Chapter

Nature of Dark Energy

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Abstract

When supernova observations in the end of the 1990s showed the cosmic expansion to be accelerating, it became necessary to reintroduce the cosmological constant Λ as a fitting parameter. Although its physical origin has remained a mystery, it has generally been interpreted as some kind of energy field referred to as "dark energy." This interpretation however implies a cosmic coincidence problem because we happen to live at a time when dark energy becomes the dominant driver of the expansion. Here we present an alternative explanation: The Λ term is induced by a global boundary constraint that ties its value to the conformal age of the universe. The cosmic coincidence problem then goes away. We illustrate how the cosmological evolution that is implied by this constraint differs from standard cosmology. Without the use of any free parameters, the theory predicts a present value of Λ that is within 2σ from the value derived from CMB observations with the Planck satellite. The universe is found to be mildly inflationary throughout the entire radiation-dominated era. This obviates the need to postulate a hypothetical, violent grand unification theory (GUT) era inflation to explain the observed large-scale homogeneity and isotropy of the universe.

Keywords: dark energy, cosmology, theory, inflation, gravitation, early universe

1. Introduction

The term "dark energy" refers to the cosmological constant Λ when interpreted as some kind of mysterious energy field that pervades space and exerts a negative pressure, which is the source of the observed accelerated expansion of the universe. Einstein [1] introduced the cosmological constant in 1917 to allow for a static universe but considered it a blunder after the cosmic expansion was discovered. It was only after the discovery of the accelerated expansion in the end of the 1990s through the use of supernovae type Ia as standard candles [2, 3] that it became necessary to reintroduce Λ as a fitting parameter to allow the observations to be modeled. Its physical nature has however been enigmatic and elusive. In particular the observed magnitude of Λ appears to make us "privileged observers," because we happen to live at a time when dark energy starts to dominate over the energy densities of matter and radiation, thereby causing the onset of an inflationary phase of the universe that will continue forever. This is often referred to as the "cosmic coincidence problem."

Dark energy is widely regarded as one of the biggest problems in contemporary physics (for a review, cf. [4]). All conceivable ways to modify gravity have been tried. Different approaches to model the observational data have been explored, e.g. [5]. Elaborate laboratory experiments have been performed in the search for new scalar fields that would modify gravity [6]. On top of this, evidence against the earlier interpretation of the supernova observations in terms of dark energy has been discovered [7].

Recently [8] it was shown that there is an alternative way to explain the need for a cosmological constant, namely, as the result of a global cosmic boundary constraint instead of through the introduction of some new physical field. This approach leads to a new cosmological framework that brings a resolution to several outstanding enigmas, including the cosmic coincidence problem. Without the use of any free parameters, Λ is predicted to have a present value that is within 2σ from the value that has been determined from CMB data with the Planck satellite [9]. The evolution of the scale factor that is derived with the new theoretical framework shows that the universe has been in a mildly accelerating, inflationary phase throughout all of the radiation-dominated era since the beginning of the Big Bang. This automatically explains the observed large-scale homogeneity and isotropy of the universe without any need to postulate a hypothetical violent inflationary phase in the grand unification theory (GUT) era of the early universe.

In Section 2 we review the arguments that have been presented in [8] for the origin of the global constraint that governs the value of Λ. These arguments depend on the participatory role of observers in the universe for the needed definition of cosmic time, with the split between past and future and the distinction between dynamic time and nonlocal (look-back) time. This will be clarified in Section 3. The mathematical equations of the new cosmological framework are derived and solved in Section 4, where we also illustrate how the cosmic evolution differs from that of standard cosmology. In Section 5 we show how inflation emerges as a natural part of cosmic history throughout the radiation-dominated era, thereby eliminating the causality problem without the assumption of any new fields. The conclusions are summarized in Section 6.

2. Resonant origin of the Λ term

In standard cosmological models, the universe is assumed to be homogeneous and isotropic on the largest scales, because this is what observations tell us. The cosmological evolution can then conveniently be described in terms of a scale factor $a(t)$ that only depends on time t. If we further assume zero spatial curvature, the metric can be expressed as

$$\mathrm{d}s^2 = -c^2\mathrm{d}t^2 + a(t)^2\left(\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega\right), \tag{1}$$

where r is the comoving distance and $\mathrm{d}\Omega$ is the surface element on the unit sphere. While observations show that there is no significant spatial curvature at the present epoch, there is also a theoretical justification for the validity of the flatness assumption, which emerges within the framework of the alternative cosmology of the present work. This will be clarified in Section 4.2.

Besides "proper time" t, we will need to make use of two other time concepts: "conformal time" η and "Euclidian conformal time" τ. The relation between them is defined by

$$\mathrm{d}\tau \equiv i\,c\,\mathrm{d}\eta \equiv i\,c\,\mathrm{d}t/a. \tag{2}$$

In terms of the temporal coordinates η and τ, the metric becomes

$$\begin{aligned} \mathrm{d}s^2 &= a(\eta)^2\left(-c^2\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\Omega\right),\\ \mathrm{d}s^2 &= a(\tau)^2\left(\mathrm{d}\tau^2 + \mathrm{d}r^2 + r^2\,\mathrm{d}\Omega\right). \end{aligned} \tag{3}$$

The conformal metric of the first of these two equations shows that the metric coefficients are proportional to $\eta_{\mu\nu}$, the Minkowski metric: $g_{\mu\nu} = a^2\eta_{\mu\nu}$. "Conformal" means that all angles and shapes of trigonometric functions are preserved in spite of the nonlinear temporal dependence of the scale factor a. Fourier decompositions are only meaningful within the conformal framework.

The word "Euclidian" as the term for the second metric in Eq. (3) does not refer to the flatness assumption but to the signature of the metric: $(++++)$ instead of the $(-+++)$ signature when using time t or η. Since the τ coordinate then formally behaves like a spatial coordinate, we have incorporated the speed of light c in the definition of τ in Eq. (2), to let τ have the dimension of space.

The transformation to Euclidian spacetime leads to remarkable advantages and insights, which have found important applications in various areas in the form of Euclidian field theory, e.g., in solid-state physics [10]. The Hamiltonian in ordinary spacetime becomes the Lagrangian in Euclidian spacetime. Quantum field theory QFT in Euclidian spacetime has the structure of statistical mechanics in ordinary spacetime. The oscillating phase factors in QFT become the Boltzmann factors, while the path integral becomes the partition function. Euclidian spacetime has long been known to provide a direct and elegant route to the derivation of the Hawking temperature of black holes, cf. [8, 11].

In the following we will show how the oscillating phase factors of the Euclidian metric field contain a resonance that fixes the value of the cosmological constant Λ. Our starting point is the Einstein equation with cosmological constant, written in the form

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right). \tag{4}$$

This is the appropriate form to be used with the weak-field approximation, because the right-hand side of Eq. (4) represents the source term for gravitational waves when making a Fourier expansion, while the left-hand side describes the evolution of the vacuum fields, cf. [12]. We have here adopted the standard sign convention with $(-+++)$ for the spacetime signature and a plus sign in front of the right-hand side.

In the weak-field approximation and the harmonic gauge, $R_{\mu\nu} \approx -\frac{1}{2}\partial^2 g_{\mu\nu}$. The d'Alembertian operator $\partial^2 \equiv \Box^2 \equiv -(1/c^2)\partial^2/\partial t^2 + \nabla^2$. For the metrics of Eq. (3) the nabla operator in the d'Alembertian vanishes, because the spatial gradients can be disregarded on cosmological scales. The vacuum fields that represent the left-hand side of Eq. (4) then have the following weak-field representations in terms of the coordinates η and τ:

$$R_{\mu\nu} - \Lambda g_{\mu\nu} \approx \frac{1}{2c^2}\frac{\partial^2 g_{\mu\nu}}{\partial\eta^2} - \Lambda g_{\mu\nu} = -\frac{1}{2}\left(\frac{\partial^2 g_{\mu\nu}}{\partial\tau^2} + \frac{\omega_\Lambda^2}{c^2}\,g_{\mu\nu}\right). \tag{5}$$

While the vacuum fields without physical sources (the $T_{\mu\nu}$ fields) describe a de Sitter exponential evolution of the scale factor a when ordinary conformal time η is used as the temporal coordinate, they describe an anti-de Sitter-like universe when the τ coordinate is instead used. In this description the exponential evolution gets replaced by oscillating phase factors. To make it explicit that the τ representation leads to oscillating solutions, we have expressed it in terms of the oscillation frequency ω_Λ to give it the form of the equation for a harmonic oscillator. We have divided the frequency with c in Eq. (5) to make it a wave number, because τ was defined in terms of spatial units for reasons of symmetry with respect to the other three Euclidian coordinates. Nevertheless it is more appropriate to refer to the resonance in terms of a temporal frequency ω_Λ rather than a wave number, because it turns out to be related to the bounded nature of the observable timeline.

With the period of the oscillation given by $\eta_\Lambda = 2\pi/\omega_\Lambda$, the relation between the cosmological constant Λ, the oscillation frequency ω_Λ, and the period η_Λ, immediately follows from Eq. (5):

$$\Lambda = \frac{1}{2}\frac{\omega_\Lambda^2}{c^2} = 2\left(\frac{\pi}{c\eta_\Lambda}\right)^2. \tag{6}$$

In standard cosmology Λ in Eq. (4) is generally moved to the right-hand side, where it can be interpreted as a mass-energy density ρ_Λ. It is convenient to describe it in terms of the dimensionless parameter Ω_Λ, which is the fraction of the critical density ρ_c that is contributed by the Λ term:

$$\rho_\Lambda \equiv \Omega_\Lambda \rho_c = \frac{c^2\Lambda}{8\pi G}. \tag{7}$$

ρ_c represents the mean mass density that defines the boundary between open and closed model universes according to the Friedmann equations.

$$\rho_c = \frac{3}{8\pi G t_H^2}, \tag{8}$$

where $t_H = 1/H$ is the Hubble time and H the Hubble constant.

From Eqs. (6)–(8) we obtain

$$\frac{\eta_\Lambda}{t_H} = \frac{2\pi}{\sqrt{6\Omega_\Lambda}}. \tag{9}$$

Inserting the value of $\Omega_\Lambda = 0.685$ determined from observations with the Planck satellite [9], we get $\eta_\Lambda/t_H \approx 3.10$. As will be explicitly confirmed by the numerical solutions in Section 4, this implies a value of η_Λ that is nearly identical to the current conformal age η_u of the universe. The distance $r_u = c\eta_u$ is the radius of the particle horizon, the maximum distance to which an observer is causally connected. η_u is the time that it would take for a photon to travel this distance if the universe would stop expanding. As the spatial points from which light is emitted continually recede from us due to the cosmic expansion, η_u is substantially larger than the "proper age" t_u of the universe.

2.1 Link between Λ and the age of the universe

In standard cosmology Λ is a constant that should have nothing to do with the current age of the universe. This is contradicted by our finding that $\eta_\Lambda \approx \eta_u$. Any other value would be in conflict with the observed magnitude of the cosmic acceleration. If Λ were independent of the age of the universe, then η_Λ would have been many orders of magnitude larger than η_u in the past and will be many orders of magnitude smaller in the future. It would then be an extraordinarily improbable coincidence if they happen to be the same in the present epoch. This gives us strong reasons to suspect that the value of Λ is indeed physically tied to the age η_u of the universe.

The existence of such a physical link means that we need to single out, among all the solutions of the oscillator equation in Eq. (5), the Fourier component with a wavelength that corresponds to the conformal age of the universe. This only makes sense if time is bounded between the Big Bang and the Now, which seems to contradict the Einsteinian view that all future times somehow "already preexist" and that the experienced split between past, present, and future is just a stubborn illusion. Here we will argue (for details, see Section 3) that the Einsteinian view only makes sense in a universe devoid of observers and that this is not the universe that we inhabit. Like in quantum physics the observer plays a fundamental role in defining the nature of reality. The split between past, present, and future is not some illusion that we need to come to terms with, but is deeply physical. As soon as we introduce an observer (which can be a test particle, without brains or consciousness!) in Einstein's universe, the split occurs. In any observable universe the future does not exist, even in principle. The only accessible region is between the Big Bang and the Now, and this region is bounded. The theory has to be applied to the observable universe, not to some idealized universe without observers. This is not just some alternative philosophical viewpoint but has profound physical consequences. It leads to a very different cosmological framework, as will be made clear in the following sections.

The existence of a metric resonance with respect to Euclidian time τ implies that

$$g_{\mu\mu} \sim a(\tau)^2 \sim e^{\pm i\,\omega_u\,\tau/c}. \tag{10}$$

Note that the Euclidian metric and scale factor have here been treated like a quantum field by allowing them to have an analytical continuation into the complex plane. When we however convert back to ordinary conformal time η by replacing τ with $ic\eta$, the oscillating phase factor transforms into an exponentially evolving factor and thereby becomes real-valued. Both exponentially decaying and increasing solutions are possible because of the $\pm$ in Eq. (10). With the initial boundary condition that the scale factor was small at early times, we can reject the decaying solution. This leaves us with the exponentially increasing de Sitter expansion of the scale factor. It is driven by the ω_u resonant parameter, which can be expressed in terms of Λ via Eq. (6). It agrees with the observed value of Λ, because as found in the previous subsection, the magnitudes of η_Λ and η_u are the same.

2.2 Resonant amplitude and the validity of the weak-field approximation

According to Euclidian field theory, the oscillating QFT phase factors in Euclidian spacetime become Boltzmann factors in ordinary spacetime, if the field has periodic boundary conditions. When interpreted as due to a cosmic resonance, our finding that $\eta_\Lambda \approx \eta_u$ implies the existence of a periodic boundary condition with period τ_u in conformal Euclidian time. Euclidian field theory then allows us to make the identification

$$e^{i\,\omega_u\,\tau_u/c} = e^{-\omega_u\,\eta_u} \equiv e^{-\hbar\,\omega_u/(k_B T_u)}. \tag{11}$$

It gives us the temperature T_u that is induced because the time string is bounded:

$$T_u = \frac{\hbar}{k_B\,\eta_u}. \tag{12}$$

The identical result can be obtained with the help of Heisenberg's uncertainty principle. For a system in thermal equilibrium at temperature T, the equipartition theorem tells us that each degree of freedom has energy $\frac{1}{2}k_BT$. We may therefore make the identifications

$$\begin{aligned}\Delta E &\equiv \frac{1}{2}k_BT_u,\\ \Delta t &\equiv \eta_u.\end{aligned} \tag{13}$$

Inserting the value for T_u from Eq. (12) then gives us the Heisenberg relation

$$\Delta E\,\Delta t = \frac{1}{2}\hbar. \tag{14}$$

Alternatively we could have started from Eqs. (13) and (14) to obtain Eq. (12).

Replacing η_u with the Planck time t_P, we obtain the Planck temperature $T_u = T_P$. Using the definitions for the Planck time and mass,

$$\begin{aligned}t_P &= \left(\frac{\hbar G}{c^5}\right)^{1/2} \approx 5.39\times 10^{-44}\,\mathrm{s},\\ m_P &= \left(\frac{\hbar c}{G}\right)^{1/2} \approx 21.8\,\mu\mathrm{g},\end{aligned} \tag{15}$$

it follows from Eq. (12) that

$$T_P = m_Pc^2/k_B \approx 1.42\times 10^{32}\,\mathrm{K}. \tag{16}$$

This comparison serves to demonstrate that the temperature T_u and the mode energy $\hbar\omega_u$ both scale with $1/\eta_u$ throughout cosmic time all the way back to the Planck era. In Planck units the present age η_u of the universe is approximately 10^{61}, which implies that the present value of T_u is only about 10^{-29} K. Energetically this is completely insignificant in comparison with the CMB temperature. The present mode energy $\hbar\omega_u$, which is about 10^{-61} in Planck units, represents the relative amplitude by which the metric is disturbed. As long as it is $\ll 1$, one may use the Newtonian limit to interpret it as a potential energy and is allowed to use the weak-field approximation to describe it. The scaling shows that it remains $\ll 1$ everywhere, except in the nonlinear regime in the immediate vicinity of the Planck era. This tells us that the weak-field approximation is valid for all times later than about 10^{-41} s (when the amplitude was about 0.005).

2.3 Nature of the global constraint for Λ

We have shown how Λ emerges as a result of a boundary condition that exists because time in the observable universe is bounded and have referred to it as a kind of cosmic resonance. At first glance one might think that this would be some sort of cosmic Casimir effect, because the Casimir effect is known to be due to a boundary condition that limits the oscillatory modes that can exist in the quantum vacuum. Thereby measurable forces get induced.

The nature of the boundary condition is however fundamentally different in our Λ theory. The resonances of the Casimir effect are due to Dirichlet boundary conditions, when the oscillations are clamped down at the boundaries. The size of the resonant cavity is then half a wavelength, or π, for the fundamental mode. In contrast, our Λ resonance is governed by a periodic boundary condition with period 2π. Agreement with the observed value of Λ is only possible if the bounded time string has a length that corresponds to 2π. Therefore Dirichlet boundary conditions can be ruled out on observational grounds alone.

The value of Λ is tied to the value ω_u of the cosmic resonance frequency. Since ω_u is due to a global constraint, it is a constant that applies to all of the observable universe at the given epoch. In particular this means that ω_u and Λ do not vary with redshift z, for the same kind of reason that the musical tones that emanate from a violin string are not functions of position along the string. Similarly the resonances of the wave function in atoms are represented by quantum numbers, which do not vary with position within the resonating cavity but characterize the system as a whole.

The choice of observer defines the observable universe and its age. The observer is by definition always located at redshift $z = 0$ and experiences (and therefore also defines) local, dynamic time. In contrast, nonlocal, look-back time (for $z \neq 0$) cannot be experienced by any observer. In our theory Λ varies with dynamic time, but it does not vary with look-back time or redshift z. This implies that there is a fundamental distinction between dynamic time and look-back time, in contrast to standard cosmology. In the next section we will clarify how the boundedness of time and the distinction between local and nonlocal time is a consequence of the participatory role of observers in the universe.

3. The participatory role of observers

Although Einstein's opinion on the split between past, present, and future seems to have been somewhat ambivalent, his most quoted statement on the subject is that this split is an illusion, "but a very stubborn one." He tended to regard all temporal instants along the infinite timeline as somehow already preexisting as part of a 4D map. This map contains both past and future, in spite of the fact that no observer is able to directly experience any other time than what we refer to as "Now." Nevertheless the physical meaning of the concept of "Now" remained elusive to him.

The Einsteinian view of a 4D spacetime that maps all times is meaningful only in a universe devoid of observers. As soon as one introduces an observer, the timeline automatically splits up, because the presence of an observer implies a "Here" and "Now." This split is profoundly physical, because we know from experience that the future is not part of the observable universe. It is not accessible to any observer, even in principle. This is the only universe in which our cosmological theories can be tested, not in some idealized universe devoid of observers, to which nobody can belong.

We are not merely dealing with an alternative philosophical viewpoint, because the introduction of observers leads to a different physical theory with different testable consequences. In the observable universe, time is always bounded, between the Big Bang as one edge and the Now as the other edge. In contrast, in the Einsteinian universe, time is unbounded in the future. The finite temporal dimension allows a global boundary constraint that leads to the emergence of a Λ term in Einstein's equations. It is the cause of the observed acceleration of the cosmic expansion.

A fundamental difference between classical and quantum physics concerns the role of observers. We can introduce test observers in classical physics, but they are not participatory in the way that they are in quantum physics. The classical world represents an objective reality that exists in a form that is independent of the presence of observers. It is the Einsteinian universe. In contrast, the quantum reality comes into existence through the participation of observers. It is the reason for the fundamental quantum fuzziness or uncertainty, the probabilistic causality, and the irreversibility through the collapse of the wave function. While the evolution of the wave function is time symmetric and deterministic, the act of "observation" or "measurement" leads to the profoundly different nature of quantum reality.

Although the role of our cosmological observers is very different from that of quantum theory, the comparison with quantum physics serves to indicate ways in which observer participation profoundly affects the nature of the theory. While abandoning the traditional classical view by allowing observer participation, we transform the theory into something that in at least this respect is closer to the nature of quantum physics. The consequence in our case is that the value of the cosmological constant gets uniquely determined in a way that leads to a very different cosmological framework.

The presence of participating observers also changes our interpretation of spacetime in a profound way by introducing a distinction between local and nonlocal time, a distinction that is absent in a universe without observers. With nonlocal time we here mean the same thing as look-back time. In contrast, dynamical time is the same as local time, because it is the only time that an observer can experience directly. The observables are redshifts, apparent brightnesses, structuring of celestial objects, etc. The observer is by definition always at redshift $z = 0$. With the help of a cosmological model, the observables may in principle be used to infer a look-back time, which represents the way that the spacetime map appears from the vantage point of the observer.

In both standard cosmology and our alternative theory, the value of Λ applies to the totality of the observable universe at the given epoch and is therefore independent of redshift. In standard theory it is also independent of epoch (age) of the universe, while in our alternative theory, it is proportional to $1/\eta_u^2$, where η_u is the conformal age. This implies a different mathematical framework for the new cosmology, which will be developed in the next section.

4. Derivation of the cosmological evolution

The choice of observer defines the age $t = t_u$ of the universe. At proper time t_u, the scale factor is $a_u = a(t_u)$, and the Hubble constant $H = \dot{a}/a$ is $H_u = H(t_u)$. In standard cosmology the evolution of the scale factor $a(t)$, which defines the cosmological model, can be deduced from the observed relation between the expansion rate $\dot{z}/(1+z)$ (Hubble constant) and the redshift z. It is then sufficient to only consider the presently observable universe. In contrast, this is not sufficient in the nonstandard cosmology that will be developed here and which we will refer to as the alternative cosmology (AC) theory in the following. The presence of the global constraint causes the nonlocal time scale (the "look-back" time when $z > 0$) to be different from the local time scale.

The dynamical time scale is the local time scale that is experienced by a comoving observer and which characterizes the age t_u of the universe. To make this distinction clear, we attach index u to all local ($z = 0$) quantities to link them to epoch t_u, i.e., to define which observable universe they refer to.

In both standard cosmology and AC theory, the expansion rate of the universe, as represented by the Hubble constant, is governed by the equation

$$H = H_u E_u(z). \tag{17}$$

z is the redshift, and

$$E_u(z) = \left[\Omega_M(a_u)(1+z)^3 + \Omega_R(a_u)(1+z)^4 + \Omega_\Lambda(a_u)\right]^{1/2} \tag{18}$$

if we assume zero spatial curvature (see Section 4.2 for a justification of this assumption). Since H_u is defined as the local Hubble constant (at $z = 0$), it follows that $\Omega_M + \Omega_R + \Omega_\Lambda = 1$, as required for flatness. $\Omega_{M,R,\Lambda}(a_u)$ represent, respectively, the matter density (including dark matter), radiation energy density, and the "dark energy" density due to the cosmological constant Λ, all in units of the critical mass-energy density. Their values in Eq. (18) refer to the epoch when the scale factor is a_u. The relation between Ω_Λ and Λ is given by

$$\Omega_\Lambda(a_u) = \frac{c^2}{3H_u^2}\,\Lambda_u \tag{19}$$

as follows from Eqs. (7) and (8). In standard cosmology Λ does not depend on a_u, but in AC theory it does.

The scale factor normalized to epoch t_u is

$$y \equiv a/a_u = 1/(1+z). \tag{20}$$

The redshifts z only have a physical meaning when they refer to an epoch t_u (because this epoch is by definition where the observer at $z = 0$ exists). In terms of parameter y, the function E in Eq. (18) becomes

$$E_u(y) = \left[\Omega_M(a_u)y^{-3} + \Omega_R(a_u)y^{-4} + \Omega_\Lambda(a_u)\right]^{1/2}, \tag{21}$$

which satisfies the requirement of Eq. (17) that $E_u = 1$ when $y = 1$ or $z = 0$.

4.1 Key difference between standard cosmology and AC theory

The values of $\Omega_{M,R,\Lambda}$ that refer to the present epoch ($t_u = t_0$) can be determined by observations. In standard cosmology the parameter Λ is a true constant, independent of both redshift and epoch t_u for all times. Eq. (17) then represents a differential equation that determines the complete evolution $a(t)$ of the scale factor, when the current values of $\Omega_{M,R,\Lambda}$ are known. In contrast, in AC theory the magnitude of Λ varies with dynamical time, tracking the radius $r_u = c\eta_u$ of the causal or particle horizon. The tracking property is governed by

$$\Lambda_u = 2(\pi/r_u)^2 \tag{22}$$

according to Eq. (6). It is the fundamental equation that sets AC theory apart from standard theory.

Because the conformal age η_u is given by an integral over all times, Ω_Λ in Eq. (18) is governed by a global integral condition in AC theory. This means that the evolution of the scale factor $a(t)$ is obtained from the solution of an integrodifferential equation. With Eqs. (19) and (22), the relation between Ω_Λ and the conformal age can be expressed in the form

$$\Omega_\Lambda(\eta_u) = \frac{2}{3}\left(\frac{\pi}{x_u}\right)^2. \tag{23}$$

Here the dimensionless parameter x_u is the conformal age in units of the Hubble time $1/H_u$ at the same epoch:

$$x_u \equiv \eta_u H_u. \tag{24}$$

4.2 Theoretical justification for the flatness assumption

While observations support our assumption of vanishing spatial curvature, AC theory requires it on theoretical grounds, in contrast to standard theory. Since curvature is induced by the presence of matter-energy sources, which may include the vacuum energy ρ_Λ from a cosmological constant, an empty universe without any such sources must have zero spatial curvature. In AC theory not only the matter and radiation energy densities go to zero in the distant future but also the energy density due to the Λ term. It vanishes when the horizon radius r_u goes to infinity according to Eq. (22). At temporal infinity the universe is therefore empty, which implies flatness. When the curvature vanishes at a temporal boundary, it will remain zero for all other epochs. In contrast, in standard cosmology the density of "dark energy" (as represented by Λ) never vanishes but dominates the future dynamics. As the universe therefore never will be empty, the curvature is not constrained to be zero.

4.3 Iterative solution of the basic equations

Because the value of the conformal age η_u depends on the value of Ω_Λ, Eq. (23) is coupled to Eq. (17) in a way that most conveniently gets solved by a straightforward iteration procedure. It is found to deliver a unique value of Ω_Λ for any given value of η_u or scale factor a_u, without numerical complications. In particular, the solution for the present epoch is $\Omega_\Lambda = 67.2\%$, which is within 2σ from the value $68.5 \pm 0.7\%$ that has recently been derived from observational data with the Planck satellite [9]. It would be strange if this remarkable agreement, obtained without the use of any free parameters, would merely be a fortuitous "coincidence."

From the relation $H = \dot{a}/a$ and Eq. (17), we obtain the conformal and proper ages η_u and t_u. For convenience we express them in terms of the dimensionless functions x_u (which was already introduced in Eqs. (23) and (24)) and g_u through normalization with the Hubble time $1/H_u$:

$$\begin{aligned} x_u &\equiv \eta_u H_u = H_u \int_0^{t_u} \frac{\mathrm{d}t}{(a/a_u)} = \int_0^1 \frac{\mathrm{d}y}{y^2 E_u(y)}, \\ g_u &\equiv t_u H_u = H_u \int_0^{t_u} \mathrm{d}t = H_u \int \frac{\mathrm{d}a}{aH} = \int_0^1 \frac{\mathrm{d}y}{y E_u(y)}. \end{aligned} \tag{25}$$

To find the x_u that is needed to determine Ω_Λ via the global constraint of Eq. (23), we need to know the correct $E_u(y)$ function to be used in Eq. (25). This function however depends on the value of Ω_Λ that we want to determine. The solution can readily be obtained by iteration as follows: (i) Assume a starting value for Ω_Λ, which allows $E_u(y)$ to be defined (as clarified below). (ii) Use this $E_u(y)$ function to solve Eq. (25) for x_u, which can be inserted in Eq. (23) to obtain a new value for Ω_Λ. Insert the result in step (i) as the new starting value, and repeat the procedure until convergence. This simple iteration procedure does not encounter any numerical problems and converges quickly.

The E_u function that is used in this iteration depends not only on the value of Ω_Λ but also on the values of $\Omega_{M,R}(a_u)$ for the chosen epoch, which we define in terms of the value of the scale factor $a_u = a(t_u)$. The starting values of $\Omega_{M,R}(a_u)$ for the iteration depend on the starting value for Ω_Λ, because the flatness condition $\Omega_M(a_u) + \Omega_R(a_u) + \Omega_\Lambda(a_u) = 1$ has to be satisfied. Let us next outline how these starting values are determined.

First of all, the value of Ω_R for the radiation energy density is directly constrained by observations, because its value at the present epoch ($a_u = a_0 = 1$) is fixed by the observed values of the CMB temperature and the Hubble constant H_0. With the assumption of zero spatial curvature, the present value of Ω_M then follows from the value of Ω_Λ, because $\Omega_M = 1 - (\Omega_R + \Omega_\Lambda)$.

Ω_R is the fraction of the critical energy density $\rho_c c^2$ that is in the form of radiation energy u_R (due to photons and neutrinos):

$$\Omega_R = \frac{u_R}{\rho_c c^2}, \tag{26}$$

where

$$u_R = a_T T^4 \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_\nu\right] = 1.681\, a_T T^4 \tag{27}$$

cf. [13]. $N_\nu = 3$ is the number of neutrino families, while $T = 2.725$ K is the measured temperature of the cosmic microwave background, and a_T is Stefan's constant.

When going to a different epoch with a different a_u, we change the a/a_u normalization in Eq. (20) for the scale factors a and the associated redshift scale z. Then the values of Ω_M and Ω_R must also change, because they refer to $z = 0$. Since $\Omega_M \rho_c \sim a_u^{-3}$ while $\Omega_R \rho_c \sim a_u^{-4}$, the ratio Ω_R/Ω_M scales as $1/a_u$.

During the iteration we enforce the correct a_u scaling of the $\Omega_R(a_u)/\Omega_M(a_u)$ ratio and the condition for spatial flatness, which together define the correct starting values for $\Omega_M(a_u)$ and $\Omega_R(a_u)$, once a starting value for $\Omega_\Lambda(a_u)$ has been chosen. In the case of standard cosmology, there is no iteration, because the scaling of Ω_Λ relative to $\Omega_{M,R}$ is already known. For instance, the ratios $\Omega_\Lambda/\Omega_M \sim a_u^3$ and $\Omega_\Lambda/\Omega_R \sim a_u^4$ both imply that the Λ term was insignificant in the past but dominates in the future. In contrast, in AC theory the relative contribution of Λ does not change much throughout cosmic history. At epoch a_u in standard theory, $\Omega_\Lambda(a_u) = \Omega_\Lambda/\left(\Omega_M a_u^{-3} + \Omega_R a_u^{-4} + \Omega_\Lambda\right)$, where the Ωs on the right-hand side refer to their values at the present epoch ($a_u = a_0 = 1$). Similarly, for the matter density, we have $\Omega_M(a_u) = \Omega_M a_u^{-3}/\left(\Omega_M a_u^{-3} + \Omega_R a_u^{-4} + \Omega_\Lambda\right)$, and correspondingly for the radiation energy density.

Besides $\Omega_{M,R,\Lambda}(a_u)$, $E_u(y)$, and x_u, the converged iterative solution gives us g_u from Eq. (25), which is needed for the completion of the derivation of the expansion history $a(t_u)$ of the universe, as we will see below. The whole procedure is repeated for whatever set of scale factors $a_u = a(t_u)$ that we have chosen. Here we have done the calculations for equidistant steps in $\log(a_u)$ from -12 to $+4$, on a scale where $\log a_u = 0$ corresponds to the present epoch.

The scale factor a_{eq} at equipartition between the energy densities of matter and radiation is given by

$$a_{eq} = \Omega_R(a_0)/\Omega_M(a_0). \tag{28}$$

We further note that the scale factor uniquely determines the temperature of the cosmic radiation background through

$$T_u = 2.725/a_u, \tag{29}$$

which is valid back to a temperature $T_u \approx 10^9$ K. Note that T_u is defined to represent the temperature of the photons. The numerical factor (in units of K) is fixed by the observed value of the CMB temperature at $a_u = 1$. Above $T_u \approx 10^{10}$ K the scaling with a_u is the same, and T_u is identical to the neutrino temperature T_ν, but the proportionality factor is about 40% smaller. Between approximately 10^{10} and 10^9 K, the positrons annihilate with the electrons, which leads to the release of energy in the form of gamma radiation that heats the photon gas without affecting the neutrino background. This is the reason why the photon temperature T_u has since been 40% larger than that of the neutrinos. The distinction between T_u and T_ν is of relevance for Big Bang nucleosynthesis (BBN) calculations.

Because T_u scales with $1/a_u$ according to Eq. (29) and u_R scales with T_u^4 according to Eq. (27), the radiative energy density $\Omega_R(a_u)\rho_c(a_u)$ scales with $1/a_u^4$ as required. By enforcing the $\Omega_R(a_u)/\Omega_M(a_u)$ ratio to scale as $1/a_u$ during the iteration, we are guaranteed to get the correct $1/a_u^3$ scaling for the energy density of matter $\Omega_M(a_u)\rho_c(a_u)$.

In **Figure 1** the parameters x_u (left panel) and g_u (right panel) have been plotted as functions of $\log a_u$ for AC theory (solid curves) and for standard cosmology (dashed curves). The left vertical dotted line in each panel marks the epoch of equipartition between matter and radiation, while the right dotted line represents the present epoch (when the scale factor is normalized to unity). Note how according to standard theory we happen to live at a special time when the x_u and g_u ratios are beginning to skyrocket. In contrast, in AC theory these ratios are constant at levels that are different when the universe is radiation and matter dominated, with a transition from one level to the other between the epoch of equipartition and the present time.

4.4 Solution for the time scale

The next step of the calculation is to use the solution for g_u to derive the functions for the epochs $t_u(a_u)$ and $\eta_u(a_u)$, the expansion rate $H_u(a_u)$, and the acceleration parameter $q_u(a_u)$.

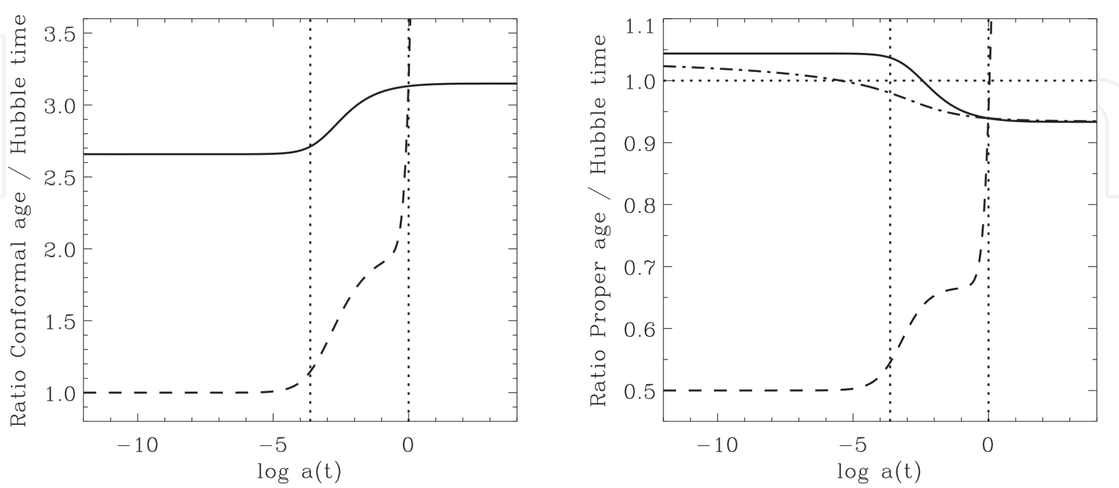

Figure 1.
The left panel shows x_u*, defined by Eq. (24) as the ratio between the conformal age* η_u *and the Hubble time* $1/H_u$*, plotted vs. log of the scale factor* $a(t)$*, while the right panel gives the corresponding plot for* g_u*, which is defined by Eq. (25) as the ratio between the proper age* t_u *and the Hubble time* $1/H_u$*. In both panels the AC theory is represented by the solid curve, the standard theory by the dashed curve. The two vertical dotted lines mark the epochs of equipartition and our present time. The dash-dotted curve in the right panel represents the exponent* α *in the power law representation of the scale factor in Eq. (33). According to standard theory, the current epoch marks the beginning of an inflationary phase that will last forever.*

The proper age t_u can be obtained through integration of the function g_u. First we realize that the defining equation for g_u in Eq. (25) can be expressed as

$$\frac{\mathrm{d}\log a_u}{\mathrm{d}\log t_u} = H_u t_u = g_u. \tag{30}$$

It can be solved by integration to obtain the proper age t_u of the universe as a function of scale factor a_u:

$$\log t_u = \int_0^{\log a_u} (1/g_{u'})\,\mathrm{d}(\log a_{u'}) + \log t_0. \tag{31}$$

The present age t_0 is obtained from the observed value H_0 of the Hubble constant and the value of $g_u(a_0) \equiv g_0$ for the present epoch through

$$\log t_0 = \log g_0 - \log H_0, \tag{32}$$

which readily follows from the definition of g_u in Eq. (25).

The left panel of **Figure 2** shows $\log a_u$ as a function of $\log t_u/t_0$. AC theory is represented by the solid curve, standard cosmology by the dashed curve. The horizontal dotted line marks the scale factor at equipartition between matter and radiation. The slope of the AC evolution in the log-log representation is nearly constant throughout all epochs, both in the past and the future. There is nothing special about our present epoch. In contrast, according to standard theory we happen to live at the start of an inflationary phase that will be everlasting, driven by some mysterious "dark energy."

Note also that the evolutionary time scales are quite different in the two theories. While both curves coincide at the present epoch, simply because they share the same normalization $a_u = 1$ at $t_u = t_0$, the age difference diverges as we go back in time or forward into the future.

Since the AC evolution is so close to linear in the log–log diagram, it is meaningful to represent it in the form of a power law:

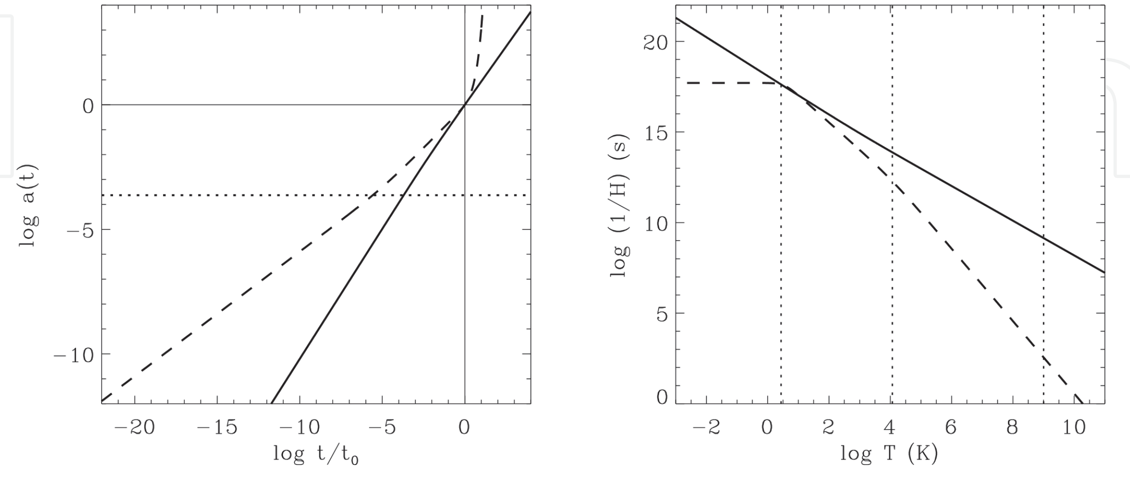

Figure 2.
In the left panel, the log of the scale factor $a(t)$ is plotted vs. log of proper time t in units of the present age t_0 of the universe. In the right panel, the log of the Hubble time in seconds is plotted vs. log of the temperature (K) of the cosmic background of electromagnetic radiation. The solid curves in both panels represent the evolution according to AC theory, while the dashed curves represent standard theory. The horizontal dotted line in the left panel marks the epoch of equipartition. The three vertical dotted lines in the right panel mark the temperatures of the present epoch, equipartition, and 1 GK (the approximate onset of nucleosynthesis). Note how in standard theory the evolution has an abrupt change at the present epoch with the onset of an inflationary phase.

$$a_u = (t_u/t_0)^{\alpha(a_u)}. \tag{33}$$

For clarity we have explicitly written the exponent α as a function of the scale factor a_u (which implies that it is also a function of time). As the function $a_u(t_u)$ is known from Eq. (31) and **Figure 2**, the functional form of α is given by

$$\alpha = \frac{\log a_u}{\log (t_u/t_0)}. \tag{34}$$

Comparison with Eq. (30) shows that α would be the same as our dimensionless function g_u if α were a true constant, independent of a_u and t_u. Since however g_u varies with a_u, the functions $\alpha(a_u)$ and $g_u(a_u)$ differ. This is illustrated in the right panel of **Figure 1**, where the function $g_u(a_u)$ as the solid curve is compared with the function $\alpha(a_u)$ as the dash-dotted curve (while g_u for standard cosmology is given by the dashed curve).

Since g_u in AC theory remains constant in the future, it coincides with the α function there, as expected. However, as we go back in time, there is a transition of g_u to a higher level, which is reached around the time of equipartition. Because of this variation, the α function initially diverges from g_u but approaches it again asymptotically as we go to ever earlier times.

Overall the temporal variations of α and g_u are very modest in AC theory, as expected from the nearly linear behavior in the left panel of **Figure 2**. In contrast, the variations are quite dramatic in standard cosmology, according to which a veritable "explosion" occurs at the present epoch, when the universe takes off in an exponential, inflationary phase.

Note that the level $\alpha = 1$, which is marked by a horizontal dotted line in the right panel of **Figure 1**, corresponds to a linear a_u vs. t_u relation with zero acceleration. Below this level we have deceleration, above it acceleration. The circumstance that g_u and $\alpha(a_u)$ in AC theory remain larger than unity in most of the radiation-dominated era of the early universe implies that the universe evolved with an accelerated expansion throughout this time. This mirrors the behavior of the acceleration parameter q_u, which will be derived and displayed in Section 5 and **Figure 3**.

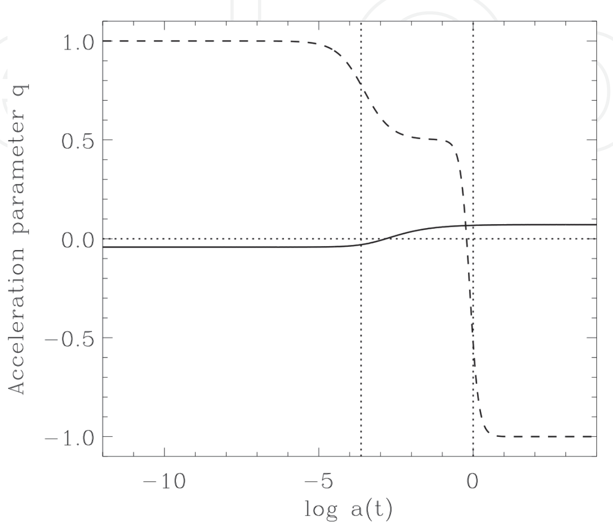

Figure 3.
Cosmic acceleration parameter q_u vs. $\log a_u$ for AC theory (solid) and standard cosmology (dashed). The vertical dotted lines mark the epochs of matter-radiation equipartition and the present time. Positive values of q imply deceleration, negative values acceleration (inflation). Note that according to AC theory the universe has been accelerating throughout the entire radiation-dominated era.

4.5 Solution for the expansion rate

Similar to Eq. (32) we obtain from the definition of g_u in Eq. (25) the expansion rate H_u as a function of $\log t_u$:

$$\log H_u = \log g_u - \log t_u. \tag{35}$$

Alternatively we may replace time t_u by the scale factor a_u via the power law description of Eq. (33), to obtain the same result in the form

$$\log H_u = -\frac{1}{\alpha(a_u)} \log a_u + \log g_u - \log t_0. \tag{36}$$

Knowing both H_u and x_u, we then get the conformal age η_u of the universe as a function of $\log a_u$ directly from Eq. (24).

In the right panel of **Figure 2**, the Hubble time $1/H_u$ is plotted vs. $\log T_u$ for AC theory (solid curve) and standard cosmology (dashed curve). The three vertical dotted lines represent, from left to right: the present epoch (T = 2.725 K), the epoch of equipartition between matter and radiation, and the BBN epoch when the radiation temperature is 10^9 K. This is the approximate temperature below which photodissociation of deuterium no longer stands in the way for Big Bang nucleosynthesis. Note how the standard theory curve has an abrupt bend at our present epoch because of the onset of an inflationary expansion. In contrast the AC theory curve remains nearly linear for all epochs, with nothing particular happening at the present epoch.

While the solid and dashed curves for $1/H_u$ coincide at the present epoch, because they obey the same observational constraint, the standard theory curve immediately diverges from the AC curve in both the future and past directions. In standard theory the expansion rate will be much faster in the future and was also much faster in the past, as compared with AC theory. This expresses the same property that was seen in the left panel of **Figure 2** for the evolution of the scale factor a. When the temperature was 10^9 K, around the BBN epoch, the age of the universe was 158 s or 2.6 min in standard cosmology, while it was 43.5 yr in AC theory, a difference by a factor of 10^7. Instead of referring to "the first 3 minutes" as the time relevant for the formation of the light elements, we would in AC theory need to refer to "the first century!"

This huge difference has major implications for our understanding of BBN physics. At a first glance, it might seem that it would make AC theory incompatible with the constraints imposed by the observed abundances of the light chemical elements, because the BBN predictions depend on the value of the expansion rate. However, a closer look at the BBN problem shows that the situation is much more complex, because we are in a totally different regime. AC theory may still be compatible with the observational constraints, but this remains an open question. At the time of writing, the required BBN modeling with AC theory is still work in progress.

Similarly the significantly slower expansion rate in AC theory around the epochs of equipartition and recombination will require a reevaluation of the processes that govern the formation of the CMB spectrum. This is needed to allow AC theory to be confronted with the constraints that are imposed by the observed CMB signatures.

5. Natural inflation without new fields

Let us next determine the cosmic acceleration parameter q_u in AC theory. The first step is to extract the time derivative $\dot{a}_u$ of the scale factor from the Hubble constant H_u:

$$\log \dot{a}_u = \log a_u + \log H_u. \tag{37}$$

Its derivative with respect to $\log t_u$ then gives us the acceleration parameter

$$q_u \equiv -\frac{\ddot{a}_u a_u}{\dot{a}_u{}^2} = -\frac{1}{g_u}\frac{\mathrm{d}\log \dot{a}_u}{\mathrm{d}\log t_u}. \tag{38}$$

In contrast, in standard cosmology q is obtained directly (without any use of index u) as a function of scale factor a via the relation $q(a) = [H_0/H(a)]^2 (0.5\Omega_M a^{-3} + \Omega_R a^{-4} - \Omega_\Lambda)$.

Figure 3 shows how standard theory (represented by the dashed curve) has three distinct levels for q. (i) In the early universe, when the universe is radiation dominated, $q = 1$. (ii) After equipartition, there is a transition to a new level ($q = 1/2$), when the universe is matter dominated. (iii) At the present epoch, there is a rapid transition to an inflationary phase that is driven by the dominating "dark energy," at the level $q = -1$.

In AC theory there is only a gentle transition around equipartition from a level of $q_u = -0.042$ when radiation dominates over matter, to a level of $q_u = +0.071$ when matter dominates over radiation. The negative level of q implies that the cosmic expansion was in an accelerated phase from the beginning of the Big Bang throughout the entire radiation-dominated era. This could be concluded already from the analysis of the right panel of **Figure 1** for g_u and $\alpha(a_u)$. Although this accelerated expansion represents a very mild form of inflation, its inflationary effect is nevertheless large, because it persists and accumulates over such a long period. It thereby accomplishes what the postulated violent inflation in the brief GUT era does. In AC theory the radiation era inflation is not postulated. Its magnitude is not a free fitting parameter but a consequence of the global resonance condition, which is the origin of the cosmological constant.

It may seem confusing that the universe is currently accelerating according to standard cosmology, while both **Figure 3** and the right panel of **Figure 1** show it to be decelerating according to AC theory. The reason is that q in standard theory refers to an apparent, nonlocal acceleration, while q_u in AC theory is the physically relevant local acceleration of the scale factor $a_u(t_u)$ in terms of the t_u time scale. This scale is different from the look-back time scale, because Λ varies with t_u.

When we throughout this chapter have referred to the "observed acceleration" of the cosmic expansion, we have implicitly meant the acceleration that is inferred when the observational data are interpreted with the Friedmann-Lemaître models, because no other framework has been available for describing the observations in terms of an evolving scale factor. The discovery with the supernova observations was that a positive cosmological constant Λ is needed to interpret the data in terms of the standard model and its magnitude could be inferred. Within this framework the inferred value of Λ means that the expansion is accelerating. However, the identical observational data with the same current value for Λ do not imply an acceleration of the local $a_u(t_u)$ scale factor within the AC theory framework. Instead the consistently derived $a_u(t_u)$ function implies a deceleration at the present epoch.

The inference of an acceleration from redshift data depends on the way in which redshift z scales with look-back time as governed by Eq. (17). AC theory does not provide any alternative definition of "look-back time." It instead explains that the physically relevant time scale is that of t_u, the age of the observable universe. This scale represents the local, dynamic time scale that is experienced by an observer (who is always located at $z = 0$). The cosmological constant in AC theory depends on t_u in contrast to standard cosmology, while being independent of redshift in both theories. Therefore the local acceleration of $a_u(t_u)$ cannot be derived from redshift observations. In summary: the acceleration inferred from supernova observations is an apparent acceleration. The physically relevant acceleration is the one that refers to the local, dynamic time scale, which is obtained with AC theory.

In standard cosmology an inflationary period in the early universe has been postulated to provide a solution to two fundamental cosmological problems: the horizon and the flatness problem [14]. The remarkable smoothness of the observed CMB tells us that the universe was homogeneous and isotropic on large scales to an extremely high degree (of order 10^{-5}) at the time of decoupling ($z \approx 10^3$). Unless one assumes extraordinarily special and improbable initial conditions, such a smoothness and isotropy can only happen, if regions in the CMB with large angular separation have been in causal contact, to allow them to interact and homogenize.

In a decelerating universe, the radius of the cosmic horizon (e.g., the Hubble radius c/H) increases faster than the expansion of space. If the dynamics of the universe were exclusively governed by matter and radiation, as in the Friedmann models without any cosmological constant, then the universe would always be decelerating. A convenient way to describe this is in terms of the "comoving Hubble radius" r_H, defined as $c/(aH)$. The temporal derivative of aH equals the acceleration of the scale factor: $\mathrm{d}(aH)/\mathrm{d}t = \mathrm{d}^2 a/\mathrm{d}t^2$. This means that when the acceleration is negative (deceleration), aH decreases, and therefore its inverse (the comoving Hubble radius) increases, and vice versa if we reverse the signs.

The described properties are illustrated in **Figure 4**, where we have plotted the comoving Hubble radius r_H as a function of log of the scale factor for standard cosmology (dashed curves) and AC theory (solid curves). Before the present epoch (marked by the dotted line), r_H in the standard theory increases steeply, as a consequence of the gravitational deceleration. This deceleration is caused by the gravitational force from the radiation energy before the epoch of equipartition (marked by the vertical dash-dotted line) and by matter afterwards. Therefore the slope of the dashed curve changes around equipartition. Near the present epoch, the negative pressure from the cosmological constant begins to dominate, which marks the beginning of a phase of eternal acceleration. This causes the dashed curve to abruptly turn over and decrease steeply. Along the entire cosmic timeline, the present epoch is singled out as the epoch when this abrupt turnover takes place.

Causal contact is only possible over distances that are smaller than the comoving horizon size. As seen by the dashed curve in **Figure 4**, the largest scales that we observe today (of order 10 Gly, the approximate present Hubble radius) only came into causal contact very recently, well after the time of recombination ($a \approx 10^{-3}$). It means that they did not have time to interact and thermalize on the Hubble time scale. This makes it a mystery to standard theory why there are such strong correlations in the CMB over regions on the sky with wide angular separations.

To solve this problem, an early inflationary phase without known physical origin was postulated [14]. With its negative slope for r_H, it had the purpose of balancing out all the enhancements of r_H that have accumulated during the decelerating history of the Friedmann-type evolution of the universe, between the end of inflation until the present time. To avoid wrecking the successful BBN predictions of the Friedmann model, it was believed that the inflationary phase had to end well before the BBN era, which in standard cosmology occurs when the age of the universe is of order minutes. Inspired by the grand unification theory endeavors in particle physics, the inflationary phase is generally postulated to occur in the era of the GUT energies, when the age of the universe was somewhere between 10^{-36} and 10^{-32} s. When the dashed curve in **Figure 4** is continued back to this early era, it has decreased by many orders of magnitude, all of which must be balanced out during

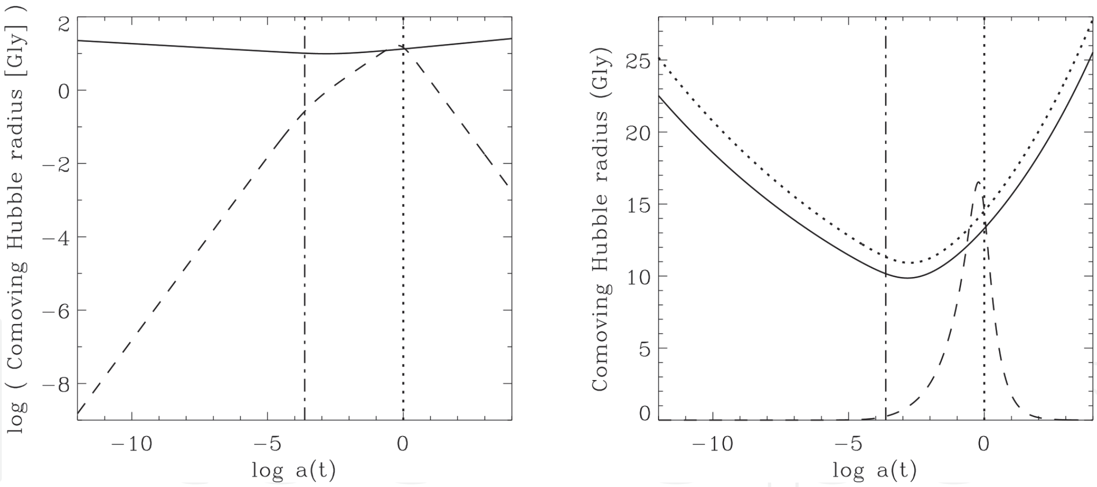

Figure 4.
Plots of the comoving Hubble radius $c/(aH)$ vs. log of the scale factor $a(t)$ for AC theory (solid curves) and standard cosmology (dashed curves). In the left panel, a log scale is used for the vertical axis, while a linear scale is used in the right panel. The vertical dash-dotted lines mark the epoch of equipartition, the dotted lines the present epoch. While H_0 based on supernova data have been used for all computations with the AC theory (and for the solid curves in this figure), the dotted curve in the right panel represents the results when H_0 from CMB data have been used instead. This serves to illustrate the degree of uncertainty that is introduced by the so-called H_0 anomaly. Note how the present epoch represents a turning point in cosmic history according to standard cosmology.

the brief inflationary phase. This is why it is generally believed that an incredibly violent inflation must have blown up the scale factor exponentially by about 60 e-foldings, which corresponds to the gigantic number of about 10^{26}.

After the inflation idea was introduced, there have been a plethora of theoretical papers on the subject, which now has a prominent place in all modern cosmology textbooks. Still, four decades after its invention, the hypothetical inflaton field that is assumed to be responsible for the phenomenon has not been identified, in spite of an abundance of searches with string theory, supersymmetric grand unified theories, or other exotic alternatives. The existence of a violently inflationary phase around the GUT era, when the universe was a tiny fraction of a second old, is often treated as a fact, while fundamental arguments against it, like in [15, 16], are largely ignored.

In contrast, the solid curve of the AC theory in **Figure 4** shows that the comoving Hubble radius r_H has never dipped below a value of 10 Gly, which represents the largest scales that are available in our present observable universe. This can be seen more clearly in the linear representation of the right panel of **Figure 4**. We already noticed in **Figures 1** and **3** that in AC theory the cosmic acceleration occurs naturally throughout the radiation-dominated phase, with a gentle transition to a decelerating phase near the epoch of recombination. Without needing to postulate or assume anything extra, without the introduction of any free parameters, we get a very extended but gentle inflationary phase that extends all the way back to the very beginning of the universe. All scales inside the horizon at the time of recombination (and CMB formation) were always inside the horizon and were therefore causally connected since the beginning. Throughout all of cosmic history until recombination, they could interact with each other and thermalize, to establish a high degree of homogeneity and isotropy. The motivation for postulating a hypothetical GUT era violent inflation does not exist in AC theory. There is no causality problem.

In the right panel of **Figure 4**, we have let AC theory be represented by two curves. The solid curve is based on the use of a value 73.5 km s^{-1} Mpc^{-1} for H_0 (the present Hubble constant) from observations of supernovae type Ia, while the dotted curve is based on 66.9 km s^{-1} Mpc^{-1} for H_0 that has been derived from the interpretation of CMB data from the Planck satellite. These two values of H_0 differ by 9.4%, a significant discrepancy that is generally referred to as the "H_0 anomaly," cf. [17]. While this anomaly is an important issue in itself, it does not affect the topics discussed in the present work. We have therefore in all our other figures elected to base all plots for the AC theory on the supernovae H_0, including the lines that mark the location of the epoch of equipartition between matter and radiation. In contrast, all the plots for the standard theory are based on H_0 from CMB data (for reasons of self-consistency of the standard framework, because all the other parameters that define standard cosmology have been determined primarily from CMB data).

The linear representation of the right panel of **Figure 4** again highlights how the present epoch is singled out by the standard model as something extraordinarily special. The comoving Hubble radius has one single narrow peak throughout all of cosmic history, and this peak is located where we happen to live in cosmic time.

6. Conclusions

The cosmological constant Λ that was needed to model the observed accelerated expansion of the universe has generally been interpreted as representing some mysterious "dark energy." However, the interpretation that dark energy is some kind of new physical field that pervades all of space leads to a cosmology (which is generally referred to as the "standard model"), in which our time in cosmic history is extraordinarily special and marks the onset of an inflationary phase that will continue forever.

Forty years ago another inflationary phase was postulated to occur in the GUT era of the very early universe, in order to answer the question why the universe is observed to be so homogeneous and isotropic on large scales [14]. The scalar inflaton field needed to drive the inflation has however not been identified in spite of a profusion of papers on this topic.

In the present work, we show that both these problems are connected and can be solved, if the Λ term that is responsible for the accelerated expansion is not a physical field but instead due to a global boundary constraint. This constraint induces a Λ term with a magnitude that tracks the conformal age η_u of the universe, such that $\Lambda \sim 1/\eta_u^2$. The density of dark energy therefore vanishes in the distant future. For the implementation of this idea, it is necessary to recognize the participatory role of observers in the universe, which has a profound effect on the nature of the theory.

We have derived and solved the mathematical equations that follow from this approach. It leads to a very different cosmological framework, which we refer to as the "AC theory" (AC for alternative cosmology). Some implications of this theory have been highlighted: The cosmic coincidence problem disappears, our epoch is not special in any way, and we are not privileged observers. The boundary constraint leads to an evolving scale factor that describes an accelerating, inflating phase from the beginning of the Big Bang throughout the entire radiation-dominated era. There is no need to postulate some early violent inflation driven by some hypothetical inflaton field, because the boundary constraint automatically causes the universe to inflate. The theory reproduces the observed value of Ω_Λ without the use of any free parameters. Because there is only one, unique solution, the possibility of parallel universes with other values of the cosmological constant does not exist.

As the cosmic expansion rate is found to have been much slower in the past than it was according to standard cosmology, the various observational data need to be reinterpreted with the new framework, in particular the BBN predictions of the abundances of the light chemical elements, and the observed signatures in the cosmic microwave background. The confrontation of the theory with such observational constraints represents work in progress that may ultimately determine the viability of the theory in its present form.

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