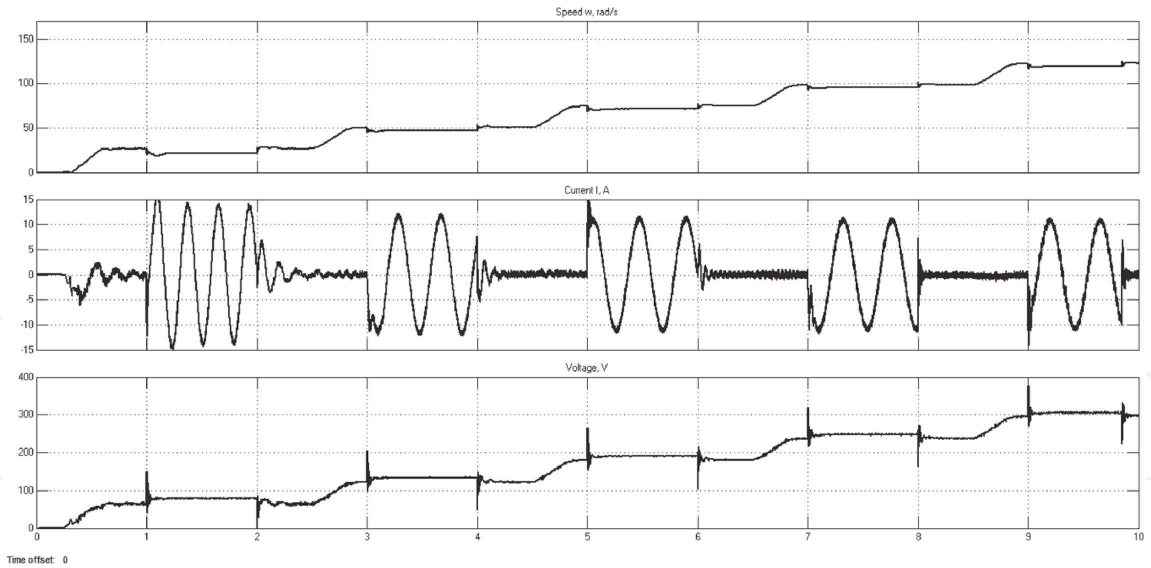

Figure 18.
Diagrams of processes in asynchronous electric drive model with speed correction and discrete elements.

Consider two points that people have been pondering over for centuries, unaware that the whole thing is in very small detail.

8. Achilles paradox

One of the most famous paradoxes of philosophy is the Zeno's Paradox about Achilles and the tortoise. The paradox goes like this:

It states that Achilles will never catch up with a hulking turtle if she begins her movement before him.

For several centuries this paradox was a "horror" of philosophers and theoretical scientists. And right now the explanations that are offered to ordinary people are very vague.

Meanwhile, it seems to me that everything is easily explained if we analyze the discreteness of time that Zeno offers and that which his interlocutors understood.

Let's try to figure out the details of Zeno's reasoning. His main position: Achilles will not catch up with the turtle, because in the interval of time for which he will reach her position, she will go further. Zeno suggests that the interlocutors consider the whole movement as a sequence of states and intervals at those points where the turtle has already visited. These intervals will be shorter and shorter until they become infinitely small. However, Zeno did not apply such concepts. Actually, turning to the concepts of infinitesimal ones, he remains himself and leaves his interlocutors in terms of finite time intervals … and space too. And he comes and leads the rest of the participants in the conversation to a clear contradiction. He tells them: "I show you my time and space, which I interrupt at any time when I want and as many times as I want. You too can tear your time, in which Achilles easily catches up with a turtle, on as many sites as you like. So our times are the same, but in mine Achilles is forever behind the turtle. So in yours, he will not catch up with her."

All ordinary people understand the discreteness of time as the same and fall into the "trap." In their head, time is unbroken, infinite, and the "time of Zeno" is only that time in which the tortoise is ahead of Achilles, and its division into an infinite number of sections—intervals. He "equates" it with the time of the interlocutor—infinite time. Zeno says "ALWAYS," but it is in his time, and he evens out times with the number of intervals. But the intervals for Zeno and his interlocutors are different.

And this is the trick of Zeno, because the number of intervals is not a length of time especially if the intervals are infinitesimal. The paradox turns into Sophism. We do not know knowingly did it Zeno ... Hardly. Otherwise, he would have created a theory of infinitesimal quantities 2000 years earlier than Descartes and Leibniz, who created higher mathematics in which discreteness and, especially, its infinitesimal values play a fundamental role. Judging by Zeno's other aporias—for example, "On the Arrow," he felt a "discrepancy" between ordinary discrete thinking, based on observations and practical experience and continuity, which scientists spoke about in his time. And he showed this problem in every way in the Achilles paradox—irresponsibly changing the discreteness of time.

9. Fermat's paradox

One of the founders of modern science is Pierre Fermat, the author of many important decisions and discoveries. But he is best known for 400 years thanks to the paradox or "Fermat's theorem," which is a very vivid illustration of the possibilities of discretization of variables of mathematical quantities, since it is precisely the discreteness of four independent variables in Fermat's theorem that leads one equation to four unknowns for a condition that cannot be fulfilled.

Fermat's theorem states that there are no positive integers that would be a solution to the equation $X^n + Y^n = Z^n$ for n greater than 2.

If any positive values of X, Y, or Z (or at least one of them) were allowed, then an equation with three unknowns for any degree would have an infinite number of solutions. This is undeniable and understandable.

But here is what happens if discreteness is introduced into an indisputable and understandable statement. It turns out that with such discreteness it is impossible to find at least one combination of three numbers and a degree corresponding to the solution of the Fermat equation.

Let us try to formulate; the theorem is a paradox with an "emphasis" on the discreteness of variables:

The sum of the natural degrees of two natural numbers is unequal to the same degree, starting from the third, no natural number.

For the first degree, this condition is not fulfilled, that is, for any two positive integers there is a third for the equality to be fulfilled.

For the second degree, there are solutions to the equation but not for any pair of numbers.

But for the third degree is no longer. Rather, there are, but some very large ones that mathematicians find once every hundred years. It is very difficult to check if there are such numbers yet. Even if there are very good computers.

About 30 years proof of Fermat's theorem was found [20]. Only reasonably good specialists, mathematicians, can understand it. And for all other people, this is not a solution to the original paradox: the simplest mathematical paradox connecting the simplest expressions to the simplest numbers.

It is as if helicopter pilots would win in mountaineering competitions. No one argues with the proof, or almost no one ... But questions remained.

What is the essence of Fermat's paradox? It may be that the discreteness of numbers turns an expression with several degrees of freedom (one equation with three unknowns) into a practically unsolvable expression. In other words, only a rigid definition of the variables involved in this condition turns excessive freedom into nonexistence.

Moreover, natural numbers are what most people see in their practical lives. And all the others are fractions. Complex vectors were for many years a "fabrication of scientists" that had no connection with reality. And these natural numbers set up such a trick. If you look at Fermat's theorem from this position, a whole series of questions will arise.

- Will there be solutions to the equation if, for the third power, the third term is added to the left side?
- How many solutions will there be if we allow fractional degrees?
- Or fractional variables?
- Is it possible for fractional numbers to find the same condition connecting several variables by impossible equations?

Is it a coincidence that for the second degree, three variables still give solutions in our three-dimensional space, and in the third degree there are no solutions already?

And so many others ...

So, to summarize this replica, it can be argued that this condition (in Fermat's theorem) connects three independent variables and their nonlinear transformations defined by the fourth variable, the degree, and it is the discreteness of all variables that makes this simple equation (or formula) impossible.

This relationship of the dimension of equations and the nonlinearity of transformations with the discreteness of variables is, in the opinion of the author of the article, the main meaning of Fermat's paradox and one more confirmation of the fundamental concept of discreteness [21].

10. Conclusion

The problem of discreteness in science and technology is one of the most interesting.

At different stages of development, either discreteness or continuity began to prevail and became a new word in science.

So, integral and differential calculi replaced arithmetic and algebra, connecting all quantities with continuous operations and conditions. Then quantum physics replaced the classical one.

The theory of stability, as the basis of cybernetics, was supplemented by discrete controls and methods, and this chapter has shown how the influence of discrete elements can be taken into account by links with continuous frequency characteristics. It is also shown how frequency analysis can well take into account the discreteness of some elements of self-propelled guns and obtain fundamentally new types of correction and new results.

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References

[1] Meerov MV, Mikhailov YN, Fridman VG. Fundamentals of Automatic Control. Moscow: Nedra; 1972

[2] Tsypkin IZ. Fundamentals of the Theory of Automatic Systems. Moscow: Science; 1977

[3] Sánchez EN, Ornelas-Tellez F. Discrete-time inverse optimal control for nonlinear systems. 2013. DOI: 10.1201/b14779

[4] Almobaied M, Eksin I, Güzelkaya M. A new inverse optimal control method for discrete-time systems. In: 2015 12th International Conference on Informatics in Control, Automation and Robotics (ICINCO), 01. 2015. pp. 275-280. DOI: 10.5220/0005562902750280

[5] Almobaied M, Eksin I, Guzelkaya M. Inverse optimal controller design based on multi-objective optimization criteria for discrete-time nonlinear systems. In: 2019 IEEE 7th Palestinian International Conference on Electrical and Computer Engineering (PICECE). 2019. pp. 1-6. DOI: 10.1109/PICECE.2019.8747189

[6] Ornelas-Tellez F, Sanchez EN, Loukianov AG. Discrete time nonlinear systems inverse optimal control: A control Lyaponov function approach. In: IEEE International Conference on Control Applications (CCA) Part of 2011 IEEE Multi-Conference on Systems and Control. 2011

[7] Kipka R, Gupta RR. The discrete-time geometric maximum principle. SIAM Journal on Control and Optimization. 2017;**57**:2939-2961. DOI: 10.1137/16M1101489

[8] Zhang M, Gan M, Chen J, Jiang Z. Adaptive dynamic programming and optimal stabilization for linear systems with time-varying uncertainty. In: 2017 11th Asian Control Conference (ASCC). 2017. pp. 1228-1233. DOI: 10.1109/ASCC.2017.8287346

[9] Zhou B. On asymptotic stability of linear time-varying systems. Automatica. 2016;**68**:266-276. DOI: 10.1016/j.automatica.2015.12.030

[10] Zhou B, Zhao T. On asymptotic stability of discrete-time linear time-varying systems. IEEE Transactions on Automatic Control. 2017;**62**:4274-4281. DOI: 10.1109/tac.2017.2689499

[11] Kodkin VL, Gafiyatullin RK, Khaibyakov ER. The suppression link, its frequency properties. In: Proceedings of the 4th International Conference on Automated Electric Drives "AEP in the 21st Century" Magnitogorsk; Russia; September 14–17. 2004

[12] Kodkin VL, Gafiyatullin RK, Khaibyakov ER. Application of frequency characteristics of the suppression link in engineering calculations. In: Proceedings of the4th International Conference on Automated Electric Drive "AEP in the 21st Century" Magnitogorsk; Russia, September 14–17. 2004

[13] Voronov AA. Fundamentals of the Theory of Automatic Control: Automatic Regulation of Continuous Linear Systems. 2nd ed. reslave ed. Moscow: Energy; 1980. pp. 312

[14] Emelyanov SV. In: Emelyanov SV, Korovin SK, editors. New Types of Feedback: Management under Uncertainty. Moscow: Science. Fizmatlit; 1997. 352 pp

[15] Kodkin VL. Methods of optimizing the speed and accuracy complex guidance systems based on equivalence of automatic control system domain of attraction and unconditional stability of

their equivalent circuits. Vestnik SUSU Series. Computer Technology, Management, Electronics. 2017;**1**(1): 23-33

[16] Rush N. In: Rush N, Abets P, Lalua M, editors. Direct Lyapunov method in the theory of stability. Moscow: Mir; 1980. pp. 300

[17] Popov VM. Hyper-Stability of Automatic Systems. Moscow: Science; 1970. pp. 456

[18] Kodkin VL. Prospects for use of AC drives in industrial robots. Identification of quality of asynchronous electric drives from spectra of rotor currents. In: Kodkin VL, Anikin AS, Baldenkov AA, editors. Proceedings—2018 Global Smart Industry Conference, GloSIC; 2018. DOI: 10.1109/GloSIC.2018. 8570111

[19] Kodkin VL, Anikin AS, Baldenkov AA. The analysis of the quality of the frequency control of induction motor carried out on the basis of the processes in the rotor circuit. Journal of Physics: Conference Series. 2018;**944**(1). DOI: 10.1088/1742-6596/ 944/1/012052

[20] Why the proof of Fermat's Great Theorem does not need improvements. Available from: https://habr.com/en/ post/461179/[Accessed: 26 July 2019]. (26 июля 2019 в 10:00 «Почему доказательство Великой теоремы Ферма не нуждается в улучшениях»)

[21] The paradox of Achilles and turtles: Meaning, interpretation of the concept. Read more on FB.ru. Available from: https://fb.ru/article/386467/paradoks-ahillesa-i-cherepahi-znachenie-rasshifrovka-ponyatiya [Accessed: 17 May 2018]