Chapter

Transitions between Stationary States and the Measurement Problem

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Abstract

Accounting for projections during measurements is the traditional measurement problem. Transitions between stationary states require measurements, posing a different measurement problem. Both are compared. Several interpretations of quantum mechanics attempting to solve the traditional measurement problem are summarized. A highly desirable aim is to account for both problems. Not every interpretation of quantum mechanics achieves this goal.

Keywords: quantum measurement problem, transitions between stationary states, interpretations of quantum theory

1. Introduction and outlook

In 1930 Paul Dirac published *The Principles of Quantum Mechanics* [1]. Two years later John von Neumann published *Mathematische Grundlagen der Quantenmechanik* [2]. These initial versions of quantum theory share two characteristics, (i) the state vector $|\psi\rangle$ (wave function ψ) describes the state of an individual system, and (ii) they involve two laws of change of the state of the system: spontaneous processes, governed by the Schrödinger equation, and measurement processes, ruled by the projection postulate ([3], pp. 5–6).

Many other versions of quantum theory followed. Those where ψ describes the state of an individual system and where the projection postulate is included among its axioms are generally called standard, ordinary, or orthodox quantum mechanics (OQM), sometimes referred to as the Copenhagen interpretation, associated to Niels Bohr.

The most relevant differences between spontaneous processes (SP) and measurement processes (MP) are as follows [4]: in SP the observer plays no role, in MP the observer (or the measuring device) plays a paramount role; in SP the state vector $|\psi(t)\rangle$ is continuous, in MP $|\psi(t)\rangle$ collapses (jumps, is projected, is reduced); in SP the superposition principle applies, in MP the superposition principle breaks down; SP are ruled by a deterministic law, MP are ruled by probability laws; in SP every action is localized, in MP there is a kind of action-at-a-distance [5]; and in SP conservation laws are strictly valid, in MP conservation laws have only a statistical sense [6–8].

Since the projection postulate contradicts the fundamental Schrödinger equation of motion, some authors rushed to the conclusion that it was defective.

Henry Margenau suggested in a manuscript sent to Albert Einstein on November 13, 1935, that this postulate should be abandoned. Einstein replied that the formalism of quantum mechanics inevitably requires the following postulate: "If a measurement performed upon a system yields a value m, then the same measurement performed immediately afterwards yields again the value m with certainty" ([3], p. 228). The projection postulate guarantees compliance with this principle.

The traditional measurement problem in quantum mechanics is how (or whether) wave function collapse occurs when a measurement is performed. Although *a similar measurement problem* is implied in transitions between stationary states (TBSS) induced by a time-dependent perturbation, *it is conspicuously absent* from the specialized literature on the subject.

The contents of this paper are as follows: time-dependent perturbation theory (TDPT) is summarized in Section 2. Section 3 shows that according to TDPT, measurements are required for TBSS to occur. Section 4 highlights the similarities and differences between the traditional measurement problem and that implied in TBSS. Section 5 includes several interpretations of quantum mechanics which attempt to solve the traditional measurement problem: Bohmian mechanics, decoherence, spontaneous localization, and spontaneous projection approach (SPA). Section 6 shows that SPA accounts for TBSS, and in cooperation with decoherence, it also accounts for the traditional measurement problem. Section 7 compiles conclusions.

2. The formulation of TDPT

TDPT was formulated by Dirac in 1930 ([1], Chapter VII). In his words: "In [TDPT] we do not consider any modification to be made in the states of the unperturbed system, but we *suppose* that the perturbed system, instead of remaining permanently in one of these states, is continually changing from one to another, or making transitions, under the influence of the perturbation" ([1], p. 167; emphasis added). The aim of TDPT is, then, to calculate the probability of TBSS which can be induced by the perturbation during a given time interval.

Dirac points out that "this method must … be used for solving *all* problems involving a consideration of time, such as those about the transient phenomena that occur when the perturbation is suddenly applied, or more generally problems in which the perturbation varies with the time in any way (i.e. in which the perturbing energy involves the time explicitly). [It must also] be used in collision problems, even though the perturbing energy does not here involve the time explicitly, if one wishes to calculate absorption and emission probabilities, since these probabilities, unlike a scattering probability, cannot be defined without reference to a state of affairs that varies with the time" ([1], p. 168; emphasis added).

TDPT is a key ingredient of OQM. It has many applications and is at the basis of quantum electrodynamics, the extension of OQM accounting for the interactions between matter and radiation ([1], Chapter X; [9], Chapter 9). Without TDPT, OQM would hardly be such a powerful and successful theory.

To develop TDPT one starts by splitting in two the total Hamiltonian $\mathrm{H}(t)$ acting on the system:

$$\mathrm{H}(t) = \mathrm{E} + \mathrm{W}(t) \tag{1}$$

E is the Hamiltonian of an unperturbed system, which can be dealt with exactly. Every dependence on time is included in $\mathrm{W}(t)$. Dirac asserts that "the perturbing energy $\mathrm{W}(t)$ can be an arbitrary function of the time" ([1], p. 172).

The eigenvalue equations of E are

$$\mathrm{E}\ |\varphi_n\rangle = E_n\ |\varphi_n\rangle \tag{2}$$

where E_n ($n = 1, 2, \ldots, N$) are the eigenvalues of E and $|\varphi_n\rangle$ the corresponding eigenvectors. For simplicity we shall consider the spectrum of E to be entirely discrete and non-degenerate. All the E_n and $|\varphi_n\rangle$ are supposed to be known.

Let $|\psi(t)\rangle$ be the state of the system at time t. We assume that at the initial time t_0, the system is in the state $|\psi(t_0)\rangle = |\varphi_j\rangle$, the eigenvector of the non-perturbed Hamiltonian E corresponding to the eigenvalue E_j. If there is no perturbation, i.e., if the Hamiltonian were E, this state would be stationary. But the perturbation causes the state to change. At time t the state of the system will be

$$|\psi(t)\rangle = U_H(t, t_0)|\psi(t_0)\rangle = U_H(t, t_0)|\varphi_j\rangle \tag{3}$$

where $U_H(t, t_0)$ is the evolution operator, a linear operator independent on $|\psi\rangle$ and depending only on H, t, and t_0 ([1], p. 109).

The probability of a transition taking place from the initial stationary state $|\varphi_j\rangle$ to the final stationary state $|\varphi_k\rangle$ (respectively corresponding to the eigenvalues E_j and E_k of E) induced by the perturbation $\mathrm{W}(t)$ during the time interval (t_0, t) is then

$$P_{jk}(t_0, t) = |\langle\varphi_k|U_H(t, t_0)|\varphi_j\rangle|^2 \tag{4}$$

See, for instance, [1], Chapter VII; [9], Chapter 9; [10], Chapter XIII; [11], Chapter IV; [12], Chapter 19; and [13], Chapter XVII. Note: symbols used by these authors may have been changed for homogeneity.

3. TBSS require measurements

TDPT includes two clearly different stages. The first governed by the Schrödinger equation and the second ruled by probability laws [14]. Concerning this issue Dirac points out: "When one makes an observation on the dynamical system, the state of the system gets changed in an unpredictable way, but in between observations causality applies, in quantum mechanics as in classical mechanics, and the system is governed by equations of motion which make the state at one time determine the state at a later time. These equations of motion … will apply so long as the dynamical system is left undisturbed by any observation or similar process … Let us consider a particular state of motion through the time during which the system is left undisturbed. We shall have the state at any time t corresponding to a certain ket which depends on t and which may be written $|\psi(t)\rangle$ … The requirement that the state at one time $[t_0]$ determines the state at another time $[t]$ means that $|\psi(t_0)\rangle$ determines $|\psi(t)\rangle$ … " ([1], p. 108).

During the first stage of TDPT the process is ruled by the Schrödinger equation:

$$i\hbar \frac{\mathrm{d}}{\mathrm{dt}}|\psi(t)\rangle = \mathrm{H}(t)\ |\psi(t)\rangle \tag{5}$$

where $\mathrm{H}(t)$ is the total Hamiltonian of the system and $\hbar$ is Planck's constant divided by 2π. The solution of Eq. (5) corresponding to the initial condition $|\psi(t_0)\rangle = |\varphi_j\rangle$ is unique; $|\psi(t)\rangle$ is completely determined by the initial state $|\psi(t_0)\rangle$ and $\mathrm{H}(t)$, which includes the perturbation $\mathrm{W}(t)$. Since $|\psi(t)\rangle$ depends only on the initial state $|\varphi_j\rangle$ and on $\mathrm{H}(t)$, or if preferred on the perturbation $\mathrm{W}(t)$, then

$$|\psi(t)\rangle \equiv |\psi_{j,H}(t)\rangle = U_H(t,t_0)|\psi(t_0)\rangle = U_H(t,t_0)|\varphi_j\rangle \tag{6}$$

The evolution from $|\varphi_j\rangle$ to $|\psi_{j,H}(t)\rangle$ given by Eq. (6) is *automatic*. No transition from the initial state $|\varphi_j\rangle$ to a stationary state $|\varphi_k\rangle$ results until time t.

In the second stage of TDPT, *it is assumed* that at a time t_f, a measurement is performed. As a consequence, a projection from $|\psi_{j,H}(t_f)\rangle$ to $|\varphi_k\rangle$ takes place. In the words of Albert Messiah: "We suppose that at the initial time t_0 the system is in an eigenstate of E, the state $|\varphi_j\rangle$ say. We wish to calculate the probability that *if a measurement is made* at a later time t_f, the system *will be found* to be in a different eigenstate of E, the state $|\varphi_k\rangle$ say. This quantity, by definition the probability of transition from $|\varphi_j\rangle$ to $|\varphi_k\rangle$, will be denoted by $P_{jk}(t_0,t_f)$" ([13], p. 725; emphases added). Clearly

$$P_{jk}(t_0,t_f) = |\langle\varphi_k|U_H(t_f,t_0)|\varphi_j\rangle|^2 \tag{7}$$

Dirac does not explicitly mention measurements. He supposes that at the initial time t_0, the system is in a state for which E has the value E_j with certainty. The ket corresponding to this state is $|\varphi_j\rangle$. At time t_f the corresponding ket will be $U_H(t_f,t_0)\ |\varphi_j\rangle$ ([1], p. 172). The probability of E then *having* the value E_k is given by Eq. (7). For $E_k \neq E_j$, $P_{jk}(t_0,t_f)$ is the probability of a transition taking place from $|\varphi_j\rangle$ to $|\varphi_k\rangle$ during the time interval (t_0,t_f), while $P_{jj}(t_0,t_f)$ is the probability of no transition taking place at all. The sum of $P_{jk}(t_0,t_f)$ for all k is unity ([1], pp. 172–173).

Note that where Messiah says "the probability that *if a measurement* [of E] *is made* … the system *will be found* to be in … the state $|\varphi_k\rangle$ … " Dirac says "the probability of E then *having* the value E_k … " Dirac's assertion, however, has exactly the same meaning as Messiah's, as shown in the following quote from Dirac's book *The Principles of Quantum Mechanics*: "The expression that an observable 'has a particular value' for a particular state is permissible in quantum mechanics in the special case when a measurement of the observable is certain to lead to the particular value, so that the state is an eigenstate of the observable … In the general case we cannot speak of an observable having a value for a particular state … [but] we can go further and speak of the probability of its having any specified value for the state, meaning *the probability of this specified value being obtained when one makes a measurement of the observable*" ([1], pp. 46–47; emphases added). Hence Dirac's statement "the probability of E then *having* the value E_k is given by Eq. (7)" should be understood as "the probability of E_k being obtained when one makes a measurement of E is given by Eq. (7)." *Both Dirac (the author of TDPT) and Messiah place measurements at the very heart of TDPT.*

The following diagram illustrates the complete process leading the system from the initial state $|\varphi_j\rangle$ to the final state $|\varphi_k\rangle$:

$$|\psi(t_0)\rangle = |\varphi_j\rangle \longrightarrow |\psi_{j,H}(t_f)\rangle = U_H(t_f,t_0)\ |\varphi_j\rangle \longrightarrow |\varphi_k\rangle$$

First stage: during the interval (t_0,t_f) the evolution of the state is ruled by the Schrödinger equation	**Second stage**: $|\psi_{j,H}(t_f)\rangle$ jumps to $|\varphi_k\rangle$ with probability $P_{jk}(t_0,t_f)$

Let ε be the non-perturbed energy represented by the operator E. *Everything happens as if at time t_f a measurement of ε is performed* [14]. If no measurement of ε is performed, OQM states that the system continues to evolve in compliance with Schrödinger's equation.

4. Two kinds of measurement problems: similarities and differences

It is often overlooked that TDPT requires a measurement of ε in order to obtain the collapse $|\psi_{j,H}(t_f)\rangle \rightarrow |\varphi_k\rangle$, suggesting that TBSS are simply the result of perturbations [14]. A perturbation is something completely different from a measurement. When the perturbation $W(t)$ is applied, the Hamiltonian changes from E to E + $W(t)$, but the Schrödinger evolution is not suspended. By contrast, a measurement interrupts the Schrödinger evolution. According to TDPT the perturbation $W(t)$ applied during the interval (t_0, t_f) as well as the measurement of ε at t_f are necessary for the transition $|\varphi_j\rangle \rightarrow |\varphi_k\rangle$ to occur.

There are, then, two kinds of measurement problems: (i) the traditional measurement problem and (ii) the measurement problem related to TBSS. Both of them are measurement problems for in both the Schrödinger evolution is interrupted and the state of the system instantaneously collapses as established by the projection postulate.

i. In the traditional measurement problem, the experimenter chooses the physical quantity to be measured. This quantity can be, in principle, any physical quantity such as the position, a component of the angular momentum, the energy, etc. Measurements of these quantities have been performed many times, with different methods, by different people, and in different circumstances.

ii. In TBSS the system jumps to an eigenstate of E, the operator representing ε. The experimenter has no choice; the only physical quantity susceptible to be "measured" is the non-perturbed energy ε. We say "measured" because it seems difficult to admit that TBSS involve measurements of any physical quantity. It seems even more difficult to admit that ε is measured every time a photon is either emitted or absorbed by an atom, as TDPT requires. TBSS could be considered "measurements" without observers or measuring devices.

"In most cases, the wave function evolves gently, in a perfectly predictable and continuous way, according to the Schrödinger equation; in some cases only (as soon as a measurement is performed), unpredictable changes take place, according to the postulate of wave packet reduction" [15]. TBSS, which are happening everywhere all the time, must also be included in *some of the cases* where unpredictable changes take place according to the projection postulate.

In previous papers we have pointed out the following contradiction: On the one hand, according to OQM *there is no room for the projection postulate* as long as we are dealing with spontaneous processes. On the other hand, to account for spontaneous processes involving a consideration of time OQM requires, through TDPT, *the application of the projection postulate*. This is a flagrant incoherence absent from the literature [14, 16].

Quantum weirdness has been associated with the traditional measurement problem. To solve it, several interpretations of quantum mechanics have been proposed. In the following section, we shall address a few of them. For a critical review of the most popular interpretations of quantum theory, see the interesting study of Franck Laloë *Do we really understand quantum mechanics?* [15].

5. Some alternative interpretations to OQM

5.1 Bohmian mechanics (BM)

It is also called the causal interpretation of quantum mechanics and the pilot-wave model. Its first version was proposed by Louis de Broglie in 1927, rapidly abandoned and forgotten, and reformulated by David Bohm in 1952 [17].

In BM it is assumed that particles are point-like. They have well-defined positions at each instant and thus describe trajectories. A system of N particles with masses m_k and *actual* positions $Q_k(t)$ ($k = 1, \dots, N$) can be described by the couple $(Q(t), \psi(t))$, where $Q(t) = (Q_1(t), \dots, Q_N(t))$ is the *actual* configuration of the system. The wave function of the system is $\psi = \psi\,(q, t) = \psi\,(q_1, \dots, q_N; t)$, a function on the space of *possible* configurations q of the system. The wave function evolves according to the Schrödinger equation:

$$\mathrm{i}\hbar\,\frac{\partial}{\partial t}\psi = \mathrm{H}\,\psi \tag{8}$$

where H is the nonrelativistic Hamiltonian. The actual positions of the particles evolve according to the guiding equation:

$$\frac{\mathrm{d}}{\mathrm{d}t}Q_k(t) = \frac{\hbar}{m_k}\;\mathrm{Im}\left[\frac{\psi^*\;\partial_k\psi}{\psi^*\;\psi}\right] \tag{9}$$

where Im [] is the imaginary part of [] and $\partial_k = (\partial/\partial \mathrm{x}_k, \partial/\partial \mathrm{y}_k, \partial/\partial \mathrm{z}_k)$ is the gradient with respect to the generic coordinates $q_k = (\mathrm{x}_k, \mathrm{y}_k, \mathrm{z}_k)$ of the kth particle. For a system of N particles, Eqs. (8) and (9) completely define BM [18]. It is worth stressing that (i) BM is a nonlocal theory and (ii) BM is a deterministic theory: the initial couple $(Q(t_0), \psi(t_0))$ determines the couple at any time $t > t_0$.

BM accounts for all of the phenomena governed by nonrelativistic quantum mechanics, from spectral lines and scattering theory to superconductivity, the quantum Hall effect and quantum computing [18]. A proposed extension of BM describes creation and annihilation events: the world lines for the particles can begin and end [19]. For any experiment the deterministic Bohmian model yields the usual quantum predictions [18].

In BM the usual measurement postulates of quantum theory emerge from an analysis of the Eqs. (8) and (9). In the collapse of the wave function, the interaction of the quantum system with the environment (air molecules, cosmic rays, internal microscopic degrees of freedom, etc.) plays a significant role. Even if the Schrödinger evolution is not interrupted, replacing the original wave function for its "collapsed" derivative is justified as a pragmatic affair [18]. In this regard BM appeals for processes of decoherence.

5.2 Decoherence

Decoherence is an interesting physical phenomenon entirely contained in the linear Schrödinger equation and does not imply any particular conceptual problem [15]. It is a consequence of the unavoidable coupling of the quantum system with the surrounding medium which "looks and smells as a collapse" [20].

Decoherence is currently the subject of a great deal of research. To grasp how it works, let us consider the following case, taken from Daniel Bes' *Quantum Mechanics* ([9], pp. 247–248).

A quantum system in the state $|\Phi_i\rangle$ (i = 1, 2) interacts with the environment, initially in the state $|\eta_0\rangle$, resulting in

$$|\Phi_i\rangle\ |\eta_0\rangle \rightarrow |\Phi_i\rangle\ |\eta_i\rangle \tag{10}$$

If the initial state of the system is $|\Phi_\pm\rangle = \left(\frac{1}{\sqrt{2}}\right)\ (|\Phi_1\rangle \pm |\Phi_2\rangle)$, the linearity of the Schrödinger equation yields entangled states:

$$|\Phi_\pm\rangle\ |\eta_0\rangle \rightarrow \left(\frac{1}{\sqrt{2}}\right)\ (|\Phi_1\rangle\ |\eta_1\rangle \pm |\Phi_2\rangle\ |\eta_2\rangle) \tag{11}$$

The corresponding pure state density matrix is

$$\begin{aligned}\rho = \frac{1}{2}\ |\Phi_1\rangle\ \langle\Phi_1|\ |\eta_1\rangle\ \langle\eta_1| \pm \frac{1}{2}\ |\Phi_1\rangle\ \langle\Phi_2|\ |\eta_1\rangle\ \langle\eta_2| \\ \pm \frac{1}{2}\ |\Phi_2\rangle\ \langle\Phi_1|\ |\eta_2\rangle\ \langle\eta_1| + \frac{1}{2}\ |\Phi_2\rangle\ \langle\Phi_2|\ |\eta_2\rangle\ \langle\eta_2|\end{aligned} \tag{12}$$

Assuming that the environment states are almost orthogonal to each other, i.e., $\langle\eta_1|\eta_2\rangle \approx 0$ ([9], p. 248), the reduced density matrix becomes

$$\rho' \approx \frac{1}{2}\ |\Phi_1\rangle\ \langle\Phi_1| + \frac{1}{2}\ |\Phi_2\rangle\ \langle\Phi_2| \tag{13}$$

"Eq. (13) does not imply that the system is in a mixture of states $|\Phi_1\rangle$ and $|\Phi_2\rangle$. Since these two states are simultaneously present in Eqs. (11) and (12), the composite system + environment displays superposition and associated interferences. However, Eq. (13) says that such quantum manifestations will not appear as long as experiments are performed only on the system" ([9], p. 248).

It has been proven that for large classical objects, decoherence would be virtually instantaneous because of the high probability of interaction of such systems with some environmental quantum. Several models illustrate the gradual cancelation of the off-diagonal elements with decoherence over time. Experiments also show that, due to the interaction with the environment, superposition states become unobservable ([9], p. 251). "These experiments provide impressive direct evidence for how the interaction with the environment gradually delocalizes the quantum coherence required for the interference effects to be observed … We find our observations to be in excellent agreement with theoretical predictions" ([21], p. 265).

5.3 Spontaneous localization

The key assumption is that each elementary constituent of any physical system is subject, at random times, to spontaneous localization processes (called hittings) around random positions. The best known mathematical model stating which modifications of the wave function are induced by localizations, where and when they occur, is usually referred to as the Ghirardi-Rimini-Weber (GRW) theory [22, 23]. It holds as follows [24]:

Let $\psi(q_1, \ldots, q_N)$ be the wave function of a system of N particles. "If a hitting occurs for the ith particle at point x, the wave function is instantaneously multiplied by a Gaussian function (appropriately normalized)" [24]:

$$G(q_i, x) = \mathrm{K}\ \exp\left[-\left(\frac{1}{2d^2}\right)(q_i - x)^2\right] \tag{14}$$

where d and K are constants. Let

$$\Phi_i(q_1, \ldots, q_N; x) = \psi(q_1, \ldots, q_N)\, G(q_i, x) \tag{15}$$

be the unnormalized wave function immediately after the localization and $P(x)$ the density probability of the hitting taking place at x. Assuming that $P(x)$ equals the integral of $|\Phi_i|^2$ over the 3N-dimensional space implies that hittings occur with higher probability at those places where, in the standard quantum description, there is a higher probability of finding the particle. The constant K appearing in Eq. (14) is chosen in such a way that the integral of $P(x)$ over the whole space equals unity. Finally, it is assumed that the hittings occur at randomly distributed times, according to a Poisson distribution, with mean frequency f. The parameters chosen in the GRW-model are f = 10^{-16} s^{-1} and d = 10^{-5} cm [24].

GRW aims to a unification of all kinds of physical evolution, including wave function reduction. On the one hand, the theory succeeds in proposing a real physical mechanism for the emergence of a single result in a single experiment, which is attractive from a physical point of view, and solves the "preferred basis problem," since the basis is that of localized states. The occurrence of superposition of far-away states is destroyed by the additional process of localization [15]. On the other hand, it fails to account for TBSS referred to in TDPT. Similar theories to GRW, like the continuous spontaneous localization, confront the same problem. The reason is simple: localizations localize (see Eqs. (14) and (15)). They do not yield the system to a stationary state.

5.4 Spontaneous projection approach (SPA)

Two kinds of processes irreducible to one another occur in nature: those strictly continuous and causal, governed by a deterministic law, and those implying discontinuities, ruled by probability laws. This is the main hypothesis of SPA [25]. Continuous and causal processes are Schrödinger's evolutions. Processes implying discontinuities are jumps to the preferential states $|\varphi_j\rangle$ $(j = 1, \ldots, N)$ belonging to the preferential set $\{N_\varphi\}$ (= $|\varphi_1\rangle, \ldots, |\varphi_N\rangle$) of the system in a given state [26, 27].

In SPA conservation laws play a paramount role. The system has the tendency to jump to the eigenstates of every constant of the motion, while the jumps must respect the statistical sense of every conservation law [25].

The preferential set may or may not exist. If the system in the state $|\psi(\mathrm{t})\rangle$ has the preferential set $\{N_\varphi\}$, we can write

$$|\psi(t)\rangle = \sum_j \gamma_j(t)\, |\varphi_j\rangle \tag{16}$$

where $\gamma_j(t)$ = $\langle\varphi_j|\psi(t)\rangle \neq 0$ for every j = 1, …, N and $N \geq 2$.

Let us stress the following characteristics of the preferential set [26, 27]:

i. It depends on the state $|\psi(t)\rangle$.

ii. If it exists, the preferential set is unique. A system in the state $|\psi(t)\rangle$ cannot have more than one preferential set.

iii. Even if in the general case the Hamiltonian of the system can be written H(t) = E + W(t), the preferential set does not depend on W(t).

iv. At least $(N-1)$ members of $\{N_\varphi\}$ are eigenstates of E. The exception, i.e., the case where a preferential state is not a stationary state, has been referred to elsewhere [28].

v. The relation

$$\langle \psi(t)|A|\psi(t)\rangle = \sum_j |\gamma_j(t)|^2 \langle \varphi_j|A|\varphi_j\rangle \tag{17}$$

must be fulfilled for every operator A representing a conserved quantity α when W(t) = 0. The validity of this relation ensures the statistical sense of the conservation of α [25].

If the system in the state $|\psi(t)\rangle$ does not have a preferential set, the Schrödinger evolution follows. By contrast, if it has the preferential set $\{N_\varphi\}$, in the small time interval $(t, t+dt)$, the system can either remain in the Schrödinger channel or jump to one of its preferential states. The probability that it jumps to the preferential state $|\varphi_k\rangle$ is

$$dP_k(t) = |\gamma_k(t)|^2 \frac{dt}{\tau(t)} = |\langle \varphi_k|\psi(t)\rangle|^2 \frac{dt}{\tau(t)} \tag{18}$$

where $\tau(t)\Delta\mathrm{E}(t) = \hbar/2$ and $[\Delta\mathrm{E}(t)]^2 = \langle\psi(t)|\mathrm{E}^2|\psi(t)\rangle - [\langle\psi(t)|\mathrm{E}|\psi(t)\rangle]^2$ [26, 27].

It is easily shown that in the interval $(t, t+dt)$, the probability that the system abandons the Schrödinger channel is $dt/\tau(t)$ and the probability that it remains in the Schrödinger channel is

$$dP_S(t) = 1 - \frac{dt}{\tau(t)} \tag{19}$$

So the dominant process in a small time interval $(t, t+dt)$ is always the Schrödinger evolution [25–27].

In cases where the system remains in the Schrödinger channel, the transformation of the state yielded by SPA exactly coincides with that yielded by OQM. It could be wrongly assumed that there is a complete correspondence (i) between OQM spontaneous processes and SPA processes where the preferential set is absent; and (ii) between OQM measurement processes and SPA processes where the system has its preferential set.

Certainly SPA processes where the preferential set is absent as well as OQM spontaneous processes are forcible Schrödinger evolutions. And unless the system is an eigenstate of the operator representing the quantity to be measured, OQM measurements entail projections. But if the system has its preferential set, according to SPA it can either be projected to a preferential state or remain in the Schrödinger channel [26, 27]. Differing from OQM, in SPA there is always room for Schrödinger evolutions.

In sum, SPA states that in general the wave function evolves gently, in a perfectly predictable and continuous way, in agreement with the Schrödinger equation; in some cases only, when the system jumps to one of its preferential states, unpredictable changes take place, according to the projection postulate. Assuming that projections are a law of nature, SPA succeeds in proposing a real physical mechanism for the emergence of a single result in a single experiment.

6. Facing both measurement problems

Measurement is a complicated and theory-laden business ([29], p. 208). When one talks about the measurement problem in quantum mechanics, one is not referring to a real and theory-laden process but just to the problem of *accounting in* *principle for projections resulting from measurements*, i.e., to the fact that the Schrödinger evolution is suspended when a measurement is performed.

SPA justifies Dirac's assertion: "in [TDPT] we do not consider any modification to be made in the states of the unperturbed system, but we suppose that the perturbed system, instead of remaining permanently in one of these states, is continually changing from one to another, or making transitions, under the influence of the perturbation" ([1], p. 167).

On the one hand, in general the preferential states of the system are the eigenstates of E, which do not depend on the perturbation $W(t)$. Hence no modification of these states should be considered. On the other hand, if the initial state of the system is $|\psi(t_0)\rangle = |\varphi_j\rangle$, an eigenstate of E, the effect of the perturbation is to gently remove the state $|\psi(t_0)\rangle$ from $|\varphi_j\rangle$, and yield it to the linear superposition $|\psi(t)\rangle$ given by Eq. (16). Once the system is in this linear superposition, it can either suddenly jump to a stationary state or remain in the Schrödinger channel. If it jumps, it can either go to a state $|\varphi_k\rangle$ (where $k \neq j$) or come back to its initial state $|\varphi_j\rangle$. The result can be described as a system continually changing from one to another stationary state or making transitions, as Dirac asserts.

In principle SPA accounts for TBSS. By contrast, decoherence has little to contribute concerning this matter.

Assuming as valid the ideal measurement scheme, in previous papers we have addressed the traditional measurement problem as follows [4, 25].

Let A be the operator representing the physical quantity α referred to the system S. We shall denote by $|a_j\rangle$ the eigenvector of A corresponding to the eigenvalue a_j ($j = 1, 2, \dots$); for simplicity we shall refer to the discrete non-degenerate case. If the initial state of S is $|a_j\rangle$ and the initial state of the measuring device M is $|m_0\rangle$, the initial state of the total system S + M (before the measurement takes place) will be denoted by $|a_j\rangle\ |m_0\rangle$. The final state of the total system (when the measurement is over) will be denoted by $|\Phi\rangle$.

According to the ideal measurement scheme the Schrödinger evolution results

$$|a_j\rangle\ |m_0\rangle \to |\Phi\rangle = |\Phi_j\rangle \tag{20}$$

This scheme is supposed to be valid in cases where the measured physical quantity is compatible with every conserved quantity referred to S + M [30].

If the initial state of S is $\sum_j \gamma_j |a_j\rangle$ (where $\gamma_j \neq 0$ for every $j = 1, \dots, N$), the linearity of the Schrödinger equation yields entangled states:

$$\left(\sum_j \gamma_j\ |a_j\rangle\right) |m_0\rangle \to |\Phi\rangle = \sum_j \gamma_j\ |\Phi_j\rangle \tag{21}$$

The set $\{N_\Phi\} = \{|\Phi_1\rangle, \dots, |\Phi_N\rangle\}$ can be considered the preferential set of S + M in the state $|\Phi\rangle$ (as a matter of fact, $\{N_\Phi\}$ clearly fulfills several of the requirements imposed to such a set). Hence, projections like $|\Phi\rangle \to |\Phi_1\rangle, \dots$. or $|\Phi\rangle \to |\Phi_N\rangle$ may result. This is SPA proposed solution to the traditional measurement problem.

Decoherence invokes an alternative solution to the traditional measurement problem. Once the expansion (21) is obtained, the density matrix corresponding to the state $|\Phi\rangle$ is replaced by the reduced density matrix as previously done in Section 5.2 (see Eqs. (12) and (13)). It is claim that "there has been a leakage of coherence from the system to the composite entity (system + environment). Since we are not able to control this entity, *the decoherence has been completed to all practical purposes*" ([9], p. 248; emphases added).

Laloë points out that "decoherence is not to be confused with the measurement process itself; it is just the process which takes place just before: during decoherence, the off-diagonal elements of the density matrix vanish … " [15]. In his view "the crux of most of our difficulties with quantum mechanics is the question: what is exactly the process that forces Nature … to make its choice among the various possibilities for the results of experiments?" [15]. SPA answers: spontaneous projections to the preferential states.

SPA and decoherence are not opposed theories competing for "an explanation" to the measurement problem but cooperating theories. Projections break down the Schrödinger evolution, but they are not frequent. If the system has its preferential set, projections can take place at the very beginning of the process or not (in SPA there is always room for Schrödinger evolutions). As long as projections do not take place, decoherence can make its work entangling the system with the environment. But nothing prevents the total, entangled system, to have its preferential set. This may be why a spontaneous projection finally breaks down the superposition of states of the total system. Nature makes its choice, and it is only then that decoherence is completed.

7. Conclusions

Carlton Caves declares: "Mention collapse of the wave function, and you are likely to encounter vague uneasiness or, in extreme cases, real discomfort. This uneasiness can usually be traced to a feeling that wave-function collapse lies 'outside' quantum mechanics: The real quantum mechanics is said to be the unitary Schrödinger evolution; wave-function collapse is regarded as an ugly duckling of questionable status, dragged in to interrupt the beautiful flow of Schrödinger evolution" [31].

If collapses implied in traditional measurement are regarded as an ugly duckling of questionable status, collapses implied in TBSS could result definitively unbearable. Neither observers nor measuring devices could be invoked to excuse their occurrence, but they are there, happening all the time, more or less everywhere, e.g., every time a photon is either emitted or absorbed by an atom.

The search for a solution to the traditional measurement problem is at the basis of most interpretations of quantum mechanics. In this paper we have summed up four of these interpretations which succeed in avoiding the quantum superposition of macroscopically distinct states, an important element of the traditional measurement problem. Every particular interpretation provides a particular point of view on the traditional measurement problem: (1) in Bohmian mechanics Schrödinger's evolution is not interrupted; replacing the original wave function for its "collapsed" derivative is just a pragmatic affair; (2) in decoherence the linear Schrödinger equation yields an unavoidable coupling of the quantum system with the surrounding medium, which is not a collapse but looks and smells as if it were; (3) in GRW collapses result from localizations; and (4) in SPA collapses result from jumps to preferential states.

By contrast, no different interpretations of quantum mechanics are invoked to account for TBSS, as if the corresponding measurement problem were immune to the different interpretations of the theory. We have shown, however, that at least one interpretation of quantum mechanics does not account for TBSS.

Every proposed solution to the measurement problem should apply to both measurement problems: the traditional and that implied in TBSS. A solution to just one of them is not good enough.

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